Parton Pseudo-Distributions from Lattice QCD

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Introduction

- Lattice calculations of Distribution functions are maturing to the point of realistic comparison with experiment
- EIC will be able to measure *more* PDFs with *more* precision than ever before
  - Not all PDFs are experimentally measurable
  - Lattice QCD should be able to complement this work.
- Combining Lattice calculations and Phenomenological fits
  - Not all PDFs are well constrained by experimental data, i.e. quark Transversity PDF
  - Lin et.al (2017) 1710.09858
- Project Goals
  - Long Term: Study methods of calculating parton distributions from ab initio Lattice QCD
  - Short Term: Understand systematic effects of the simplest and most precise case, the iso-vector quark unpolarized PDF

PDFs from the lattice

- "Twist-2" Local Operators
  - Restricted to low moments by reduced rotational symmetry
- Two current Correlations
  - "Light-like" separated Hadronic Tensor

    K-F Liu et al Phys. Rev. Lett. 72 1790 (1994) , Phys. Rev. D62 (2000) 074501
  - Good lattice cross sections

    Y.-Q. Ma J.-W. Qiu (2014) 1404.6860, Y.-Q. Ma, J.-W. Qiu (2017) 1709.03018
- Wilson Line Quark Bilinears
  - Quasi PDF

    X. Ji, Phys.Rev.Lett. 110, (2013), J.-W. Chen et.al. (2018) 1803.04393, C Alexandrou et.al. (2018) 1803.02685
  - Pseudo PDF

    A. Radyushkin Phys.Lett. B767 (2017), K. Orginos, A Radyushkin, JK, S Zafeiropoulos (2017) 1706.05373

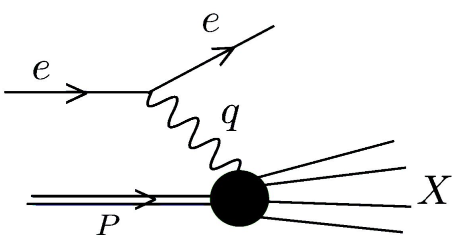
Outline

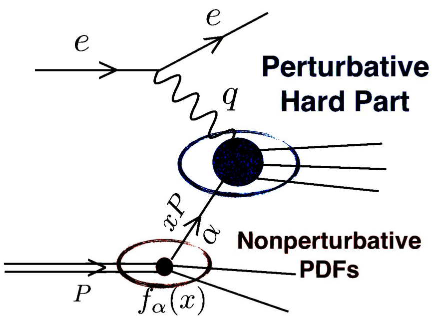
- Reminder of PDF definitions
  - Lattice PDF methods are based off of different definitions
- Matrix elements from Lattice QCD
  - Use of new Feynman-Hellman summation method
- Methods for Lattice PDFs
  - Many methods are interrelated, but focus of Pseudo-PDF method
- Results for Matrix elements
- Extraction of from PDFs data
  - What to do when you get the data

Parton Distribution Functions

- Cross section factorization
  - DIS example

$$d\sigma_h = f_{h/q} \otimes d\sigma_q$$





- Light cone matrix element definition

$$f_{h/q}(x,\mu^2) = \int d(\xi^- p^+) e^{\pm ix(\xi^- p^+)} \qquad p = \left(p^+, \frac{m^2}{2p^+}, 0_T\right)$$

$$\langle h(p)|\bar{\psi}_q(0,\xi^-,0_T)\gamma^+ W((0,\xi^-,0_T);0)\psi_q(0)|h(p)\rangle_{\mu^2}$$

- Divergences in the Hard parton cross section are canceled by PDF
  - Gives rise to Q^2 dependence of the PDF

PDF Moments and the OPE

- Mellin moments $a_n(\mu^2) = \int dx x^{n-1} f(x, \mu^2)$

- Local matrix element

$$\langle h(p)|\bar{\psi}\gamma^{\{\mu_1} D^{\mu_2} \ldots D^{\mu_n\}}\psi|h(p)\rangle_\mu = a_n(\mu)p^{\{\mu_1} \ldots p^{\mu_n\}}$$

- Symmetrized indices give only lowest twist contribution
- Lorentz invariant definition doesn't require light cone

Ioffe Time distribution

$\nu = p \cdot z$ B. L. Ioffe, Phys. Lett. 30B, 123 (1969)

V. Braun, et. al Phys. Rev. D 51, 6036 (1995)
I.I. Balitsky and V.M. Braun, Nucl. Phys. B311, 541 (1988)

- $I\,(\nu, \mu^2) = \int_{-1}^{1} dx\; e^{i\nu x}\; f(x, \mu^2)$
- CP Even/Odd combinations
  - Even: $q_-(x) = f(x) + f(-x) = q(x) - \bar{q}(x) \equiv q_V(x)$
  - Odd: $q_+(x) = f(x) - f(-x) = q(x) + \bar{q}(x) = 2\,q_V(x) + \bar{q}(x)$

$$Re\,[I(\nu)] = \int_0^1 dx\;\cos(\nu x)\;q_-(x) = \int_0^1 dx\;\cos(\nu x) q_V(x) \equiv I_V(\nu)$$

$$Im\,[I(\nu)] = \int_0^1 dx\;\sin(\nu x) q_+(x) = \int_0^1 dx\;\sin(\nu x)(q(x) + \bar{q}(x))$$

- Perturbative position space DGLAP evolution

$$I_V(\nu, \mu_2^2) = I_V(\nu, \mu_1^2) - \frac{C_F \alpha_S}{2\,\pi} \log\left(\frac{\mu_2^2}{\mu_1^2}\right) \int_0^1 du\;\left[\frac{1}{2}\delta(1-u) - (1-u) - 2\left(\frac{u}{1-u}\right)_+\right] I_V(u\nu, \mu_1^2)$$

Matrix Elements from Lattice Field Theory

- Importance sampling of path integral

$$\langle O(\bar{\psi}, \psi, A_\mu)\rangle = \frac{1}{Z}\int D[\bar{\psi}]D[\psi]D[A_\mu]O(\bar{\psi}, \psi, A_\mu)e^{-S(\bar{\psi},\psi,A_\mu)}$$

$$\approx \frac{1}{N}\sum_i^N F_O(U_\mu^{(i)})$$

- Correlation functions

$$C_2(\vec{p}, T) = \langle O_N(-\vec{p}, T)\bar{O}_N(\vec{p}, 0)\rangle$$

$$C_{op}(O_{op}; \vec{p}, T) = \sum_t \sum_{\vec{x}} \langle O_N(-\vec{p}, T) O_{op}(\vec{x}, t)\bar{O}_N(\vec{p}, 0)\rangle$$

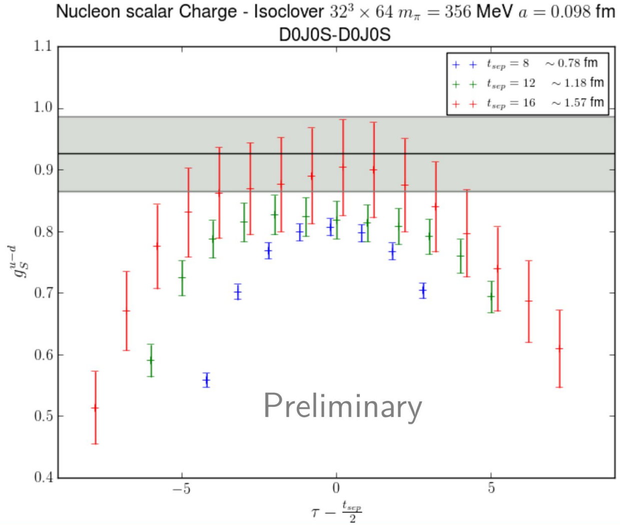
- Feynman-Hellman extraction C. Bouchard et.al Phys. Rev. D 96, no. 1, 014504 (2017)

$$\frac{\langle N(p)|O_{op}|N(p)\rangle}{2E_{N(p)}} = \lim_{T\to\infty}\frac{1}{\tau}(R(T+\tau) - R(T)) \qquad R(T) = \frac{C_{op}(O_{op}; \vec{p}, T)}{C_2(\vec{p}, T)}$$

$$O_q^\alpha(z; T) = \sum_{\vec{x}} \bar{\psi}_q(\vec{x}+\vec{z}, T)\lambda^3\gamma^\alpha W((\vec{x}+\vec{z}, T); (\vec{x}, T))\psi_q(\vec{x}, T)$$

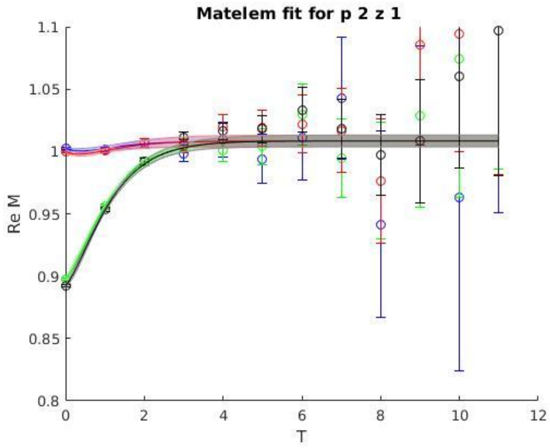
Feynman Hellman matrix element extraction

- 2 parameter extraction
- Questionable Excited state effects



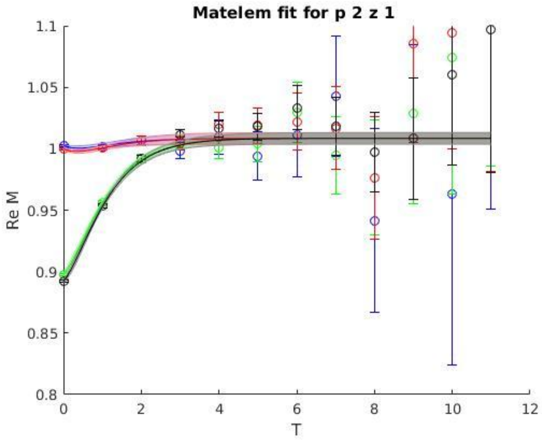
Thanks to Colin Egerer for plot from his LightCone 2018 talk

- 1 parameter extraction
- More clear distinction between excited state, plateau, noise regions



See Axial coupling calculation in Chang et.al.
arXiv:1710.06523

Technical Lattice difficulties

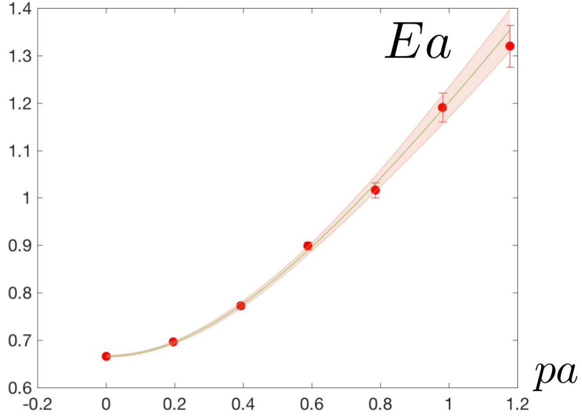


- Excited states contamination
- Signal to noise
  - $C_2(p,T) = \langle O_h(p,T) O_h(p,0)^\dagger \rangle \propto e^{-E_h(p)T}$

    $var\left[C_2(p,T)\right] = \langle O_h(p,T) O_h(p,T)^\dagger O_h(p,0) O_h(p,0)^\dagger \rangle \propto e^{-n_q m_\pi T}$

    $\frac{var\left[C_2(p,T)\right]^{\frac{1}{2}}}{C_2(p,T)} \propto e^{(E_h(p) - n_q m_\pi /2)T}$
    - Momentum smearing Bali et.al. Phys. Rev. D 93, 094515 (2016)
- Use of heavy pions
- Reduced Symmetries
- Connected and disconnected
- Restriction to low momenta

$$apmax = \frac{2\pi}{L}\left(\frac{L}{4}\right) = \frac{\pi}{2} \sim O(1)$$



Two changes make PDFs unattainable

- Wick Rotation
  - 4 D vs 3+1 D
  - Euclidean space lacks a light cone

- Rotational Symmetry breaking
  - O(4) has infinite irreducible representations while H_4 has 13 irreducible representations
  - Mixing between operators in the same irreducible representation will begin to mix with each other
  - Only lowest 3 moments are calculable directly with local matrix elements.

LaMET and Quasi-Distributions

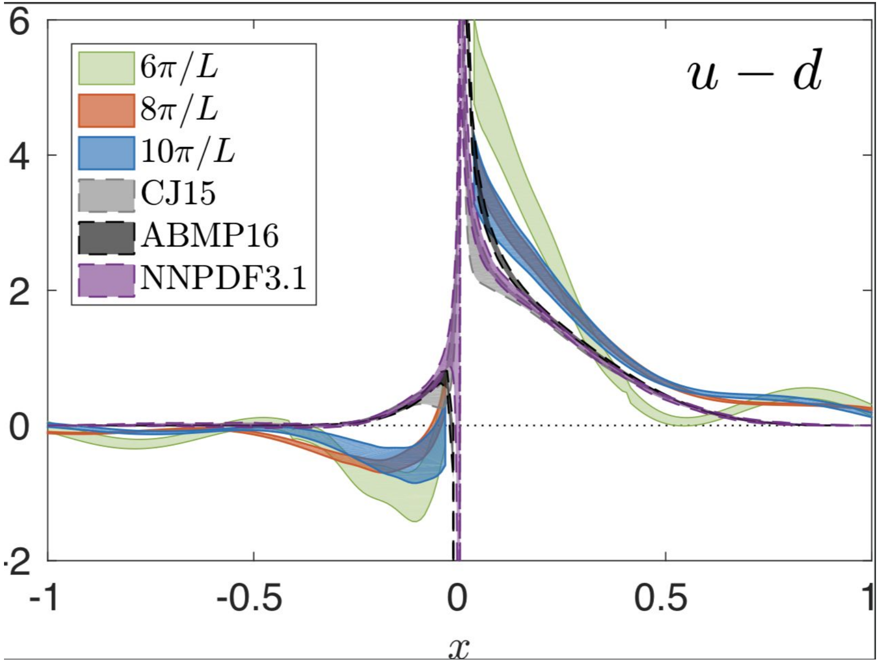
- Feynman's original descriptions of PDFs
  - Approach light cone from large boosts
- Large Momentum Effective Theory
  - Not Typical EFT, but expansion in powers of inverse momentum
- Domain of quasi-PDF is larger than PDFs
- Matrix elements of quark fields in high momentum hadronic states can be directly matched to the light cone physics.

$$q(y, P_z) = \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{iyP_z z} \langle h(P)|\bar{\psi}(z)\gamma_t W(z;0)\psi(0)|h(P)\rangle$$

- Violate Lorentz invariance to create a scale to match with PDF in Large momentum limit

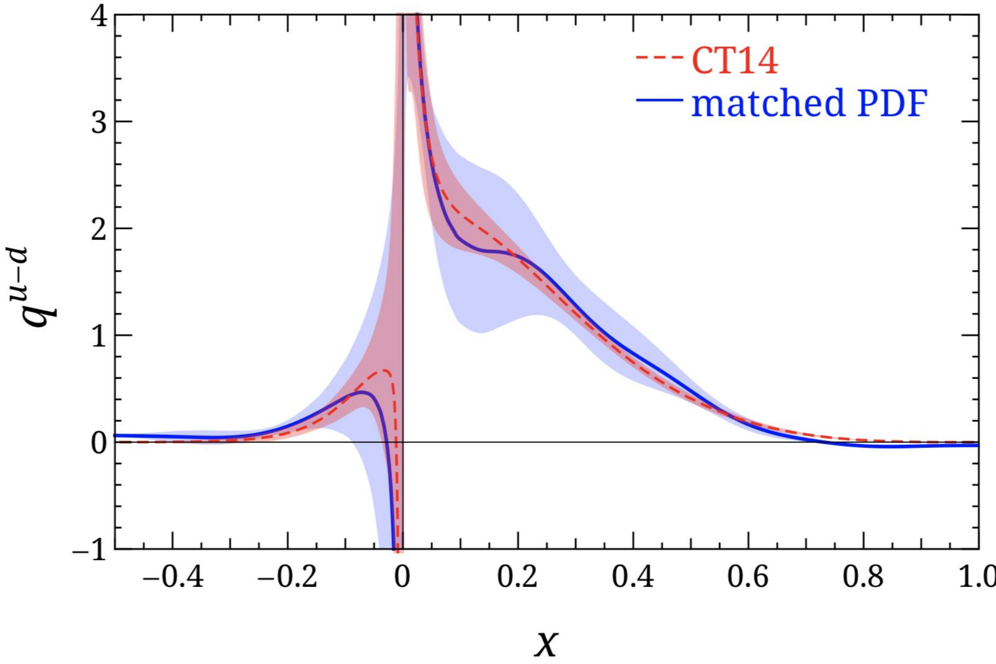
X. Ji (2013) 1305.1539

Quasi-PDF calculations



ETM Collaboration

C. Alexandrou et al, arxiv:1803.02685 (2018)



LP3 Collaboration

J.-W. Chen et al, arxiv:1803.04393 (2018)

Pseudo-Distributions

- Standard partonic distributions, particularly collinear distributions, are defined via matrix elements with light-like separations
  - Describe probability distribution of quark states
  - Not suitable for lattice calculation
- Pseudo-distributions are Lorentz invariant generalizations of partonic distributions defined via matrix elements with space like separations
  - Do not have probabilistic interpretation
  - Acceptable for lattice calculation
- In the limit that the space like separation goes to 0, the standard distribution is recovered
  - Limit is made difficult by the collinear divergence

Ioffe Time Pseudo-Distributions

- A general matrix element of interest

$$M^{\alpha}(z,p) = \langle h(p)|\bar{\psi}(z)\gamma^{\alpha}W(z;0)\psi(0)|\,h(p)\rangle$$

- Lorentz decomposition
  - Physicists love to use of symmetries
  - Choice of p, z, and α can remove higher twist term

$$M^{\alpha}(z,p) = 2\,p^{\alpha}\,M_p(\nu,z^2) + z^{\alpha}M_z(\nu,z^2)$$

- Factorizable Relationship to ITDF
  - Perturbatively calculable Wilson coefficients for each parton

$$M(\nu,-z^2) = \sum_i C_i(\,z^2\,\mu^2,\alpha_S) \otimes I_i(\nu,\mu^2) + H.T.$$

A. Radyushkin (2017) 1710.08813
J.-H. Zhang (2018) 1801.03023
T. Izubuchi (2018) 1801.03917

$$M^\alpha(z,p) = \langle h(p)|\bar{\psi}(z)\gamma^\alpha W(z;0)\psi(0)|\, h(p)\rangle$$

Special Cases

$$M^\alpha(z,p) = 2\, p^\alpha\, M_p(\nu, z^2) + z^\alpha M_z(\nu, z^2)$$

- Light cone PDF

$$p = (p^+, p^-, 0_T) \quad z = (0, z^-, 0_T) \quad \alpha = +$$

$$M_p((p^+z^-), 0) = \int_{-1}^{1} dx\, e^{ix(p^+z^-)} f(x)$$

- Straight Link “Primordial” TMD

$$p = (p^+, p^-, 0_T) \quad z = (0, z^-, z_T) \quad \alpha = +$$

$$M_p\left((p^+z^-), -z_T^2\right) = \int_{-1}^{1} dx\, e^{ix(p^+z^-)} \int d^2k_T\;\, e^{ik_T\cdot z_T} F(x, k_T^2)$$

- Pseudo-PDF

$$p = (E, 0{,}0, p\_3\,) \quad z = (0{,}0{,}0, z_3\,) \quad \alpha = 0$$

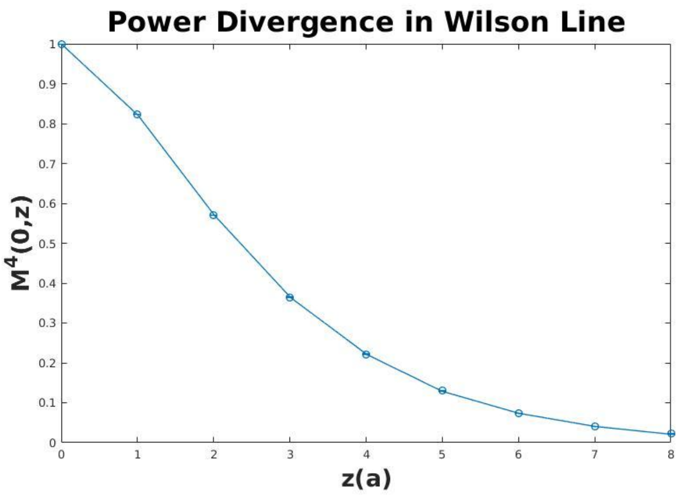
$$M_P\left((-z_3p_3), -z_3^2\right) = \int_{-1}^{1} dx\, e^{ix(-z_3p_3)} P(x, -z_3^2)$$

A. Radyushkin (2017) 1612.05170

Renormalization and the Reduced distribution

- Vector current $Z_p^{-1} = M^4(0,p)$
  - Forces matrix elements to give unit charge
- Reduced distribution $\mathfrak{M}(\nu, z^2) = \dfrac{\mathcal{M}(\nu, z^2)}{\mathcal{M}(0, z^2)}$
  - TMDs and suppression of polynomial corrections

    $$F(x,k_T^2) = f(x)g(k_T^2) \Rightarrow M(\nu,z^2) = M(\nu,0)M(0,z^2)$$
  - BONUS: Multiplicative UV power divergent corrections from Wilson line cancel away



Matching Lattice data to Ioffe distribution

- Matching between pseudo ITDF and MS bar scheme ITDF via factorization of IR divergences.
- At 1-loop, scale evolution and matching can be simultaneous
- Allows for a direct relationship between ITDF/PDF and pseudo ITDF
    - No more need for extrapolations in the scale
    - Does require scale to be in regime dominated by perturbative effects
- No real need for pseudo PDFs. Go directly from pseudo ITDF to PDF
- Only perturbative correction proportional to α_S (around 10%)

$$I(\nu,\mu^2) = \mathcal{M}(\nu,z^2) + \frac{C_F\alpha_S}{2\pi}\int_0^1 du\left[B(u)\left(\log\left(z^2\mu^2\frac{e^{2\gamma_E}}{4}\right)+1\right)+\left(\frac{4\log(1-u)}{1-u}-2(1-u)\right)_+\right]\mathcal{M}(u\nu,z^2)$$

The Sort-of PDFs

$$0.2GeV \approx 1fm^{-1}$$

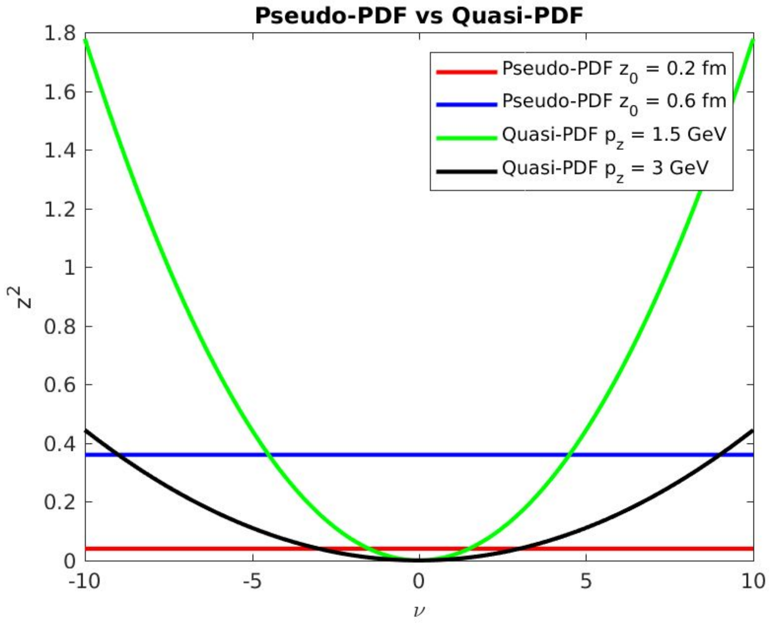
- Both are integrals of pseudo ITDF
    - **Pseudo PDF** has fixed invariant scale dependence

$$P(x, z_0^2) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\nu e^{-i\nu x}\mathcal{M}(\nu, z_0^2)$$

    - **Quasi PDF** mixes invariant scales

$$Q(x, p_z^2) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\nu e^{-i\nu x}\mathcal{M}(\nu, \frac{\nu^2}{p_z^2})$$

- To expedite desired limit of $z^2 \longrightarrow 0$
    - Pseudo-PDFs use reduced distributions
    - Quasi-PDFs use LaMET



Good Lattice Cross Sections

- Good Experimental Cross Section - An experiment whose results, Form Factors, asymmetries, etc., are factorizable to a particular PDF.
    - DIS, SIDIS, DY, ….

- Good Lattice Cross Section - A lattice QCD calculable matrix element whose result is sensitive to a particular PDF (Matrix element and not actually a cross section)
    - Vector-vector currents, Axial-vector currents, Ioffe Time Distribution Functions, ….

Numerical Study

Both use Wilson-Clover Stout smeared Fermions

Quenched Wilson plaquette gauge action

K. Orginos, A Radyushkin, JK, S Zafeiropoulos (2017) 1706.05373

- $\beta = 6.0 \quad m_\pi = 600$ MeV $\quad 32^3 \times 64 \quad a = 0.1$ fm

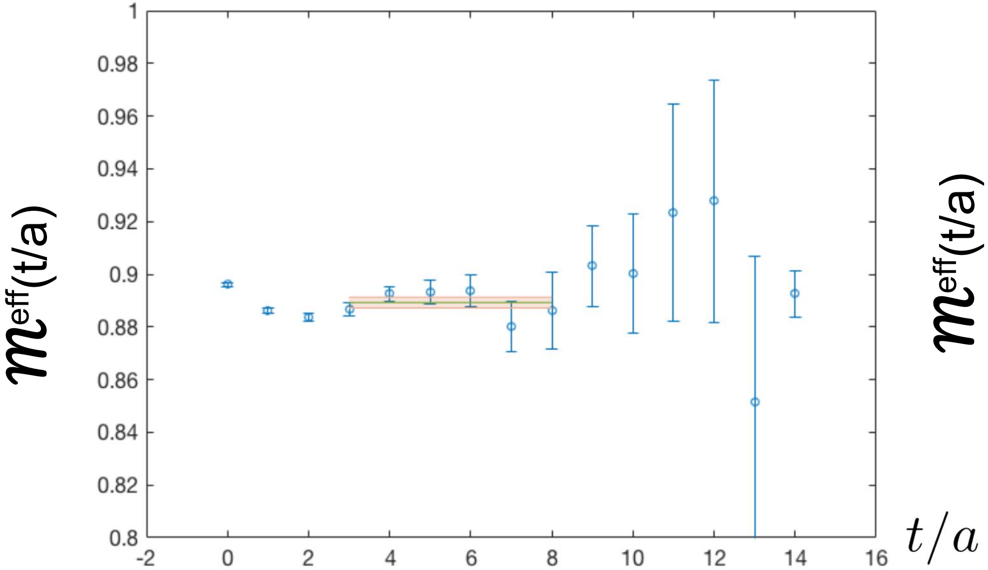
Dynamical Tree level tadpole Symanzik improved gauge action (Preliminary)

Unpublished

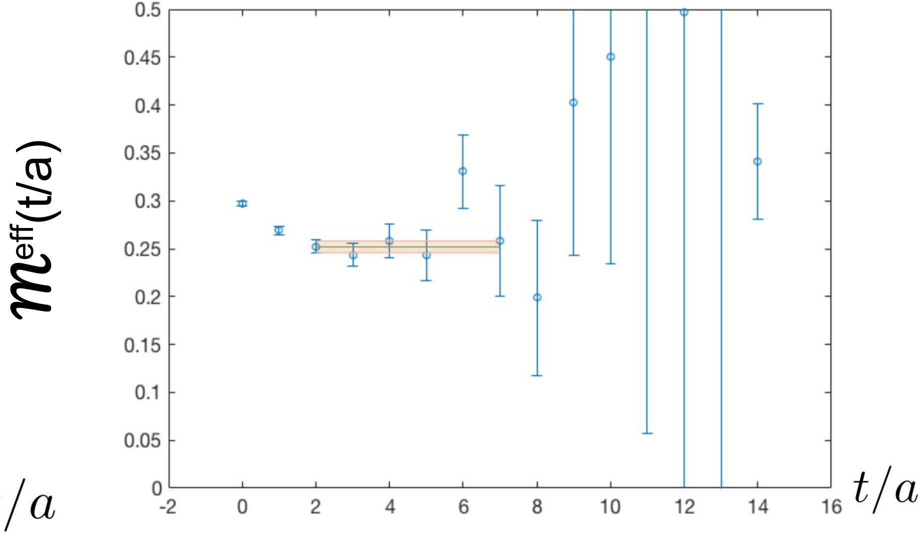
- $a127m440: \quad \beta = 6.1 \quad m_\pi = 440$ MeV $\quad 24^3 \times 64 \quad a = 0.127$ fm
- $a127m440L: \quad \beta = 6.1 \quad m_\pi = 440$ MeV $\quad 32^3 \times 96 \quad a = 0.127$ fm
- $a094m400: \quad \beta = 6.3 \quad m_\pi = 400$ MeV $\quad 32^3 \times 64 \quad a = 0.094$ fm

Matrix element extraction

p=2 z=4

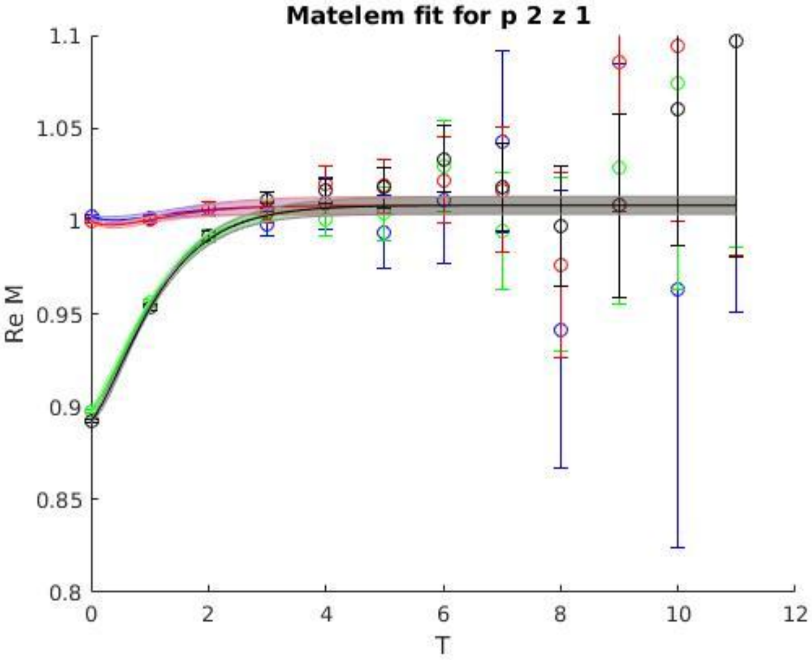


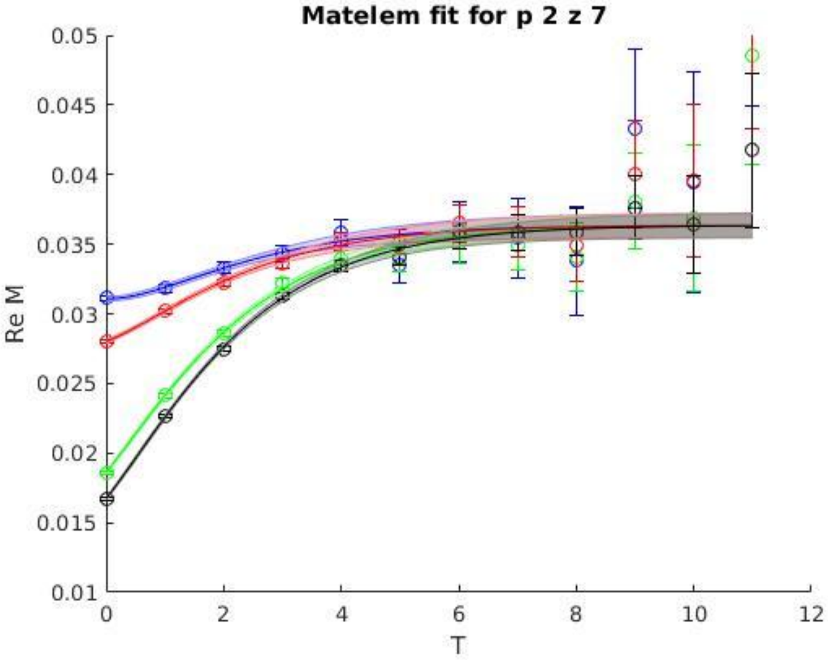
p=3 z=8



Some good extractions

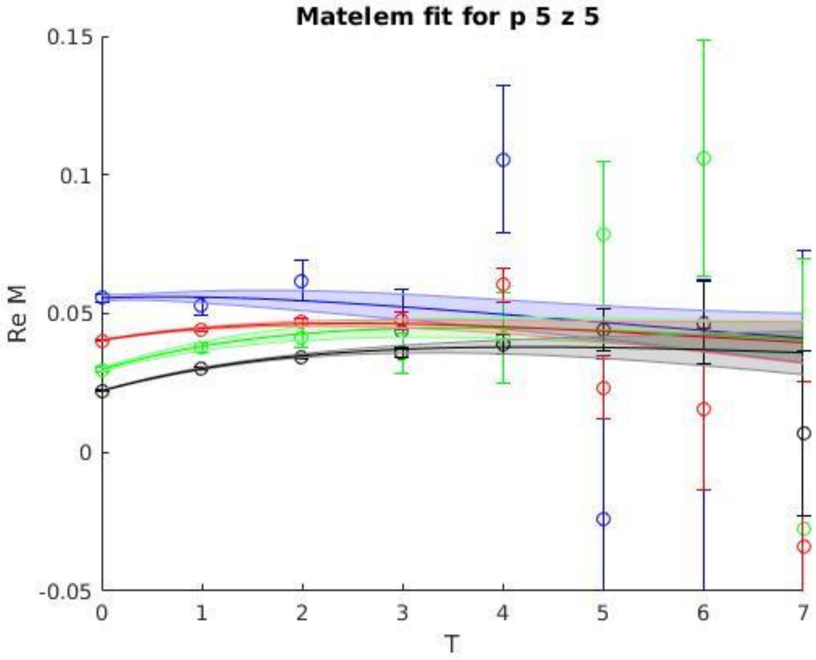
Preliminary

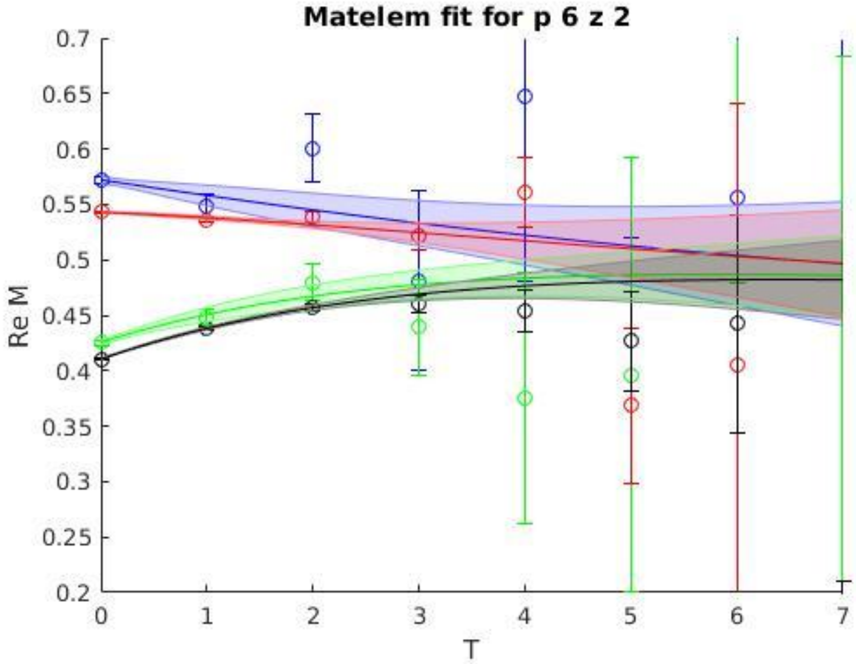




Less good extractions

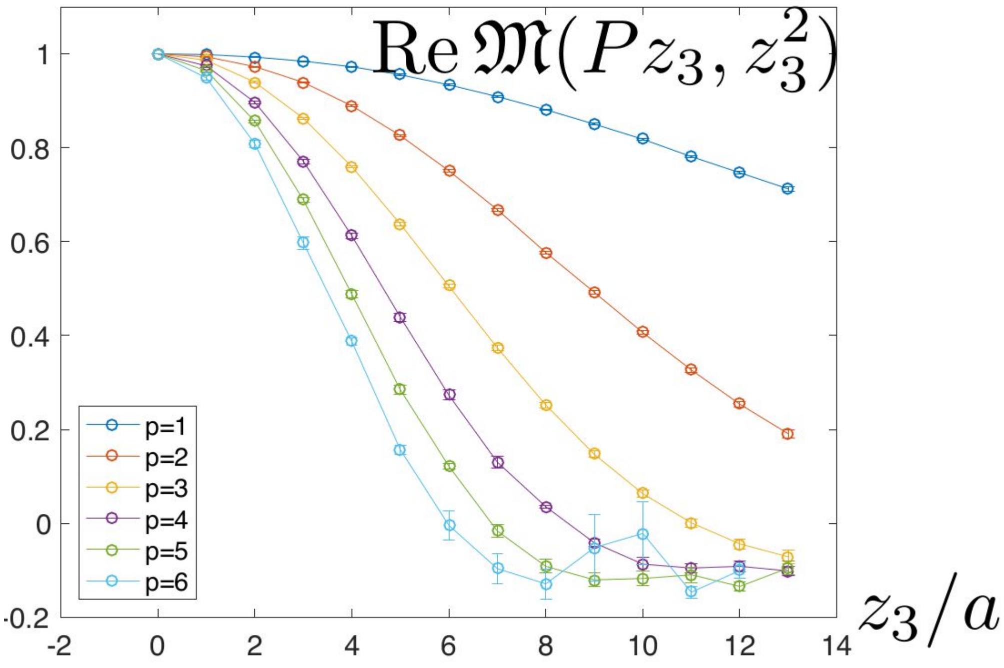
Preliminary

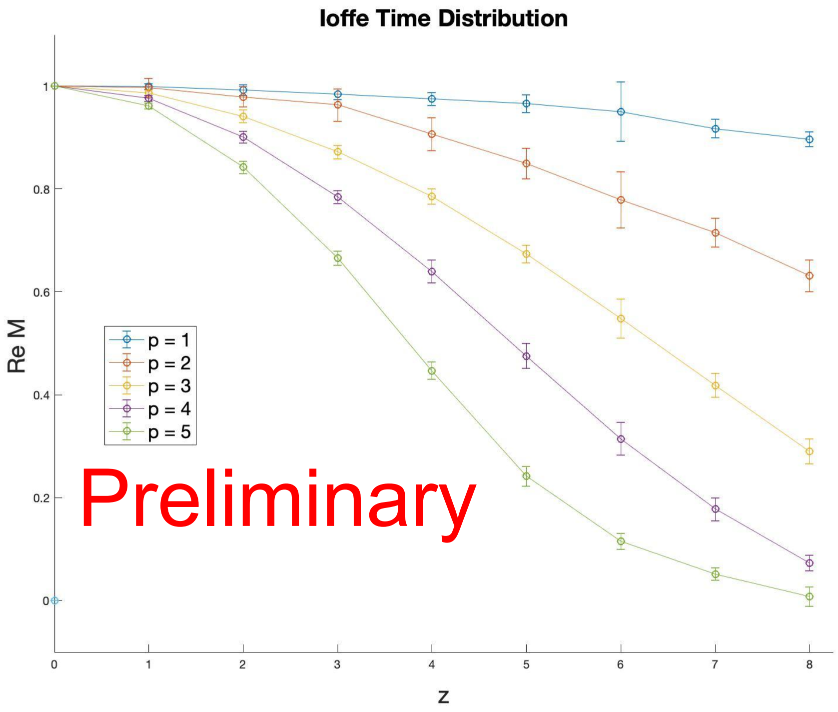




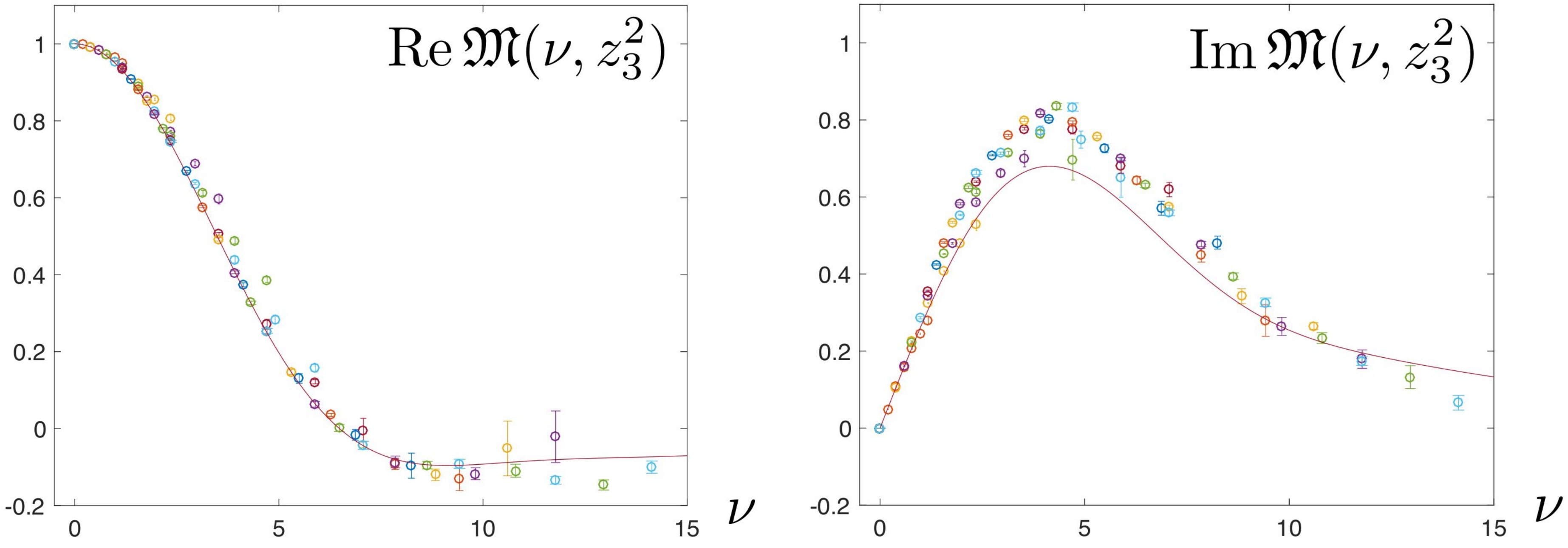
Quenched Results

Dynamical Results



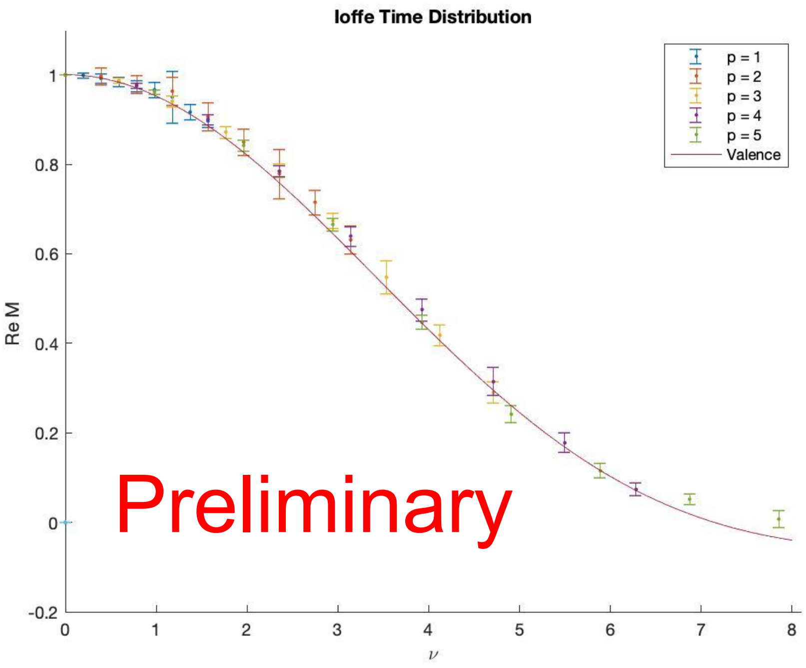


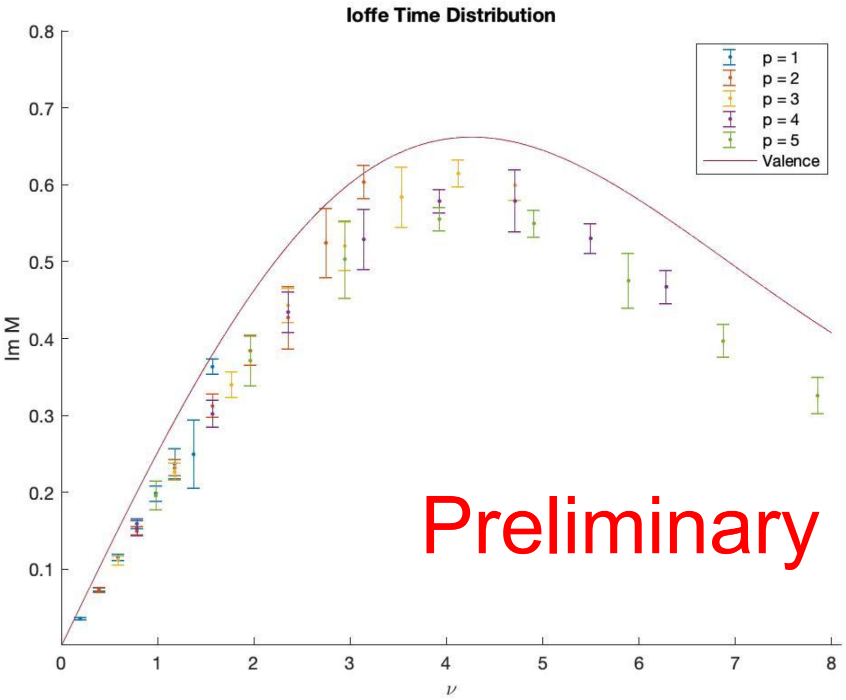
Quenched Results



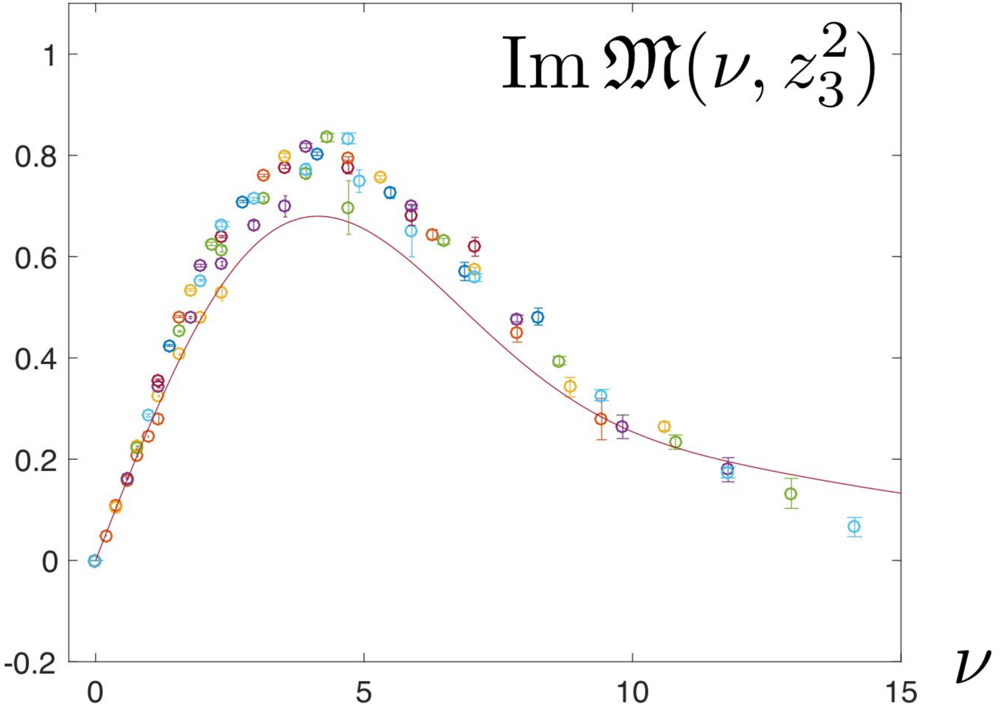
K. Orginos, A Radyushkin, JK, S Zafeiropoulos (2017) 1706.05373

Pseudo-ITDF Results a127m440L





Imaginary Component and AntiQuarks



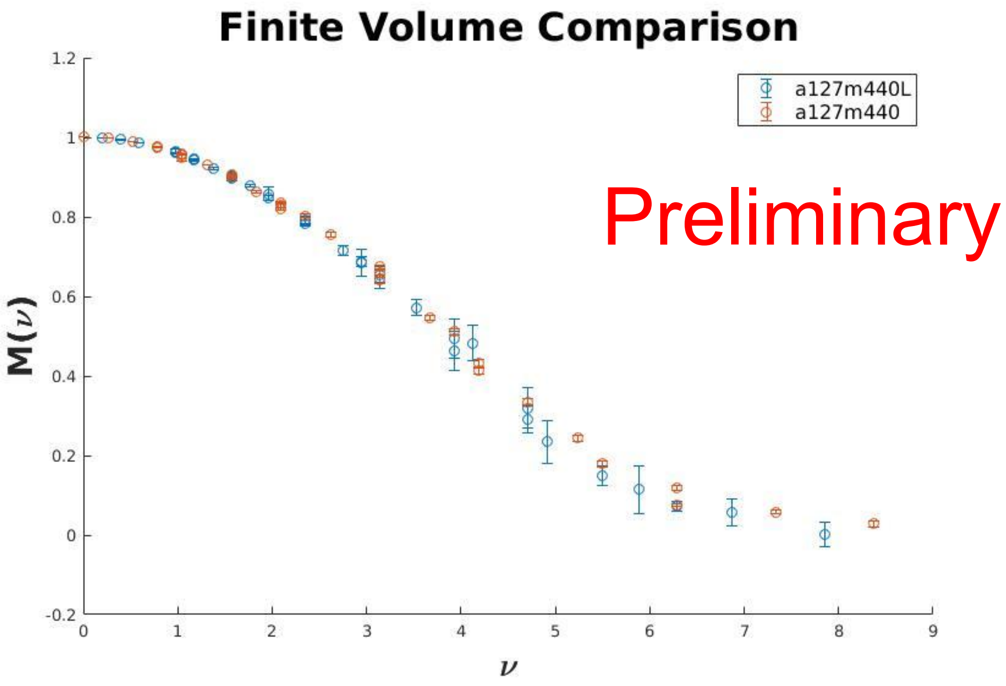
- Imaginary component mixes valence, sea, and anti-quark distributions
- Use real component to find valence contribution, the rest is the sea and anti-quark distribution
- Identify anti-quark distribution without need of needing to perform inaccurate Fourier transforms and requiring the unreliable low x region
- Qualitatively it gives proper sign for both quenched and dynamical iso-vector quarks

Comparison of Volumes

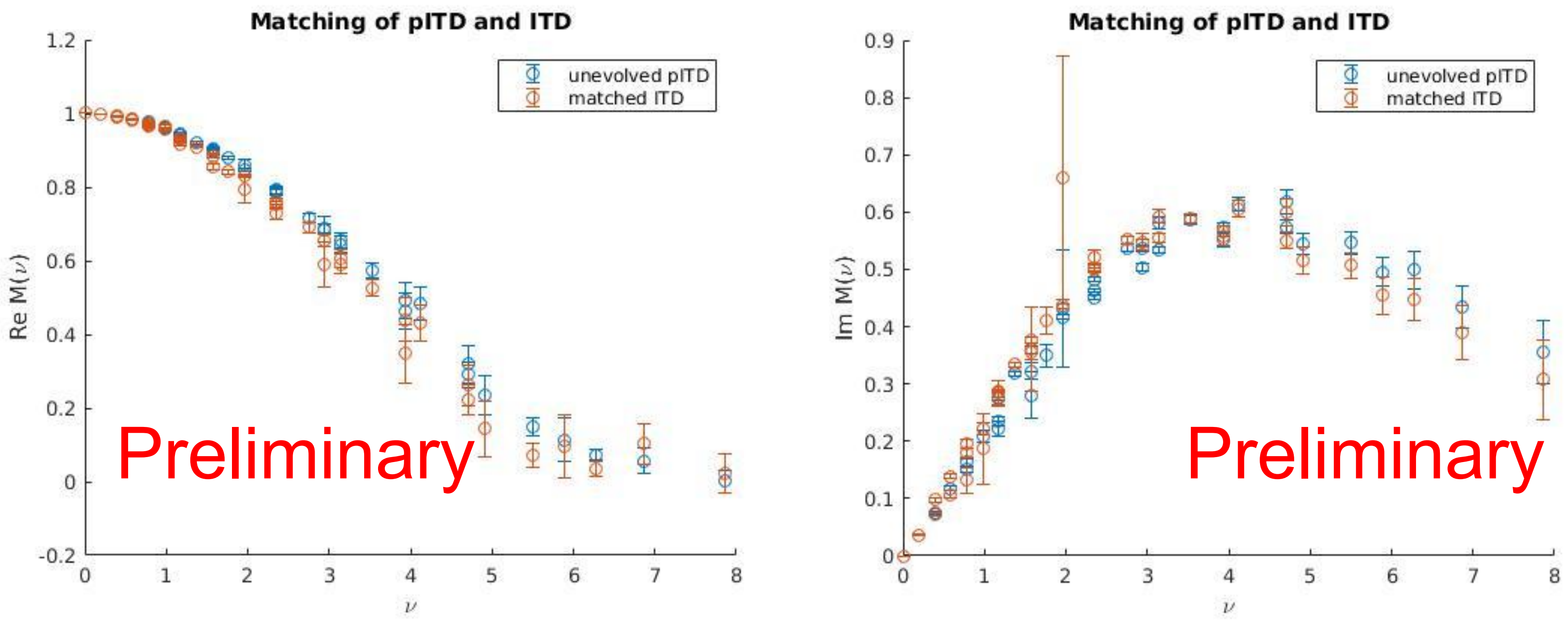
- Two Current matrix elements can have very large finite volume corrections

  Briceno et al. (2018) 1805.01034
  Bali et al. (2018) 1807.03073

- Finite volume effects of Wilson line operator has been unstudied

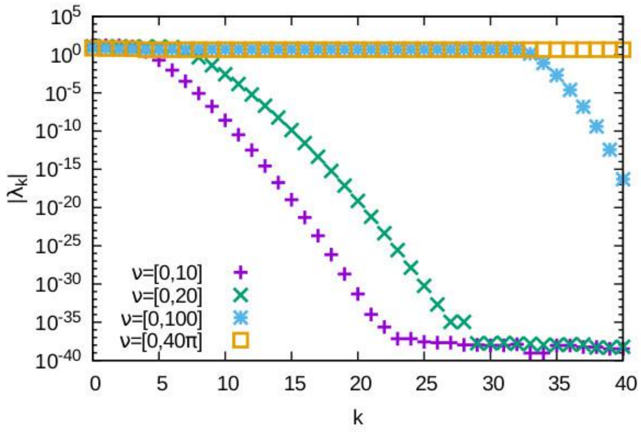


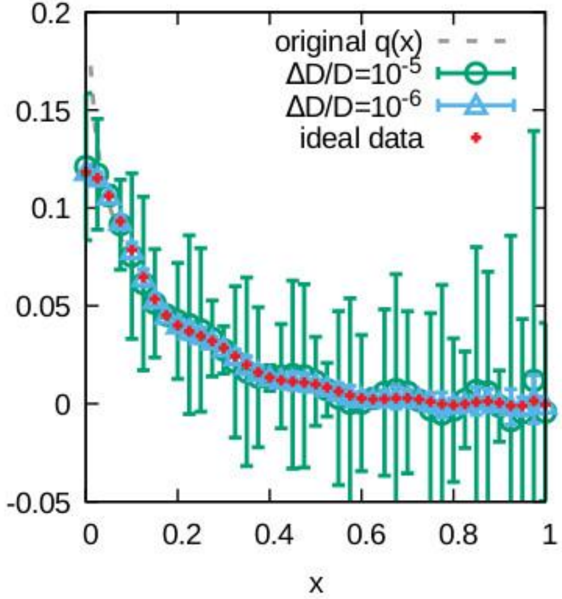
ITDF MS bar matched Results a127m440L



PDF Reconstruction is hard

- Truncated discretized Fourier (cosine) transform is unreliable for realistic lattice data
  - Ill posed inverse problem
  - Consider problem as matrix equation $\mathcal{M}_i = \sum_j C_{ij} q_j$
- Mock test to reconstruct PDF from 40 evenly spacing Ioffe time PDF points given Gaussian noise.
  - Noisy data requires either unreasonably large ranges of Ioffe Time unreasonably precise data to reproduce model PDFs.





Real component and the Valence Quark distribution

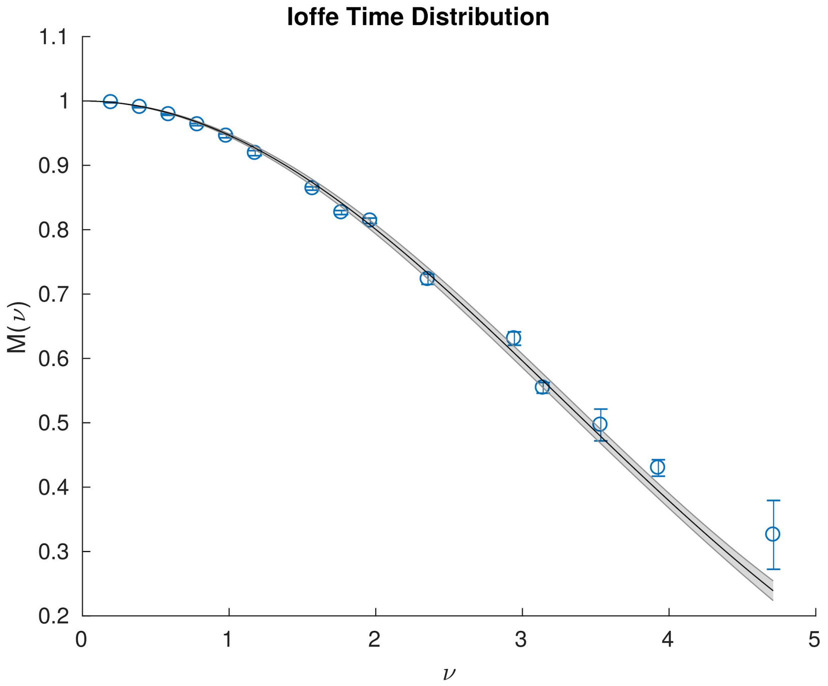
- In first attempt to avoid ill posed inverse Fourier transform
- A general model PDF used by JAM collaboration for fitting

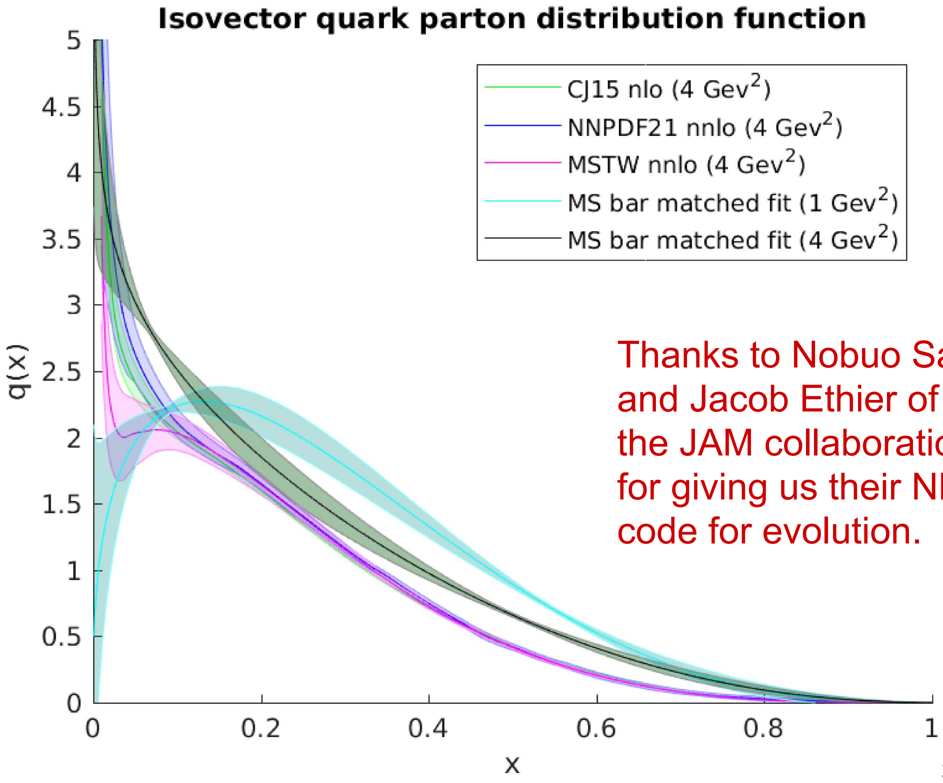
$$f_{abcd}(x) = N_{abcd}\ x^a(1-x)^b(1+c\sqrt{x}+d\,x)$$

$$N_{abcd} = B(a+1, b+1) + cB(a+\frac{3}{2}, b+1) + dB(a+2, b+1)$$

- Expected lowest order behaviors
  - Regge $a = -\frac{1}{2}$
  - Quark counting $b = 3$
  - Small Corrections $c \sim 0 \sim d$

Quenched Pseudo PDF Matched to MS bar Compared to Global fit PDFs



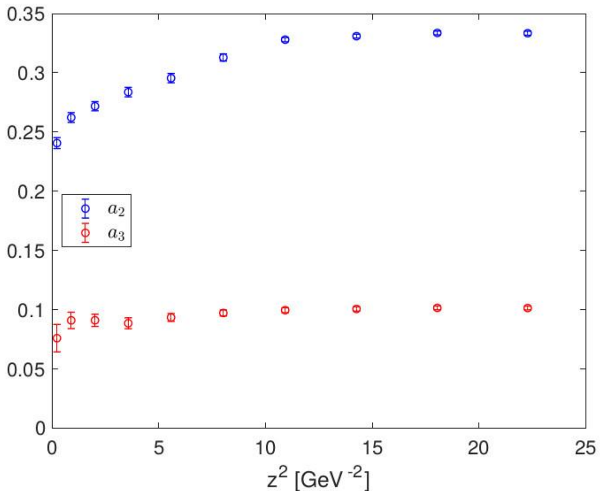


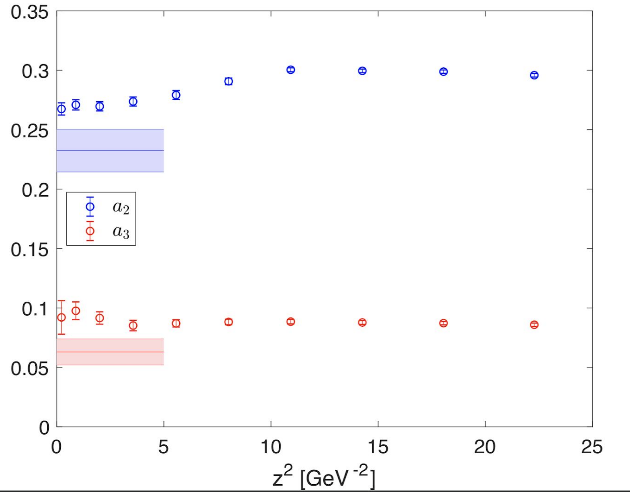
Thanks to Nobuo Sato and Jacob Ethier of the JAM collaboration for giving us their NLO code for evolution.

Moments of PDFs

- Taylor Expansion coefficients of Ioffe Time Distributions are the moments of PDFs
  - Even moments from Real component of ITDF
  - Odd moments from Imaginary component of ITDF
- With enough data there is no limit on what moments can be calculated
  - The reduced matrix element is renormalization group invariant and could not have any divergent lattice spacing dependence
  - Power divergences in the local matrix elements of the OPE are canceled by power divergences in the Wilson Coefficients

JK, K. Orginos, S. Zafeiropoulos 1807.10933





Backus Gilbert Reconstruction

J. Liang, K-F. Liu, Y-B. Yang (2017) 1710.11145

- Finds “most stable”, lowest variance, solution to inverse
- Used in wide range of engineering and physics applications
- Used to extract PDF from Lattice calculation of Hadronic Tensor
- Create “Delta function” $\Delta(x-\bar{x}) = \sum_j q_j(\bar{x}) K_j(x)$ and minimize its width

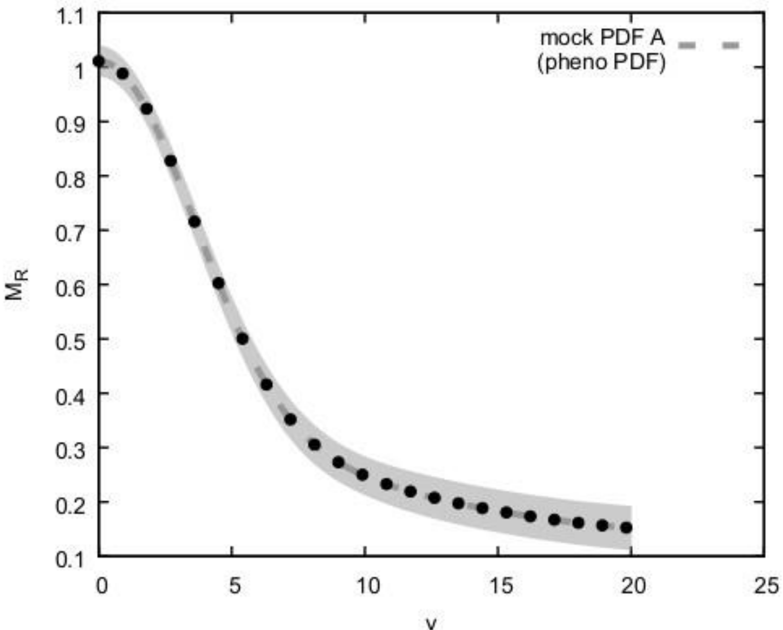
  $$\sigma = \int dx (x-\bar{x})^2 \Delta(x-\bar{x})$$

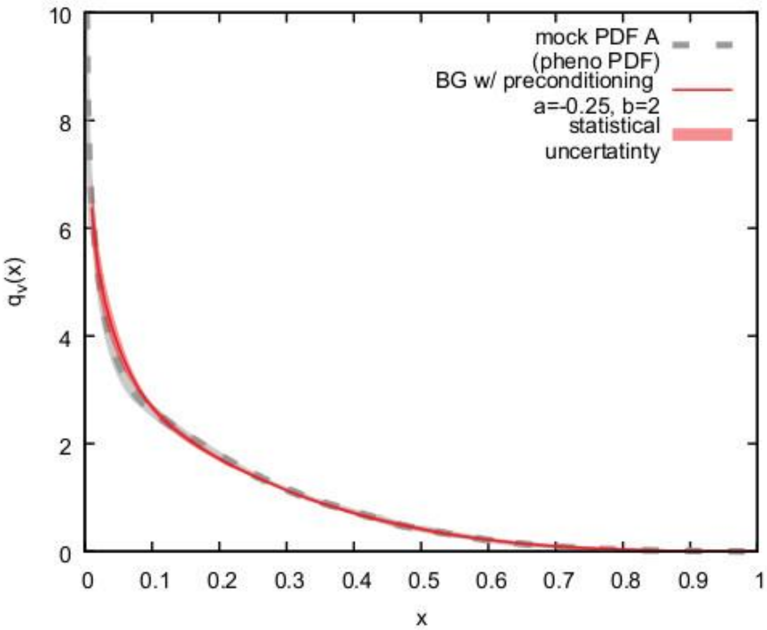
- In limit of width to 0, the Backus Gilbert method would reconstruct exact unknown function

Mock Tests of Backus Gilbert Reconstruction

- Tests use NNPDF31_nnlo_as_0118 data set with artificial errors.
- Reconstructions are more stable and reliable than direct inversion or fits.

  Backus Gilbert





Bayesian Reconstruction

Y. Burnier and A. Rothkopf (2013) 1307.6106

- Technique based upon Bayes Theorem

$$P[q|M,I] = \frac{P[M|q,I]P[q|I]}{P[M|I]}$$

- Acknowledging the ill posed nature of the problem and that a unique solution require addition of further information
- Parameterize the probabilities and extremize the posterior probability
- Developed for extraction of quark spectral function which is a much harder application

$$P[M|q,I] = exp[-L]$$

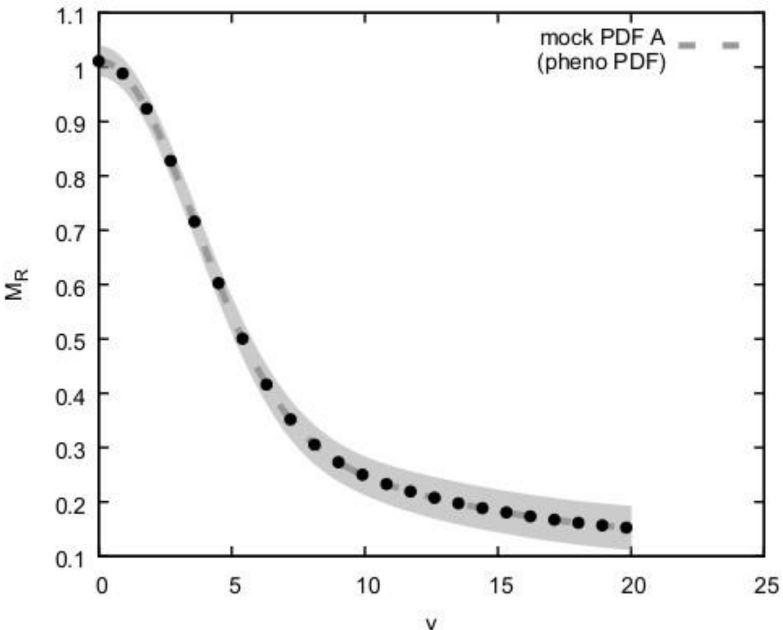
$$P[q|I] = exp[-S] \qquad S = \sum_n \Delta x_n (1 - \frac{q_n}{m_n} + \log \frac{q_n}{m_n})$$

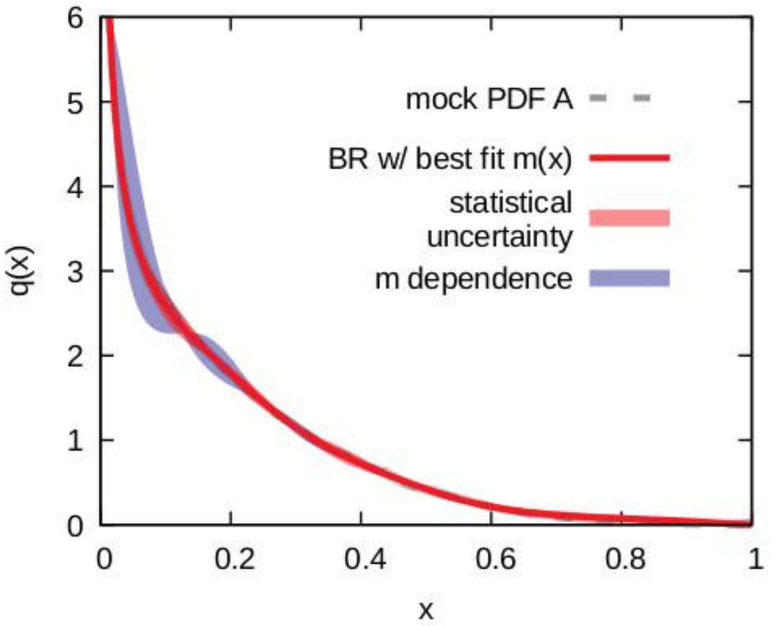
$$P[M|I] = N$$

Mock Tests of Bayesian Reconstruction

- Tests use NNPDF31_nnlo_as_0118 data set with artificial errors.
- Reconstructions are more stable and reliable than direct inversion or fits.

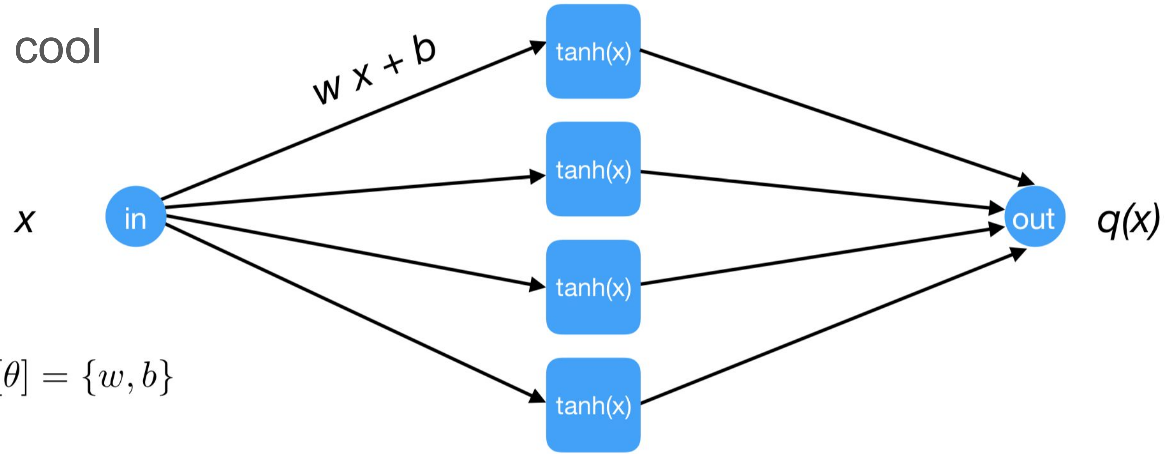
  Bayesian Reconstruction





Neural Network Reconstruction

- In the style of NNPDF, a series of neural networks can be constructed to represent the ill posed inverse transformation.
  - Many choices of Network geometry and activation functions need to be explored
- Even a small Neural net can be used to reconstruct PDF to accuracy of other methods.
- Machine Learning is cool



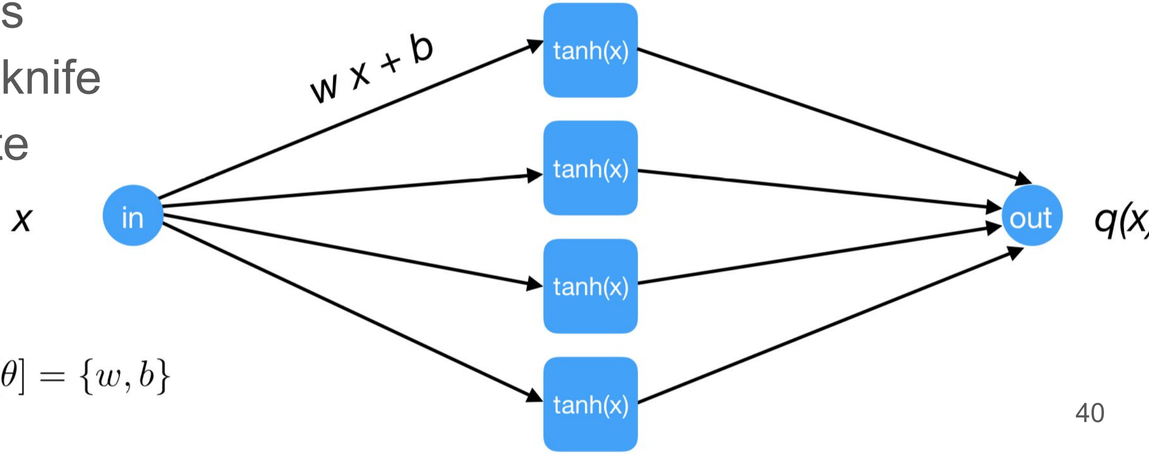
$$[\theta] = \{w, b\}$$

Neural Network Procedure

- Choose network geometry and activation function
- Using the full dataset, minimize network with respect to

$$\chi^2 = (M_k - \int_0^1 dx cos(\nu_k x) q_{[\theta]}(x)) C^{-1} (M_k - \int_0^1 dx cos(\nu_k x) q_{[\theta]}(x))^T$$

  several times, removing networks with largest value
- Repeat for a few generations
- Retrain each replica on jackknife samples to get error estimate

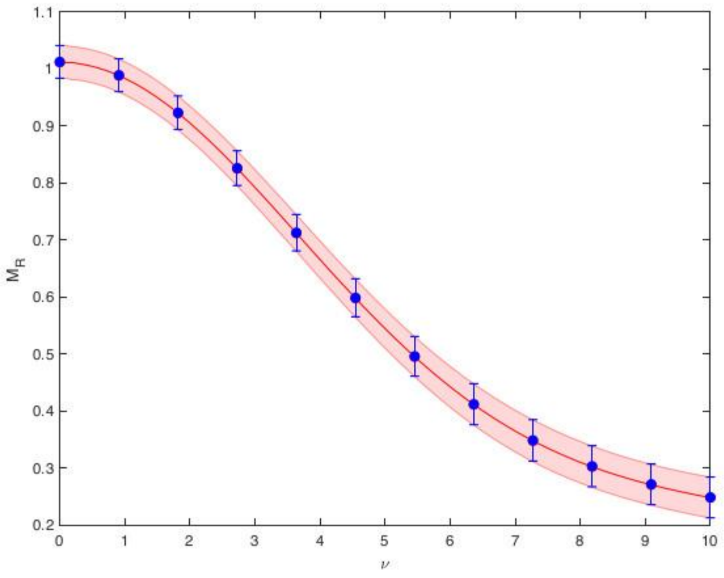


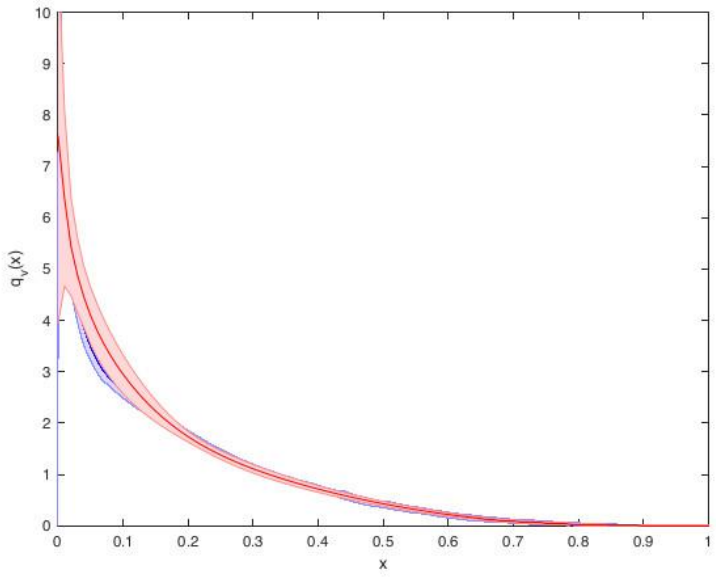
$$[\theta] = \{w, b\}$$

Mock Tests of Neural Network Reconstructions

- Tests use modified NNPDF data set with artificial errors.
- Reconstructions are more stable and reliable than direct inversion or fits.

  Neural Network





Summary

- Extraction of any PDF moment is possible directly from lattice data
- Qualitative agreement with PDFs despite few systematics under control
  - Evolution of data to a perturbative regime is important for recovery of divergent behavior
- Systematic issues with Fourier transform are controllable with inverse problem techniques
- Once these techniques are understood and controlled then any light cone distribution is within reach of the lattice.
- To Do List:
  - Systematics left to be thoroughly studied (pion mass, lattice spacing, ...)
  - Study PDF reconstruction methods on real lattice data
  - Extend to Spin dependent distributions and other hadrons of interest

Thank you for your attention!