Sudakov suppression of jets in dense QCD matter

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@ Jefferson Laboratory, November 26, 2018

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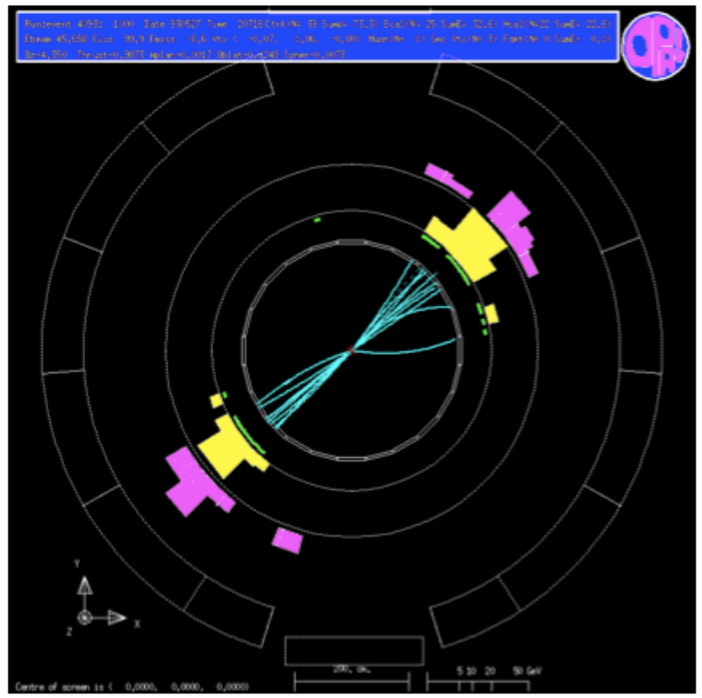
QCD jets

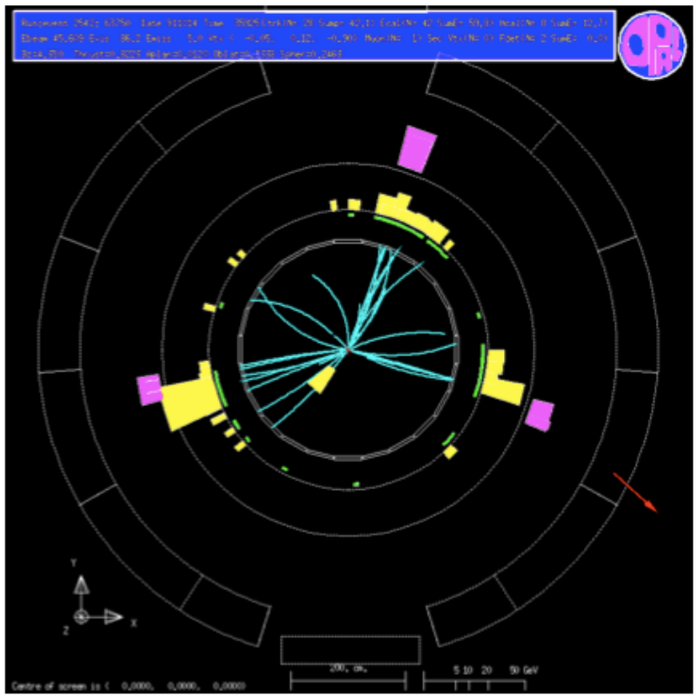
QCD Jet: collimated bunch of energetic particles

LEP (OPAL)

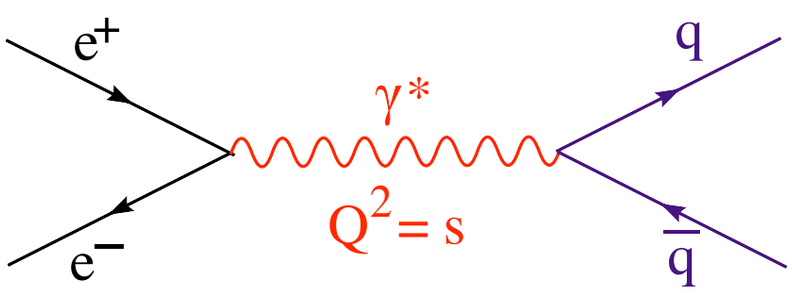
events

1993

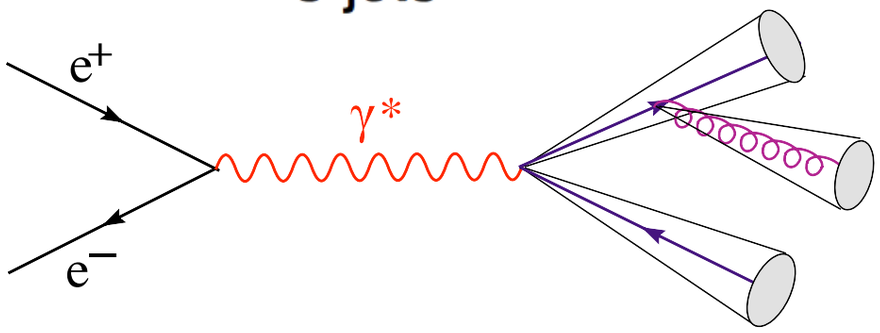




2 jets



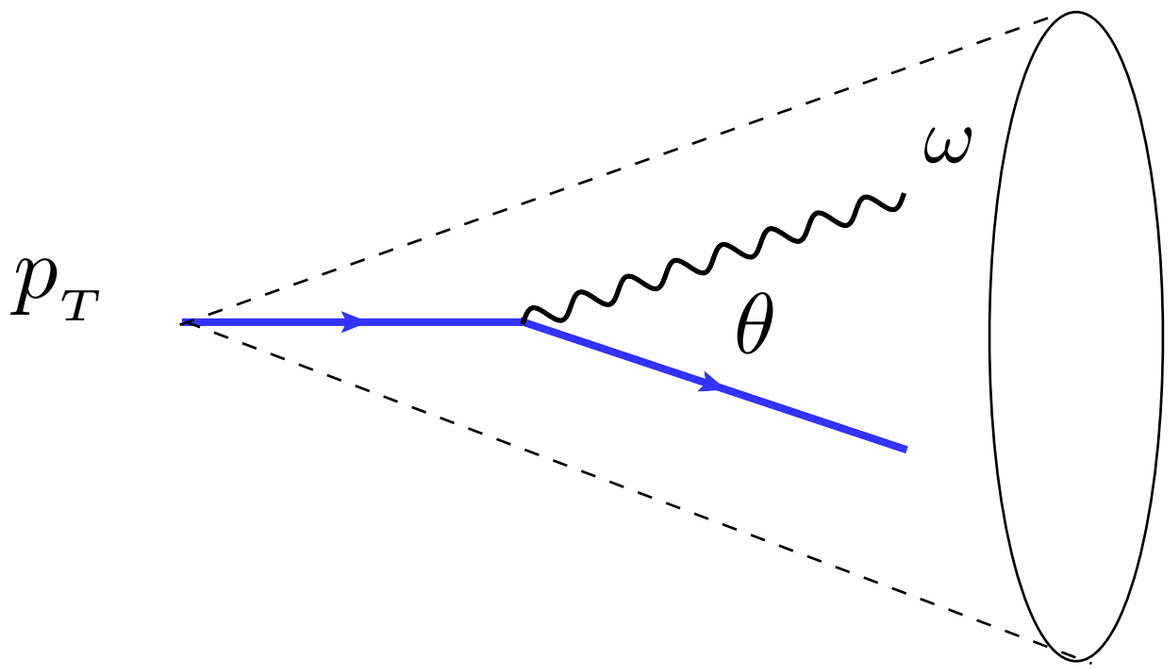
3 jets



→ Test of QCD, BMS search, **probe of the QGP**

QCD jets

Building block probability for parton cascades in vacuum



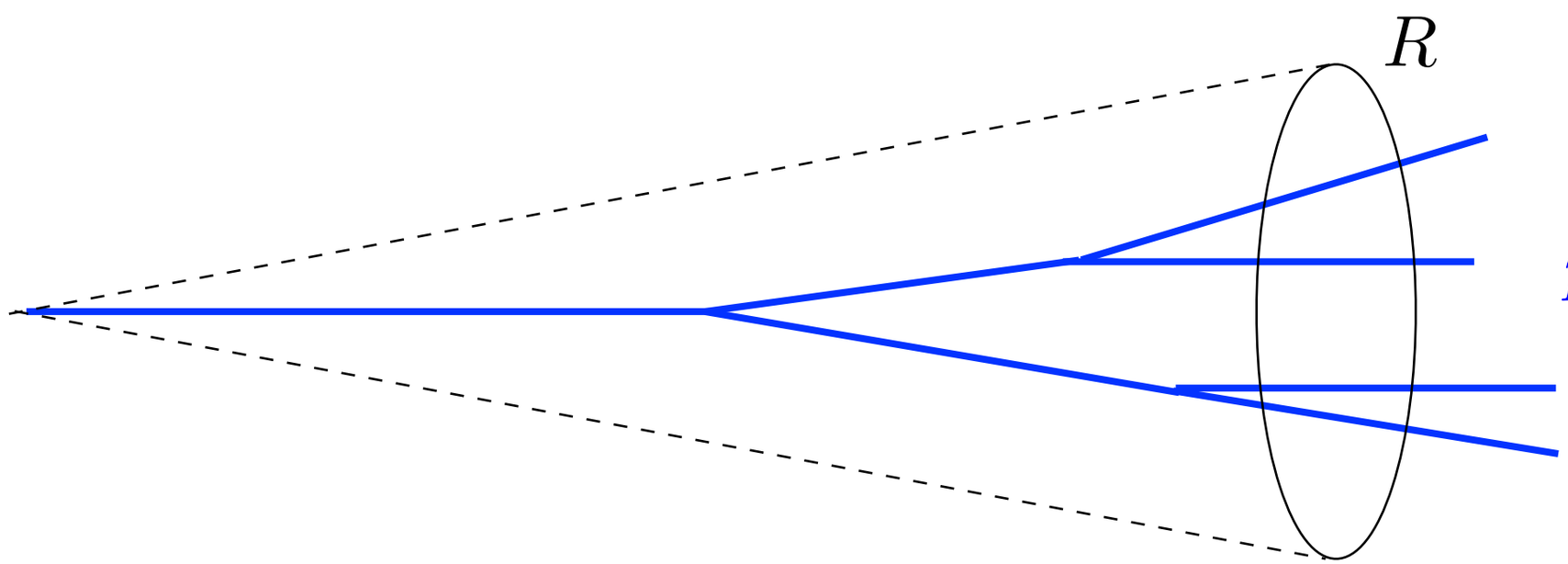
$$\mathrm{d}P \sim \alpha_s C_R \frac{\mathrm{d}\theta}{\theta} \frac{\mathrm{d}\omega}{\omega}$$

$\Longrightarrow$ collimation of jets

Large phase-space for multiple branching: many particles produced (implemented in Event Generators such as PYTHIA, HERWIG, SHERPA, etc.)

QCD jets

- Soft & Collinear divergences (resummation)
- **Color coherence:** angular ordered shower, interjet activity
- Not uniquely defined: cone size **R**, reconst. algo, …



$$p_T = \sum_i p_{Ti}$$

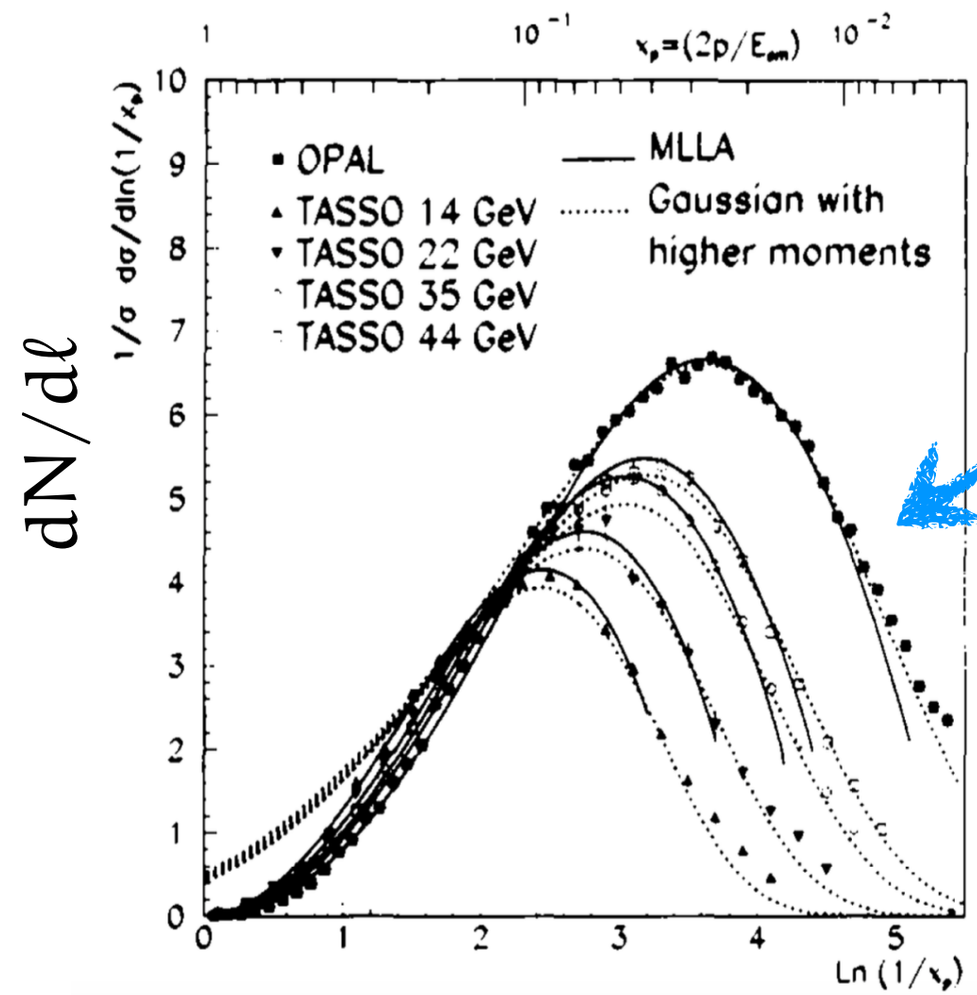
$$\frac{1}{E} \quad \ll \quad t_{\text{form}} \sim \frac{k_{||}}{k_\perp^2} \quad \ll \quad \frac{E}{\Lambda_{\text{QCD}}}$$

$$p_T \equiv E$$

Large separation of time scales

QCD jets

- Soft & Collinear divergences (resummation)
- **Color coherence:** angular ordered shower, interjet activity
- Not uniquely defined: cone size **R**, reconst. algo, …



$dN/d\ell$

Suppression of soft hadrons due to color coherence

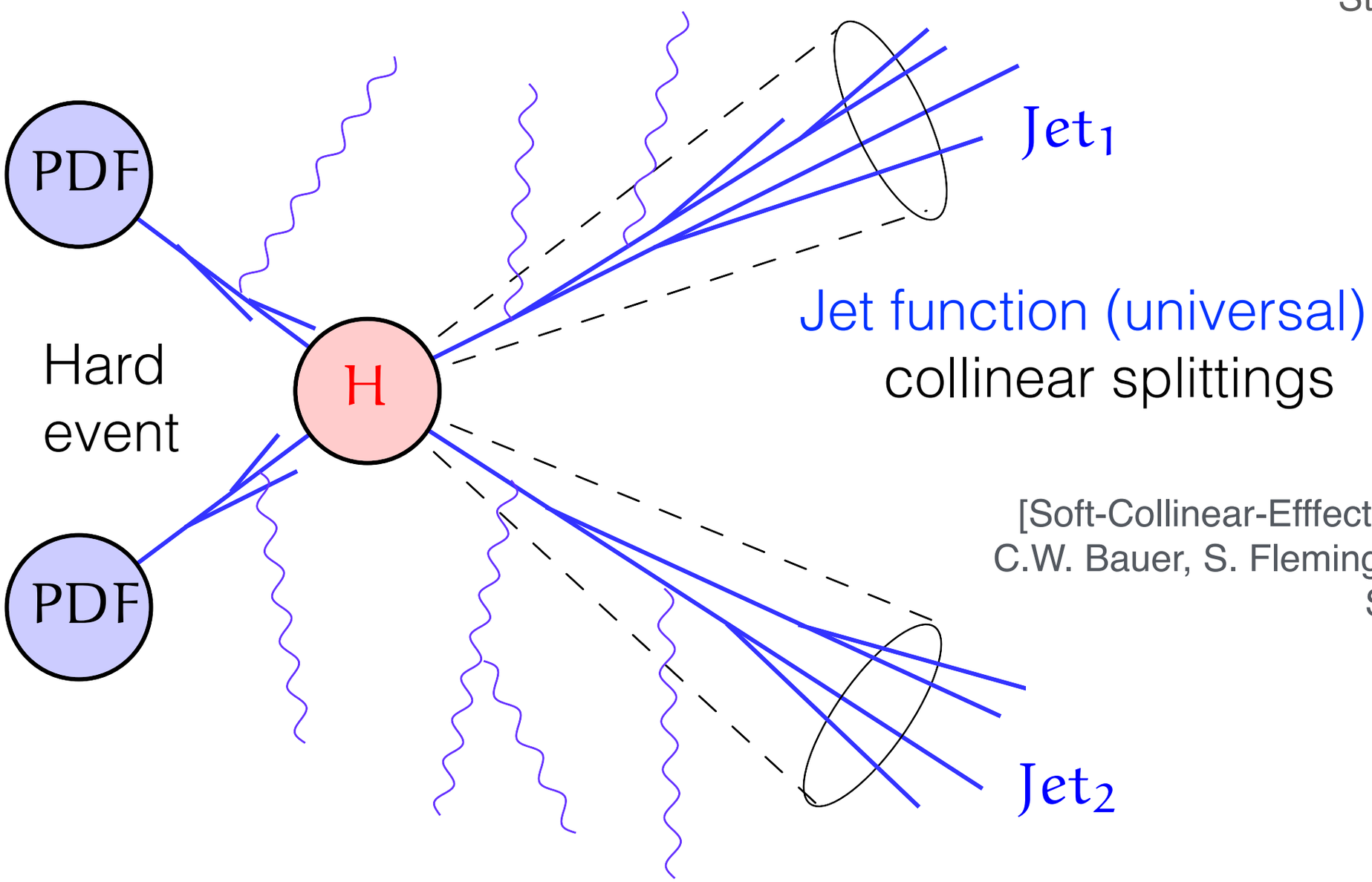
$$\ell = \ln \frac{1}{z}$$

Fragmentation function

Jets in pQCD and color coherence

coherent soft radiation

[Bassetto, Ciafaloni, Marchesini, Mueller, Dokshitzer, Khoze, Toyan, Collins, Soper, Sterman … 1980's]



Jet function (universal)
collinear splittings

[Soft-Collinear-Efffective Theory (SCET): C.W. Bauer, S. Fleming, D. Pirjol, and I.W. Stewart 2000-2002]

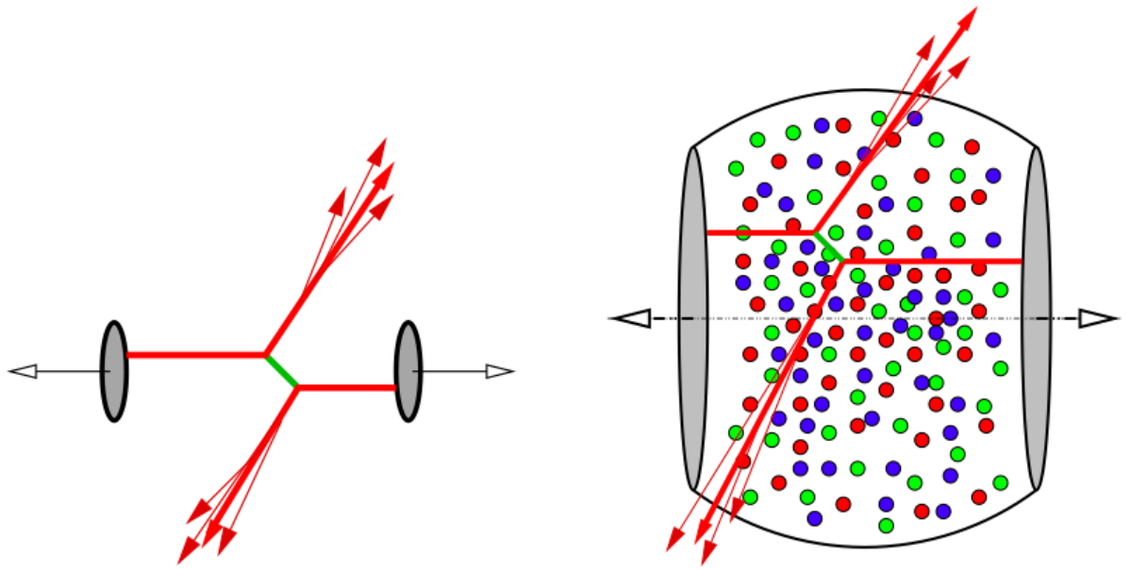
Factorization of the cross-section

$$\sigma \sim \mathrm{PDF} \times H \times J_1 \times J_2 \times S$$

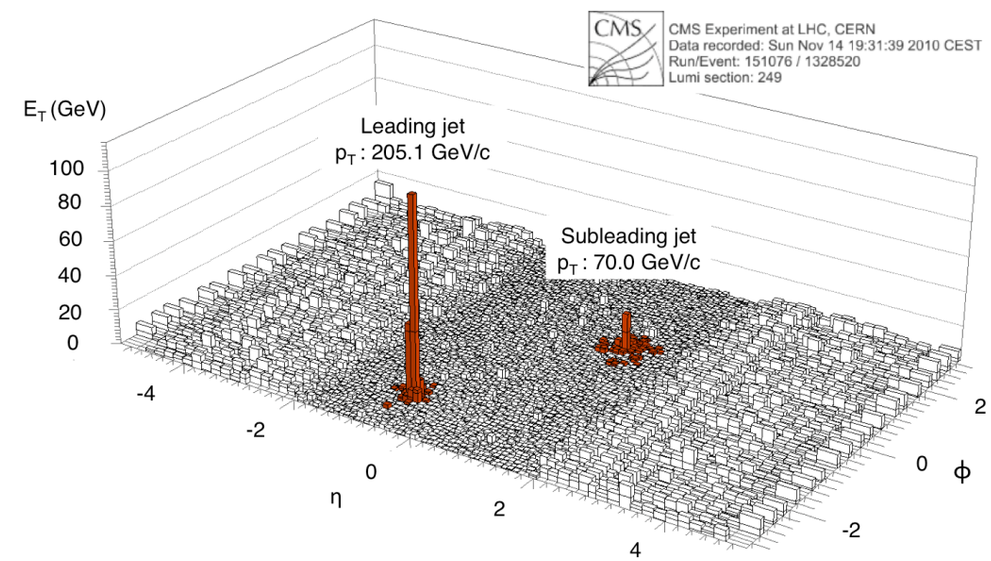
Jet observables of two types

- Infrared-Collinear (IRC) safe observables: sum over final state hadrons $\longrightarrow$ cancellations of divergences. Ex: event shape: thrust, jet mass, jet spectra, etc. Resummation of large logs, e.g. log R, log Q/M, can be necessary
- Collinear sensitive observables: pQCD still predictive (factorization theorems). Ex: Fragmentation Functions

Jets in Heavy Ion Collisions



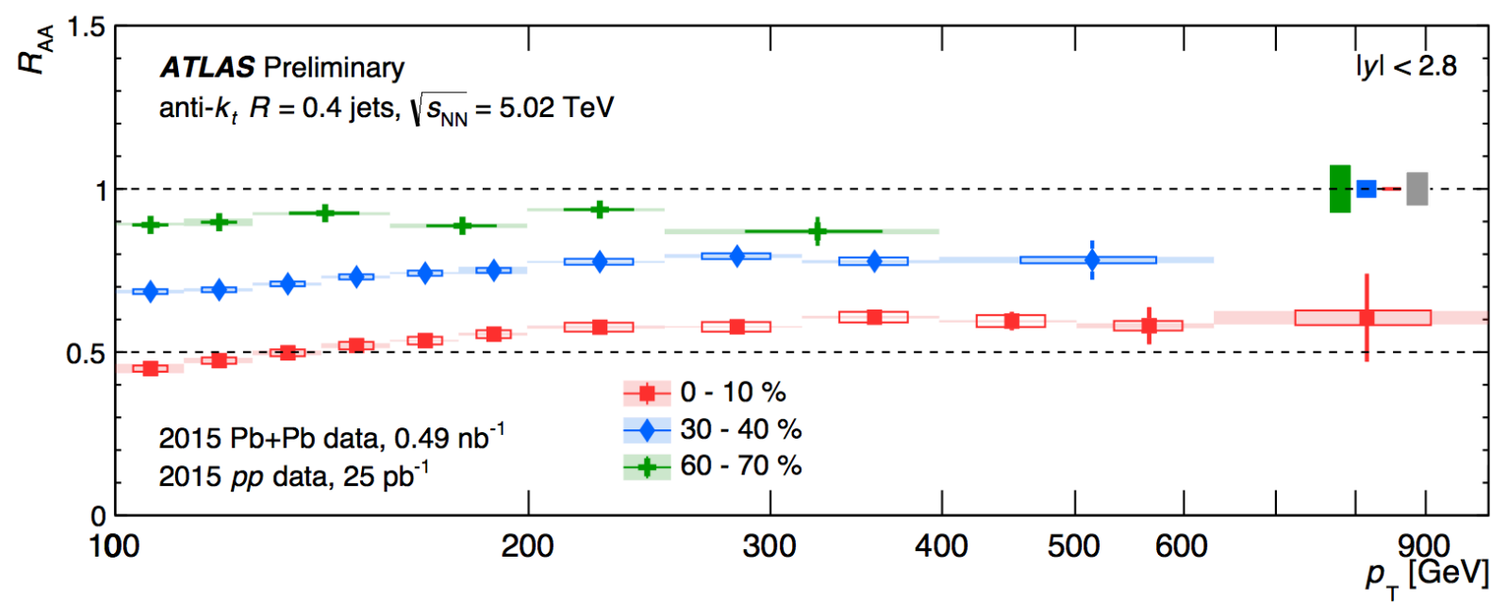
quark-gluon plasma



Strong jet suppression (**up to 1 TeV!**) observed in ultra-relativistic heavy ion collisions at LHC

$$R_{\mathrm{AA}} = \frac{\mathrm{d}N_{\mathrm{AA}}/\mathrm{d}^2p_T}{N_{\mathrm{coll}} \times \mathrm{d}N_{\mathrm{pp}}/\mathrm{d}^2p_T}$$

Inclusive jet spectra ratio



How much energy is lost?

- **A rough estimate:** consider a constant energy loss **ϵ**
- using a power spectrum $$\frac{\mathrm{d}N}{\mathrm{d}^2 p_T} \sim p_T^{-n}$$

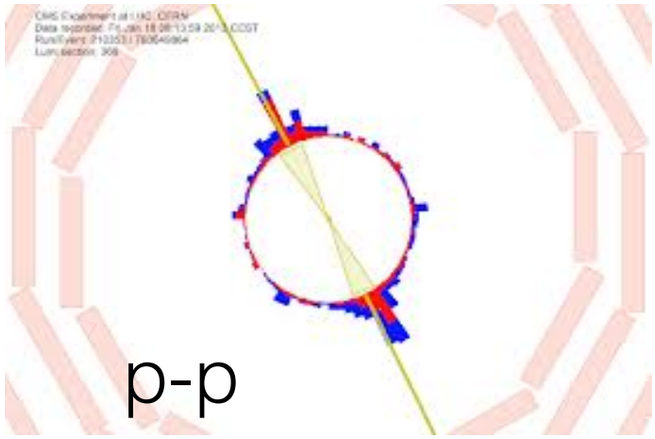
we have

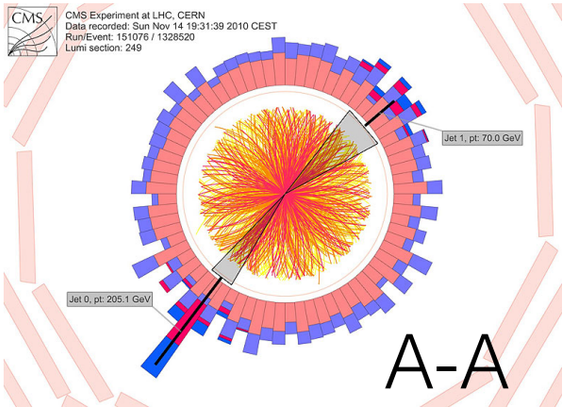
$$R_{\mathrm{AA}} \sim \frac{p_T^n}{(p_T + \epsilon)^n} \simeq 1 - \frac{n\epsilon}{p_T}$$

Hence, for **R_{AA} ~ 0.5** and **n = 6**, one finds that jets with **p_T ~ 300 GeV** lose typically about **ϵ ~ 25 GeV**

Extending pQCD Toolbox

- In Heavy Ion Collisions, standard perturbative techniques break down because of **final state interactions** **(large particle density)**
- **Thermalization, hydrodynamization, jet-quenching, …**
- Many scales in the problem: from the hard partons to the soft constituents of the thermal bath
- Standard approach to jet-quenching: treat jets as perturbations on top of a **classical background field** (QGP)





Outline

- Medium-induced radiation
- Single quark radiative energy loss
- NLO: two-pronged energy loss
- Resummation of large double-logarithms in the jet spectrum

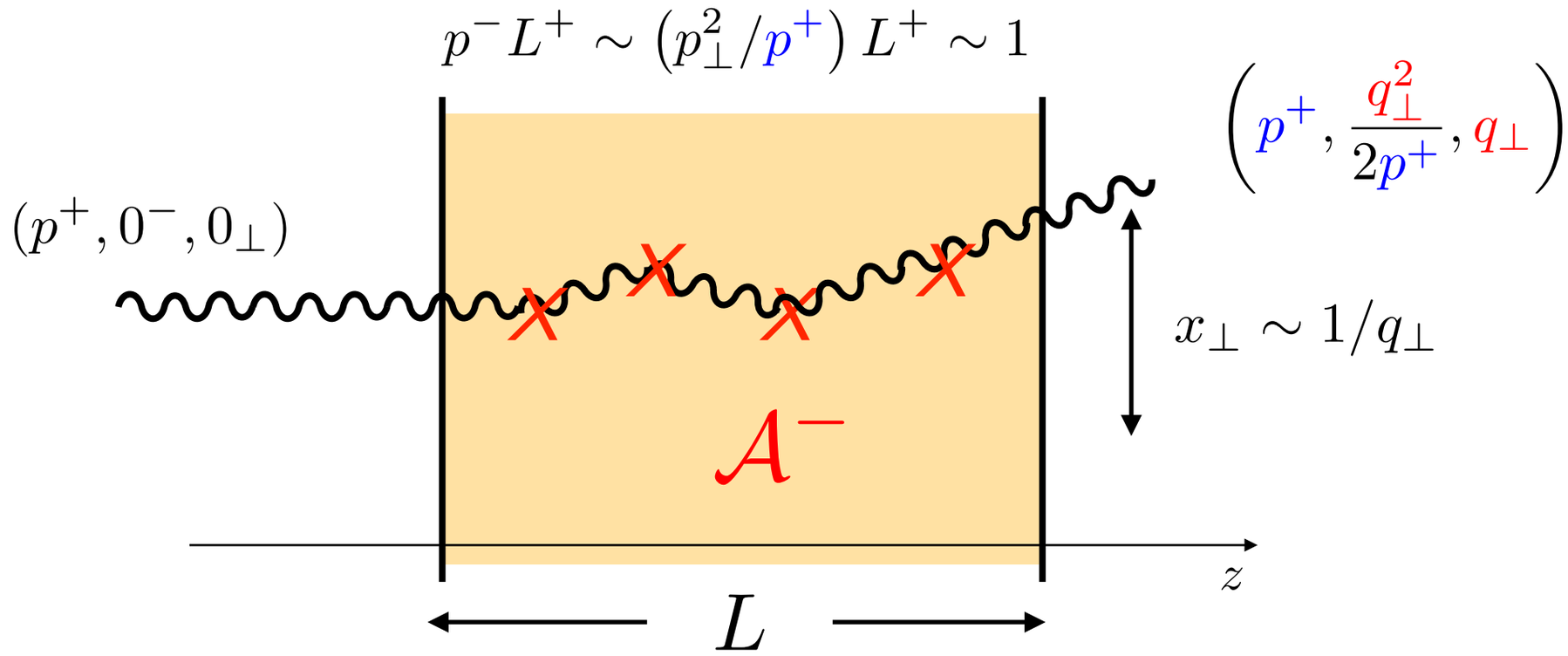
Medium-induced radiation

Non-eikonal propagation

- **The scattering remains eikonal:** neglect power corrections of the **small momentum transfer** $q^+ \ll p^+$:

$$\text{eikonal vertex} \sim \delta(q^+)\, p^\mu \;\Leftrightarrow\; \mathcal{A}^-(x^+, x_\perp)$$

- **Large medium:** allow the gluon to **explore the transverse plan** between two scatterings



Medium average

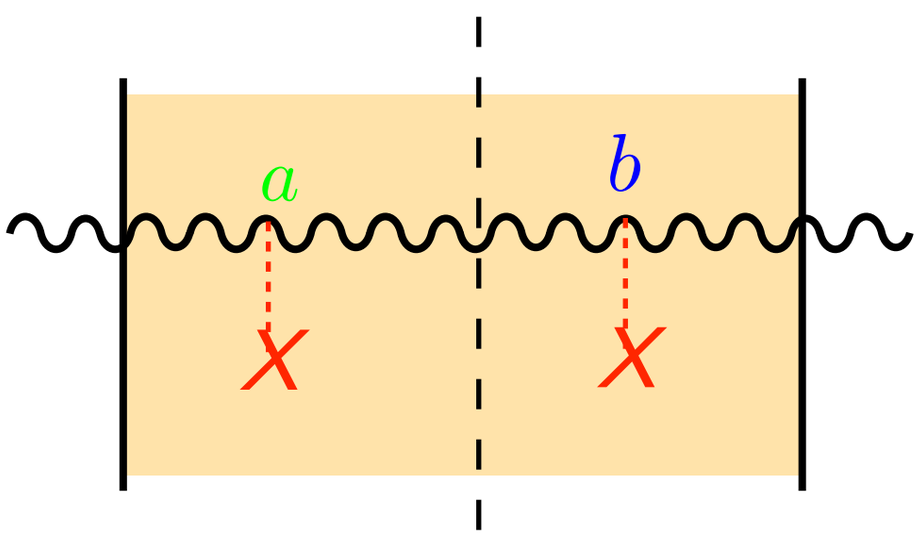
- Non-eikonal propagator described by a path integral:

$$\mathcal{G}_{p^+}(\boldsymbol{x}_\perp, t \,|\, \boldsymbol{x}'_\perp, t') = \int_{\boldsymbol{x}}^{\boldsymbol{x}'} \mathcal{D}\boldsymbol{r}\, \mathrm{e}^{i\int_t^{t'} \mathrm{d}\xi (\dot{\boldsymbol{r}}(\xi))^2}\; U[\mathcal{A}(\xi, \boldsymbol{r}(\xi))]$$

Wilson line

- Medium average: assume **Gaussian random variable**

$$\langle\, \mathcal{A}^a(\boldsymbol{p}, t)\mathcal{A}^b(\boldsymbol{p}', t')\,\rangle \equiv \delta^{ab}\delta(t-t')\delta(\boldsymbol{p}-\boldsymbol{p}')\gamma(\boldsymbol{p})$$



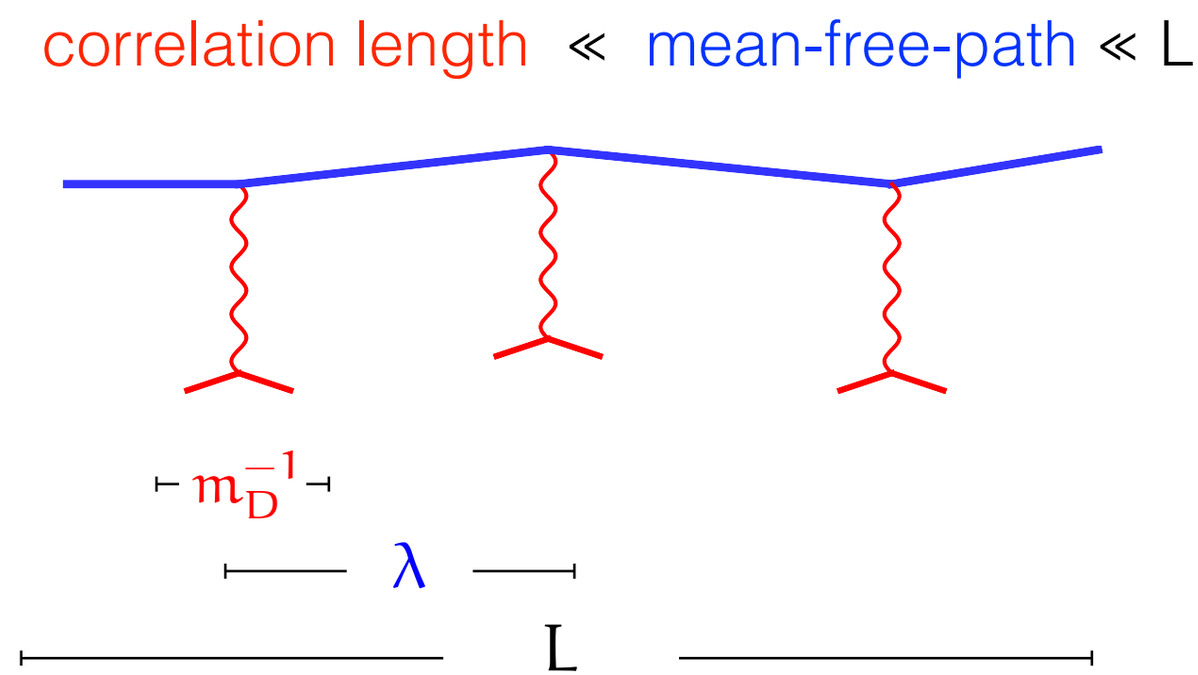
related to the (2 to 2) elastic cross-section

$$\gamma(\boldsymbol{q}) \sim \frac{\mathrm{d}\sigma}{\mathrm{d}^2\boldsymbol{q}} = \frac{\alpha_s^2}{\boldsymbol{q}^4}$$

The jet-quenching parameter

Momentum broadening (diffusion in transverse momentum space):

$$\langle k_\perp^2 \rangle \equiv \hat{q} L$$



- the jet-quenching $\hat{q}$ parameter encodes **medium properties** (LO: 2 to 2 elastic scattering):

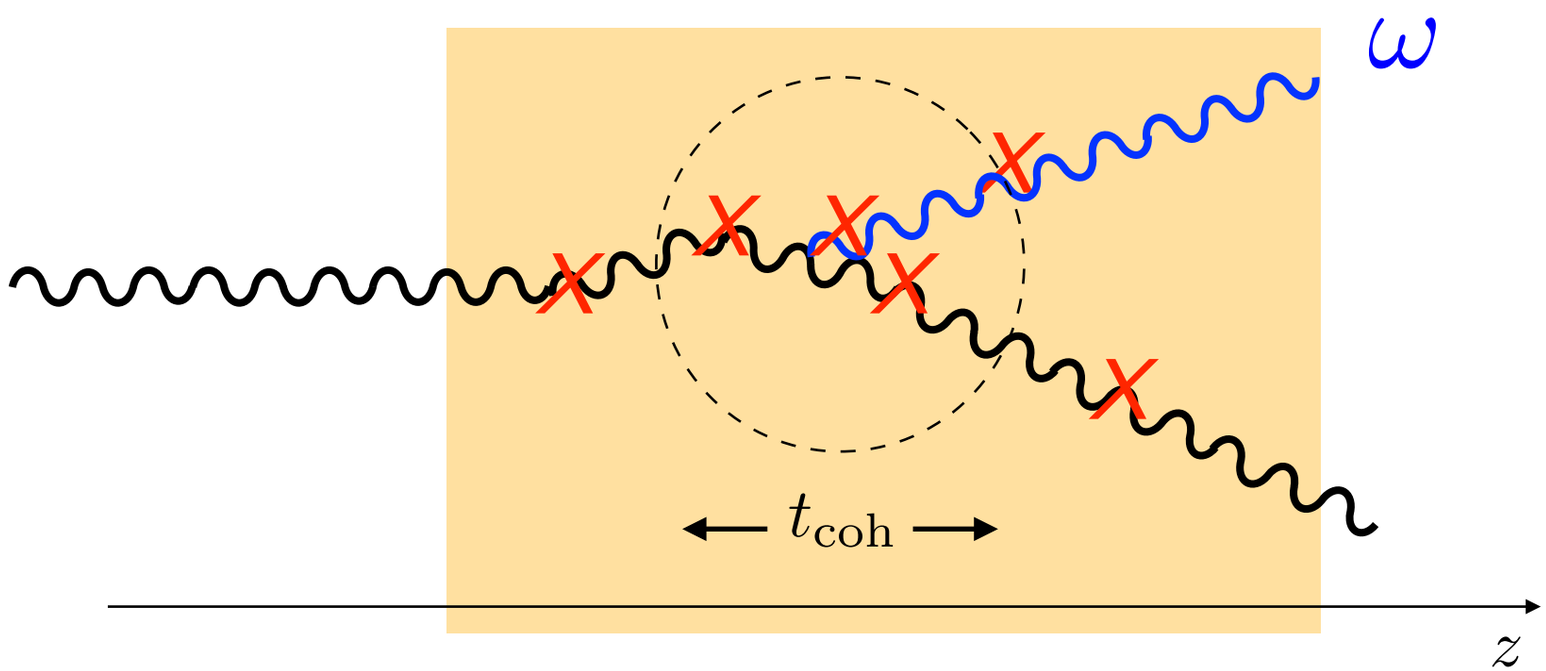
$$\hat{q} \equiv n \int_{q_\perp} q_\perp^2 \frac{\mathrm{d}\sigma_{\mathrm{el}}}{\mathrm{d}q_\perp} \sim \alpha_s^2 C_R \, n \, \ln \frac{Q^2}{m_D^2}$$

estimate: $Q^2 \sim \hat{q} L \sim 10\,\mathrm{GeV}^2$

Medium-induced splittings

- Multiple scattering can trigger gluon radiation
- **Laudau-Pomeranchuk-Migdal effect:** during the splitting time many scattering centers act coherently as a single one and thus, suppressing the radiation rate

$$t_{\rm coh} = \frac{\omega}{k_\perp^2} \sim \frac{\omega}{\hat{q}\, t_{\rm coh}} \qquad \Rightarrow \qquad t_{\rm coh} \sim \sqrt{\frac{\omega}{\hat{q}}}$$



Radiative spectrum

$$\omega \frac{{\rm d}I}{{\rm d}\omega} \sim \bar{\alpha} \sqrt{\frac{\hat{q} L^2}{\omega}}$$

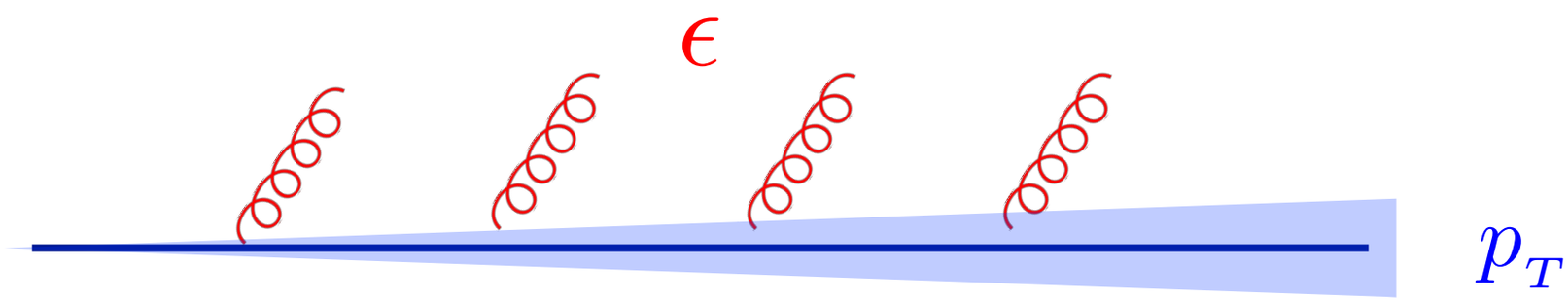
[Baier, Dokshitzer, Mueller, Peigné, Schiff (1995-2000) Zakharov (1996) Wiedemann (2000) Arnold, Moore, Yaffe (2002)]

[Dilute medium limit. N=1 opacity expansion: Gyulassy, Levai, Vitev (2000) Guo, Wang (2000)]

Single quark energy loss

Single quark energy loss

- **Standard energy loss picture:** medium-induced radiation off a single parton [Baier, Dokshitzer, Mueller, Schiff, JHEP (2001)]



- **Jet spectrum:** convolution of the energy loss probability with the spectrum in vacuum

$$\frac{\mathrm{d}\sigma(p_T)}{\mathrm{d}^2 p_T \mathrm{d}y} = \int_0^\infty \mathrm{d}\epsilon\, \mathcal{P}(\epsilon)\, \frac{\mathrm{d}\sigma^{\mathrm{vac}}(p_T+\epsilon)}{\mathrm{d}^2 p_T \mathrm{d}y}$$

Single quark energy loss

Because the **jet spectrum** is **steeply falling (n >> 1)** one can make the following approximation

$$\frac{\mathrm{d}\sigma^{\mathrm{vac}}(p_T+\epsilon)}{\mathrm{d}^2 p_T \mathrm{d}y} \sim \frac{1}{(p_T+\epsilon)^n} \simeq \frac{\mathrm{e}^{-\frac{n\epsilon}{p_T}}}{{p_T}^n}$$

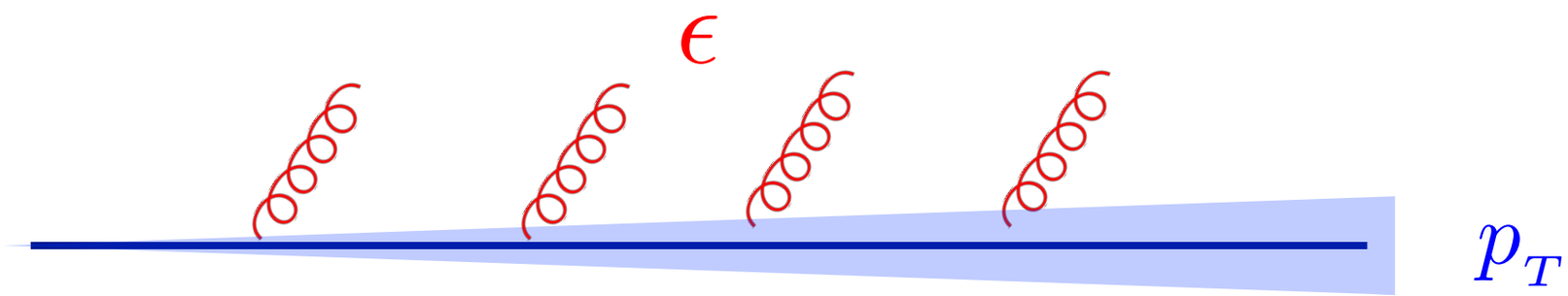
This allows to relate the **jet spectrum** to the **Laplace Transform** of the quenching probability

$$R_{\mathrm{AA}} \sim Q(p_T) \equiv \tilde{\mathcal{P}}(\nu = n/p_T)$$

where

$$\tilde{\mathcal{P}}(\nu) = \int \mathrm{d}\epsilon\, \mathcal{P}(\epsilon)\, \mathrm{e}^{-\nu\epsilon}$$

Single quark energy loss



In the **short formation time** approximation soft radiations can be treaded as independent yielding a **Poisson-like Distribution:**

$$\mathcal{P}(\epsilon) = \mathrm{e}^{-\int \frac{\mathrm{d}I}{\mathrm{d}\omega}\mathrm{d}\omega} \left\{ 1 + \sum_{n=1}^{\infty} \frac{1}{n!} \prod_{i=1}^{n} \int \mathrm{d}\omega_i \frac{\mathrm{d}I}{\mathrm{d}\omega_i} \delta(\epsilon - \sum_i^n \omega_i) \right\}$$

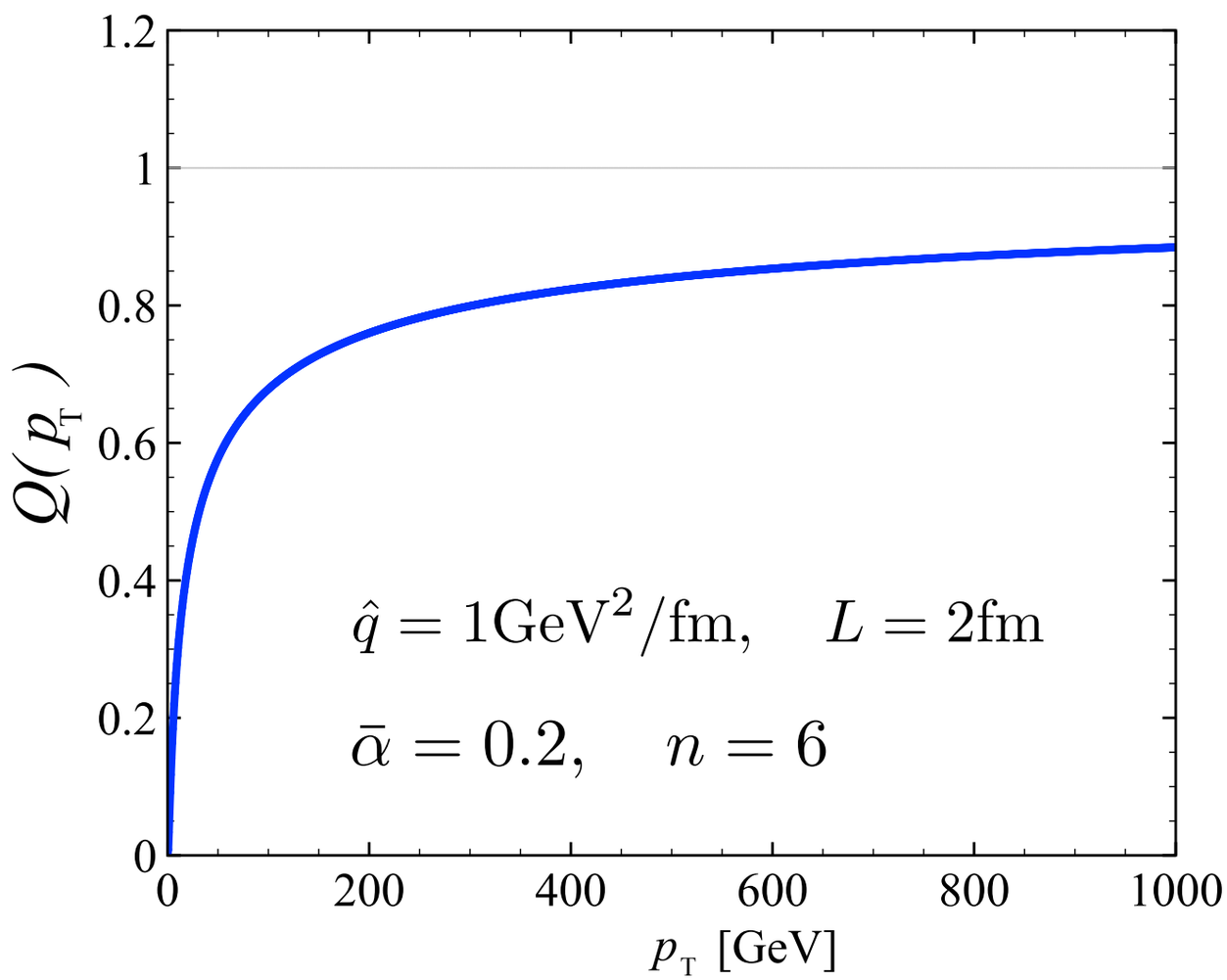
In **Laplace space** it reduces to

$$\tilde{\mathcal{P}}(\nu) = \exp\left[- \int \mathrm{d}\omega \frac{\mathrm{d}I}{\mathrm{d}\omega} \left(1 - \mathrm{e}^{-\nu\omega}\right) \right]$$

Single quark energy loss

- Neglecting **finite size effects** one obtains a simple analytic formula for the quenching factor

$$\mathcal{Q}(p_T) \simeq \exp\left(-\bar{\alpha}\, L \sqrt{\frac{\pi \hat{q}\, n}{p_T}}\right)$$



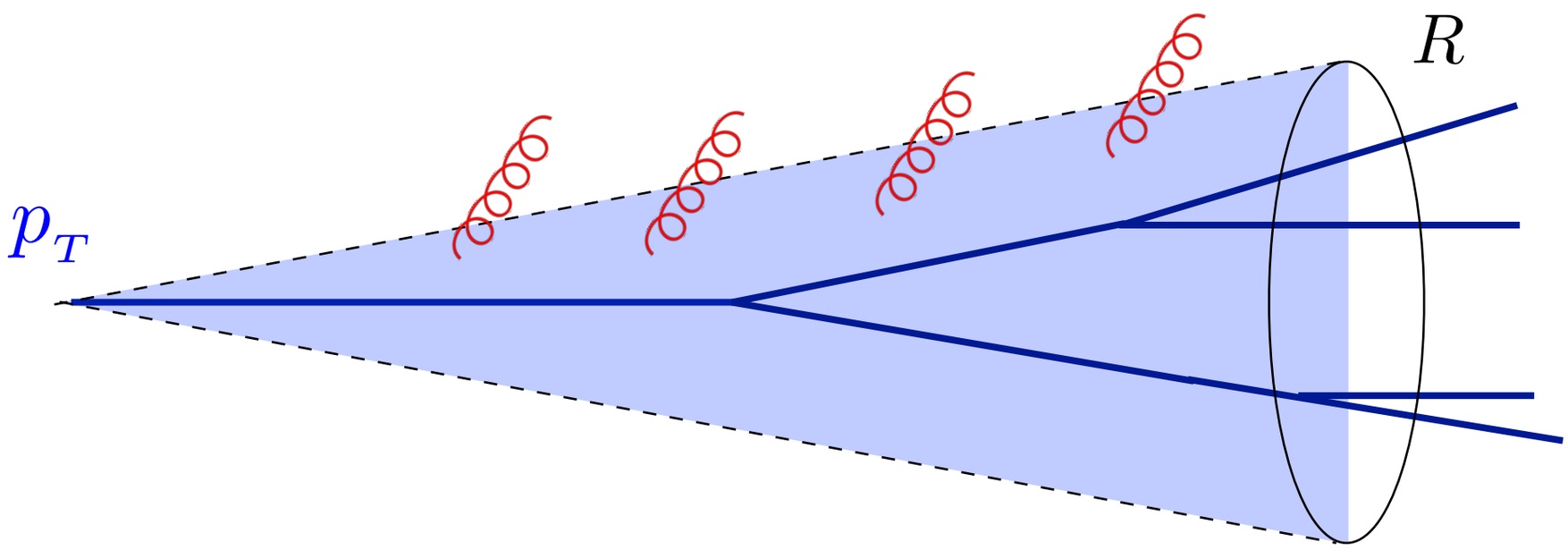
Strong quenching

$$p_T \ll \pi\, n\, \bar{\alpha}^2 \hat{q} L^2$$

$$Q(p_T) \ll 1$$

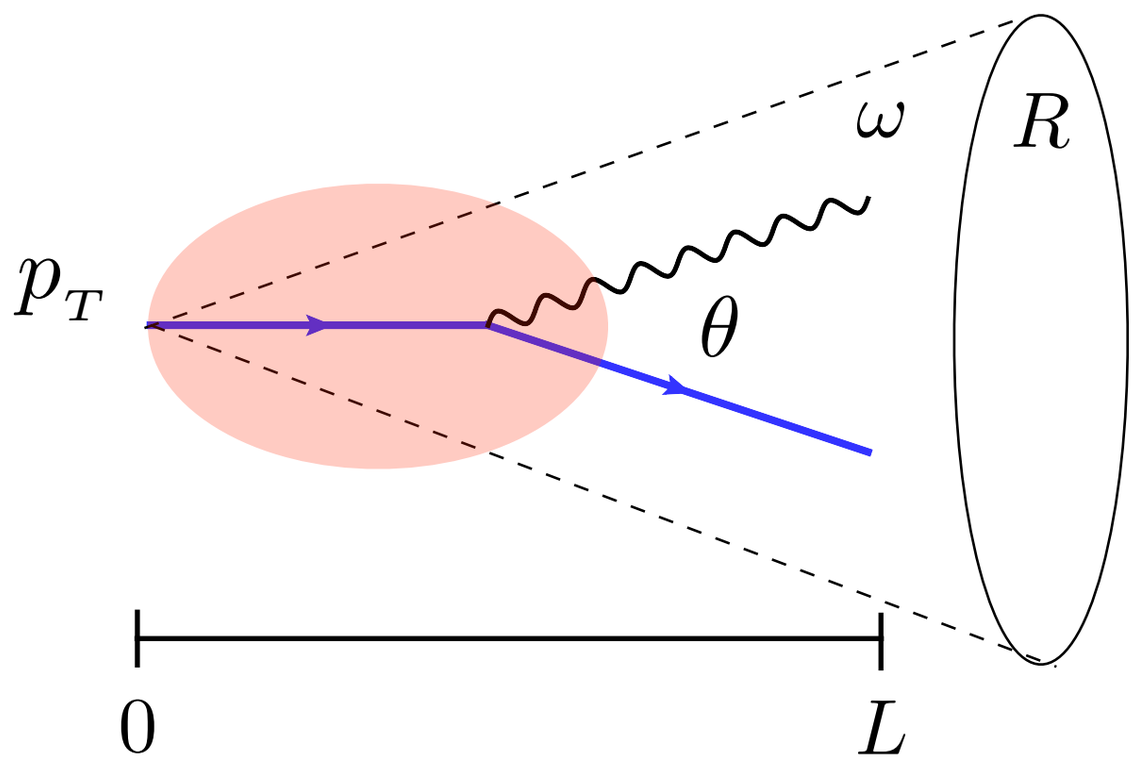
Jet quenching and fluctuations

- Energy is lost mainly via radiation **but how does a jet as a muti-parton system lose energy to the medium?**
- Does one need to account for fluctuations of energy loss due to **fluctuations of the jet substructure**?



Phase-space analysis

- How large are next-to-leading order contributions?



$$t_{\mathrm{f}} = \frac{1}{\omega\theta^2}$$

- Probability for a virtual quark to split inside the medium:

$$\mathrm{PS} = \bar{\alpha} \int_0^{p_T} \frac{\mathrm{d}\omega}{\omega} \int_0^R \frac{\mathrm{d}\theta}{\theta} \; \Theta(t_{\mathrm{f}} < L) = \frac{\bar{\alpha}}{4} \log^2 \left(p_T R^2 L \right)$$

Phase-space analysis

- Large double-logarithmic phase-space at high pT:

$$\frac{1}{p_T R^2} \quad \ll \quad t_{\rm f} \quad \ll \quad L$$

- When $\bar{\alpha}\log^2(p_T R^2 L) \gtrsim 1$ higher-orders are not negligible

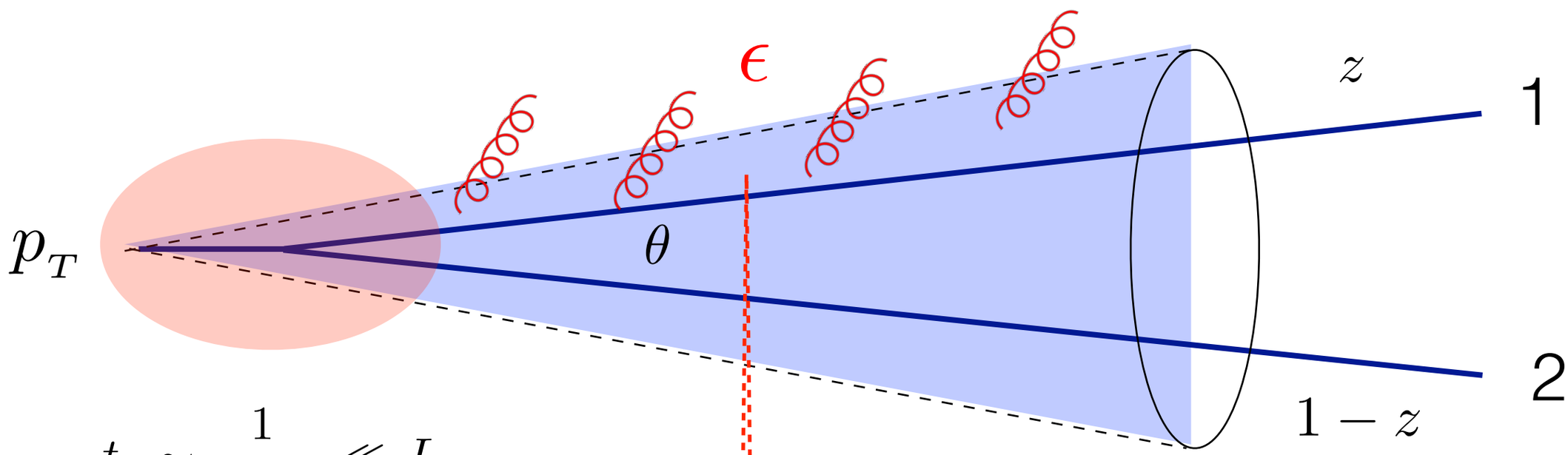
$\Rightarrow$ **double-logs (DL) need to be resummed**

- **Estimate:** for R=0.3, L=2 fm and pT=500 GeV, one finds **$\mathrm{Log}^2 \sim 40$**

Two-pronged energy loss

Two-pronged energy loss

- Consider a high energy parton that splits rapidly into two hard subjects within the jet cone
- At high p_T the branching time is shorter than the length of the medium ⇒ factorization



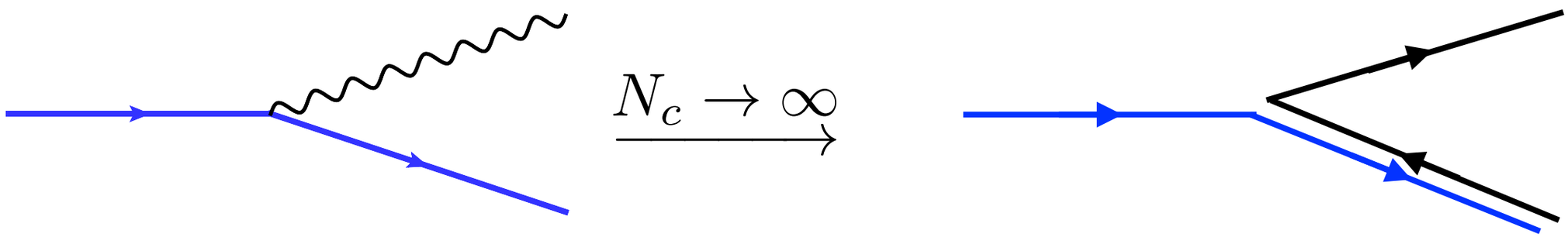
$$t_f \sim \frac{1}{p_T \theta^2} \ll L$$

- Two pronged inclusive spectrum:

$$\theta \frac{\mathrm{d}N}{\mathrm{d}\theta \mathrm{d}z \mathrm{d}p_T} = \int_0^\infty \mathrm{d}\epsilon \, P_2(\epsilon) \, \bar{\alpha} \, P(z) \, \frac{\mathrm{d}N^{\mathrm{vac}}(p_T + \epsilon)}{\mathrm{d}p_T}$$

Two-pronged energy loss

- In the large-Nc approximation



- The two-pronged energy loss probability factorizes into the total charge probability convoluted with the color singlet antenna probability distribution

$$\mathcal{P}(\epsilon) = \int_{\epsilon_1,\epsilon_2} \mathcal{P}_{\rm tot}(\epsilon_1)\,\mathcal{P}_{\rm sing}(\epsilon_2)\;\delta(\epsilon - \epsilon_1 - \epsilon_2)$$

Two-pronged energy loss

arXiv:1706.06047 [hep-ph]

- The **color singlet antenna** probability distribution reads:

$$\mathcal{P}_{\rm sing}(\epsilon, L) = \int_{\epsilon_1,\epsilon_2} \mathcal{P}_q(\epsilon_1, L)\, \mathcal{P}_q(\epsilon_2, L)\ \delta(\epsilon - \epsilon_1 - \epsilon_2)$$
$$+\, 2\int_0^L {\rm d}t \int_{\epsilon_1,\epsilon_2,\omega} \mathcal{P}_q(\epsilon_1, L-t)\mathcal{P}_q(\epsilon_2, L-t)\, \Gamma(\omega)\, S(t)\delta(\epsilon - \epsilon_1 - \epsilon_2 - \omega)$$

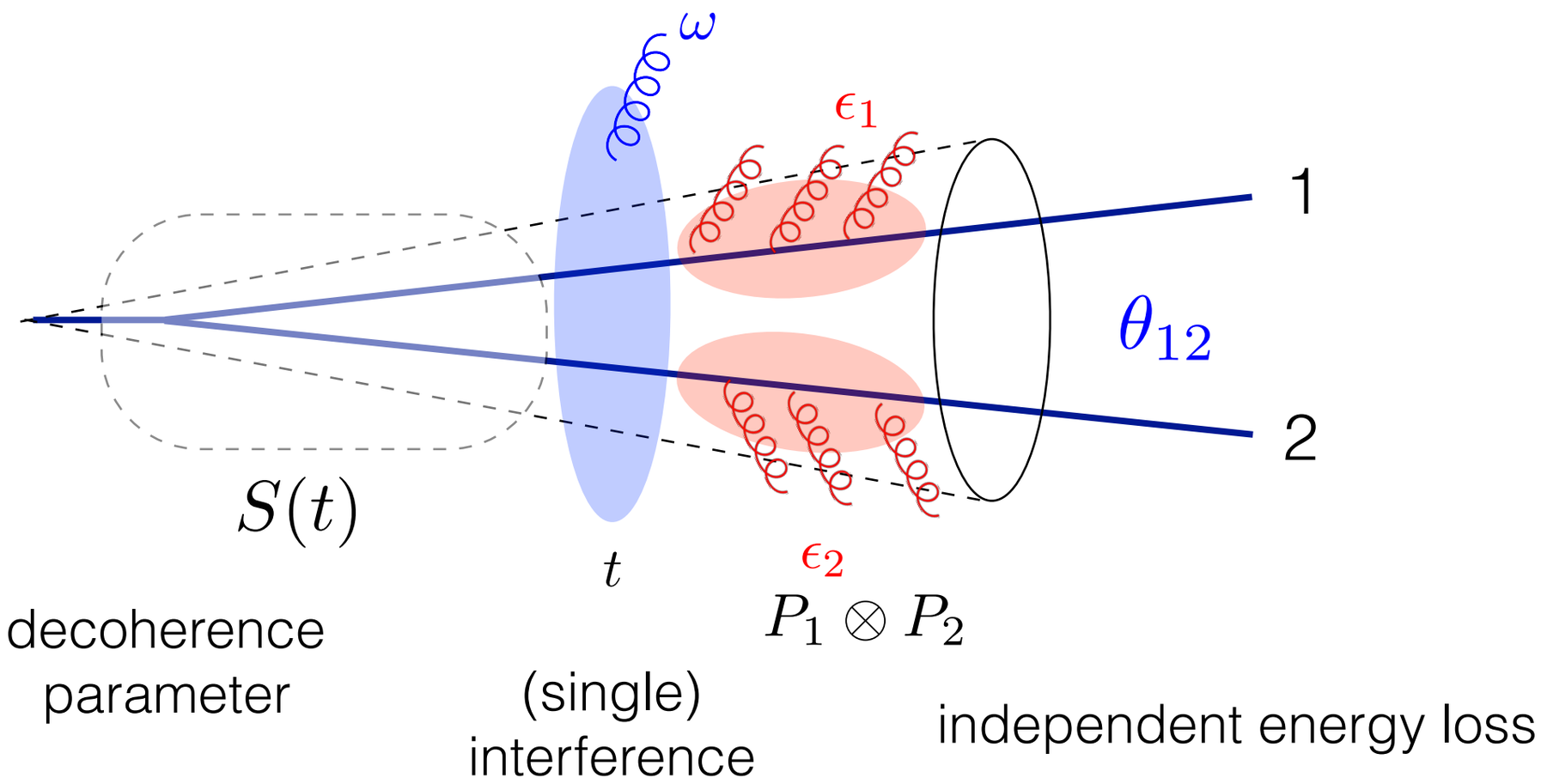
- with $$\Gamma(\omega) = \frac{{\rm d}I}{{\rm d}\omega {\rm d}t} - \delta(\omega)\int_0^\infty {\rm d}\omega' \frac{{\rm d}I}{{\rm d}\omega' {\rm d}t}$$

- **Decoherence** parameter

$$S(L) = \exp\left[-\frac{1}{12}(L/t_{\rm d})^3\right] \qquad t_{\rm d} \equiv (\hat{q}\,\theta^2)^{1/3}$$

- Two terms: **independent energy loss** + **interferences**

Two-pronged energy loss



- Propagation of two color charges at fixed angle
- Up to the decoherence time $t_d \sim (\hat{q}\,\theta_{12}^2)^{-1/3}$ radiation off the total charge
- At large angle: suppression of neighboring jets

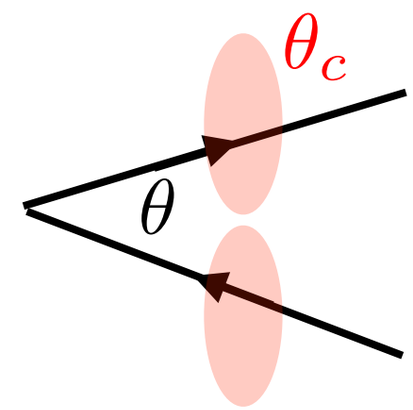
Two-pronged energy loss

Two limiting cases:

I - the medium resolves the antenna: $t_{\rm d} \gg L \quad (\theta \gg \theta_c \equiv 1/\sqrt{\hat{q}L^3})$

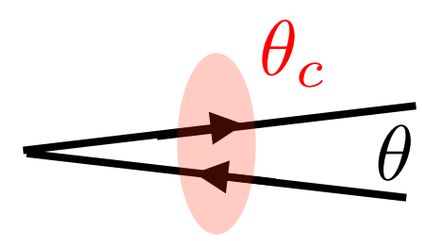
$S(L) \to 0$

$\mathcal{P}_{\rm sing}(\epsilon) \to \int_1 \mathcal{P}_q(\epsilon_1)\mathcal{P}_q(\epsilon - \epsilon_1)$



II - the medium does not resolve the antenna: $t_{\rm d} \gg L \quad (\theta \gg \theta_c)$

$S(L) \to 1$ $\qquad \mathcal{P}_{\rm sing}(\epsilon) \to \delta(\epsilon)$



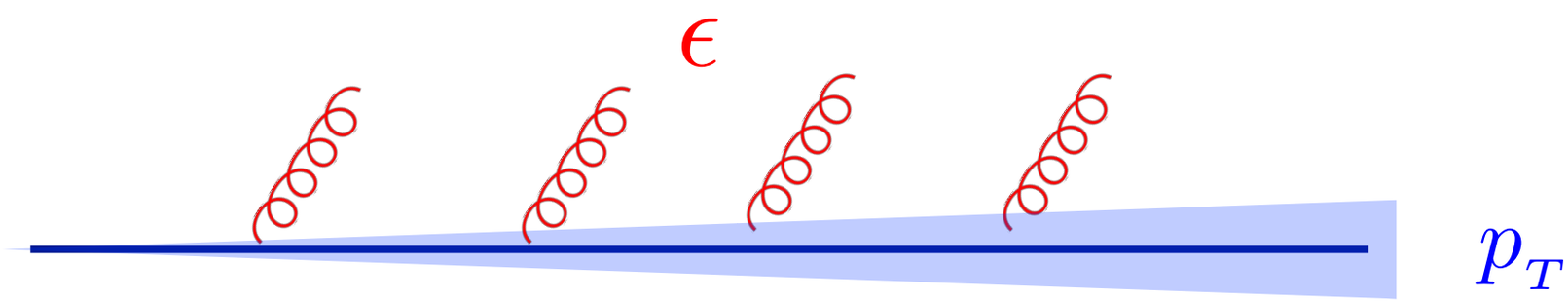
MT, Salgado, Tywoniuk PRL (2011), PLB (2012), JHEP (2011-2012)
Casalderrey-Solana, Iancu JHEP (2012)
Casalderrey-Solana, MT, Salgado, Tywoniuk PLB (2013)

Jet spectrum

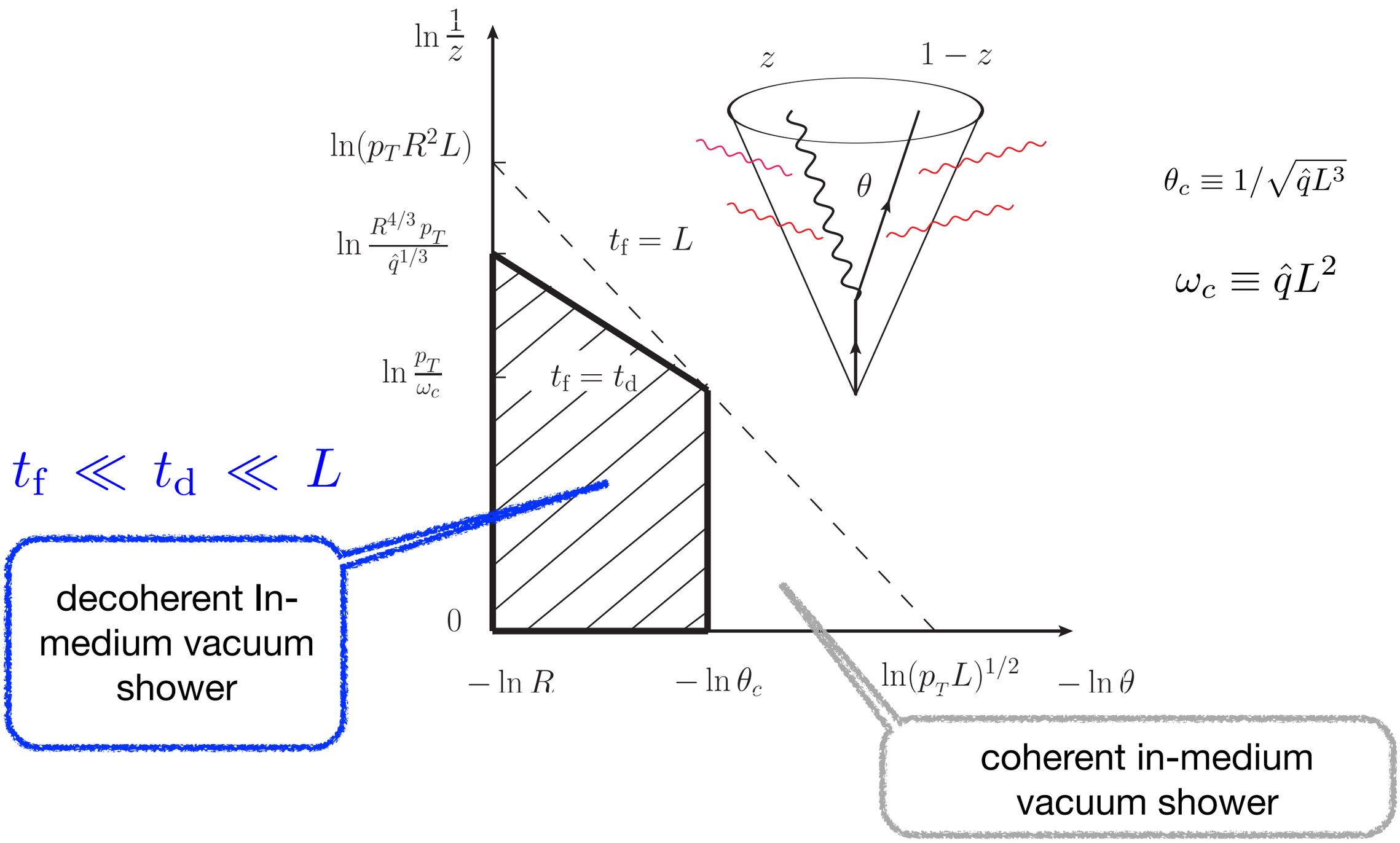
First correction to the jet spectrum

- To LO the quenching factor is that of the total charge (primary quark)

$$Q^{(0)}(p_T) = Q_{\rm tot}(p_T) \equiv Q_{\rm q}(p_T)$$



Jet evolution: phase-space analysis

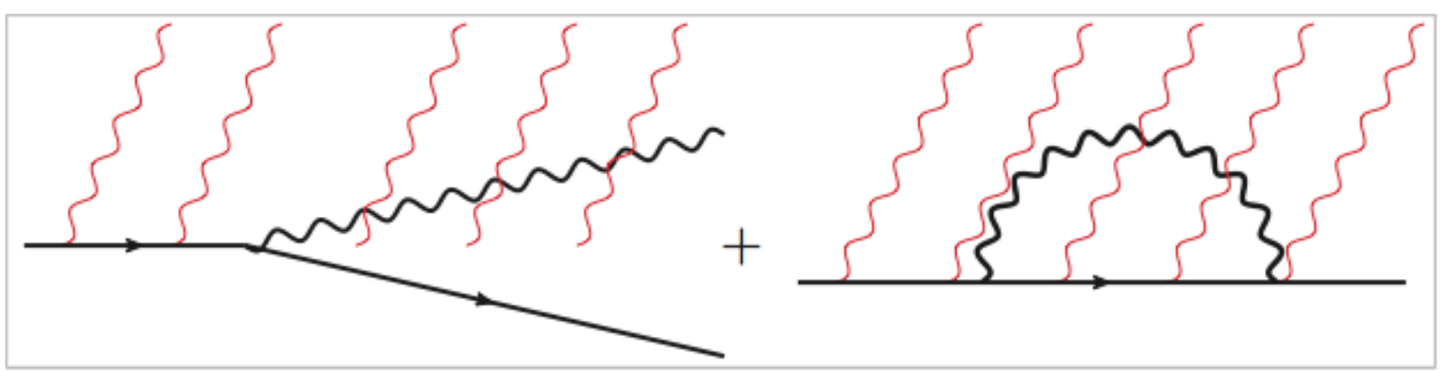


NLO corrections to the jet spectrum

$$R_{\mathrm{AA}} = Q^{(0)} + \bar{\alpha}\, Q^{(1)} + \mathcal{O}(\bar{\alpha}^2)$$

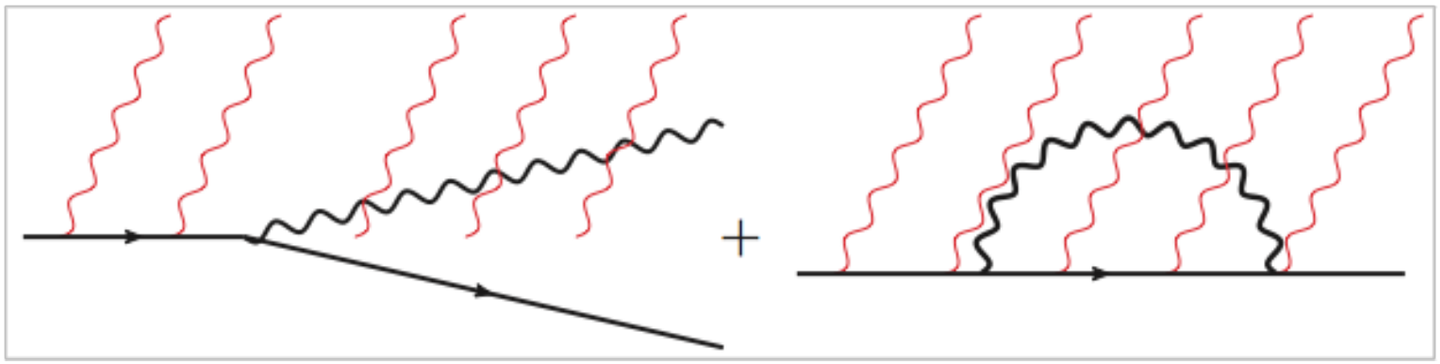
- To leading-log (LL) accuracy: cancellation between real and virtual contributions (KLN theorem) except in the region

$$t_{\mathrm{f}} \ll t_{\mathrm{d}} \ll L$$



NLO corrections to the jet spectrum

Mismatch between real and virtual



gluon + quark energy loss $\gg$ quark energy loss

$$Q^{(1)}(p_T) = \bar{\alpha} \int_{\theta_c}^{R} \frac{\mathrm{d}\theta}{\theta} \int_{(\hat{q}/\theta^4)^{1/3}}^{p_T} \frac{\mathrm{d}\omega}{\omega} \left[Q_q^2(p_T) - 1 \right] \; Q_{\mathrm{tot}}(p_T)$$

Leading Log approximation to the jet spectrum

- LL contributions exponentiate in the **strong quenching limit**: only the leading particle survives albeit suppressed by a **Sudakov form factor** in addition to its total charge energy loss

$$C(p_T) = \exp\left[-2\bar{\alpha}\ln\frac{R}{\theta_c}\left(\ln\frac{p_T}{\omega_c} + \frac{2}{3}\ln\frac{R}{\theta_c}\right)\right]$$

- Now we have

$$R_{\mathrm{AA}} \sim Q(p_T) = Q_{\mathrm{tot}}(p_T) \times C(p_T)$$

- Increasing suppression with R (at large R energy must be recovered, not included here). Effect observed on groomed jets with JEWEL event generator [Andrews et al (2018)]

Leading Log approximation to the jet spectrum

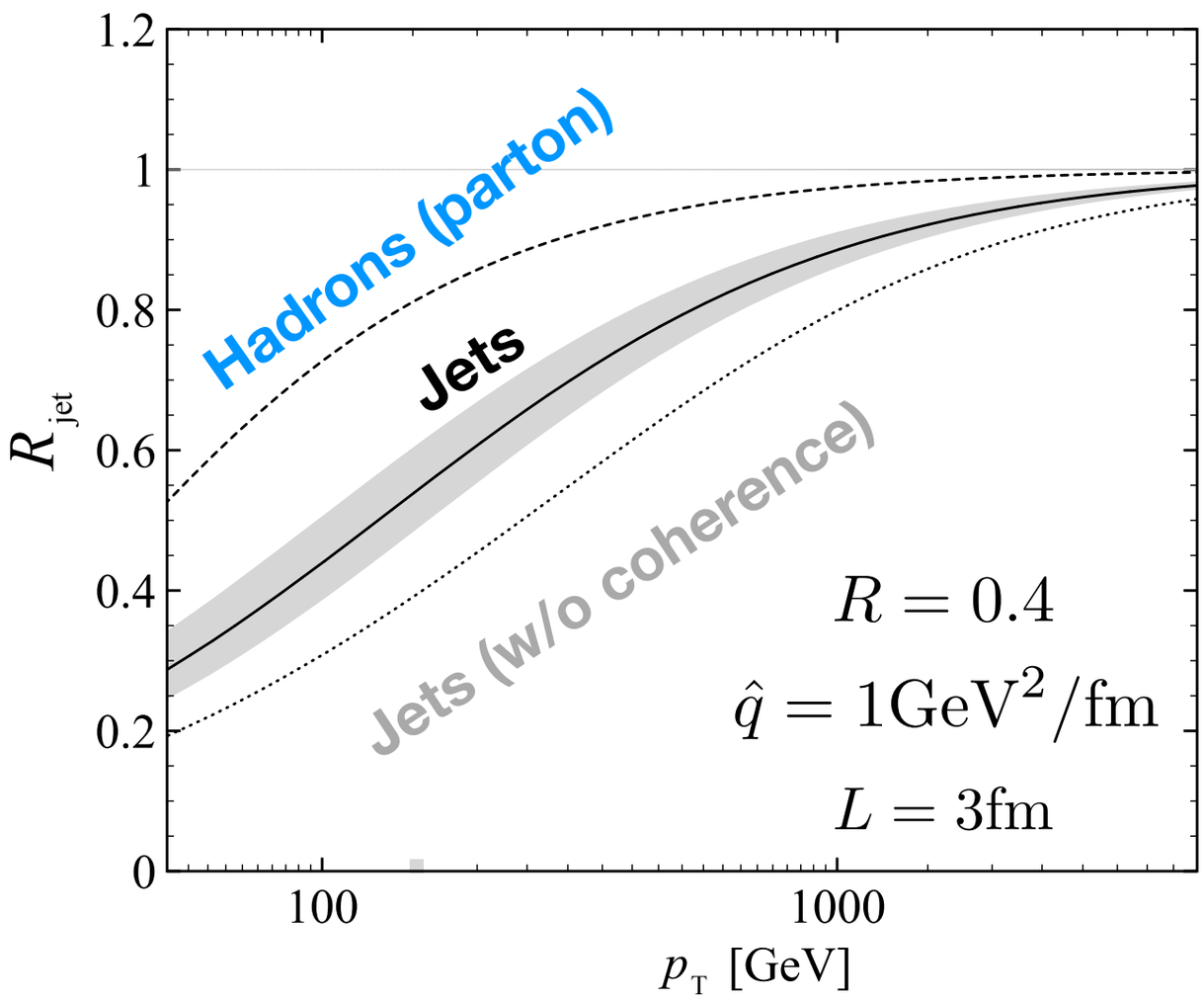
- Including running coupling, full LO splitting function and finite quenching, we obtain a non-linear evolution equation:

$$C_q(p_T, R) = 1 + \int_0^1 \mathrm{d}z \int_0^R \frac{\mathrm{d}\theta}{\theta} \frac{\alpha_s(k_\perp)}{\pi} P_{qg}(z)\ \Theta(t_\mathrm{f} < t_\mathrm{d} < L)$$
$$\times \left[\ C_q(zp_T, \theta)\ C_g(zp_T, \theta)\ \mathcal{Q}_q^2(p_T) - \ C_q(zp_T, \theta)\ \right]$$

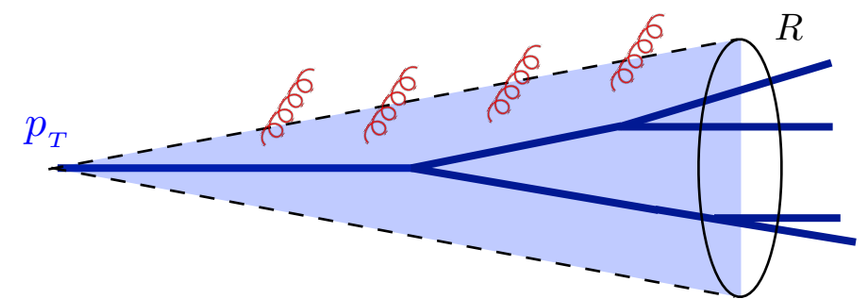
- Caveats: pT-broadening, mini-jet absorption, in-cone medium-induced radiation, medium back-reaction neglected

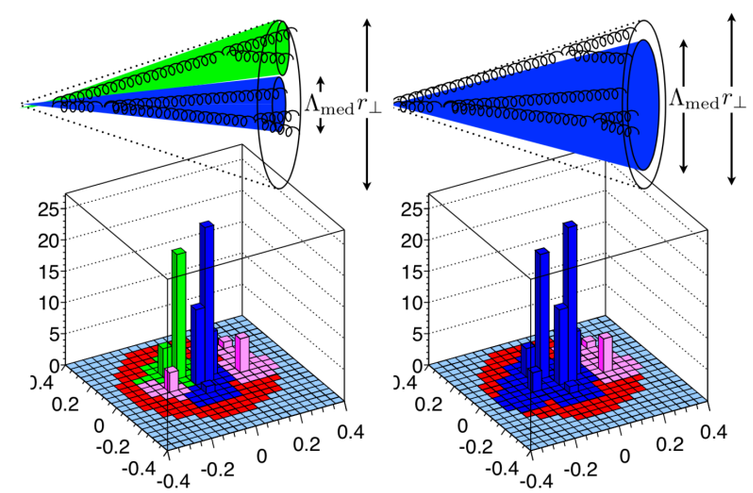
The nuclear modification factor (numerics)

$$R_{\rm jet} = Q_{\rm tot}(p_T) \times C(p_T)$$



$$\theta_c \sim (\hat{q}L^3)^{1/3} \sim 0.1$$





Qualitatively encoded in MC event generators: Hybrid Model, JEWEL and MARTINI

Summary

- Important high order corrections to jet quenching: sensitivity of the inclusive jet spectrum to fluctuations of jet substructure (encoded qualitatively in MC's not in analytic approaches)

- To leading log accuracy in the strong quenching limit we obtain a **Sudakov suppression factor**

- **Probabilistic picture to LL accuracy:** (i) early in-medium vacuum shower generated by medium-modified Sudakov. (ii) Unresolved vacuum shower for $\theta < \theta_c$ (iii) Resolved partons for $\theta > \theta_c$ undergo medium-induced cascade over a distance of order the medium length. (iv) Final stage: fragmentation in vacuum with modified angular ordering

- **Outlook:** Investigate jet substructure observables, combine analytic and MC studies, …

Backup

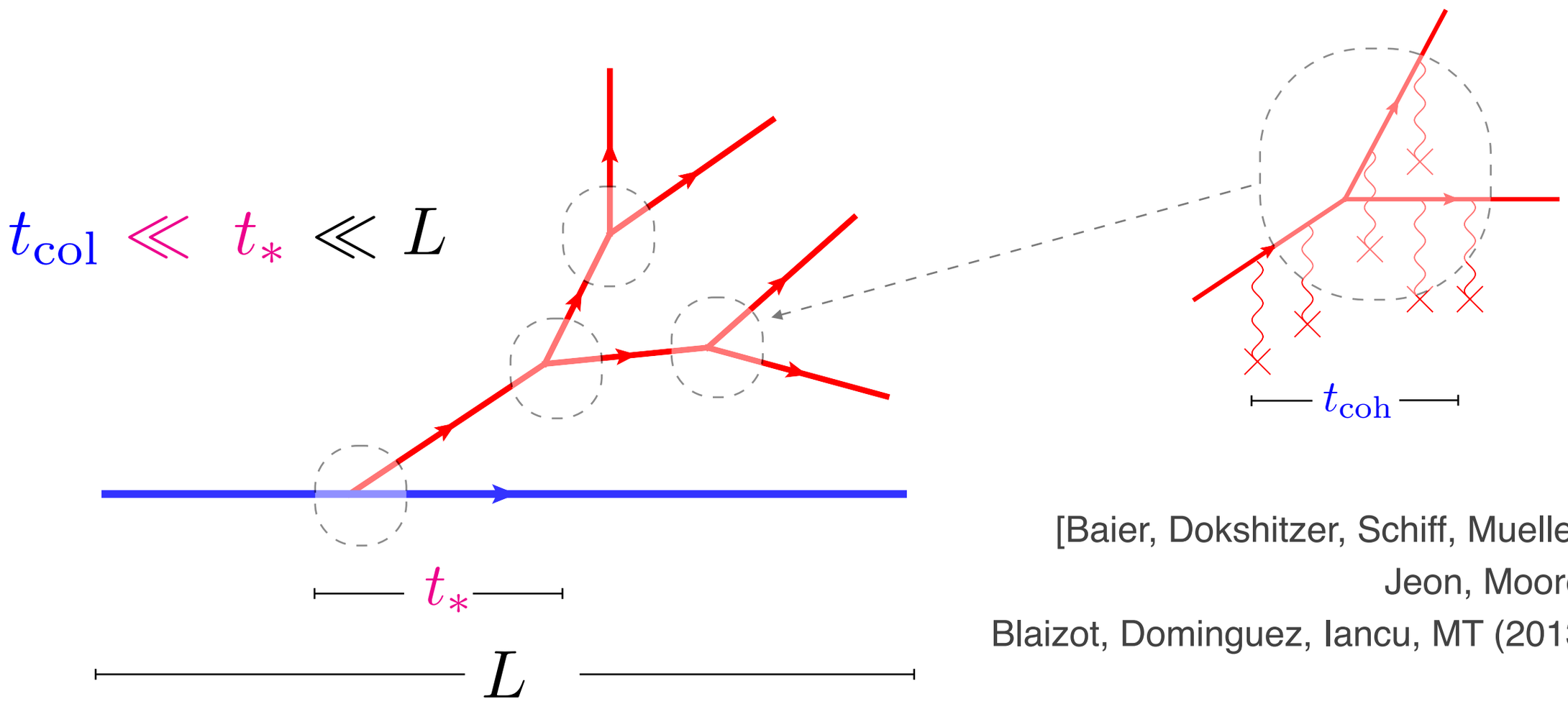
In-medium gluon cascade

- **Probabilistic picture:** large probability for **soft, rapid and independent multiple gluon branching**

$$\omega \frac{\mathrm{d}P}{\mathrm{d}\omega \mathrm{d}t} \equiv \frac{\alpha_s}{t_{\mathrm{coh}}} \equiv \frac{1}{t_*}$$

branching time:

$$t_*(\omega) = \frac{1}{\alpha_s}\sqrt{\frac{\omega}{\hat{q}}}$$



[Baier, Dokshitzer, Schiff, Mueller (2001)
Jeon, Moore (2003)
Blaizot, Dominguez, Iancu, MT (2013-2014)]

Energy flow at large angle

[Blaizot, Iancu, Fister, Torres, MT (2013-2014) Kurkela, Wiedemann (2014)]

- Multiple branchings at parametrically large angle

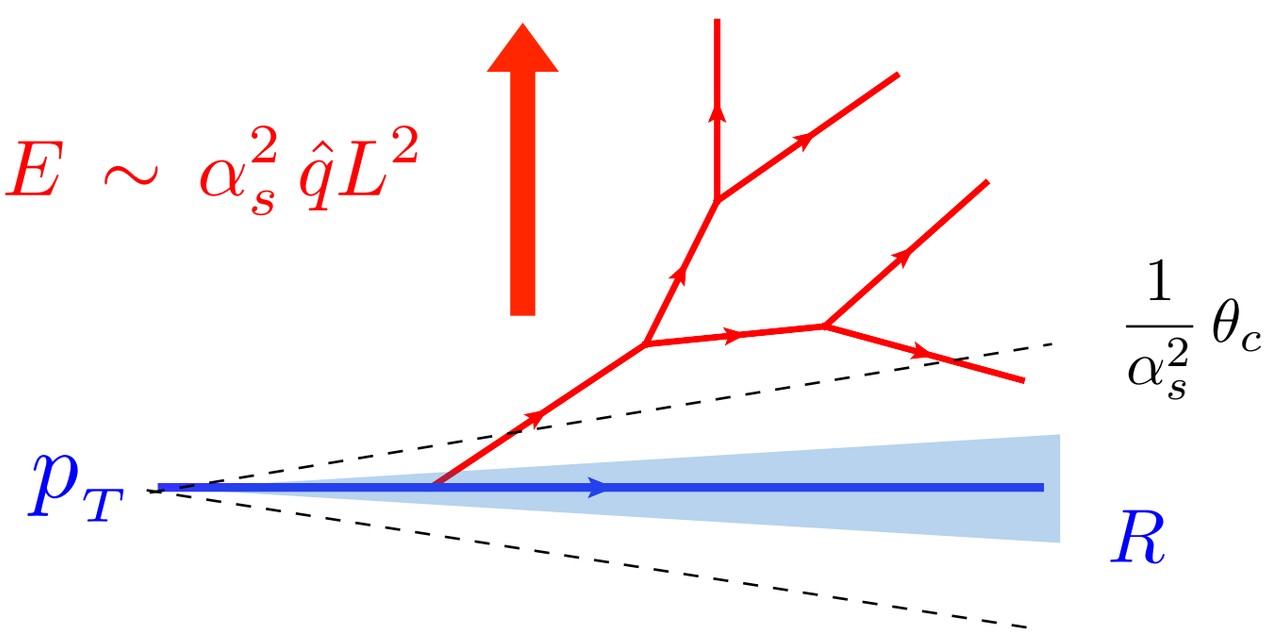
$$\theta_{\rm br} \gg \frac{1}{\alpha_s^2}\theta_c \gg R$$

- Constant energy flow from jet energy scale p_T energy down to the medium temperature scale $\omega \sim T$

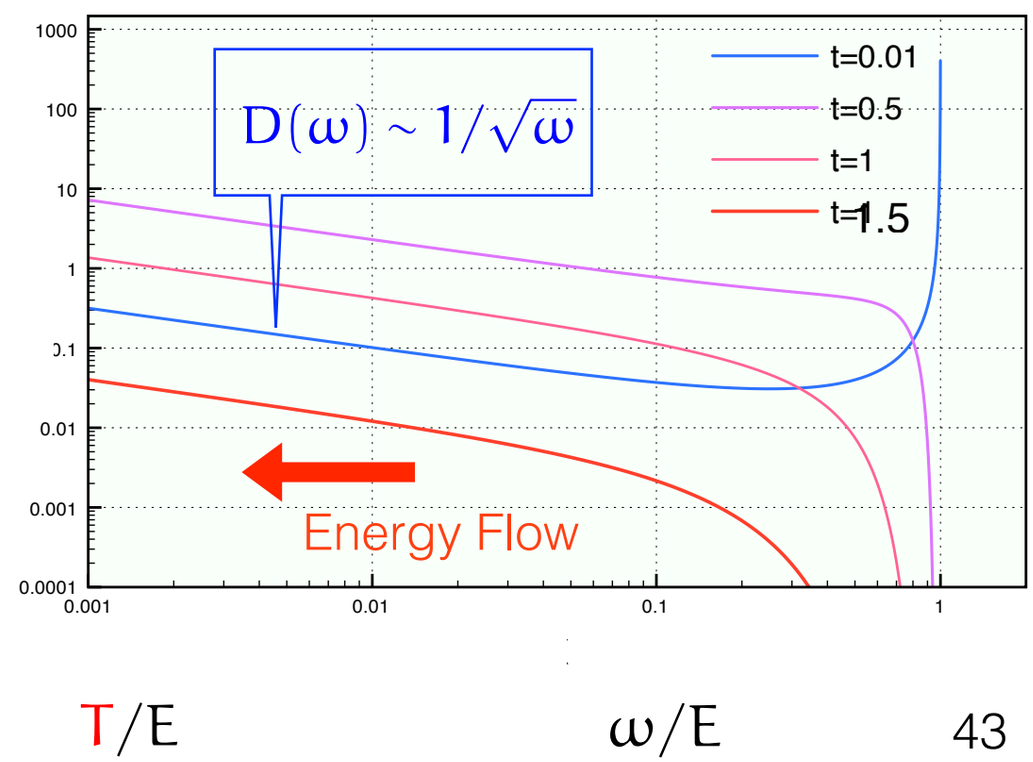
[Iancu, Wu (2015)]

Energy lost to the medium:

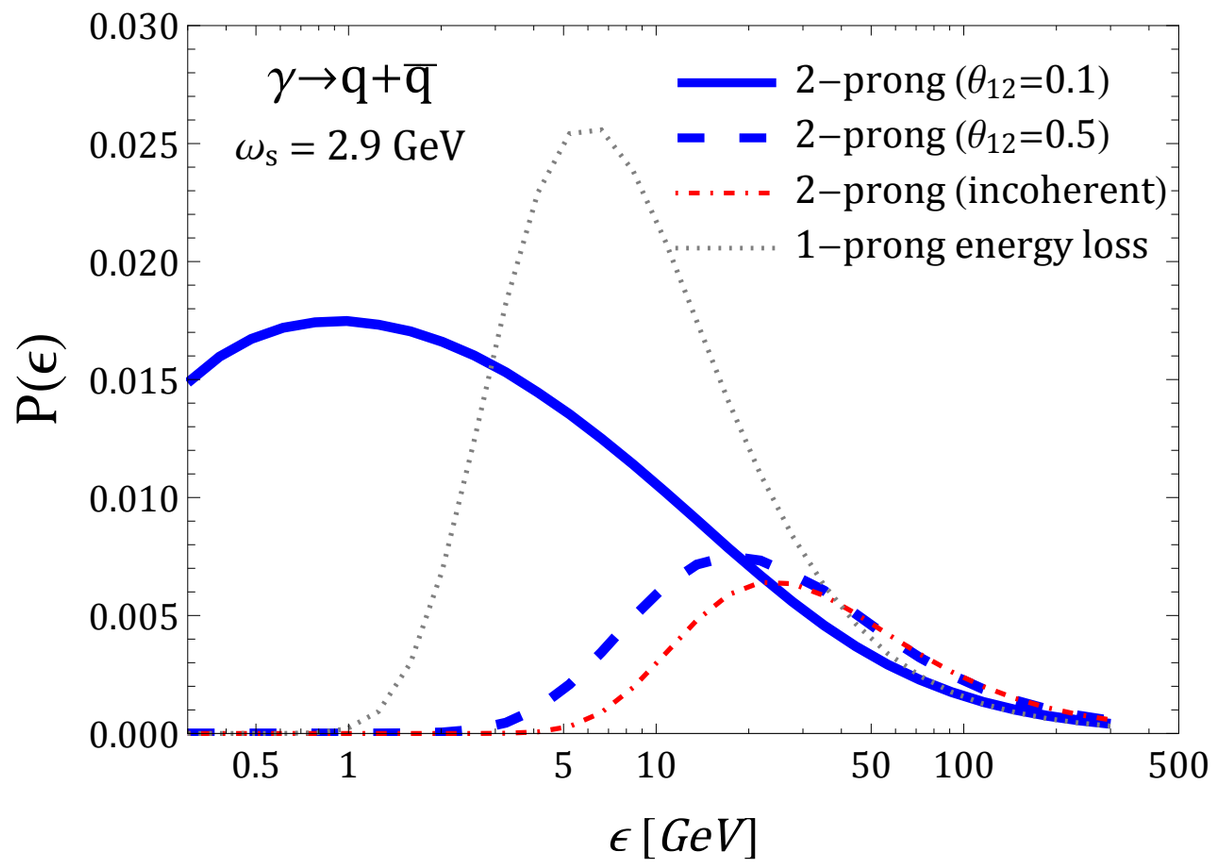
$$\Delta E \sim \alpha_s^2 \hat{q} L^2$$



Energy distribution as function of time



Neighboring jet energy loss



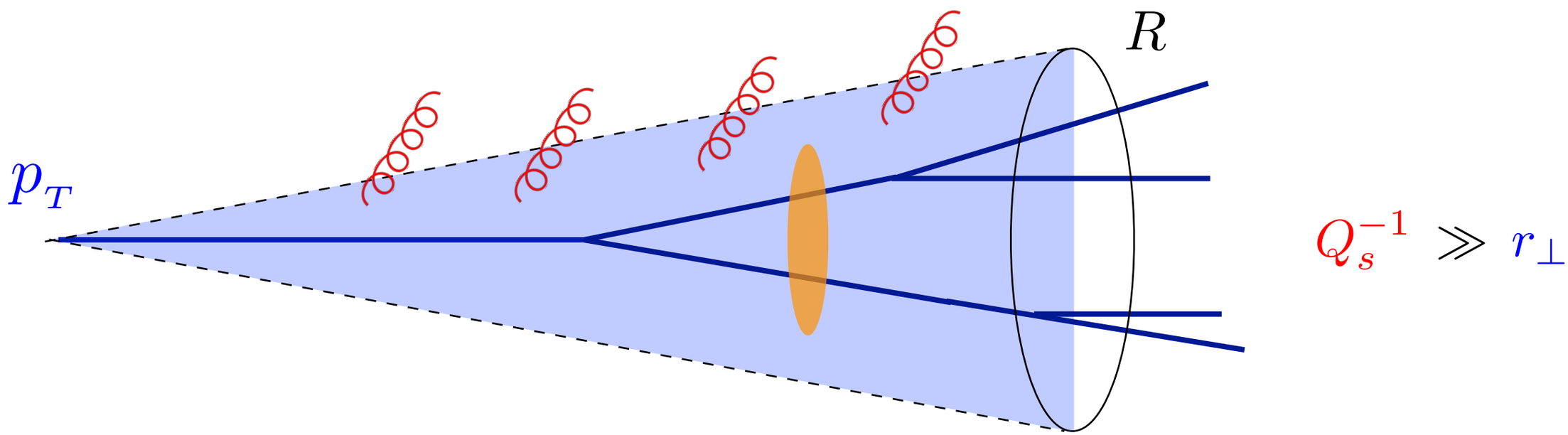
$$\hat{q} = 1\,\mathrm{GeV}^2/\mathrm{fm}$$

$$L = 5\,\mathrm{fm}$$

- Up to the decoherence time $t_d \sim (\hat{q}\,\theta_{12}^2)^{-1/3}$ radiation off the total charge
- At large angle: suppression of neighboring jets

Jet quenching and fluctuations

- How does a jet as a muti-parton system lose energy to the medium?
- Need to account for fluctuations of energy loss due to fluctuation in the jet substructure



Two limiting cases

(i) coherent Eloss, as a single color charge (parton)

(ii) incoherently as multiple charges

Jet quenching and fluctuations

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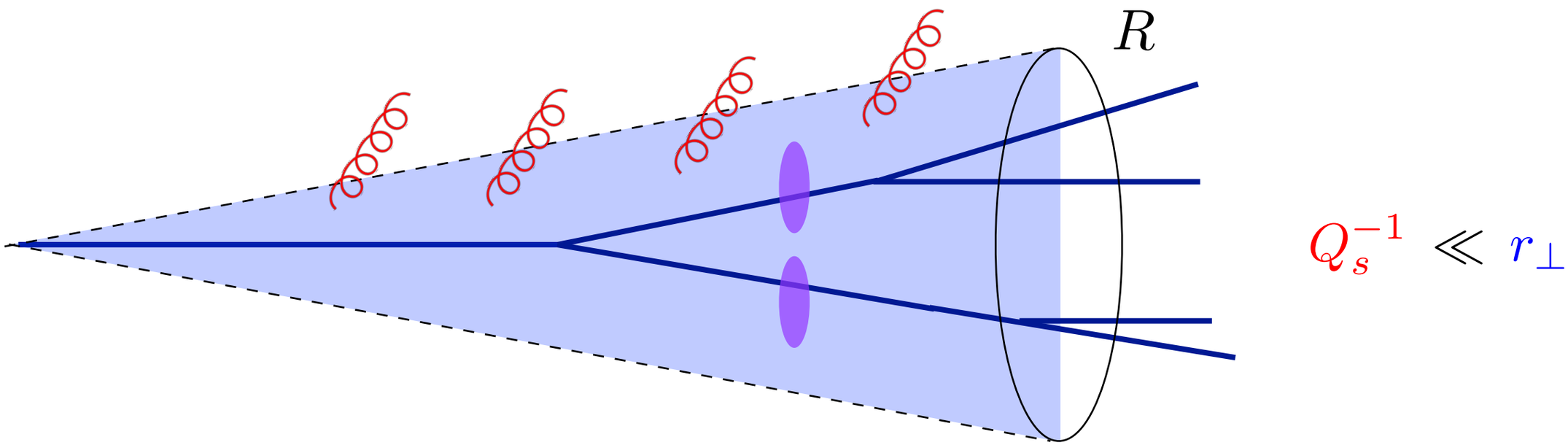


Two limiting cases

(i) coherent Eloss, as a single color charge (parton)

(ii) incoherently as multiple charges

Singlet antenna energy loss

- Leading radiative corrections to the **decoherence parameter** can be absorbed in the renormalization of the jet quenching parameter:

$$\Delta_{\rm med}(t) = 1 - [S_1(t)]^2 = 1 - \exp\left[-\frac{1}{4}\int {\rm d}t'\, \hat{q}(t')\, \boldsymbol{x}_{12}^2(t')\right]$$

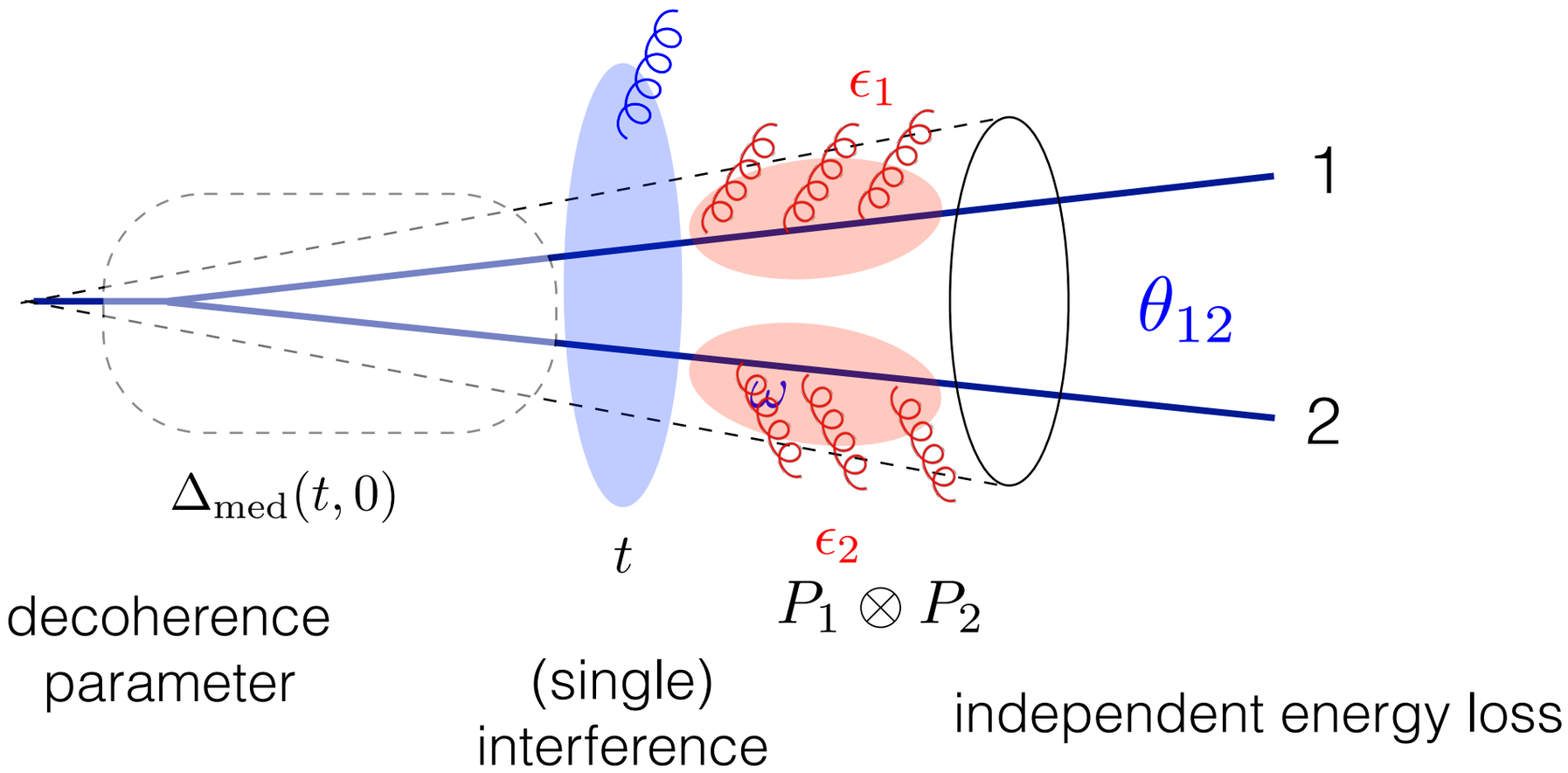
$$\hat{q} \simeq \hat{q}_0 \left(1 + \frac{\bar{\alpha}}{2} \ln^2 \frac{1}{\boldsymbol{x}_{12}^2 m_D^2}\right)$$

coherence $\quad \Delta_{\rm med} \to 0 \quad (\boldsymbol{x}_{12} \to 0)$

decoherence $\quad \Delta_{\rm med} \to 1$

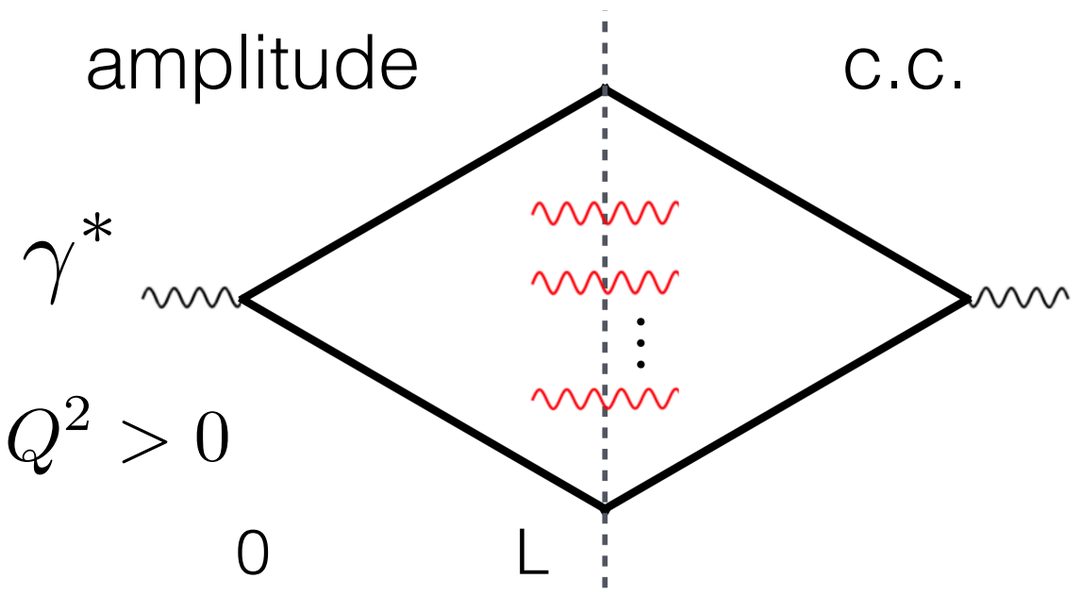
Result and physical picture

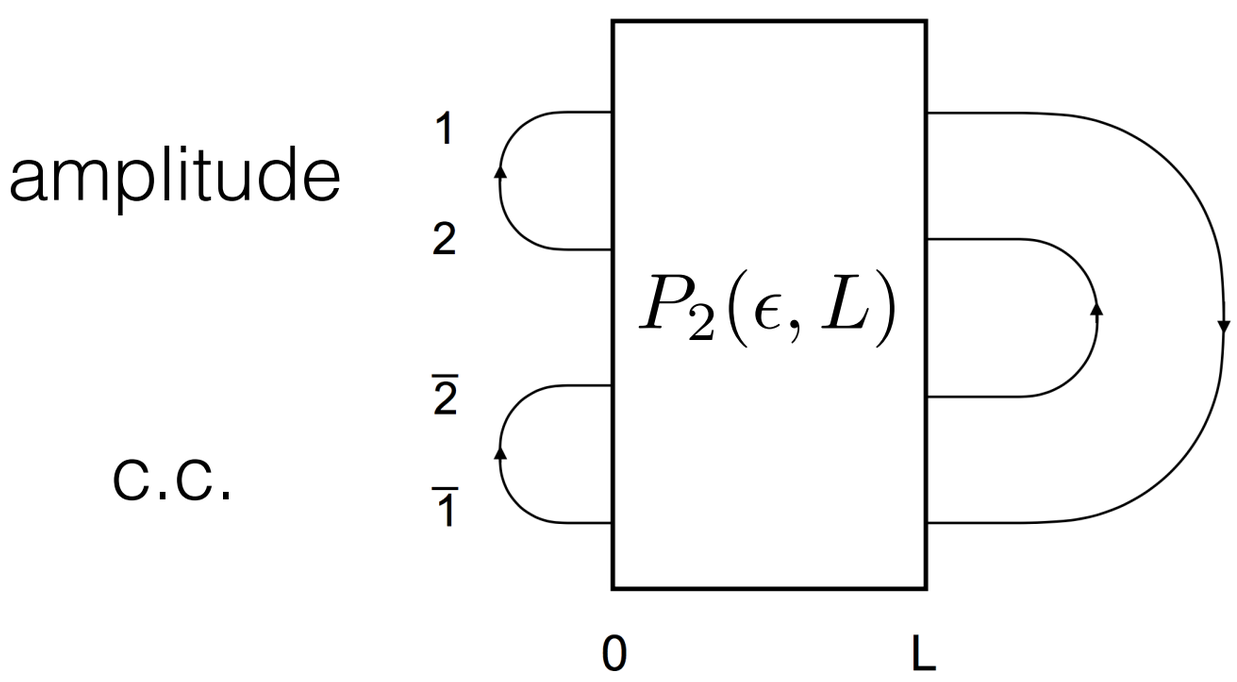
[YMT, K. Tywoniuk (in preparation)]



- Propagation of two color charges at fixed angle
- Up to the decoherence time $t_d \sim (\hat{q}\,\theta_{12}^2)^{-1/3}$ radiation off the total charge
- At large angle: suppression of neighboring jets

Two-pronged energy loss



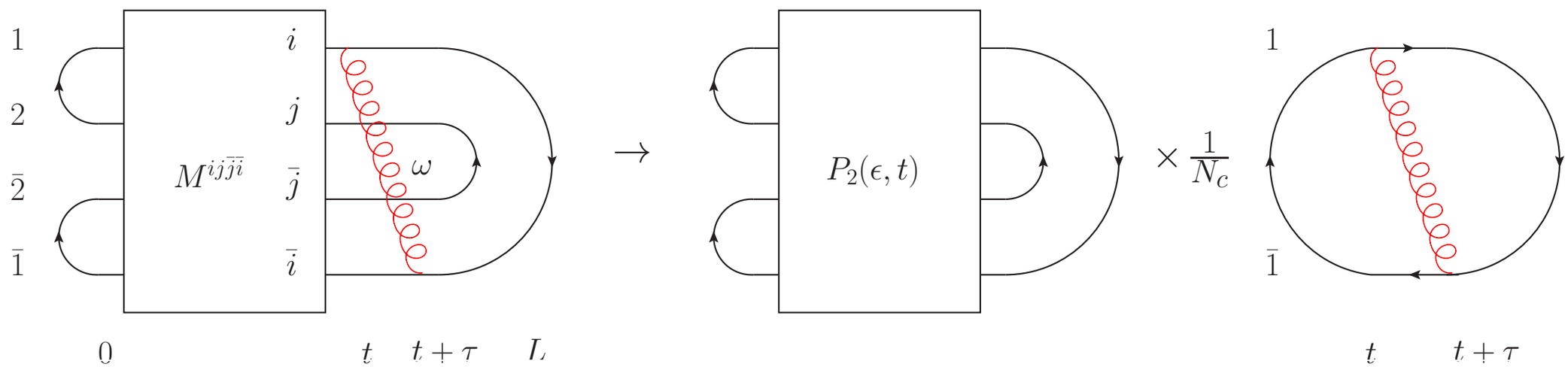


Caveats:

- large N_c
- resum medium-induced soft emissions
- short formation times
- color singlet: straightforward generalization to triplet and octet configurations

Two-pronged energy loss

- Direct emission term (diagonal contribution)



$$\Delta P_2(\epsilon, L) = \int_0^L \mathrm{d}t \int_0^\infty \mathrm{d}\omega\, \Gamma_{11}(\omega, t)\, P_2(\epsilon - \omega, t)$$

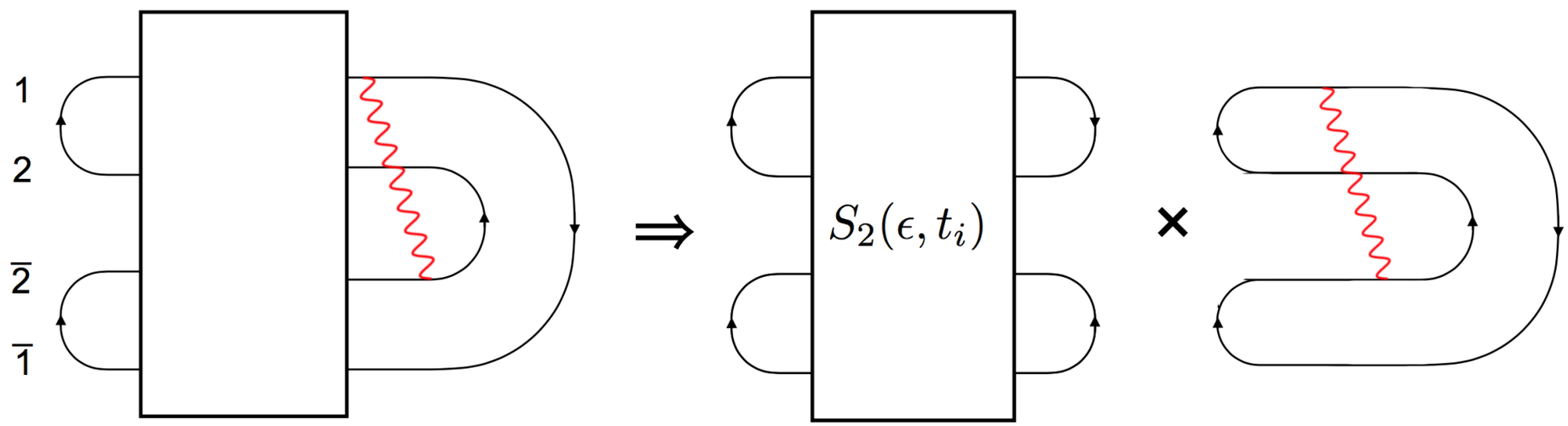
- Correction identical to single particle case:

$$\Gamma_{11}(\omega, t) \equiv \frac{\mathrm{d}I_{11}}{\mathrm{d}\omega \mathrm{d}t} - \delta(\omega) \int_0^\infty \mathrm{d}\omega' \frac{\mathrm{d}I_{11}}{\mathrm{d}\omega' \mathrm{d}t}$$

real virtual

Two-pronged energy loss

- Interferences and color flip (recall that all propagators are evaluated in the medium background field)



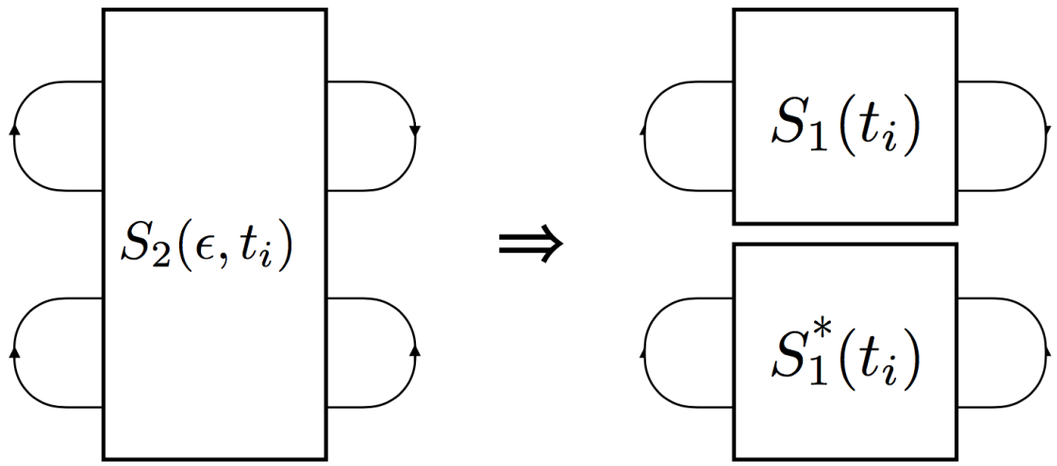
$$\Delta P_2(\epsilon, L) = \int_0^L \mathrm{d}t \int_0^\infty \mathrm{d}\omega\, \Gamma_{12}(\omega, t)\; S_2(\epsilon - \omega, t)$$

- Involves new color structure

$$S_2 \sim \langle \mathrm{tr}(V_2^\dagger V_1)\, \mathrm{tr}(V_1^\dagger V_2) \rangle$$

Two-pronged energy loss

- In the Large N_c approximation



$$+ \quad \mathcal{O}\left(\frac{1}{N_c}\right)$$

- Amplitude and c.c. are disconnected $\Rightarrow$ only virtual emissions contribute

- In the absence of radiation we recover the decoherence parameter: $\Delta_{\rm med} \equiv 1 - S_1^2$

antenna transverse size

$$S_1(t) \equiv \langle {\rm tr} V_2^\dagger V_1 \rangle_{\rm med} \sim \exp\left[-\frac{1}{4}\,\hat{q}\int_0^t {\rm d}t'\, \boldsymbol{x}_{12}^2(t')\right]$$

[MT, Salgado, Tywoniuk, arXiv:1105.1346, MT, Salgado, Tywoniuk arXiv:1205.5739, Casalderrey-Solana, Iancu arXiv:1105:1760]