Particle-vortex statistics and dense QCD

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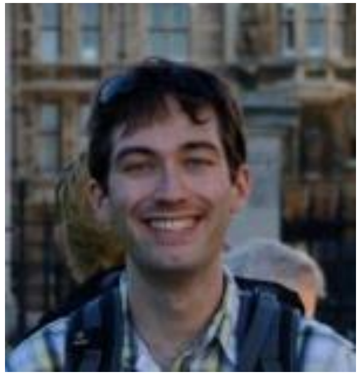
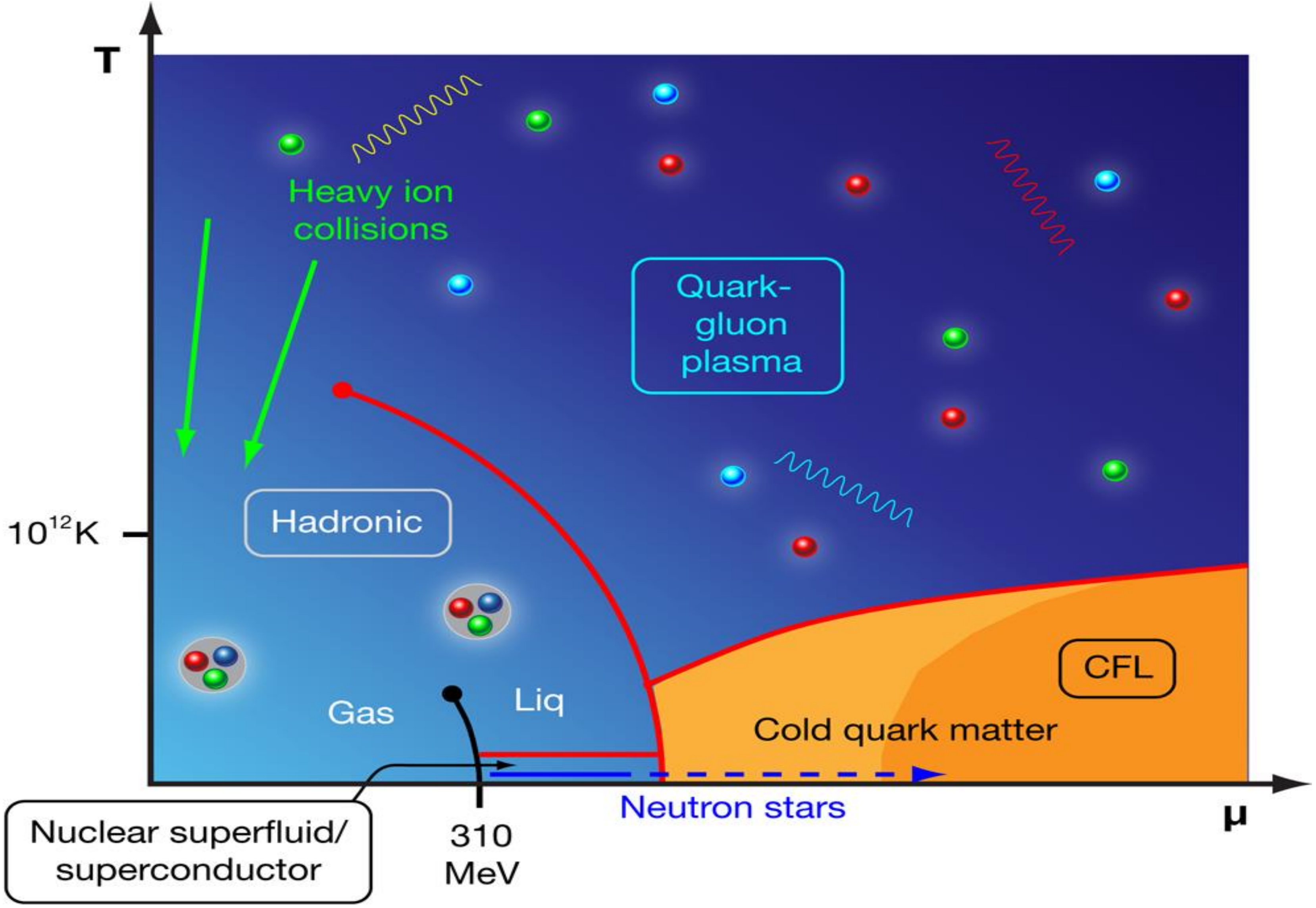


arXiv:1808.04827

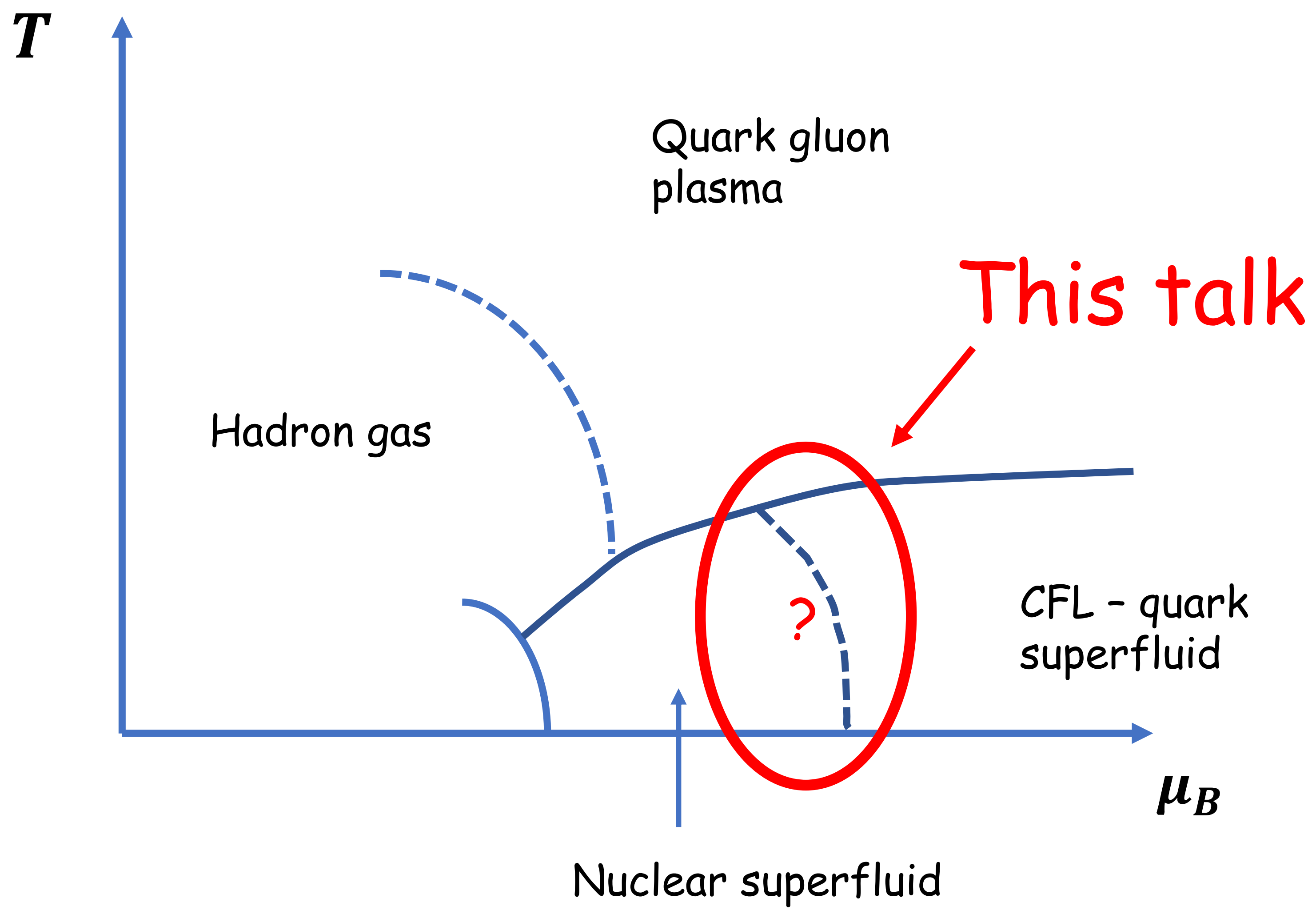
JLAB 2019

QCD phase diagram



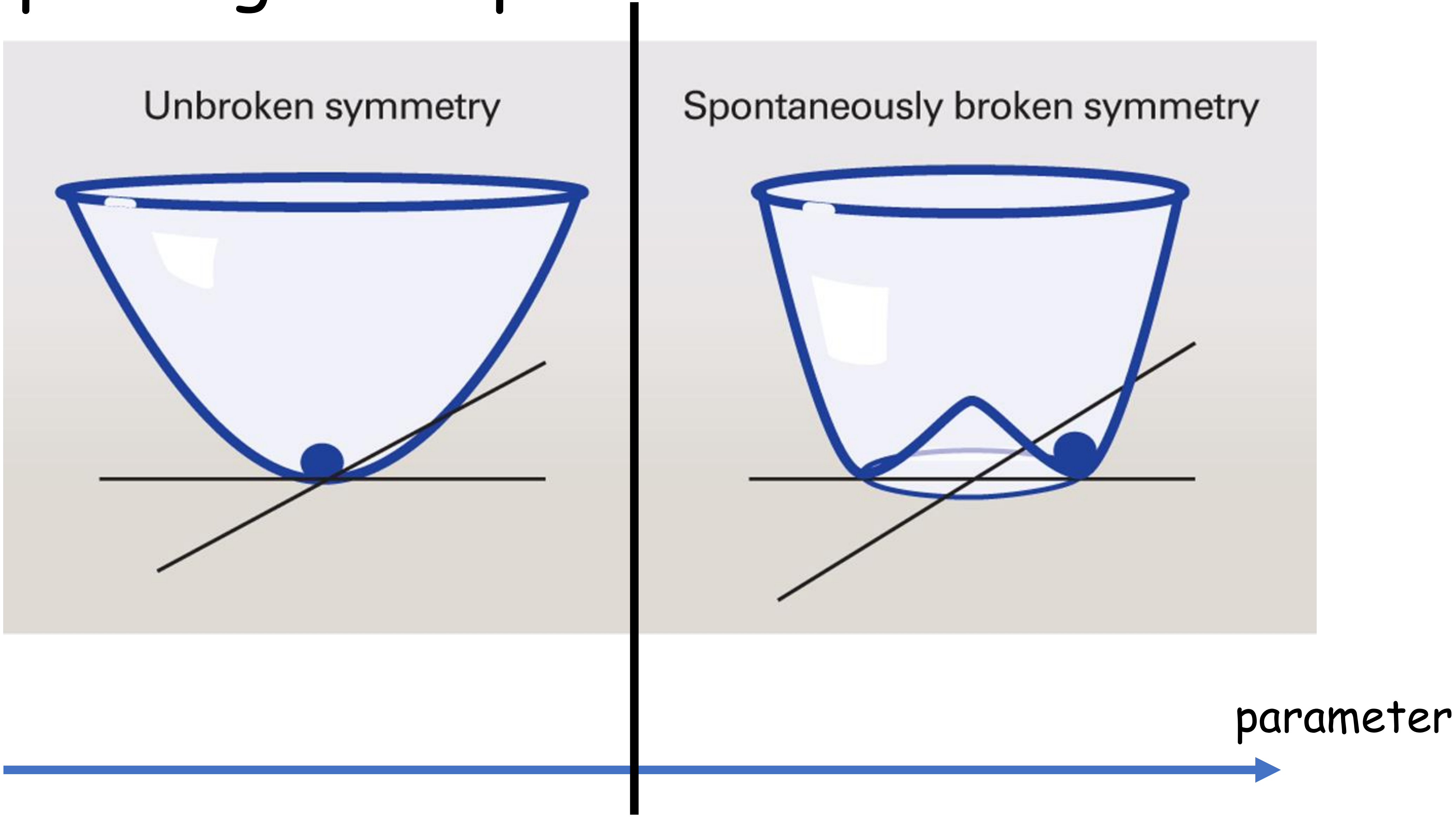
Credit : APS

An idealization pretty close to nature (?) : 3-color 3-flavor QCD : equal quark masses.

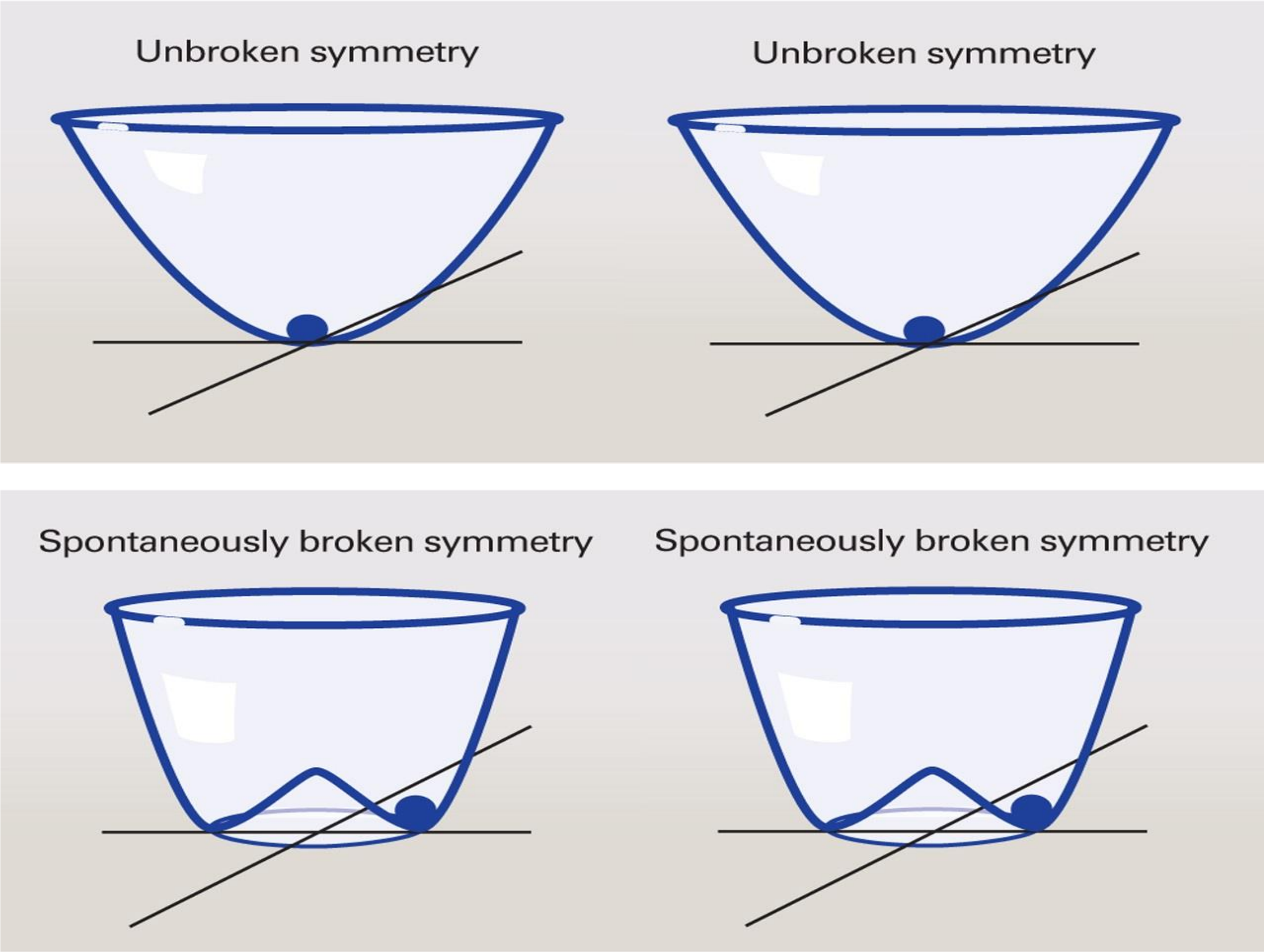


Wilczek, Schaefer, Son, Alford, Rajagopal in 1990 s

Standard approach : symmetry/Landau paradigm for phase transition

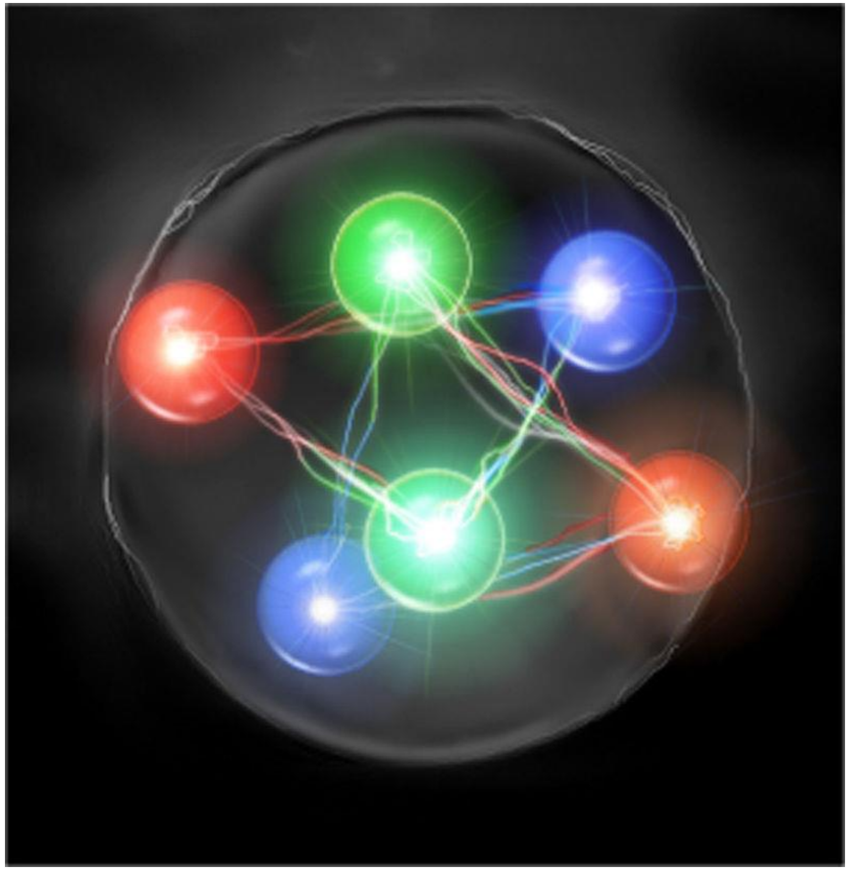


Standard approach : symmetry/Landau paradigm for continuity



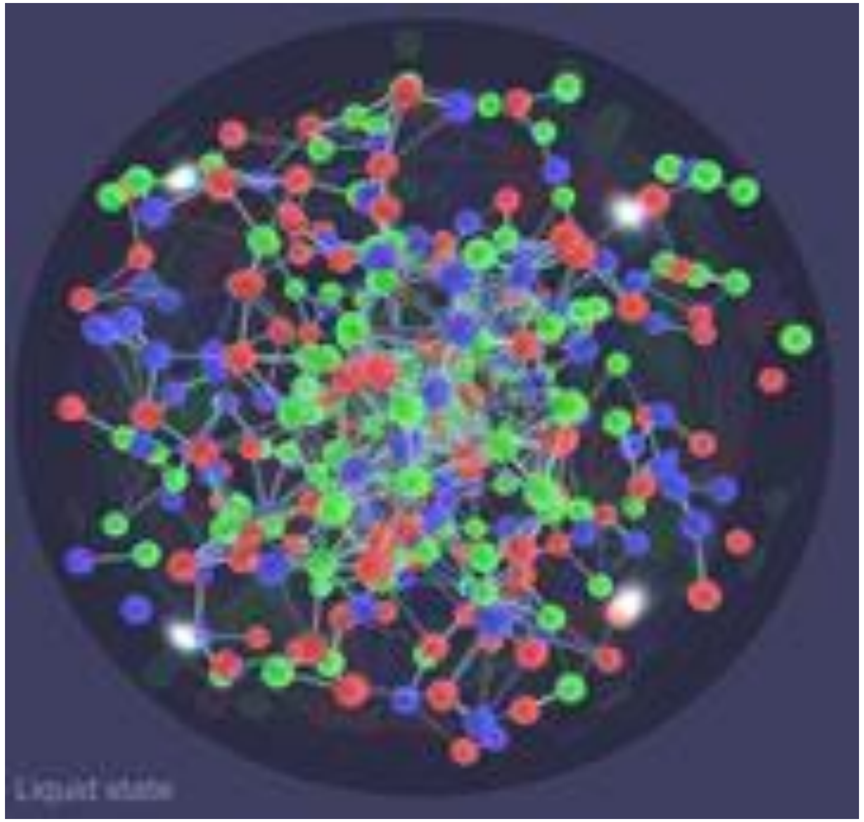
Nuclear matter (low density):

- Condensation of H dibaryons.
- Breaks $U(1)_B$ baryon number spontaneously : superfluidity.
- In the chiral limit : spontaneous breaking of chiral symmetry : $SU(3)_L \times SU(3)_R \to SU(3)_V$.



Asymptotically high density

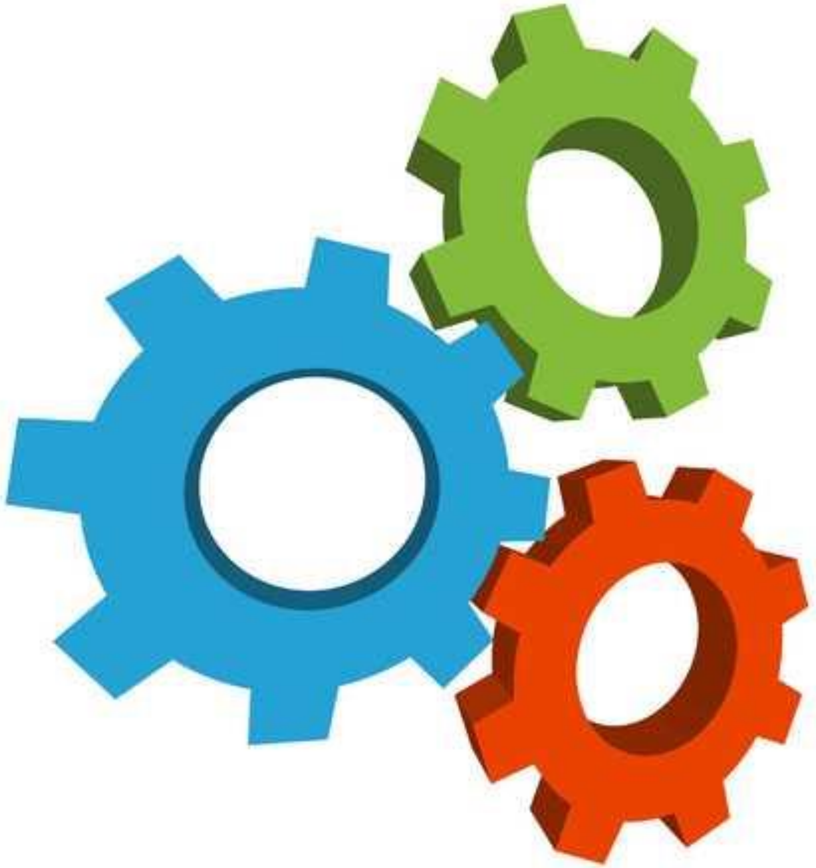
- Fermi sphere of quarks.
- BCS instability at the Fermi surface.
- Cooper pairs of quarks.



Asymptotically high density : color-flavor locking

- Color-Meissner effect ⟶ weakly coupled at all length scales.

- The form of the condensate $\left\langle q_c^i \, C \, q_b^j \right\rangle \propto \Delta \, \epsilon^{ijk} \epsilon_{abk} \equiv \Phi_{ab}^{ij}$. i, j color and a, b flavor indices.

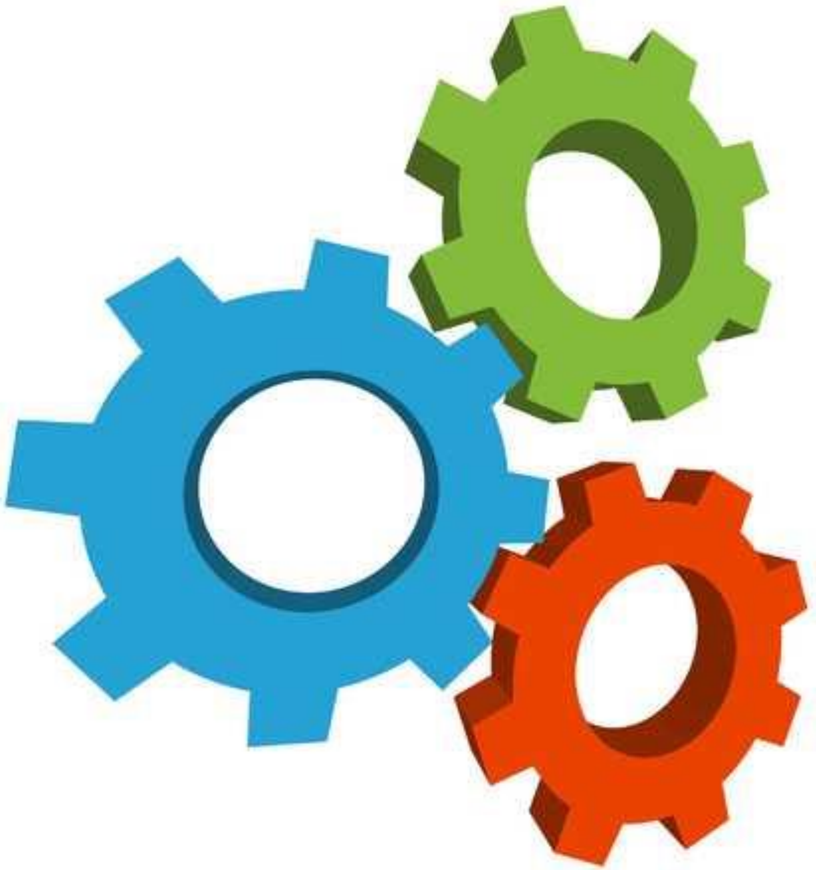


Asymptotically high density : color-flavor locking

Write the condensate as a color antifundamental

$$\epsilon^{ijl}\epsilon_{abm}\Phi^{ij}_{ab} \equiv \phi^{l}_{m}$$

$\psi = -i\log(\det(\phi))$ is the Nambu-Goldstone (NG) mode for baryon number breaking.



Gauge invariant order parameter ➡ Continuity

v.e.v for the diquark condensate (schematic)

$$\langle qq \rangle \neq 0$$

$$\langle (qq)^3 \rangle \neq 0 \qquad \langle \bar{q}\bar{q}qq \rangle \neq 0$$

Like pairing of baryons.

$$U(1)_B \to Z_2$$

In the chiral limit
$$SU(3)_L \times SU(3)_R \to SU(3)_V$$

Continuity conjecture

Schaefer-Wilczek, 1999

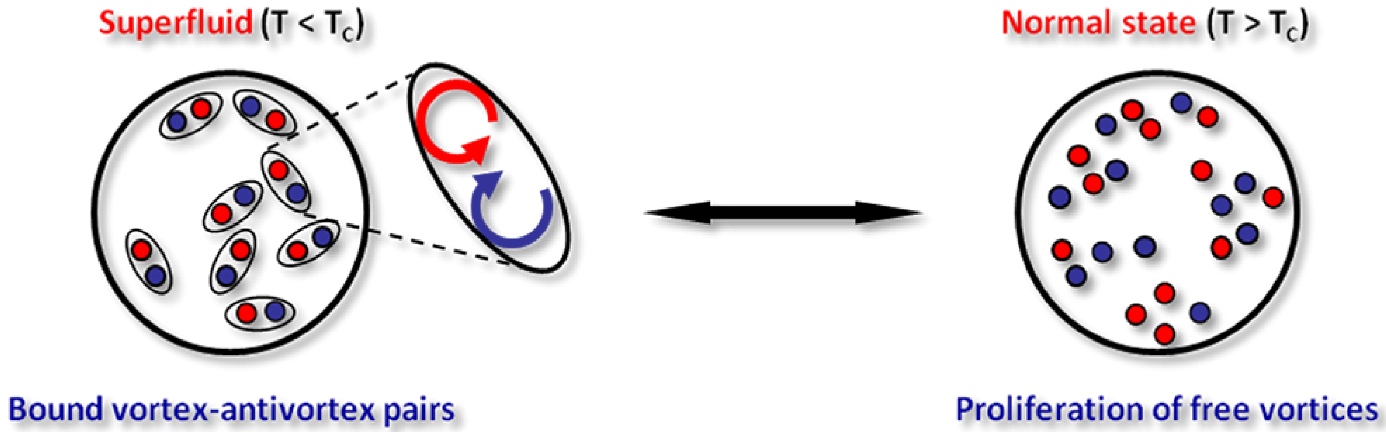
Low energy EFT

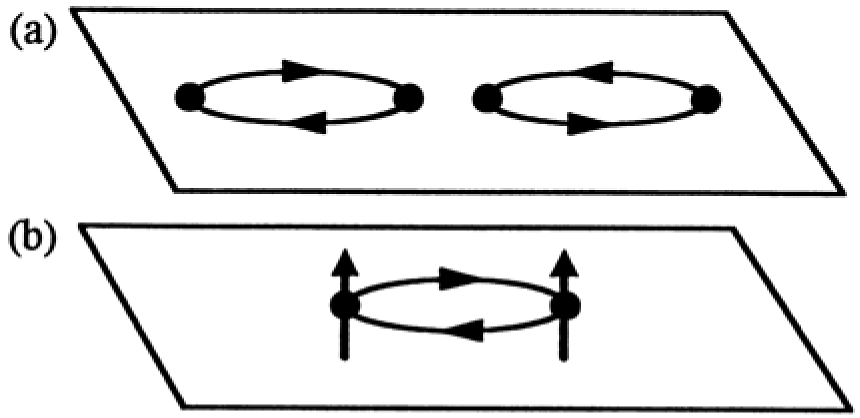
$$S_{U(1)} = \int d^4x \left(\frac{1}{2} 6 \frac{\mu^2}{\pi^2} ((\partial_t \psi)^2 + \frac{1}{3} (\partial_i \psi)^2) \right) + \dots$$

In the limit of $m_q = 0$, there are extra NGB s coming from chiral symmetry breaking.

This is conventional wisdom, but is incomplete as we will see.

Phase transition = change in symmetry ? Not necessarily..





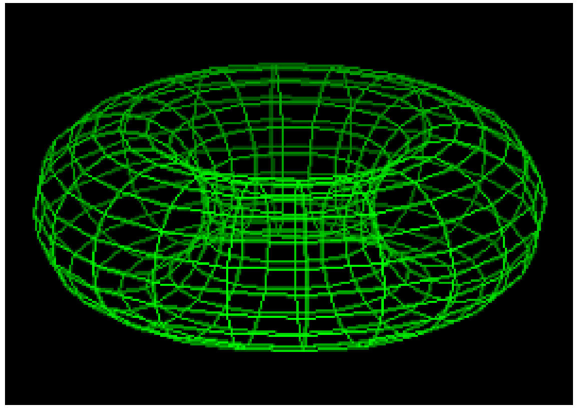
B-K-T transition

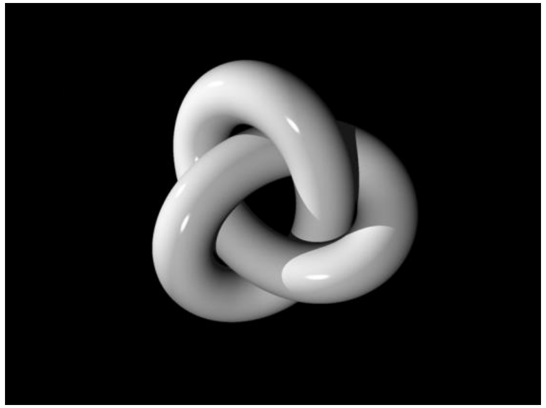
Fractional quantum hall effect

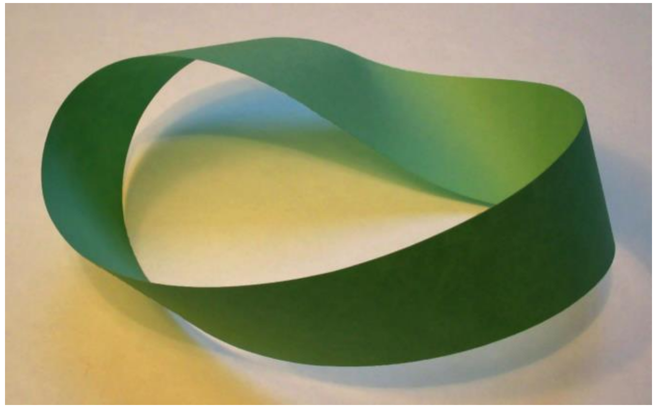
Credit: http://www-amop.phy.cam.ac.uk/amop-zh/Research3.html, http://www.pnas.org/content/96/16/8821

Phase transition detected by probing topology

Analyze theory on spatially compact manifolds



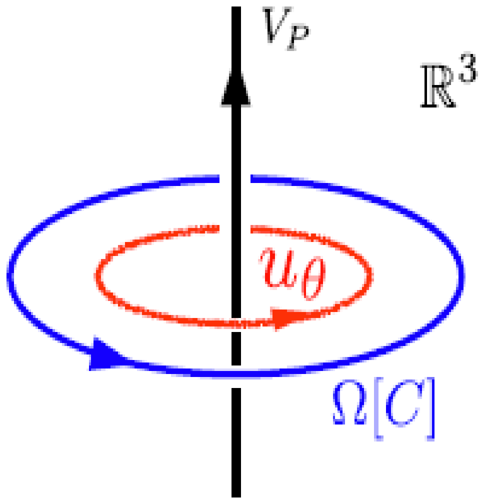




Or equivalently

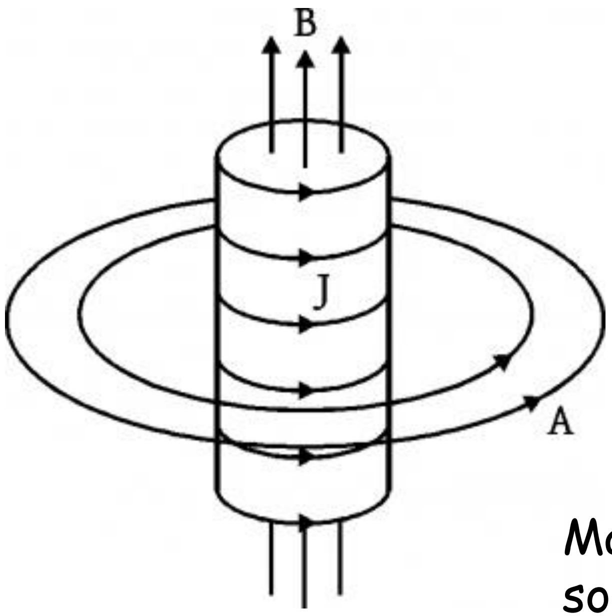
Look into topological field configurations in ordinary space-time : like vortices, flux tubes etc.

Check Aharonov-Bohm phases.



We'll take this route for this talk.

Standard Aharonov-Bohm (to become the topological order parameter)



Magnetic flux through solenoid

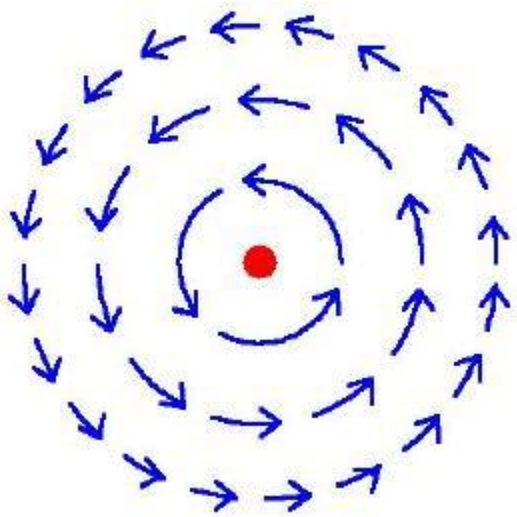
$$\Omega \equiv e^{i\, e \int_C A} = e^{i\phi}$$

What other than a solenoid has a "confined" flux tube ?

Ans: superconductors.

To understand superconducting flux tubes first remember superfluid vortices.

Toy example ordinary single component superfluid vortex.

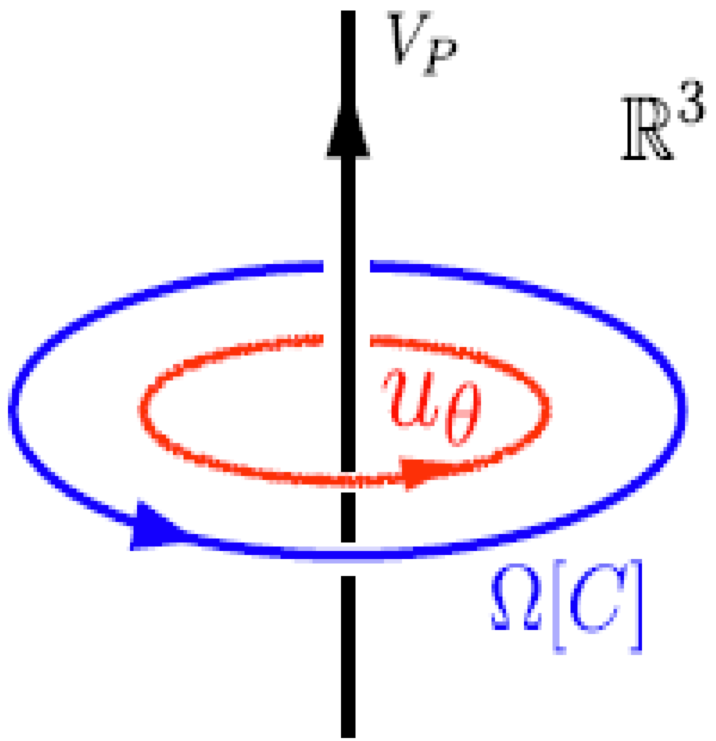


Credit: ucdavis

Phase of order parameter ψ winds around the vortex axis by one $\sim e^{i\theta}$, θ = azimuthal angle, r = distance from the core.

$$E_{\psi} \sim |\nabla\psi|^2 \sim \frac{1}{r^2}$$

Toy example ordinary superconducting flux-tube/vortex.

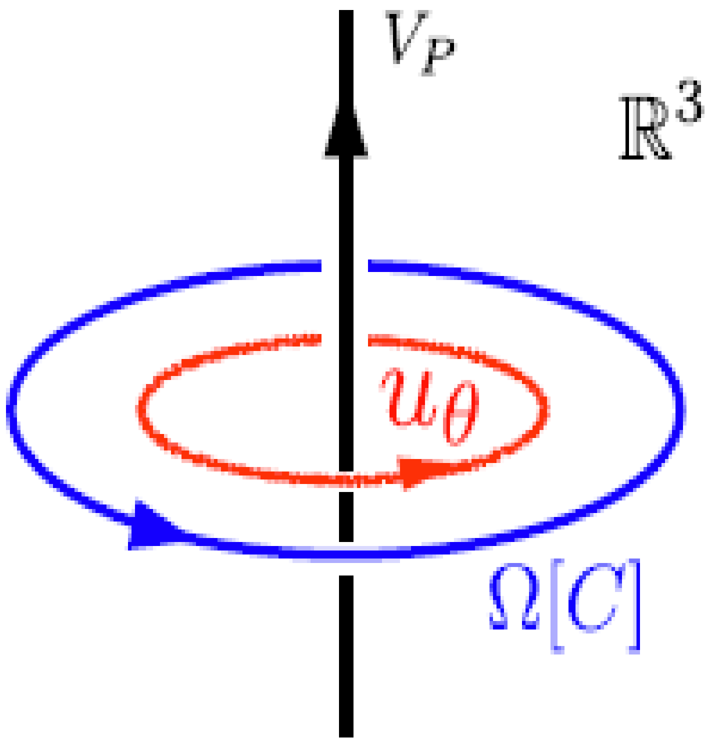


Two electron Cooper pair : $\langle ee \rangle = \psi$ winds around the vortex axis by one unit: $\psi \sim e^{i\theta}$.

$$E_\psi \sim |\mathrm{D_i}\psi|^2$$

Covariant derivative $\partial_i + i\ 2\ e\ A_i$

Toy example ordinary superconducting flux-tube.



Minimize energy density with the gauge field ansatz $A = \frac{b}{r}\hat{\theta}$.

Result : $b = \frac{1}{2}$.

$$E_\psi \sim 0$$

Aharonov – Bohm phase of $e^{i\,e\int A} = e^{i\pi} = -1.$

Quark matter … color Aharonov-Bohm (AB)

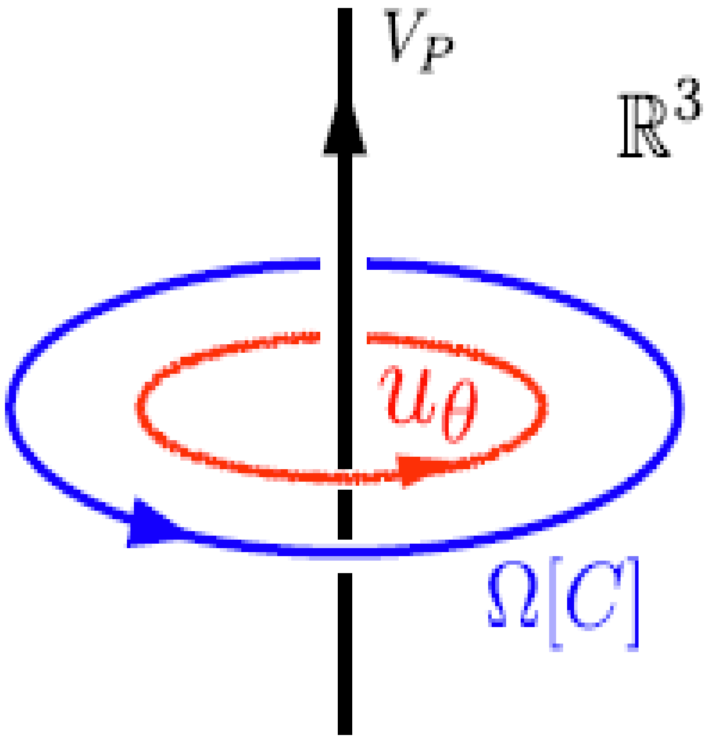
- Color Aharonov-Bohm phase is a gauge invariant quantity.

$$\Omega \equiv \mathrm{Tr}[e^{i\, g \int_C A}]$$

- This phase around vortices is going to serve as a new tool for detecting new phases.

Vortices

- Solve for vortex profiles in CFL phase.
- The energy density



$$E_\phi \sim \mathrm{Tr}|D_i\phi|^2, \qquad D_i = \partial_i + i\,A_i$$

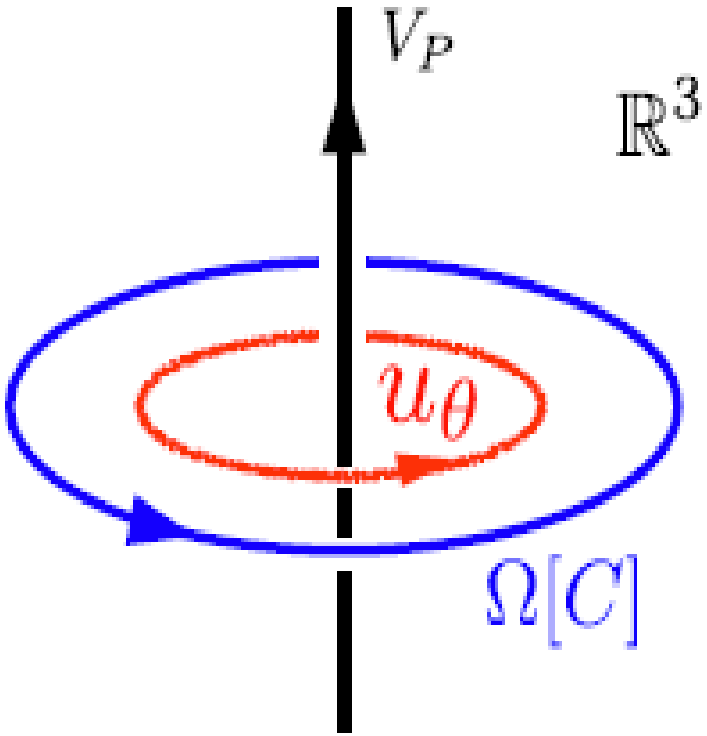
Vortices

- Minimal circulation (cheapest) vortices

$$\frac{\phi}{\Delta} = \mathrm{diag}\left[e^{i\theta} f(r), g(r), g(r)\right].$$

$$A_\theta = \frac{h(r)}{r}\mathrm{diag}\left[-2a, a, a\right]$$

$$f(r), g(r), h(r) \to 1 \quad \text{with} \quad r \to \infty.$$



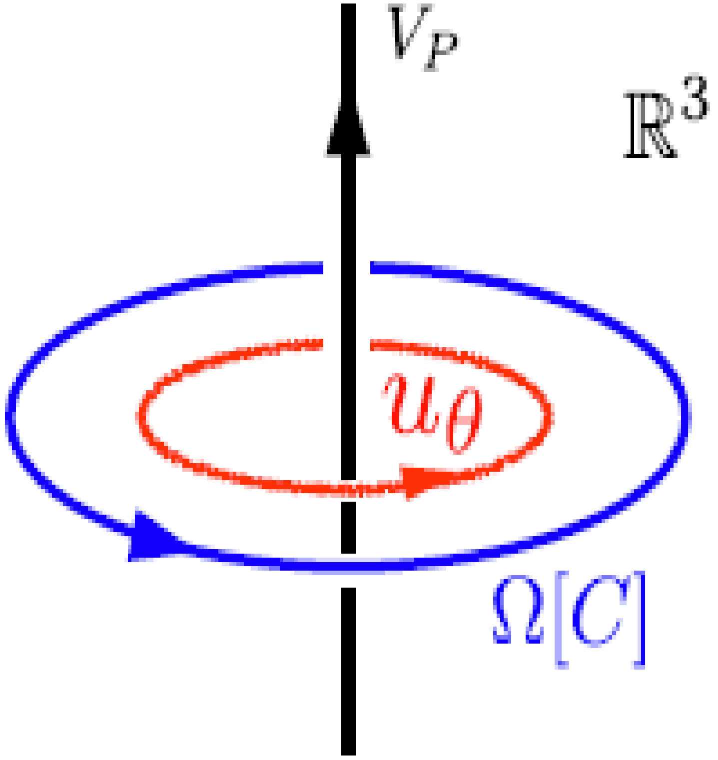
Far away from the core

- $E \sim \frac{1}{r^2}\mathrm{Tr}\left(\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} -2a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{pmatrix}\right)$

- Minimizing the energy density

  $\sim \frac{1}{r^2}\left((1-2a)^2 + 2a^2\right)$

  ➡ $a = \frac{1}{3}.$

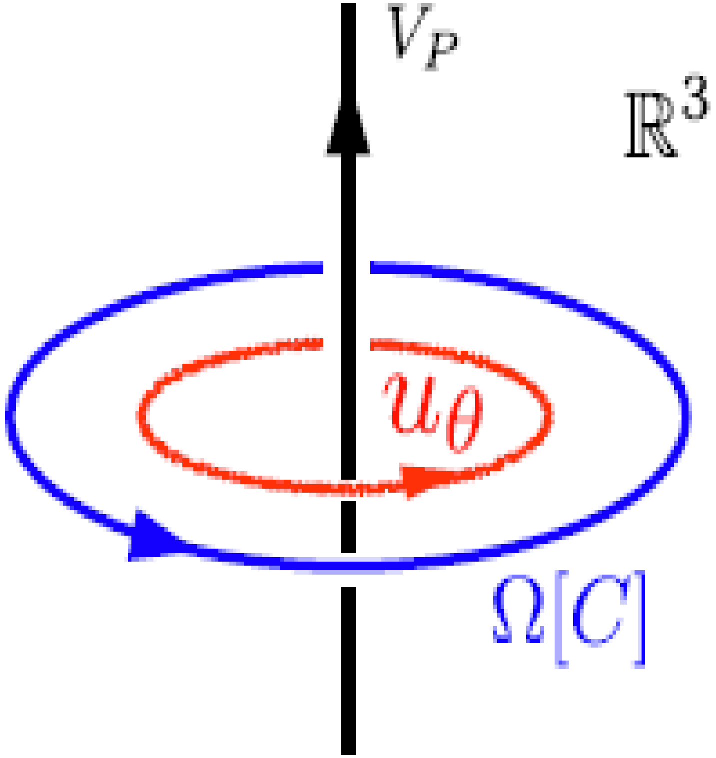


Points to note (contrast with ordinary superconducting and superfluid vortex)

- $\mathrm{Det}\left(\frac{\phi}{\Delta}\right) \to e^{i\theta}$ for $r \to \infty$

$\longrightarrow$ superfluid vortex with minimal nontrivial $U(1)_B$ winding.

- **Global** vortices.



Necessarily present in rotating neutron star for example.

Aharonov - Bohm around vortices :

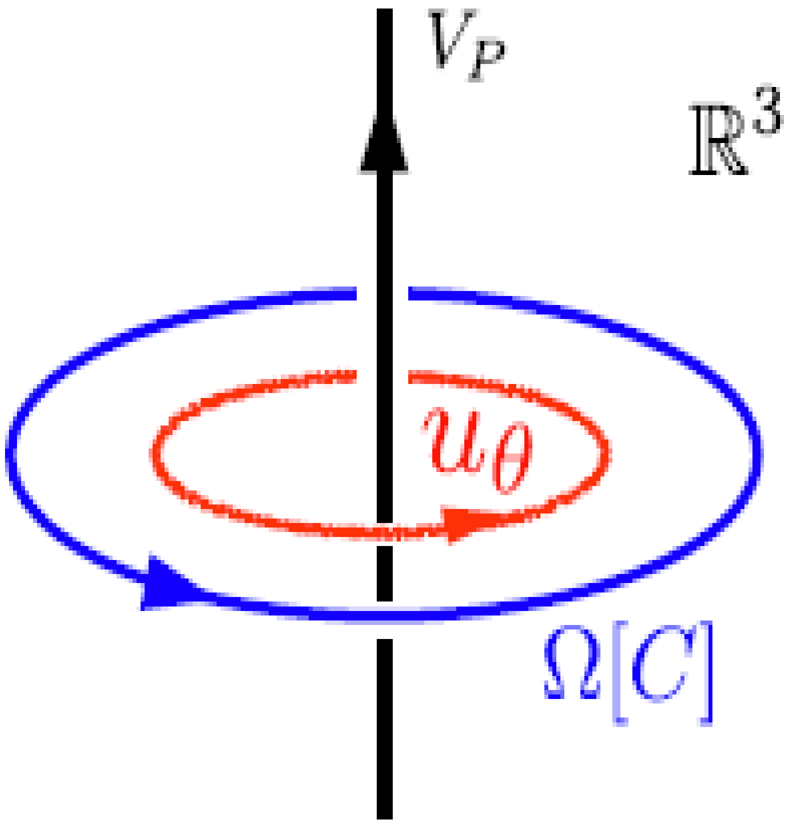
AB phase along a path (P) encircling the vortex C

$$\Omega \equiv e^{i\int_C A} = e^{\frac{2\pi i}{3}}$$

In other words

$$\Omega = e^{\frac{2\pi i}{3} l(C,P)}$$

where l is the linking number of paths C and P.



Screening and **fractionalization** :

- How do nontrivial AB phase affect *physical* gauge invariant quasiparticle excitations ?

- How does this relate to color screening in color superconductor ?

The nontrivial Z_3 phases correspond to the Aharonov-Bohm phase acquired by a quark going around a minimal vortex.

Dressing and **fractionalization** :

Remember : CFL condensate ϕ is a color-antifundamental.

Quark q

Condensate ϕ

$q\,\phi$ color singlet : “baryon”

Orbital angular momentum : $L_z \equiv \hat{z}.\bar{r} \times (\bar{p} - \bar{A})$ shifted by 1/3 with respect to $\mathbb{Z}$ in the presence of a vortex.

Dressing and **fractionalization** :

- The angular velocity $\frac{d\theta}{dt} = \frac{L_z}{m\,r^2}$ is shifted by the presence of the AB phases.

- A $q\phi$ excitation in the presence of a vortex (far from its core) moves as a free particle with fractional orbital angular momentum $L_z \in \mathbb{Z} + 1/3$.

Quasiparticle excitations :

ΔL_z	0	$+\hbar/3$	$-\hbar/3$
bosons	$q\bar{q}$	$\bar{q}\bar{q}\phi$	$qq\phi^*$
fermions	$qqq,\ \bar{q}\bar{q}\bar{q}$	$q\phi$	$\bar{q}\phi^*$

q ($\bar{q}$) is quark (hole) quasiparticle excited above the fermi surface.

ϕ is the condensate v.e.v.

Takeaways : 

- Conventional low energy EFT of CFL (high density quark matter) incomplete.

- EFT for the $U(1)_B$ NGB needs to be coupled to a topological QFT to produce the right particle-vortex statistics.

- Coupling to TQFT can be made explicit using BF formulation of Z_3 discrete gauge theory.

Implications part 2:
low density nuclear matter (Expectation)

- In hadronic regime gauge field fluctuates strongly → no perturbative control.

- No coherent di-quark condensate over macroscopic length scales.

- Trivial AB phase : $\Omega = 1$ around vortices.

Although not calculable, this is the most likely scenario.

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Quark-hadron discontinuity

- If the AB phase around minimal nontrivial vortices in superfluid nuclear matter is trivial :

Phase transition between quark and hadronic matter.

Continuity ?

- If there is no phase transition between the two regimes, the vortices in nuclear matter have to exhibit the same AB phase as in quark matter.

- The low energy EFT of Goldstone mode then couples to a TQFT in nuclear matter just as in quark matter.

- Orbital angular momentum in the presence of $U(1)_B$ superfluid vortices would be fractionalized in nuclear matter.

→ Hard to believe.

Work in progress

- Look for simpler models that produce the same low energy EFT.. Terrestrial superfluids ? Lattice models ?

- Move away from the flavor degenerate limit and repeat the analysis.

- Implications for neutron star physics, nuclear experiments ?

The EFT :

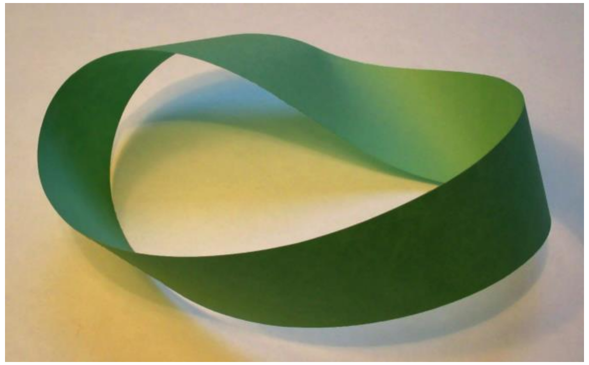
- $S_{eff} = \int_{M_4} \frac{f^2}{2} (\partial_\mu \psi)^2 + \frac{i}{2\pi} \int_{M_5} \epsilon^{\mu\nu\sigma\lambda} b^3_{\mu\nu} (3\partial_\sigma a_\lambda - \partial_\sigma \partial_\lambda \psi)$

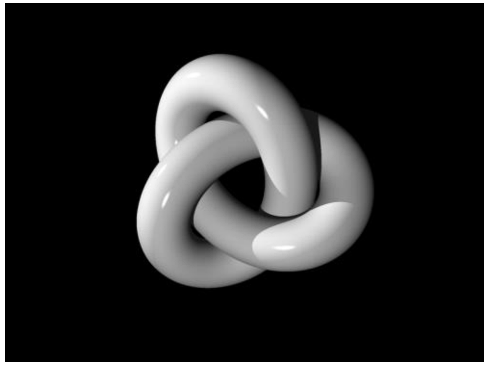
This 3 is important to produce the holonomy or AB phase.

The goldstone mode : phonon

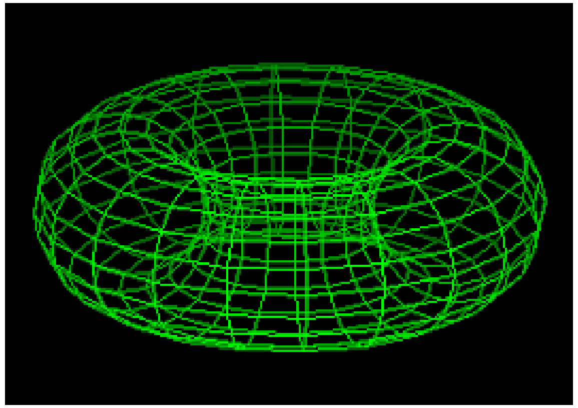
The discrete gauge field

Probing topology





- The results obtained with vortices obtainable on a compact manifold.



- Helps make connection with standard techniques of detecting topological order / ground state degeneracy.

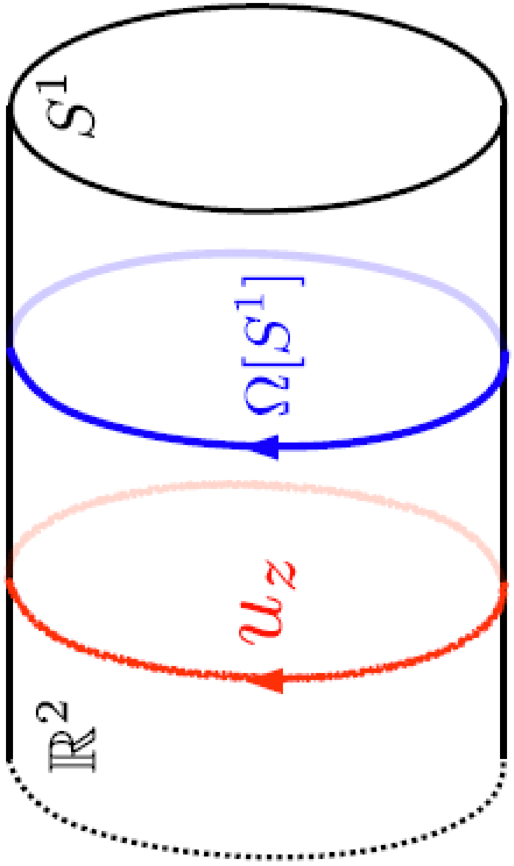
Imagine QCD of nontrivial manifold.

- Compactify one spatial dimension.

- length L : Larger than all other length scales in the problem.

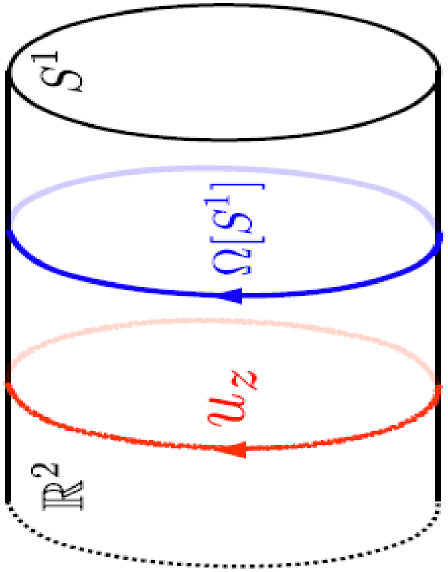
- Use periodic boundary conditions (b.c.) for all fields.

Goal : To compute : $\Omega \equiv e^{i \oint A}$ along S_1.



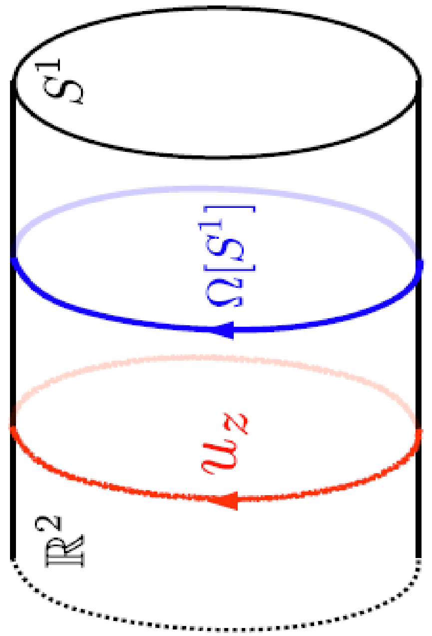
- Without loss of generality write the condensate as a 3×3 diagonal color-flavor matrix with windings along the compact direction given by k_1, k_2, k_3.

- The gradient term then generates an effective potential for Ω given by



$$V_{eff}(\Omega) = \kappa \frac{\Delta^2 v_s^2}{L^2} \min_{k \in Z^3} \sum_{i=1,2,3} (2\pi k_i + \theta_i)^2 + ..$$

with $\Omega = \begin{pmatrix} e^{i\theta_1} & 0 & 0 \\ 0 & e^{i\theta_2} & 0 \\ 0 & 0 & e^{i\theta_3} \end{pmatrix}$: $\theta_3 = -\theta_1 - \theta_2$



Minimize the effective potential.

Results :

One global minimum at $\theta = \{0,0,0\}$ with $k = \{0,0,0\}$. → $\Omega = 1$.

Two local minima at $\theta = \{\frac{2\pi}{3}, \frac{2\pi}{3}, -\frac{4\pi}{3}\}$, $k = \{0,0,1\}$ → $\Omega = e^{\frac{2\pi i}{3}}$.

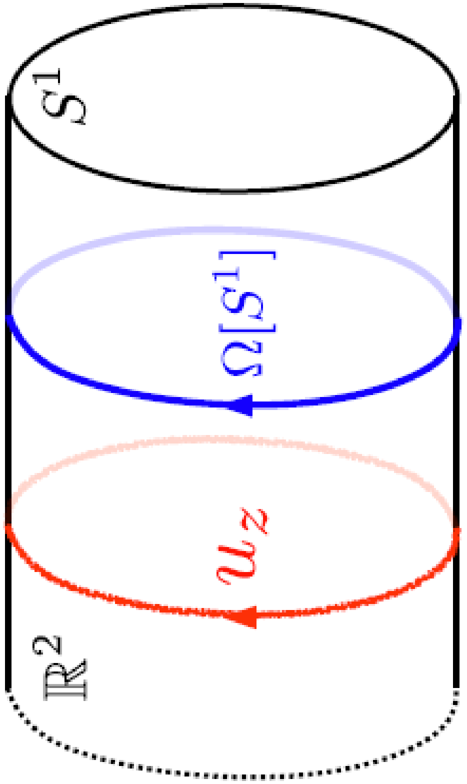
and $\theta = \{\frac{4\pi}{3}, -\frac{2\pi}{3}, -\frac{2\pi}{3}\}$, $k = \{-1,0,0\}$. → $\Omega = e^{-\frac{2\pi i}{3}}$.

Interpretation of the minima

- superfluid flow velocity along S_1 : $u_\mu = \frac{1}{2\mu_B} \operatorname{tr}\left(\phi^{-1} D_\mu \phi\right).$
- superflow exists along S_1

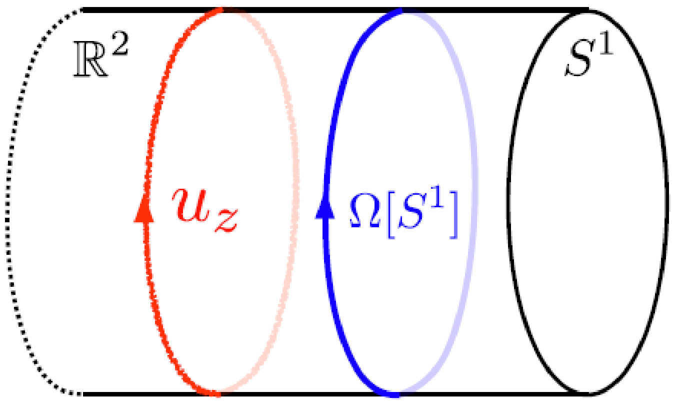
$$u_z = \frac{\pi}{\mu_B L}(k_1 + k_2 + k_3)\ .$$

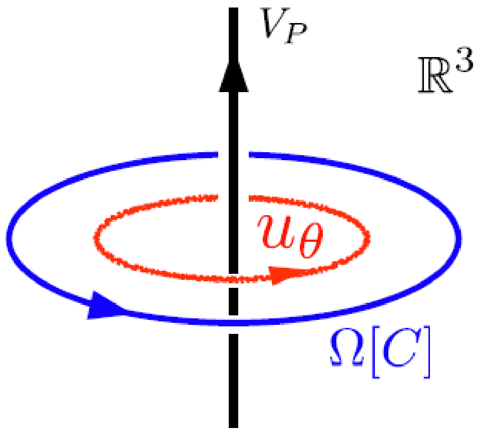
- For the global minimum, $u_z = 0.$
- But, for the local minima : $\sum_i k_i = \pm 1$. So, nonzero superfluid flow (and minimal circulation) along compact direction for the local minima.



Interpretation :

- The circle compactified theory is mimicking an annular region surrounding a superfluid vortex on R_4.





- Fundamental rep quarks have Z_3 statistics with minimal vortices in high density quark matter.

Comment on Higgs-confinement :

- Our story not in conflict with Higgs-confinement continuity proved in Fradkin-Shenker.
- The example in Fradkin-Shenker involves no $U(1)_B$ global symmetry.
- Over generalizations of the theorem may fail.
- Our results may well be one of the many scenarios where Higgs-confinement continuity fails.