Complementarity in Joint Lattice-Experiment Analyses of Parton Distributions

APS Topical Group on Hadronic Physics
GHP

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Jefferson Lab

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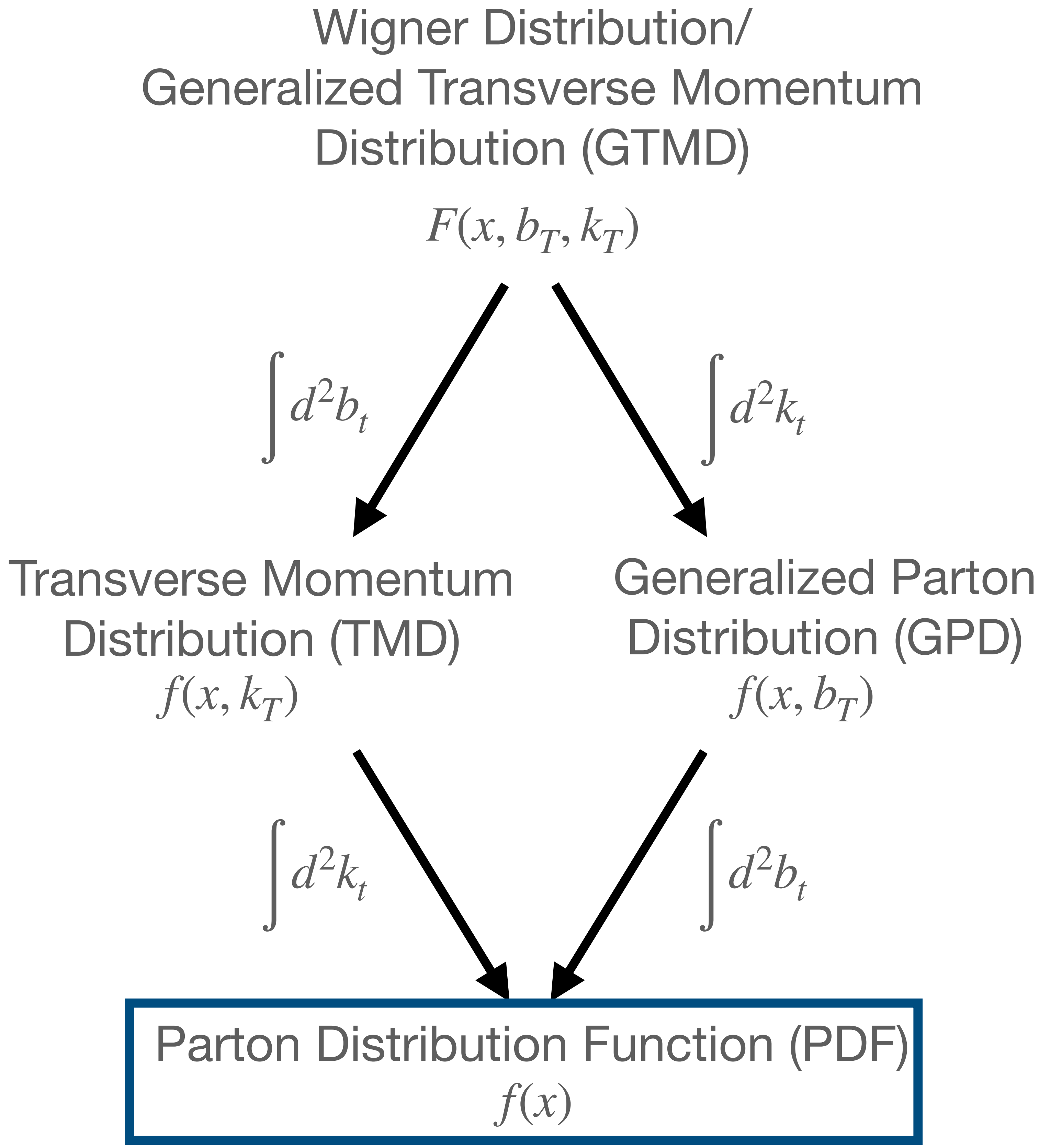
Parton Structure

For various flavors and spin combinations



Wigner Distribution/
Generalized Transverse Momentum Distribution (GTMD)
$F(x, b_T, k_T)$

$\int d^2 b_t$

$\int d^2 k_t$

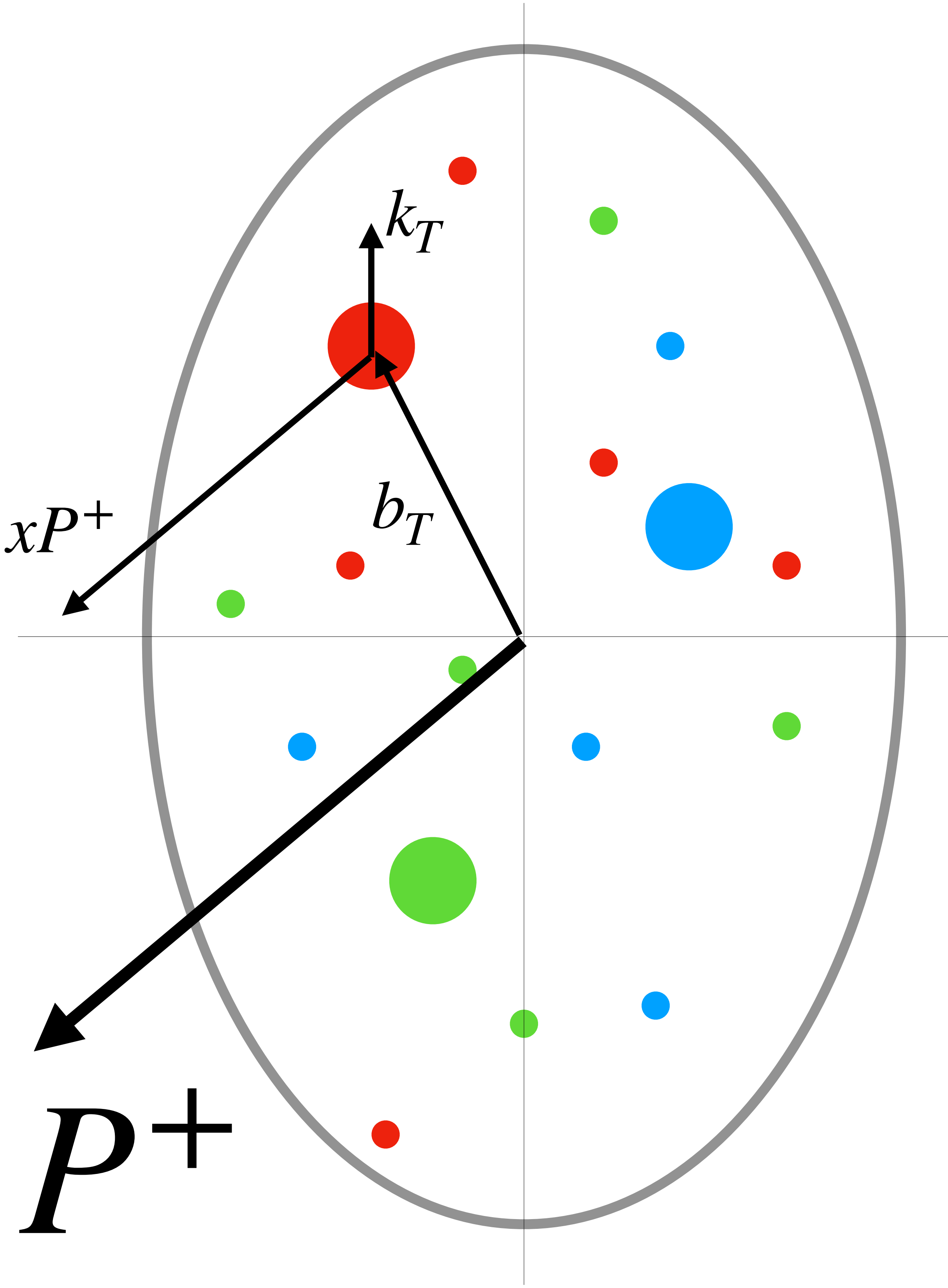
Transverse Momentum Distribution (TMD)
$f(x, k_T)$

Generalized Parton Distribution (GPD)
$f(x, b_T)$

$\int d^2 k_t$

$\int d^2 b_t$

Parton Distribution Function (PDF)
$f(x)$



If PDFs are universal, then….

If the same universal PDFs are factorizable from lattice and experiment, and if power corrections can be controlled for both

Why not analyze both simultaneously?

- Factorization of hadronic cross sections

$$d\sigma_h = d\sigma_q \otimes f_{h/q} + P.C.$$

- Factorization of Lattice observables

$$M_h = M_q \otimes f_{h/q} + P.C.$$

Consider Lattice as a theoretical prior to the experimental Global Fit

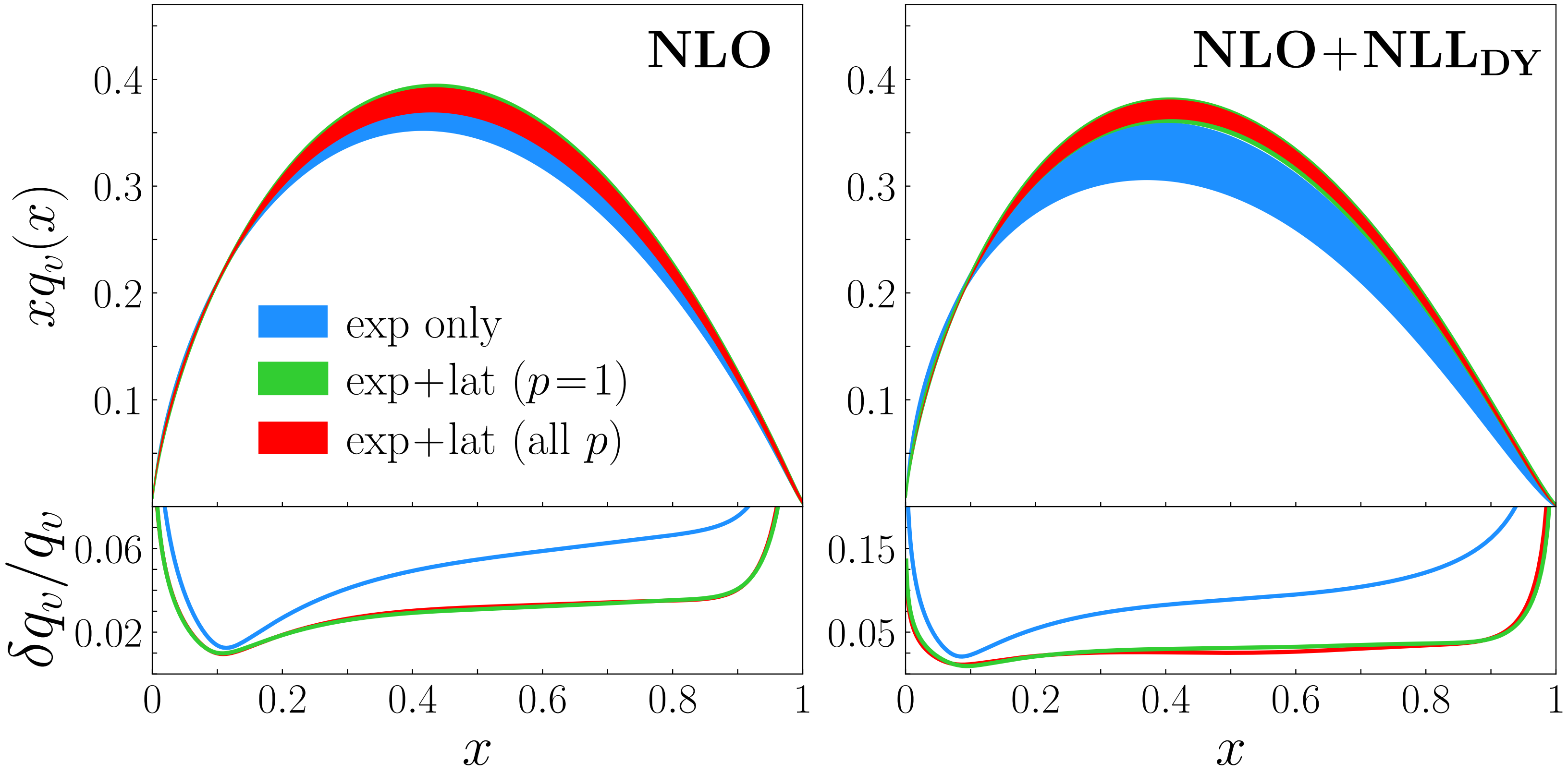
Complementarity in Lattice and Experiment

- Lattice limited to low ν, sensitive to $x > \sim 0.2$
- Lattice matching relation is integral over all x
- Low p_z data can reach high signal-to-noise compared to experiment
- Lattice can make many hadrons with equal difficulty

- Cross Sections limited to specific max x_B
- Cross Section matching is integral from x_B to 1
  - Creates sensitively to hard kernel in large x region

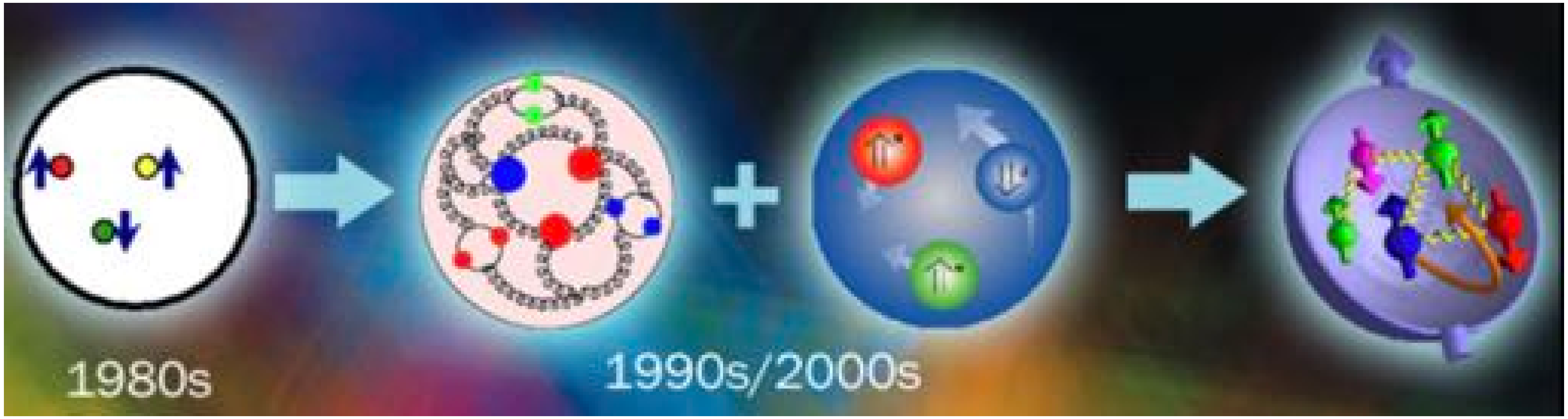
Example: unpolarized valence quark PDF of pion

P. Barry et al, *Phys. Rev. D* 105 (2022) 11, 114051



Gluon Structure

- Why do we want to know gluon distributions?
  - Understanding hadronic properties such as mass, spin, ….



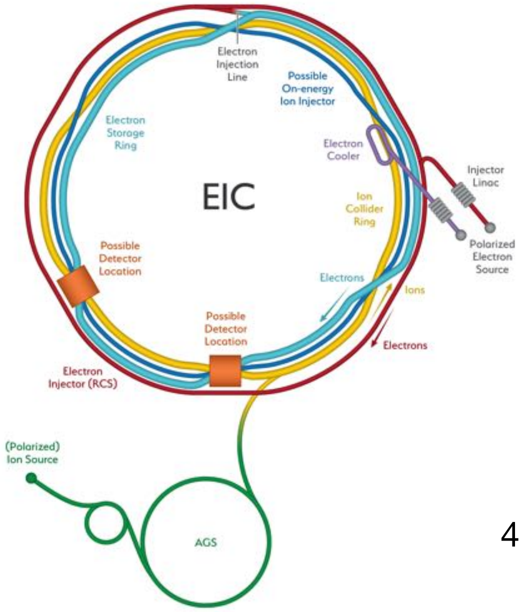
  - Understanding Higgs or top production in high energy collisions
  - Understanding low x physics, gluon saturation

Nuclear Experiment

[Submitted on 7 Dec 2012 (v1), last revised 30 Nov 2014 (this version, v3)]

Electron Ion Collider: The Next QCD Frontier - Understanding the glue that binds us all

A. Accardi, J.L. Albacete, M. Anselmino, N. Armesto, E.C. Aschenauer, A. Bacchetta, D. Boer, W.K. Brooks, T. Burton, N.-B. Chang, W.-T. Deng, A. Deshpande, M. Diehl, A. Dumitru, R. Dupré, R. Ent, S. Fazio, H. Gao, V. Guzey, H. Hakobyan, Y. Hao, D. Hasch, R. Holt, T. Horn, M. Huang, A. Hutton, C. Hyde, J. Jalilian-Marian, S. Klein, B. Kopeliovich, Y. Kovchegov, K. Kumar, K. Kumerički, M.A.C. Lamont, T. Lappi, J.-H. Lee, Y. Lee, E.M. Levin, F.-L. Lin, V. Litvinenko, T.W. Ludlam, C. Marquet, Z.-E. Meziani, R. McKeown, A. Metz, R. Milner, V.S. Morozov, A.H. Mueller, B. Müller, D. Müller, P. Nadel-Turonski, H. Paukkunen, A. Prokudin, V. Ptitsyn, X. Qian, J.-W. Qiu, M. Ramsey-Musolf, T. Roser, F. Sabatié, R. Sassot, G. Schnell, P. Schweitzer, E. Sichtermann, M. Stratmann, M. Strikman, M. Sullivan, S. Taneja, T. Toll, D. Trbojevic, T. Ullrich, R. Venugopalan, S. Vigdor, W. Vogelsang, C. Weiss, B.-W. Xiao, F. Yuan, Y.-H. Zhang, L. Zheng



R. Jaffe and A. Manohar, Nucl. Phys. B 337, 509 (1990)

Spinning gluons

$$J = \frac{1}{2}\Delta\Sigma + L_q + L_G + \Delta G$$

$$\Delta G = \int dx\, \Delta g(x)$$

- Positivity constraints are invalid

$$|\Delta g| \leq g$$

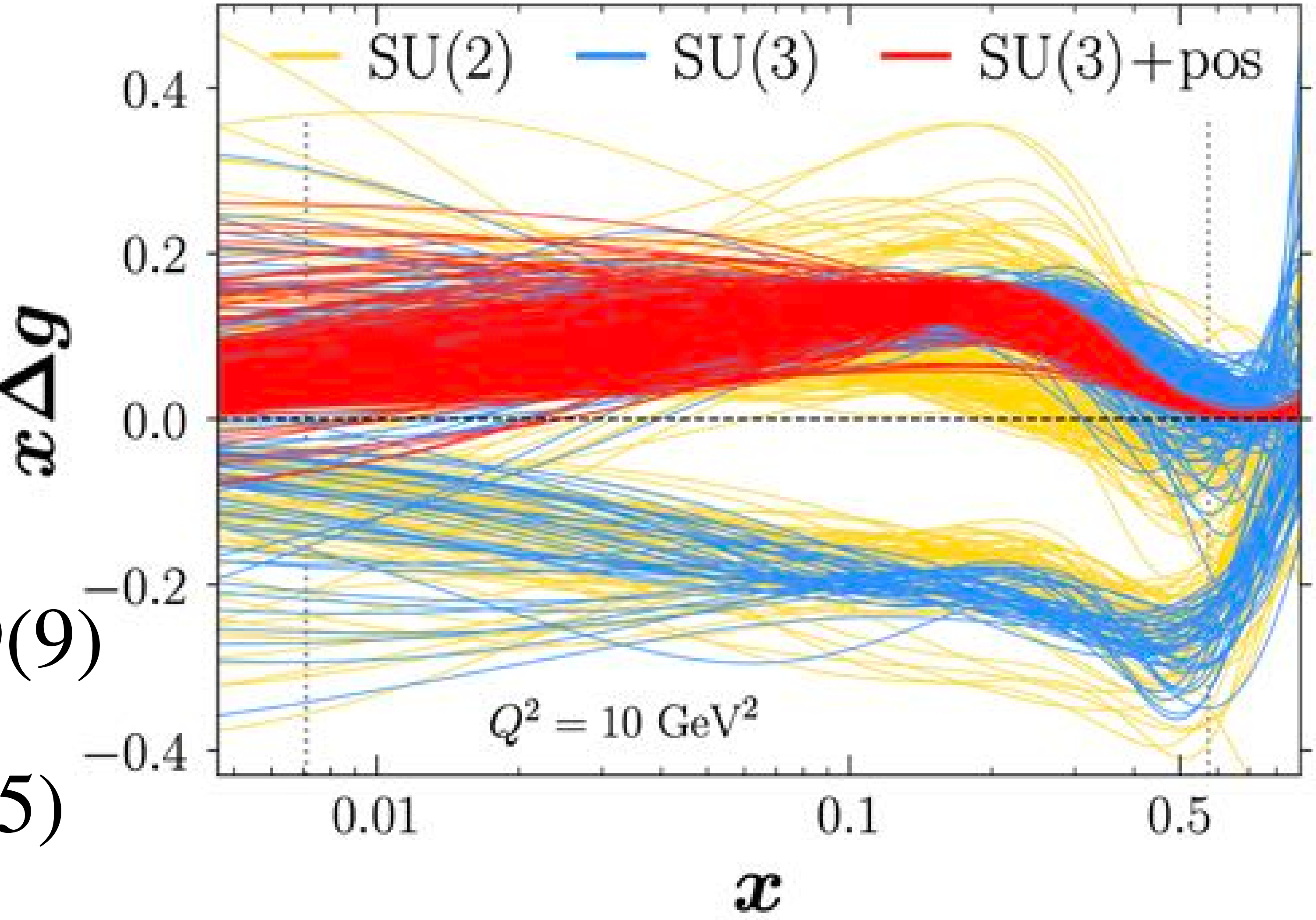
J. Collins, T. Rogers, N. Sato,
Phys Rev D 105 (2022) 7,076010

- Removal reveals new band of solutions

- With constraint: $\Delta G = 0.39(9)$

- Without constraint: $\Delta G = 0.3(5)$

- Lattice: $\Delta G = 0.251(47)(16)$



Y. Zhou et al (JAM) Phys. Rev. D 105, 074022 (2022)

Y-B. Yang et al (χ-QCD) Phys. Rev. Lett. 118, 102001 (2017)
K-F. Liu arXiv: 2112.08416

Parton Distributions and the Lattice

- Parton Distributions are defined by operators with light-like separations

- Use space-like separations

  X. Ji *Phys Rev Lett* 110 (2013) 262002

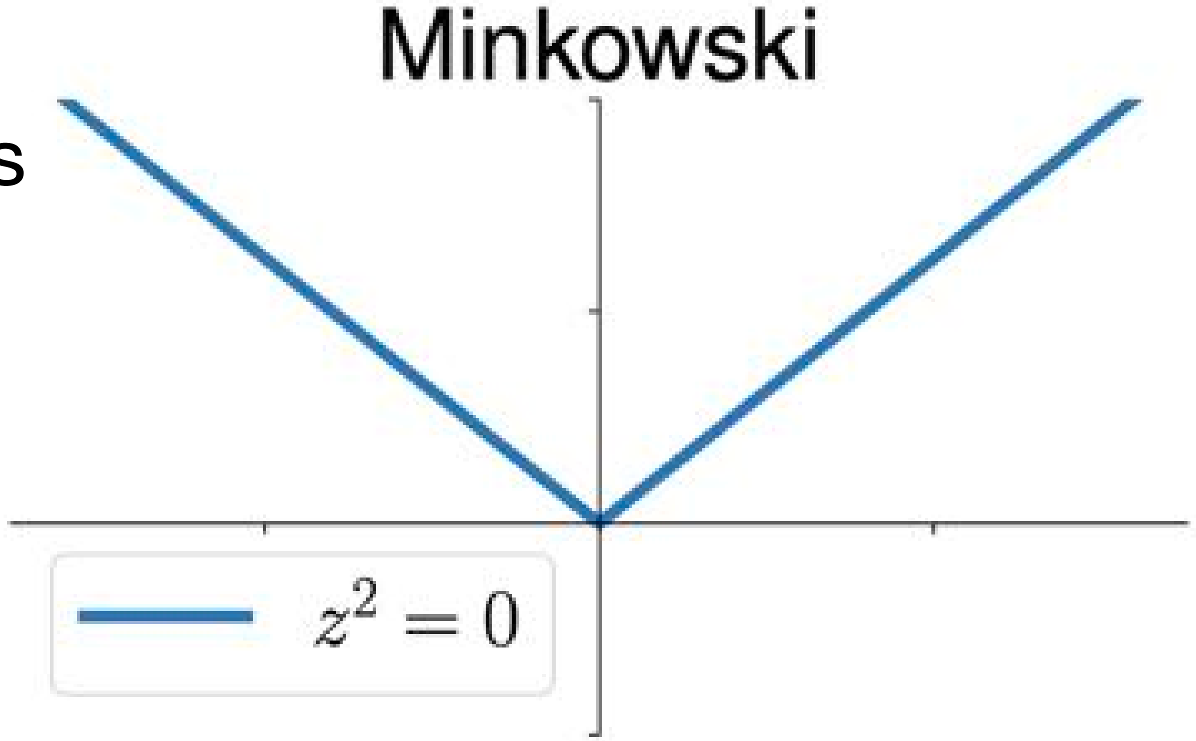
  - Wilson line operators

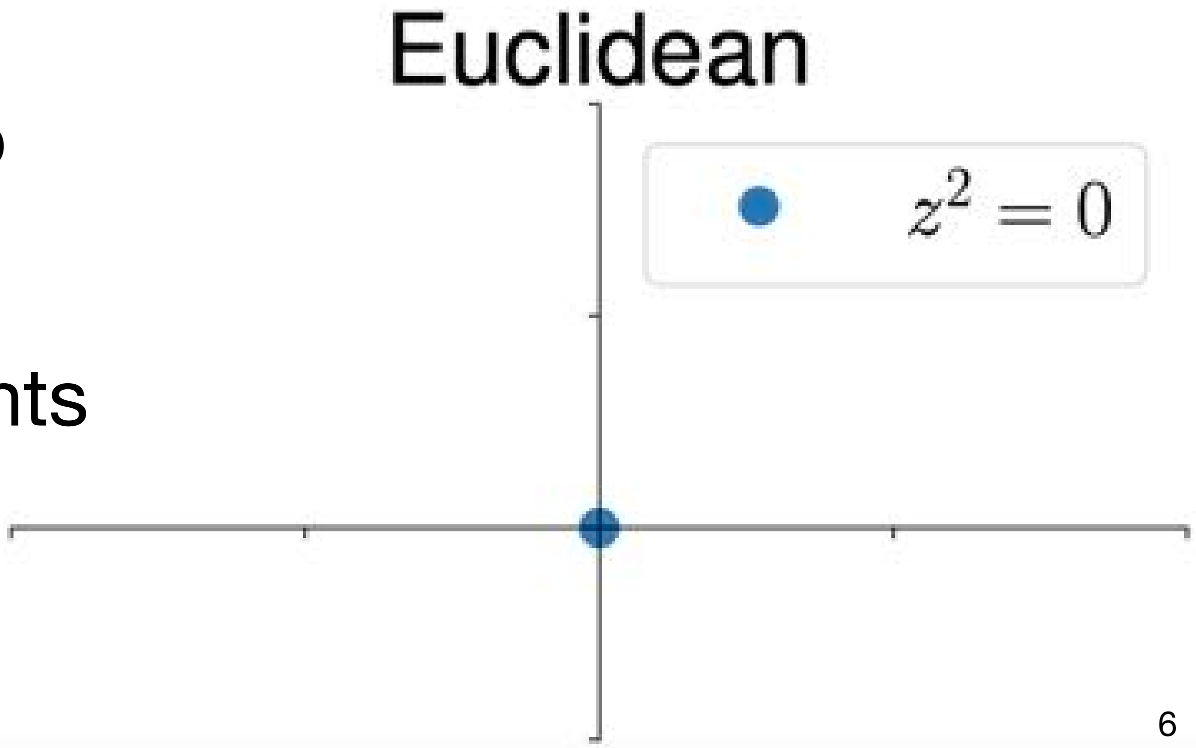
$$O_{\Gamma}^{\mathrm{WL}}(z) = \bar{\psi}(z)\Gamma W(z;0)\psi(0)$$

$$z^2 \neq 0$$

  - Factorizations exist analogous to cross sections

- Fourier transforms of matrix elements give PDF in certain limits





The Role of Separation and Momentum

- In quasi-PDF/LaMET and pseudo-PDF/Short distance, separation and momentum swap roles

Factorization Scale:

$$p_z^2 \,/\, z^2$$

- Describes scale in hard part
- Scale for factorization to PDF
- Scale in power expansion
- Keep away from Λ_{QCD}^2
- Technically only requires single value

NEED!

Dynamical variable:

$$z \,/\, p_z \text{ , or } \nu = p \cdot z$$

- Describes interesting partonic structure
- Variable for inverse Fourier Transform
- Can take large or small value
- Want as many as are available
- Wider range improves the inverse problem if you really want x-space

WANT!

Helicity Gluon matrix element

I. Balitsky, W. Morris, A. Radyushkin JHEP 02 (2022) 193
C. Egerer et al (HadStruc) arXiv:2207.08733

- Helicity Gluon Matrix Element:

$$\widetilde{M}_{\mu\alpha;\nu\beta}(z,p,s) = \frac{1}{2}\epsilon_{\nu\beta\rho\sigma}M_{\mu\alpha;\rho\sigma} = \langle p,s\,|\,\mathrm{Tr}\left[F^{\mu\alpha}(z)W(z;0)\,\widetilde{F}^{\nu\beta}(0)\right]|\,p,s\rangle$$

- Useful Combination $\widetilde{\mathscr{M}}(z,p) = \left[\widetilde{M}_{ti;it} + \widetilde{M}_{ij;ij}\right]$
  - Gives **two** amplitudes, one has no leading twist contribution

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- Useful Combination $\widetilde{\mathcal{M}}(z,p) = \left[\widetilde{M}_{ti;it} + \widetilde{M}_{ij;ij}\right]$
  - Gives **two** amplitudes, one has no leading twist contribution
- Use ratio with finite continuum limit

$$\widetilde{\mathfrak{M}}(\nu, z^2) = i\,\frac{\left[\widetilde{\mathcal{M}}(z,p)/p_z p_0\right]/Z_L(z/a)}{\mathcal{M}(0,z^2)/m^2}$$

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- Relation to gluon and quark singlet ITD

$$\langle x\rangle_g\widetilde{\mathfrak{M}}(\nu,z^2) = \int_0^1 \widetilde{C}^{gg}(x\nu,\mu^2z^2)\Delta g(x,\mu^2) + \widetilde{C}^{qg}(x\nu,\mu^2z^2)\Delta q_S(x,\mu^2)$$

Lorentz decomposition

I. Balitsky, W. Morris, A. Radyushkin
JHEP 02 (2022) 193

$$
\begin{aligned}
\widetilde{M}^{(2)}_{\mu\alpha;\lambda\beta}(z,p) = {} & (sz)\big(g_{\mu\lambda}p_\alpha p_\beta - g_{\mu\beta}p_\alpha p_\lambda - g_{\alpha\lambda}p_\mu p_\beta + g_{\alpha\beta}p_\mu p_\lambda\big)\widetilde{\mathcal{M}}_{pp} \\
& +(sz)\big(g_{\mu\lambda}z_\alpha z_\beta - g_{\mu\beta}z_\alpha z_\lambda - g_{\alpha\lambda}z_\mu z_\beta + g_{\alpha\beta}z_\mu z_\lambda\big)\widetilde{\mathcal{M}}_{zz} \\
& +(sz)\big(g_{\mu\lambda}z_\alpha p_\beta - g_{\mu\beta}z_\alpha p_\lambda - g_{\alpha\lambda}z_\mu p_\beta + g_{\alpha\beta}z_\mu p_\lambda\big)\widetilde{\mathcal{M}}_{zp} \\
& +(sz)\big(g_{\mu\lambda}p_\alpha z_\beta - g_{\mu\beta}p_\alpha z_\lambda - g_{\alpha\lambda}p_\mu z_\beta + g_{\alpha\beta}p_\mu z_\lambda\big)\widetilde{\mathcal{M}}_{pz} \\
& +(sz)\big(p_\mu z_\alpha - p_\alpha z_\mu\big)\big(p_\lambda z_\beta - p_\beta z_\lambda\big)\widetilde{\mathcal{M}}_{ppzz} \\
& +(sz)\big(g_{\mu\lambda}g_{\alpha\beta} - g_{\mu\beta}g_{\alpha\lambda}\big)\widetilde{\mathcal{M}}_{gg}
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{M}^{(1)}_{\mu\alpha;\lambda\beta}(z,p) = {} & \Big(g_{\mu\lambda}s_\alpha p_\beta - g_{\mu\beta}s_\alpha p_\lambda - g_{\alpha\lambda}s_\mu p_\beta + g_{\alpha\beta}s_\mu p_\lambda\Big)\widetilde{\mathcal{M}}_{sp} \\
& +\Big(g_{\mu\lambda}p_\alpha s_\beta - g_{\mu\beta}p_\alpha s_\lambda - g_{\alpha\lambda}p_\mu s_\beta + g_{\alpha\beta}p_\mu s_\lambda\Big)\widetilde{\mathcal{M}}_{ps} \\
& +\Big(g_{\mu\lambda}s_\alpha z_\beta - g_{\mu\beta}s_\alpha z_\lambda - g_{\alpha\lambda}s_\mu z_\beta + g_{\alpha\beta}s_\mu z_\lambda\Big)\widetilde{\mathcal{M}}_{sz} \\
& +\Big(g_{\mu\lambda}z_\alpha s_\beta - g_{\mu\beta}z_\alpha s_\lambda - g_{\alpha\lambda}z_\mu s_\beta + g_{\alpha\beta}z_\mu s_\lambda\Big)\widetilde{\mathcal{M}}_{zs} \\
& +(p_\mu s_\alpha - p_\alpha s_\mu)(p_\lambda z_\beta - p_\beta z_\lambda)\widetilde{\mathcal{M}}_{pspz} \\
& +(p_\mu z_\alpha - p_\alpha z_\mu)(p_\lambda s_\beta - p_\beta s_\lambda)\widetilde{\mathcal{M}}_{pzps} \\
& +(s_\mu z_\alpha - s_\alpha z_\mu)(p_\lambda z_\beta - p_\beta z_\lambda)\widetilde{\mathcal{M}}_{szpz} \\
& +(p_\mu z_\alpha - p_\alpha z_\mu)(s_\lambda z_\beta - s_\beta z_\lambda)\widetilde{\mathcal{M}}_{pzsz}
\end{aligned}
$$

Want: $M_{\Delta g}(\nu, z^2) = \left[\widetilde{\mathcal{M}}^{(+)}_{sp} - \nu\,\widetilde{\mathcal{M}}_{pp}\right]$

Can get: $\widetilde{\mathcal{M}}(z,p) = \left[\widetilde{M}_{ti;it} + \widetilde{M}_{ij;ij}\right]$

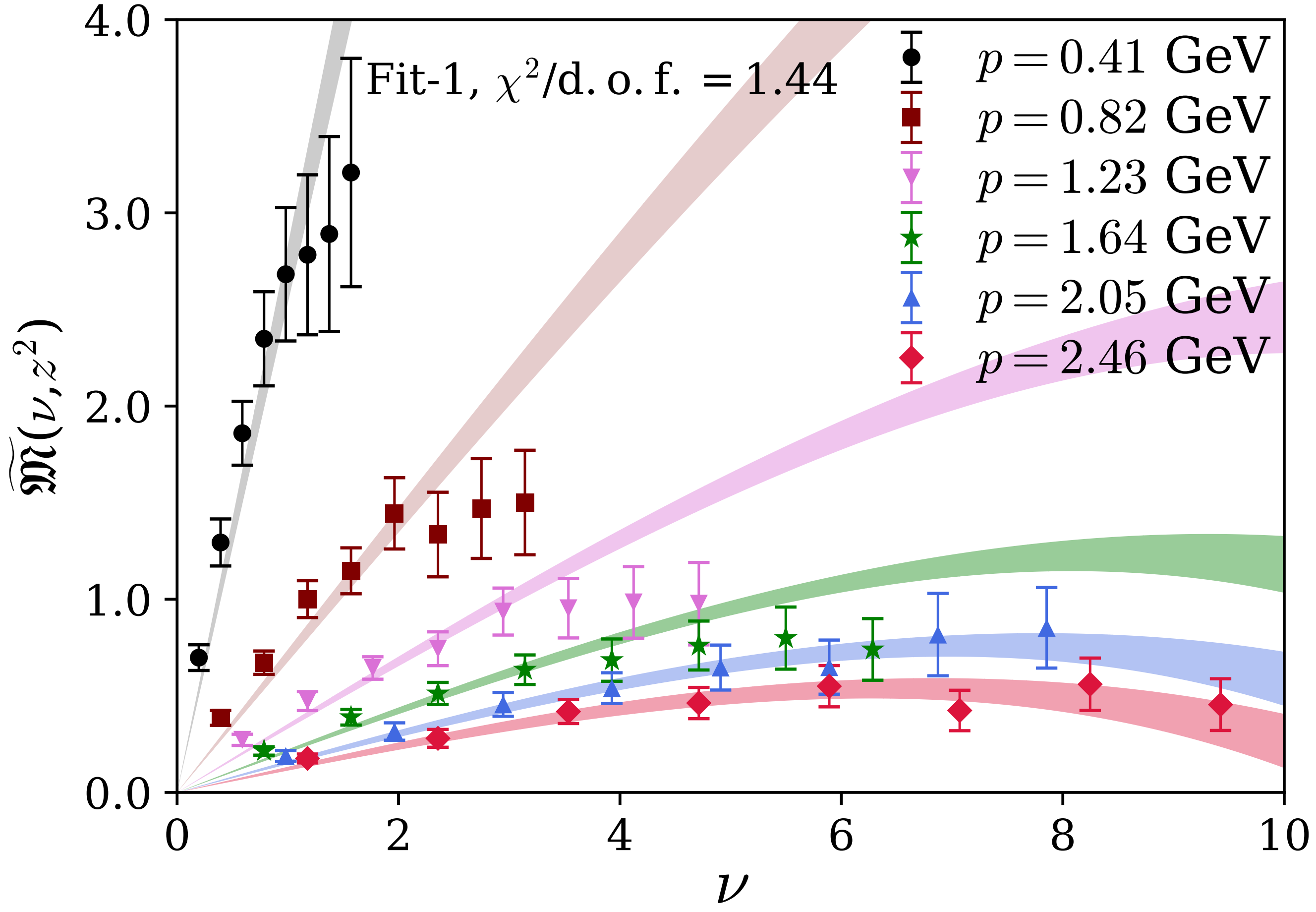
$$
\begin{aligned}
&= M_{\Delta g} - \frac{m^2 z^2}{\nu}\widetilde{\mathcal{M}}_{pp} \\
&= M_{\Delta g} - \frac{m^2}{p_z^2}\nu\,\widetilde{\mathcal{M}}_{pp}
\end{aligned}
$$

Helicity Gluon Matrix Element

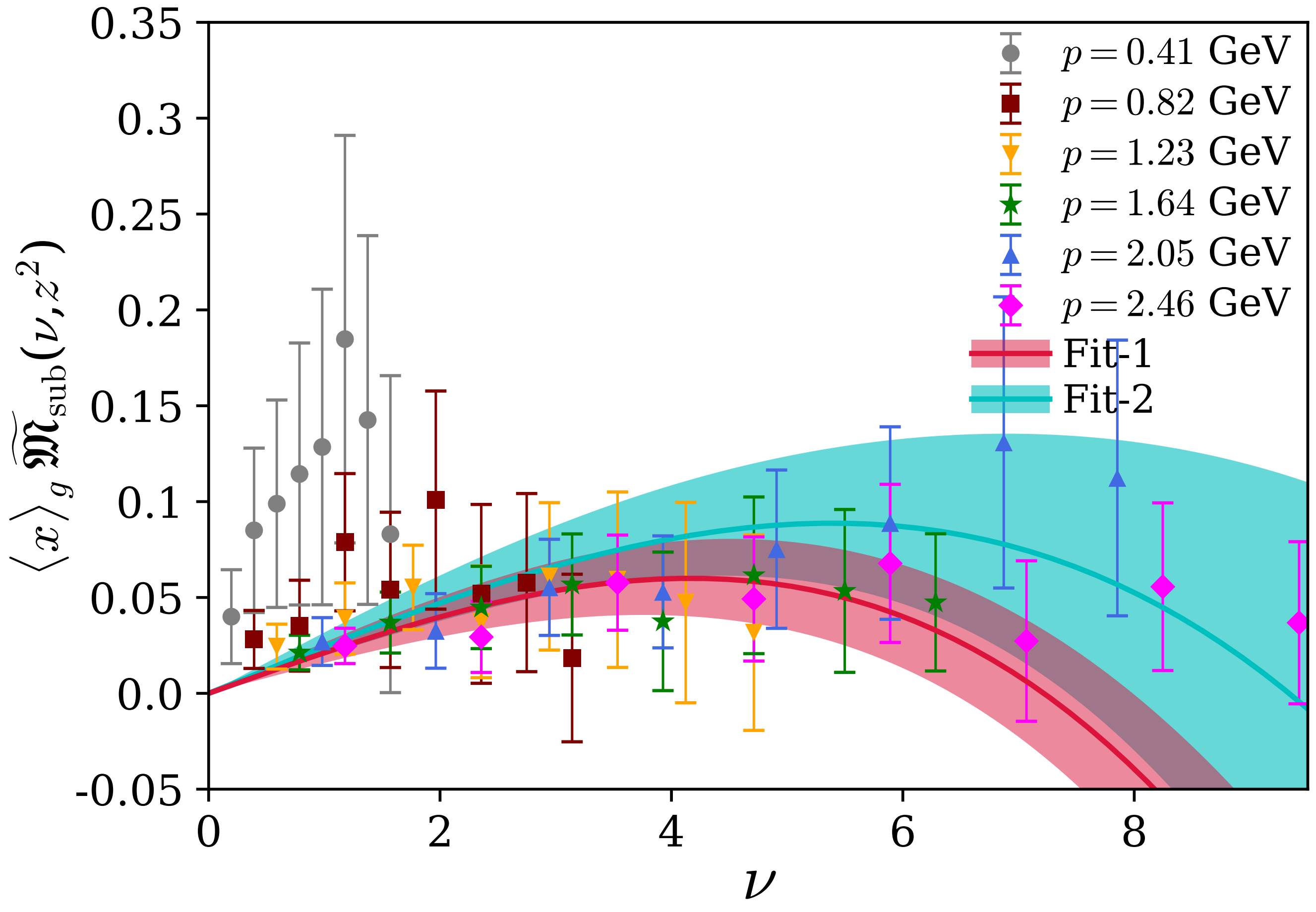
- Large contamination from $\frac{m^2}{p_z^2}\nu\widetilde{\mathscr{M}}_{pp}$ will need to be removed

$$\widetilde{\mathfrak{M}}(z,p) = M_{\Delta g} - \frac{m^2}{p_z^2}\nu\widetilde{\mathscr{M}}_{pp}$$

- Model both terms



- Subtract rest frame



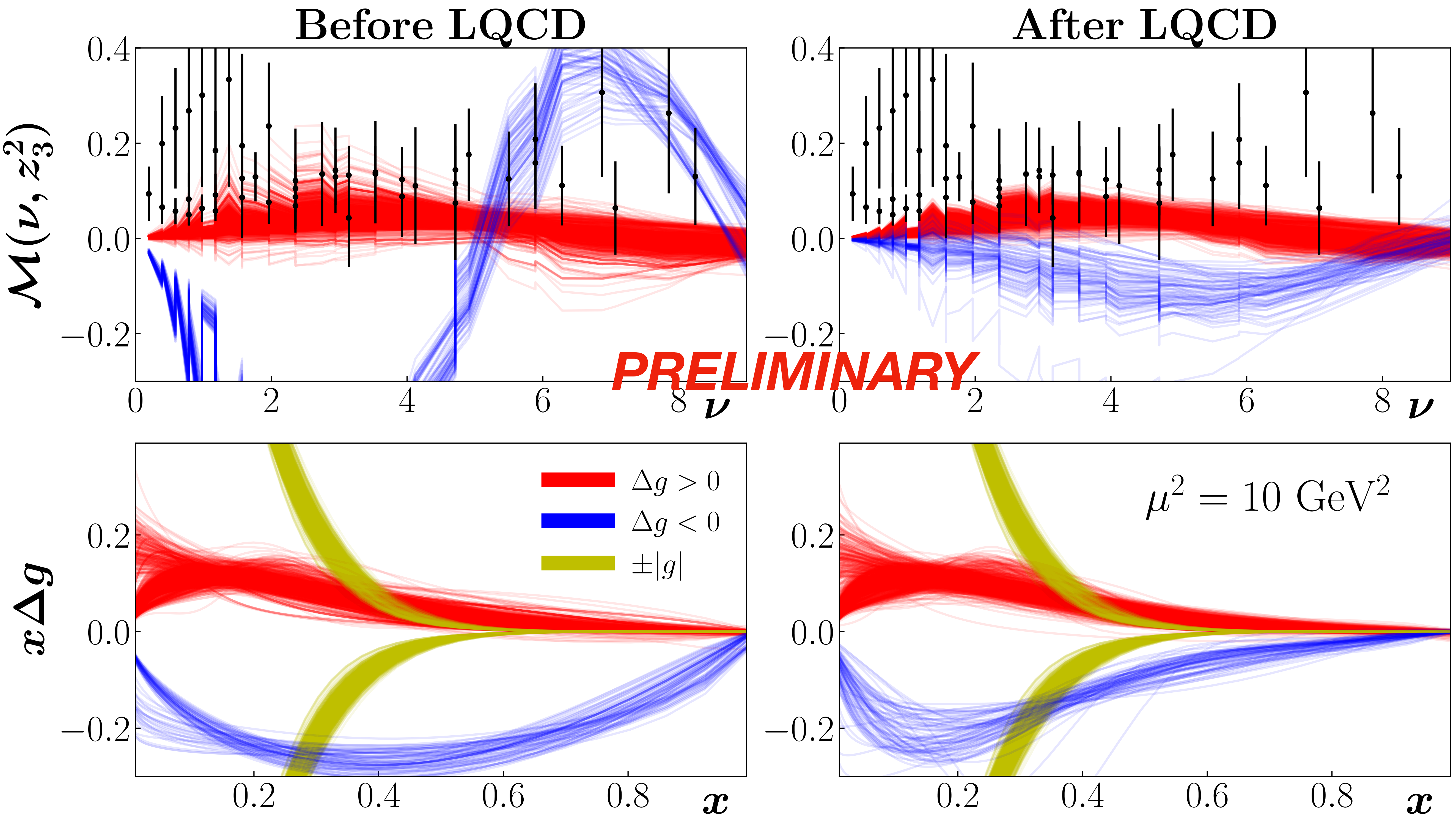
C. Egerer et al (HadStruc) arXiv: 2207.08733

- Model with Neural Network

T. Khan, T. Liu, R. Sufian arXiv:2211.15587

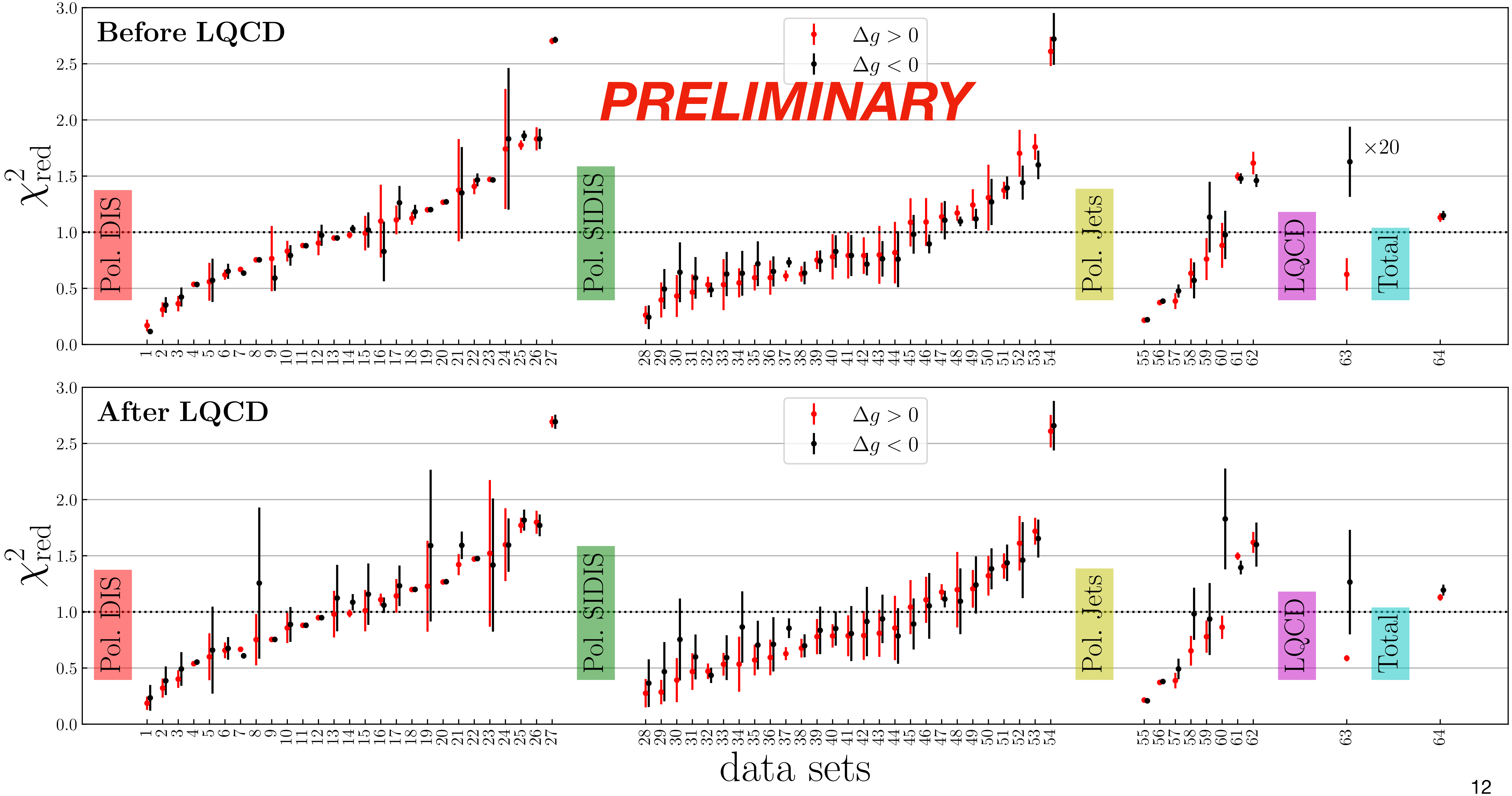
See slides Raza Sufian Wed 2:20pm

Helicity Gluon PDF with LQCD



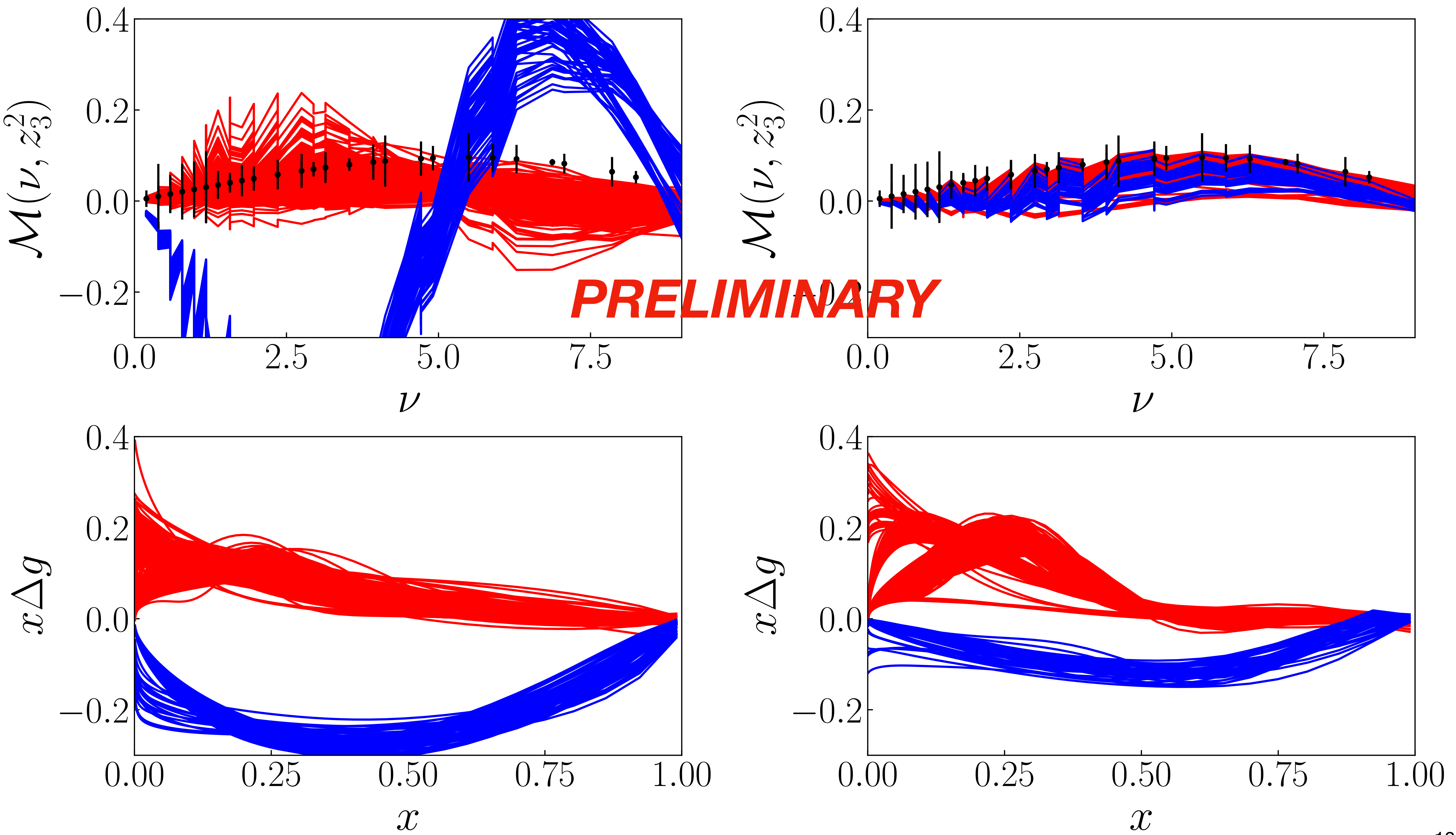
Helicity Gluon PDF with LQCD

- Negative and positive Δg appear consistent with experiment and lattice



What about at higher precision?

- Using fits, estimate impact of higher statistics (10x number of samples)



Conclusions

- Many PDFs lack the amount of data and the precision which unpolarized quark PDFs obtain
- Lattice data is sensitive to higher x while experiment is sensitive to lower x
- Even imprecise lattice data can impact the gluon helicity of the nucleon
    - Though, negative Δg is compatible with modern lattice results
- Future combined lattice-experimental analyses can obtain high precision PDFs

Thank you and the organizers!

Joe Karpie acknowledges financial support from The Gordon and Betty Moore Foundation and the American Physical Society to present this work at the GHP 2023 workshop.