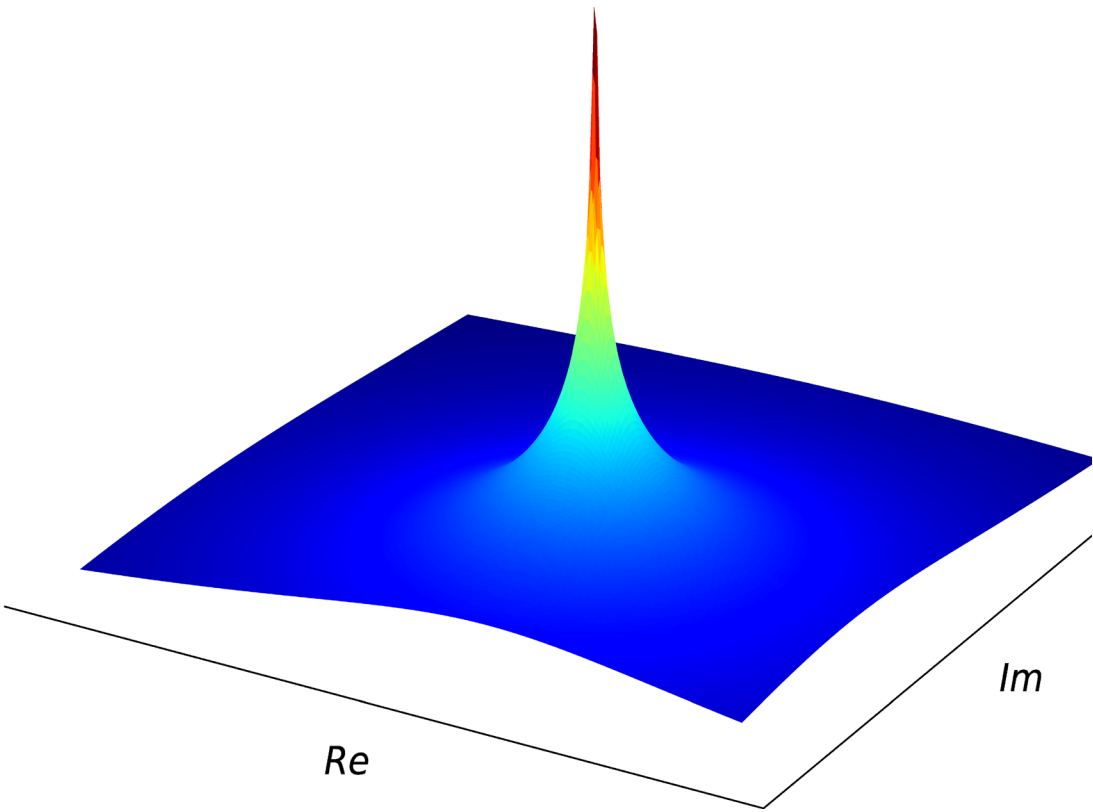
A lattice QCD study of the ρ meson

Luka Leskovec

The Cake Seminar,
December 5, 2018



based on

PHYSICAL REVIEW D **96,** 034525 (2017)

P-wave $\pi\pi$ scattering and the ρ resonance from lattice QCD

Constantia Alexandrou,[1,2] Luka Leskovec,[3,*] Stefan Meinel,[3,4,†] John Negele,[5] Srijit Paul,[2] Marcus Petschlies,[6,‡] Andrew Pochinsky,[5] Gumaro Rendon,[3] and Sergey Syritsyn[4,7]

PHYSICAL REVIEW D **98,** 074502 (2018)

$\pi\gamma \to \pi\pi$ transition and the ρ radiative decay width from lattice QCD

Constantia Alexandrou,[1,2] Luka Leskovec,[3,*] Stefan Meinel,[3,4,†] John Negele,[5] Srijit Paul,[2] Marcus Petschlies,[6,‡] Andrew Pochinsky,[5] Gumaro Rendon,[3] and Sergey Syritsyn[4,7]

Outline

- Motivation
- About the lattice methods we emply
- Strong $\pi\pi$ scattering
- The radiative transition $\pi\gamma \to \pi\pi$
- ρ photoproduction

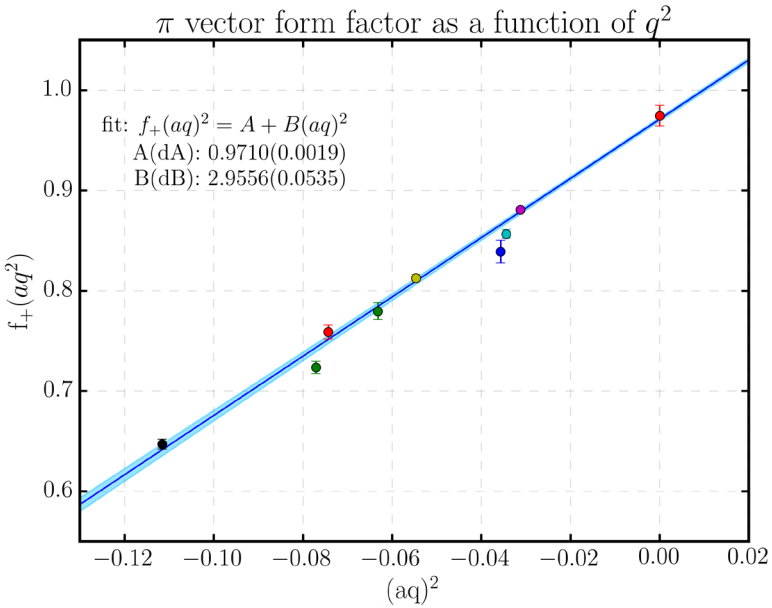
Motivation

Motivation

- What do we want to know?

	Quantity
x	mass
x	decay width
x	cross section(s)
?	how big it is?
…	how does it look like?
…	what is it made of?
…	…

- Why do we want to know it?
  (knowing is understanding)

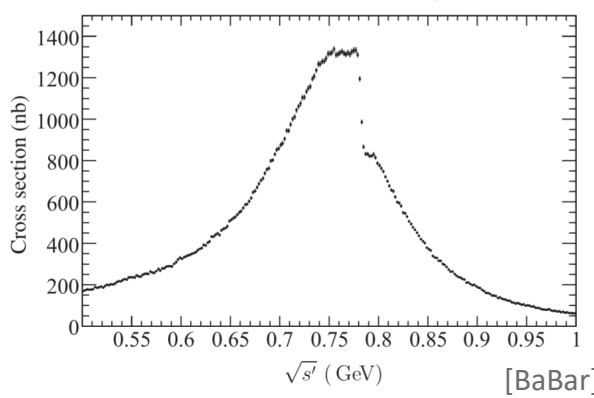


https://www.jlab.org/jleic/index.html

what do we know (from PDG and literature)

- Studied a lot on the lattice
  [Feng et al., Lang et al., Aoki et al., Pelissier et al., HadSpec, HadSpec, RQCD, Bulava et al., Guo et al., Fu et al., Bulava et al., …]
- Vanilla resonance
  - Breit-Wigner
  - no background phase
  - isospin triplet :: ρ^-, ρ^0, ρ^+
  - vector particle :: $J^P = 1^-$
  - elastic P-wave $\pi\pi$ scattering

$e^+e^- \to \pi^+\pi^-(\gamma)$



[BaBar]

$\rho(770)$ $I^G(J^{PC}) = 1^+(1^{--})$

See related review:
$\rho(770)$

$\rho(770)$ MASS	
NEUTRAL ONLY, e^+e^-	775.26 ± 0.25 MeV
CHARGED ONLY, τ DECAYS and e^+e^-	775.11 ± 0.34 MeV
MIXED CHARGES, OTHER REACTIONS	763.0 ± 1.2 MeV
Mass m	
CHARGED ONLY, HADROPRODUCED	766.5 ± 1.1 MeV
NEUTRAL ONLY, PHOTOPRODUCED	769.0 ± 1.0 MeV
NEUTRAL ONLY, OTHER REACTIONS	769.0 ± 0.9 MeV (S = 1.4)
$m_{\rho(770)^0} - m_{\rho(770)^\pm}$	-0.7 ± 0.8 MeV (S = 1.5)
$m_{\rho(770)^+} - m_{\rho(770)^-}$	
$\rho(770)$ RANGE PARAMETER	$5.3^{+0.9}_{-0.7}$ GeV^{-1}
$\rho(770)$ WIDTH	
NEUTRAL ONLY, e^+e^-	147.8 ± 0.9 MeV (S = 2.0)
CHARGED ONLY, τ DECAYS and e^+e^-	149.1 ± 0.8 MeV
MIXED CHARGES, OTHER REACTIONS	149.5 ± 1.3 MeV
CHARGED ONLY, HADROPRODUCED	150.2 ± 2.4 MeV
NEUTRAL ONLY, PHOTOPRODUCED	151.7 ± 2.6 MeV
NEUTRAL ONLY, OTHER REACTIONS	150.9 ± 1.7 MeV (S = 1.1)
$\Gamma_{\rho(770)^0} - \Gamma_{\rho(770)^\pm}$	0.3 ± 1.3 MeV (S = 1.4)
$\Gamma_{\rho(770)^+} - \Gamma_{\rho(770)^-}$	1.8 ± 2.1

Decay Modes ▸ Expand all decays

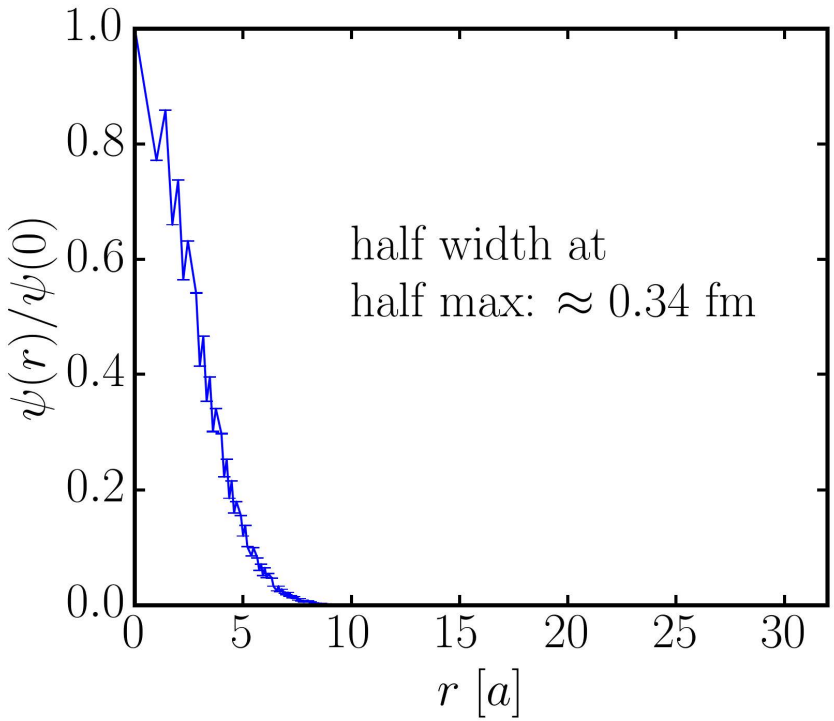
Mode			Fraction (Γ_i / Γ)	Scale Factor/ Conf. Level	P (MeV/c)
Γ_1	$\pi\pi$		$(100)\%$		363
▾ $\rho(770)^+$ **decays**					
Γ_2	$\pi^\pm\pi^0$		$(100)\%$		363
Γ_3	$\pi^\pm\gamma$		$(4.5 \pm 0.5) \times 10^{-4}$	S=2.2	375
Γ_4	$\pi^\pm\eta$		$< 6 \times 10^{-3}$	CL=84%	152
Γ_5	$\pi^\pm\pi^+\pi^-\pi^0$		$< 2.0 \times 10^{-3}$	CL=84%	254
▾ $\rho(770)^0$ **decays**					
Γ_6	$\pi^+\pi^-$		$(100)\%$		362
Γ_7	$\pi^+\pi^-\gamma$		$(9.9 \pm 1.6) \times 10^{-3}$		362
Γ_8	$\pi^0\gamma$		$(4.7 \pm 0.6) \times 10^{-4}$	S=1.4	376
Γ_9	$\eta\gamma$		$(3.00 \pm 0.21) \times 10^{-4}$	S=1.0	194
Γ_{10}	$\pi^0\pi^0\gamma$		$(4.5 \pm 0.8) \times 10^{-5}$	S=1.0	363
Γ_{11}	$\mu^+\mu^-$	[1]	$(4.55 \pm 0.28) \times 10^{-5}$	S=1.0	373
Γ_{12}	e^+e^-	[1]	$(4.72 \pm 0.05) \times 10^{-5}$	S=1.0	388
Γ_{13}	$\pi^+\pi^-\pi^0$		$(1.01^{+0.54}_{-0.36} \pm 0.34) \times 10^{-4}$		323
Γ_{14}	$\pi^+\pi^-\pi^+\pi^-$		$(1.8 \pm 0.9) \times 10^{-5}$		251
Γ_{15}	$\pi^+\pi^-\pi^0\pi^0$		$(1.6 \pm 0.8) \times 10^{-5}$		257
Γ_{16}	$\pi^0e^+e^-$		$< 1.2 \times 10^{-5}$	CL=90%	376
Γ_{17}	ηe^+e^-				194

About the Lattice

Lattice Gauge Ensemble

Label	$N_s^3 \times N_t$	a [fm]	L [fm]	m_π [MeV]	N_{config}
C13	$32^3 \times 96$	0.11403(77)	3.649(25)	317	1041

- $N_f = 2 + 1$ clover-Wilson fermions
- Isotropic lattice by: K. Orginos et al.
- $m_\pi L \approx 5.8$
- ρ unstable
- forward, sequential and stochastic propagators
- 8 sources/config
- Wuppertal smearing (N,α) = (20,3.0)



Angular momentum on the lattice

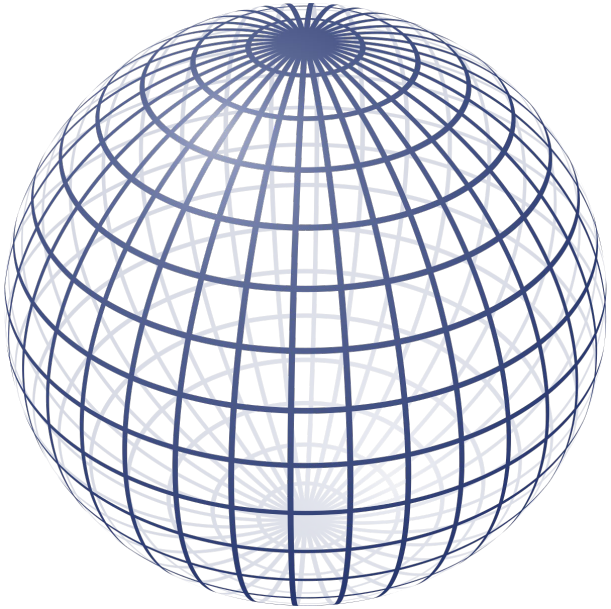
Many-to-one mapping

Infinite volume:

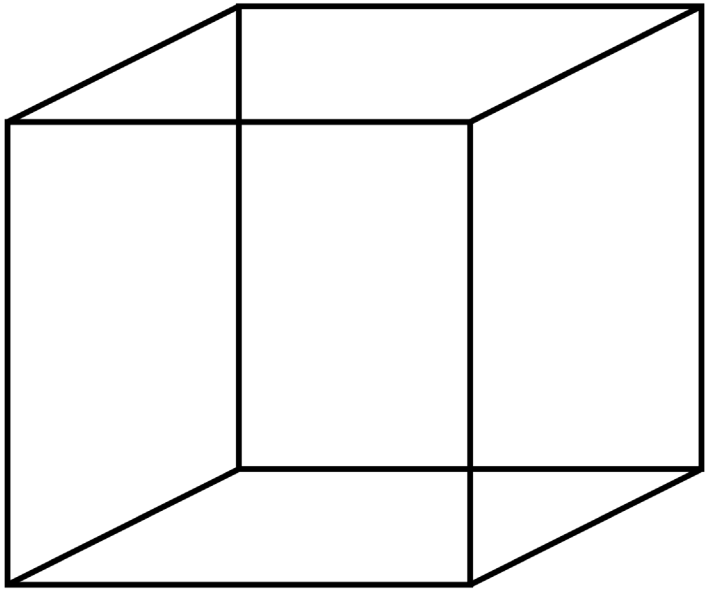
- $O(3)$ symmetry
- Infinite irreps $J^P = 0^{\pm}, 1^{\pm}, \ldots$

Finite volume box:

- O_h symmetry
- 10 irreducible representations

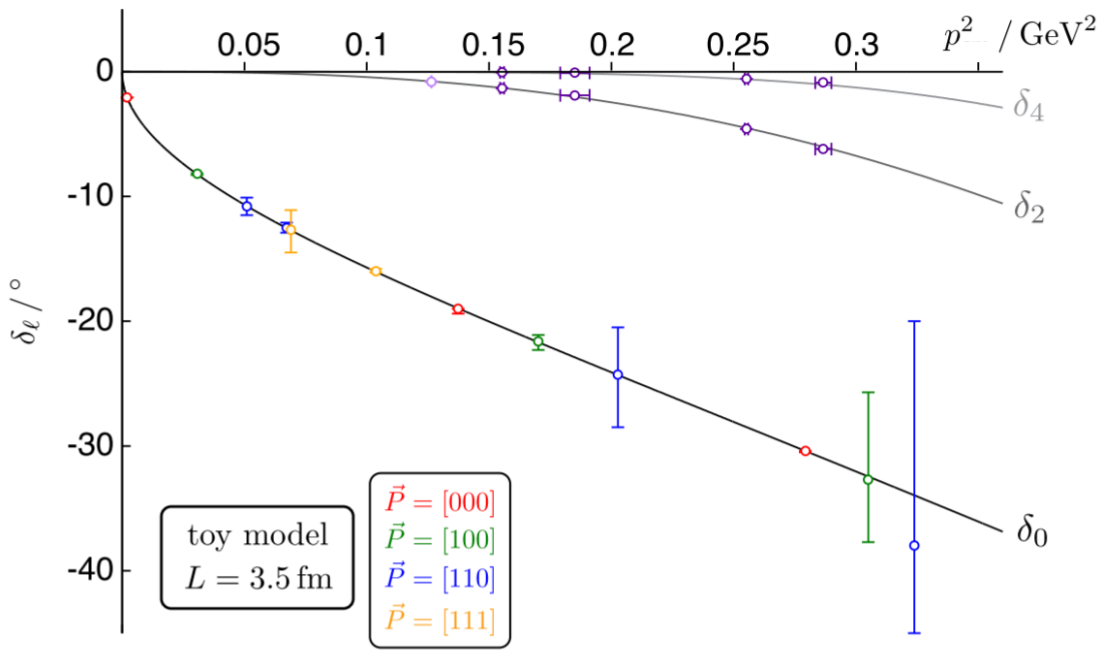


[R.C Johnson]



Angular momentum on the lattice

[HadSpec]



- Phase shift for mesons:

$$\delta_l(p) \propto p^{2l+1}$$

- Higher partial waves heavily suppressed
- Many-to-one is not an issue

Projection to lattice irreducible representations

- Single hadron interpolator projections:

$$O_{qq}^{\Lambda,\vec{P}} = \frac{\dim(\Lambda)}{N} \sum_{R} \boxed{\chi_\Lambda(R)} \boxed{R} O_{qq}(\vec{P})$$

Group properties and group elements

- Multi hadron interpolator projections:

$$O_{\pi\pi}^{\Lambda,\vec{P}} = \frac{\dim(\Lambda)}{N} \sum_{R} \boxed{\chi_\Lambda(R)} \left[\pi^+ \left(\frac{\vec{P}}{2} + \boxed{R}\vec{p}_1 \right) \pi^0 \left(\frac{\vec{P}}{2} - \boxed{R}\vec{p}_2 \right) - \pi^0 \left(\frac{\vec{P}}{2} + \boxed{R}\vec{p}_1 \right) \pi^+ \left(\frac{\vec{P}}{2} - \boxed{R}\vec{p}_2 \right) \right]$$

Setup

- interpolators with only spin structure:

- $O_1(\vec{P}) = \sum_{\vec{x}} \bar{u}\gamma_i d\ e^{i\,\vec{P}.\vec{x}}$
- $O_2(\vec{P}) = \sum_{\vec{x}} \bar{u}\gamma_t\gamma_i d\ e^{i\,\vec{P}.\vec{x}}$
- $O_3(\vec{P}) = \pi^+(\vec{p}_1)\pi^0(\vec{p}_2) - \pi^0(\vec{p}_1)\pi^+(\vec{p}_2)$
- $O_4(\vec{P}) = \pi^+(\vec{p}_1{}')\pi^0(\vec{p}_2{}') - \pi^0(\vec{p}_1{}')\pi^+(\vec{p}_2{}')$
- $\pi(\vec{p}) = \sum_{\vec{x}} \bar{q}\gamma_5 q\ e^{i\vec{p}.\vec{x}}$

- $\vec{p}_1^{(\prime)} + \vec{p}_2^{(\prime)} = \vec{P}$

- use:
  - “all” moving frames
  - all irreducible representations

$\vec{P}\frac{L}{2\pi}$	Λ
(0,0,0)	T_1^-
(0,0,1) + permutations	A_1^-, E^-
(0,1,1) + permutations	A_1^-, B_1^-, B_2^-
(1,1,1)	A_1^-, E^-

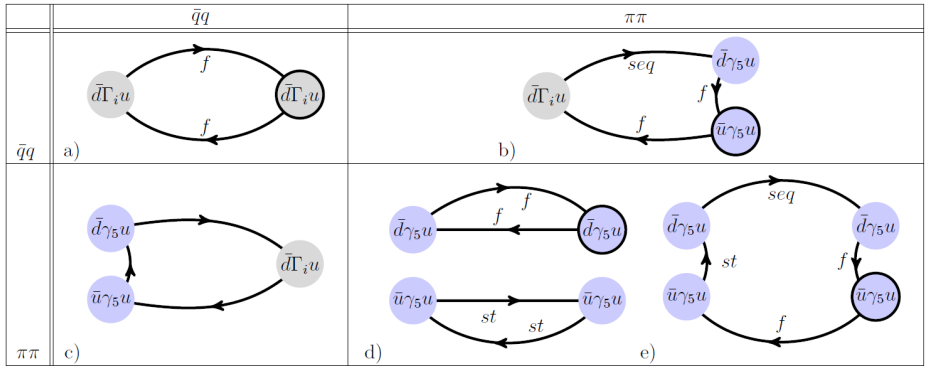
Strong $\pi\pi$ scattering

Spectrum

- Use the variational approach:

$$C_{ij}^{\Lambda,\vec{P}}(t)v_j^{n,\Lambda,\vec{P}} = \lambda_n^{\Lambda,\vec{P}}(t,t_0)C_{ij}^{\Lambda,\vec{P}}(t_0)v_j^{n,\Lambda,\vec{P}}$$

- Wick contractions

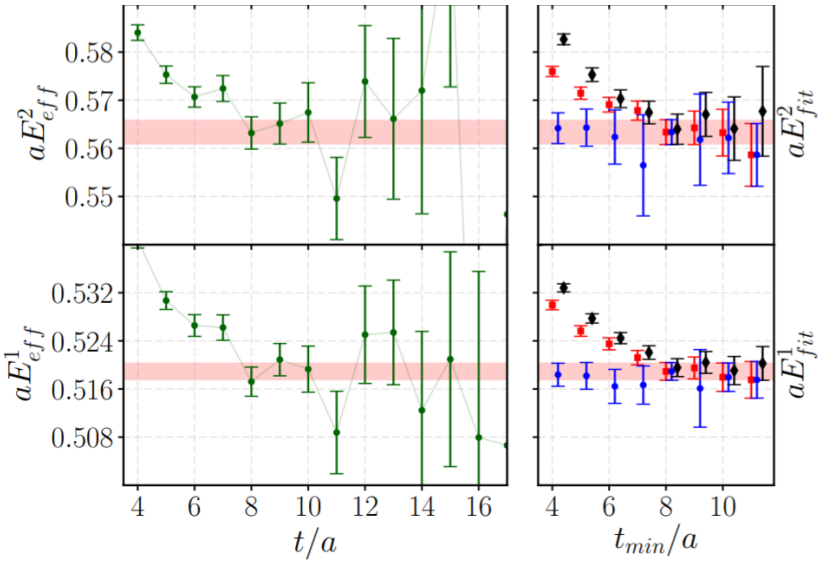


- Optimized interpolating operators:

$$\boldsymbol{o}_n^{\Lambda,\vec{P}} = v_j^{n,\Lambda,\vec{P}}\, O_j(\vec{P})$$

- $\left\langle \boldsymbol{o}_n^{\Lambda,\vec{P}}(t)\boldsymbol{o}_n^{\Lambda,\vec{P}}(0)\right\rangle \propto e^{-E_n^{\Lambda,\vec{P}}t}$

- $\sqrt{s_n^{\Lambda,\vec{P}}} = \sqrt{E_n^2 - \left(\vec{P}\right)^2}$



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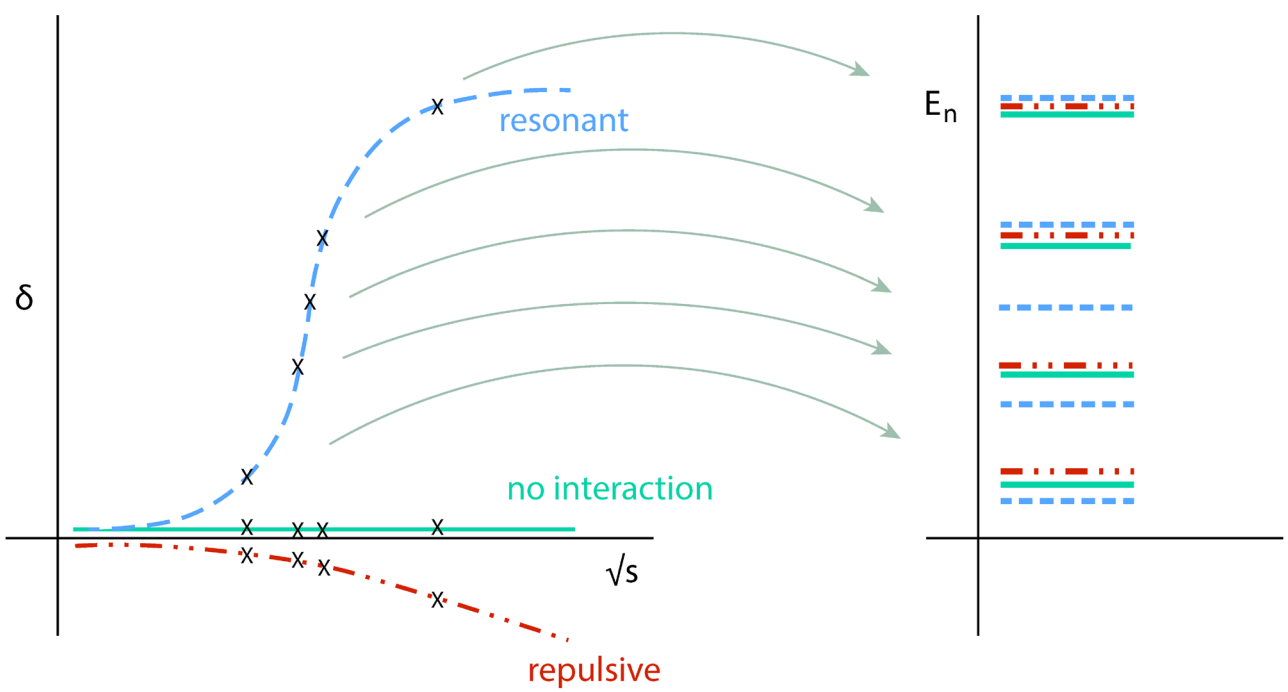
Mapping the spectrum to the strong phase shift

[Luescher, Rummukainen et al., Sharpe et al., Christ et al., Hansen et al., Gockeler et al., Briceno, Dudek et al.,…]

$$\det\left[\ \boxed{\cot\delta_l}\ -\ \boxed{M^{\Lambda,\vec{P}}\left(\sqrt{s}\right)}\ \right]=0$$

Infinite volume scattering phase

Finite volume spectrum



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The strong scattering phase shift

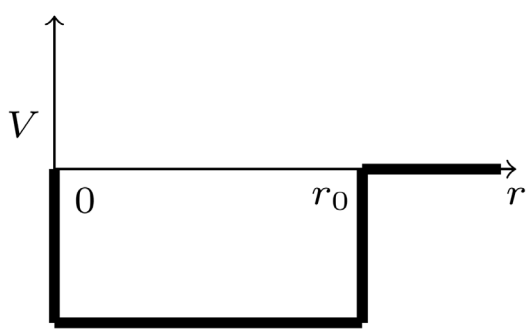
- Breit-Wigner (BW I):

$$\frac{1}{\sqrt{s}\,\Gamma}$$

$$\cot\delta_1 \propto \cot\delta_1(s_P) + (s - s_P)\left.\frac{d\cot\delta_1(s)}{d\,s}\right|_{s=s_P}$$

$$T_{\pi\pi\to\pi\pi}(s) = \frac{1}{\cot\delta - i} = \frac{\sqrt{s}\,\Gamma}{m_R^2 - s - i\sqrt{s}\,\Gamma}$$

- Modified B-W with barrier factor (BW II):

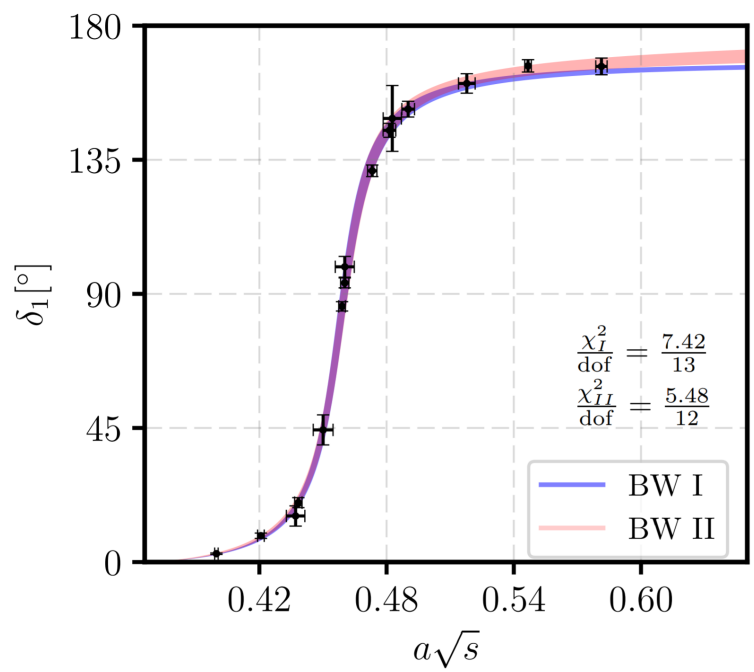


- BW I:

$$\Gamma_I = \frac{g_{\rho\pi\pi}^2}{6\,\pi}\,\frac{p^3}{s}$$

- BW II:

$$\Gamma_I = \frac{g_{\rho\pi\pi}^2}{6\,\pi}\,\frac{p^3}{s}\,\frac{1+(p_R r_0)^2}{1+(p r_0)^2}$$

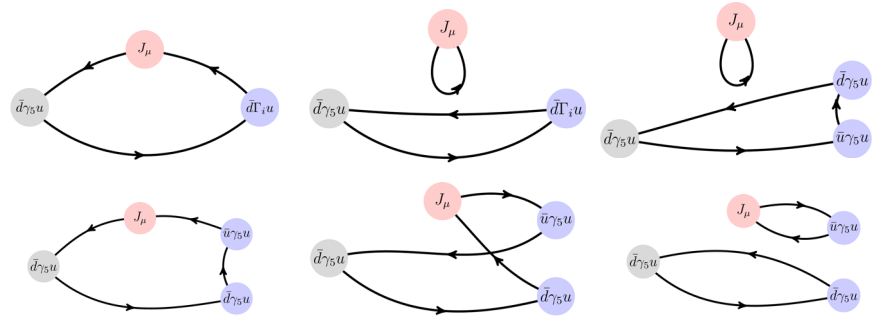


The $\pi\gamma \to \pi\pi$ transition

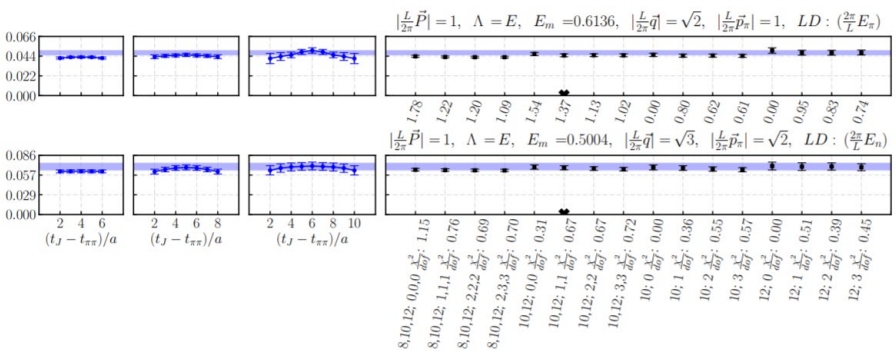
Matrix elements

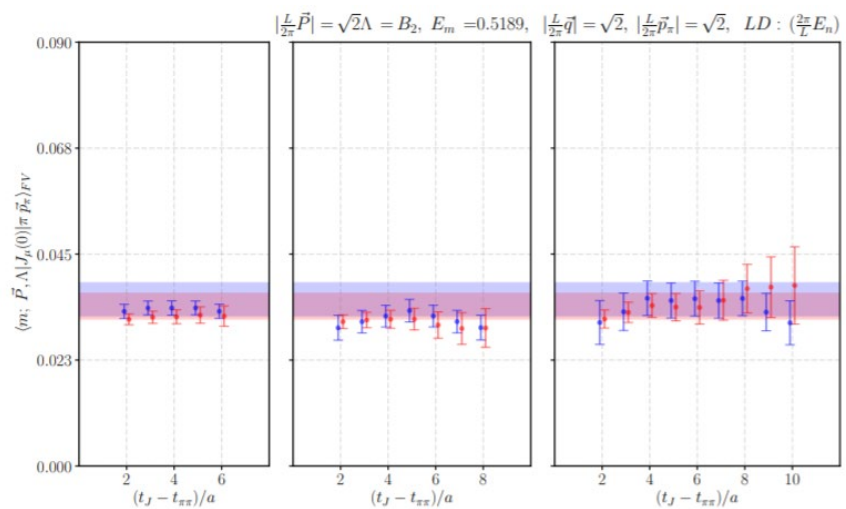
$$\Omega_{3,\mu,n}^{\vec{p}_\pi,\vec{P},\Lambda,r}\left(t_\pi, t_J, t_{\pi\pi}, t_0\right) = v_{\mathrm{i}}^{\mathrm{n},\vec{P},\Lambda}\, C_{3,\mu,i}^{\vec{p}_\pi,\vec{P},\Lambda,r}\left(t_\pi, t_J, t_{\pi\pi}\right)$$

[Dudek et al.,
Becirevic et al.,
Shultz et al.]



$$\left|\left\langle \pi, \vec{p}_\pi \middle| J_\mu(0,\vec{q}) \middle| n, \vec{P}, \Lambda, r \right\rangle_{FV}\right|^2 = 4\, E_n^{\vec{P},\Lambda}\, E_\pi^{\vec{p}_\pi}\, \frac{\Omega_{3,\mu,n}^{\vec{p}_\pi,\vec{P},\Lambda,r}\left(t_\pi,t_J,t_{\pi\pi},t_0\right)\Omega_{3,\mu,n}^{\vec{p}_\pi,\vec{P},\Lambda,r\,+}\left(t_\pi,t_{\pi\pi}+t_\pi-t_J,t_{\pi\pi},t_0\right)}{C_\pi^{\vec{p}_\pi}(\Delta t)\,\lambda_n^{\vec{P},\Lambda}(\Delta t,t_0)}$$





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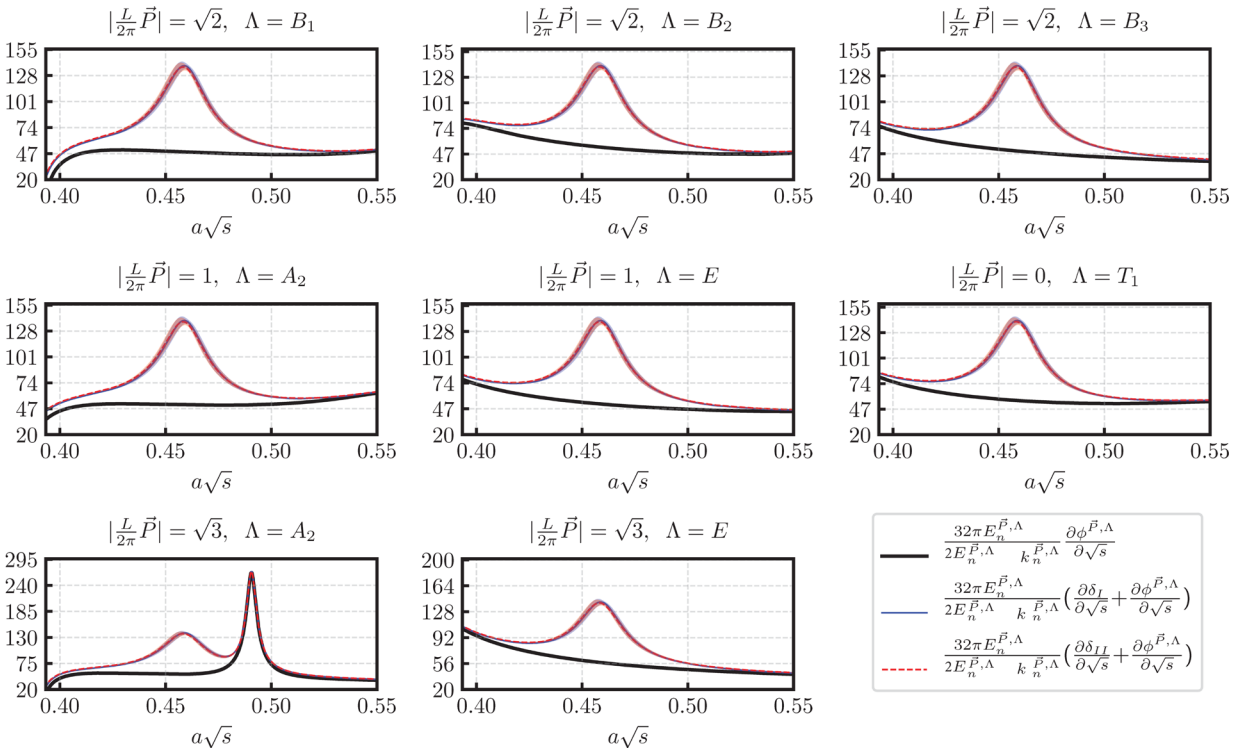
Mapping the matrix elements to the transition amplitude

$$\frac{\left|\left\langle \pi, \vec{p}_\pi \middle| J_\mu(0) \middle| s, q^2; \vec{P}, \Lambda, r \right\rangle_{IV}\right|^2}{\left|\left\langle \pi, \vec{p}_\pi \middle| J_\mu(0, \vec{q}) \middle| n, \vec{P}, \Lambda, r \right\rangle_{FV}\right|^2} = \frac{1}{2\, E_n^{\vec{P},\Lambda}} \frac{16\,\pi \sqrt{s_n^{\vec{P},\Lambda}}}{p_n^{\vec{P},\Lambda}} \left(\frac{\partial \delta}{\partial E} + \frac{\partial \phi^{\vec{P},\Lambda}}{\partial E} \right)\Bigg|_{E=E_n^{\vec{P},\Lambda}}$$

$$\frac{m_R^2 - s}{\sqrt{s}\,\Gamma}$$

$$M^{\Lambda,\vec{P}}$$

[Lellouch and Lüscher, Lin et al., Briceno et al., Briceno and Hansen, Meyer, Bernard et al., Christ et al., Agadjanov et al., Detmold et al.]



The infinite volume matrix element

multi-hadron transition matrix element:

$$\langle \pi\, \pi \,|\, J_\mu(0) \,|\, \pi \,\rangle = \frac{2\, i\, V_{\pi\gamma\to\pi\pi}(q^2, s)}{m_\pi}\, \varepsilon^{\nu\mu\alpha\beta}\, \epsilon_\nu(P, m)\, (p_\pi)_\alpha\; P_\beta$$

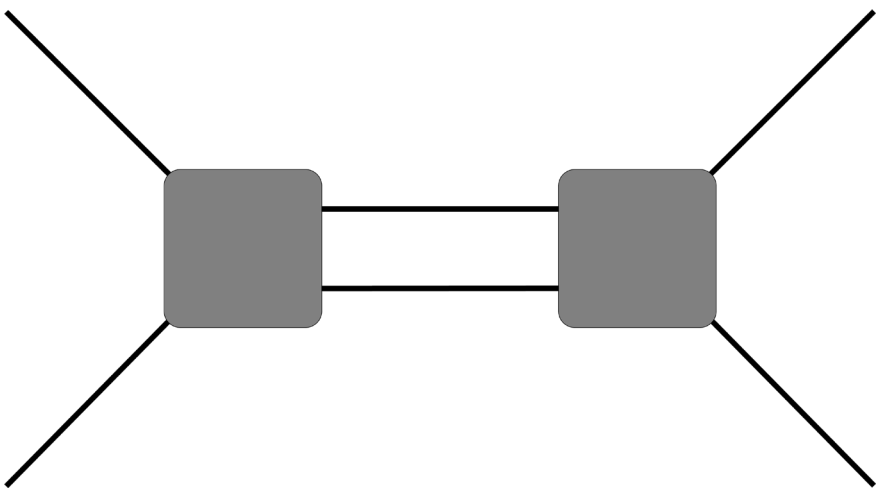
Momentum transfer: $q^2 = (p_\pi - P)^2$

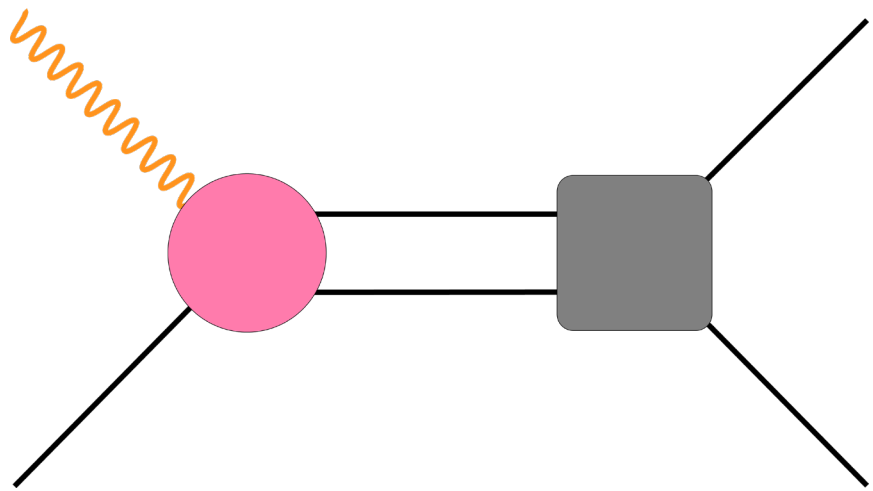
$\pi\pi \to \pi\pi$ scattering in vicinity of s_P:

$$T_{\pi\pi\to\pi\pi} = \frac{G^2_{\rho\pi\pi}}{s_P - s}$$

$\pi\gamma \to \pi\pi$ scattering in vicinity of s_P:

$$V_{\pi\gamma\to\pi\pi} = \frac{G_{\rho\pi\pi}\, G_{\rho\pi\gamma}}{s_P - s}$$





The infinite volume matrix element

Photoproduction amplitude (Watson):

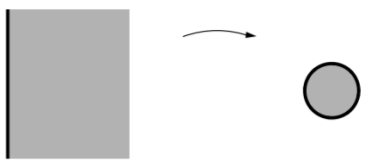
$$V_{\pi\gamma\to\pi\pi} = \sqrt{\frac{16\,\pi}{k\,\Gamma}}\,\frac{F(q^2,s)}{\cot\delta - i}$$

Expand F in :

[Boyd et al., Bourelly et al.]

- $\boldsymbol{S} = \frac{s-m_R^2}{m_R^2}$
- $z = \frac{\sqrt{t_+-q^2}-\sqrt{t_+-t_0}}{\sqrt{t_+-q^2}+\sqrt{t_+-t_0}}$

$t_+ = (2m_\pi)^2, t_0 = 0$



Factor out the pole, located at m_P, in q^2:

$$F(q^2,s) = \frac{\sum_{n,m} A_{nm}\, z^n\, \boldsymbol{S}^m}{1 - \frac{q^2}{m_P^2}}$$

Systematic truncation in sum:

- F1: $n + m \leq K$
- F2: $n \leq N,\ m \leq K - n$
- F3: $n \leq N,\ m \leq M$

parameterization	
BWI F1 K2	BWII F1 K2
BWI F1 K3	
BWI F2 N1 K2	BWII F2 N1 K2
BWI F2 N1 K3	BWII F2 N1 K3
BWI F3 N1 M1	
BWI F3 N1 M2	BWII F3 N1 M2
	BWII F3 N1 M3
BWI F3 N2 M2	
BWI F3 N2 M3	BWII F3 N2 M3

Keep only good parametrizations

- $\frac{\chi^2}{dof} \leq 1.1$
- σ_p is of sane value
  (i.e. we are not probing a flat direction)

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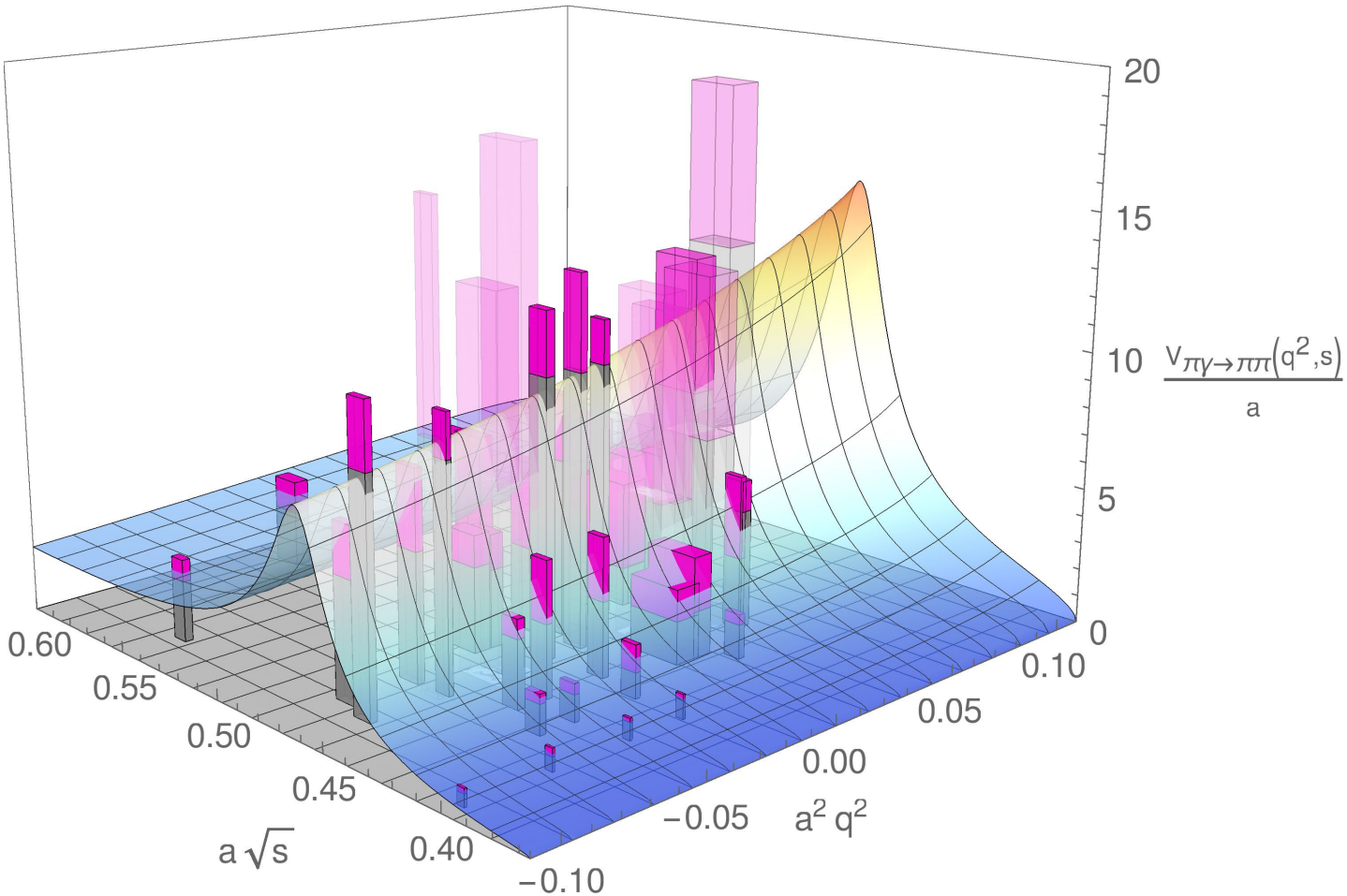
Nominal transition amplitude: BW II F1 K2



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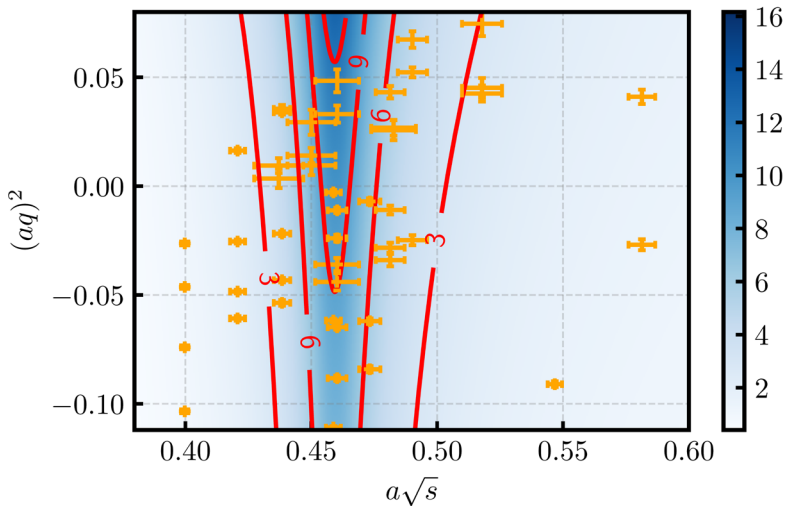
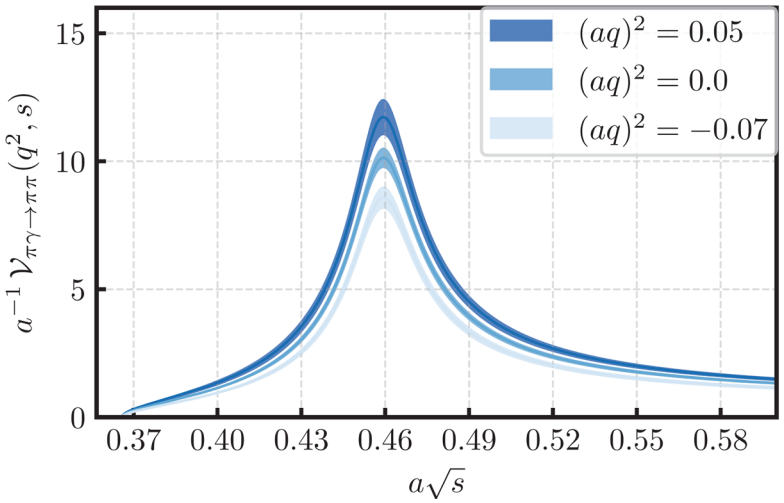
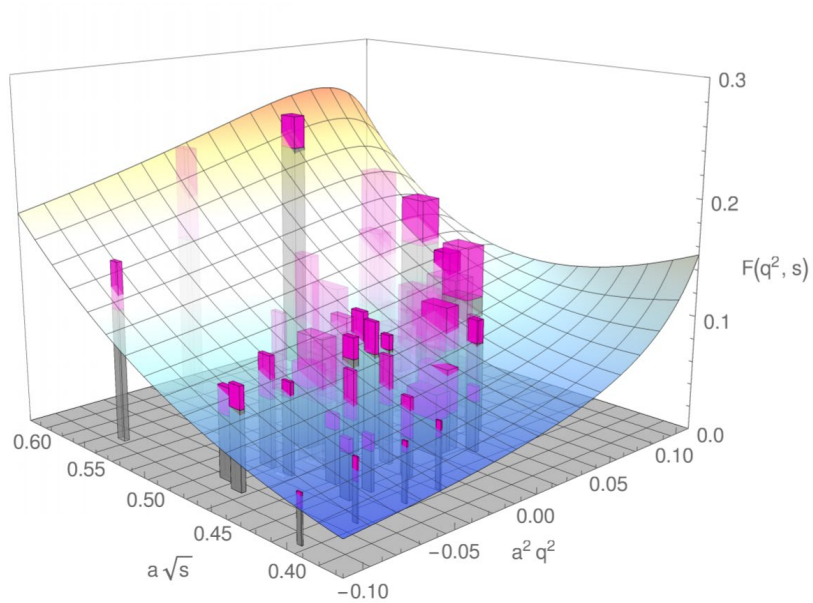
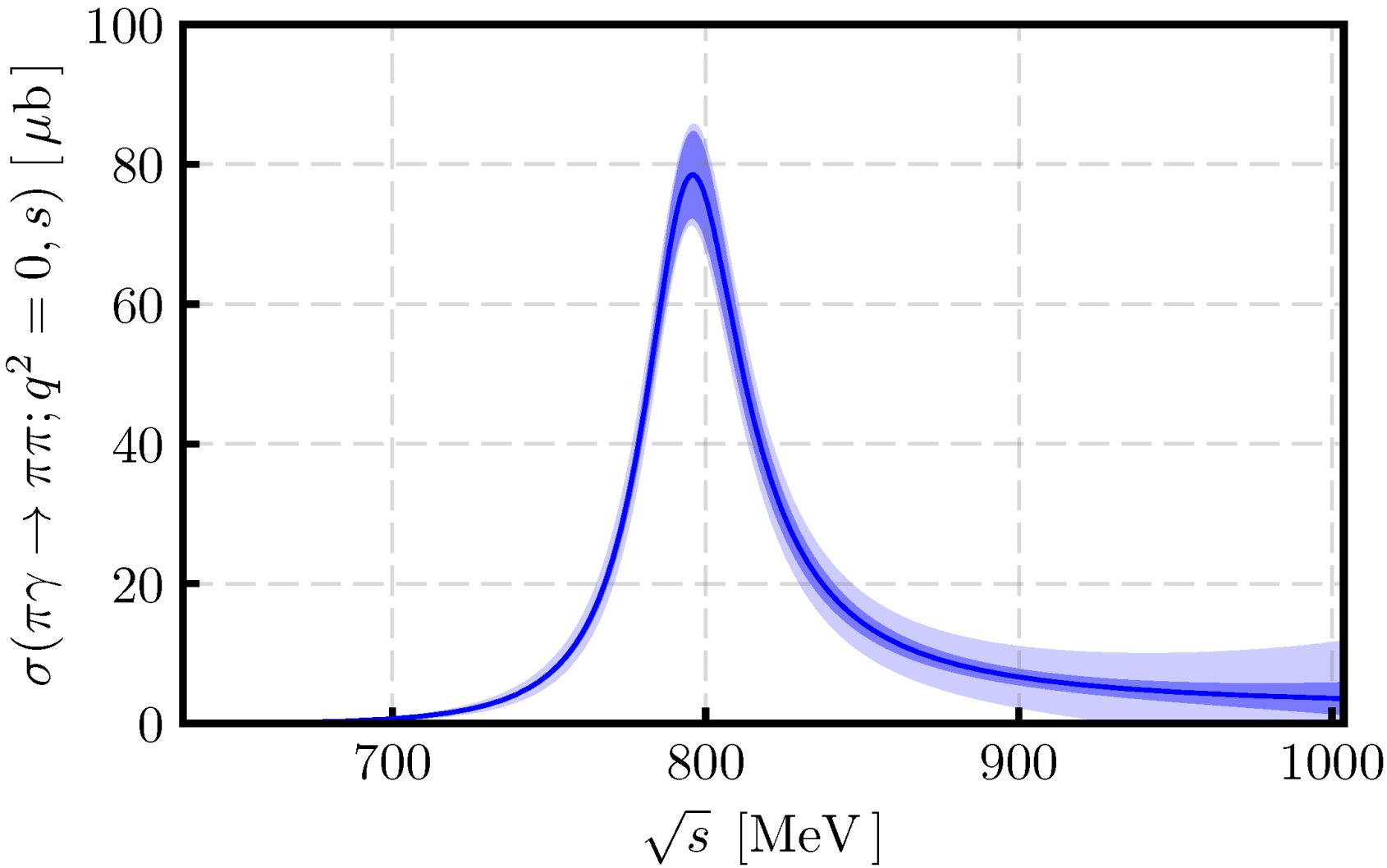


- Use 48 kinematic points
- Covered region
  - $a\sqrt{s} \in (0.39, 0.58)$
  - $(aq)^2 \in (-0.12, 0.07)$
  - $\sqrt{s} \in (675, 1003)$ MeV
  - $(q)^2 \in (-0.36, 0.21)$ GeV^2





The $\pi\gamma \to \pi\pi$ cross section at $m_\pi = 317$ MeV



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ρ photoproduction

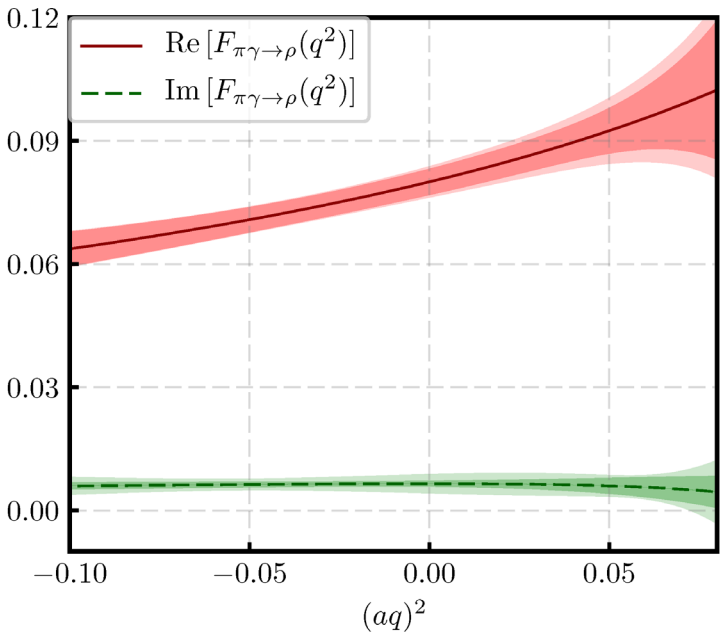
The resonance form factor and photocoupling

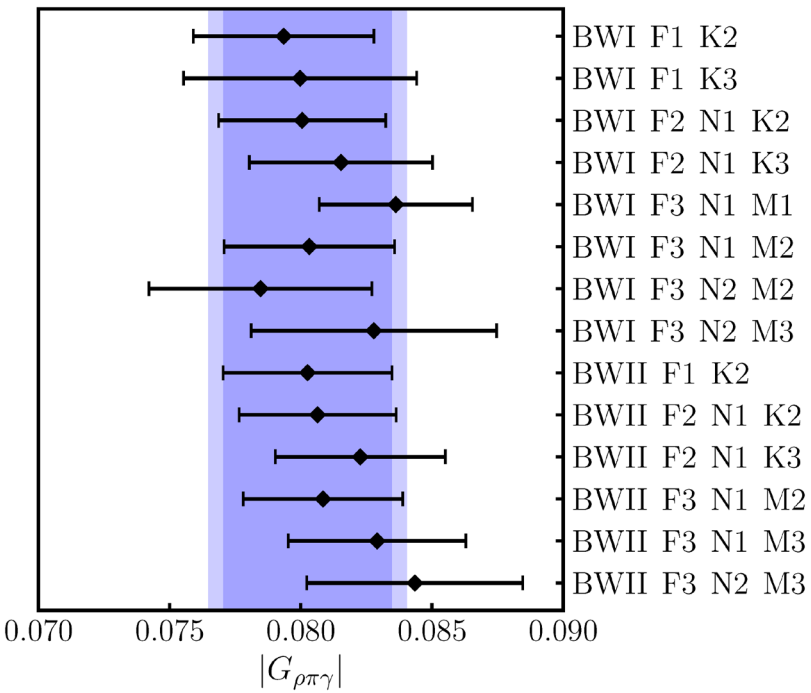
- Analytic continuation:

$$F_{\pi\gamma\to\rho}(q^2) = F(q^2, s = s_P)$$

- Photocoupling:

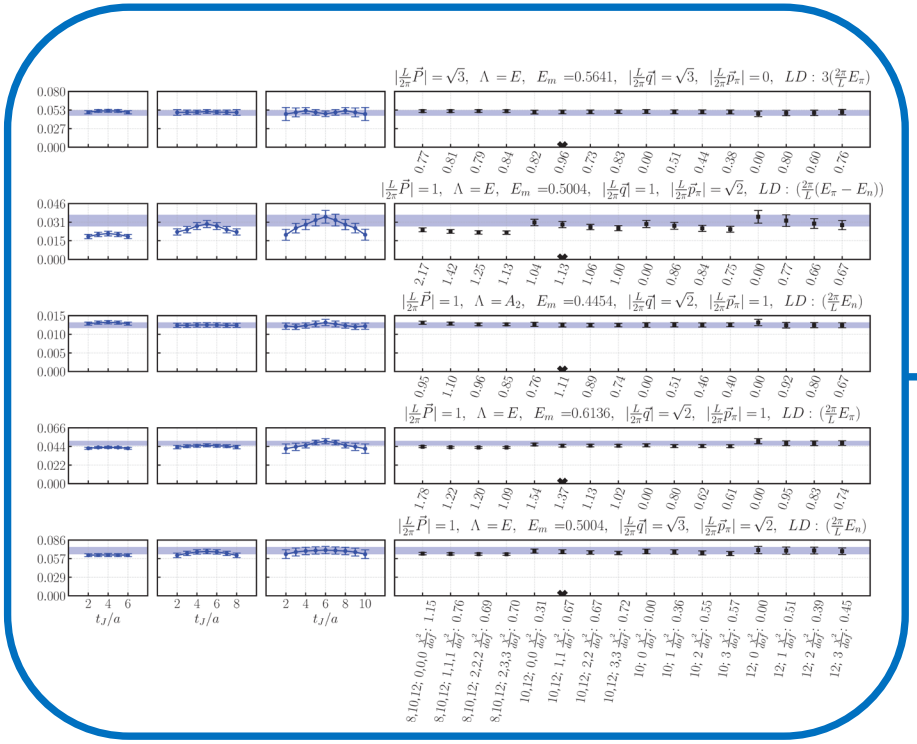
$$G_{\rho\pi\gamma} = F_{\pi\gamma\to\rho}(0)$$

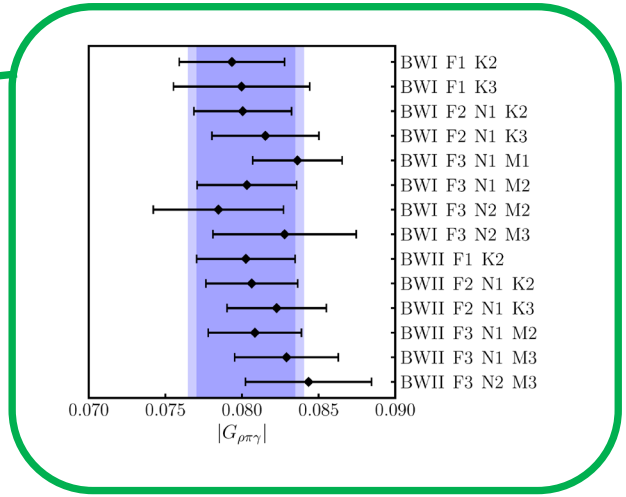


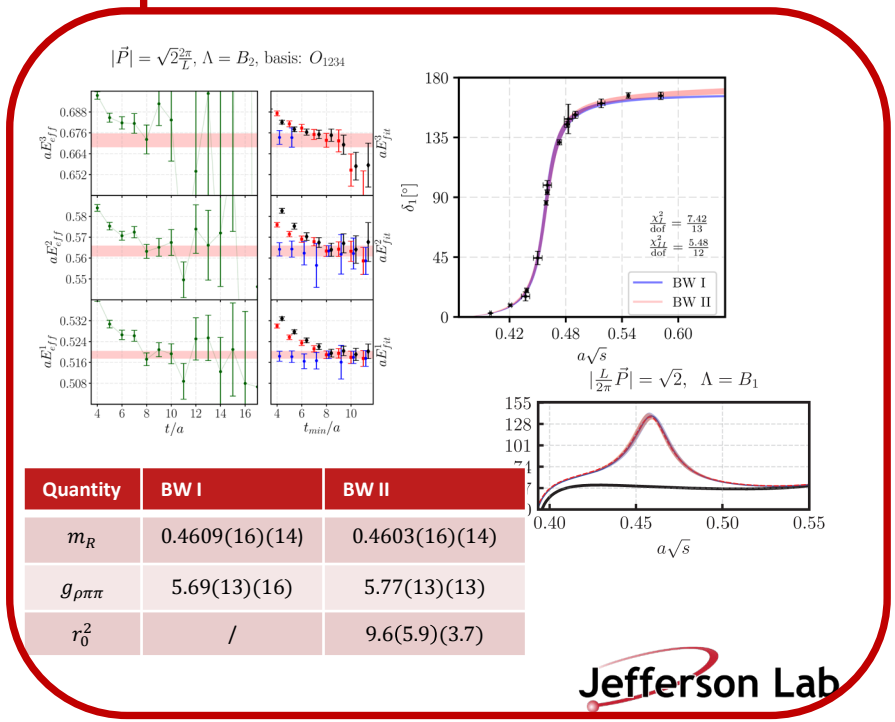


Photocoupling uncertainties

$G_{\rho\pi\gamma}$ = 0.802 (32) (20)







Quantity	BW I	BW II
m_R	0.4609(16)(14)	0.4603(16)(14)
$g_{\rho\pi\pi}$	5.69(13)(16)	5.77(13)(13)
r_0^2	/	9.6(5.9)(3.7)

Jefferson Lab

Observables

- Calculation at unphysical m_π
- Broken thresholds
- Assume couplings $g_{\rho\pi\pi}$ and $G_{\rho\pi\gamma}$ pion mass independent (*)
- Use physical thresholds

$$\Gamma(\rho \to \pi\pi) = \frac{g_{\rho\pi\pi}^2}{6\,\pi}\,\frac{p^3}{s}$$

$$\Gamma(\rho \to \pi\gamma) = \frac{2\,\alpha}{3}\left(\frac{m_\rho^2 - m_\pi^2}{2\,m_\rho}\right)^3 \frac{|G_{\rho\pi\gamma}|^2}{m_\pi^2}$$

Quantity	Lattice (these calc)	Experiment (PDG)
m_ρ	$797.6(3.7)(5.4)$ MeV	$766.5(1.1)$ MeV
$\Gamma(\rho \to \pi\pi)$	$133.6(6.1)(7.5)$ MeV	$150.2(2.4)$ MeV
$\Gamma(\rho \to \pi\gamma)$	$84.2\ (6.7)\ (4.3)$ keV	$68\ (7)$ keV

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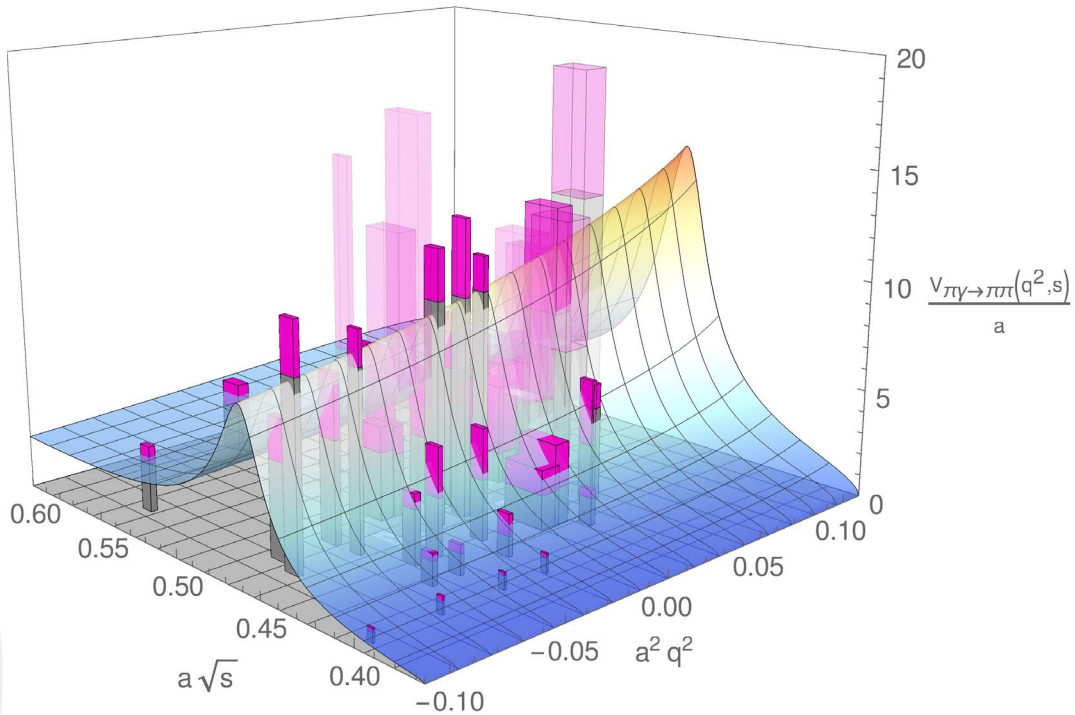
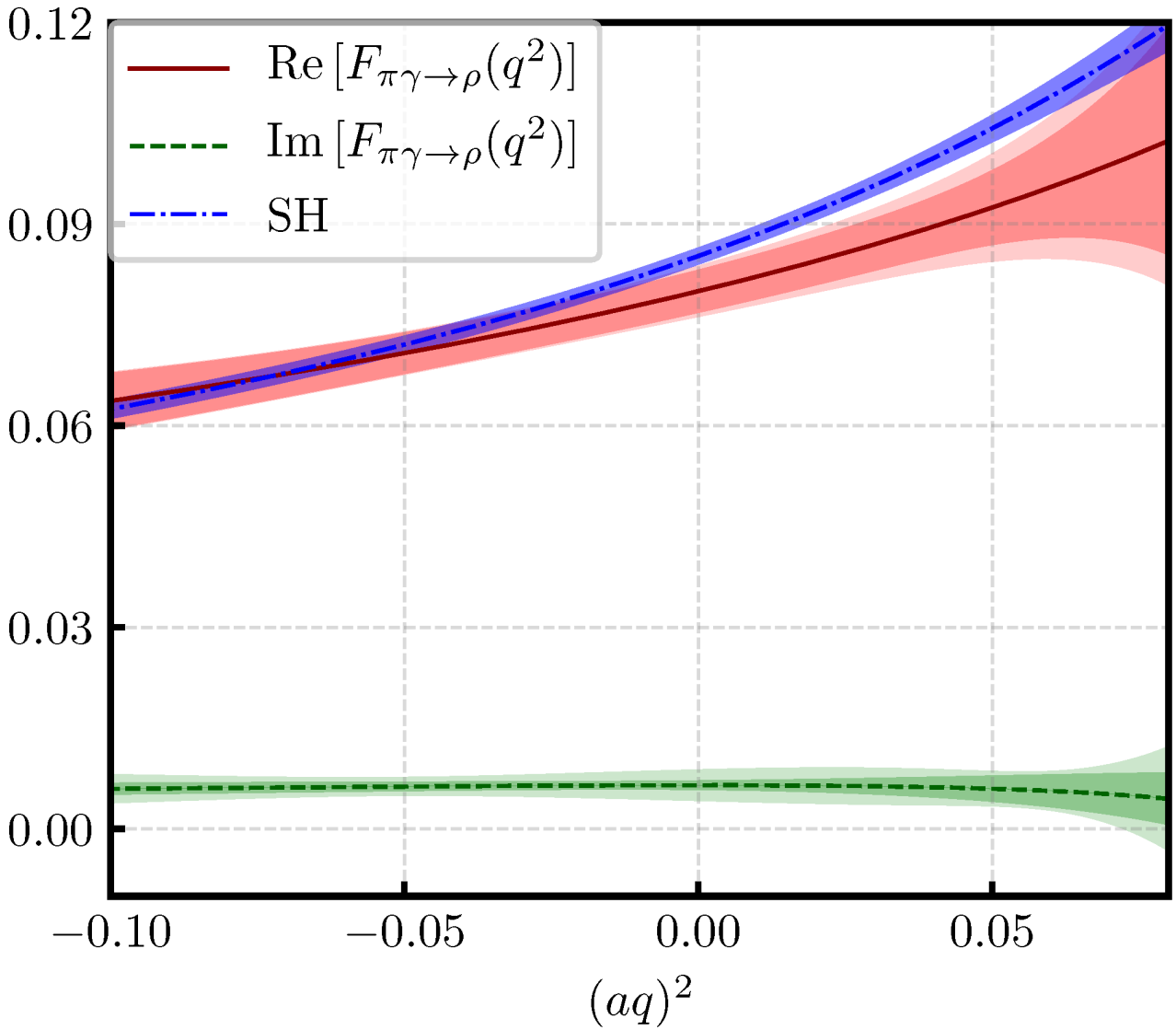
- Strong $\pi\pi$ scattering
- ρ total decay width
- ρ photoproduction
- Tremendous progress in last decade
- Interesting future ahead
  [see Briceno, Hansen, …]



BONUS: single-hadron vs multi-hadron