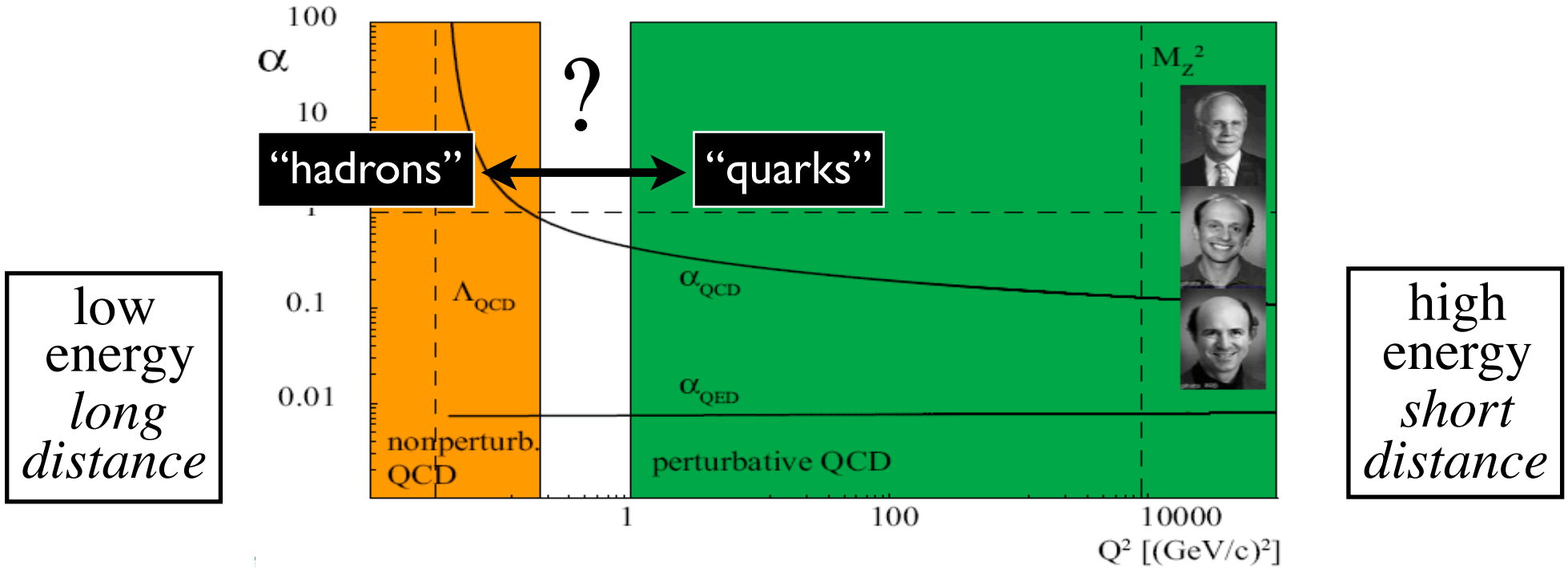
F2/EMC Collaboration Meeting
Jefferson Lab, May 2, 2019

Quark-hadron duality in electron scattering

Wally Melnitchouk

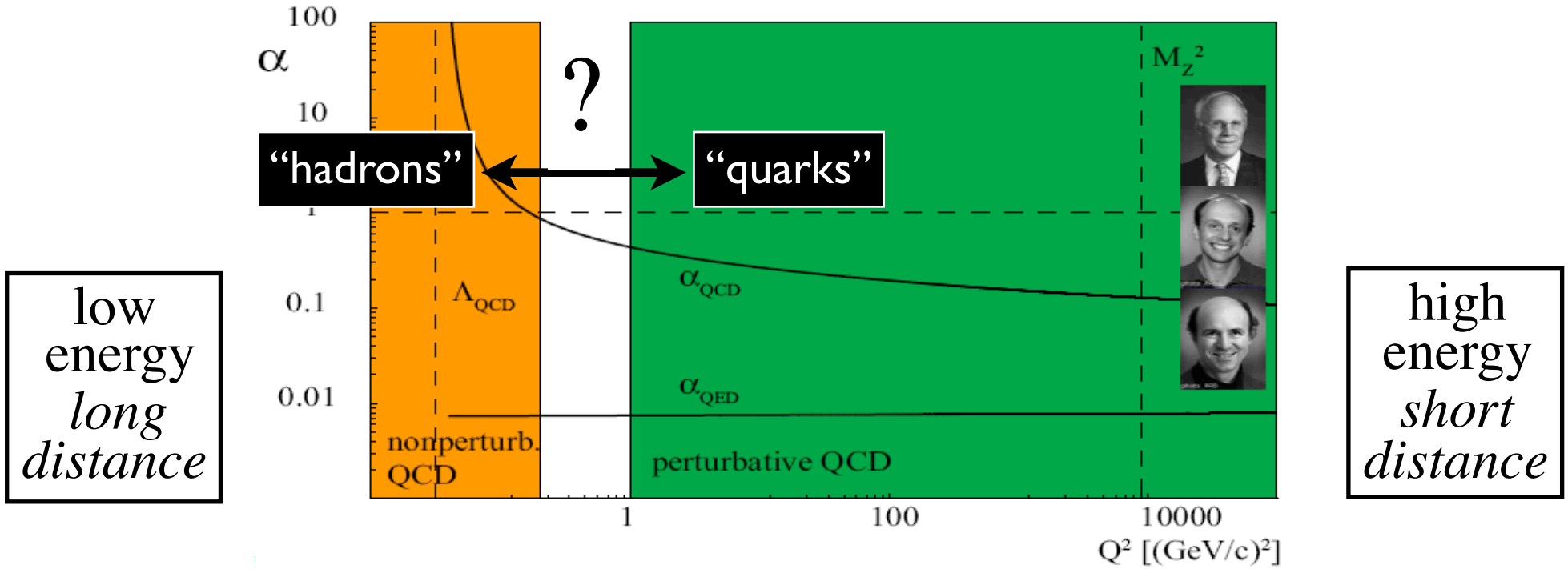
Jefferson Lab



- Duality hypothesis: complementarity between *quark* and *hadron* descriptions of observables

$$\sum_{hadrons} = \sum_{quarks}$$

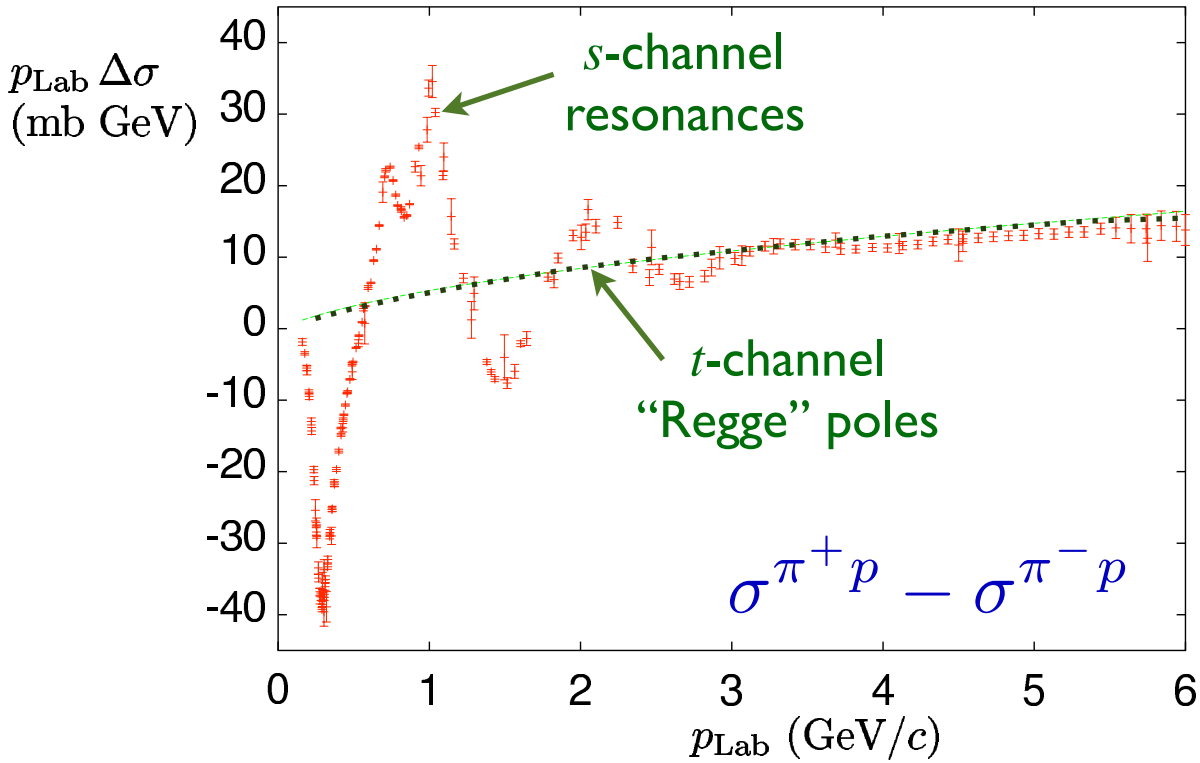
→ can use either set of *complete* basis states to describe physical phenomena



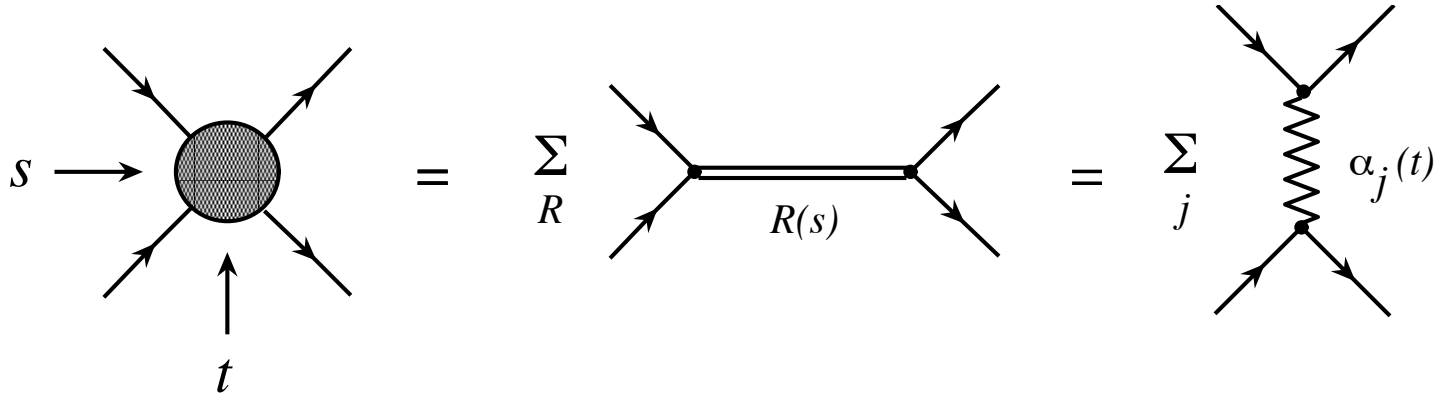
- In practice, at *finite energy* typically access only limited set of basis states

- Question is not "*why* duality exists", but
  — *how* it arises?
  — how can we make use of it (in a controlled way)?

Duality in hadron-hadron scattering



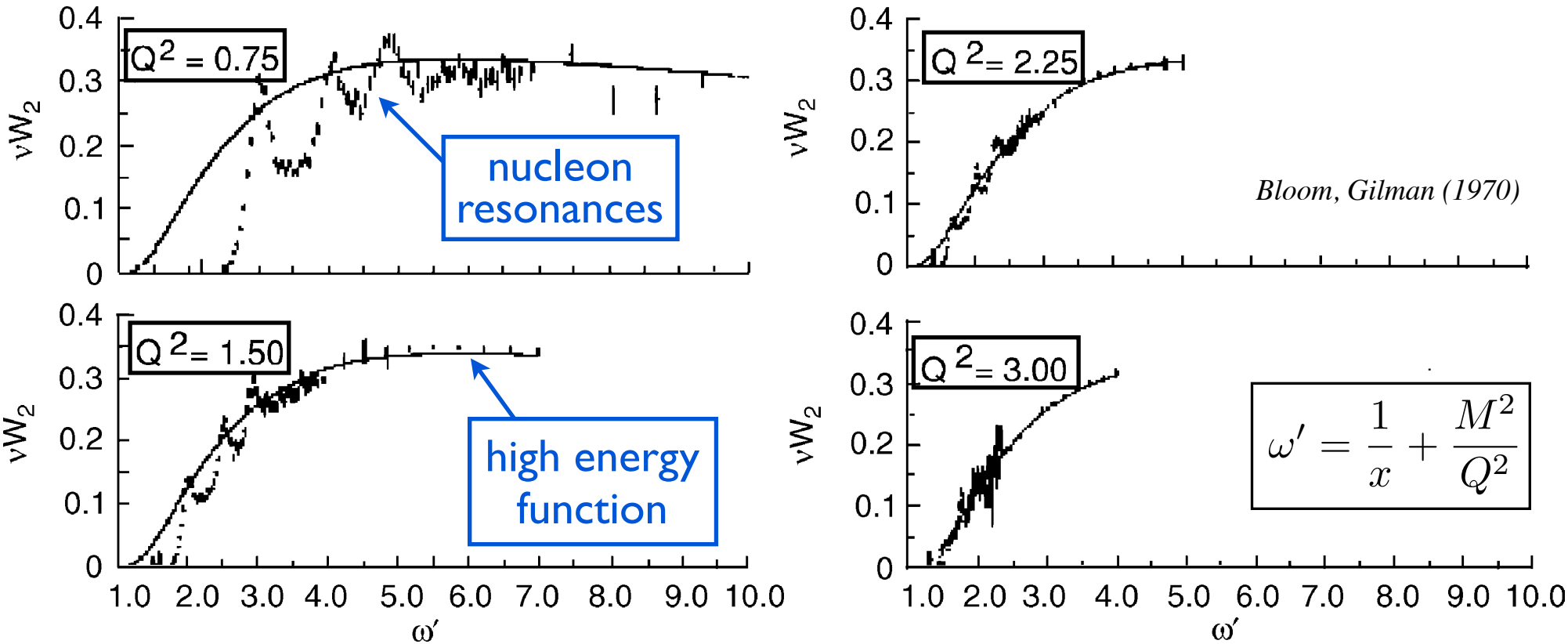
Igi (1962)
Dolen, Horn, Schmidt (1968)



$s - t$ channel duality

Duality in electron-proton scattering

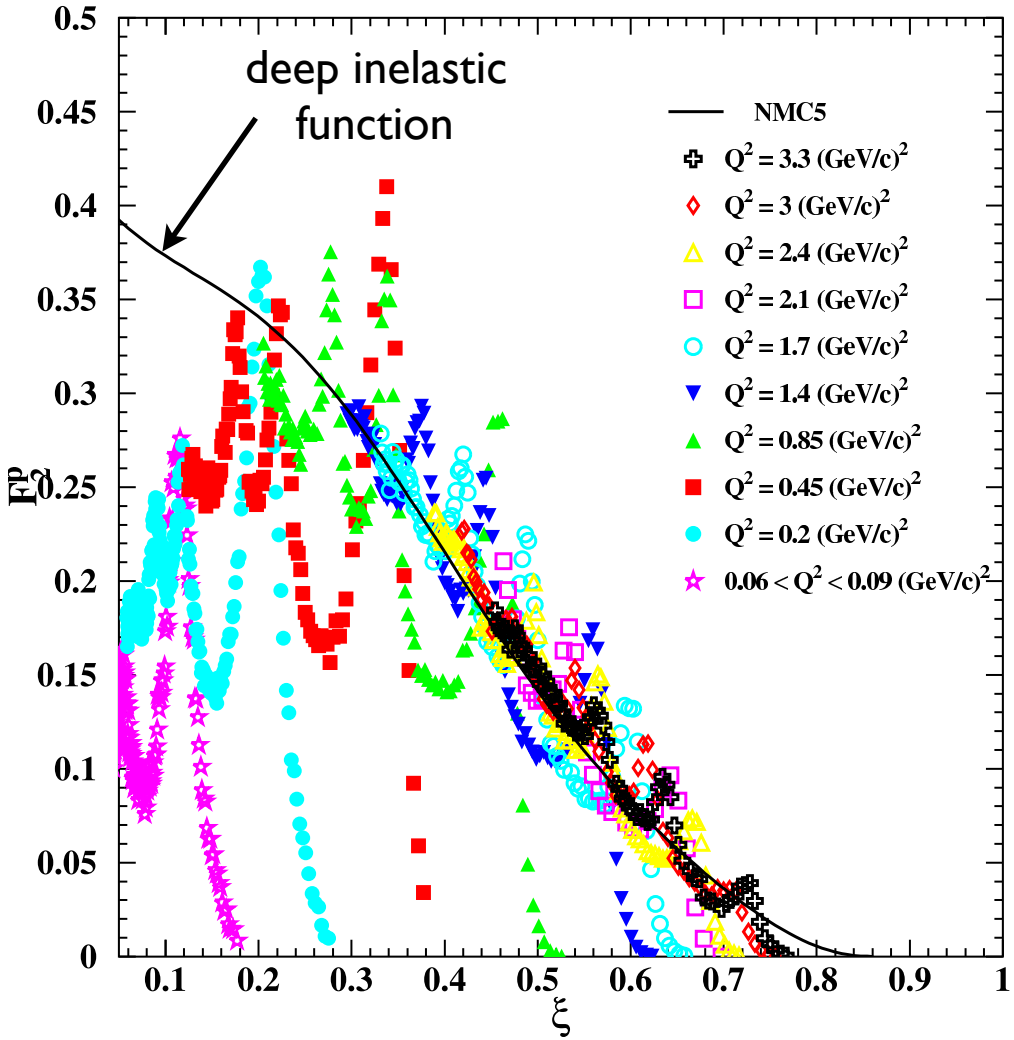
"Bloom-Gilman duality"



"hadrons" $$\frac{2M}{Q^2}\int_0^{\nu_m} d\nu\ \nu W_2(\nu, Q^2) = \int_1^{\omega'_m} d\omega'\ \nu W_2(\omega')$$ "quarks"

finite-energy sum rules

Duality in electron-proton scattering



Niculescu et al. (2000)

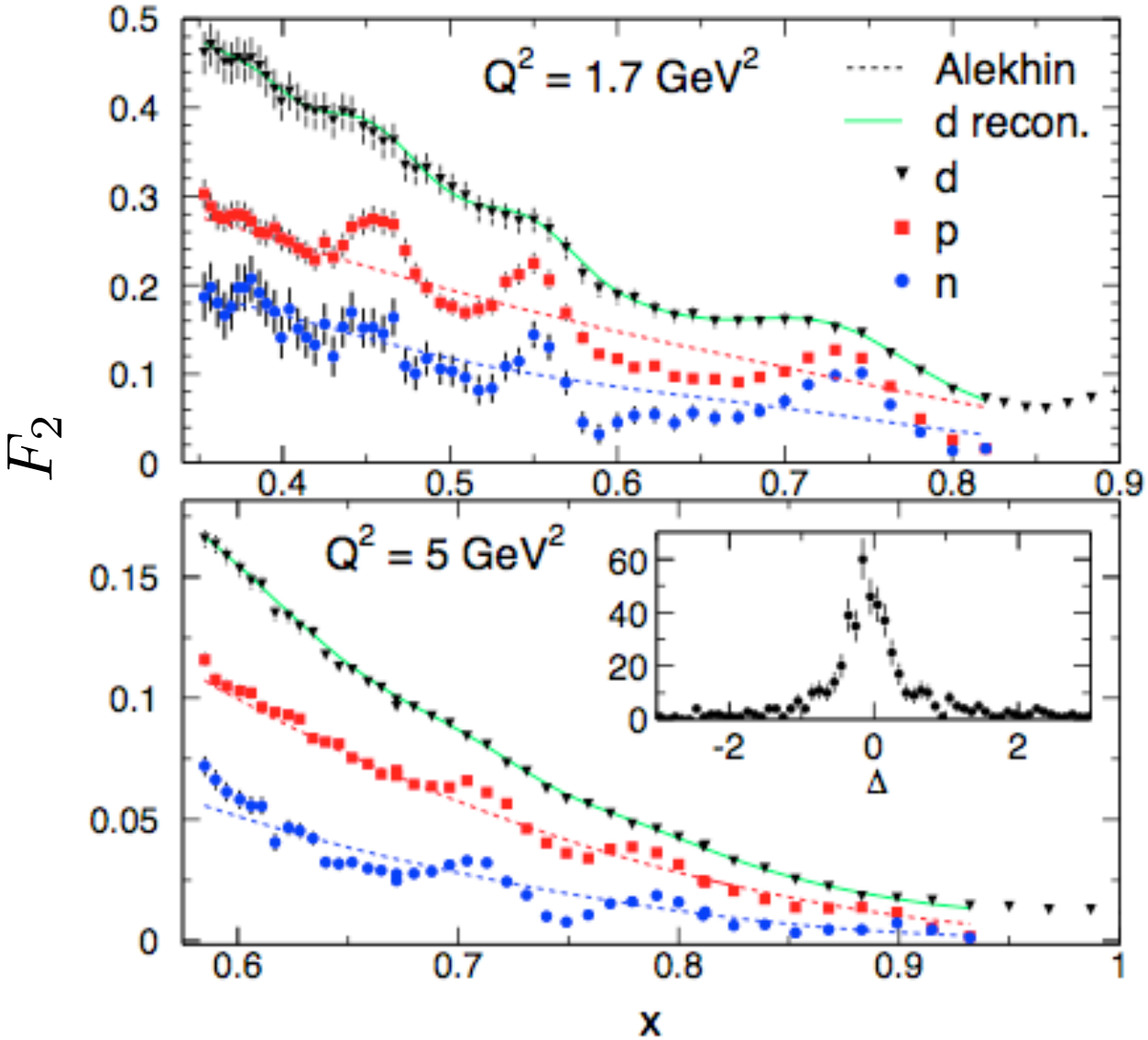
$$\xi = \frac{2x}{1 + \sqrt{1 + 4M^2x^2/Q^2}}$$

- average over resonances
  (strongly Q^2 dependent)

  $\approx$ Q^2 independent scaling function

Duality in electron-neutron scattering

No free neutron targets, but (new) iterative method allows neutron resonance structure function to be extracted



Malace, Kahn, WM, Keppel (2010)

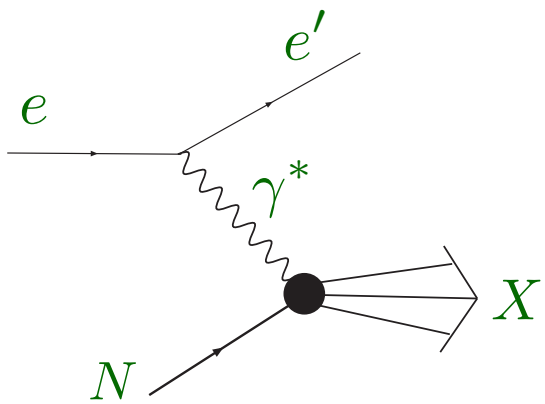
- evidence for duality also in neutron!

Duality in QCD

— global duality —

Duality and QCD

- Kinematics of inclusive deep-inelastic scattering (DIS)



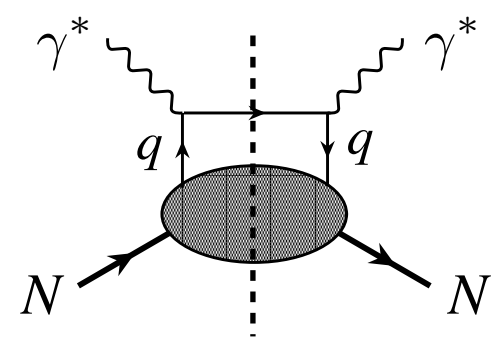
$$\frac{d^2\sigma}{d\Omega dE'} = \frac{4\alpha^2 E'^2 \cos^2\frac{\theta}{2}}{Q^4}\left(2\tan^2\frac{\theta}{2}\frac{F_1}{M} + \frac{F_2}{\nu}\right)$$

$$\nu = E - E'$$
$$Q^2 = \vec{q}^{\,2} - \nu^2$$
$$W^2 = M^2 + Q^2\frac{(1-x)}{x}$$

$$x = \frac{Q^2}{2M\nu}$$

- In *deep-inelastic* region $(W \gtrsim 2\ \mathrm{GeV},\ Q^2 \gtrsim 1\ \mathrm{GeV}^2)$ structure functions given by parton distributions

$$F_2(x,Q^2) \overset{\mathrm{LO}}{=} x\sum_q e_q^2\, q(x,Q^2)$$



Duality and QCD

- Operator product expansion in QCD

$\longrightarrow$ expand *moments* of structure functions in powers of $1/Q^2$

$$M_n(Q^2) = \int_0^1 dx\; x^{n-2}\; F_2(x, Q^2)$$

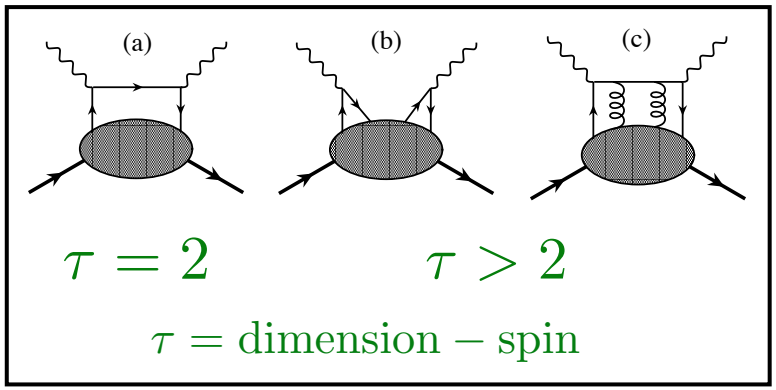
$$= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots$$

matrix elements of operators
with specific "twist" τ

e.g. $\langle N | \overline{\psi}\, \gamma^+ \psi\, | N \rangle$

$\langle N | \overline{\psi}\, \widetilde{G}^{+\nu} \gamma_\nu\, \psi\, | N \rangle$

etc.



Duality and QCD

- Operator product expansion in QCD

  $\longrightarrow$ expand *moments* of structure functions in powers of $1/Q^2$

$$M_n(Q^2) = \int_0^1 dx\; x^{n-2}\; F_2(x, Q^2)$$
$$= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots$$

- If moment $\approx$ independent of Q^2

  $\longrightarrow$ "higher twist" terms $A_n^{(\tau>2)}$ small

- Duality $\longleftrightarrow$ suppression of higher twists

Nonsinglet moments

- Nonsinglet nucleon structure function

$$(F_2^p - F_2^n)(x) = \frac{1}{3}\mathcal{H}^{\rm NS} \otimes \left[u + \bar{u} - d - \bar{d}\right](x) + \mathcal{O}(1/Q^2)$$

- Moments of nonsinglet distributions

$$M_n^{\rm NS}(Q^2) = \frac{1}{3}\mathcal{H}_n^{\rm NS}(Q^2)\ \langle x^n \rangle_{u-d}\ +\ \mathcal{O}(1/Q^2)$$

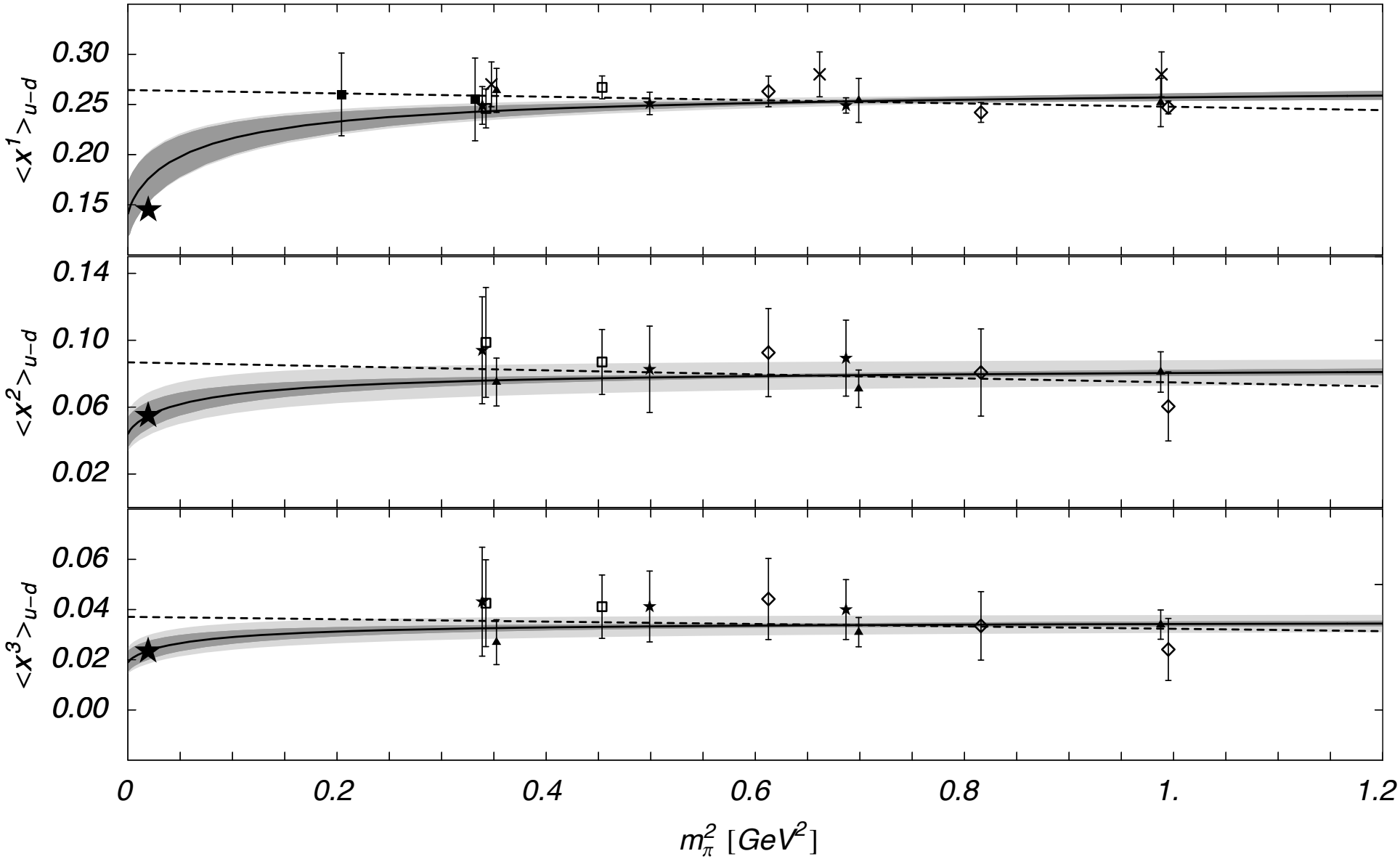
Gottfried sum rule for n=1

where

$$\langle x^n \rangle_q = \int_0^1 dx\, x^n\, [q - (-1)^n \bar{q}](x)$$

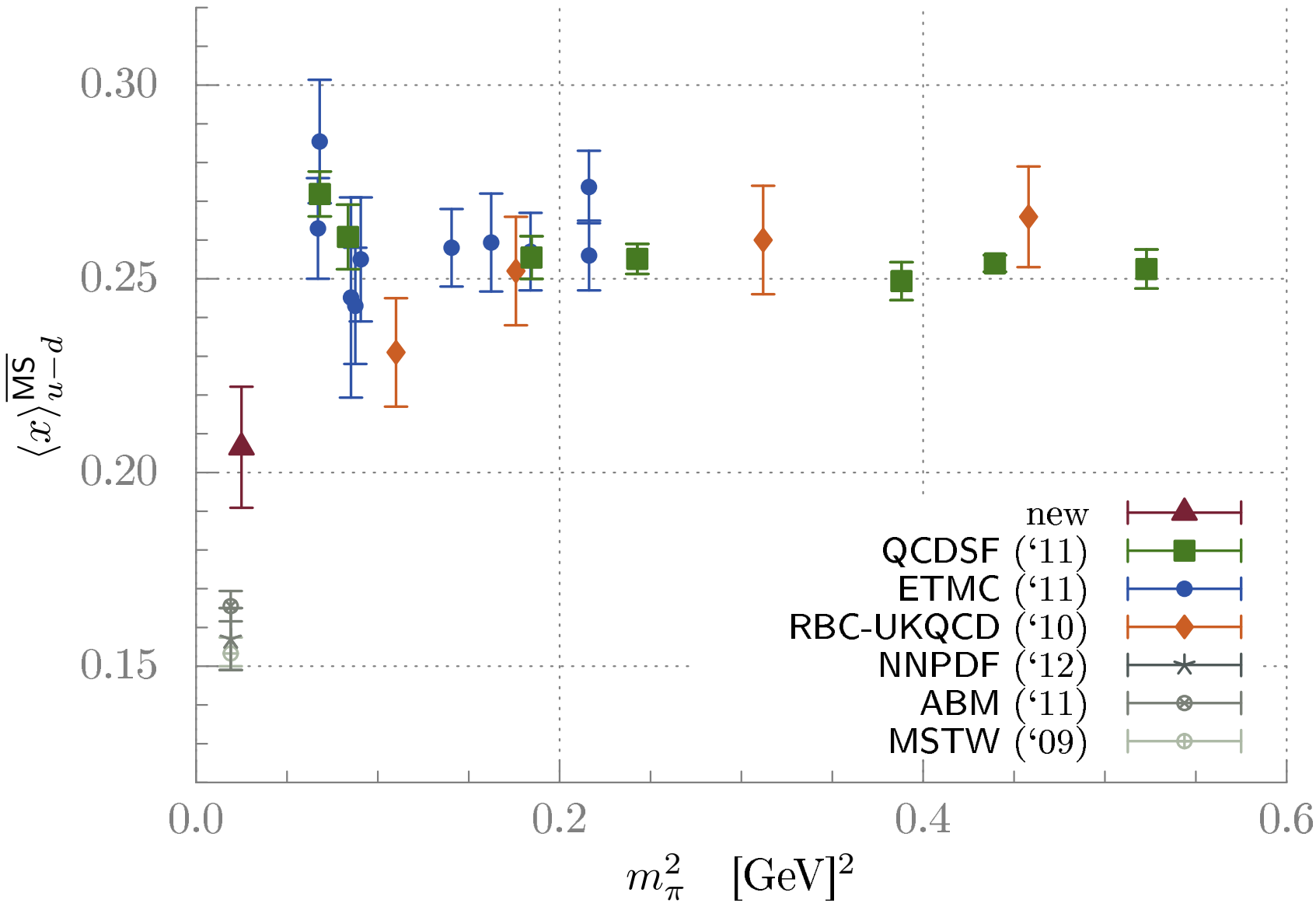
leading twist PDF
(from global analysis, lattice, …)

Nonsinglet moments



Detmold, WM, Thomas, Mod. Phys. Lett. A18 (2003) 2681

Nonsinglet moments



Bali et al., Phys. Rev. D86 (2012) 054504

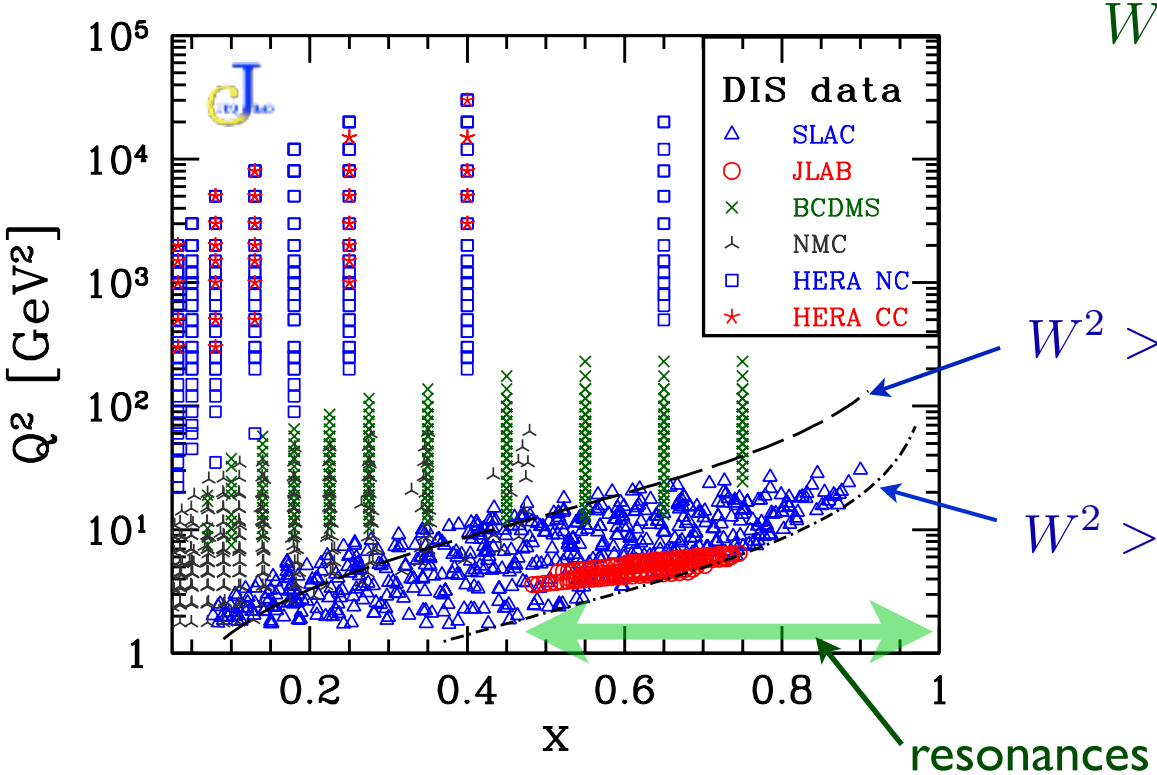
Duality and QCD

- Note: at finite Q^2, from kinematics *any* moment of *any* structure function (of *any* twist) must, by definition, include the resonance region

$$W^2 = M^2 + Q^2 \frac{(1-x)}{x} \longrightarrow x_{\rm res} = \frac{Q^2}{W_{\rm res}^2 - M^2 + Q^2}$$

$$W_{\rm res} = 2 \text{ GeV} \implies x_{\rm res} \approx 0.24$$

$$\text{at } Q^2 = 1 \text{ GeV}^2$$



Duality and QCD

- Note: at finite Q^2, from kinematics *any* moment of *any* structure function (of *any* twist) must, by definition, include the resonance region

- Resonance and DIS regions are intimately connected
  $\longrightarrow$ resonances an *integral* part of scaling structure function

  e.g. in large-N_c limit, spectrum of zero-width resonances is "maximally dual" to quark-level (smooth) structure function

Local Duality

— truncated moments —

Truncated moments

- Complete moments can be studied via twist expansion

  $\longrightarrow$ Bloom–Gilman duality has a precise meaning

  ($i.e.$, duality violation = higher twists)

- Rigorous connection between local duality & QCD difficult

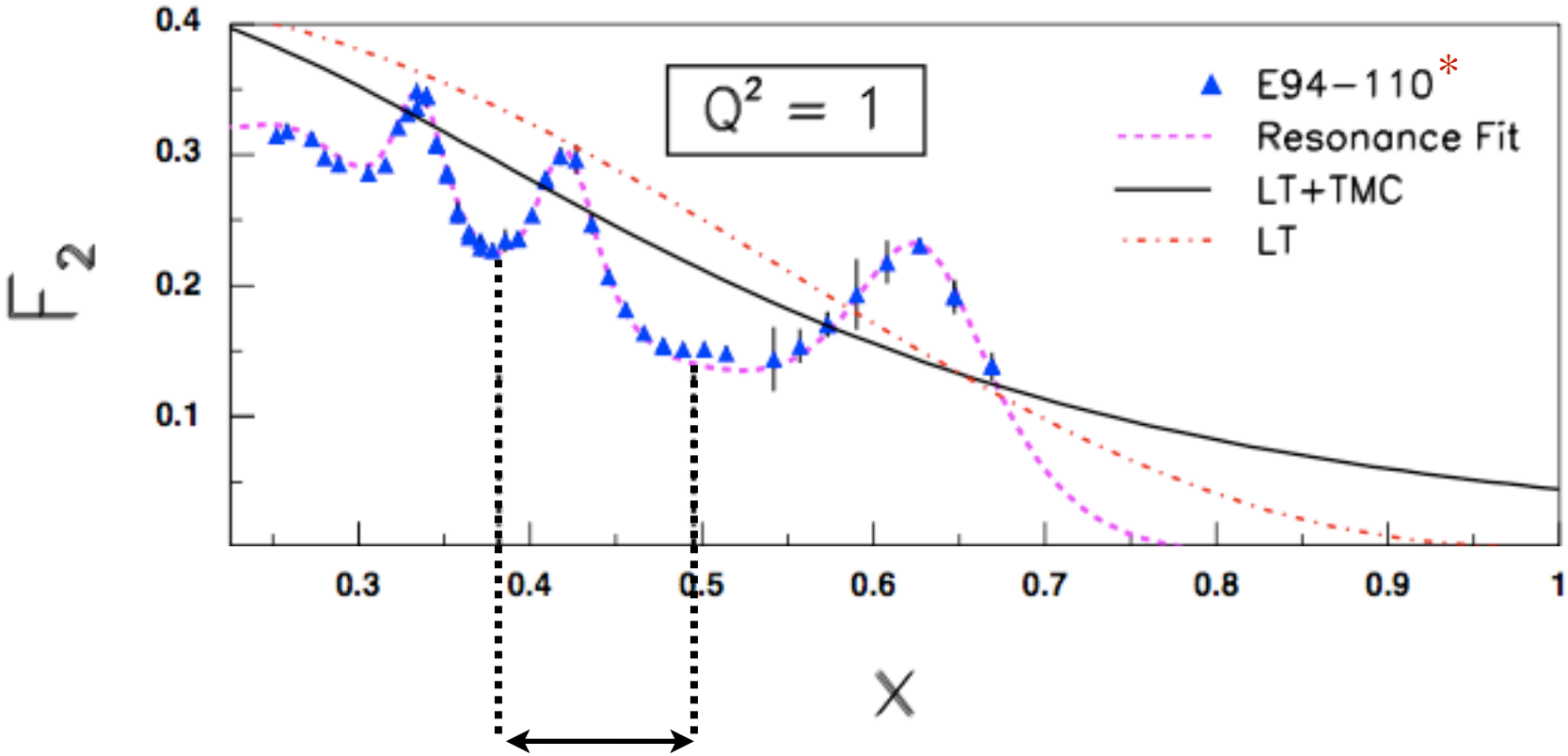
  $\longrightarrow$ need prescription for how to average over resonances

- *Truncated* moments allow study of restricted regions in x (or W) within pQCD in well-defined, systematic way

$$\overline{M}_n(\Delta x, Q^2) = \int_{\Delta x} dx \; x^{n-2} \; F_2(x, Q^2)$$

Forte, Magnea (1999)
Psaker, Malace, Keppel, WM (2008)

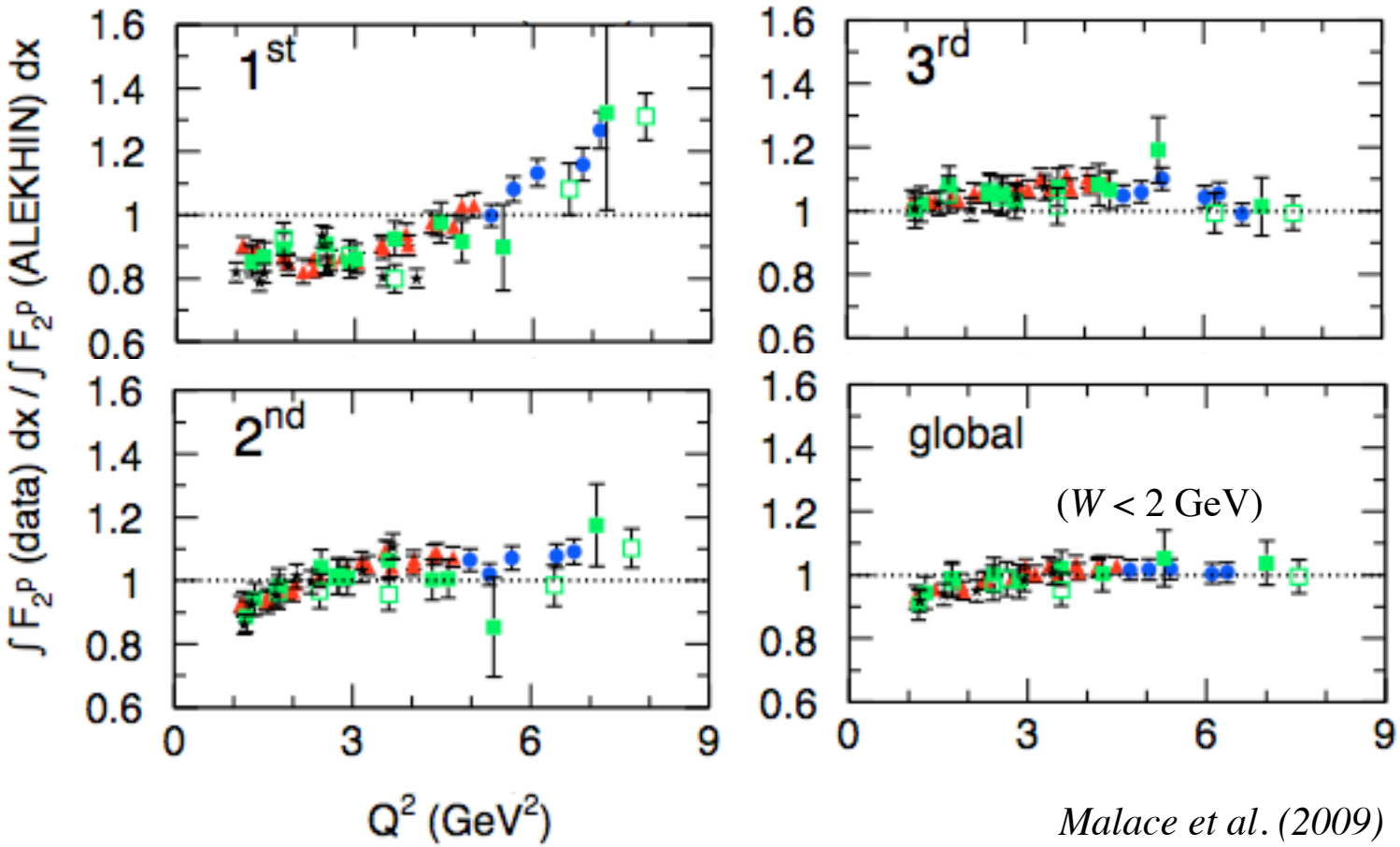
Truncated moments



* JLab Hall C

how much of this region is *leading twist* ?

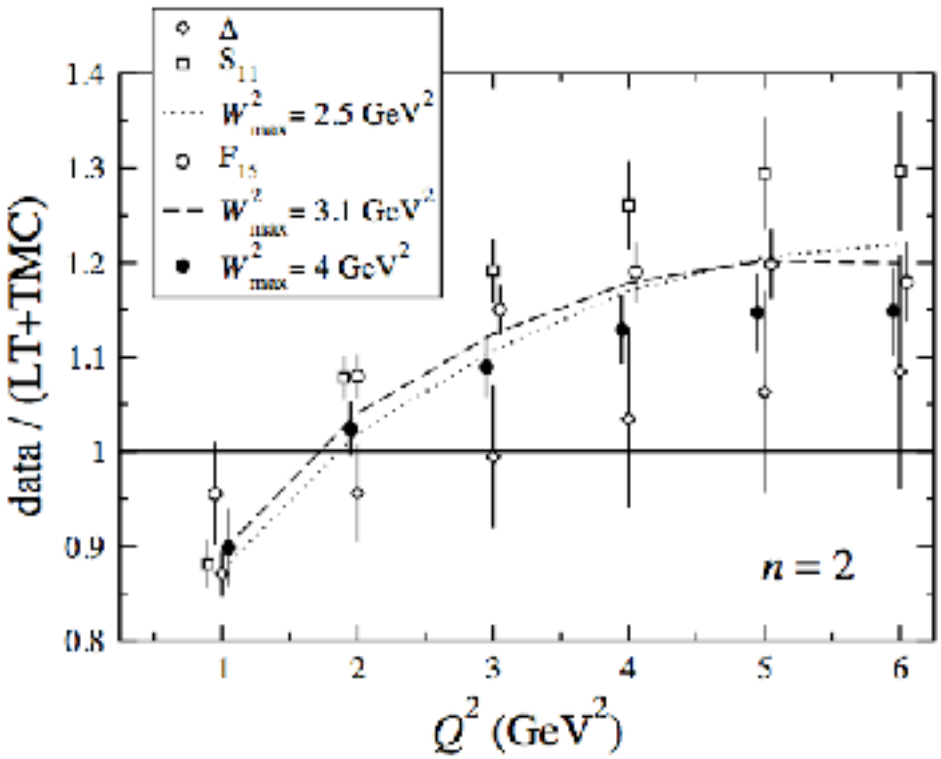
Truncated moments

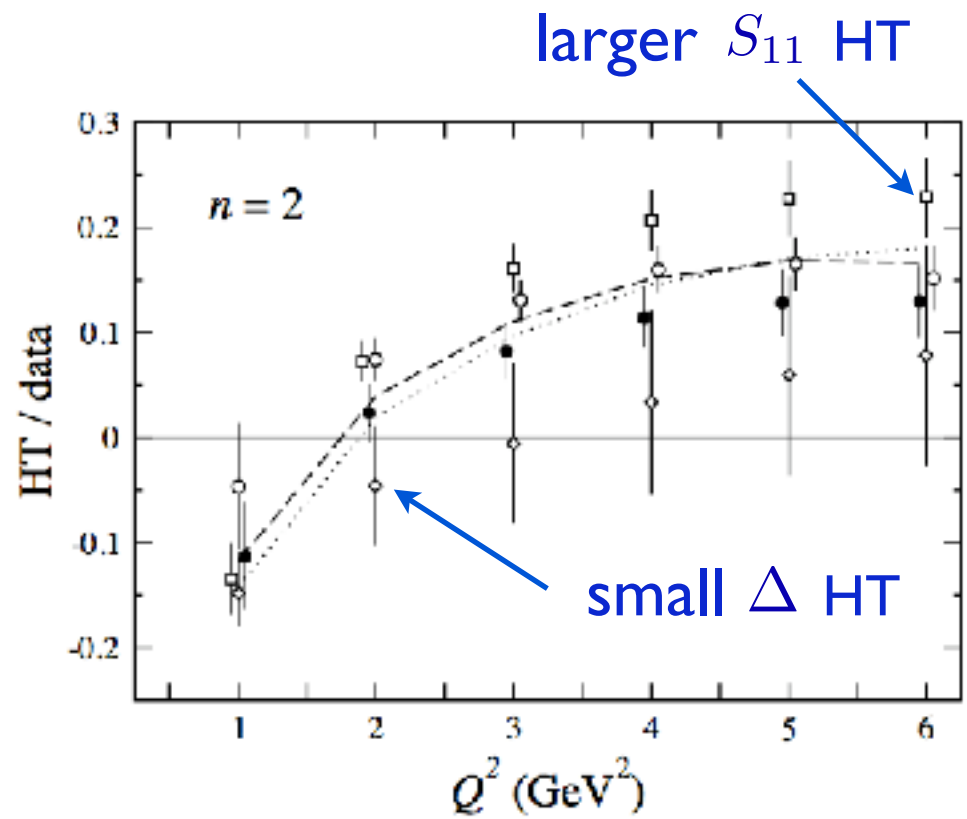


Malace et al. (2009)

→ duality appears in various resonance regions

Truncated moments





$\longrightarrow$ higher twists $< 10\text{–}15\%$ for $Q^2 > 1\ \mathrm{GeV}^2$

Resonances & twists

- Total "higher twist" is *small* at scales $Q^2 \sim \mathcal{O}(1\ \mathrm{GeV}^2)$

- On average, nonperturbative interactions between quarks and gluons not dominant (at these scales)

  $\longrightarrow$ nontrivial interference between resonances?

- Can we understand this dynamically, at quark level?

- Can we use resonance region data to learn about *leading twist* structure functions (and *vice versa*)?

  $\longrightarrow$ expanded data set has potentially significant implications for global quark distribution studies

Applications of Duality

CTEQ-JLab (CJ) global PDF analysis

- Global QCD analysis of high-energy scattering data, including large-x, low-Q^2 region

- Systematically study effects of Q^2 & W cuts

cut0: $Q^2 > 4\ \mathrm{GeV}^2, \quad W^2 > 12.25\ \mathrm{GeV}^2$

cut1: $Q^2 > 3\ \mathrm{GeV}^2, \quad W^2 > 8\ \mathrm{GeV}^2$

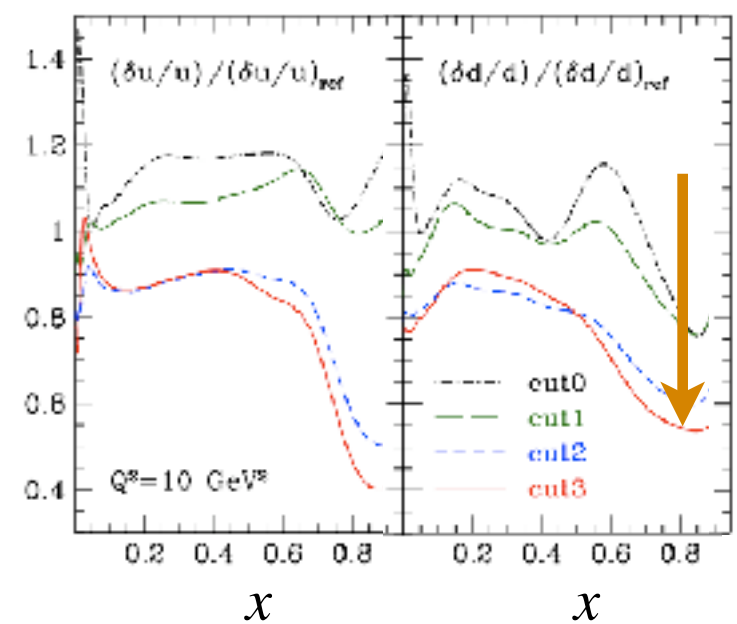
cut2: $Q^2 > 2\ \mathrm{GeV}^2, \quad W^2 > 4\ \mathrm{GeV}^2$

cut3: $Q^2 > m_c^2\ , \quad W^2 > 3\ \mathrm{GeV}^2$

factor 2 increase in DIS data from cut0 → cut3

→ larger database with weaker cuts significantly reduced errors, especially at large x

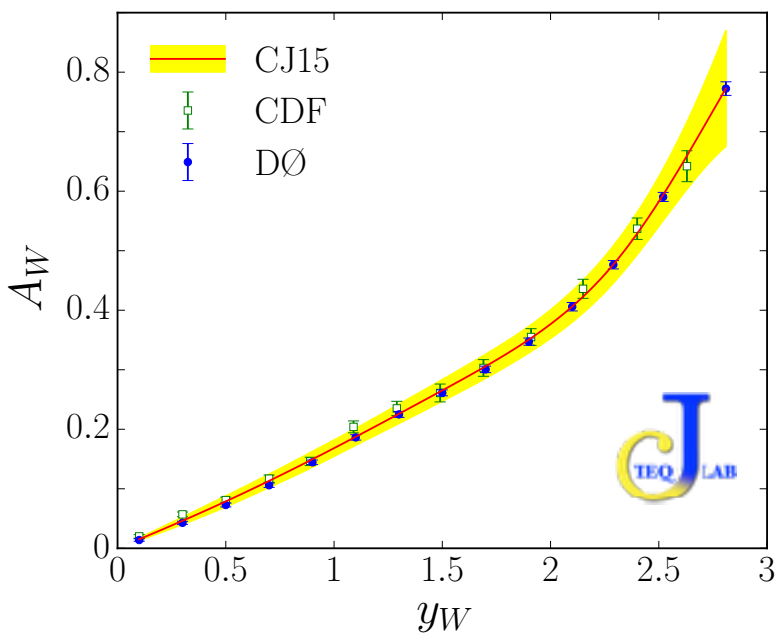
→ up to ~ 40–60% error reduction when cuts extended into near-resonance region



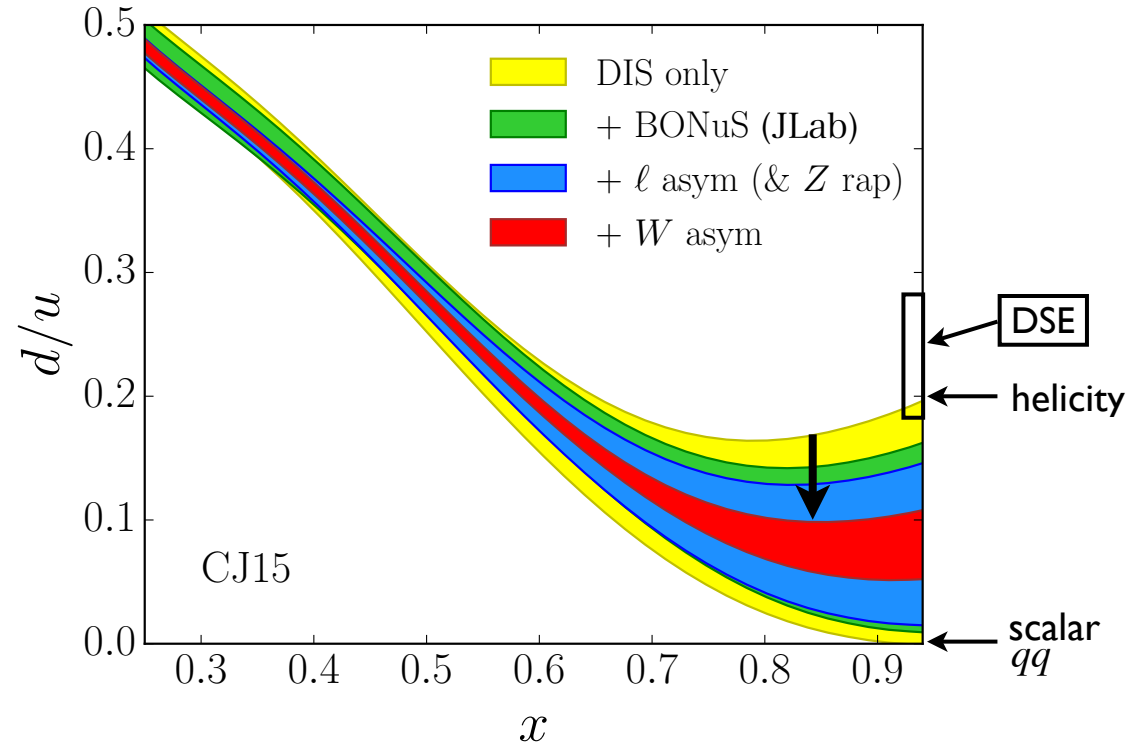
CTEQ-JLab (CJ) global PDF analysis

- Valence d/u ratio at high x

→ significant reduction of PDF errors with new JLab tagged neutron & FNAL W-asymmetry data



Accardi, WM, Owens (2016)



→ extrapolated ratio at $x = 1$

$$d/u \;\rightarrow\; 0.09 \pm 0.03$$

→ upcoming experiments at JLab (MARATHON, BONuS, SoLID) will determine d/u up to $x \sim 0.85$

Outlook and open questions

- Confirmation of duality (experimentally & theoretically) suggests origin in dynamical cancelations between resonances

  $\longrightarrow$ explore more realistic descriptions based on phenomenological $\gamma^* N N^*$ form factors

- Era of "quantitative duality" — need to define the extent to which duality "works"

- Is duality between (high energy) continuum and resonances, or between total (resonance + background)?

  $\longrightarrow$ "resonance region" vs. "resonances"

  $\longrightarrow$ incorporate nonresonant background in same framework + quantum mechanics

- Where does duality <u>*not*</u> work (and why)?