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Simultaneous QCD analysis of quantum probability distributions

Wally Melnitchouk

— Jefferson Lab —

JLab Angular Momentum Collaboration

https://www.jlab.org/jam

Overview

- JAM (Jefferson Lab Angular Momentum) collaboration studies the parton structure of hadrons through extraction of "quantum probability distributions" (PDFs, FFs, TMDs) via global QCD analysis using Monte Carlo-based methods

- Methodology is based on Bayesian statistics and Monte Carlo sampling of the parameter space

- Inter-dependence of observables on distributions requires simultaneous extraction of unpolarized and polarized PDFs & fragmentation functions

Bibliography

- *Strange quark suppression from a simultaneous Monte Carlo analysis of PDFs and fragmentation functions*
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- *First Monte Carlo global QCD analysis of pion parton distributions*
  Patrick Barry, Nobuo Sato, WM, Chueng Ji: Phys. Rev. Lett. **121**, 152001 (2018)

- *First Monte Carlo global analysis of nucleon transversity with lattice QCD constraints*
  Huey-Wen Lin, WM, Alexei Prokudin, Nobuo Sato, Harvey Shows: Phys. Rev. Lett. **120**, 152502 (2018)

- *First simultaneous extraction of spin-dependent PDFs and fragmentation functions from a global QCD analysis*
  Jake Ethier, Nobuo Sato, WM: Phys. Rev. Lett. **119**, 132001 (2017)

- *First Monte Carlo analysis of fragmentation functions from e^+e^- annihilation*
  Nobuo Sato, Jake Ethier, WM, Hirai, Kumano, Alberto Accardi: Phys. Rev. D **94**, 114004 (2016)

- *Iterative Monte Carlo analysis of spin-dependent parton distributions*
  Nobuo Sato, WM, Sebastian Kuhn, Jake Ethier, Alberto Accardi: Phys. Rev. D **93**, 074005 (2016)

- *Constraints on spin-dependent parton distributions at large x from global QCD analysis*
  Pedro Jimenez-Delgado, Harut Avakian, WM: Phys. Lett. B **738**, 263 (2014)

- *Impact of hadronic and nuclear corrections on global analysis of spin-dependent PDFs*
  Pedro Jimenez-Delgado, Alberto Accardi, WM: Phys. Rev. D **89**, 034025 (2014)

Parton distributions in hadrons

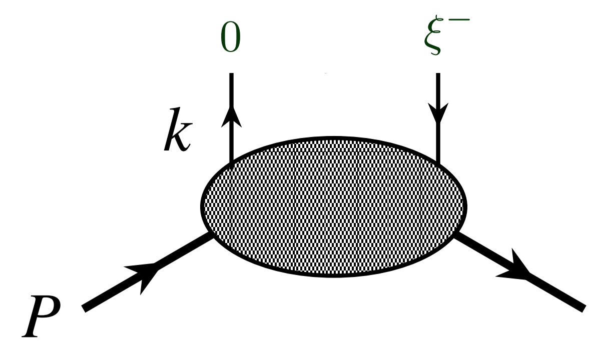
- Parton distribution functions (PDFs) are light-cone correlation functions

$$q(x) = \int_{-\infty}^{\infty} d\xi^- e^{-ixP^+\xi^-} \langle\, P \,|\, \overline{\psi}(\xi^-)\, \gamma^+ \mathcal{W}(\xi^-,0)\, \psi(0) \,|\, P\,\rangle$$

→ light cone momentum fraction $x = \dfrac{k^+}{P^+}$

→ Wilson line (gauge invariance)

$$\mathcal{W}(\xi^-,0) = \exp\left\{ - ig \int_0^{\xi^-} d\eta^- \mathcal{A}^+(\eta^-) \right\}$$



- In $\mathcal{A}^+ = 0$ gauge, in fast-moving frame PDF has a probabilistic interpretation as a particle density

$$\int_{-1}^{1} dx\, q(x) \;=\; \langle\, P \,|\, \overline{\psi}(0)\, \gamma^+\, \psi(0) \,|\, P\,\rangle \;\approx\; \langle\, P \,|\, \psi^\dagger(0)\, \psi(0) \,|\, P\,\rangle$$

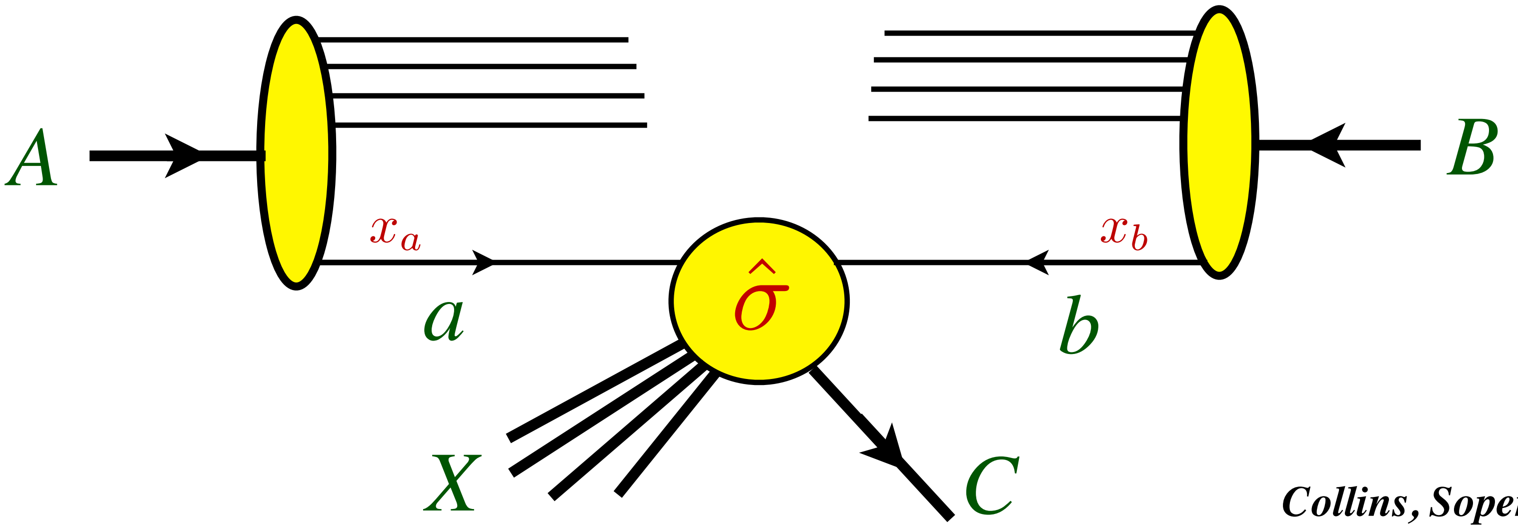
number density

$\overline{\psi}\, \gamma^0\, \psi \approx \overline{\psi}\, \gamma^z\, \psi$

number operator

Parton distributions in hadrons

- Inclusive high-energy particle production $A\,B \to C\,X$



Collins, Soper, Sterman (1980s)

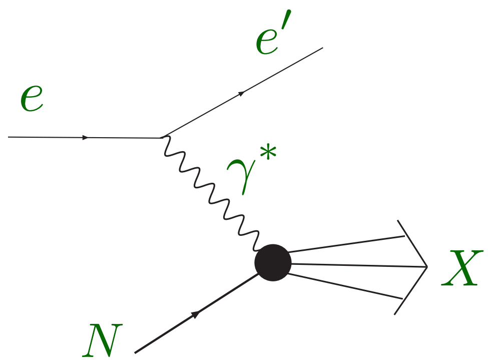
⟶ QCD factorization: separation of hard (perturbative, calculable) from soft (nonperturbative, parametrized) physics

$$\sigma_{AB\to CX}(p_A,p_B) = \sum_{a,b}\int dx_a\,dx_b\,f_{a/A}(x_a,\mu)\,f_{b/B}(x_b,\mu) \times \sum_n \alpha_s^n(\mu)\,\hat{\sigma}^{(n)}_{ab\to CX}(x_a p_A, x_b p_B, Q/\mu)$$

⟶ process-independent parton distribution functions $f_{a/A}$ characterizing structure of bound state A

Parton distributions in hadrons

- Most information on PDFs obtained from lepton-hadron deep-inelastic scattering (DIS)



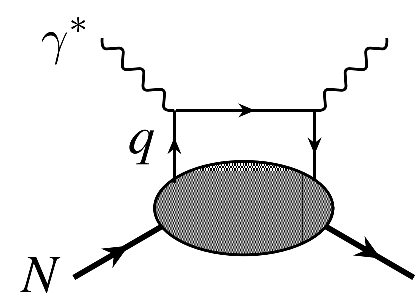
$$\frac{d^2\sigma}{d\Omega dE'} = \frac{4\alpha^2 E'^2 \cos^2\frac{\theta}{2}}{Q^4}\left(2\tan^2\frac{\theta}{2}\frac{F_1}{M} + \frac{F_2}{\nu}\right)$$

$$x_B = \frac{Q^2}{2M\nu} \qquad Q^2 = \vec{q}^{\,2} - \nu^2 \qquad \nu = E - E'$$

$\longrightarrow$ structure function given as convolution of hard Wilson coefficient with PDF

$$F_2(x_B, Q^2) = x_B \sum_q e_q^2 \int_{x_B}^1 \frac{dx}{x} C_q\left(\frac{x_B}{x}, \alpha_s\right) q(x, Q^2)$$

$$\rightarrow x_B \sum_q e_q^2\, q(x_B, Q^2)$$



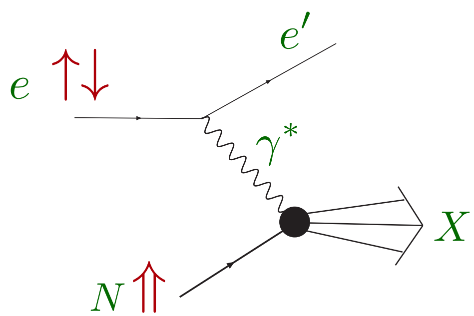
for leading order approximation $C_q \longrightarrow \delta\left(1 - \frac{x_B}{x}\right)$

Parton distributions in hadrons

- Spin-dependent PDFs are defined similarly

$$\Delta q(x) = q^{\uparrow}(x) - q^{\downarrow}(x)$$
$$= \int_{-\infty}^{\infty} d\xi^- e^{-ixP^+\xi^-} \langle P \,|\, \overline{\psi}(\xi^-)\, \gamma^+ \gamma_5\, \mathcal{W}(\xi^-, 0)\, \psi(0) \,|\, P \rangle$$

$\longrightarrow$ measured in polarized lepton-nucleon DIS



$$A_{\parallel} = \frac{\sigma^{\uparrow\Downarrow} - \sigma^{\uparrow\Uparrow}}{\sigma^{\uparrow\Downarrow} + \sigma^{\uparrow\Uparrow}} = D\,(A_1 + \eta\, A_2)$$

$$A_1 \approx \frac{2x_B\, g_1(x_B, Q^2)}{F_2(x_B, Q^2)}$$

$\longrightarrow$ polarized structure function in terms of spin-dependent PDFs

$$g_1(x_B, Q^2) = \frac{1}{2} \sum_q e_q^2 \int_{x_B}^{1} \frac{dx}{x}\, \Delta C_q\left(\frac{x_B}{x}, \alpha_s\right) \Delta q(x, Q^2)$$

$$\rightarrow \frac{1}{2} \sum_q e_q^2\, \Delta q(x_B, Q^2)$$ at leading order

Global PDF analysis

- Universality of PDFs allows data from many different processes (DIS, SIDIS, weak boson/jet production in pp, Drell-Yan …) to be analyzed simultaneously

  → distributions parametrized using a specific functional form, with parameters fitted to data

- Extraction of PDFs is challenging because usually there exist multiple solutions — "inverse problem"

  → PDFs are not directly measured, but inferred from observables involving convolutions with other functions

Bayesian approach to global analysis

- Analysis of data requires estimating expectation values E and variances V of "observables" $\mathcal{O}$ (functions of PDFs) which are functions of parameters

$$E[\mathcal{O}] = \int d^n a\, \mathcal{P}(\vec{a}|\mathrm{data})\, \mathcal{O}(\vec{a})$$

$$V[\mathcal{O}] = \int d^n a\, \mathcal{P}(\vec{a}|\mathrm{data})\, \left[\mathcal{O}(\vec{a}) - E[\mathcal{O}]\right]^2$$

"Bayesian master formulas"

- Using Bayes' theorem, probability distribution $\mathcal{P}$ given by

$$\mathcal{P}(\vec{a}|\mathrm{data}) = \frac{1}{Z}\, \mathcal{L}(\mathrm{data}|\vec{a})\, \pi(\vec{a})$$

in terms of the likelihood function $\mathcal{L}$ and priors π

Bayesian approach to global analysis

- Likelihood function

$$\mathcal{L}(\text{data}|\vec{a}) = \exp\left(-\frac{1}{2}\chi^2(\vec{a})\right)$$

is a Gaussian form in the data, with χ^2 function

$$\chi^2(\vec{a}) = \sum_i \left(\frac{\text{data}_i - \text{theory}_i(\vec{a})}{\delta(\text{data})}\right)^2$$

with priors $\pi(\vec{a})$ and evidence Z

$$Z = \int d^n a\, \mathcal{L}(\text{data}|\vec{a})\, \pi(\vec{a})$$

$\longrightarrow$ Z tests if $e.g.$ an n-parameter fit is statistically different from $(n+1)$-parameter fit

Bayesian approach to global analysis

- Standard method for evaluating E, V via maximum likelihood

$\longrightarrow$ maximize probability distribution

$$\mathcal{P}(\vec{a}|\text{data}) \;\to\; \vec{a}_0$$

$\longrightarrow$ if $\mathcal{O}$ is linear in parameters, and if probability is symmetric in all parameters

$$E[\mathcal{O}(\vec{a})] = \mathcal{O}(\vec{a}_0)\,, \quad V[\mathcal{O}(\vec{a})] \to \text{Hessian} \qquad H_{ij} = \frac{1}{2}\frac{\partial\chi^2(\vec{a})}{\partial a_i \partial a_j}\Big|_{\vec{a}=\vec{a}_0}$$

- In practice, since in general $E[f(\vec{a})] \neq f(E[\vec{a}])$, maximum likelihood method sometimes fails

$\longrightarrow$ need more robust (Monte Carlo) approach

$$E[\mathcal{O}] \;\approx\; \frac{1}{N}\sum_k \mathcal{O}(\vec{a}_k)\,, \qquad V[\mathcal{O}] \;\approx\; \frac{1}{N}\sum_k \left[\mathcal{O}(\vec{a}_k) - E[\mathcal{O}]\right]^2$$

Bayesian approach to global analysis

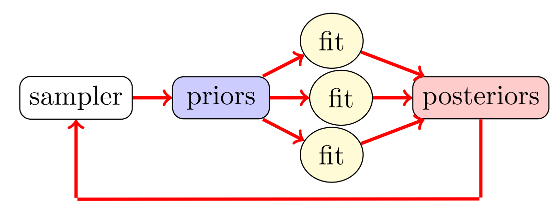
- First group to use MC for global PDF analysis was NNPDF, using neural network to parametrize $P(x)$ in *Ball, Forte et al. (2002)*

$$f(x) = N\, x^{\alpha} (1-x)^{\beta}\, P(x)$$

→ "unbiased"? not really… "pre-processing" coefficients…
there is no such thing as an unbiased PDF fit!

Accardi, WM, Nocera, Sato et al. (2019)

- JAM — iterative, multi-step Monte Carlo

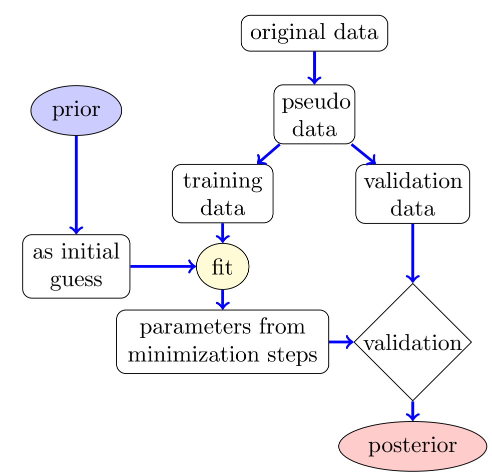


→ traditional functional form for distributions, but sample much larger parameter space

→ no assumptions for exponents

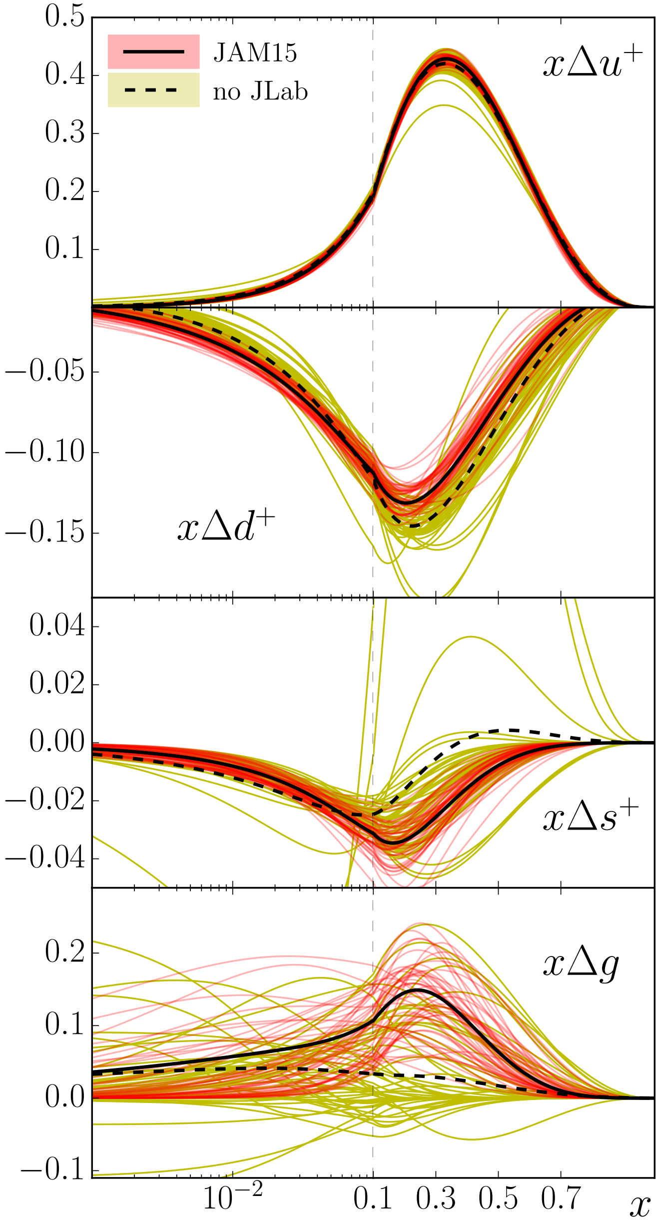
→ iterate until convergence (posteriors = priors)

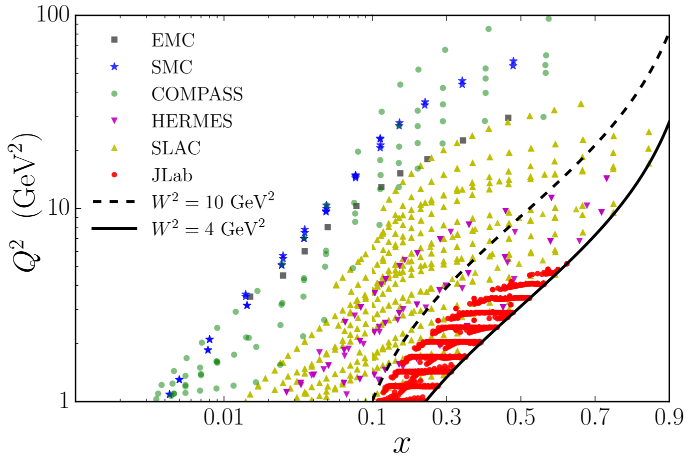
→ robust determination of PDF uncertainties



First application of IMC — spin structure

- First JAM MC analysis studied impact of JLab data on spin structure of the nucleon





Sato, WM, Kuhn, Ethier, Accardi (2016)

→ inclusion of JLab data increases # data points by factor ~ 2

→ reduced uncertainty in Δs^+, Δg through Q^2 evolution

→ <u>s-quark polarization *negative*</u> from inclusive DIS data (assuming SU(3) symmetry)

First application of IMC — spin structure

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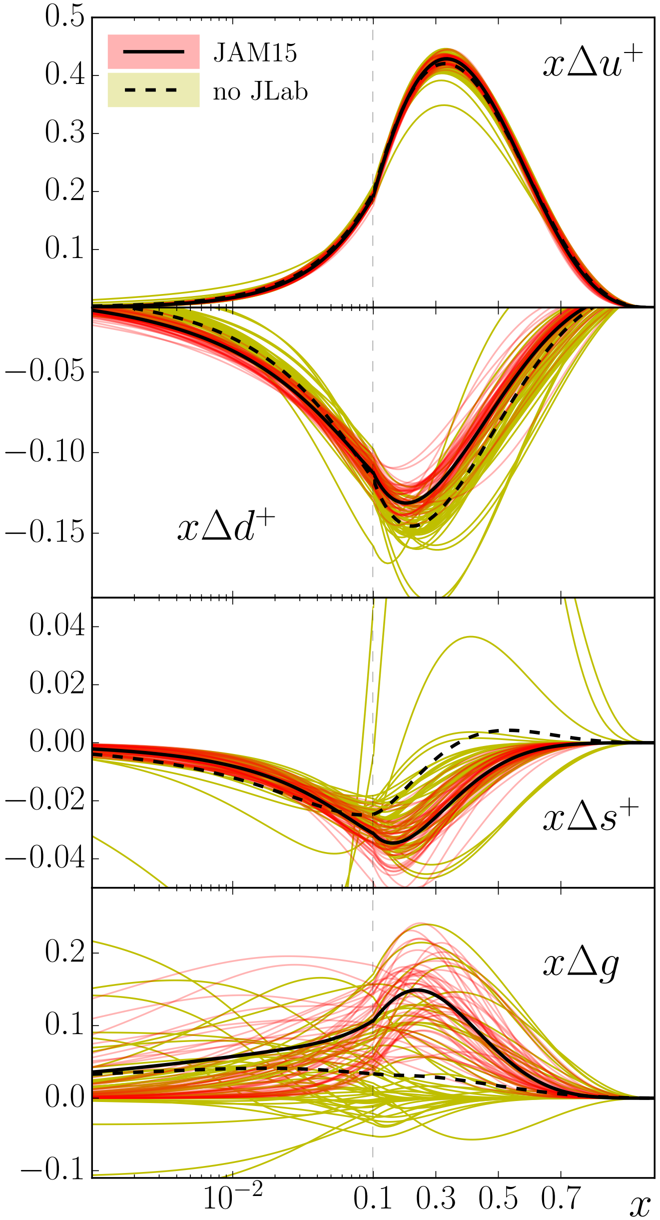
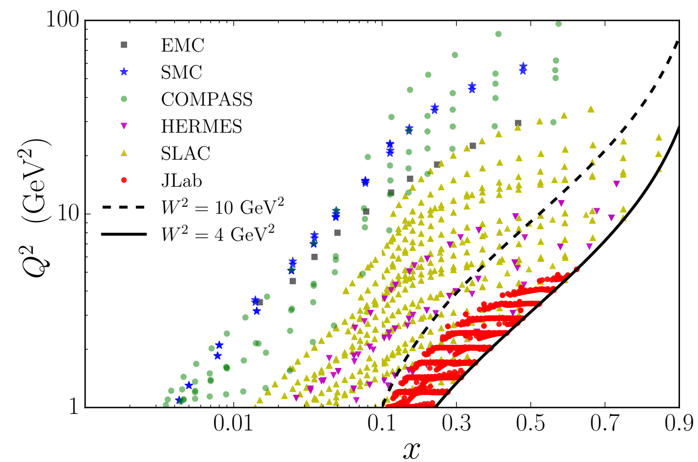




Sato, WM, Kuhn, Ethier, Accardi (2016)

→ inclusion of JLab data increases # data points by factor ~ 2

→ reduced uncertainty in Δs^+, Δg through Q^2 evolution

→ *s*-quark polarization *negative* from inclusive DIS data (assuming SU(3) symmetry)

First application of IMC — spin structure

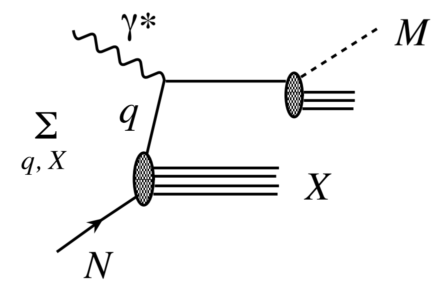
- Inclusive DIS data cannot distinguish between q and $\bar{q}$

  → 2 observables $(g_1^p,\ g_1^n)$ can determine up to 2 unknowns, $e.g.\ \ \Delta u + \Delta\bar{u},\ \Delta d + \Delta\bar{d}$ — sea quarks from Q^2 dependence

  → semi-inclusive DIS sensitive to $\Delta q\ \&\ \Delta\bar{q}$

$$\sim \sum_q e_q^2\ \left[\Delta q(x)\, D_q^h(z) + \Delta\bar{q}(x)\, D_{\bar{q}}^h(z)\right]$$

fragmentation functions



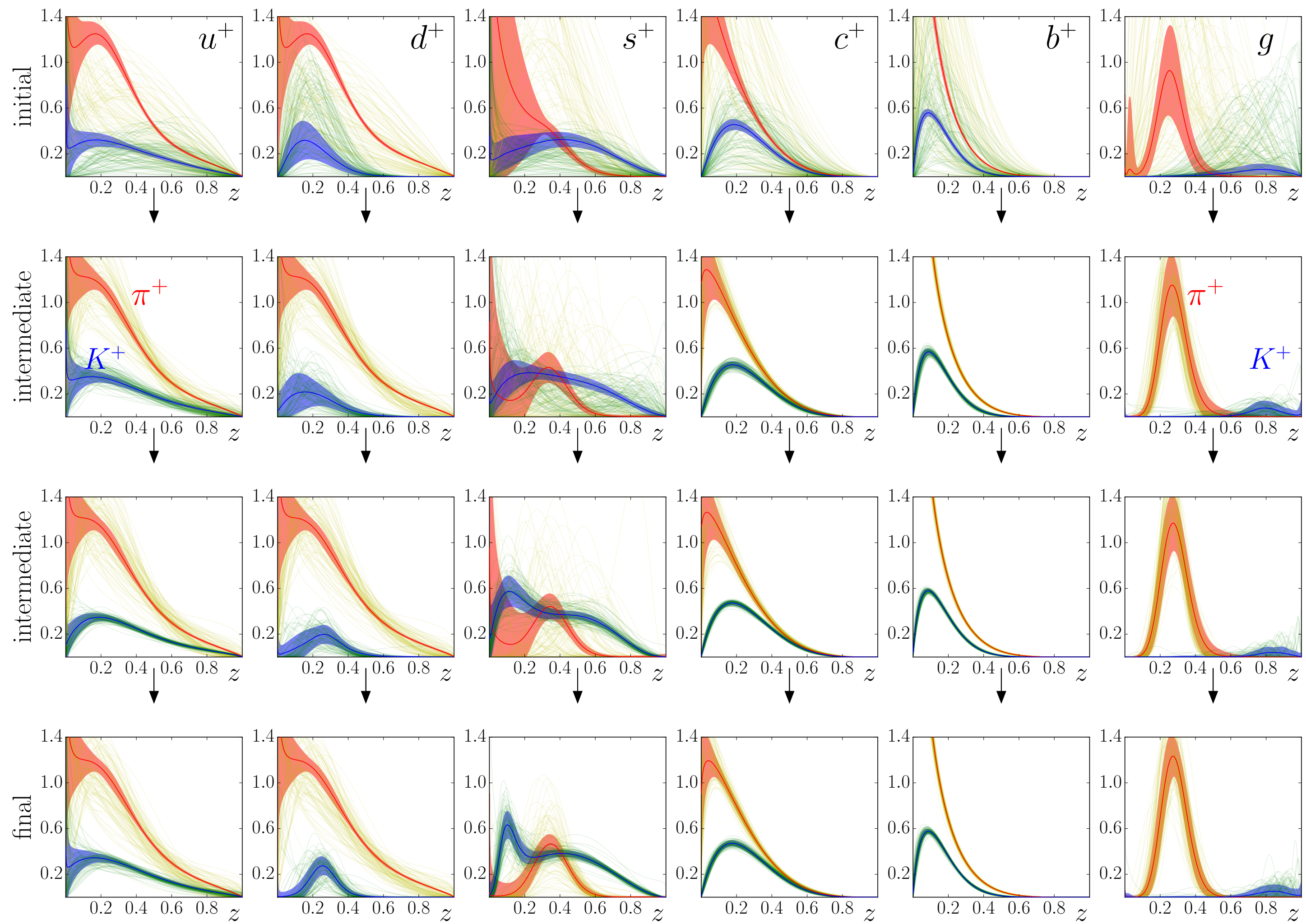
- Global analysis of DIS + SIDIS data gives different *sign* for strange quark polarization for different fragmentation functions!

  → $\Delta s > 0$ for "DSS" FFs, but $\Delta s < 0$ for "HKNS" FFs

  → need to understand origin of differences in fragmentation functions

IMC analysis of fragmentation functions

- Analysis of single-inclusive e^+e^- annihilation data for π, K production (from DESY, CERN, SLAC & KEK) from $Q \sim 10$ GeV to M_Z



$$e^+e^- \to h\,X$$

single-inclusive annihilation (SIA)

Sato, Ethier, WM, Hirai, Kumano, Accardi (2016)

$\longrightarrow$ convergence after ~ 20 iterations

IMC analysis of fragmentation functions

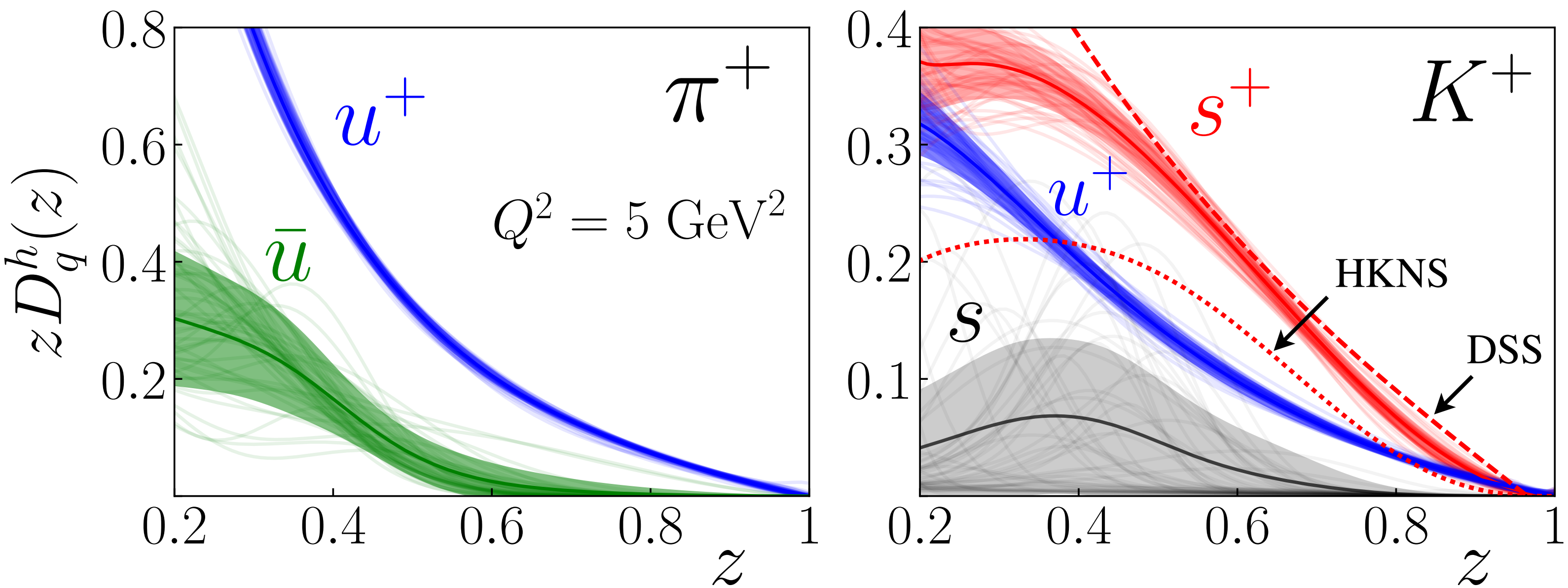
- Analysis of single-inclusive e^+e^- annihilation data for π, K production (from DESY, CERN, SLAC & KEK) from $Q \sim 10$ GeV to M_Z

prompt (conventional)

Experiment	Ref.	Observable	Q (GeV)	Pions N_{dat}	Pions norm.	Pions χ^2	Kaons N_{dat}	Kaons norm.	Kaons χ^2
ARGUS	[26]	Inclusive	9.98	35	1.024 (1.058)	51.1 (55.8)	15	1.007	8.5
Belle	[38,39]	Inclusive	10.52	78	0.900 (0.919)	37.6 (21.7)	78	0.988	10.9
BABAR	[40]	Inclusive	10.54	39	0.993 (0.948)	31.6 (70.7)	30	0.992	4.9
TASSO	[23–25]	Inclusive	12-44	29	(*)	37.0 (38.8)	18	(*)	14.3
TPC	[27–29]	Inclusive	29.00	18	1	36.3 (57.8)	16	1	47.8
		uds tag	29.00	6	1	3.7 (4.6)			
		b tag	29.00	6	1	8.7 (8.6)			
		c tag	29.00	6	1	3.3 (3.0)			
HRS	[30]	Inclusive	29.00	2	1	4.2 (6.2)	3	1	0.3
TOPAZ	[37]	Inclusive	58.00	4	1	4.8 (6.3)	3	1	0.9
OPAL	[32,33]	Inclusive	91.20	22	1	33.3 (37.2)	10	1	6.3
		u tag	91.20	5	1.203 (1.203)	6.6 (8.1)	5	1.185	2.1
		d tag	91.20	5	1.204 (1.203)	6.1 (7.6)	5	1.075	0.6
		s tag	91.20	5	1.126 (1.200)	14.4 (11.0)	5	1.173	1.5
		c tag	91.20	5	1.174 (1.323)	10.7 (6.1)	5	1.169	13.2
		b tag	91.20	5	1.218 (1.209)	34.2 (36.6)	4	1.177	10.9
ALEPH	[34]	Inclusive	91.20	22	0.987 (0.989)	15.6 (20.4)	18	1.008	6.1
DELPHI	[35,36]	Inclusive	91.20	17	1	21.0 (20.2)	27	1	3.9
		uds tag	91.20	17	1	13.3 (13.4)	17	1	22.5
		b tag	91.20	17	1	41.9 (42.9)	17	1	9.1
SLD	[31]	Inclusive	91.28	29	1.002 (1.004)	27.3 (36.3)	29	0.994	14.3
		uds tag	91.28	29	1.003 (1.004)	51.7 (55.6)	29	0.994	42.6
		c tag	91.28	29	0.998 (1.001)	30.2 (40.4)	29	1.000	31.7
		b tag	91.28	29	1.005 (1.005)	74.6 (61.9)	28	0.992	134.1
Total:				459		599.3 (671.2)	391		395.0
					$\chi^2/N_{dat} = 1.31$ (1.46)				$\chi^2/N_{dat} = 1.01$

IMC analysis of fragmentation functions

- Analysis of single-inclusive e^+e^- annihilation data for π, K production (from DESY, CERN, SLAC & KEK) from $Q \sim 10$ GeV to M_Z



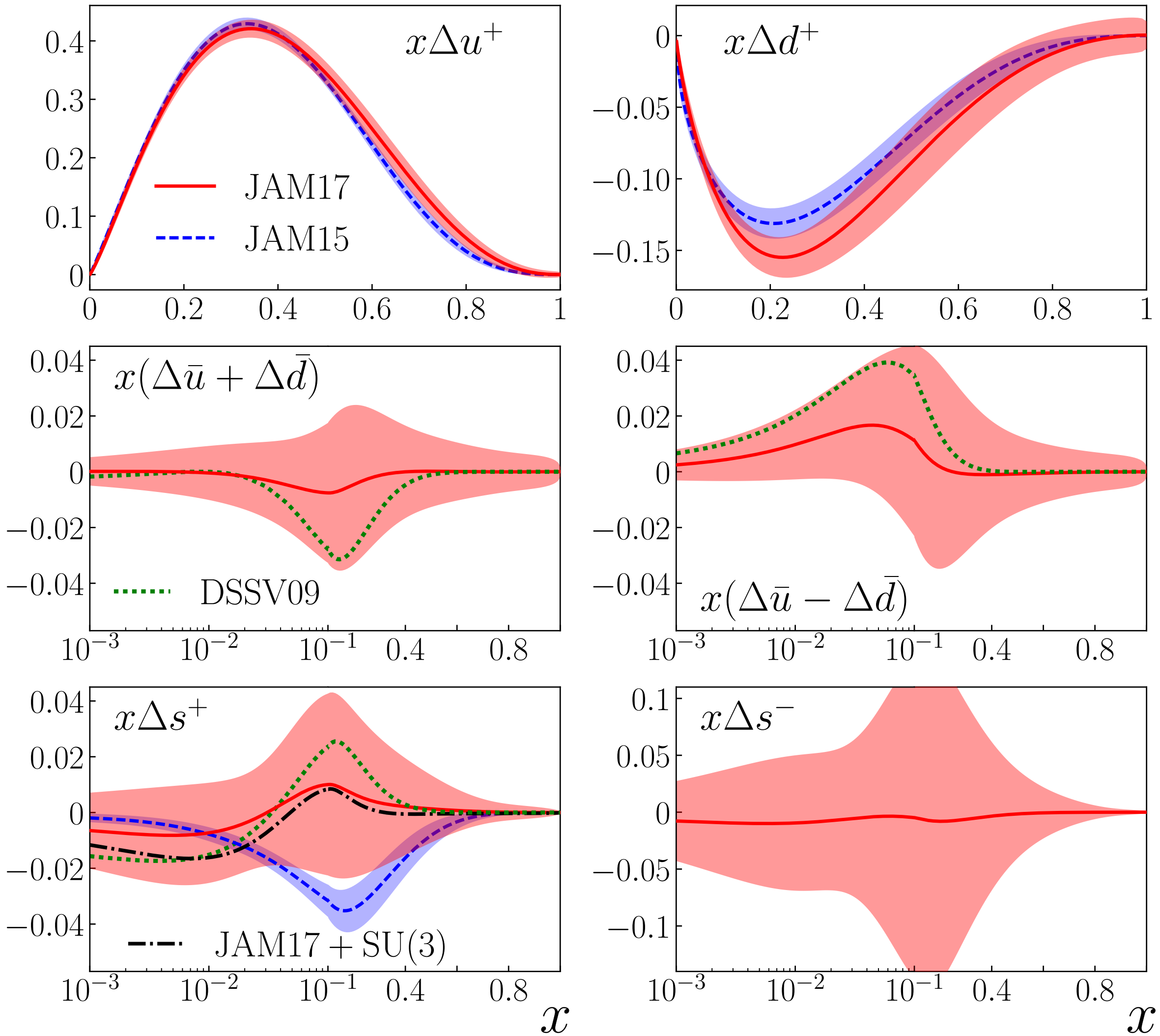
Ethier, Sato, WM (2017)

→ favored $u^+ = u + \bar{u}$ & $s^+ = s + \bar{s}$ FFs well constrained

→ larger $s \to K$ fragmentation *cf*. HKNS suggests <u>less negative Δs</u>

Simultaneous spin PDF + FF analysis

- First simultaneous extraction of spin PDFs and FFs, fitting polarized DIS + SIDIS (HERMES, COMPASS) and SIA data



→ no assumption of SU(3) symmetry

$$\Delta u^+ - \Delta d^+ \stackrel{?}{=} g_A \equiv F + D$$

$$\Delta u^+ + \Delta d^+ - 2\Delta s^+ \stackrel{?}{=} a_8 \equiv 3F - D$$

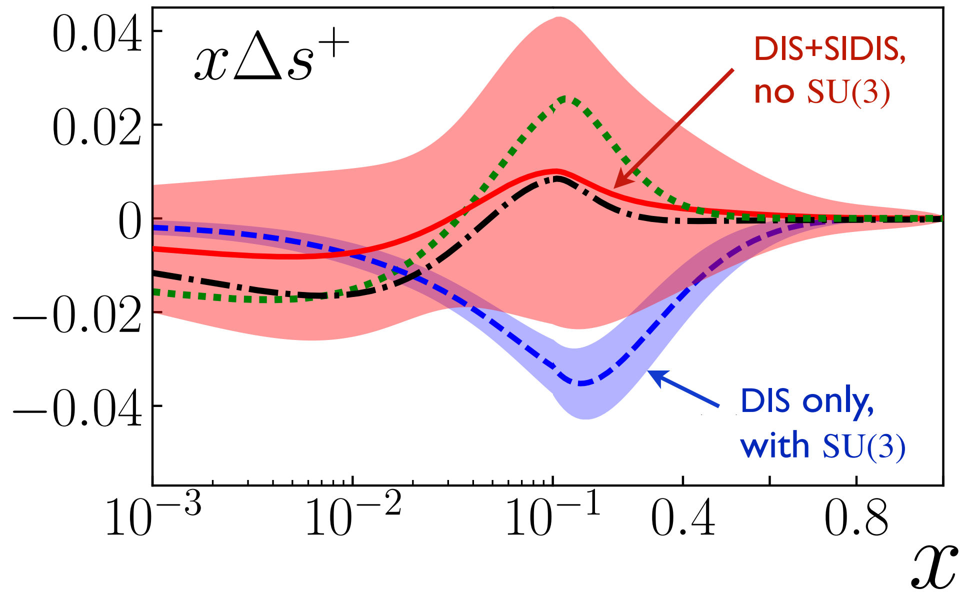
→ Δs slightly > 0 at high x, consistent with zero

→ $\Delta s - \Delta\bar{s}$ & $\Delta\bar{u} - \Delta\bar{d}$ consistent with zero

Ethier, Sato, WM (2017)

Simultaneous spin PDF + FF analysis

- Polarized strangeness in previous, DIS-only analyses was negative at $x \sim 0.1$, induced by SU(3) and parametrization bias



Ethier, Sato, WM (2017)

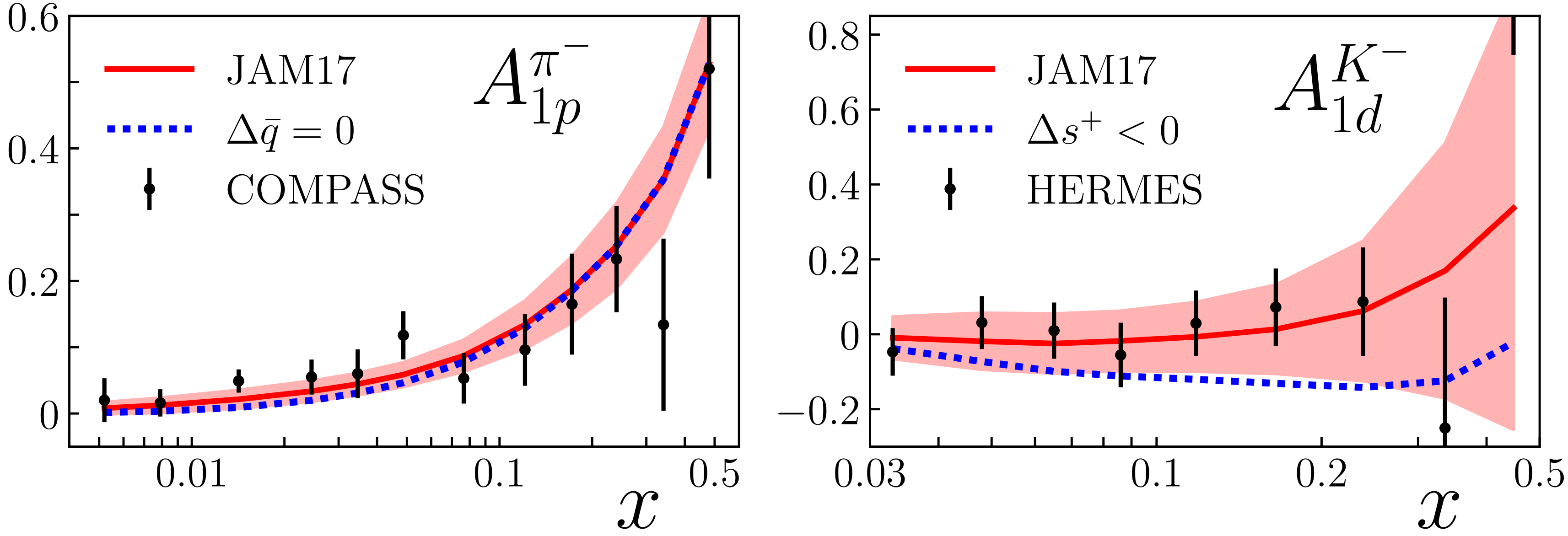
→ weak sensitivity to Δs^+ from DIS data & evolution

— SU(3) pulls Δs^+ to generate moment ~ −0.1

— negative peak at $x \sim 0.1$ induced by fixing $b \sim 6-8$

Simultaneous spin PDF + FF analysis

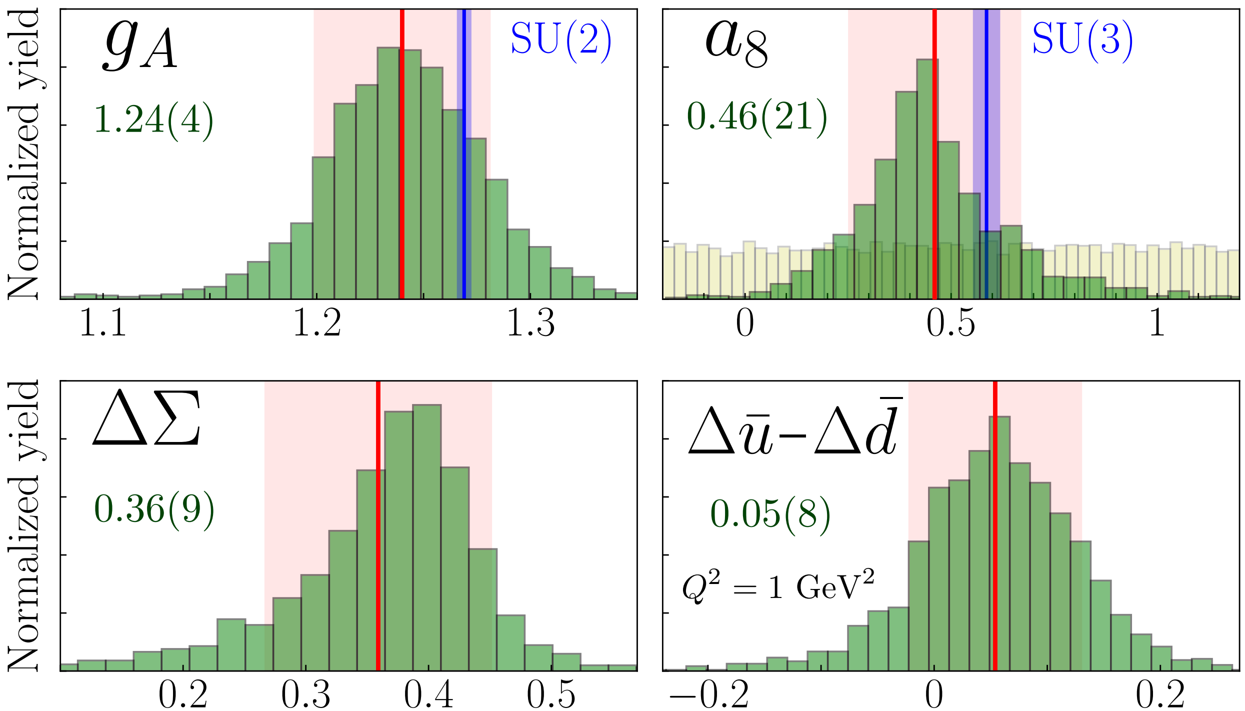
- SIDIS data, especially for K production, clearly prefer a less negative Δs



Ethier, Sato, WM (2017)

Simultaneous spin PDF + FF analysis

■ Statistical distribution of lowest moments (axial charges)



Ethier, Sato, WM (2017)

$\longrightarrow$ triplet charge g_A consistent with $\mathrm{SU(2)}$ value

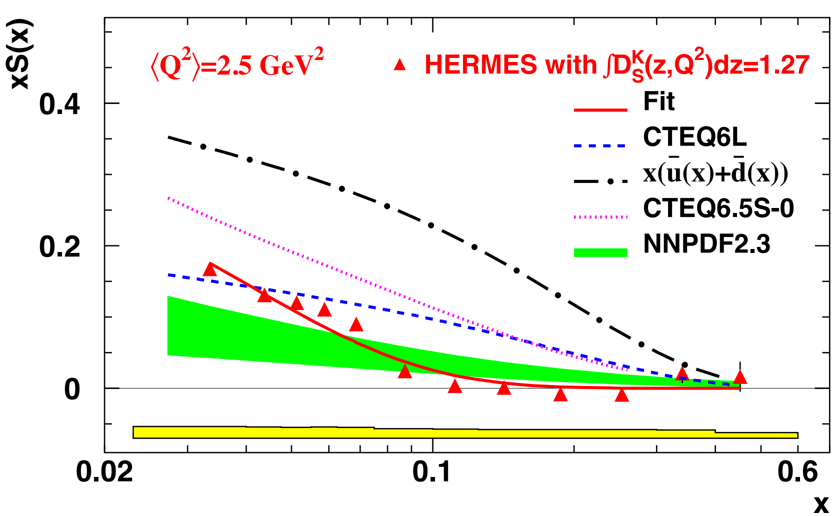
$\longrightarrow$ hint of $\mathrm{SU(3)}$ breaking in octet charge a_8 ***Bass, Thomas (2010)***

$\longrightarrow$ less negative $\Delta s = -0.03(10)$ gives larger total helicity $\Delta\Sigma = 0.36(9)$

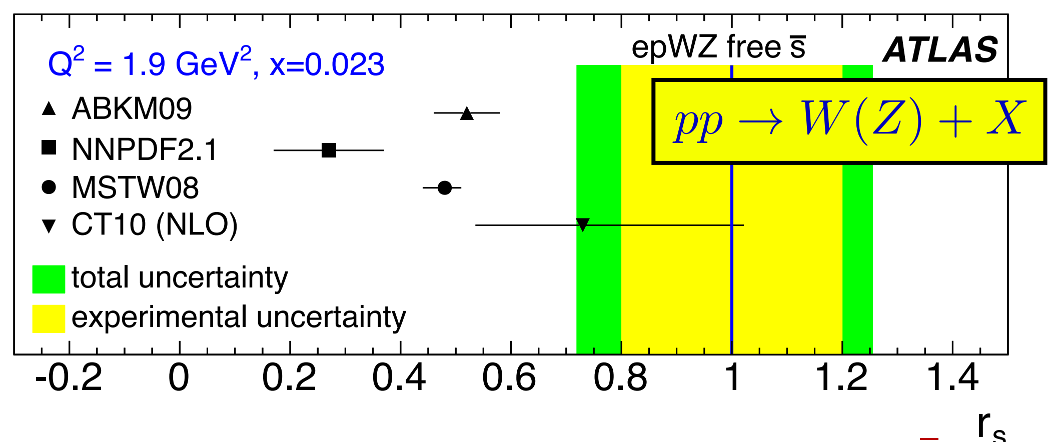
Simultaneous spin PDF + FF analysis

- What impact does unpolarized strange PDF have on the extraction of polarized strange?
  - → only systematic way is to fit unpolarized PDFs, polarized PDFs and fragmentation functions simultaneously…

- Shape of unpolarized strange PDF is interesting (and controversial) in its own right!
  - → historically, strange to nonstrange ratio $R_s = \frac{s+\bar{s}}{\bar{u}+\bar{d}} \sim 0.4$



PRD 89 (2014) 097101



PRL 109 (2012) 012001

$$r_s = (s+\bar{s})/2\bar{d} = 1.00^{+0.25}_{-0.28}$$

JAM 2019 analysis

- Study the impact of SIDIS data on <u>unpolarized</u> PDFs

  → unpolarized fixed-target DIS on p, d (SLAC, BCDMS, NMC), HERA collider data (runs I & II)

  → Drell-Yan (Fermilab E866), jet production (CDF, D0)

  → SIDIS pion & kaon multiplicities for deuteron (COMPASS)

  → e^+e^- annihilation (DESY, LEP/CERN, SLAC, KEK)

  PDFs

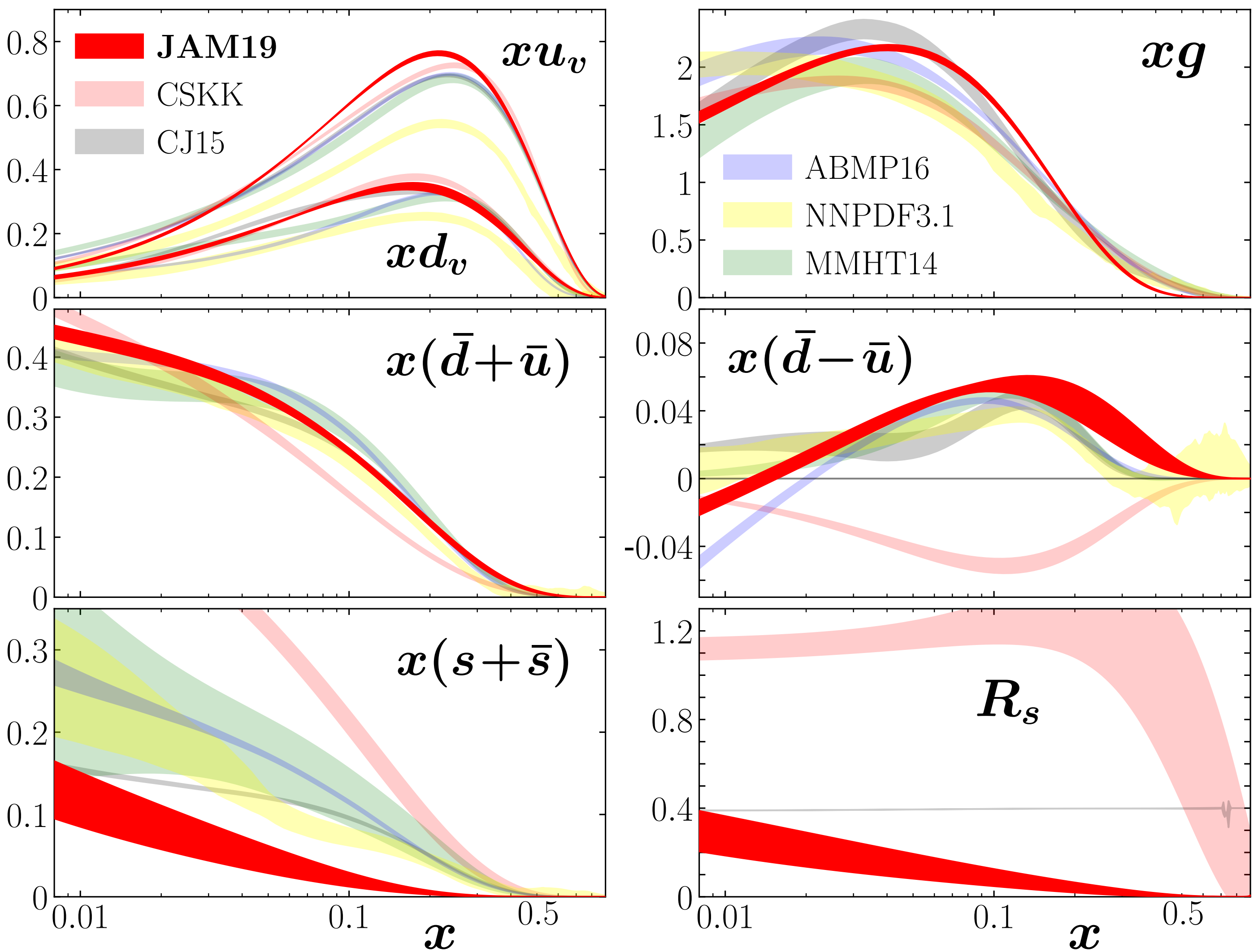
  FFs

- 52 shape parameters + 41 "nuisance" parameters for systematic uncertainties (data normalizations)

- 953 fits to 4366 data points (2680 DIS, 992 SIDIS, 250 DY, 444 SIA)

  → such an analysis has never been attempted before…

JAM 2019 analysis



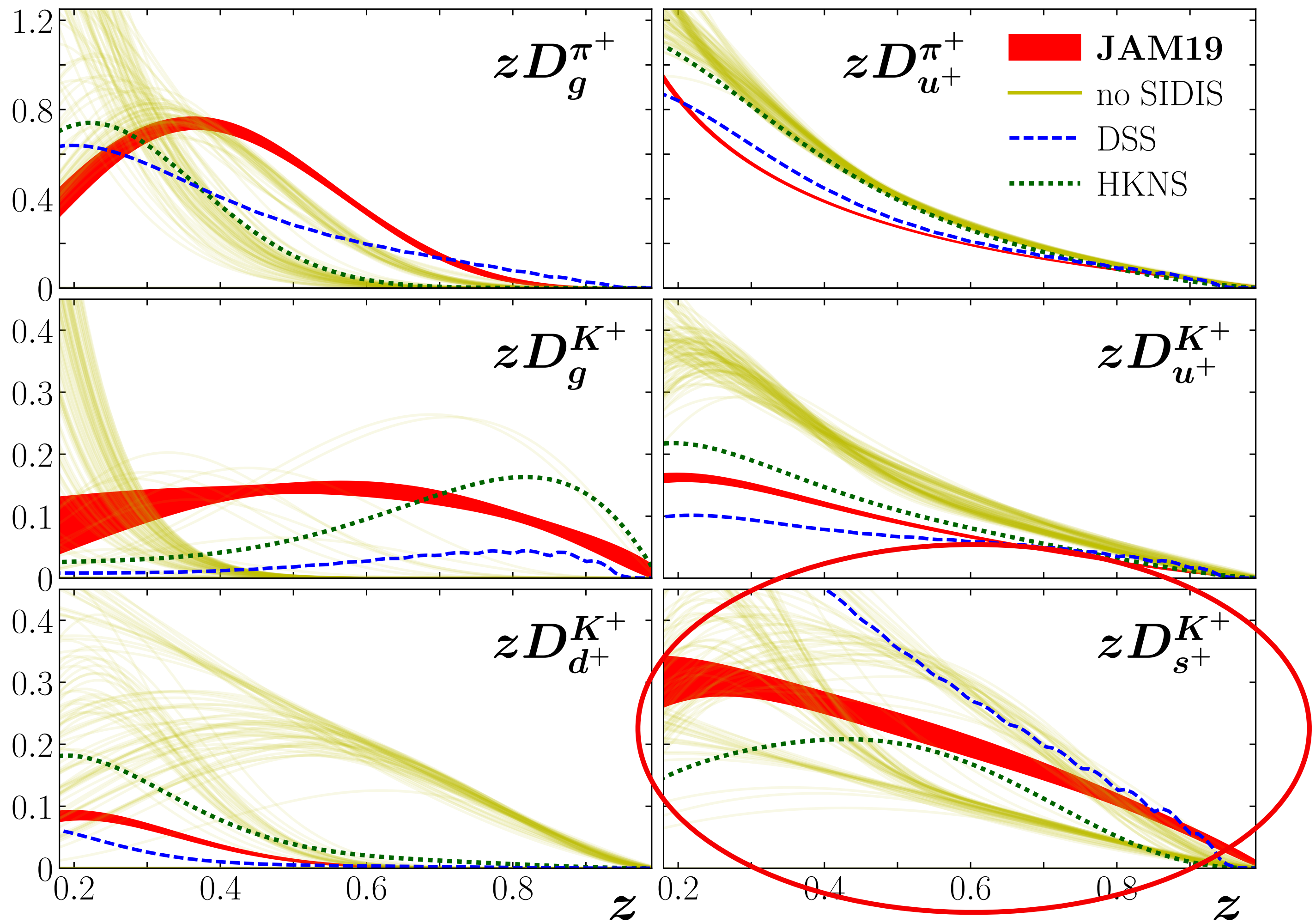
mean reduced
$\chi^2 = 1.3$
for all data

Sato, Andres, Ethier, WM (2019)

→ valence & light sea quark broadly in agreement with other groups

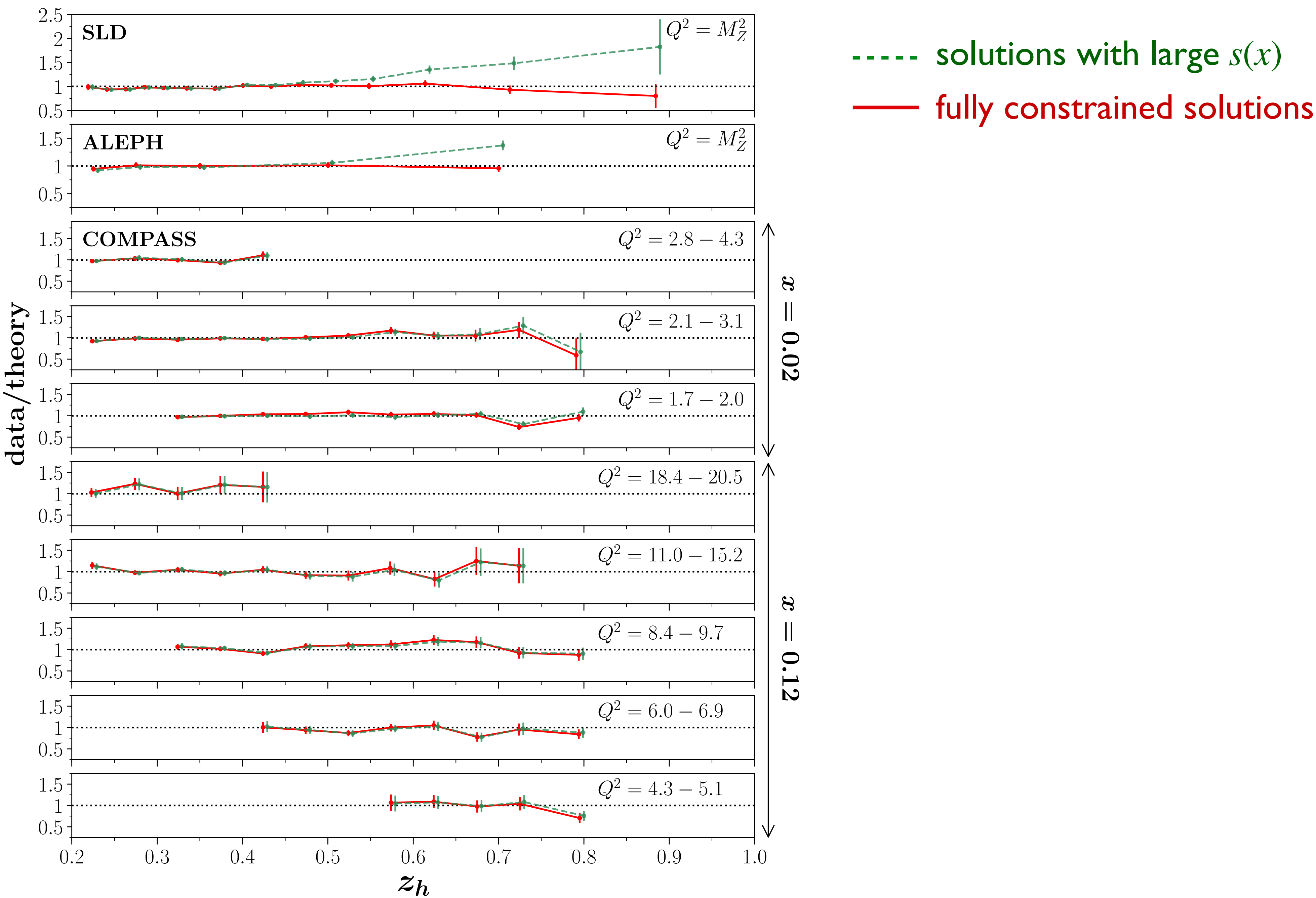
→ striking <u>suppression of strange</u> PDF compared to ATLAS extraction

JAM 2019 analysis

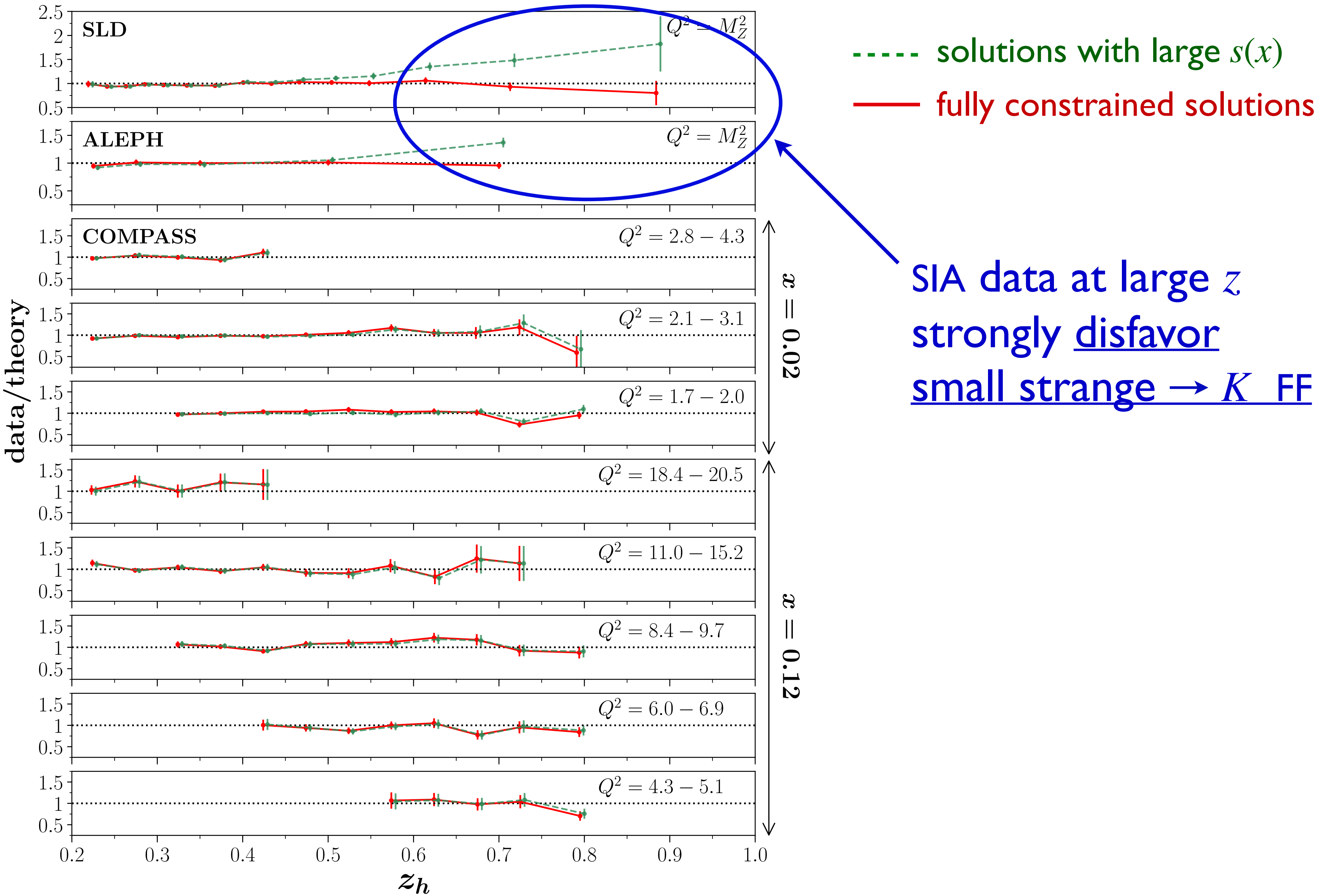


→ SIDIS + SIA data force strange to kaon FF to be larger

JAM 2019 analysis

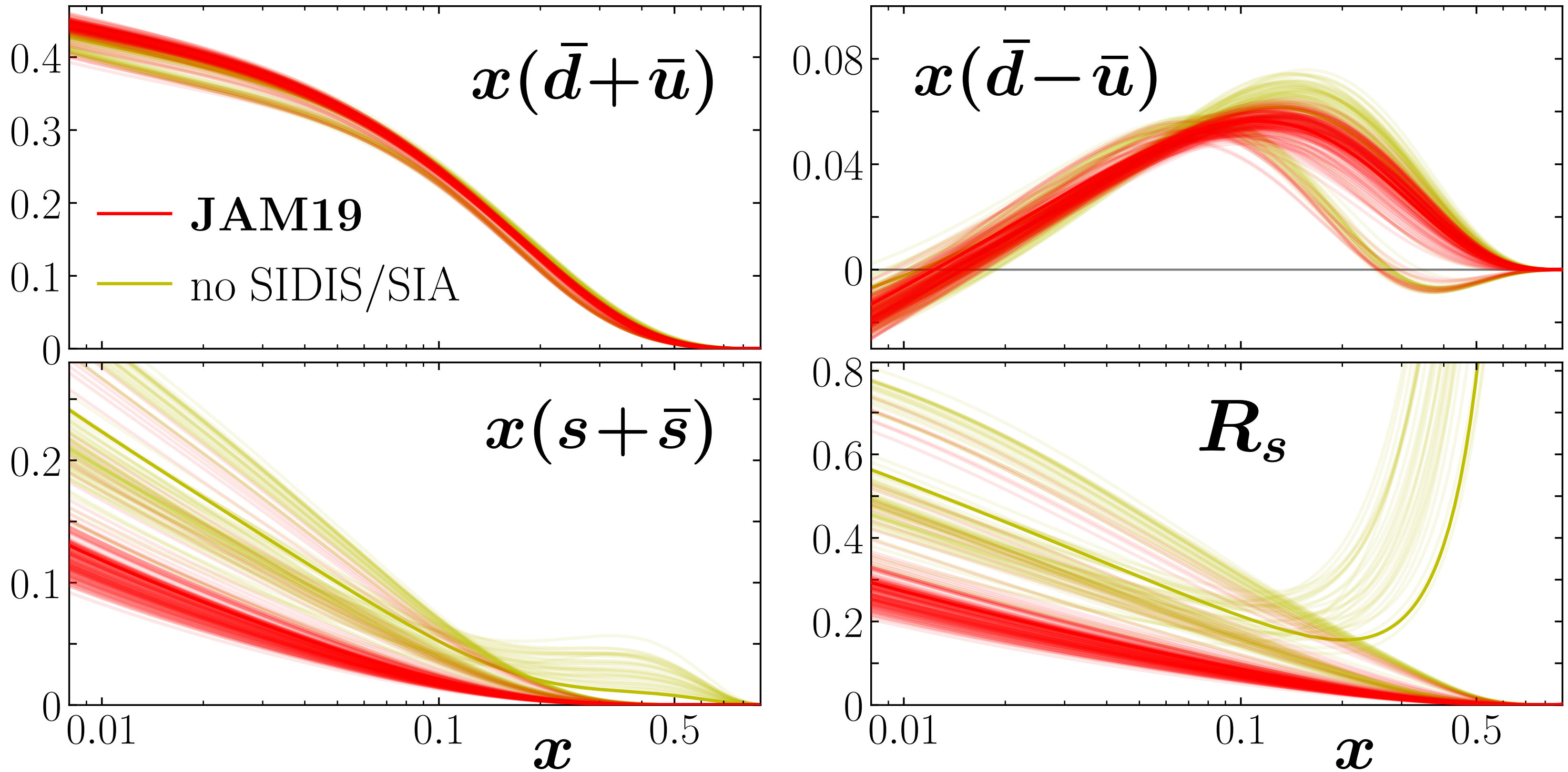


JAM 2019 analysis



SIA data at large z strongly <u>disfavor</u>
<u>small strange → K FF</u>

JAM 2019 analysis



⟶ vital role played by SIDIS + SIA data in constraining strange PDF

PDFs in lattice QCD

- Recent progress in extracting x dependence of PDFs in lattice QCD from matrix element of nonlocal operator

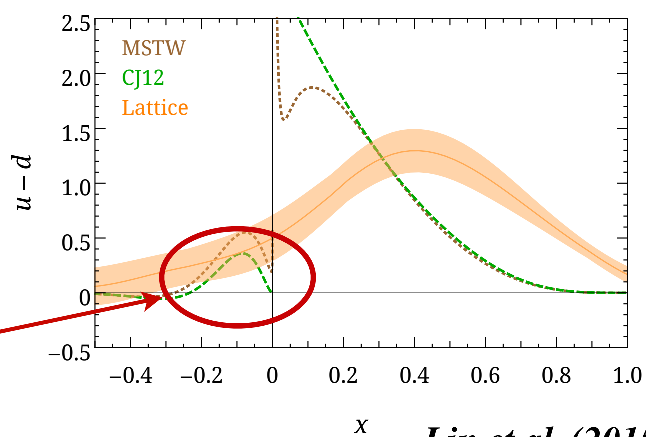
$$h(z,P_z) = \langle\, P\,|\,\overline{\psi}(0,z)\,\gamma_z \mathcal{W}(z,0)\,\psi(0,0)\,|\,P\,\rangle$$
$$= \int_{-\infty}^{\infty} dy\; e^{iyP_z z}\; \widetilde{q}(y,P_z)$$

$\longrightarrow$ quasi-PDF $\widetilde{q}$ related to light-cone PDF via matching kernel $\widetilde{C}$

$$q(x,\mu) = \int_{-\infty}^{\infty} \frac{dy}{|y|}\; \widetilde{C}\Big(\frac{x}{y},\mu,P_z\Big)\; \widetilde{q}(y,P_z,\mu)$$

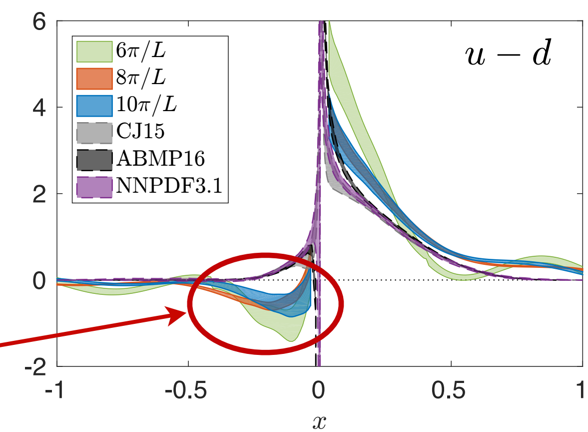
- Conflicting results on sign of $\bar{d}-\bar{u}$ asymmetry

$\bar{d} > \bar{u}$



Lin et al. (2015)

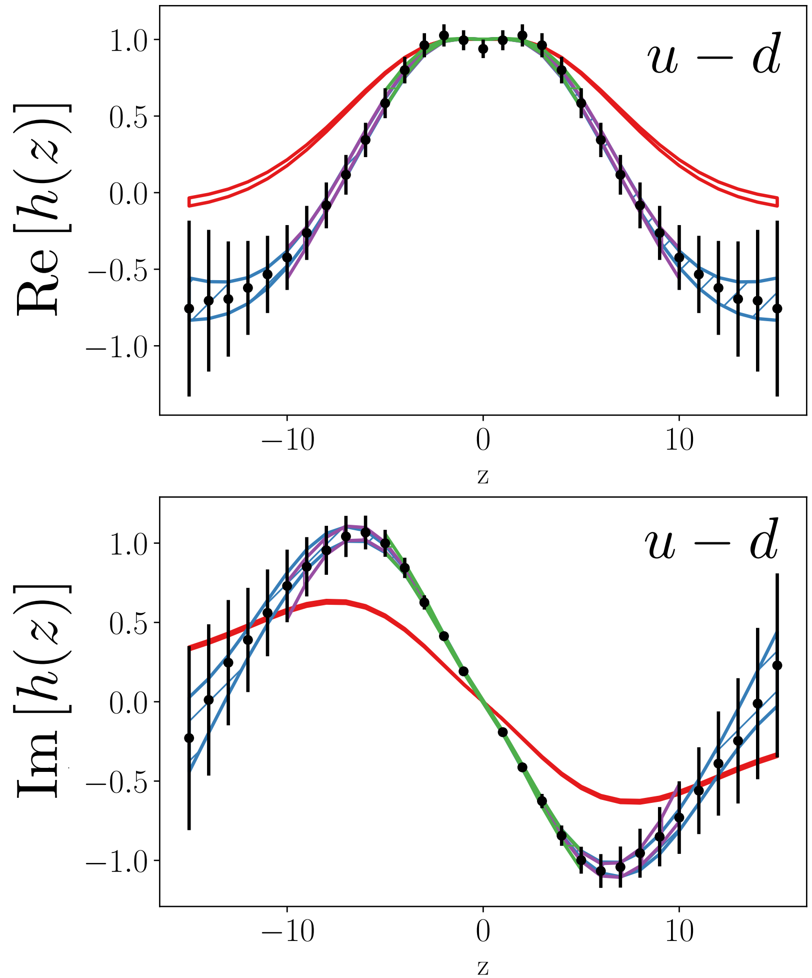
$\bar{d} < \bar{u}$

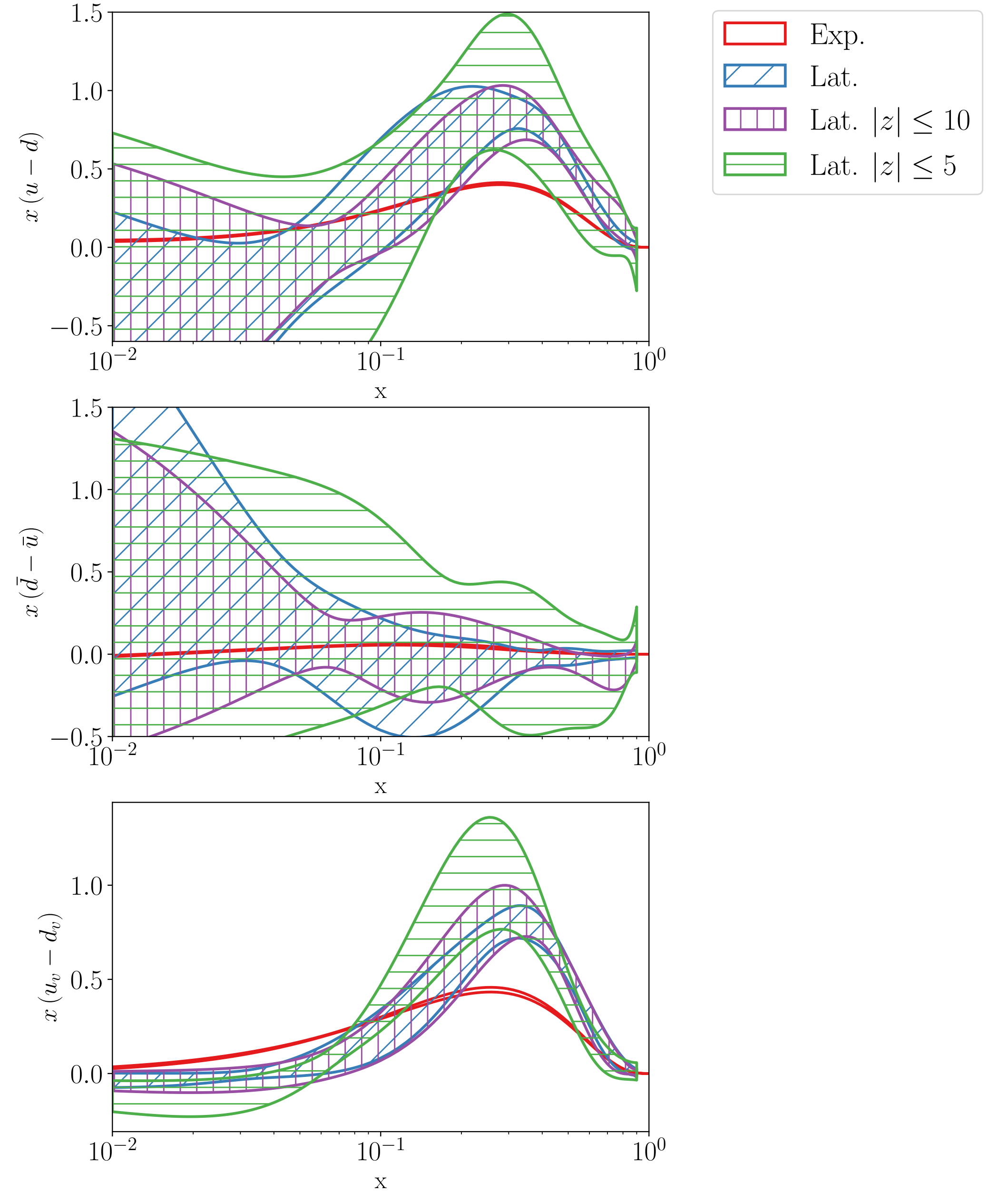


Alexandrou et al. (2018)

PDFs in lattice QCD

- Fit lattice observable directly within JAM framework



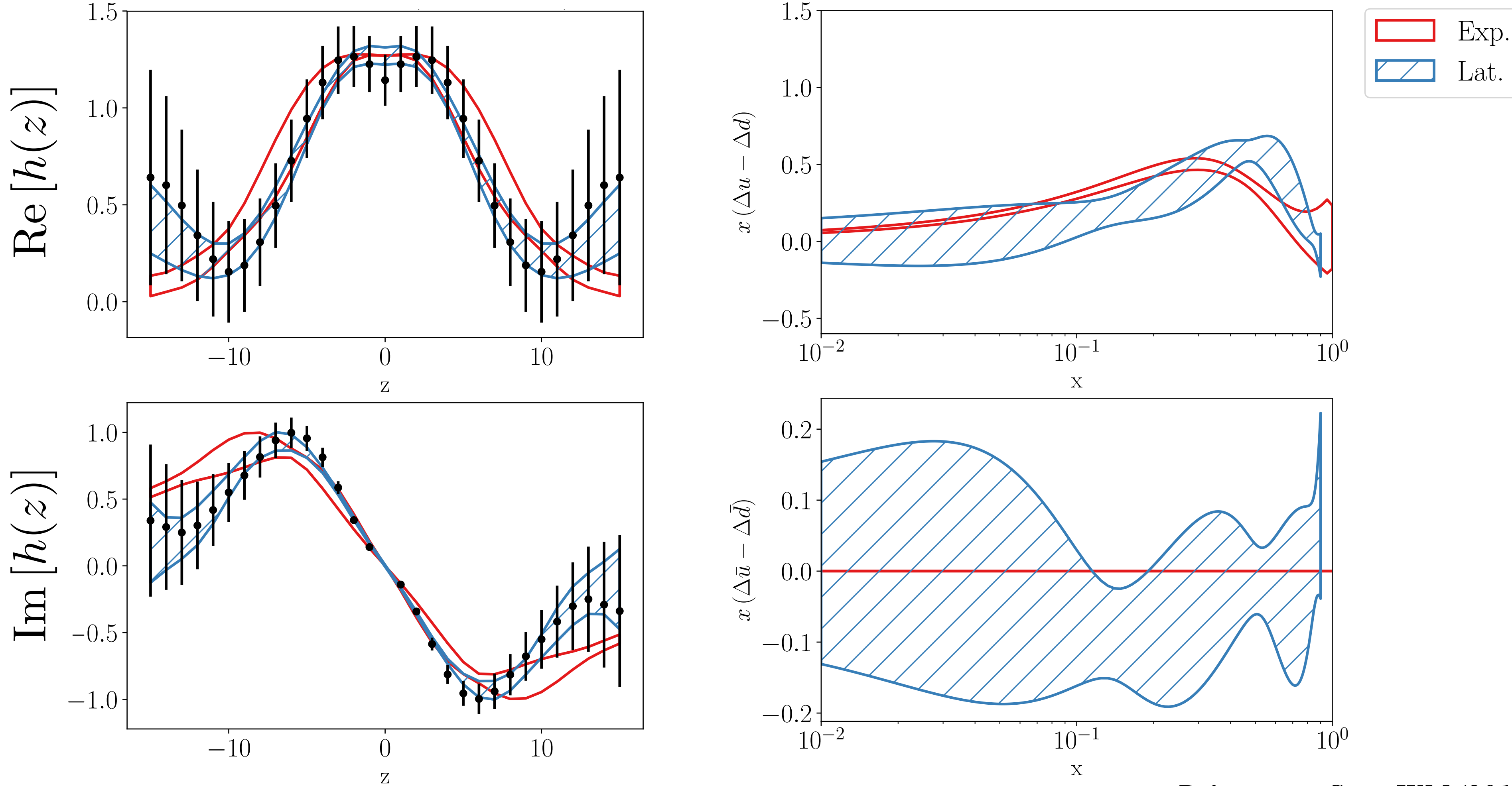


→ cannot determine $\bar{d} - \bar{u}$ from present lattice data

Bringewatt, Sato, WM (2019)

PDFs in lattice QCD

- Fit lattice observable directly within JAM framework

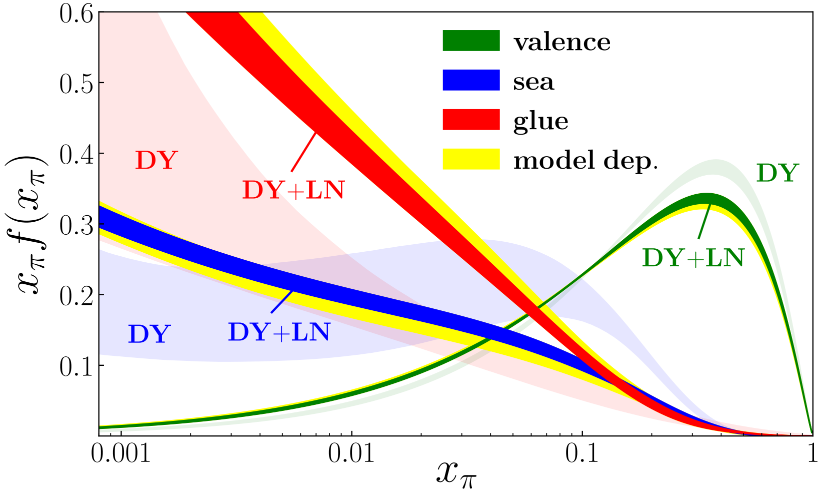


Bringewatt, Sato, WM (2019)

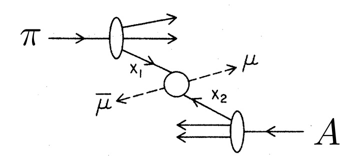
⟶ better agreement between lattice and experiment for polarized PDFs (within larger uncertainties)

PDFs in the pion

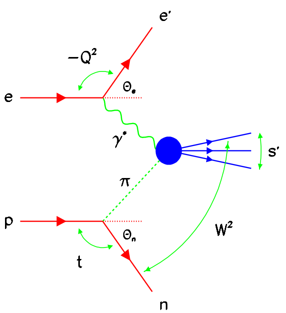
- MC analysis combining pQCD with chiral EFT to fit πN Drell-Yan + leading neutron electroproduction data from HERA



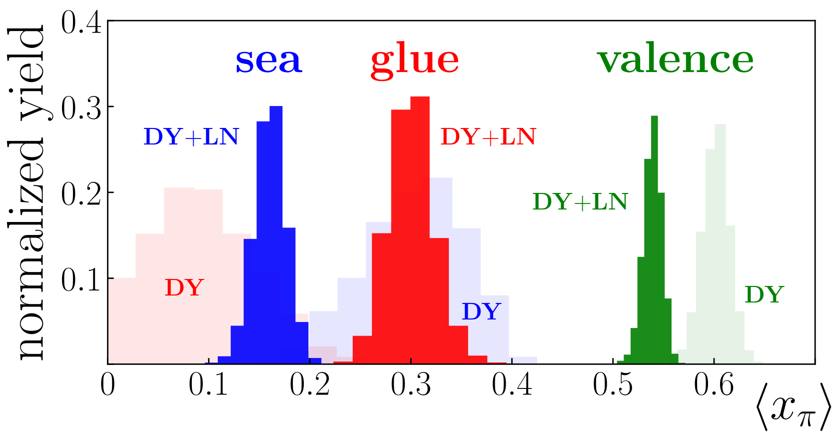
DY = πN Drell-Yan data (medium/high x)



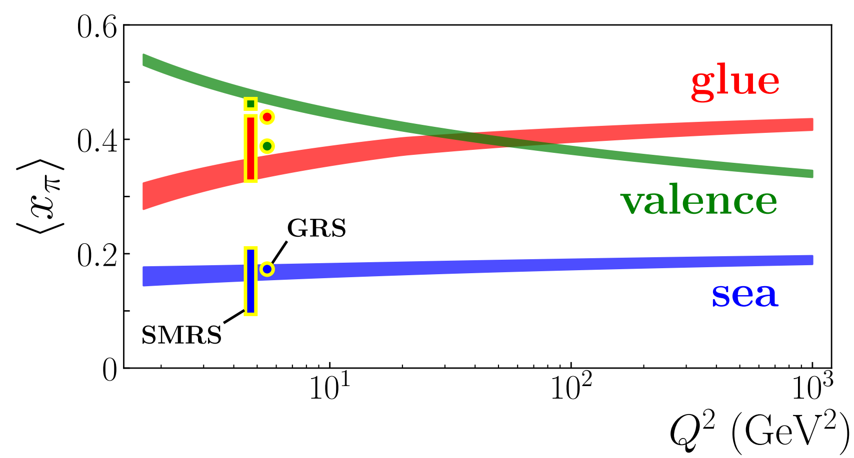
LN = leading neutron electroproduction (low x)



- Larger gluon fraction in the pion than without LN constraint

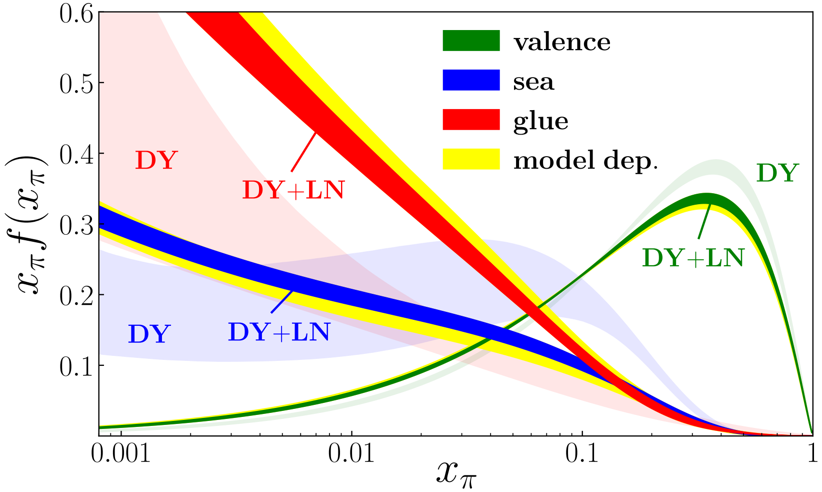


Barry, Sato, WM, C.-R. Ji (2018)



PDFs in the pion

- MC analysis combining pQCD with chiral EFT to fit πN Drell-Yan + leading neutron electroproduction data from HERA



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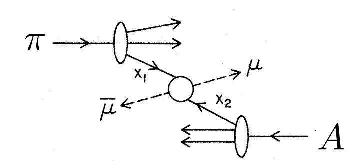

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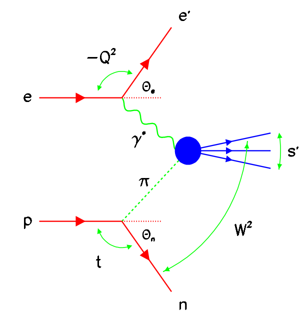
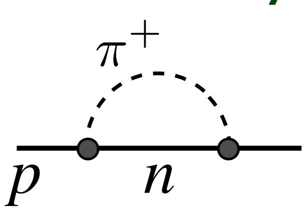
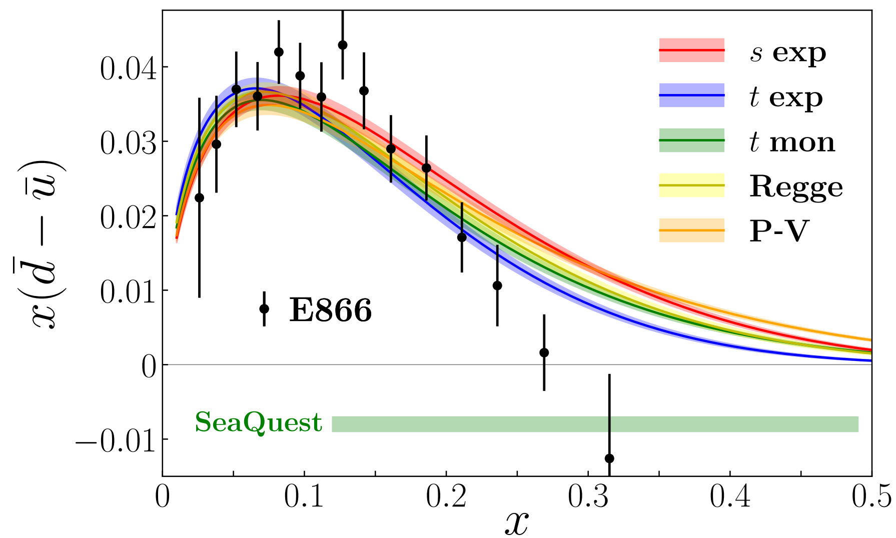


- Provides new insights into the origin of the $\bar{d} - \bar{u}$ asymmetry in the proton

$\longrightarrow$ chiral effective theory relates asymmetry to structure of pion



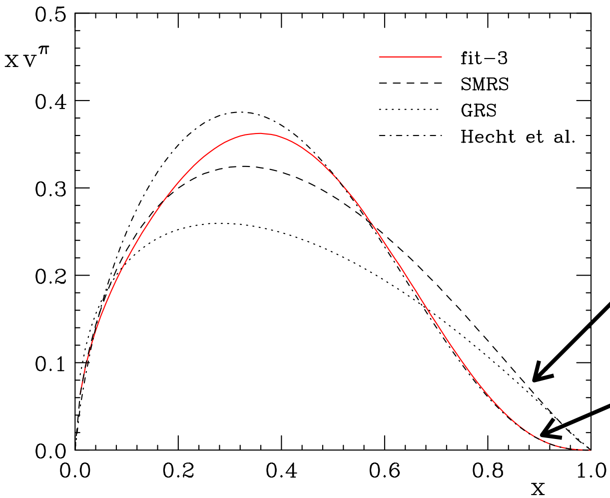
$$(\bar{d} - \bar{u})(x) = \int \frac{dy}{y} f_{\pi^+ n}(y)\, \bar{q}^\pi(x/y)$$



Barry, Sato, WM, C.-R. Ji (2018)

PDFs in the pion

- $x \to 1$ behavior of pion PDF is controversial: $\sim (1-x)$ or $(1-x)^2$?

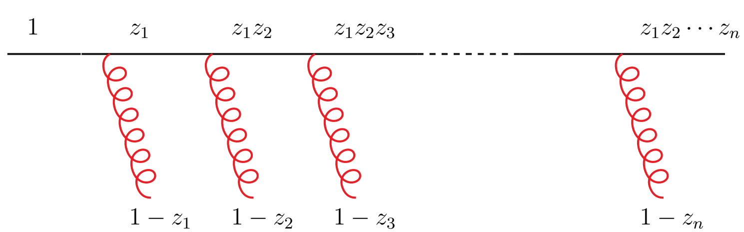


Aicher, Schafer, Vogelsang (2010)

no resummation: more consistent with $\sim (1-x)$

with resummation: more consistent with $\sim (1-x)^2$

- Hard scattering coefficient function kinematically enhanced when $z \to 1$ because of gluon emissions

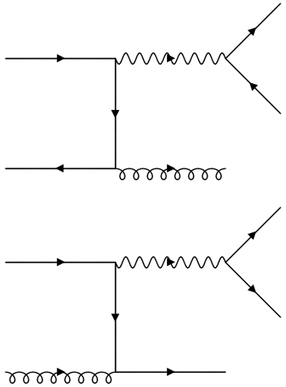


Patrick Barry et al. (2019)

$\longrightarrow$ effect of resummation on phenomenology?

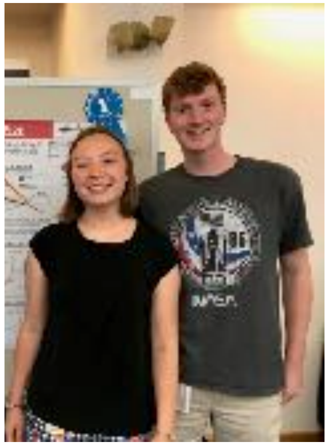
PDFs in the pion

- New analysis examines whether large-q_T DY data can be simultaneously described with q_T-integrated DY + HERA LN data

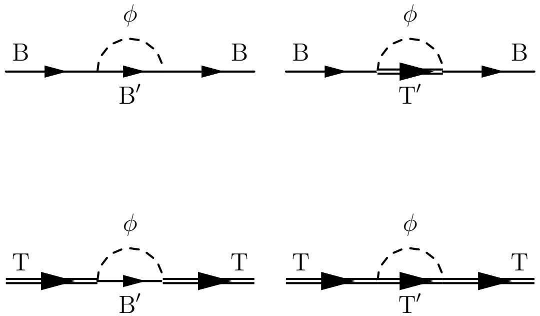


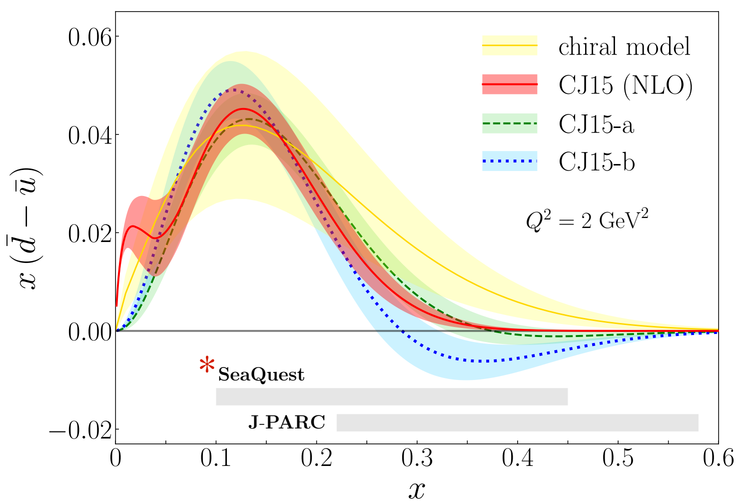
→ large-q_T photon requires hard gluon to recoil against — sensitivity to <u>gluon PDF</u> in pion at <u>large x</u>!

Nina Cao et al. (201



- Generalization of chiral model to SU(3) octet & decuplet baryons





Marston Copeland et al. (2019)

* expected Oct. 2019

Outlook

- New paradigm in global analysis — simultaneous determination of collinear distributions using MC sampling of parameter space

- Next steps: simultaneous analysis of all collinear distributions — unpolarized & polarized PDFs and FFs
  (including jet, W production, … data)

- Longer-term: technology developed here will be applied to global QCD analysis of transverse momentum dependent (TMD) distributions — map out full 3-d image of hadrons