*TNT Seminar, NCSU*
*October 4, 2023*

# Unraveling the 3D structure of the pion

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# Outline

- Motivation(s) for studying pion structure

- Simultaneously fit pion PDFs to Drell-Yan and "leading neutron" DIS data
  ⟶ first constraints on gluon & sea quark PDFs at low $x$

- Global QCD analysis with threshold resummation
  ⟶ high-$x$ pion PDFs: supports $\sim (1-x)$ behavior large $x$

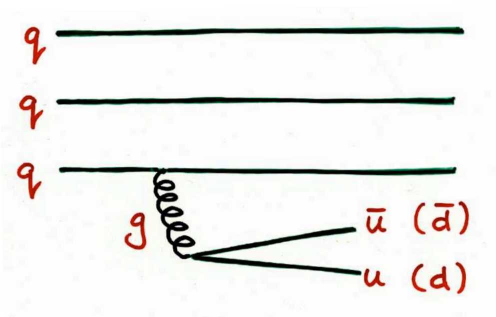
- Impact of lattice QCD data

- Extend analysis to incorporate transverse momentum
  ⟶ large $p_T$ — sensitivity to gluon PDF at high $x$
  ⟶ small $p_T$ — pion TMDs

- Outlook

# Sea of the proton

- From text-books: perturbative QCD expected to generate symmetric $q\bar{q}$ sea via gluon radiation into $q\bar{q}$ pairs

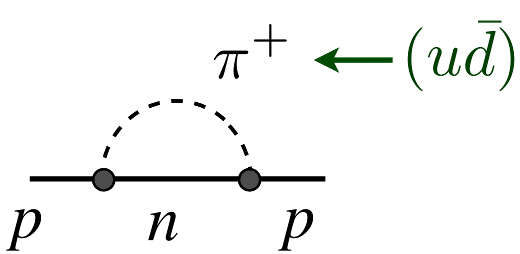



→ since $u$ and $d$ quarks nearly degenerate, expect flavor-symmetric light-quark sea

$$\bar{d} \approx \bar{u}$$

*Ross, Sachrajda (1979)*

- (Almost) from text-books: Thomas suggested that chiral symmetry of QCD ("low energy") should have consequences for antiquark PDFs in the nucleon ("high energy")

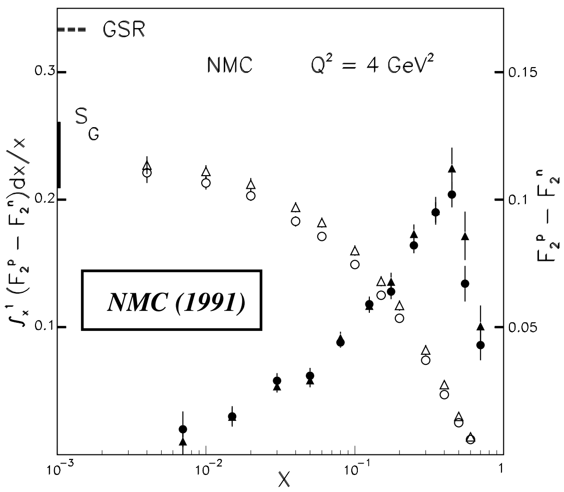



$$\rightarrow \quad \bar{d} > \bar{u}$$

*Thomas (1984)*

# Sea of the proton

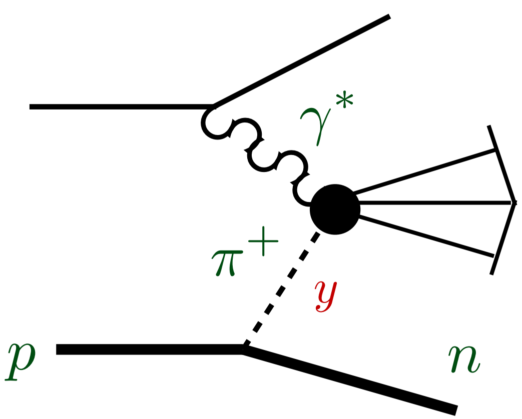
- First clear experimental support for $\bar{d} \neq \bar{u}$ came from measurement of Gottfried sum observed by NMC




$$\int_0^1 \frac{dx}{x}(F_2^p - F_2^n) = \frac{1}{3} - \frac{2}{3}\int_0^1 dx\,(\bar{d} - \bar{u})$$
$$= 0.235(26)$$

$\longrightarrow$ violation of "Gottfried sum rule"

- Sullivan process — DIS from pion cloud of the nucleon



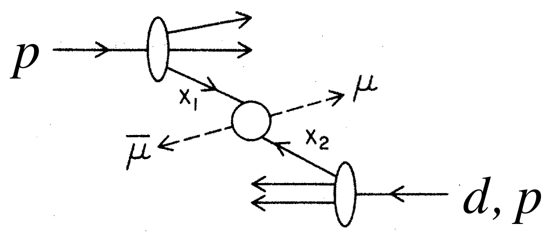

*Sullivan (1972)*

$$(\bar{d} - \bar{u})(x) = \int_x^1 \frac{dy}{y} f_{\pi^+ n}(y)\, \bar{q}_v^{\pi}(x/y)$$

$p \to \pi^+ n$ splitting function ("flux factor") computed from chiral effective theory

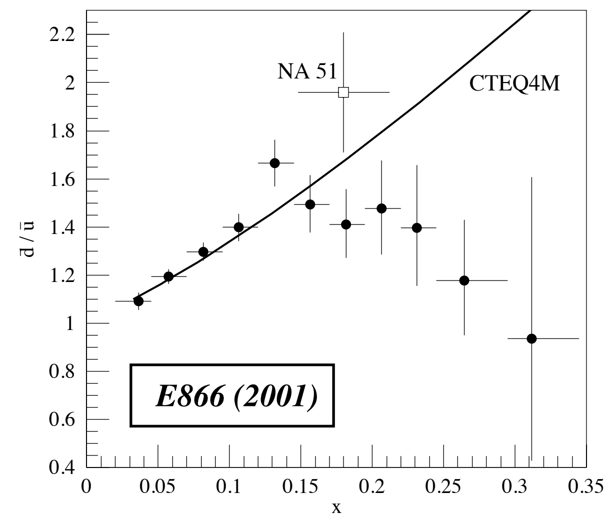
# Sea of the proton

- $x$ dependence of $\bar{d} - \bar{u}$ asymmetry established in Fermilab E866 *pp/pd* Drell-Yan experiment



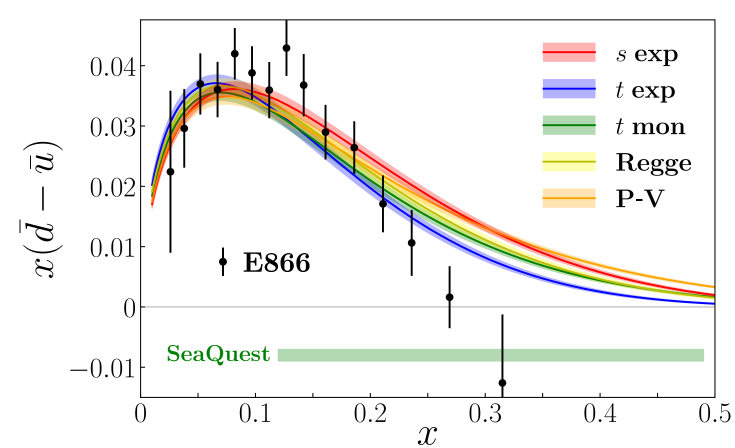

$$\frac{d\sigma}{dx_1 dx_2} \sim \sum_q e_q^2\, q(x_1)\, \bar{q}(x_2) \;+\; (x_1 \leftrightarrow x_2)$$

$$\frac{\sigma^{pd}}{\sigma^{pp}} \approx 1 + \frac{\bar{d}(x_2)}{\bar{u}(x_2)} \quad \text{for } x_1 \gg x_2$$




→ data can be well described within chiral EFT / pion cloud framework

→ need to know pion PDF!

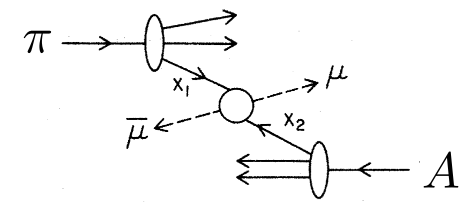



***Barry, Sato, WM, C.-R. Ji***
***PRL 121, 152001 (2018)***

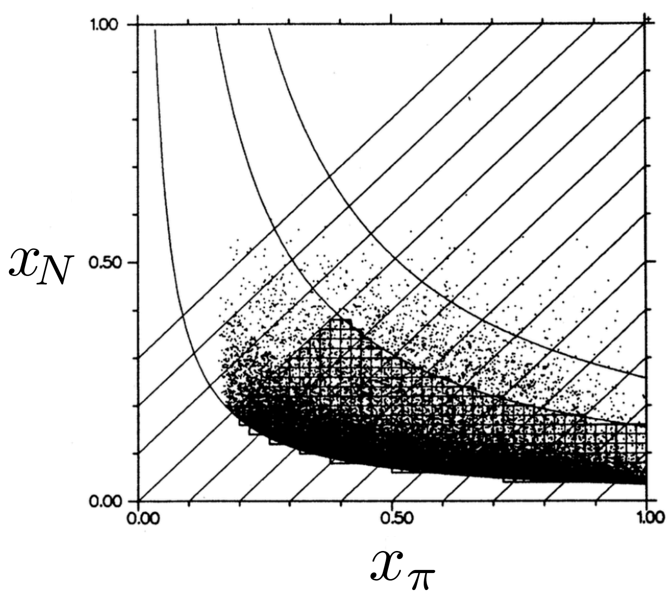
# PDFs in the pion — Drell-Yan

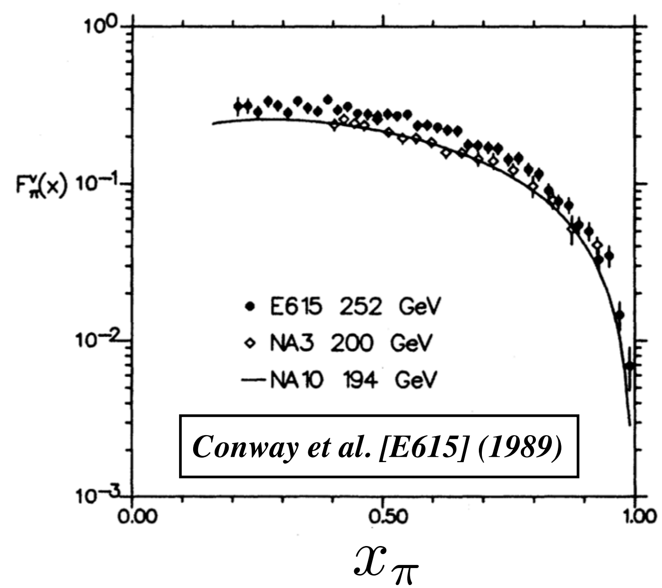
- PDFs in the pion difficult to study experimentally

  → most information has come from pion-tungsten Drell-Yan data (CERN, Fermilab)




  → constrains valence PDFs at $x \gg 0$

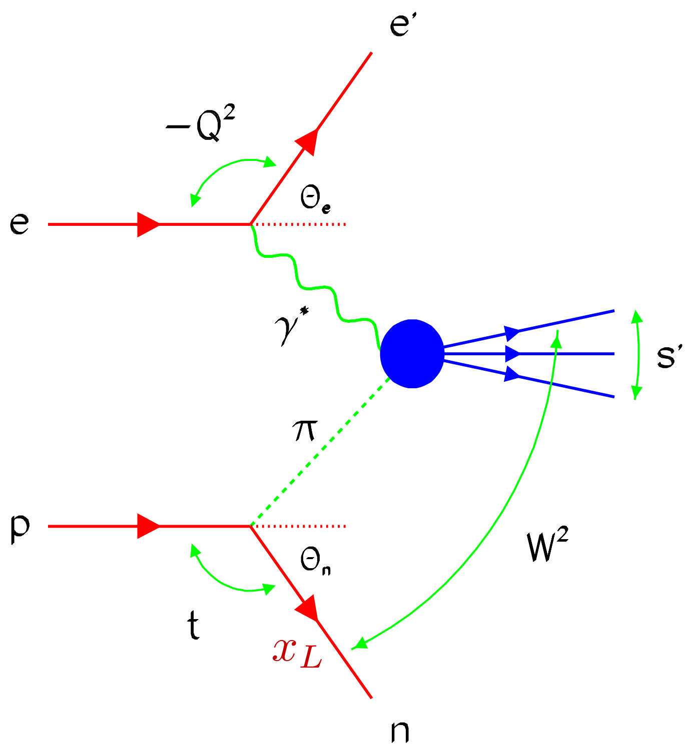


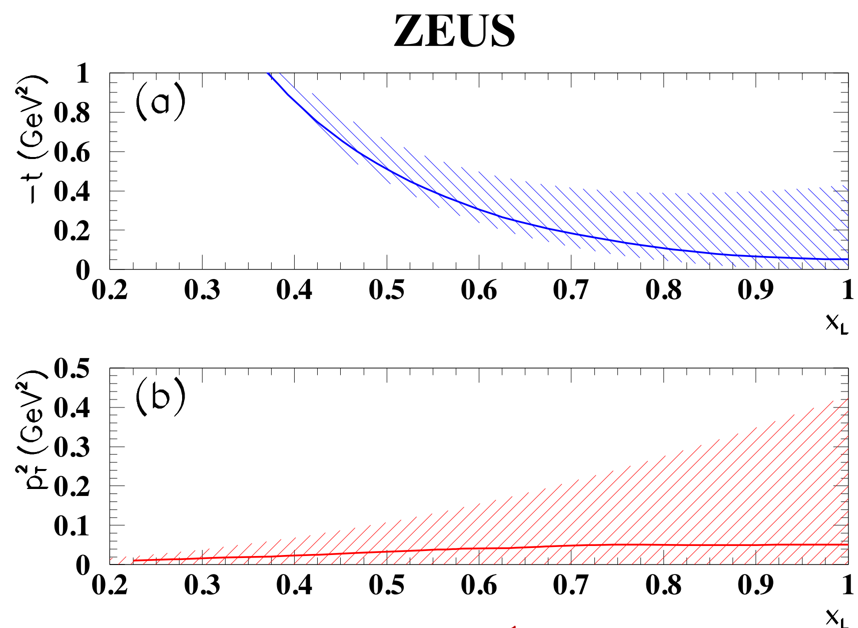




… but pion sea quark & gluon PDFs at small $x$ unconstrained

# PDFs in the pion — leading neutrons

- ZEUS & H1 collaborations at HERA measured spectra of neutrons produced at very forward angles, $\theta_n < 0.8$ mrad



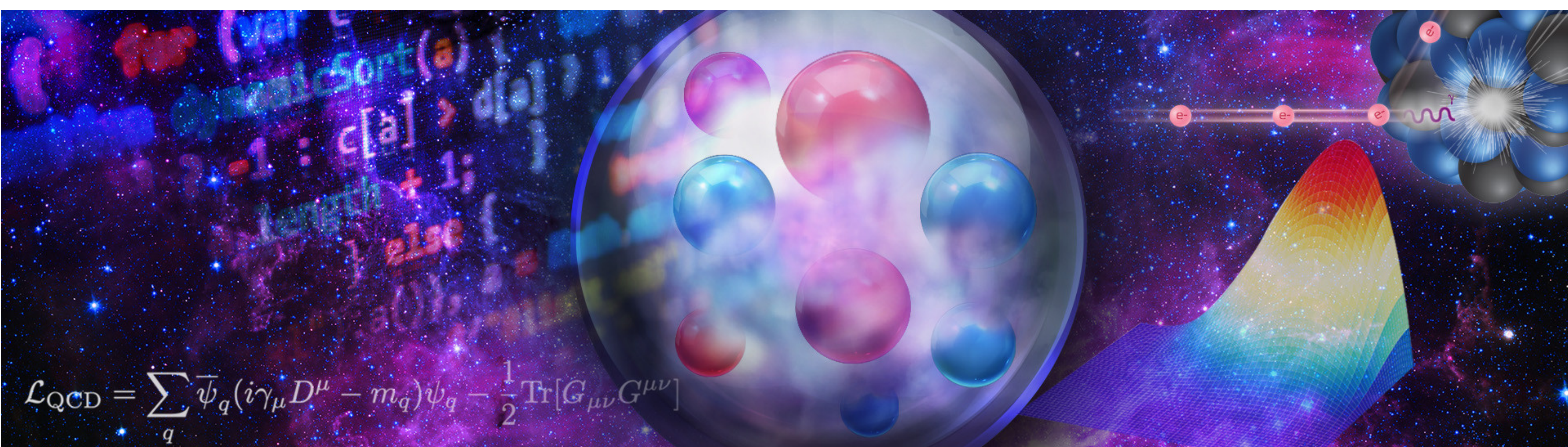

$x_L$

$x_L \approx 1 - y$

→ can data be described within Sullivan process?

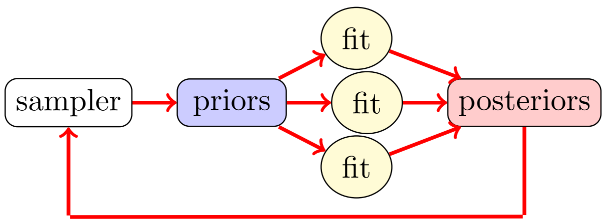
→ first simultaneous fit performed by JAM Collaboration

# Global QCD analysis



# JAM global QCD analysis

- **Theoretical framework**
  - — collinear factorization (NLO)
  - — iterative Monte Carlo
  - — Bayesian sampling of parameter space




- **Traditional functional form for PDFs**

$$f(x) = N\, x^{\alpha} (1-x)^{\beta}\, P(x)$$

polynomial, neural net, …

→ iterate until convergence (posteriors = priors)

- **"Bayesian master formulas" for expectation values and variances for $\mathcal{O}$ with parameters $\vec{a}$**

$$E[\mathcal{O}] = \int d^n a\, \mathcal{P}(\vec{a}|\mathrm{data})\, \mathcal{O}(\vec{a})$$

$$V[\mathcal{O}] = \int d^n a\, \mathcal{P}(\vec{a}|\mathrm{data})\, \left[\mathcal{O}(\vec{a}) - E[\mathcal{O}]\right]^2$$

probability distribution $\mathcal{P}(\vec{a}|\mathrm{data}) \propto \mathcal{L}(\mathrm{data}|\vec{a})\, \pi(\vec{a})$

likelihood function

prior distribution

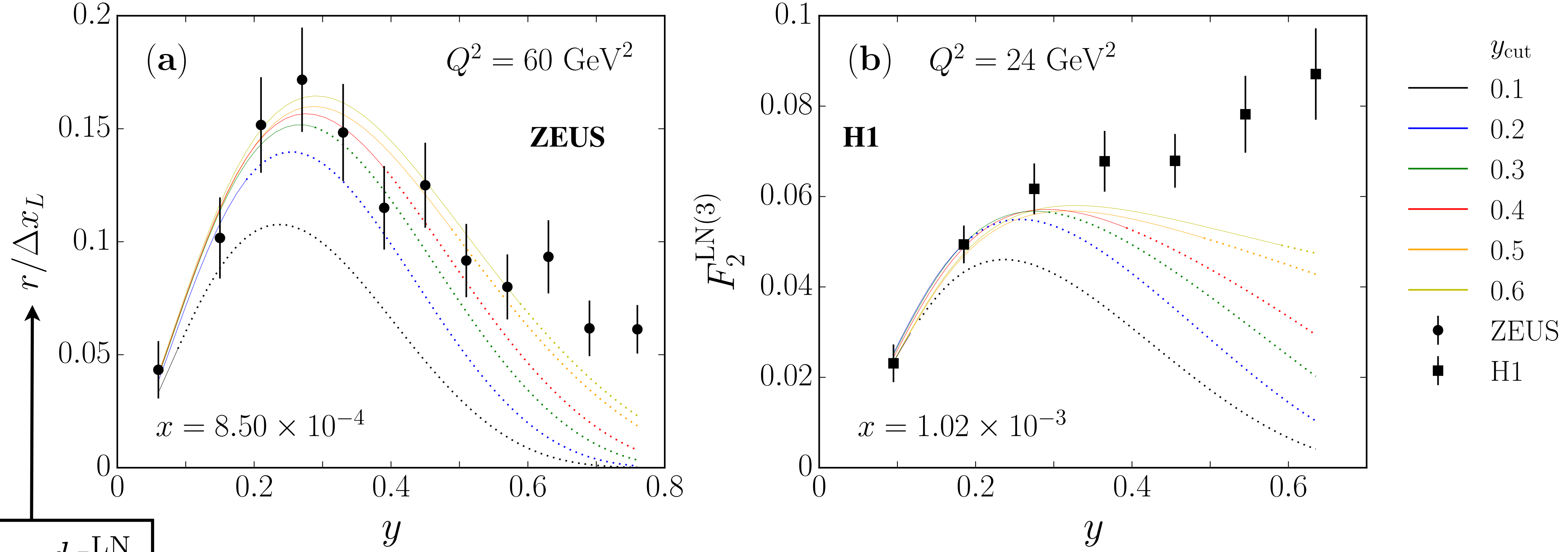
$$\mathcal{L}(\mathrm{data}|\vec{a}) = \exp\left[-\tfrac{1}{2}\chi^2(\vec{a})\right]$$

# Impact of leading neutrons

- Measured LN differential cross section (integrated over $p_T$)

$$\frac{d^3\sigma^{\rm LN}}{dx\,dQ^2\,dy} \sim F_2^{\rm LN(3)}(x,Q^2,y)$$

$2f_{\pi N}(y)\,F_2^{\pi}(x/y,Q^2)$ for $\pi$ exchange



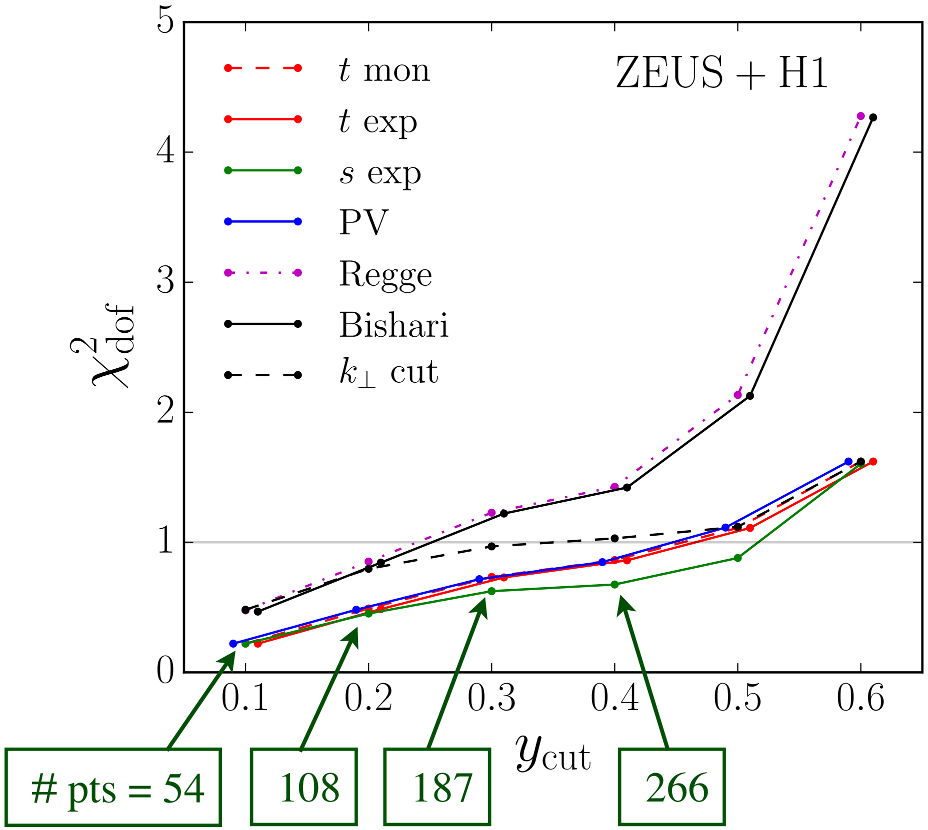

$$r = \frac{d\sigma^{\rm LN}}{d\sigma^{\rm inc}}$$

→ quality of fit depends on range of $y$ fitted

→ expect more non-pionic contributions at larger $y$

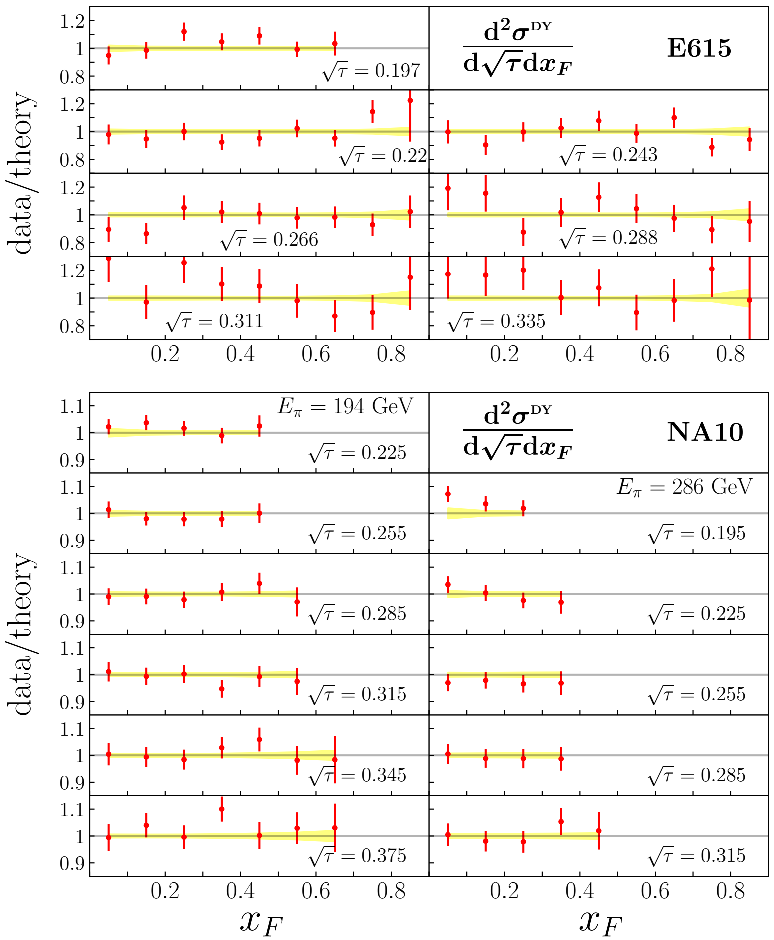
# Impact of leading neutrons

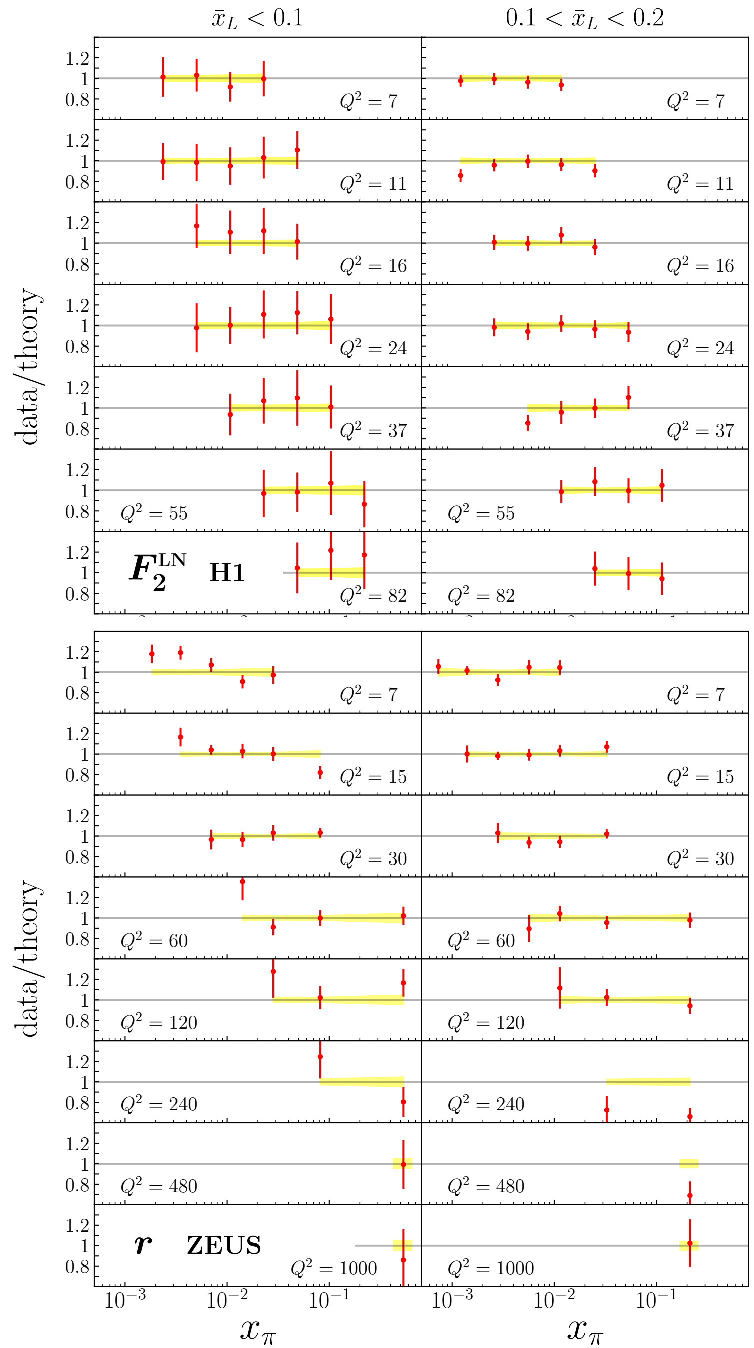
- Combined fit to HERA LN and Drell-Yan data




→ optimize $\chi^2_{\rm dof}$ with maximum number of points that can be described

# Impact of leading neutrons



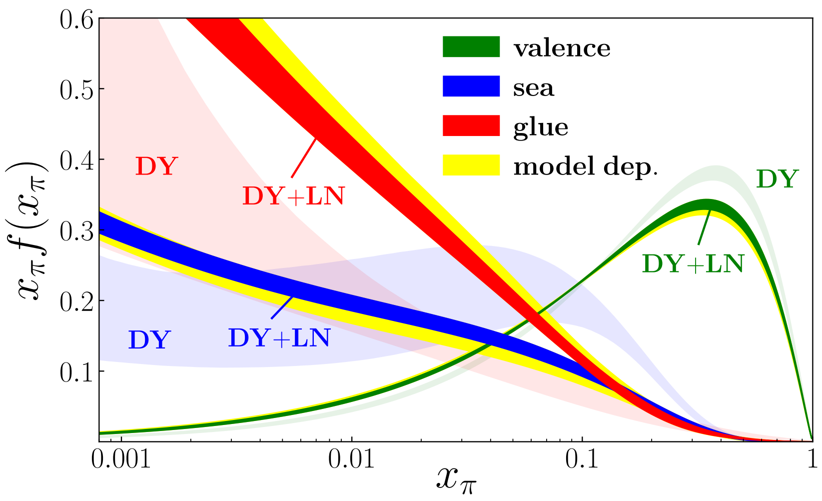




$\bar{x}_L = 1 - x_L$
$= y$

| Process | Experiment (observables) | $N_{dat}$ | $\chi^2_{dat}$ | $n_e$ |
|---|---|---|---|---|
| DY | E615 $(x_F, Q)$ | 61 | 0.85 | 1.08 |
| | NA10 (194 GeV) $(x_F, Q)$ | 36 | 0.52 | 0.88 |
| | NA10 (286 GeV) $(x_F, Q)$ | 20 | 0.78 | 0.83 |
| | E615 $(Q, p_T)$ | 34 | 1.08 | 0.83 |
| | E615 $(x_F, p_T)$ | 49 | 0.85 | 0.50 |
| LN | H1 | 58 | 0.38 | 1.26 |
| | ZEUS | 50 | 1.51 | 0.95 |
| Total | | 308 | 0.85 | |

# Impact of leading neutrons

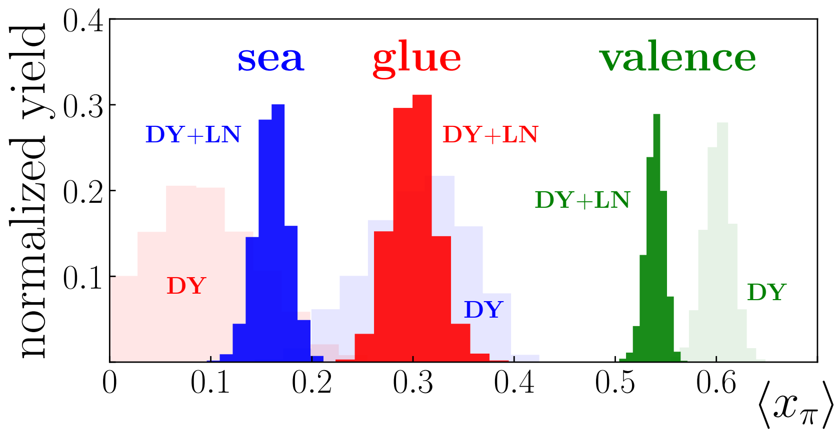
- MC analysis combining pQCD with chiral EFT to fit $\pi N$ Drell-Yan + leading neutron electroproduction data from HERA



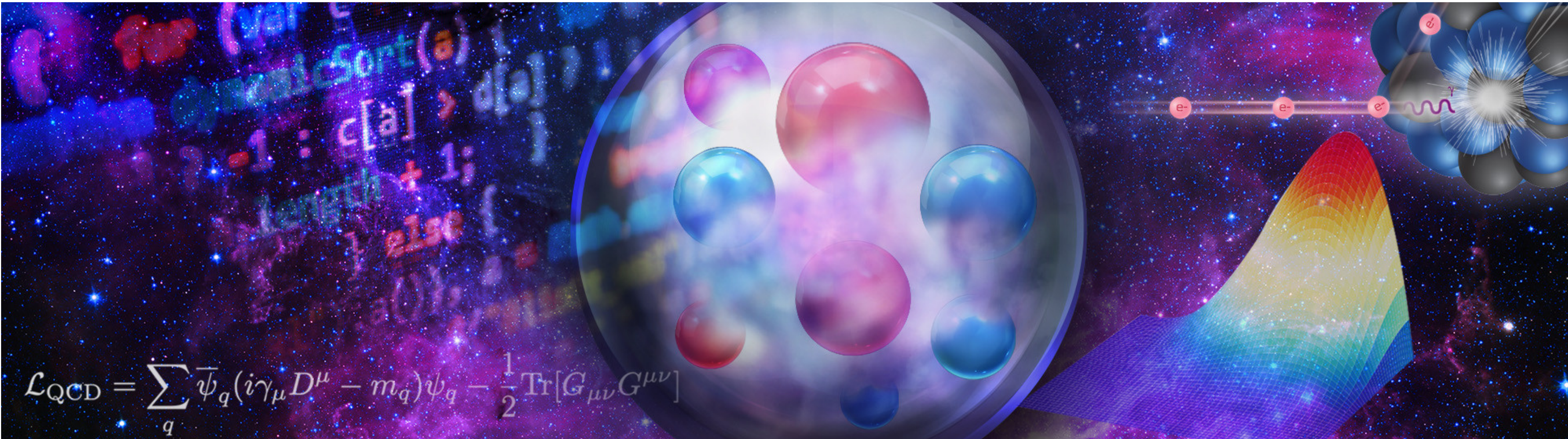

***Barry, Sato, WM, C.-R. Ji***
***PRL 121, 152001 (2018)***

→ significant reduction of glue and sea quark PDF uncertainties

→ larger gluon fraction in the pion than without LN constraint

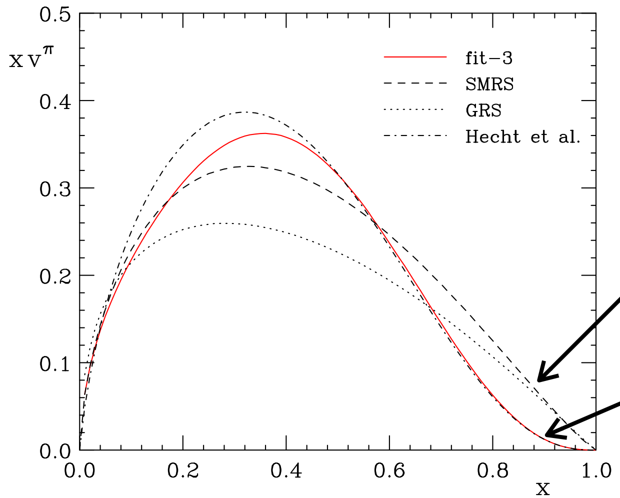


# Large-$x$ resummation



# Pion PDFs with threshold resummation

- $x \to 1$ behavior of pion PDF is controversial: $\sim (1-x)$ or $(1-x)^2$ ?

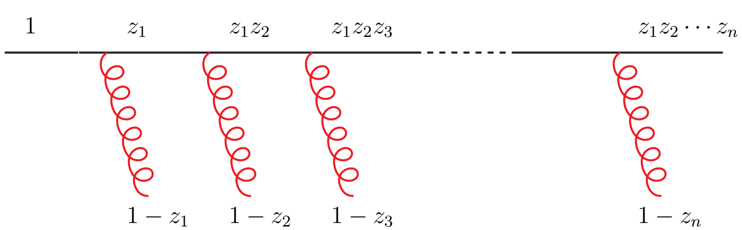



*Aicher, Schafer, Vogelsang (2010)*

no resummation: more consistent with $\sim (1-x)$

with resummation: more consistent with $\sim (1-x)^2$

- Hard scattering coefficient function kinematically enhanced when $z \to 1$ because of (soft) gluon emissions




$\longrightarrow$ effect of resummation on phenomenology?

# Pion PDFs with threshold resummation

- Two ways to construct Mellin moments of differential cross section

→ **Mellin-Fourier**

*Mukherjee, Vogelsang (2006)*
*Bolzoni (2006)*
*Bonvini, Forte, Ridolfi (2011)*

$$\sigma_{\mathrm{MF}}(N,M) \equiv \int_0^1 \mathrm{d}\tau \tau^{N-1} \int_{\log\sqrt{\tau}}^{\log\frac{1}{\sqrt{\tau}}} \mathrm{d}Y e^{iMY} \frac{\mathrm{d}^2\sigma}{\mathrm{d}\tau \mathrm{d}Y}$$

$\tau = Q^2/s$ $\qquad x^0_{\pi,A} = \sqrt{\tau}\, e^{\pm Y}$

→ **double Mellin**

*Westmark, Owens (2017)*
*Lustermans, Michel, Tackmann (2019)*

$$\sigma_{\mathrm{DM}}(N,M) \equiv \int_0^1 \mathrm{d}x^0_\pi \, (x^0_\pi)^{N-1} \int_0^1 \mathrm{d}x^0_A \, (x^0_A)^{M-1} \frac{\mathrm{d}^2\sigma}{\mathrm{d}\tau \mathrm{d}Y}$$

- For MF method, Fourier transform of threshold log $\delta(\hat{Y} - \frac{1}{2}\log(x_\pi/x_A))$ gives factor $\cos\left(M\log(1/\sqrt{z})\right)$

$\hat{Y} = Y - \frac{1}{2}\log(x_\pi/x_A)$
$z = Q^2/x_\pi x_A S$

→ expand cosine $\cos \to 1$ "expansion method"

→ keep cosine factor "cosine method"

used in Aicher, Schafer, Vogelsang (2010) analysis

# Pion PDFs with threshold resummation

## Deriving resummation expressions – MF

Claim: yellow terms give rise to the resummation expressions

$$\frac{C_{q\bar{q}}}{e_q^2} = \delta(1-z)\,\frac{\delta(y)+\delta(1-y)}{2}\left[1+\frac{C_F\alpha_s}{\pi}\left(\frac{3}{2}\ln\frac{M^2}{\mu_f^2}+\frac{2\pi^2}{3}-4\right)\right]$$
$$+\frac{C_F\alpha_s}{\pi}\left\{\frac{\delta(y)+\delta(1-y)}{2}\left[(1+z^2)\left[\frac{1}{1-z}\ln\frac{M^2(1-z)^2}{\mu_f^2 z}\right]_+ +1-z\right]\right.$$
$$\left.+\frac{1}{2}\left[1+\frac{(1-z)^2}{z}y(1-y)\right]\left[\frac{1+z^2}{1-z}\left(\left[\frac{1}{y}\right]_+ + \left[\frac{1}{1-y}\right]_+\right)-2(1-z)\right]\right\}$$

Claim: Red terms are power suppressed in $(1-z)$ and don't contribute to the same order as the yellow terms

## Generalized threshold resummation

Rewrite the $(z,y)$ coefficients in terms of $(z_a,z_b)$, and for the red term:

$$z_{a,b} = \frac{\sqrt{\tau}e^{\pm Y}}{x_{a,b}}$$

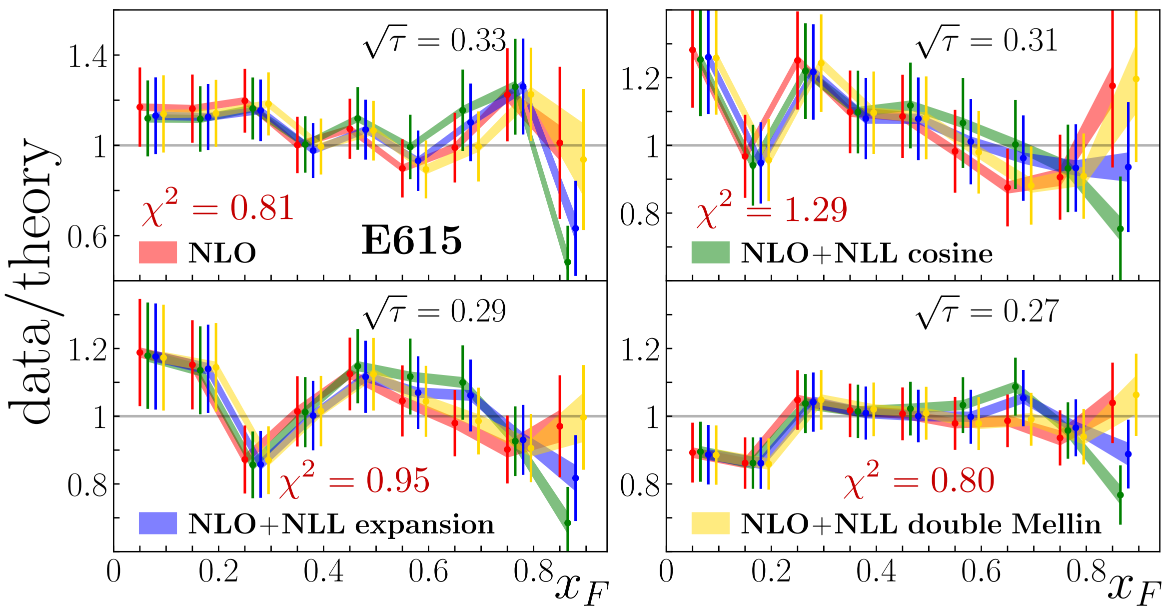
$$\mathrm{d}z\,\mathrm{d}y\,\frac{1}{1-z}\left(\frac{1}{y}+\frac{1}{1-y}\right) = \mathrm{d}z_a\mathrm{d}z_b\,\frac{1}{(1-z_a)(1-z_b)}\left[1+\mathcal{O}(1-z_a,1-z_b)\right].$$

This is *not* power suppressed in $(1-z_a)$ or $(1-z_b)$ - cannot disentangle $(z,y)$

Double Mellin method, however, includes these terms

*Lustermans, Michel, Tackmann (2019)*

# Pion PDFs with threshold resummation




→ good fits to data for most resummation methods (slightly worse for "cosine" method)

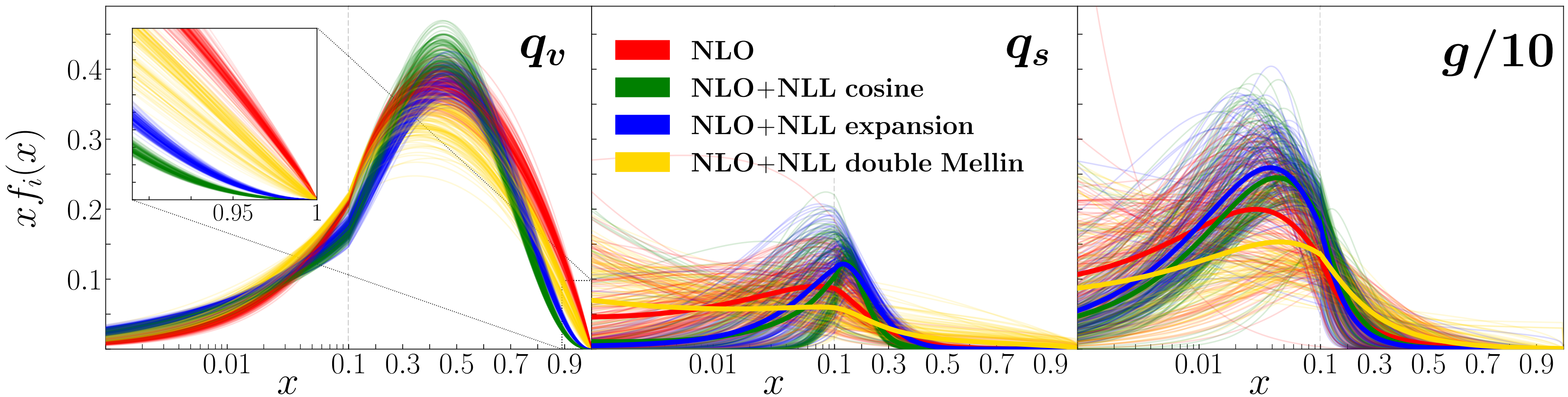
→ valence quarks give ~ 5% momentum fractions to gluons after resummation (for all methods)

| resummation method | $\langle x \rangle_v$ | $\langle x \rangle_s$ | $\langle x \rangle_g$ |
|---|---|---|---|
| NLO | 0.53(2) | 0.14(4) | 0.34(6) |
| NLO+NLL cosine | 0.47(2) | 0.14(5) | 0.39(6) |
| NLO+NLL expansion | 0.46(2) | 0.16(5) | 0.38(6) |
| NLO+NLL double Mellin | 0.46(3) | 0.15(7) | 0.40(5) |

*Barry, Ji, Sato, WM*
*PRL 127, 232001 (2021)*

# Pion PDFs with threshold resummation

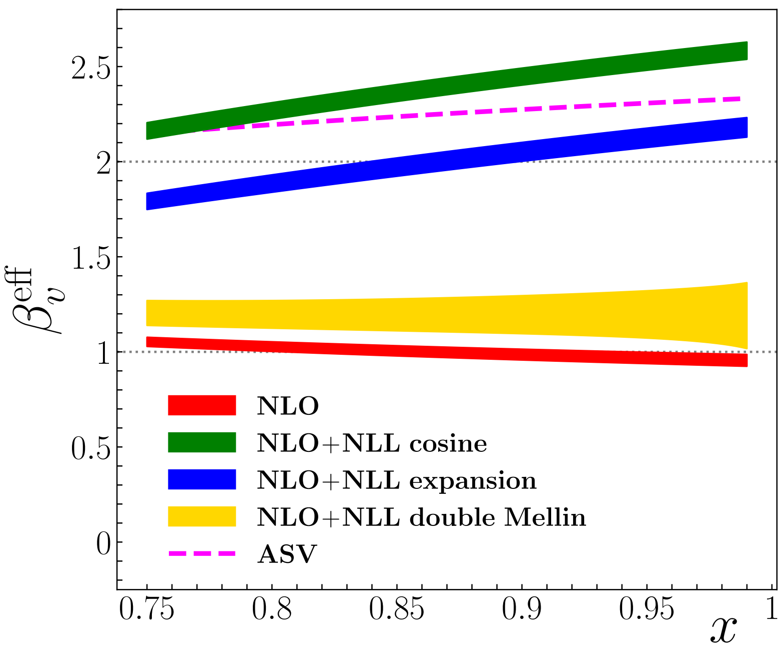
→ redistribution of $x$ dependence




→ effective exponent

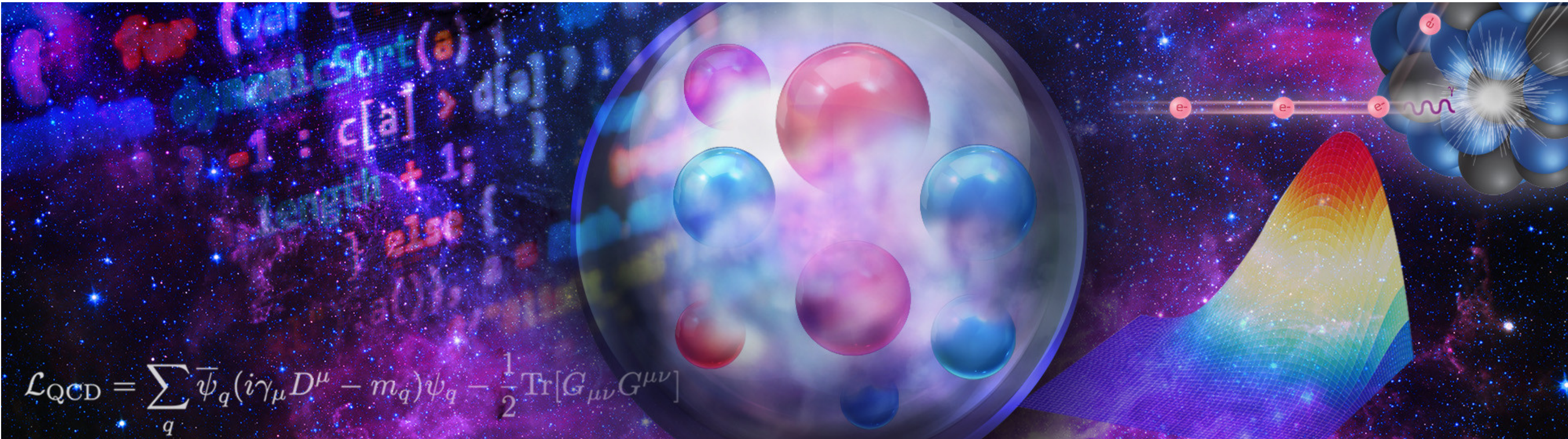
$$\beta_v^{\text{eff}}(x,Q) = \frac{\partial \log |q_v(x,Q)|}{\partial \log(1-x)}$$

→ double Mellin method (theoretically preferred) gives similar results to fixed-order NLO — more consistent with $\beta_v^{\text{eff}} \sim 1$




*Barry, Ji, Sato, WM*
*PRL 127, 232001 (2021)*

# Impact of lattice QCD data



# Pseudo Ioffe-time distributions

## Reduced pseudo Ioffe-time distribution

$$\mathfrak{M}(\nu, z^2) = \frac{\mathcal{M}(\nu, z^2)}{\mathcal{M}(0, z^2)}$$

where

$$\mathcal{M}(\nu, z^2) = \frac{1}{2p^0} \langle p|\bar{\psi}(0)\gamma^0 \mathcal{W}(z;0)\psi(z)|p\rangle$$

UV divergences cancel in ratio

Ioffe time $\nu = p \cdot z$

gauge link of length $z$

*Radyushkin PRD 96, 034025 (2017)*

## Factorization of Rp-ITD via OPE into PDF and hard coefficient function

$$\mathrm{Re}\mathfrak{M}(\nu, z^2) = \int_0^1 \mathrm{d}x q_v(x, \mu_{\mathrm{lat}}) \mathcal{C}^{\text{Rp-ITD}}(x\nu, z^2, \mu_{\mathrm{lat}})$$

← leading twist

$$+ z^2 B_1(\nu) + \frac{a}{|z|} P_1(\nu) + e^{-m_\pi (L-z)} F_1(\nu) + \ldots$$

power corrections — lattice spacing — finite volume

# Pseudo Ioffe-time distributions

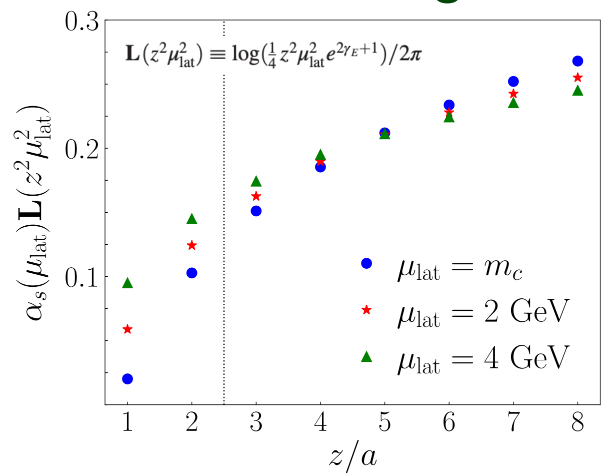
## Lattice data from JLab HadStruc collaboration

→ isotropic clover Fermion action

| ID | $a$ (fm) | $m_\pi$ (MeV) | $\beta$ | $L^3 \times T$ | $p_1$ (MeV) |
|---|---|---|---|---|---|
| a127m413 | 0.127(2) | 413(4) | 6.1 | $24^3 \times 64$ | 406(6) |
| a127m413L | 0.127(2) | 413(5) | 6.1 | $32^3 \times 96$ | 305(5) |

***Sufian et al., PRD 102, 054508 (2020)***

scale setting




## Explore several scenarios

→ "A" experiment only

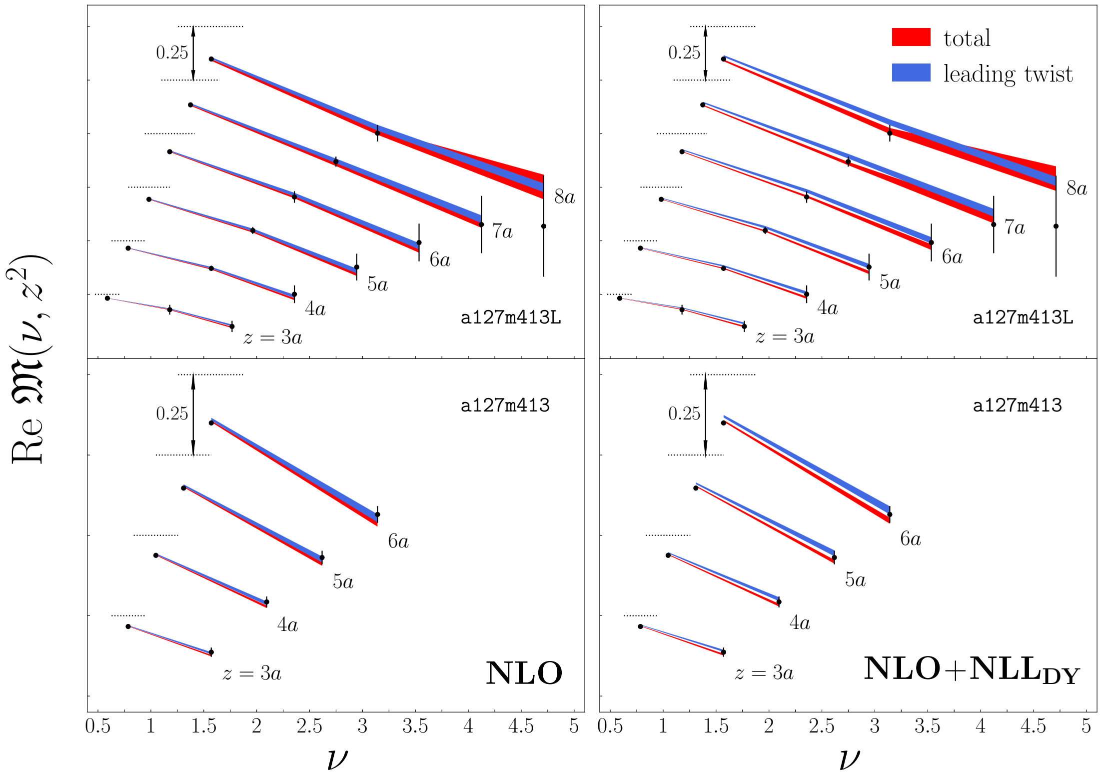
→ "B" experiment + lattice (leading twist only)

→ "C" experiment + lattice (full fit)

| | | | Scenario A | | Scenario B | | Scenario C | |
|---|---|---|---|---|---|---|---|---|
| | | | NLO | +NLL$_{\rm DY}$ | NLO | +NLL$_{\rm DY}$ | NLO | +NLL$_{\rm DY}$ |
| Process | Experiment | $N_{\rm dat}$ | $\bar{\chi}^2$ | | $\bar{\chi}^2$ | | $\bar{\chi}^2$ | |
| DY | E615 | 61 | 0.84 | 0.82 | 0.83 | 0.82 | **0.84** | **0.82** |
| | NA10 (194 GeV) | 36 | 0.53 | 0.53 | 0.52 | 0.54 | **0.51** | **0.53** |
| | NA10 (286 GeV) | 20 | 0.80 | 0.81 | 0.78 | 0.79 | **0.74** | **0.81** |
| LN | H1 | 58 | 0.36 | 0.35 | 0.39 | 0.39 | **0.38** | **0.37** |
| | ZEUS | 50 | 1.56 | 1.48 | 1.62 | 1.69 | **1.59** | **1.62** |
| Rp-ITD | a127m413L | 18 | ... | ... | 1.04 | 1.06 | **1.05** | **1.04** |
| | a127m413 | 8 | ... | ... | 1.98 | 2.63 | **1.00** | **1.18** |
| Total | | 251 | 0.82 | 0.80 | 0.89 | 0.92 | **0.85** | **0.86** |

***Barry et al., PRD 105, 114051 (2022)***

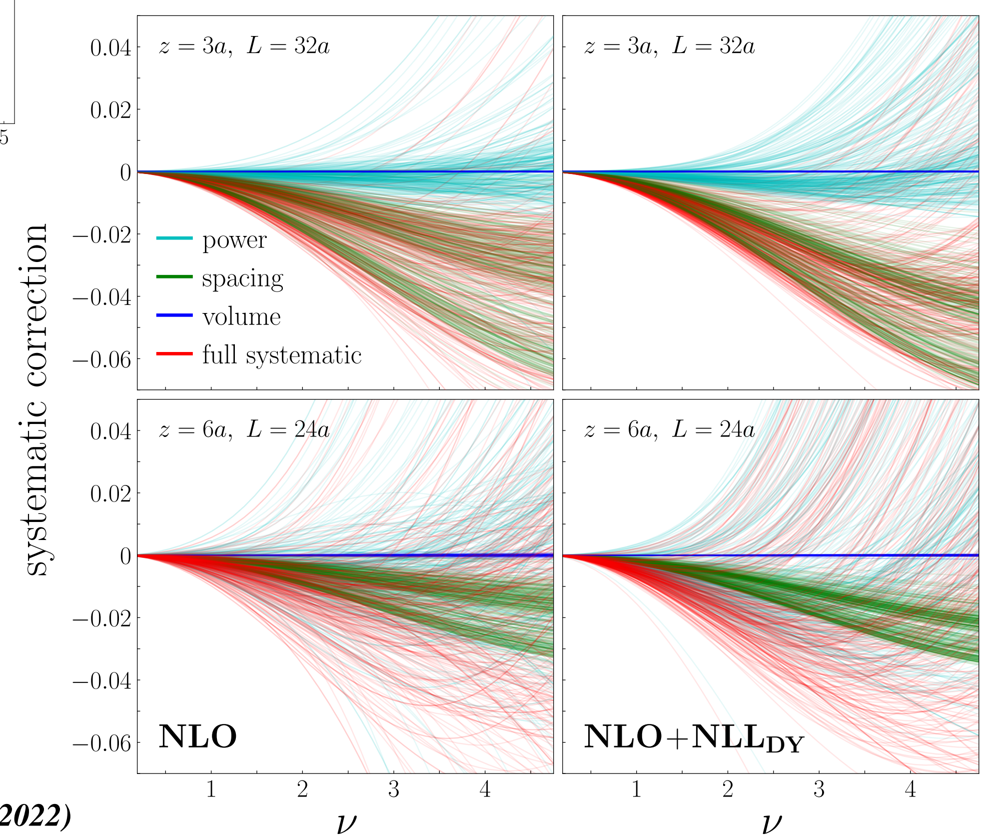
# Pseudo Ioffe-time distributions




→ systematic corrections generally negative, with magnitude growing with increasing $z$ and $\nu$

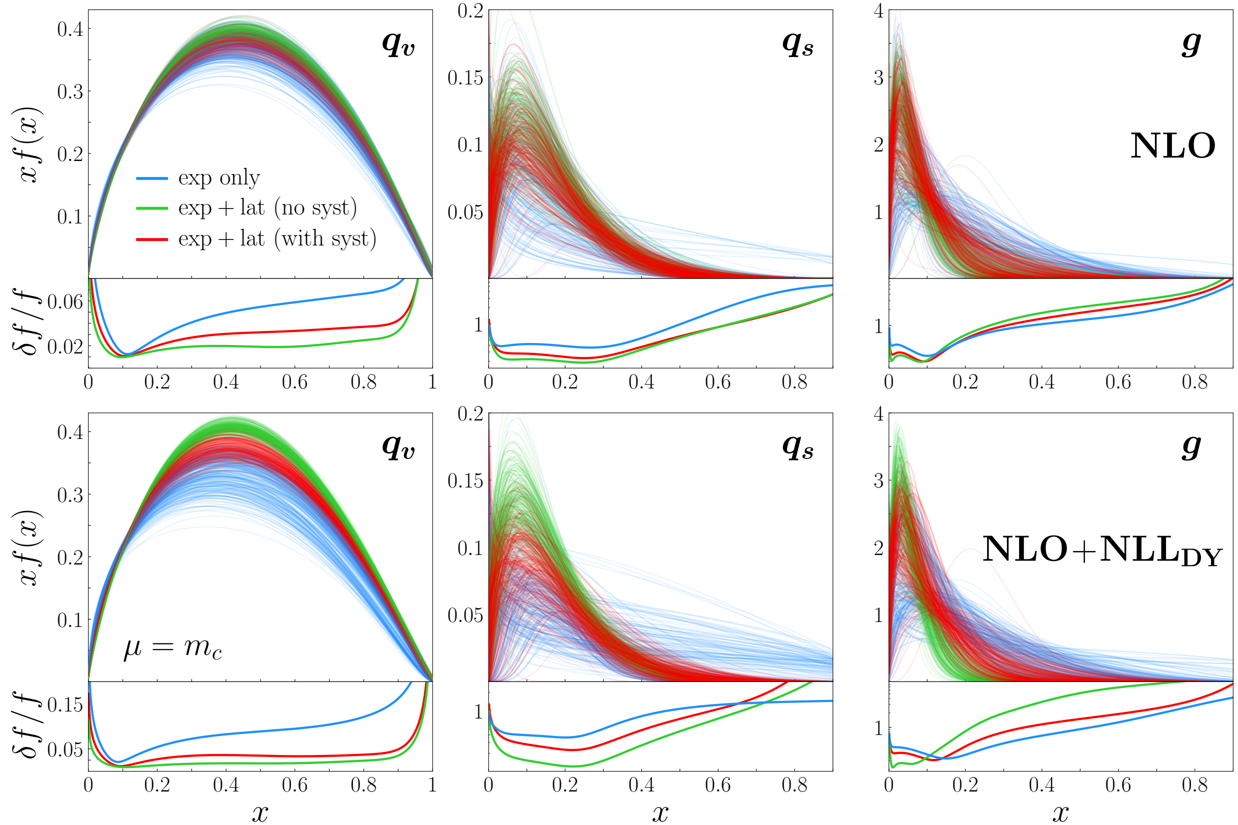
→ systematic contributions need to be included for reliable extraction of leading twist PDFs

→ dependence on order of calculation may suggest issue with convergence of perturbative expansion




***Barry et al., PRD 105, 114051 (2022)***

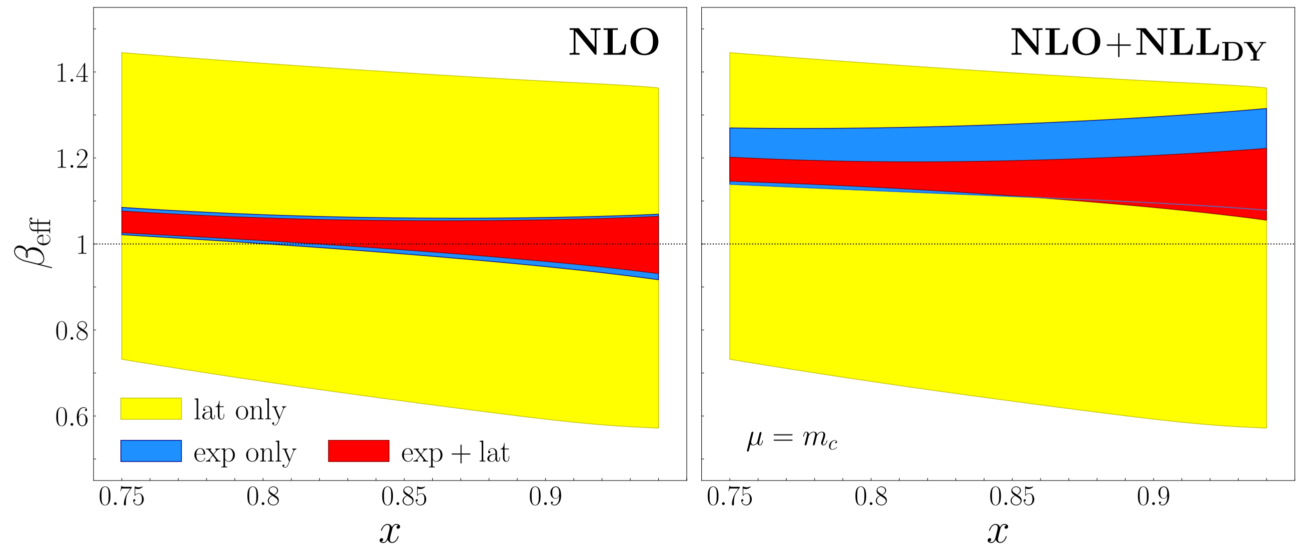
# Pseudo Ioffe-time distributions



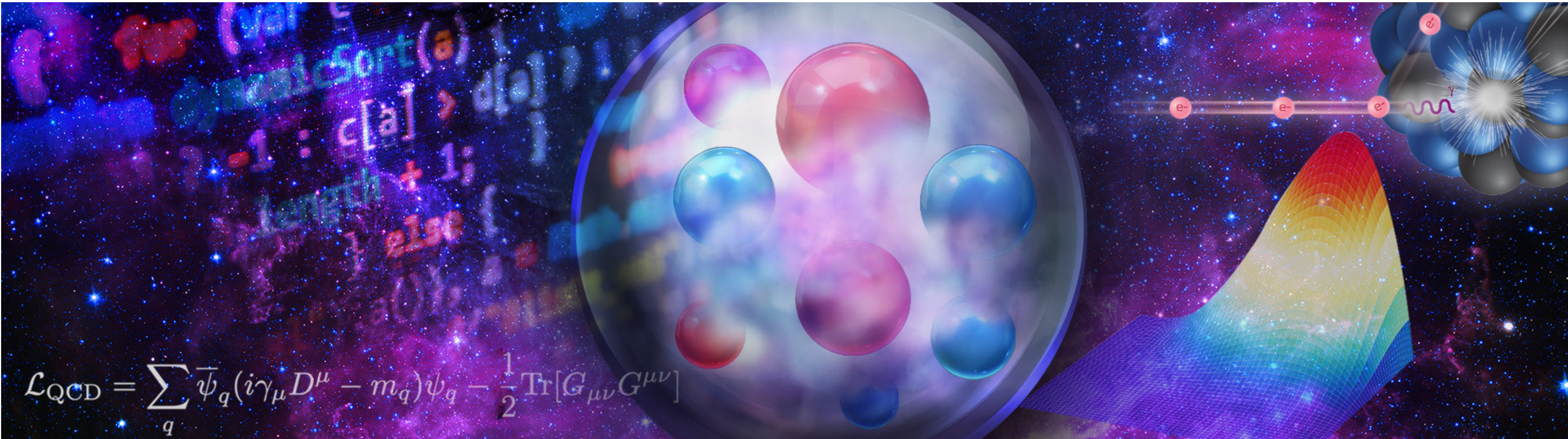

→ PDFs from "exp only" more consistent with "exp+lat" including systematic corrections

→ including lattice data reduces PDF uncertainty for quark sea and glue at high $x$ (indirectly through momentum sum rule)

→ lattice data consistent with experiment, and with $\beta^{\rm eff} \sim 1$



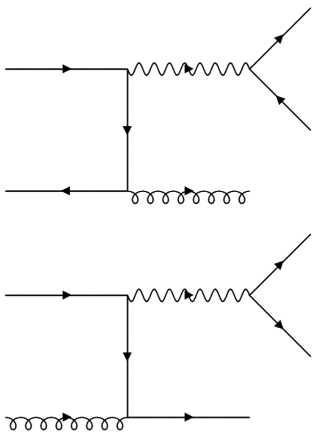
# Transverse momentum



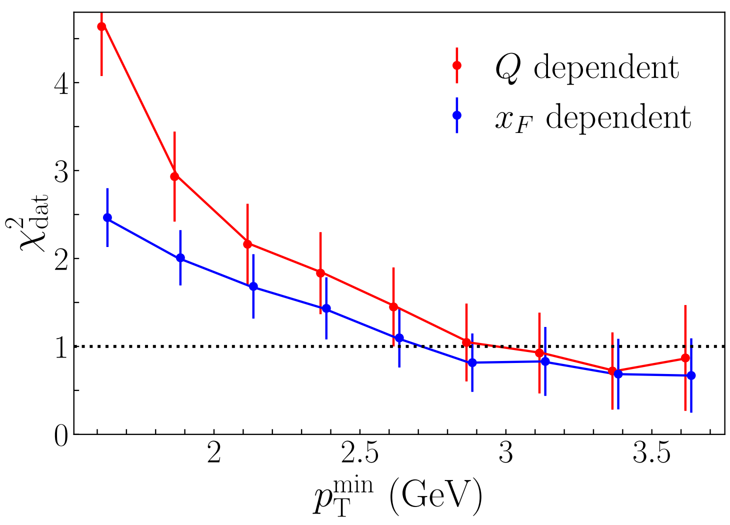
# Transverse momentum dependence

- E615 also collected data differential in transverse momentum — never before included in global QCD analysis

→ large-$p_T$ photon requires hard gluon to recoil against, offering sensitivity to gluon PDF in pion at high $x$

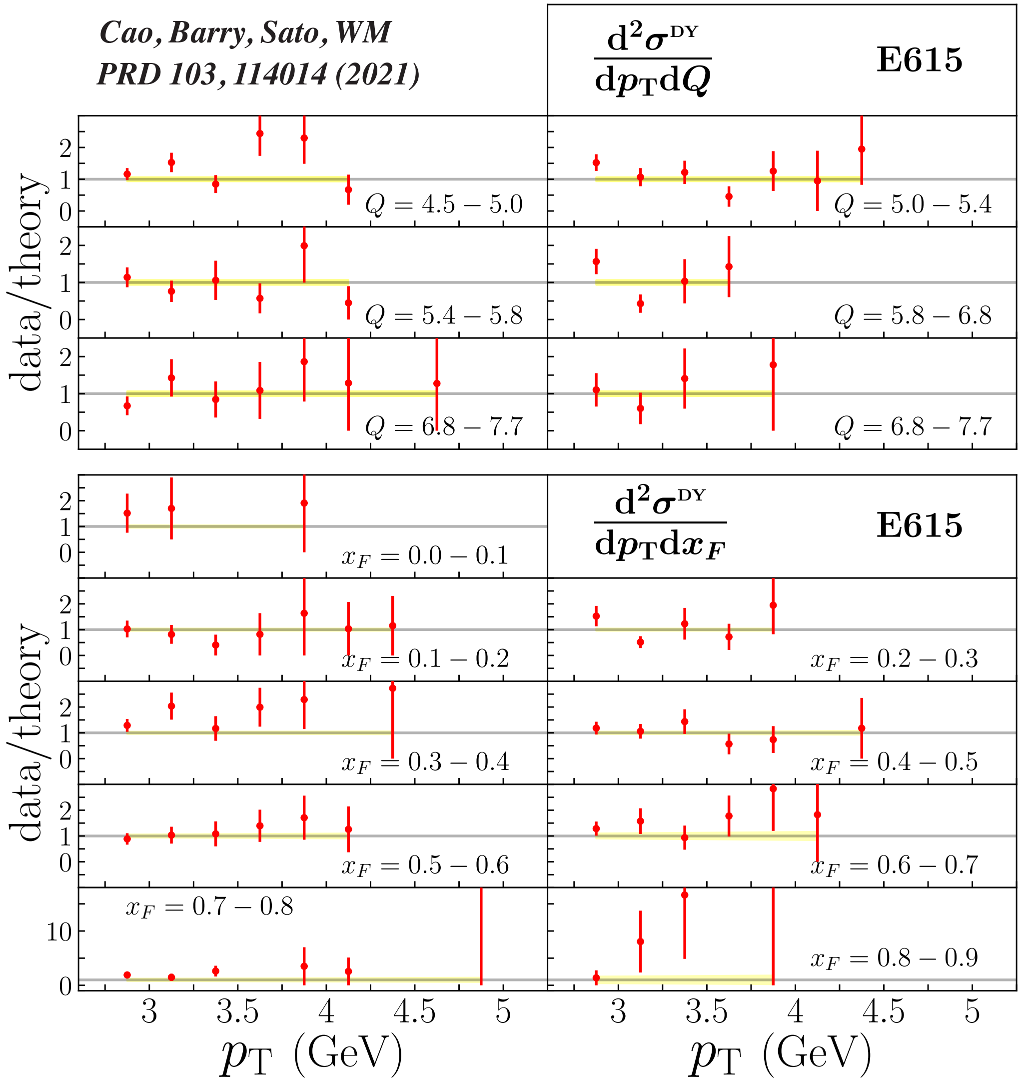


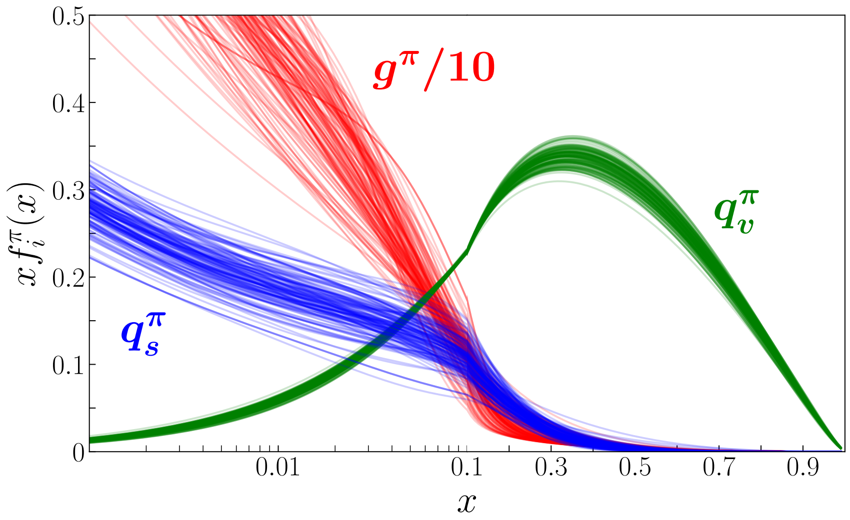
→ what is range of applicability of pQCD description of $p_T$ distribution?

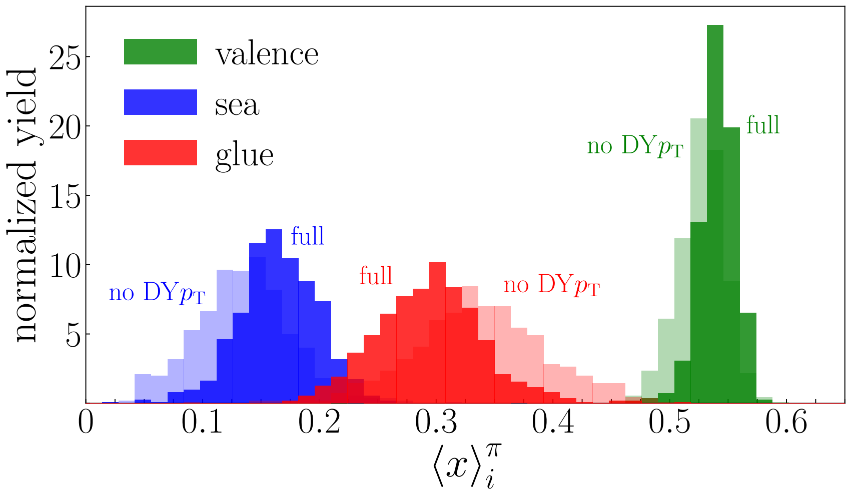



*Cao, Barry, Sato, WM*
*PRD 103, 114014 (2021)*

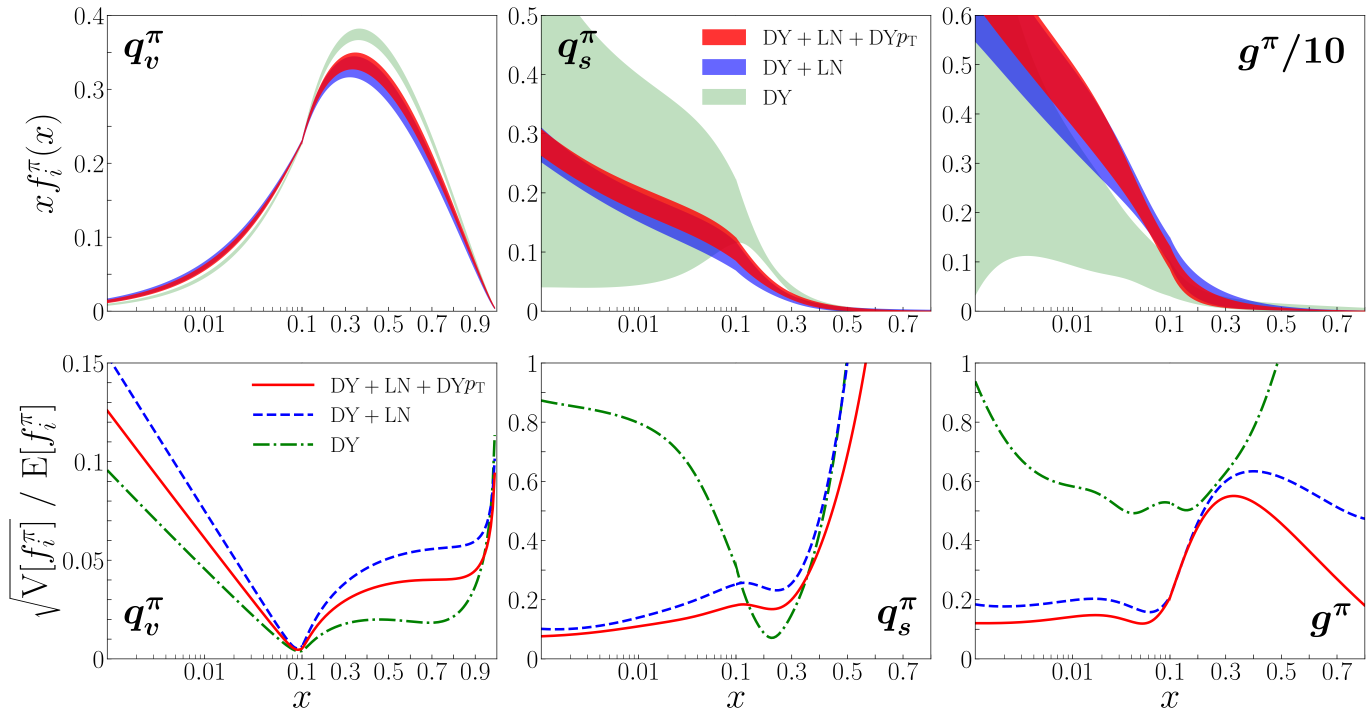
# Transverse momentum dependence



# Transverse momentum dependence




*Cao, Barry, Sato, WM*
*PRD 103, 114014 (2021)*

⟶ some reduction in gluon PDF uncertainty
— need higher precision data

⟶ first time that one has been able to describe $p_T$ spectra $(p_T > 2.7\,\mathrm{GeV})$ spectra in terms of collinear PDFs

⟶ opens path to pion TMD studies

# Pion TMDs

- Unpolarized TMD PDF is $k_T$-space Fourier transform of light-cone correlator (with momentum $P$)

$$\tilde{f}_{q/\mathcal{N}}(x, b_T) = \int \frac{\mathrm{d}b^-}{4\pi}\, e^{-ixP^+b^-}\,\mathrm{Tr}\big[\langle \mathcal{N}\,|\,\bar{\psi}_q(b)\gamma^+\mathcal{W}(b,0)\psi_q(0)\,|\,\mathcal{N}\,\rangle\big]$$

impact parameter
$b \equiv (b^-, 0^+, \boldsymbol{b}_T)$

⟶ small-$b_T$ behavior given in terms of collinear PDFs

- At small $q_T$ TMD factorization allows one to write cross section as

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}Q^2\mathrm{d}y\mathrm{d}q_T^2} = \sum_q \mathcal{H}_{q\bar{q}}^{\mathrm{DY}}(Q,\mu_Q) \int \frac{\mathrm{d}^2\boldsymbol{b}_T}{(2\pi)^2}\, e^{i\boldsymbol{b}_T\cdot\boldsymbol{q}_T}\;\; \tilde{f}_{q/\mathcal{N}}(x_\mathcal{N}, b_T;\mu_Q, Q^2)\, \tilde{f}_{\bar{q}/A}(x_A, b_T;\mu_Q, Q^2)$$

factorization scale ($\mu_Q$) — rapidity scale ($Q^2$)

where process-independent TMD PDF is

$$\tilde{f}_{q/\mathcal{N}(A)}(x, b_T;\mu_Q, Q^2) = (C\otimes f)_{q/\mathcal{N}(A)}(x;b_*)\;\exp\left\{-g_{q/\mathcal{N}(A)}(x,b_T) - g_K(b_T)\ln\frac{Q}{Q_0} - S(b_*, Q, \mu_Q)\right\}$$

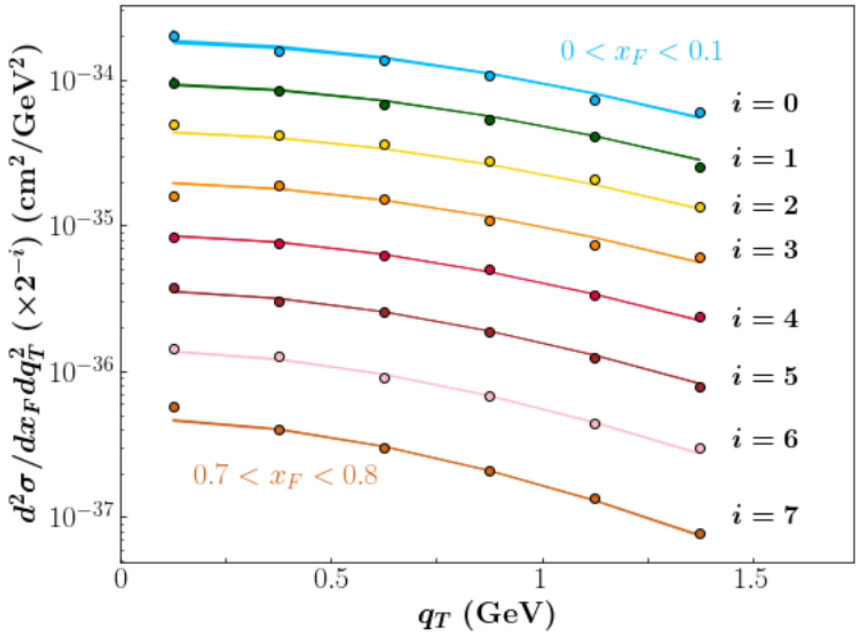
"$b_*$ prescription" $\quad b_* = b_T/\sqrt{1+b_T^2/b_{\max}^2}$

*Collins, Soper, Sterman*
*NPB 250, 199 (1985)*

# Pion TMDs

- Simultaneous fit of pion collinear PDF and TMD PDF

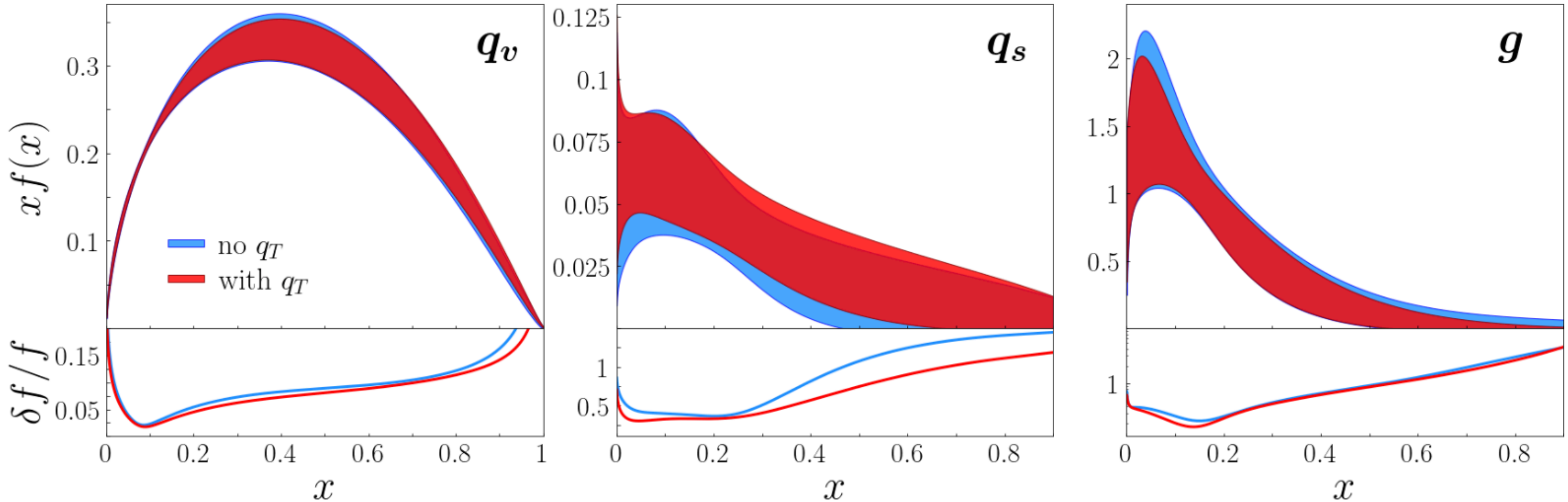
| Process | Experiment | $\sqrt{s}$ (GeV) | $\chi^2/N$ | $Z$-score |
|---|---|---|---|---|
| **TMD** | | | | |
| $q_T$-dep. $pA$ DY | E288 [89] | 19.4 | 0.93 | 0.25 |
| $pA \to \mu^+\mu^- X$ | E288 [89] | 23.8 | 1.33 | 1.54 |
| | E288 [89] | 24.7 | 0.95 | 0.23 |
| | E605 [90] | 38.8 | 1.07 | 0.39 |
| | E772 [91] | 38.8 | 2.41 | 5.74 |
| (Fe/Be) | E866 [92] | 38.8 | 1.07 | 0.29 |
| (W/Be) | E866 [92] | 38.8 | 0.89 | 0.11 |
| $q_T$-dep. $\pi A$ DY | E615 [93] | 21.8 | 1.61 | 2.58 |
| $\pi W \to \mu^+\mu^- X$ | E537 [94] | 15.3 | 1.11 | 0.57 |
| **collinear** | | | | |
| $q_T$-integr. DY | E615 [93] | 21.8 | 0.86 | 0.76 |
| $\pi W \to \mu^+\mu^- X$ | NA10 [95] | 19.1 | 0.54 | 2.27 |
| | NA10 [95] | 23.2 | 0.91 | 0.18 |
| leading neutron | H1 [96] | 318.7 | 0.36 | 4.61 |
| $ep \to enX$ | ZEUS [97] | 300.3 | 1.48 | 2.16 |
| Total | | | 1.15 | 2.55 |




→ ~ 600 data points (~ 400 transverse momentum dependent)

# Pion TMDs

- Simultaneous fit of pion collinear PDF and TMD PDF

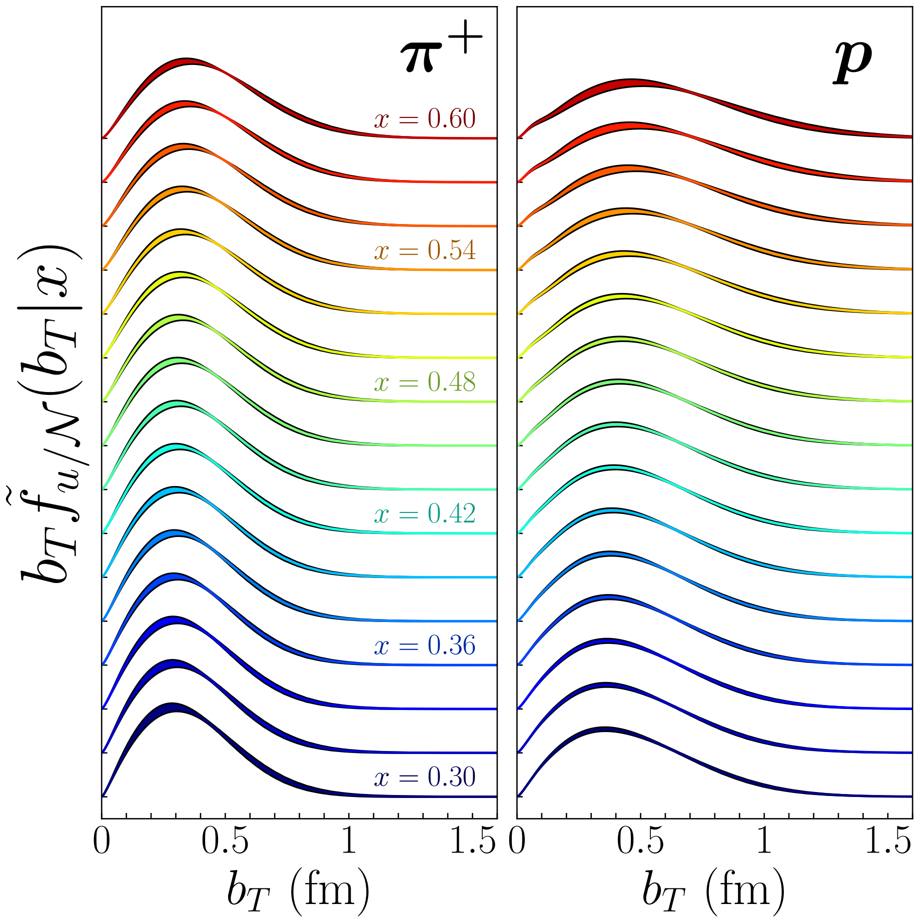



⟶ little impact of transverse momentum data on collinear PDFs

# Pion TMDs

- From Bayes' theorem, define "conditional density" dependent on "$b_T$ given $x$"

$$\tilde{f}_{q/\mathcal{N}}(b_T|x;Q,Q^2) \equiv \frac{\tilde{f}_{q/\mathcal{N}}(x,b_T;Q,Q^2)}{\int \mathrm{d}^2\boldsymbol{b}_T\, \tilde{f}_{q/\mathcal{N}}(x,b_T;Q,Q^2)}$$




⟶ $u$ quark in pion narrower than $u$ quark in proton

⟶ broadening distribution with increasing $x$

*Barry, Gamberg, WM, Moffat, Pitonyak, Prokudin, Sato*
*arXiv:2302.01192*

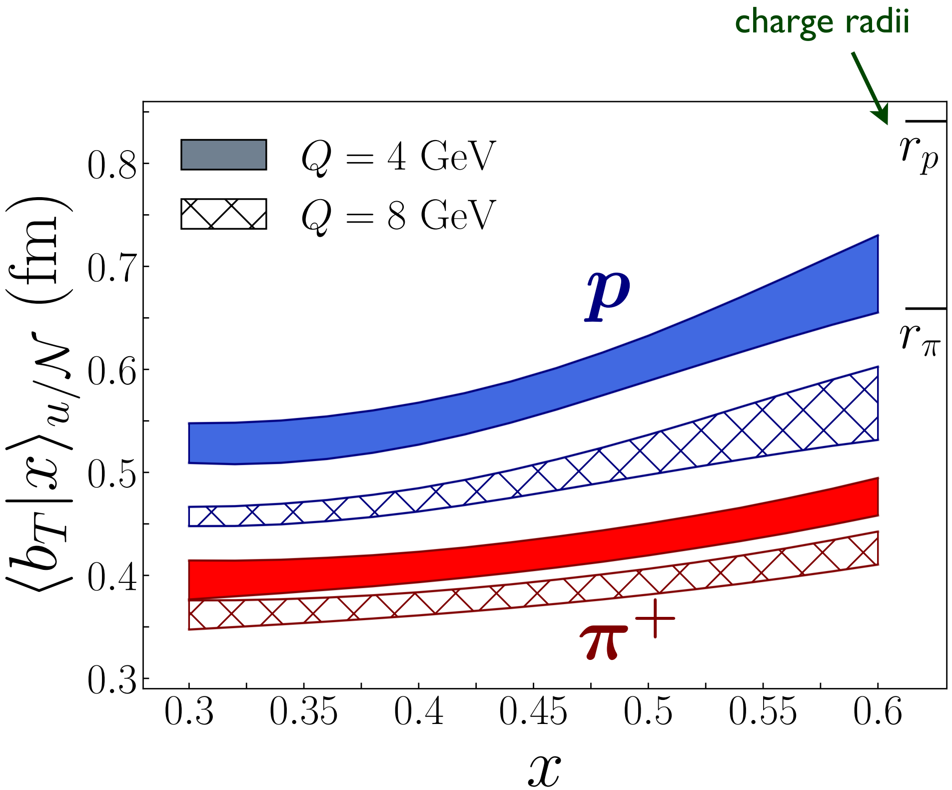
# Pion TMDs

- Transverse separation of quarks in pion

⟶ conditional average $b_T$ of quark $q$ in hadron $\mathcal{N}$

$$\langle b_T|x\rangle_{q/\mathcal{N}} = \int \mathrm{d}^2\boldsymbol{b}_T\, b_T\, \tilde{f}_{q/\mathcal{N}}(b_T|x;Q,Q^2)$$

⟶ on average, find ~ 20% <u>reduction</u> of $u$-quark transverse correlations in pions relative to protons
(at ~ 5σ –7σ level)

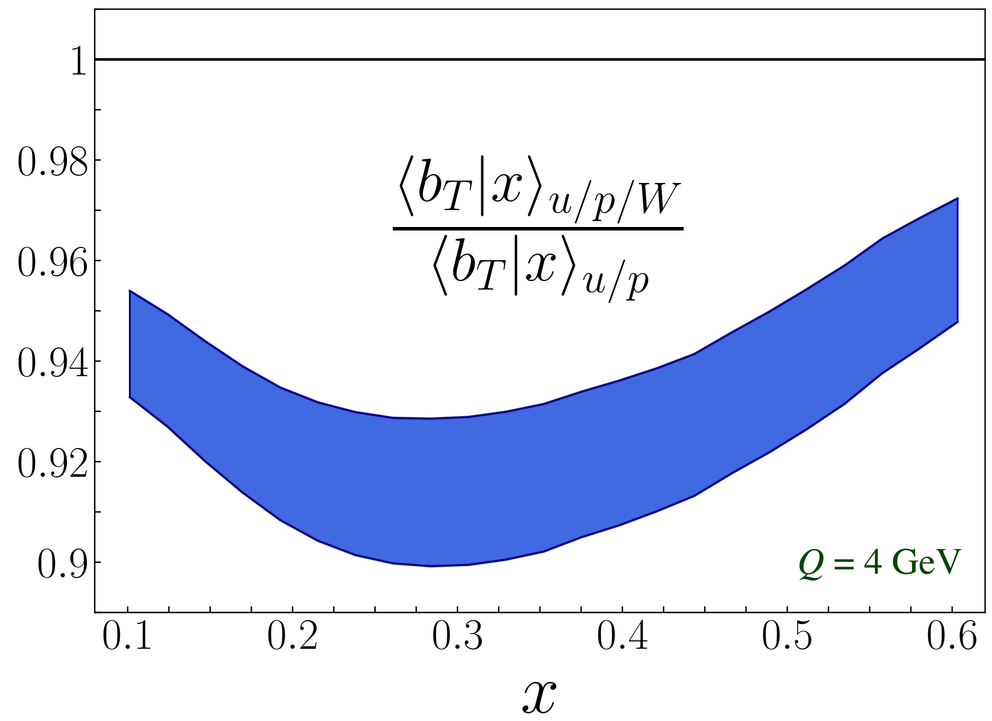



*Barry, Gamberg, WM, Moffat, Pitonyak, Prokudin, Sato*
*arXiv:2302.01192*

# Pion TMDs

## Transverse EMC effect

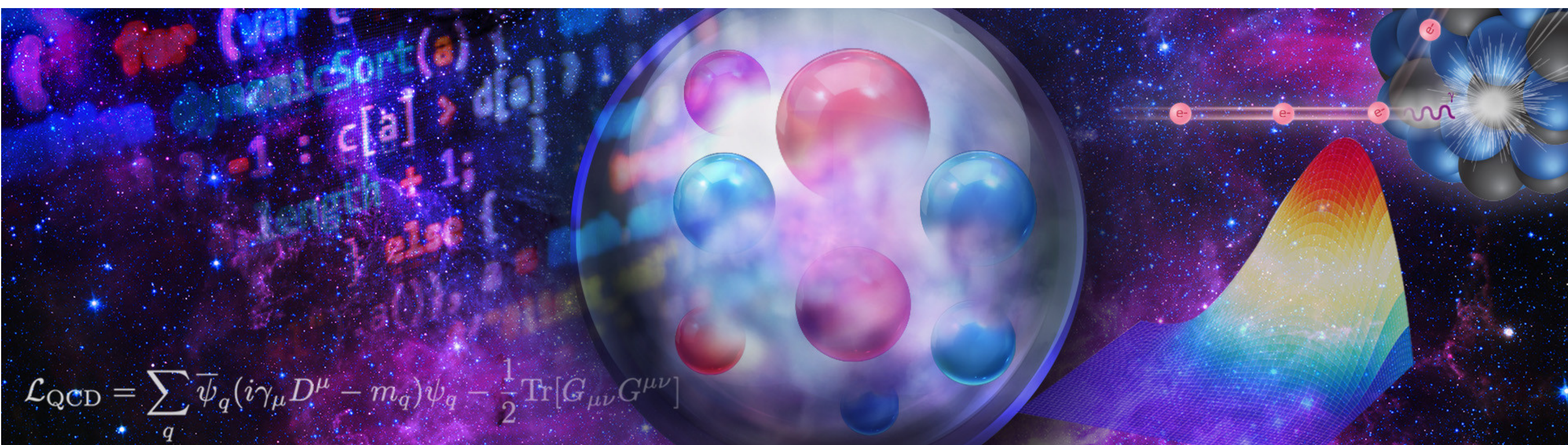
→ ratio of conditional average $b_T$ of $u$-quark in proton in tungsten relative to that in free proton




→ ratio ~ 5% - 10% below unity for entire range of $x$ covered

*Barry, Gamberg, WM, Moffat, Pitonyak, Prokudin, Sato*
*arXiv:2302.01192*

# Outlook


$\mathcal{L}_{\mathrm{QCD}} = \sum_q \overline{\psi}_q (i\gamma_\mu D^\mu - m_q)\psi_q - \frac{1}{2}\mathrm{Tr}[G_{\mu\nu}G^{\mu\nu}]$

# Outlook

- JAM global QCD analysis allows simultaneous description of Drell-Yan ($p_T$-integrated and differential) and leading neutron electroproduction data in terms of universal set of pion PDFs

  → map out pion structure from low $x$ to high $x$

- Global QCD analysis with threshold resummation

  → suggests ~ $(1-x)$ behavior at large $x$

  → consistent with lattice QCD data

- Successful extension to incorporate transverse momentum

  → (high $p_T$) more precise data needed to constrain gluon PDF at high $x$

  → (low $p_T$) new avenue for study of pion TMDs / 3D structure

- Future experimental avenues

  → TDIS (Tagged DIS) experiment at JLab $(e\, d \longrightarrow e\, p\, X)$

  → AMBER (pion & kaon beams) experiment at CERN