The Axial Form Factor Extracted from Elementary Targets

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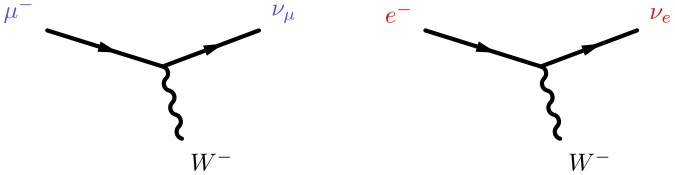
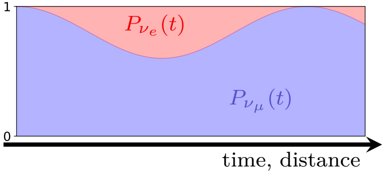
Outline

- Neutrino Oscillation Introduction
- Neutrino Cross Sections
- Quasielastic Scattering from Experiment
  - Constraints from Deuterium Scattering
  - Preliminary Hydrogen Scattering
- LQCD Results
  - Introduction
  - Preliminary CalLat Calculation
- LQCD Survey of $F_A(Q^2)$
  - Summary of $F_A(Q^2)$ Calculations
  - T2K/DUNE Implications
- Future Prospects

Neutrino Oscillation

Neutrino Oscillation (in a slide)



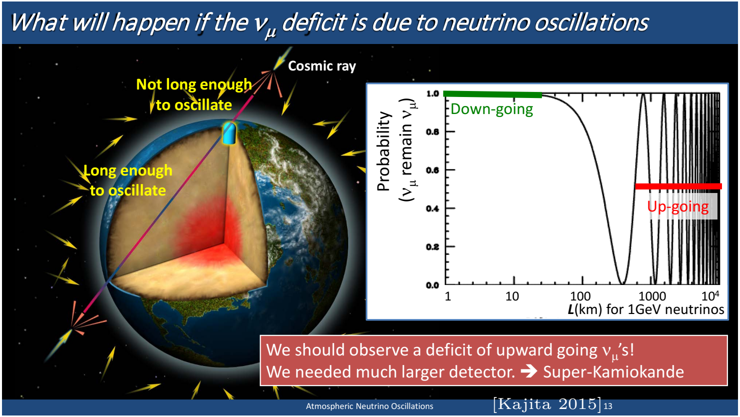


Neutrinos created during fusion reactions in sun, nuclear beta decay

“Flavor” defined by associated charged lepton during weak interaction

Flavor changes spontaneously during near light-speed propagation: $\nu_\mu \to \nu_e$

Discovery of Neutrino Oscillation



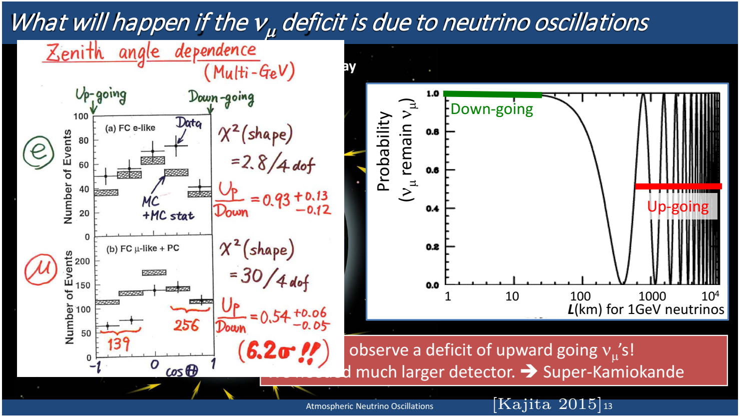
[Kajita 2015]

Difference between upward vs downward ν at Super-Kamiokande (1998)

$\implies$ Neutrinos have mass and oscillate!

2015 Nobel prize to Arthur McDonald, Takaaki Kajita

Discovery of Neutrino Oscillation



[Kajita 2015]

Difference between upward vs downward ν at Super-Kamiokande (1998)

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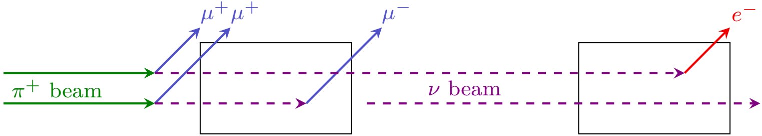
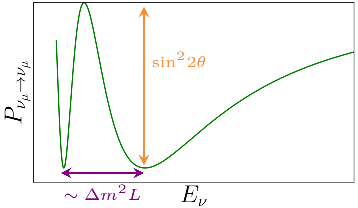
Neutrino Oscillation (with math)



$$\underbrace{|\nu_\ell\rangle}_{\substack{\text{flavor}\\ \text{eigenstate}}} = \sum_i U^*_{\ell i} \underbrace{|\nu_i\rangle}_{\substack{\textit{mass}\\ \textit{eigenstate}}} \qquad |\nu_i\rangle \to e^{-iE_i t}|\nu_i\rangle$$

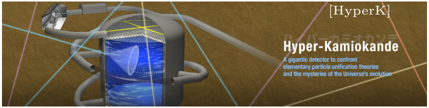
mass eigenstates – propagation
flavor eigenstates – interaction

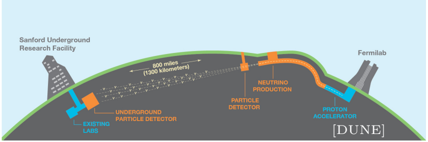


2 flavor model: $P_{\nu_\mu \to \nu_e} = \sin^2 2\theta \sin^2 \left(1.27 \frac{\Delta m^2 [\mathrm{eV}^2] L[\mathrm{km}]}{E_\nu [\mathrm{GeV}]} \right)$

Oscillation probability is function of L/E_ν at fixed L

Neutrino Physics Goals





Flagship long baseline oscillation experiments

DUNE: USA, HyperK: Japan

Answer fundamental questions about neutrinos:

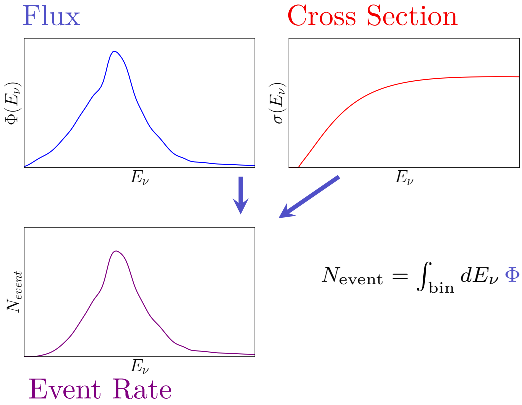
- mass ordering ($\Delta m^2_{32} > 0$?)
- octant ($\sin^2\theta_{23} = 0.5$?)
- CP violation ($\delta_{\rm CP} =$?)
- PMNS unitarity?
- 3 ν flavors?
- precision constraints

Measurements of solar, supernova ν

Data collection starts 2028–2029 $\Longrightarrow$ need support from theory!

Neutrino Cross Sections

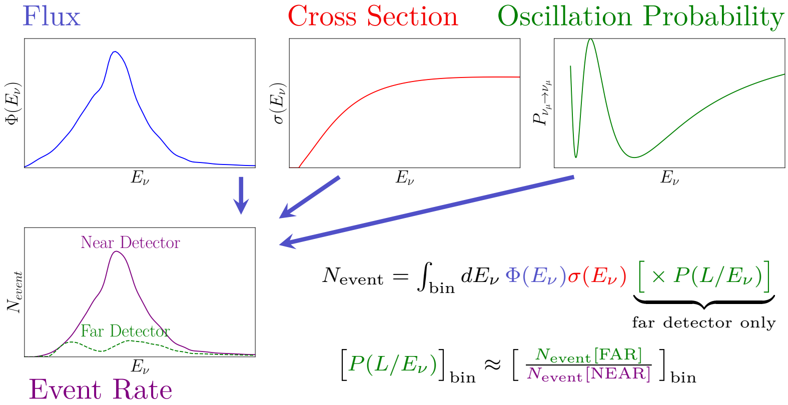
Measuring Oscillation Probability



$$N_{\text{event}} = \int_{\text{bin}} dE_\nu \, \Phi(E_\nu) \sigma(E_\nu)$$

Broad flux & distribution of event E_ν

Measuring Oscillation Probability



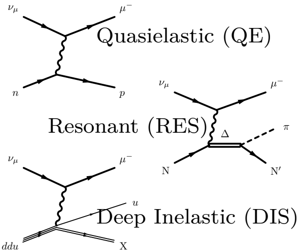
$$N_{\text{event}} = \int_{\text{bin}} dE_\nu\, \Phi(E_\nu)\sigma(E_\nu) \underbrace{\left[\times P(L/E_\nu)\right]}_{\text{far detector only}}$$

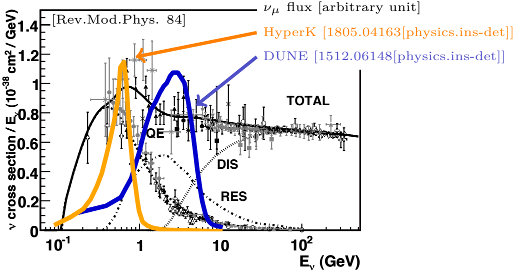
$$\left[P(L/E_\nu)\right]_{\text{bin}} \approx \left[\frac{N_{\text{event}}[\text{FAR}]}{N_{\text{event}}[\text{NEAR}]}\right]_{\text{bin}}$$

Broad flux & distribution of event E_ν

far/near $\implies$ oscillation probability, but picture too simplified...

Neutrino Cross Sections





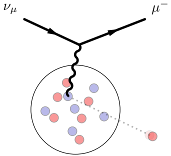
E_ν spans several kinematic regimes

Different interaction channels contributing to event rates

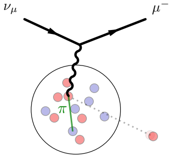
Need precise, theoretically robust cross sections for multiple event topologies

Neutrino Event Topologies

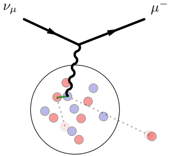
Quasielastic



π production



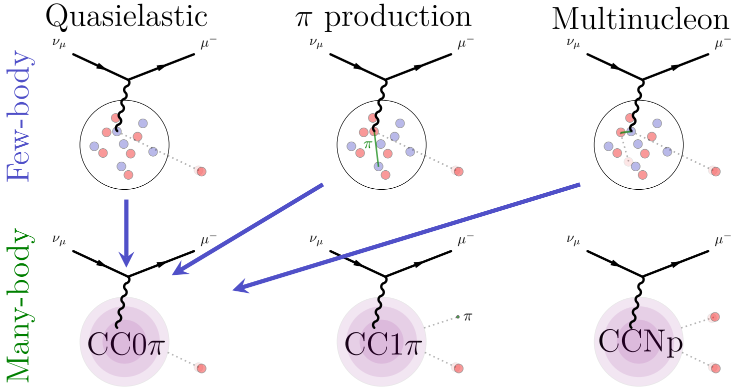
Multinucleon



Reinteractions within nucleus change kinematics

Only particles that escape are detectable

Neutrino Event Topologies



Reinteractions within nucleus change kinematics

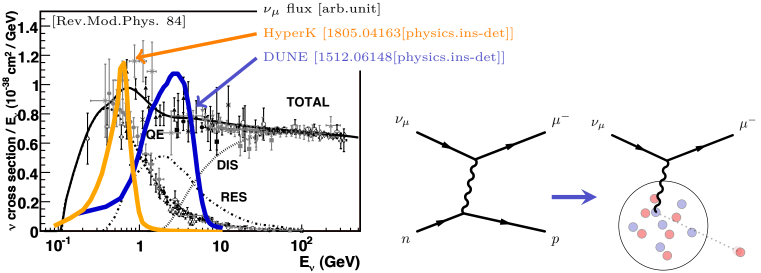
Only particles that escape are detectable

Mismatch between *nucleon* amplitudes & *nuclear* cross sections...

$\Longrightarrow$ Event-by-event E_ν measurements are not possible

$\Longrightarrow$ Reconstruct E_ν distributions from measured event rates

Neutrino Oscillation and Quasielastic



Compute *nucleon* amplitudes, ingredients for *nuclear* models

Quasielastic is lowest E_ν, simplest

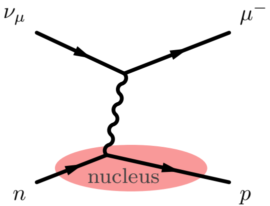
Question:

How well do we know nucleon quasielastic cross section from elementary target sources?

- Hydrogen/Deuterium scattering
- Lattice QCD

QE Experimental Constraints

Quasielastic Form Factors



$$\mathcal{M}_{\text{nucleon}} = \langle \ell | \mathcal{J}^\mu | \nu_\ell \rangle \langle N' | \mathcal{J}_\mu | N \rangle$$

$$\langle N'(p') | (V-A)_\mu(q) | N(p) \rangle$$
$$= \bar{u}(p') \Big[\gamma_\mu F_1(q^2) + \frac{i}{2M_N} \sigma_{\mu\nu} q^\nu F_2(q^2) + \gamma_\mu \gamma_5 F_A(q^2) + \frac{1}{2M_N} q_\mu \gamma_5 F_P(q^2) \Big] u(p)$$

Quasi-free nucleon inside nucleus —

- F_1, F_2: constrained by eN scattering
- F_P: subleading in cross section,
  $\propto F_A$ from pion pole dominance constraint

Leading contribution to nucleon cross section uncertainty is axial form factor F_A

Form Factor Parameterizations

Dipole ansatz — $$F_A(Q^2) = g_A \left(1 + \frac{Q^2}{m_A^2}\right)^{-2}$$

- Overconstrained by both experimental and LQCD data (revisit later)
- Inconsistent with QCD, requirements from unitarity bounds
- Motivated by $Q^2 \to \infty$ limit, data restricted to low Q^2

Model independent alternative: z expansion [Phys.Rev.D 84 (2011)] —

$$F_A(z) = \sum_{k=0}^{\infty} a_k z^k \qquad z(Q^2; t_0, t_{\text{cut}}) = \frac{\sqrt{t_{\text{cut}} + Q^2} - \sqrt{t_{\text{cut}} - t_0}}{\sqrt{t_{\text{cut}} + Q^2} + \sqrt{t_{\text{cut}} - t_0}} \qquad t_{\text{cut}} \leq (3M_\pi)^2$$

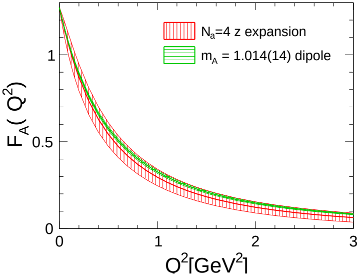
- Rapidly converging expansion
- Controlled procedure for introducing new parameters

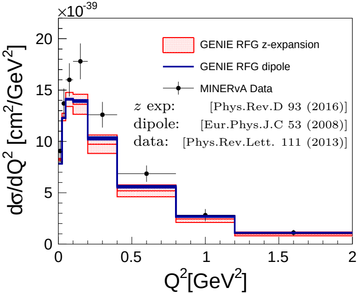
Deuterium Constraints on F_A

- Outdated bubble chamber experiments:
  - Total $O(10^3)$ ν_μQE events
  - Digitized event distributions only
  - Unknown corrections to data
  - Deficient deuterium correction
- Dipole overconstrained by data
  underestimated uncertainty $\times O(10)$
- Prediction discrepancies could be from
  nucleon and/or nuclear origins

Coming soon:

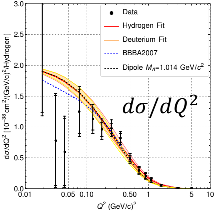
Updated joint fit with
MINERνA $\bar{\nu}_\mu p \to \mu^+ n$ dataset

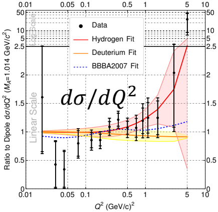


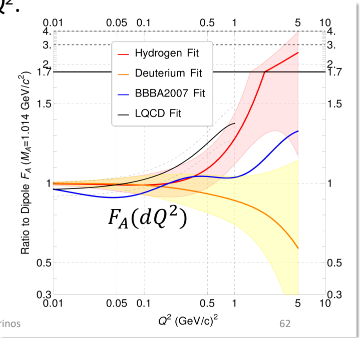


Free Nucleon Axial Form Factor

- We have ~5800 such events on a background of ~12500.
- Shape is not a great fit to a dipole at high Q^2.
- LQCD prediction at high Q^2 is close to this result, but maybe not at moderate Q^2.





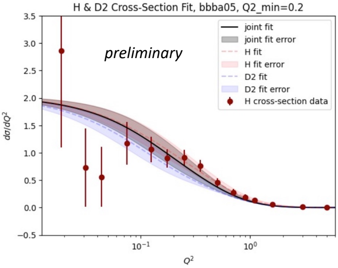


28 September 2023 K. McFarland, Measuring Protons with Neutrinos 62

See also [Nature 614 (2023)]

Compatible with D_2 Data? Mmmmmmaybe?

- We have some progress on joint fits with neutrino-deuterium analysis (*Phys.Rev.D* 93 (2016) 11, 113015), including comprehensive analysis of compatibility.
  - Note that compatibility depends on the choice of vector form factors, since vector-axial vector interference flips sign.
  - We see that compatibility also depends strongly on how low in Q^2 we use the D_2 data, which might suggest low Q^2 nuclear effects?
- With BBBA05 vector form factors and Q^2>0.2 GeV^2, $\delta\chi^2$~5.5, or p-value of ~2%.



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See also [Nature 614 (2023)]

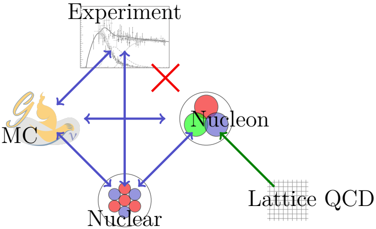
LQCD as Disruptive Technology

How can we improve precision?

Ideal: Modern high stats ν-D_2 scattering bubble chamber experiment

$\Longrightarrow$ LQCD as a complement to experiment

- ✓ No nuclear effects
- ✓ Realistic uncertainty estimates
- ✓ Systematically improvable
- ✓ Computers are (relatively) inexpensive



Build from the ground up:

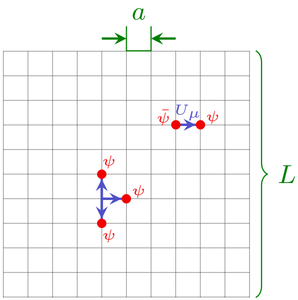
Nucleon amplitudes from first principles

Robust uncertainty quantification

Well motivated theory inputs to nuclear models/EFTs

Matrix Elements from LQCD

Lattice QCD Formalism



Numerical evaluation of path integral

Quark, gluon DOFs —

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}\psi \, \mathcal{D}\overline{\psi} \, \mathcal{D}U \, \exp(-S) \, \mathcal{O}_\psi \, [U]$$

Parameters: $am_{(u,d),\mathrm{bare}}$
$am_{s,\mathrm{bare}}$
$\beta = 6/g^2_{\mathrm{bare}}$

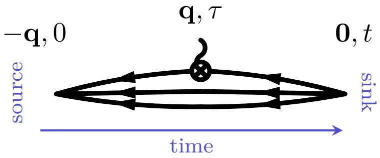
Matching: e.g. $\frac{M_\pi}{M_\Omega}$, $\frac{M_K}{M_\Omega}$, M_Ω
1 per parameter

Results — first principles predictions from QCD,
gluons to all orders

"Complete" error budget $\implies$ extrapolation in a, L, M_π guided by EFT, FVχPT

- $a \to 0$ (continuum limit)
- $L \to \infty$ (infinite volume limit)
- $M_\pi \to M_\pi^{\mathrm{phys}}$ (chiral limit)

Fit Setup



Fit exponential dependence of axial "3-point" functions:

$$C^{3\text{pt}}_{\mathcal{A}_z}(t,\tau,\mathbf{q}) = \langle \mathcal{N}(\mathbf{0},t) \mathcal{A}_z(\mathbf{q},\tau) \overline{\mathcal{N}}(-\mathbf{q},0)\rangle$$

$$\sim \sum_{mn} z_n^{\mathbf{0}} \, A^{\mathbf{q}}_{nm} \, z_m^{\mathbf{q}\dagger} \, e^{-E_n^{\mathbf{0}}(t-\tau)} e^{-E_m^{\mathbf{q}}\tau}$$

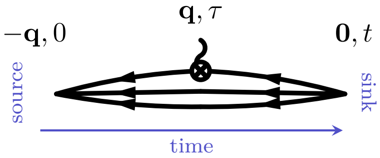
Towers of excited states m, n depend on momenta injected

Current $\mathcal{A}_z$ couples to axial, induced pseudoscalar form factors

Overlaps, energies constrained by "2-point" functions

$$C^{2\text{pt}}(t,\mathbf{q}) = \langle \mathcal{N}(\mathbf{q},t) \overline{\mathcal{N}}(-\mathbf{q},0)\rangle \sim \sum_m z_m^{\mathbf{q}} \, z_m^{\mathbf{q}\dagger} \, e^{-E_m^{\mathbf{q}} t}$$

Fit Setup



Plot ratio correlator:

$$\mathcal{R}_{\mathcal{A}_z}(t,\tau,\mathbf{q}) = \frac{C^{3\text{pt}}_{\mathcal{A}_z}(t,\tau,\mathbf{q})}{\sqrt{C^{2\text{pt}}(t-\tau,\mathbf{0})C^{2\text{pt}}(\tau,\mathbf{q})}} \sqrt{\frac{C^{2\text{pt}}(\tau,\mathbf{0})}{C^{2\text{pt}}(t,\mathbf{0})} \frac{C^{2\text{pt}}(t-\tau,\mathbf{q})}{C^{2\text{pt}}(t,\mathbf{q})}}$$

$$\xrightarrow[t-\tau,\tau\to\infty]{} \frac{1}{\sqrt{2E_0^{\mathbf{q}}(E_0^{\mathbf{q}}+M)}} \left[-\frac{q_z^2}{2M} \mathring{F}_P(Q^2) + (E_0^{\mathbf{q}}+M)\mathring{F}_A(Q^2) \right]$$

$$Q^2 = |\mathbf{q}|^2 - (E_0^{\mathbf{q}} - M)^2$$

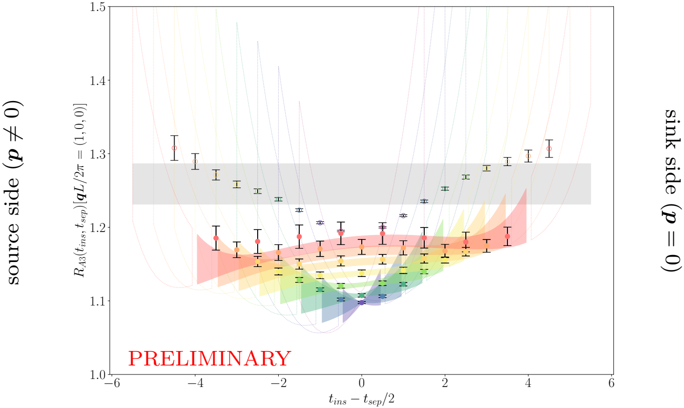
$\mathcal{A}_z$ with $q_z = 0 \quad \Longrightarrow \quad \mathcal{R}_{\mathcal{A}_z}(t,\tau,\mathbf{q}) \to \sqrt{\frac{E_0^{\mathbf{q}}+M}{2E_0^{\mathbf{q}}}}\mathring{g}_A(Q^2)$

$\Longrightarrow$ No induced pseudoscalar

$\Longrightarrow$ Simplified analysis of $\mathring{F}_A(Q^2) = \mathring{g}_A(Q^2)$

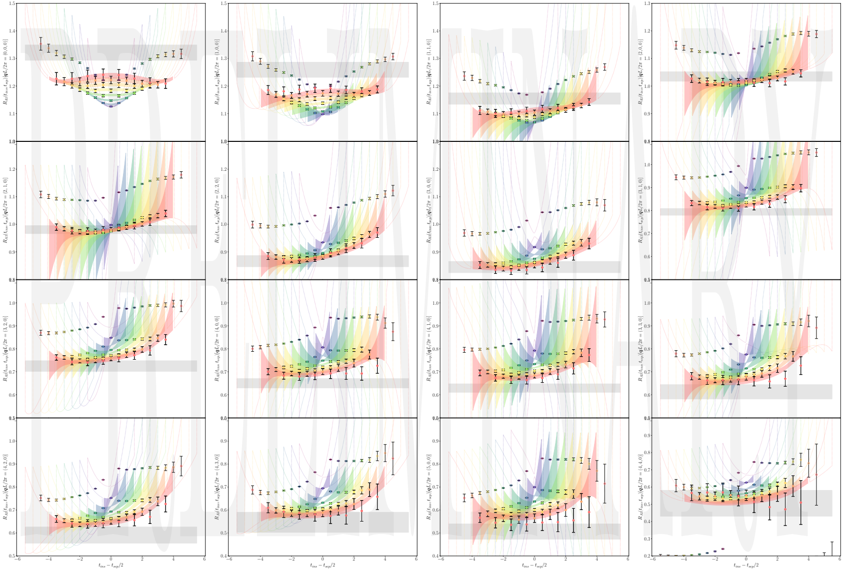
$\Longrightarrow$ `a12m130` ensemble only, $N_{state} = 3$ only

Correlation Function Ratio

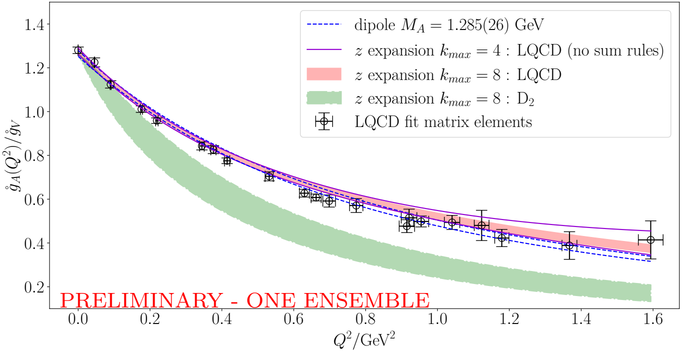


- Horizontal: source-insertion time, centered about midpoint
- Vertical: correlator ratio $\sim$ axial matrix element
- Color: source-sink separation time
- Colored bands: fit range
- Gray band: $\mathring{g}_A$ posterior value

$\mathring{g}_A(Q^2)$ Correlators



Axial Form Factor Fit



Trend of high-Q^2 enhancement seen in other LQCD results

2–4% LQCD uncertainty vs 10% uncertainty on D_2 result

TODO list:

$\boldsymbol{q}L/2\pi = (1, 0, 0)$ matrix element larger than expectation

Deep dive into excited states systematics, prior dependence

More momenta, $q_z \neq 0$, full set of ensembles

LQCD Survey and Implications

Status of Lattice QCD Determination of Nucleon Form Factors and Their Relevance for the Few-GeV Neutrino Program

Annual Review of Nuclear and Particle Science

Vol. 72:- (Volume publication date September 2022)
Review in Advance first posted online on July 8, 2022. (Changes may still occur before final publication.)
https://doi.org/10.1146/annurev-nucl-010622-120608

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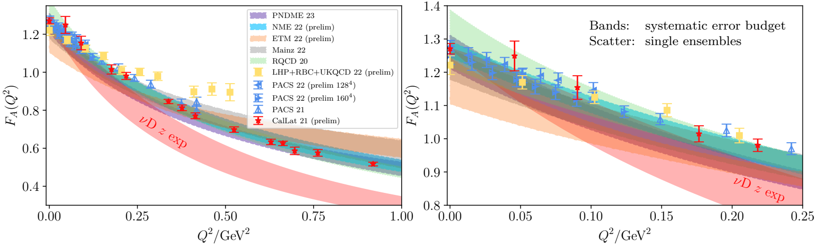
Abstract

Calculations of neutrino–nucleus cross sections begin with the neutrino–nucleon interaction, making the latter critically important to flagship neutrino oscillation experiments despite limited measurements with poor statistics. Alternatively, lattice quantum chromodynamics (LQCD) can be used to determine these interactions from the Standard Model with quantifiable theoretical uncertainties. Recent LQCD results of g_A are in excellent agreement with data, and results for the (quasi-)elastic nucleon form factors with full uncertainty budgets are expected within a few years. We review the status of the field and LQCD results for the nucleon axial form factor, $F_A(Q^2)$, a major source of uncertainty in modeling sub-GeV neutrino–nucleon interactions. Results from different LQCD calculations are consistent but collectively disagree with existing models, with potential implications for current and future neutrino oscillation experiments. We describe a road map to solidify confidence in the LQCD results and discuss future calculations of more complicated processes, which are important to few-GeV neutrino oscillation experiments.

Expected final online publication date for the *Annual Review of Nuclear and Particle Science*, Volume 72 is September 2022. Please see http://www.annualreviews.org/page/journal/pubdates for revised estimates.

Nucleon Axial Form Factor

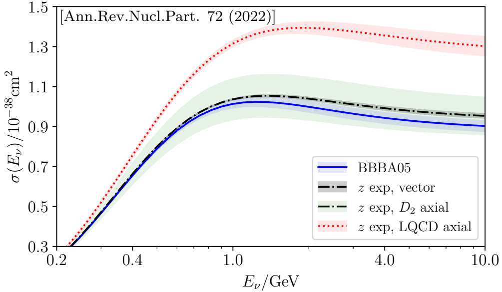


LQCD results maturing:

- Many results, all physical M_π: *independent data & different methods*
- Small systematic effects observed (expectation: largest at $Q^2 \to 0$)
- Nontrivial consistency checks from PCAC

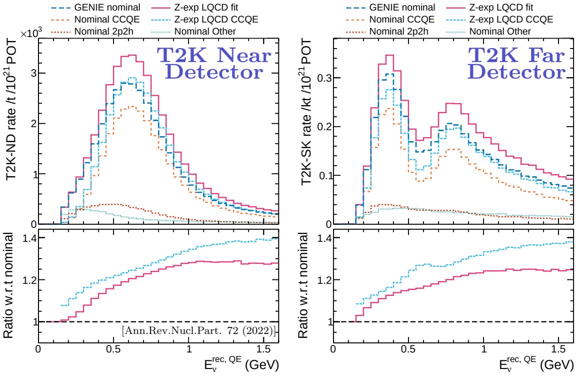
Evidence of slow Q^2 falloff, situation unlikely to change drastically

Free Nucleon Cross Section



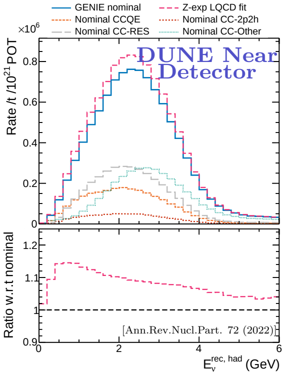
- LQCD prefers 30-40% enhancement of ν_μ CCQE cross section
- recent Monte Carlo tunes require 20% enhancement of QE
  [Phys.Rev.D 105 (2022)] [2206.11050 [hep-ph]]
  similar trend with continuum Schwinger function methods
  [Phys.Rev.D 105 (2022)] [2206.12518 [hep-ph]]
- With improved precision, sensitive to vector FF tension (black vs blue)
  [Phys.Rev.D 102 (2020)] vs [Nucl.Phys.B Proc.Suppl. 159 (2006)]

T2K Implications



- Dashed dark blue (GENIE nominal) vs solid magenta (z exp LQCD fit)
- QE enhancements produce 10-20% event rate enhancement, E_ν-dependent
- cross section changes at ND $\neq$ *effective* cross section changes at FD:
  insufficient CCQE model freedom $\rightarrow$ bias in FD prediction
- Monte Carlo tuning invalidates more sophisticated comparisons

DUNE Implications



- Solid dark blue (GENIE nominal) vs dashed magenta (z exp LQCD fit)
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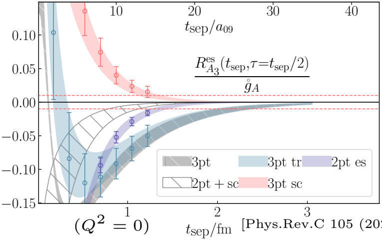
- cross section changes at ND $\neq$ *effective* cross section changes at FD: insufficient CCQE model freedom $\rightarrow$ bias in FD prediction
- Monte Carlo tuning invalidates more sophisticated comparisons

Future Directions

LQCD Excited States — Empirical



$(Q^2 = 0)$ [Phys.Rev.C 105 (2022)]

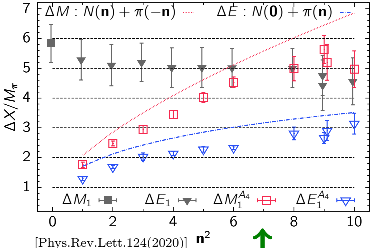
Compare fit to correlator data ratio

Contamination dominated by "transition" states $(0 \to n$, blue)

Typically signal below $\lesssim 1$ fm, contamination $\gtrsim 2$ fm

Excited states present in practically-achievable large time limit

$(Q^2 \geq 0)$



[Phys.Rev.Lett.124(2020)]

NOTE: expect only approx agreement between data/curves

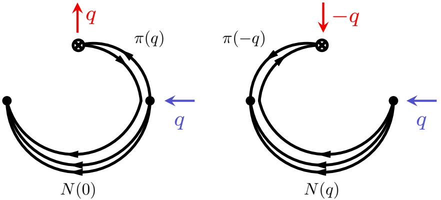
NME collab:

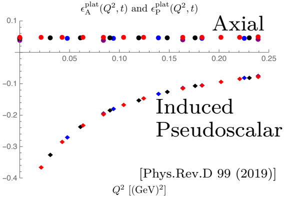
Q^2 contamination from $N \to N\pi$

Dominant contribution agrees with χPT expectation

$N\pi$ is important for $F_A(Q^2)$

LQCD Excited States — χPT and $N\pi$





Contamination in $g_A(Q^2)$ primarily from enhanced $N\pi$, mostly from induced pseudoscalar

Correlator fits without axial current not sensitive to $N\pi$
[Phys.Rev.C 105 (2022)] [Phys.Rev.D 105 (2022)]

Alternate fit strategies:

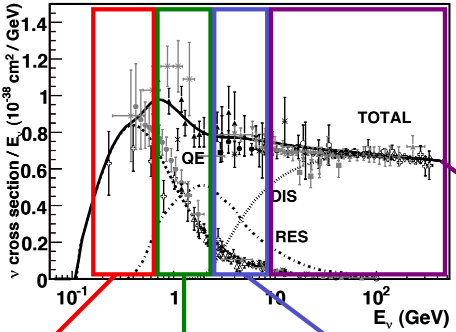
- explicit $N\pi$ operators
- include $\mathcal{A}_4$ (strong $N\pi$ coupling)
- Kinematic constraints ($F_P = 0$)

Prediction from χPT: [Phys.Rev.D 99 (2019)]

First demonstration of $N\pi$: [Phys.Rev.Lett. 124 (2020)]

χPT-inspired fit methods for fitting form factor data
[Phys.Rev.D 105 (2022)] [JHEP 05 (2020) 126]

Energy Regimes



Deep Inelastic Scattering

–Axial quasi/pseudo PDF

Quasielastic

–Nucleon Form Factors

–Full Error Budgets

$N \to \Delta, N \to N^*$

–Transition Matrix Elements

–Multiparticle Operators

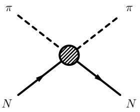
"Shallow Inelastic Scattering" (SIS)

–Hadronic Tensor

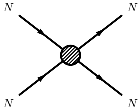
–Four Point Functions

LQCD Target Calculations

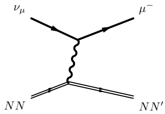
$N\pi$ Scattering



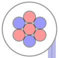
NN Scattering



NN Quasielastic

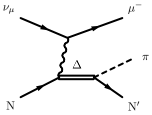


(incomplete list!)

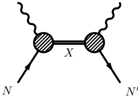


Nuclear

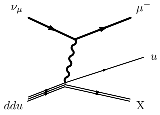
Resonant $N\pi$



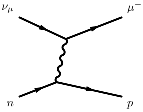
Four-point Inclusive



Deep Inelastic

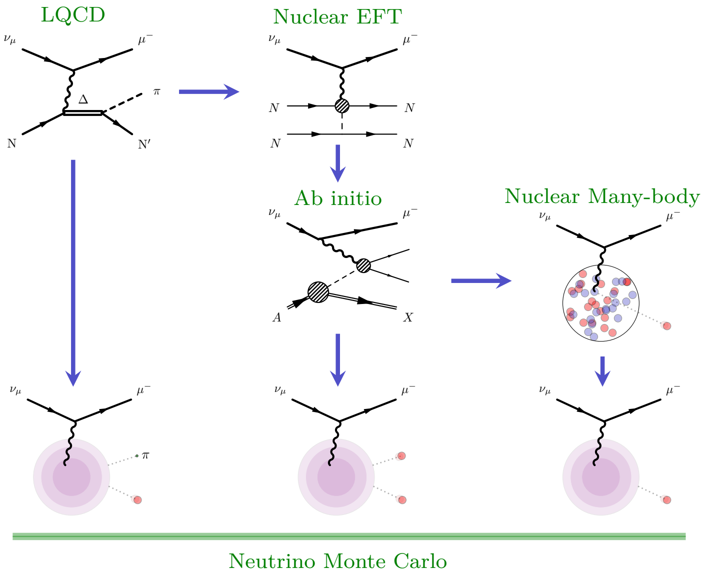


Quasielastic



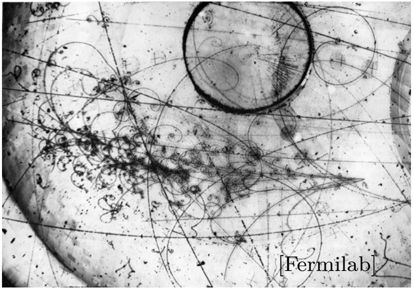
Nucleon

Roadmap To Nuclear



Concluding Remarks

Outlook

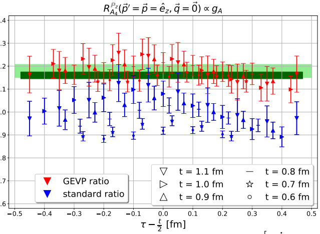


- Nucleon form factor uncertainty significantly underestimated
- Mounting evidence that QE ν cross section underestimated
  $\implies$ Attention needed to avoid biased results
- LQCD is a proxy for missing experimental data
- Nucleon-pion effects are the next frontier...
  - Transition form factors
  - Low-energy constants for meson exchange
  - Pion production
- Exciting results upcoming: hydrogen scattering, LQCD

Thank you for your attention!

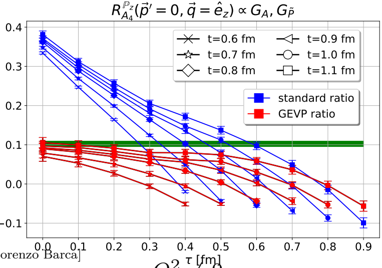
Backup

Axial FF - $N\pi$ Interpolating Operators



$Q^2 = 0$

[priv.comm. Lorenzo Barca]



$Q^2 \neq 0$

2×2 operator basis, explicit 3- & 5-quark interpolating operators

Significantly flatter ratios, simplified analysis

Will analysis with only 3-quark operators be consistent?