Extracting parton distribution functions from lattice quantum chromodynamics

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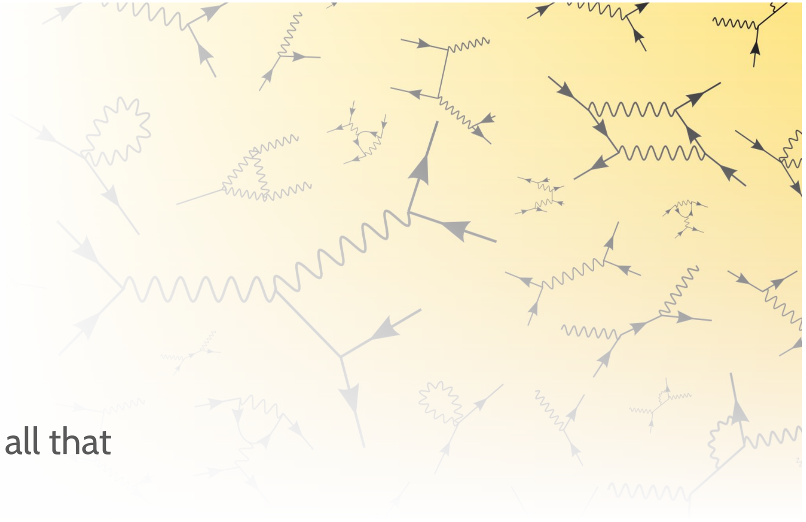
SPIN 2023, Duke University, September 26 2023

WILLIAM & MARY
CHARTERED 1693

QUARK-GLUON TOMOGRAPHY COLLABORATION

DOE Early Career Award DE-SC0023047

Overview



1. Introduction:
    - Parton distribution functions
    - Lattice quantum chromodynamics
    - LaMET, quasi-distributions, pseudo-distributions and all that

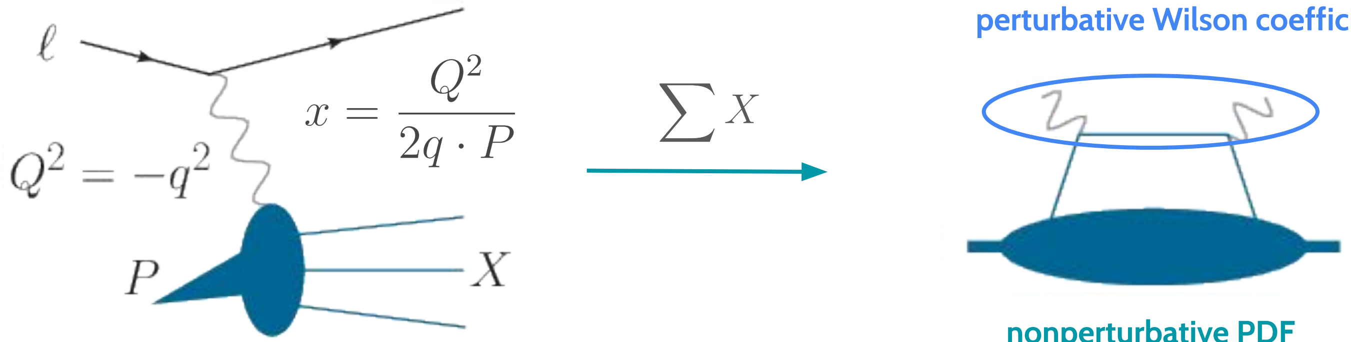
2. Recent progress and results
    - Quark distributions
    - Gluon distributions

Parton distribution functions and inclusive scattering processes

Parton distribution functions (PDFs) encode longitudinal partonic structure of hadrons

- Directly connect the standard model to nuclear physics
- Important source of systematic uncertainties at hadron colliders

Deep inelastic scattering is the primary theoretical and experimental probe of PDFs



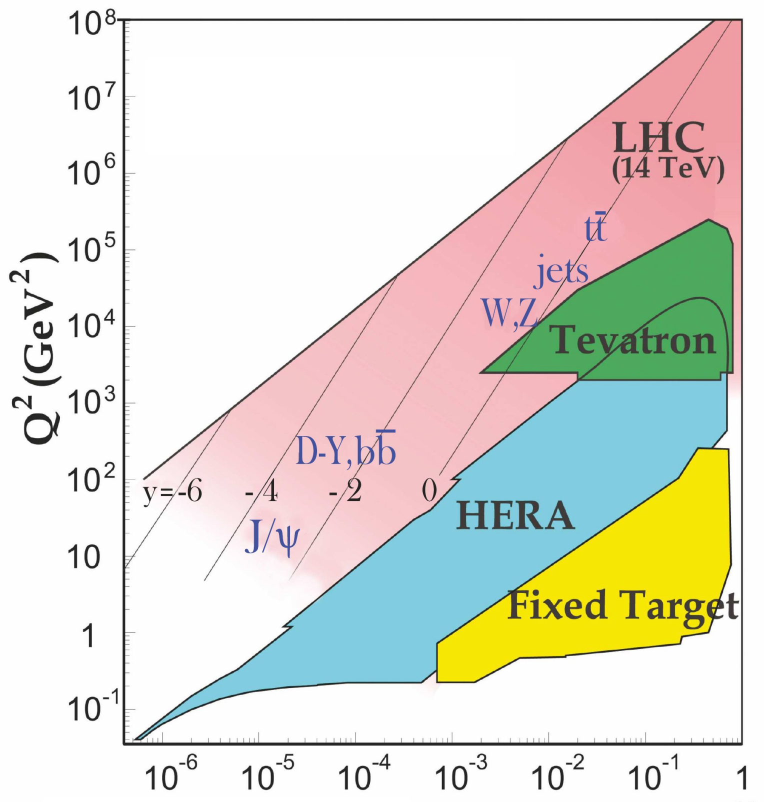
perturbative Wilson coefficient

nonperturbative PDF

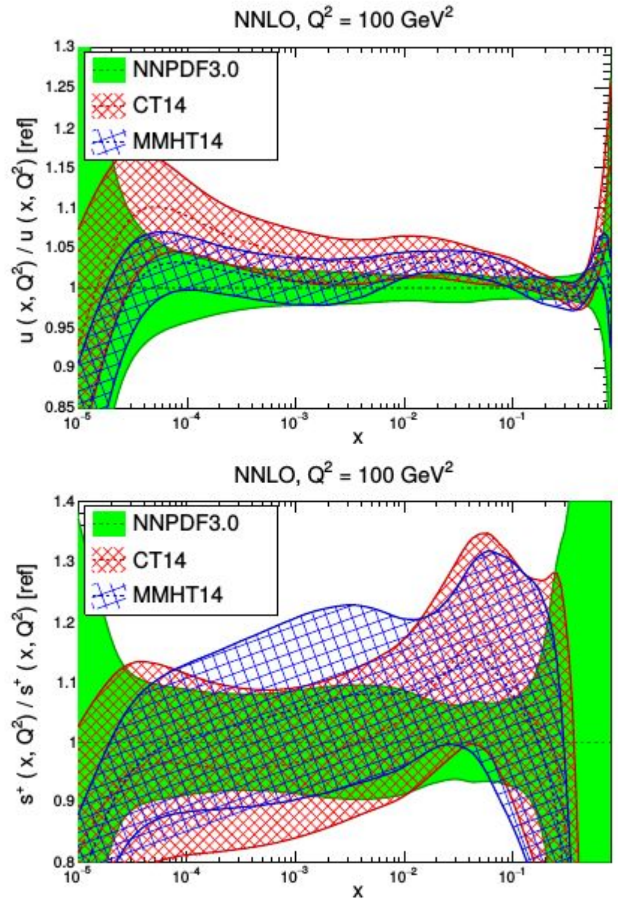
In QFT, PDFs are defined through matrix elements of fields at light-like separations

- Lattice QCD provides the primary method for systematic, first-principles calculations of QCD
- Lattice can only compute matrix elements of fields at space-like separations
- Until a decade ago, first principles' calculations of parton distribution functions were impractical

Phenomenological solution

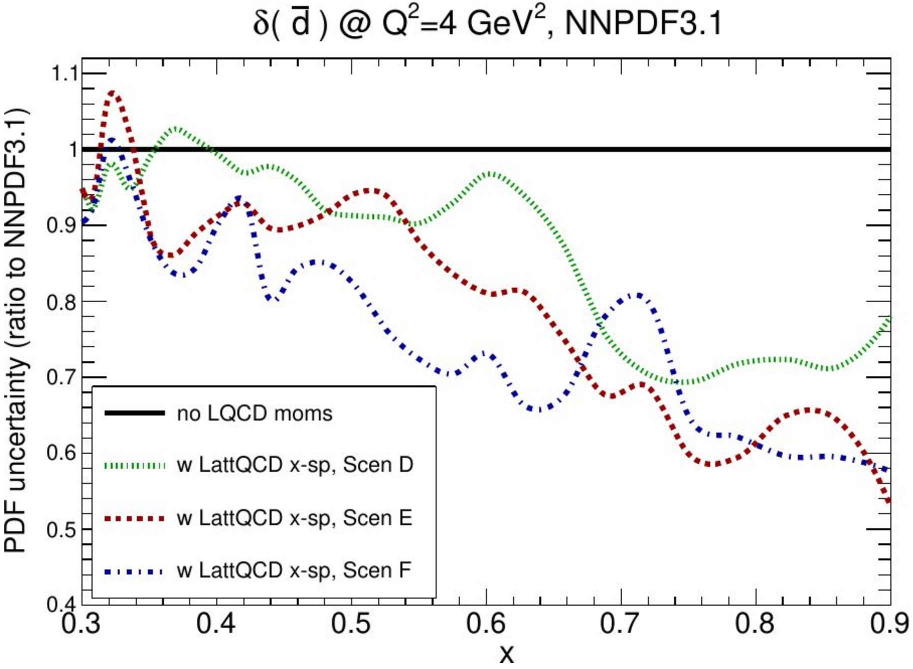


PDG, PTEP (2022) 083C01

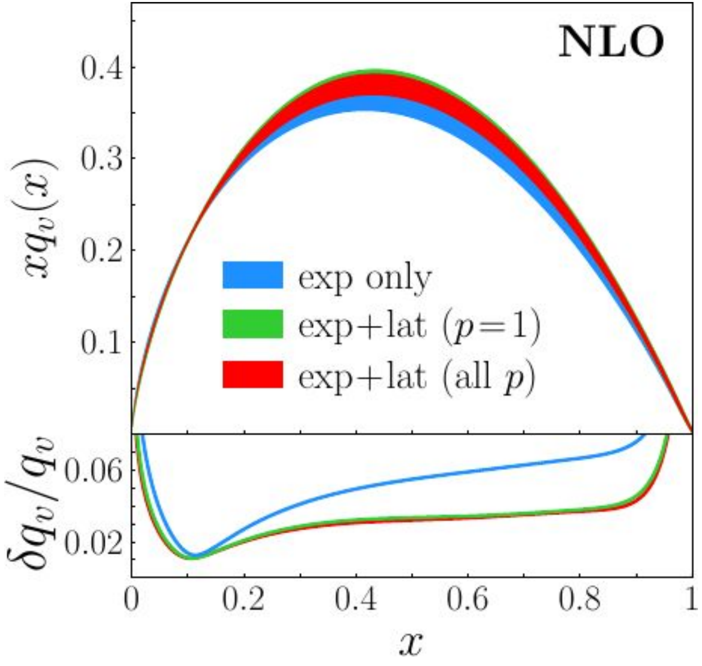


PDFs from QCD

First principles calculations complement, and inform, JLab 12 GeV, the LHC and the EIC



Lin et al., Prog. Nucl. Part. Phys. 100 (2018) 107



Barry et al., PRD 105 (2022) 114051

Lattice QCD

Provides a rigorous, nonperturbative, gauge-invariant definition of QCD

Formulated in the path integral representation of quantum field theory

$$\langle\mathcal{O}\rangle = \frac{1}{\mathcal{Z}}\int \mathcal{D}\left[\overline{\psi},\psi,A\right]\ \mathcal{O}\left[\overline{\psi},\psi,A\right]\ e^{iS_{\mathrm{QCD}}[\overline{\psi},\psi,A]}$$

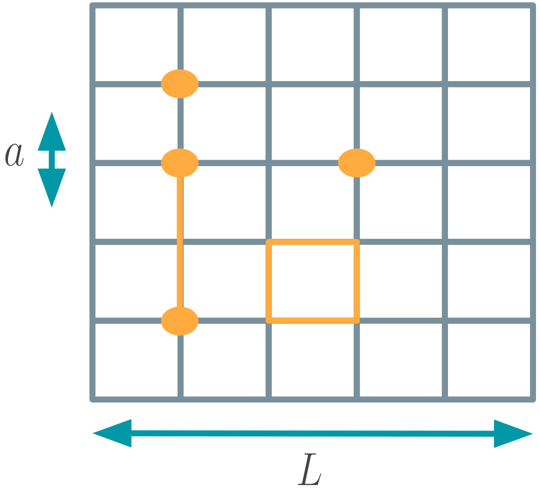
“Wick rotate” to Euclidean spacetime

$$\langle\mathcal{O}\rangle = \frac{1}{\mathcal{Z}}\int \mathcal{D}\left[\overline{\psi},\psi,A\right]\ \mathcal{O}\left[\overline{\psi},\psi,A\right]\ e^{-S_{\mathrm{QCD}}[\overline{\psi},\psi,A]}$$

Integral computed stochastically using Markov Chain Monte Carlo methods

$$\langle\mathcal{O}\rangle \approx \frac{1}{\mathcal{Z}}\sum \mathcal{O}\left[\overline{\psi},\psi,A\right]\Big|_{e^{-S_{\mathrm{QCD}}[\overline{\psi},\psi,A]}}$$

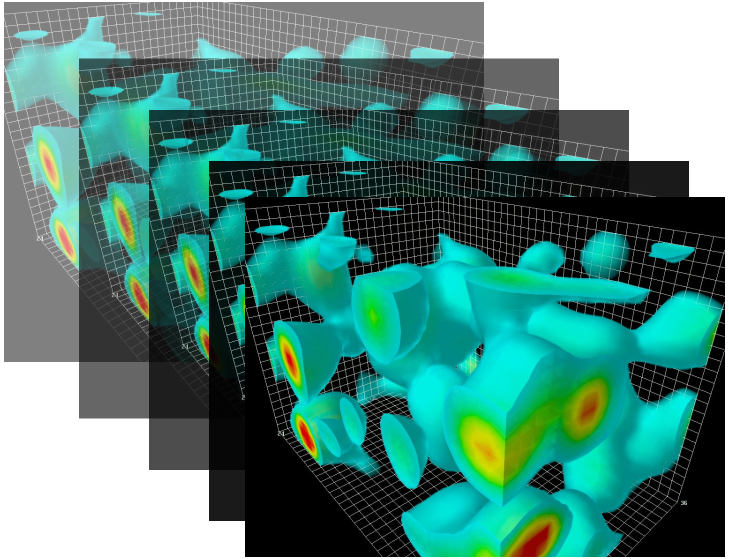
Lattice QCD: the numerical recipe



1. Take a small box of Euclidean spacetime and discretise the box to form a hypercubic spacetime lattice

2. Distribute quarks and gluons in the box - quarks on the nodes, gluons on the links

Lattice QCD: the numerical recipe



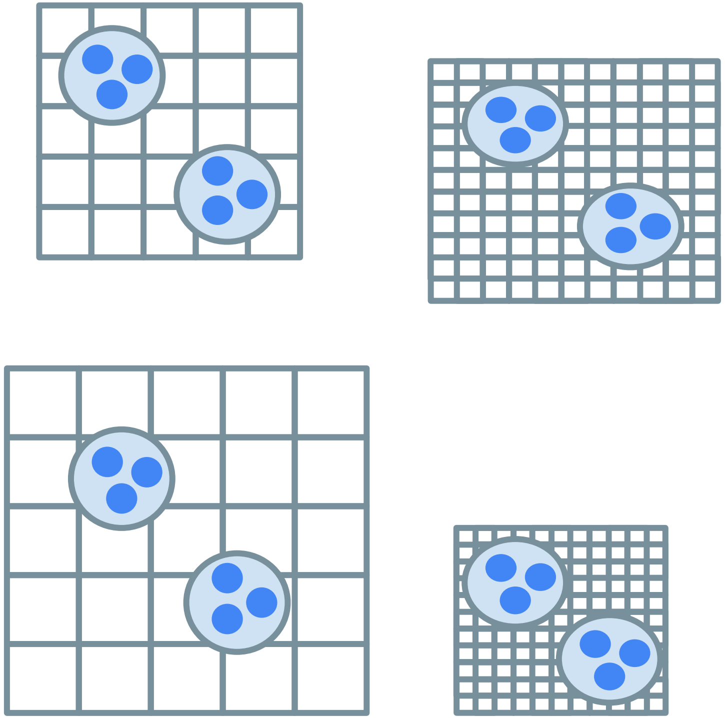
Leinweber et al., CSSM, Adelaide

1. Take a small box of Euclidean spacetime and discretise the box to form a hypercubic spacetime lattice

2. Distribute quarks and gluons in the box - quarks on the nodes, gluons on the links

3. Generate many copies (ensemble) of this QCD vacuum, with Boltzmann probability distribution exp[-S_{QCD}]

Lattice QCD: the numerical recipe



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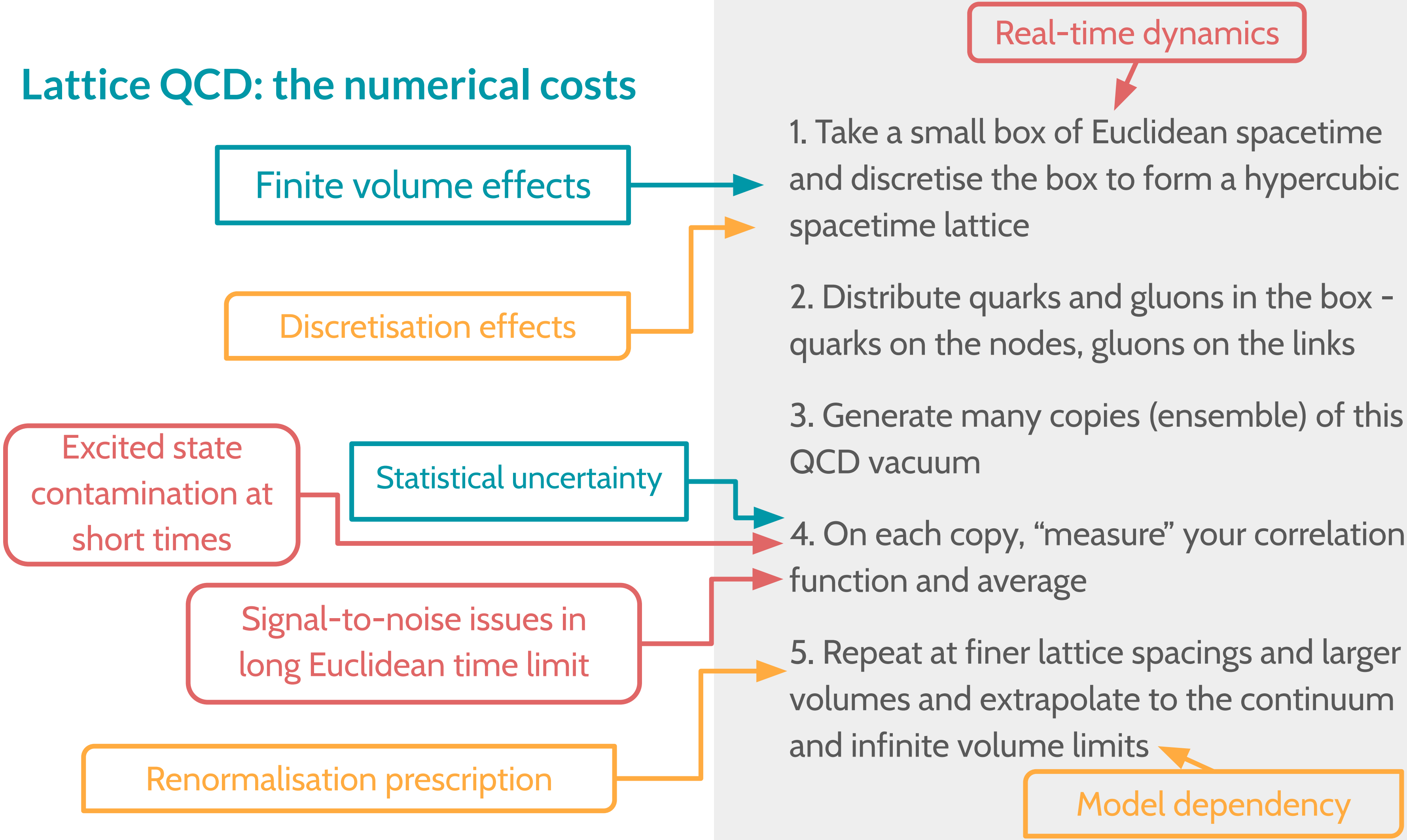
4. On each copy, “measure” your correlation function and average

5. Repeat at finer lattice spacings and larger volumes and extrapolate to the continuum and infinite volume limits

Lattice QCD: the numerical costs



Real-time dynamics

Finite volume effects

Discretisation effects

Excited state contamination at short times

Statistical uncertainty

Signal-to-noise issues in long Euclidean time limit

Renormalisation prescription

Model dependency

1. Take a small box of Euclidean spacetime and discretise the box to form a hypercubic spacetime lattice

2. Distribute quarks and gluons in the box - quarks on the nodes, gluons on the links

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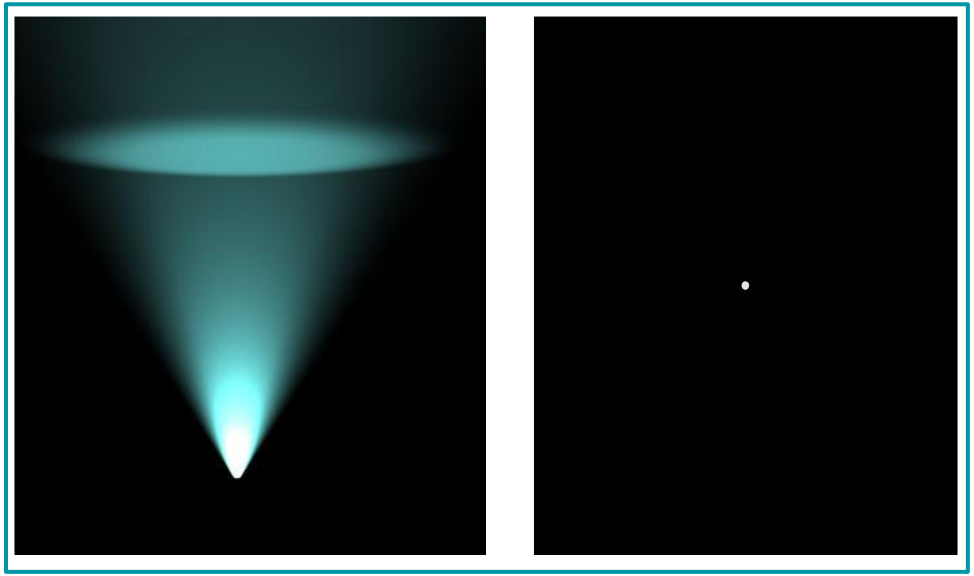
Parton distribution functions: the challenge

Lattice QCD is formulated in a finite volume on a discretised Euclidean spacetime lattice

Lightcone not accessible



Parton distribution functions: the challenge

Lattice QCD is formulated in a finite volume on a discretised Euclidean spacetime lattice

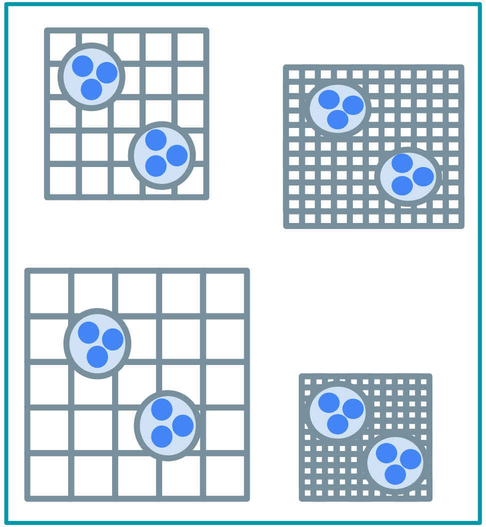
- Light cone not accessible

Path integral is sampled stochastically via Markov chain Monte Carlo

- Infinite momentum not accessible numerically!

Static quantities extracted in the long Euclidean-time limit

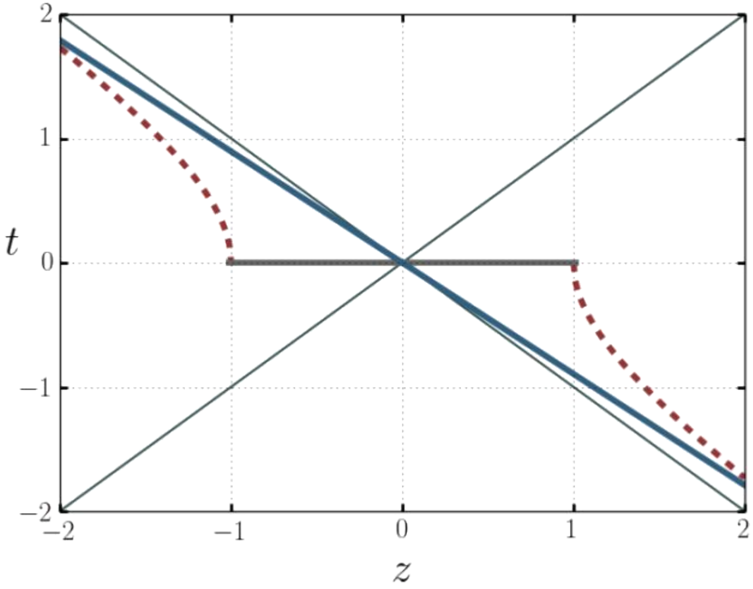
- Noise-to-signal ratio increases exponentially with Euclidean time
- Particularly challenging for gluons



Ji et al., PRL 110 (2013) 262002
Ji, Sci. Ch. Phys. Mech. Ast. 57 (2014) 1407
Ji et al., RMP 93 (2021) 35005

Large-momentum effective theory (LaMET)

Intuitive picture for PDFs - defined through operators of light-like separated fields



Ji et al., PRL 110 (2013) 262002
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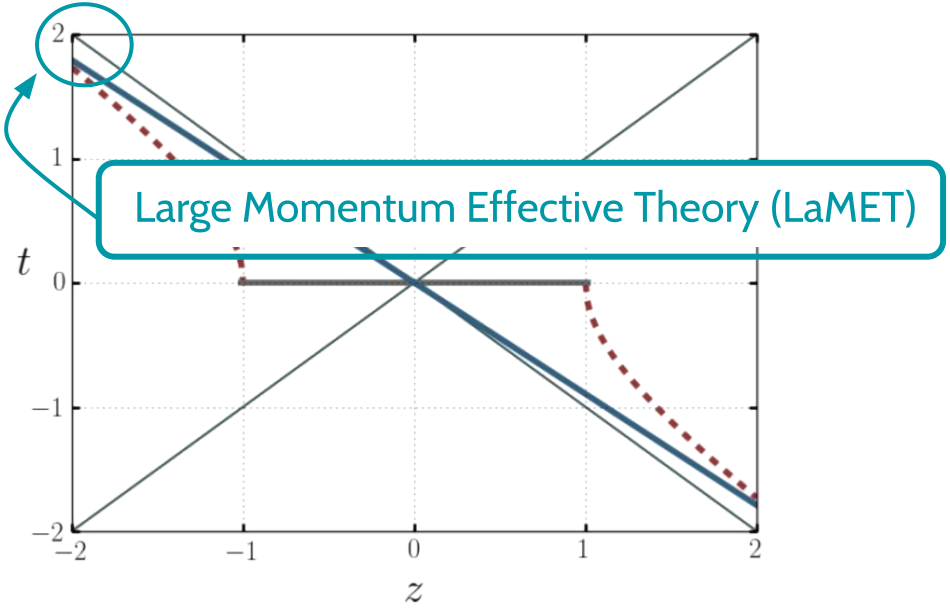


Large Momentum Effective Theory (LaMET)

Large momentum effective theory (LaMET)

Extensive review in
Ji et al., Rev. Mod. Phys. 93 (2021) 35005

Framework to relate lattice-calculable to infinite-momentum quantities

Ji et al., PRL 111 (2013) 112002

- originally introduced to enable the calculation of the gluon spin contribution
- now a general framework for first principles' calculations

Effective theory: infinite momentum limit does not commute with removing the regulator

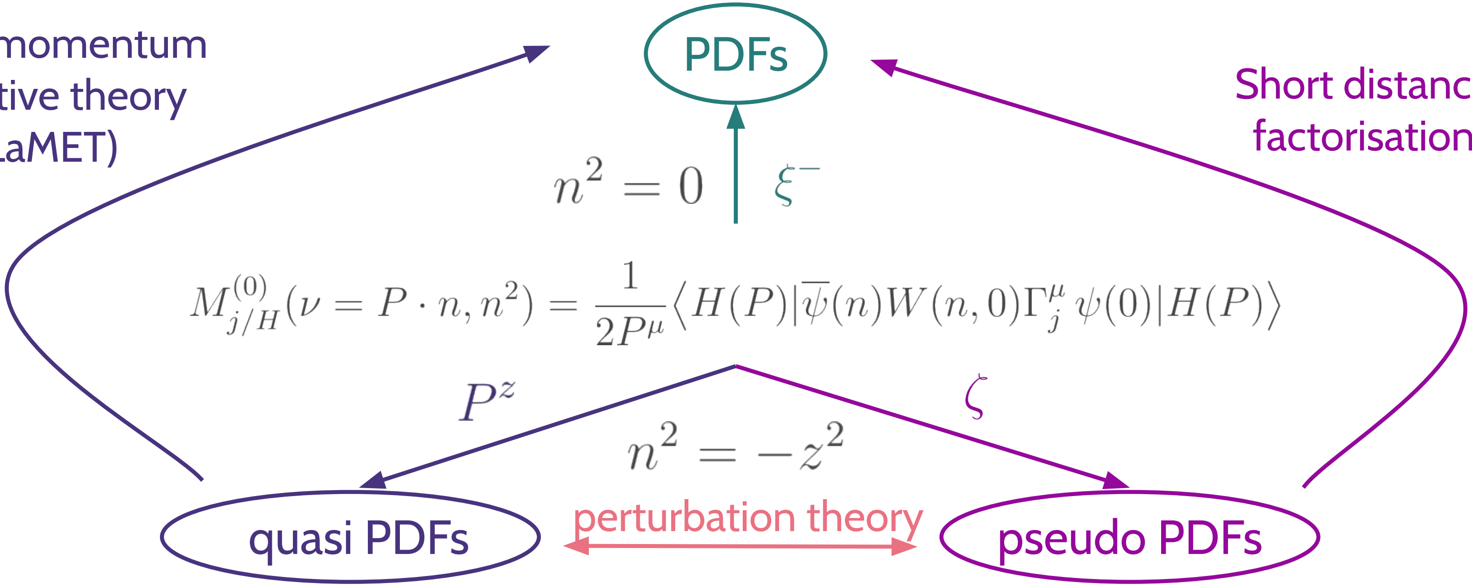
Relies on perturbative matching

- Quantities required to have the same infrared behaviour
- For example, at one loop in the MS-bar scheme

$$\Delta G^{\overline{\mathrm{MS}}}(\mu) = \frac{\alpha_s}{3\pi}\left[3\ln\frac{\mu^2}{m^2} + 7\right] \qquad \Delta G^{\overline{\mathrm{MS}}}(\mu, P_z) = \frac{\alpha_s}{3\pi}\left[\frac{5}{3}\ln\frac{\mu^2}{m^2} - \frac{1}{9} + \frac{4}{3}\ln\frac{P_z^2}{m^2}\right]$$

A panoply of distributions

$$f^{(0)}_{j/H}(\xi) = \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega^-}{4\pi} e^{-i\xi P^+\omega^-} \langle H(P)\,|\overline{\psi}(0,\omega^-,\mathbf{0}_\mathrm{T})W(\omega^-,0)\Gamma_j\psi(0)|\,H(P)\rangle$$



Large momentum effective theory (LaMET)

PDFs

$n^2 = 0$ ξ^-

Short distance factorisation

$$M^{(0)}_{j/H}(\nu = P\cdot n, n^2) = \frac{1}{2P^\mu}\langle H(P)|\overline{\psi}(n)W(n,0)\Gamma^\mu_j\,\psi(0)|H(P)\rangle$$

P^z ζ

$n^2 = -z^2$

perturbation theory

quasi PDFs

pseudo PDFs

$$\widetilde{f}^{(0)}_{j/H}(\xi, P^z) = \int_{-\infty}^{\infty} \frac{\mathrm{d}z}{4\pi} e^{i\xi P_z z} \langle H(P)\,|\overline{\psi}(0,z,\mathbf{0}_\mathrm{T})W(z,0)\Gamma_j\psi(0)|\,H(P)\rangle$$

$$\widetilde{p}^{(0)}_{j/H}(\xi, z^2) = \int_{-\infty}^{\infty} \frac{\mathrm{d}\zeta}{4\pi} e^{i\xi\zeta} \langle H(P)\,|\overline{\psi}(0,z,\mathbf{0}_\mathrm{T})W(z,0)\Gamma_j\psi(0)|\,H(P)\rangle$$

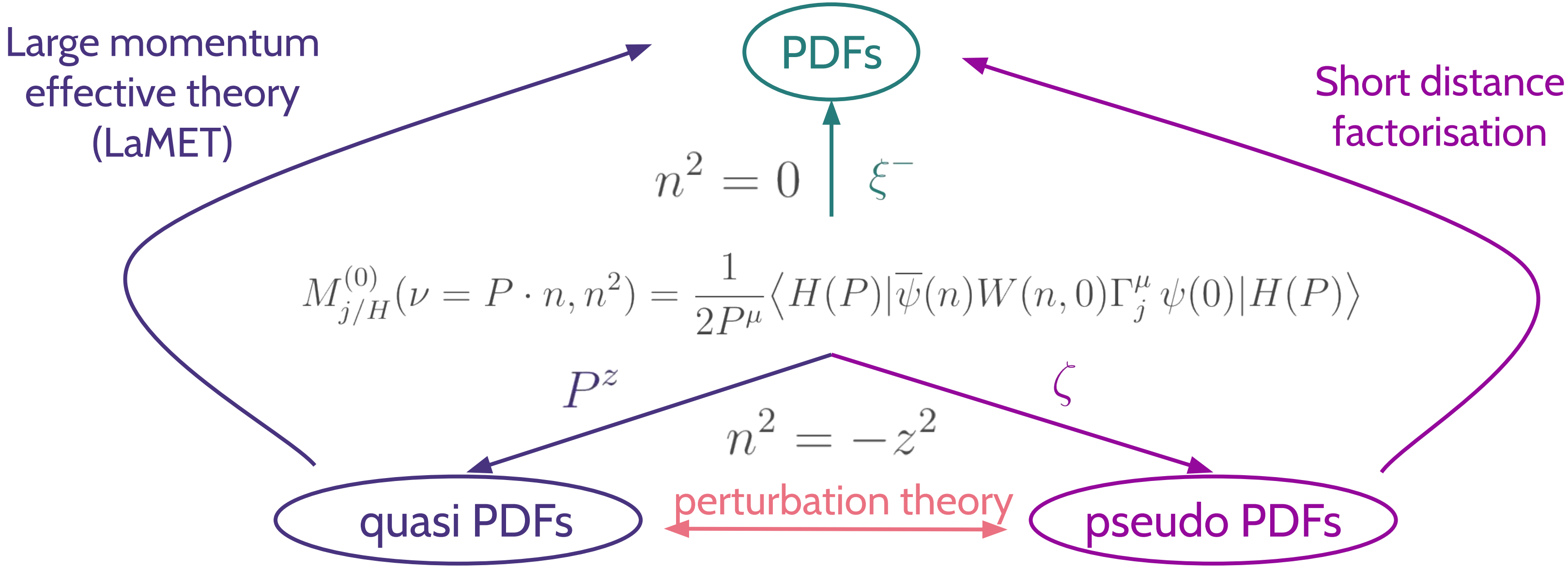
Ji, PRL 110 (2013) 262002

Radyushkin, PRD 96 (2017) 034025

A panoply of distributions

Lin et al., PPNP 100 (2018) 107
CJM, POS(LATTICE2018) 018
Zhao, IMJPA 33 (2019) 1830033

Cichy & Constantinou, AHEP (2019) 3036904
Constantinou et al., PPNP 121 (2021) 130908
Ji et al., RMP 93 (2021) 035005
Constantinou et al., 2202.07193



Large momentum effective theory (LaMET)

PDFs

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P^z ζ

$n^2 = -z^2$

quasi PDFs

perturbation theory

pseudo PDFs

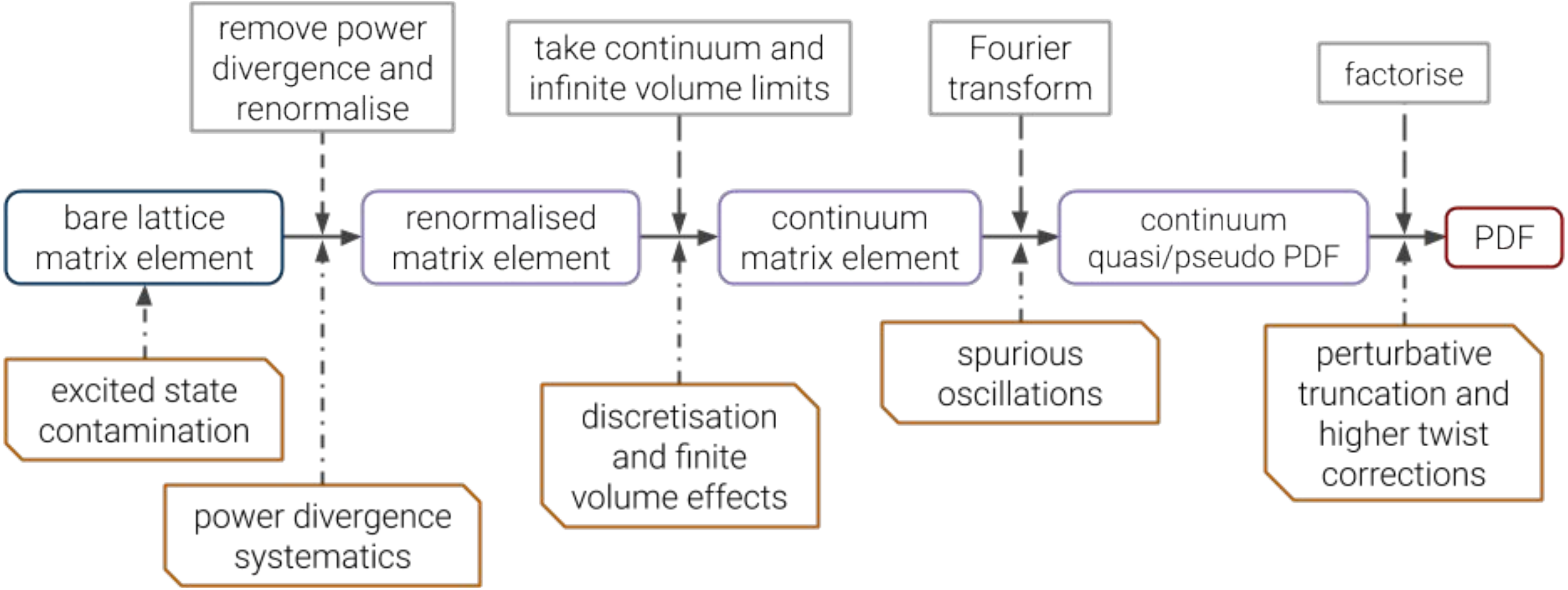
“Lattice ~~cross-sections~~” factorisable matrix elements

$$\sigma_{j/H}(\xi,\cdots)\in\left\{\widetilde{f}^{(0)}_{j/H}(\xi,P^z),\widetilde{p}^{(0)}_{j/H}(\xi,z^2),\cdots\right\}$$

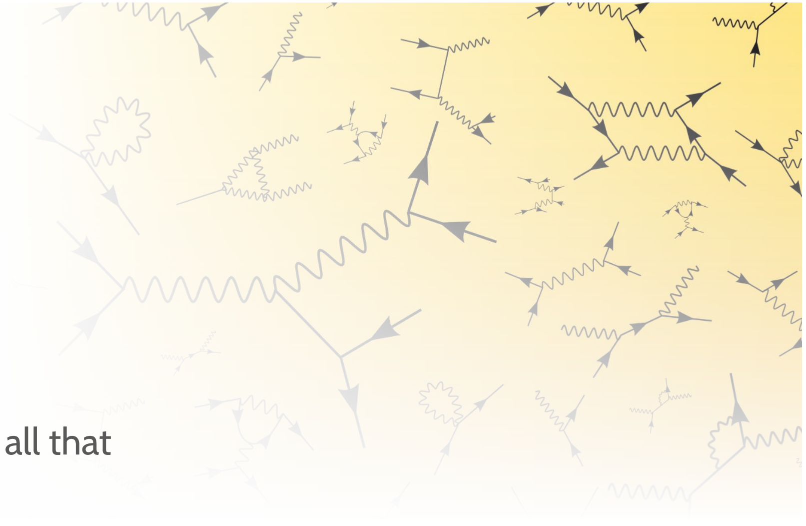
Note: Davoudi & Savage, PRD 86 (2012) 054505
Musch et al., PRD 83 (2011) 094507
Braun & Müller, EPJC 55 (2008) 349
Detmold & Lin, PRD 73 (2006) 014501
Liu & Dong, PRL 72 (1994) 1790

In practice

CJM, POS(LATTICE2018) 018



Overview



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    - Quark distributions
    - Gluon distributions

Current landscape of lattice calculations

Lattice determinations currently innovative but on the cusp of becoming industrial

Multiple collaborations investigating multiple theoretical approaches

Gao et al., PRD 107 (2023) 074509
Edwards et al., JHEP 03 (2023) 086
Bhat et al., PRD 106 (2022) 054504

- Benchmark calculations of well-known valence quark PDFs of nucleons and pions
  - state-of-the-art calculations with multiple lattice spacings and volumes at physical pion masses
  - identifying, quantifying and reducing systematic uncertainties

Gao et al., PRD 106 (2022) 074505
Gao et al., PRD 106 (2022) 114510
Gao et al., PRL 128 (2022) 142003
Egerer et al., PRD 105 (2022) 034507

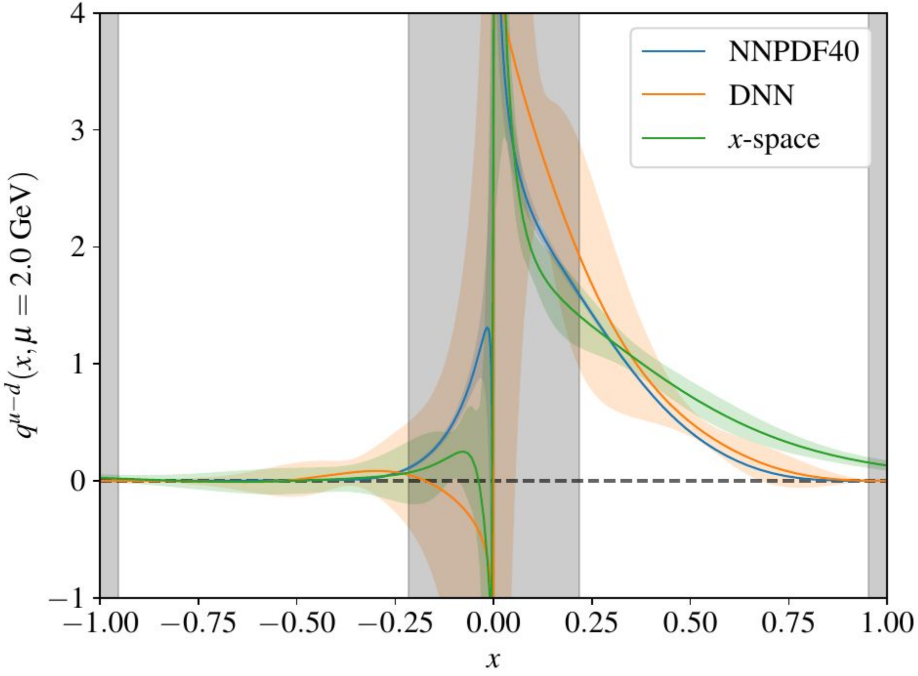
- Exploratory calculations of poorly-known parton distribution functions
  - gluon and quark-singlet PDFs of nucleons
  - quark parton distribution functions of other mesons and baryons
  - calculations of distributions beyond leading twist
  - systematic uncertainties of 20% will have significant phenomenological impact

Fan et al., PRD 108 (2023) 014508
Salas-Chavira et al., PRD 106 (2022) 0945110
Bhattarcharya et al., PRD 104 (2021) 114510

- Exploratory calculations of poorly-known three-dimensional structure
  - valence quark GPDs
  - Collins-Soper kernel for rapidity evolution of TMDs
  - systematic uncertainties of 50% will have very significant phenomenological impact

Battarcharya et al., PRD 106 (2022) 114512
Lin, PLB 824 (2022) 136821

Isovector unpolarised PDF of the nucleon



Gao et al., PRD 107 (2023) 074509

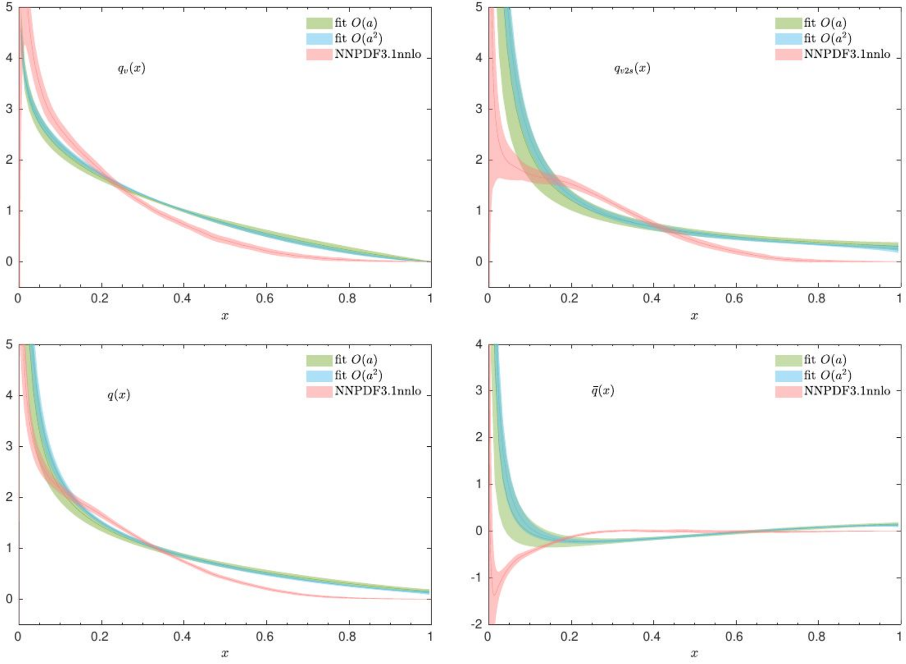
First determination using NNLO matching at physical pion mass

- a = 0.076 fm, pion mass of 140 MeV
- Nf = 2+1 HISQ fermions
- maximum momentum of 1.53 GeV
- NNLO perturbative matching

Comparison of methods:

- fit to first four moments
- pseudo-PDF with deep neural network
- quasi-PDF with hybrid renormalization

Isovector unpolarised PDF of the nucleon



Bhat et al., PRD 106 (2022) 5 054504

First determination using NNLO matching with continuum extrapolation

- a = 0.064, 0.082, 0.093 fm,
- pion mass of 370 MeV
- Nf = 2+1+1 twisted mass fermions
- maximum momentum of 1.8 GeV
- NNLO perturbative matching
- pseudo-PDF approach

Comparison of reconstruction methods

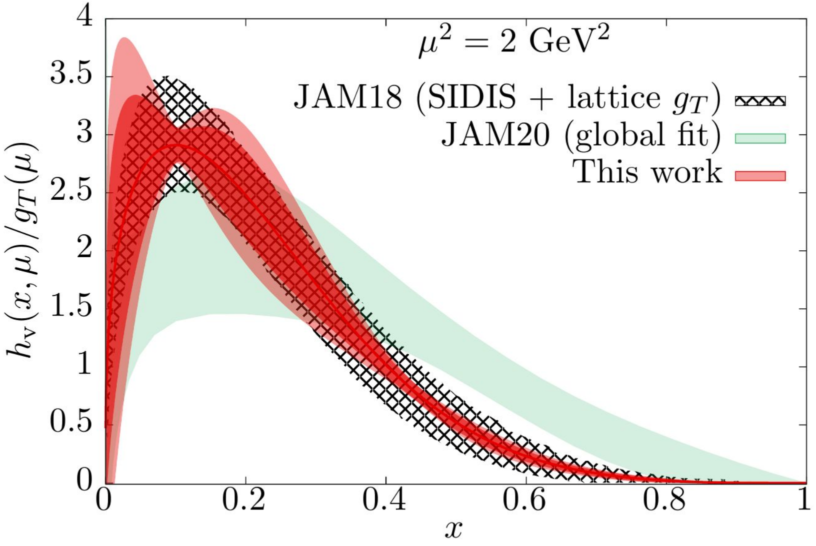
- naive Fourier transform
- Backus-Gilbert method
- model ansatz

Investigation of continuum limit

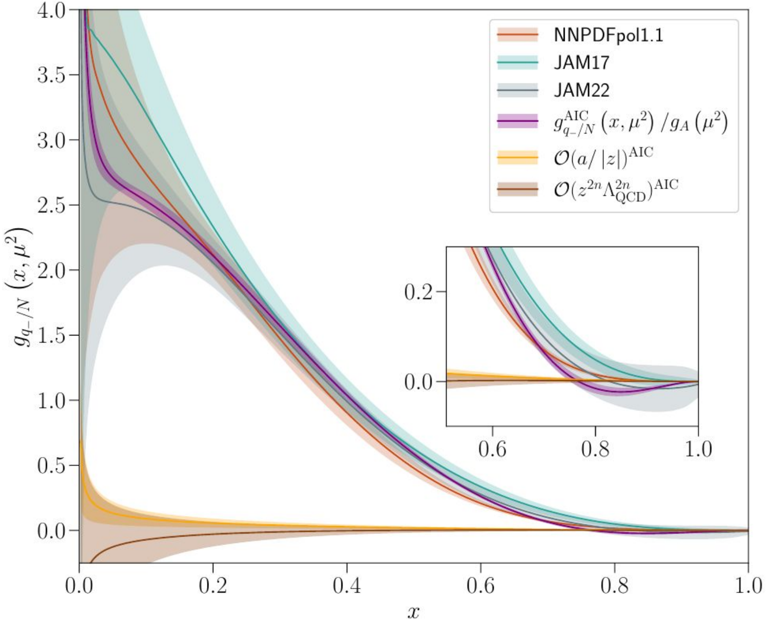
Tranversity and helicity distributions of the nucleon

Determined via pseudo-PDF approach

- a = 0.094 fm and pion mass of 358 MeV
- Nf = 2+1 clover-Wilson fermions
- maximum momentum of 2.47 GeV

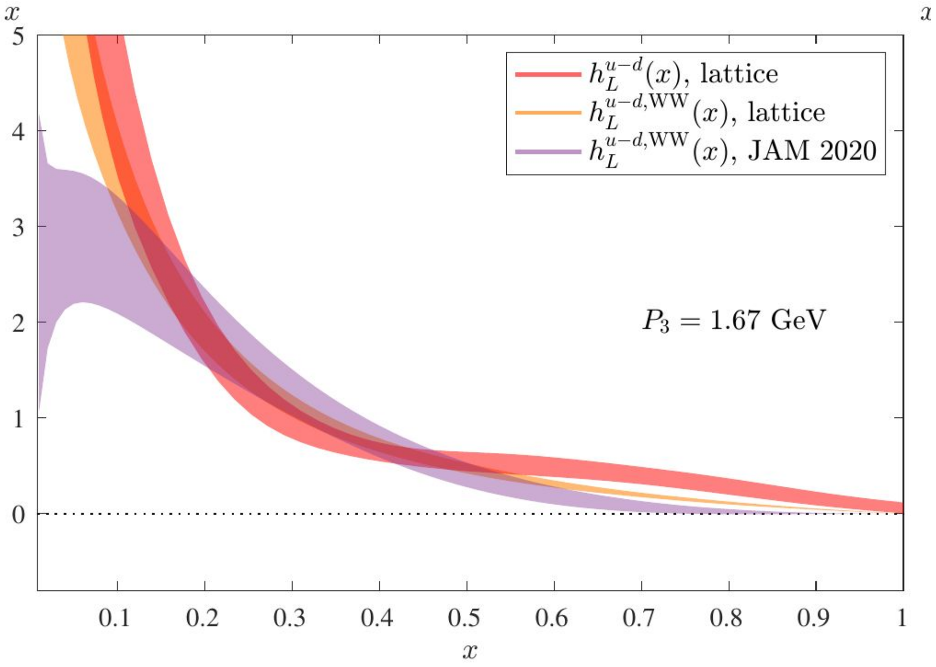


Egerer et al. (HadStruc), PRD 105 (2022) 034507



Edwards et al. (HadStruc), JHEP 03 (2023) 086

Isovector chiral-odd twist-3 PDF of the nucleon



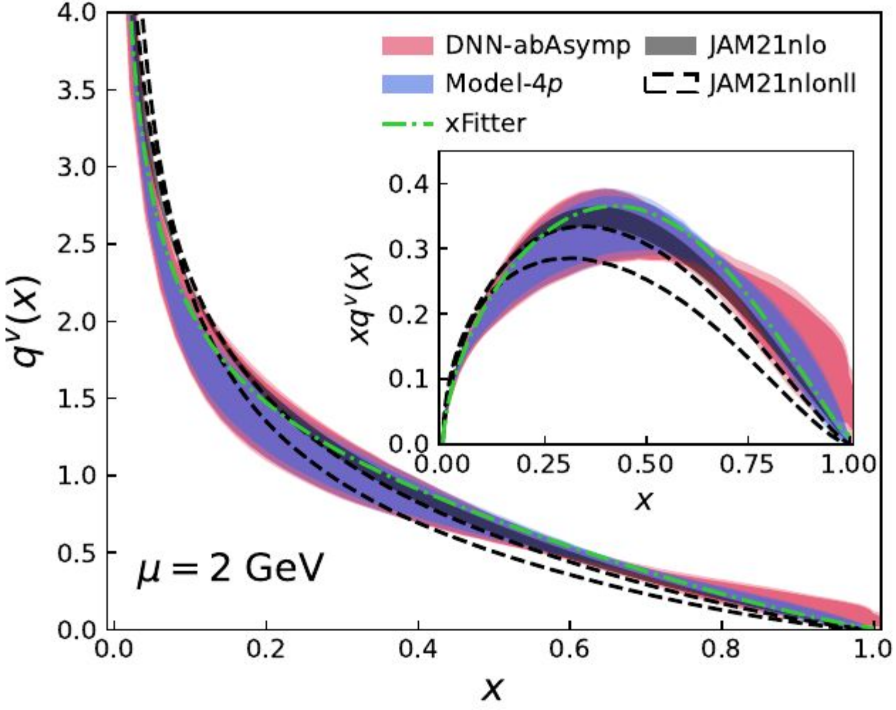
First determination of a twist-3 PDF

- a = 0.093 fm, pion mass of 260 MeV
- Nf = 2+1+1 twisted mass fermions
- maximum pion momentum of 1.67 GeV
- quasi-PDF method

Comparison of Wandzura-Wilczek approximation to lattice data and global fit

Bhattarcharya et al., PRD 104 (2021) 114510

Isovector unpolarised light quark PDFs of the pion



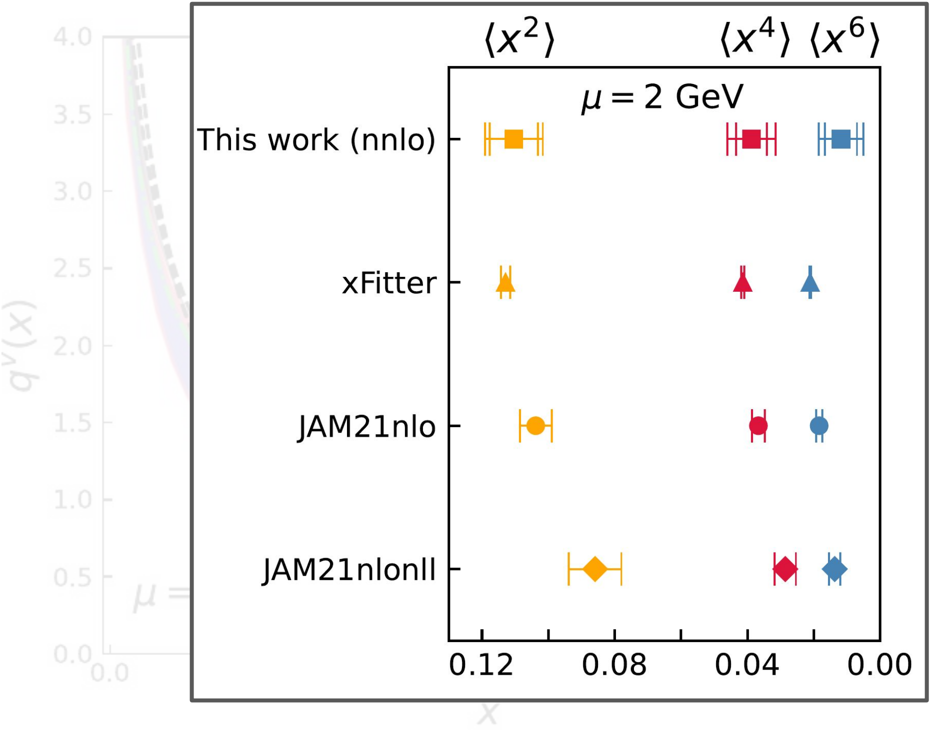
Gao et al., PRD 106 (2022) 114510

First determination of continuum-extrapolation at physical pion mass

- a = 0.04, 0.06, 0.075 fm,
- pion masses of 140 and 300 MeV
- Nf = 2+1 HISQ fermions
- maximum pion momentum of 2.42 GeV
- NNLO perturbative matching

See also: Gao et al., PRL 128 (2022) 14 142003

Isovector unpolarised light quark PDFs of the pion



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Gluons!

Exploratory calculations (one lattice spacing and pion mass)

Nucleons:	Fan et al., PRL 121 (2018) 242001 Fan et al., IJMP A 36 (2021) 2150080 Khan et al. (HadStruc), PRD 104 (2021) 094516 Egerer et al. (HadStruc), PRD 106 (2022) 094511
Mesons:	Fan et al., PLB 823 (2021) 136778 Salas-Chavira et al., PRD 106 (2022) 9 0945110

Gluons are key to understanding the visible universe

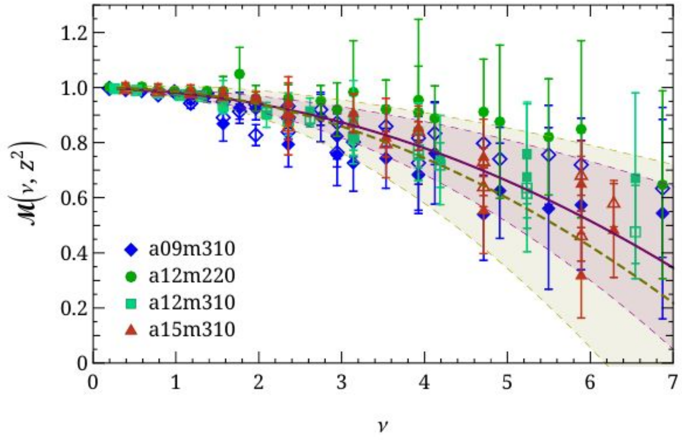
- Dominant contribution to mass of the visible universe
- Significant contribution to the spin of hadrons
- Fundamental to understanding a new form of matter: color glass condensate

Complete tomography of hadrons* needs detailed understanding of gluon structure

*a central goal of the largest US-based collider effort (EIC) for the next 20 years

Opportunity for collaboration and interplay between theory and experiment

Unolarised gluon distribution of the nucleon



First determination of unpolarized gluon distribution in continuum and physical pion mass limits

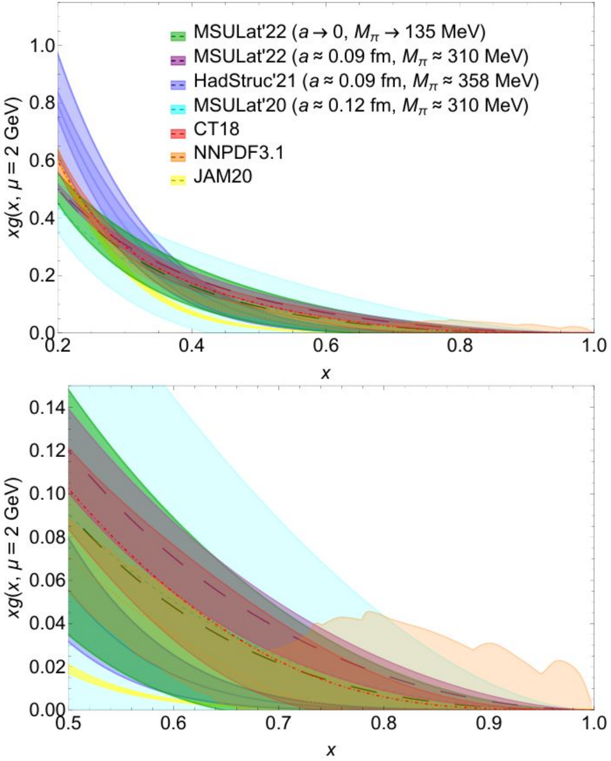
Steps towards a precision calculation

- pseudo-distribution approach
- three lattice spacings and three pion masses
- clover-Wilson on Nf = 2+1+1 HISQ
- maximum nucleon momentum of 2.14 GeV

Demonstrates steps towards full quantitative control of systematic uncertainties for lattice calculations of gluon distributions

Fan et al., PRD 108 (2023) 014508

Unolarised gluon distribution of the nucleon



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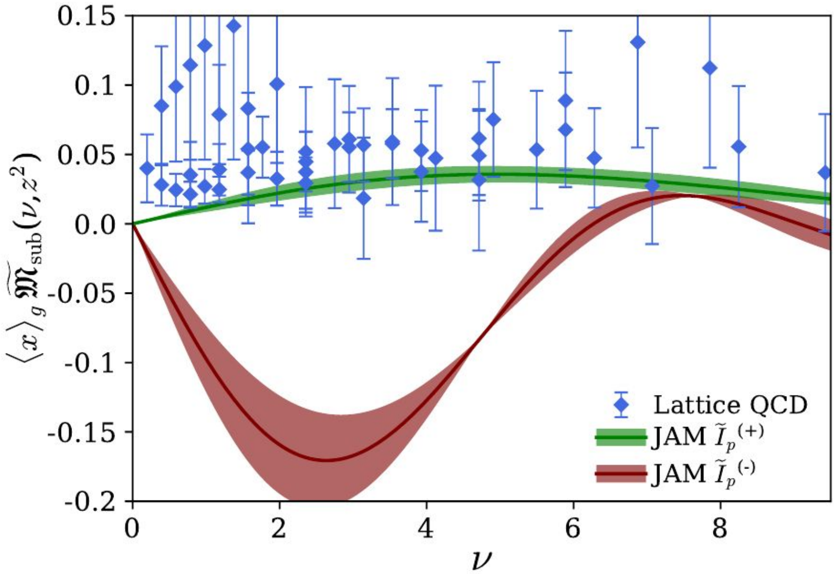
Polarised gluon distribution of the nucleon



Egerer et al. (HadStruc), PRD 106 (2022) 094511

First determination of polarized gluon distribution

Very poorly constrained from experiment

Proof-of-principle calculation

- one lattice spacing and one pion mass
- Nf = 2+1 clover-Wilson
- maximum nucleon momentum of 2.29 GeV

Demonstrates potential of lattice calculations, at the level of 20-50% precision, to understand poorly-constrained distributions

Lattice inputs for global analyses

QCD factorization: PDFs are universal hadronic quantities, independent of probe

- Extract from experimental cross-sections
- Extract from lattice matrix elements

JAM-HadStruc approach:

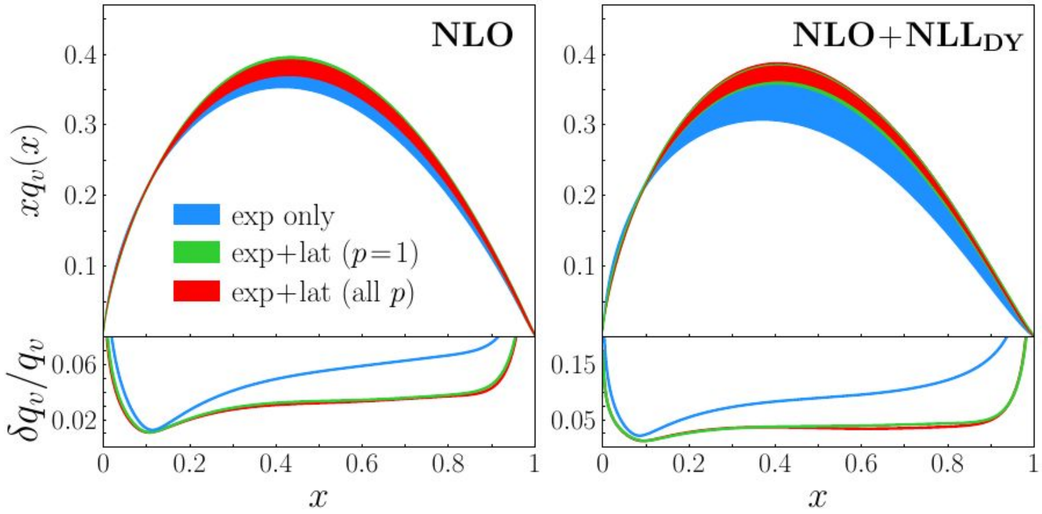
- Lattice matrix elements provide first principles' priors for global analysis framework

Lattice results and experimental data complementary

- For example, lattice QCD sensitive to $x > 0.2$, where experimental data are less constraining

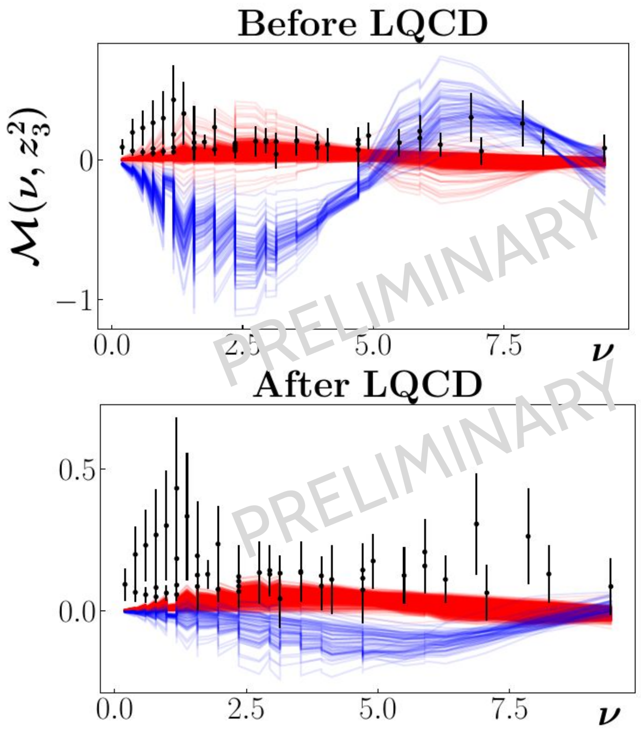
Lattice inputs for global analyses

Isovector quark PDF of the pion



Barry et al. (HadStruc & JAM), PRD 105 (2022) 114051

Polarised gluon PDF of the nucleon



(HadStruc & JAM), in progress

Summary

Multiple efforts worldwide to extract PDFs from lattice QCD calculations

Challenging calculations, but significant theoretical and computational progress
Benchmark calculations of well-known valence quark PDFs of nucleons and pions

- state-of-the-art calculations with multiple lattice spacings and volumes at physical pion masses
- identifying, quantifying and reducing systematic uncertainties

Exploratory calculations of poorly-known parton distribution functions and three-dimensional structure

- gluon and quark-singlet PDFs of nucleons
- quark parton distribution functions of other mesons and baryons
- valence quark GPDs
- Collins-Soper kernel for rapidity evolution of TMDs

Ongoing efforts to incorporate lattice data as Bayesian priors in global analysis

Outlook

Current and near-term: 0-5 years

- Fully-controlled calculations of isovector quark PDFs at large *x*
- Calculations of isoscalar quark and unpolarised gluon PDFs
- Proof-of-principle calculations of TMDs
- Calculations of GPDs mapped over a range of momentum transfers and skewness
- Continued theoretical and algorithmic development to enable long-term goals
- Pipelines for global analyses of 3D structure incorporating lattice data

Longer term: 5-10 years

- Fully-controlled precision calculations of isovector quark PDFs at large *x*
- Fully-controlled calculations of isoscalar quark and unpolarised gluon PDFs
- Calculations of TMDs at a range of transverse momenta
- Controlled calculations of GPDs over a broader kinematic range
- Global analyses of 3D structure that include lattice data, tightly coupled to EIC program

e.g. Quark-Gluon Tomography Topical Collaboration

Thank you!

Chris Monahan

cjmonahan@wm.edu

WILLIAM & MARY
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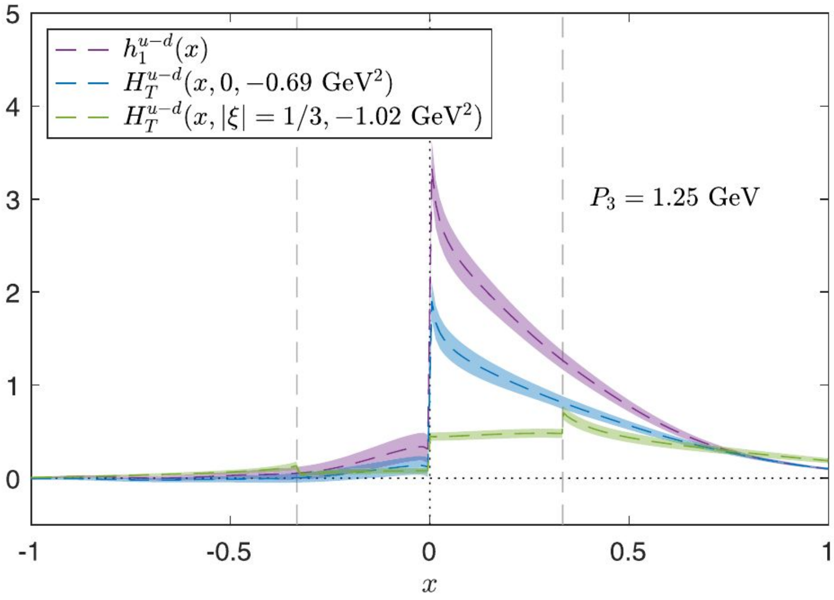
QUARK-GLUON TOMOGRAPHY COLLABORATION

DOE Early Career Award DE-SC0023047

Back-up slides

GPDs of the nucleon



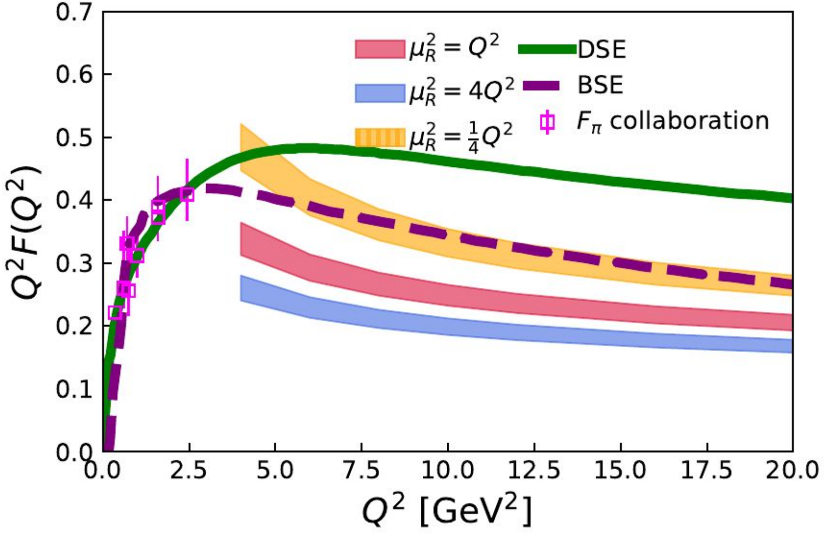
Alexandrou et al. (ETM), PRD 105 (2022) 034501

First determination of quark transversity GPDs of the nucleon

- Nf = 2+1+1 twisted mass clover-Wilson fermions with one lattice spacing and pion mass
- Maximum nucleon momentum = 1.67 GeV
- Maximum momentum transfer = -1.02 GeV^2
- Zero and nonzero skewness

Demonstrates feasibility for lattice QCD calculations to map the three-dimensional structure of hadrons!

Distribution amplitude of the pion



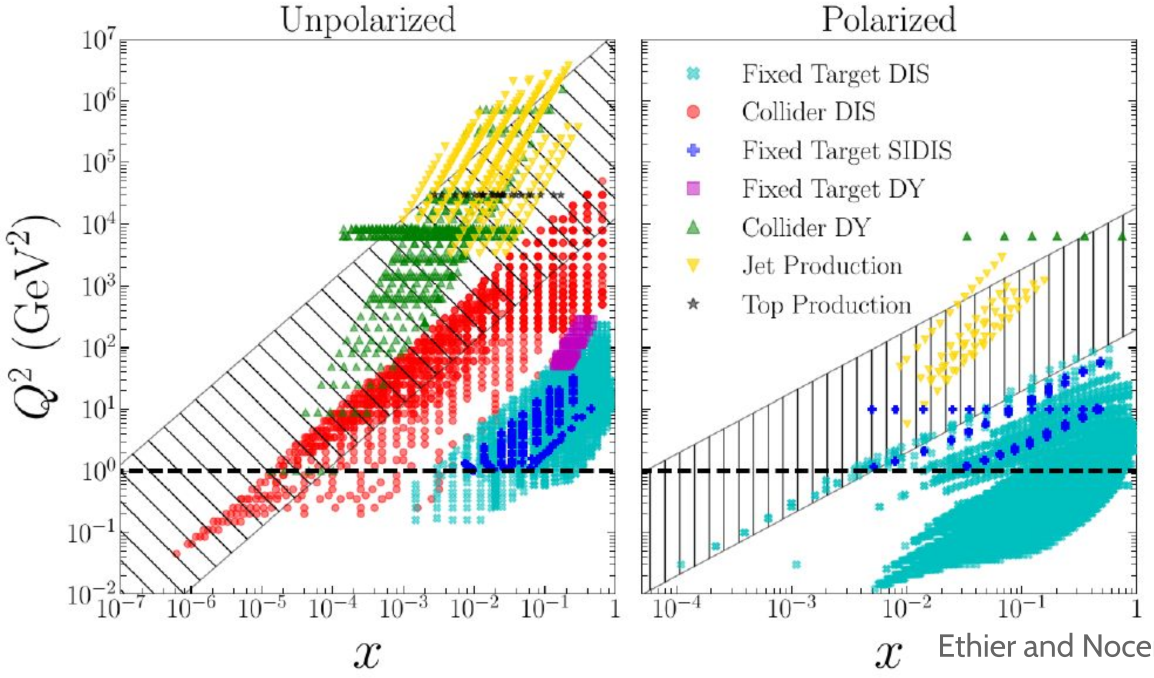
Gao et al., PRD 106 (2022) 074505

Determination of pion distribution amplitude

- a = 0.076 fm and pion mass of 140 MeV
- Wilson-clover on Nf = 2+1 HISQ fermions
- maximum pion momentum of 1.78 GeV
- NNLO perturbative matching

Gluon PDFs from experimental data

PDFs: How much of the momentum of a fast-moving hadron is carried by its constituent gluons?



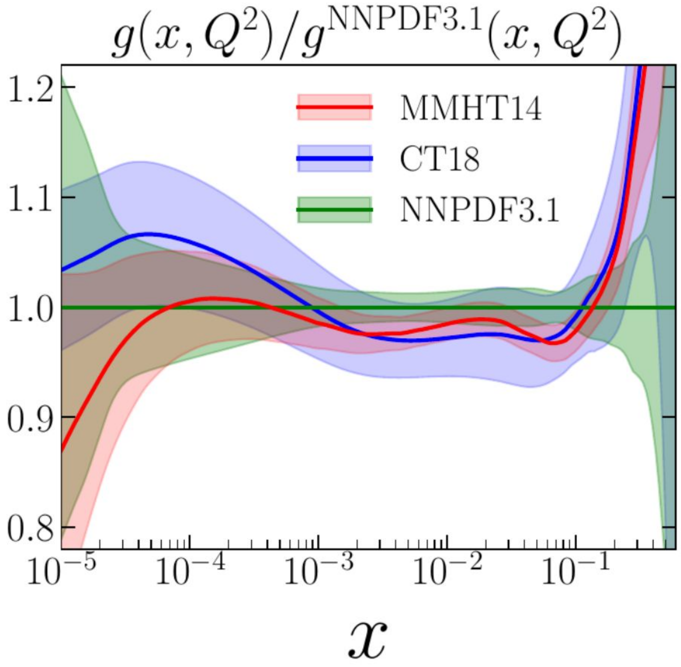
Ethier and Nocera, ARNPS 70 (2020) 43

LHC has considerably improved our knowledge of gluon PDFs

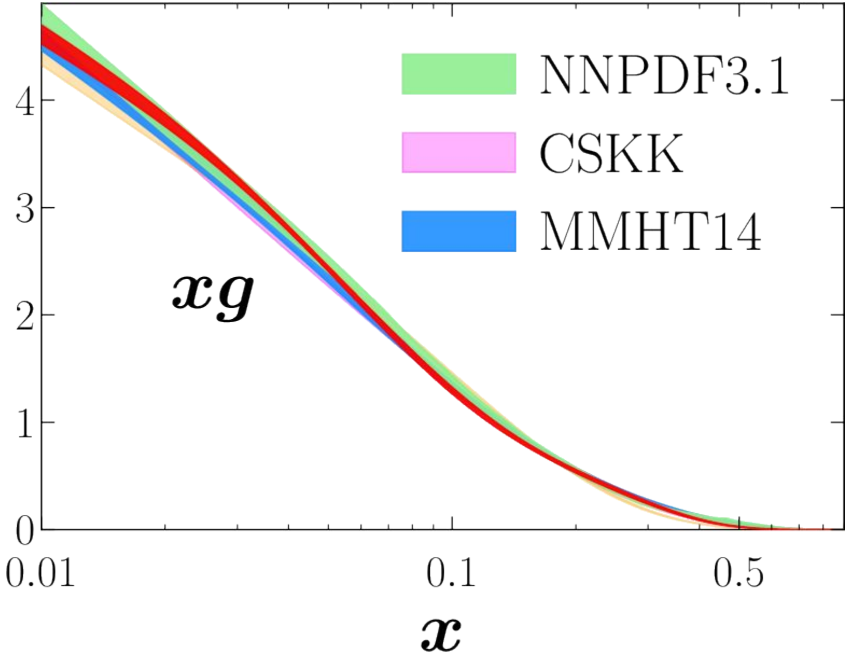
EIC and LHeC will expand this significantly

Gluon PDFs from experimental data

PDFs: How much of the momentum of a fast-moving hadron is carried by its constituent gluons?



Ethier and Nocera, Ann.Rev.Nucl.Part.Sci. 70 (2020) 43



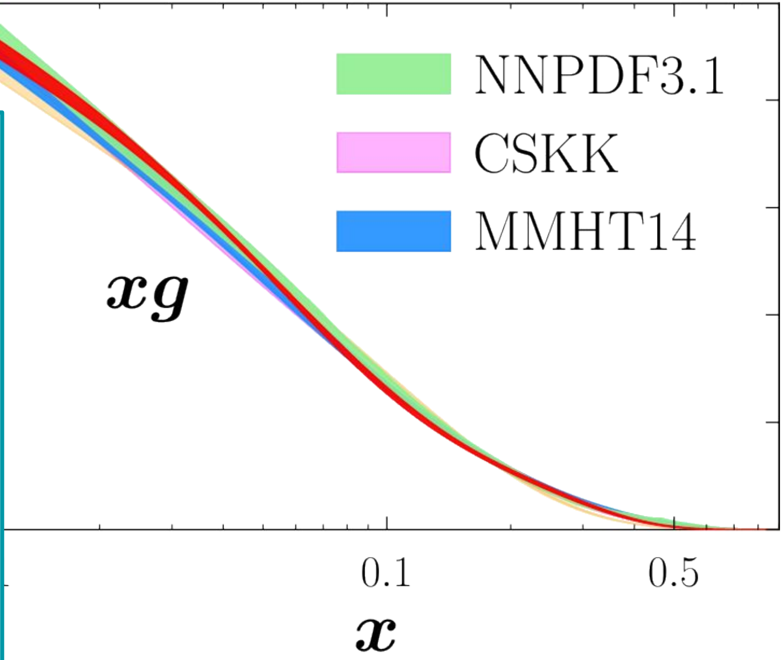
Zhou et al., PRD 105 (2022) 074022

Large uncertainties remain at large and small Bjorken-x

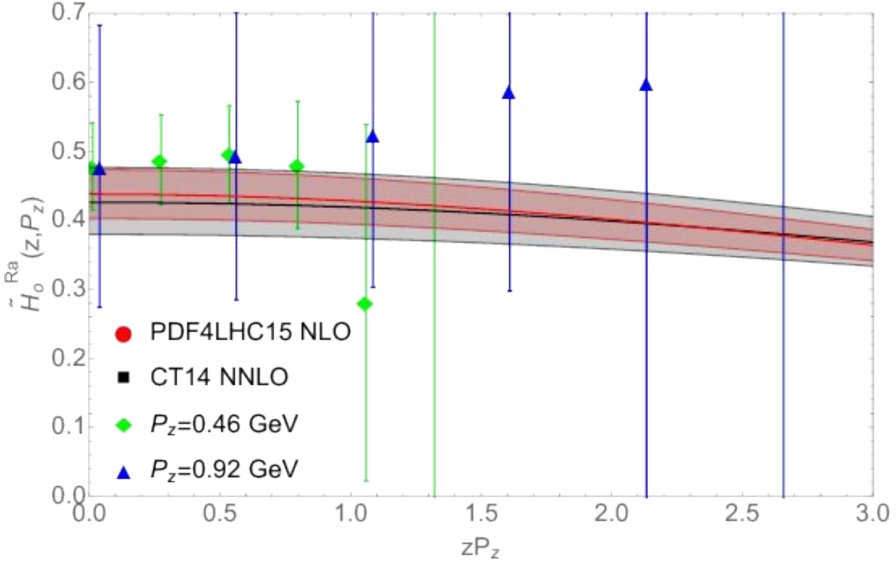
See also: Fan et al., Int.J.Mod.Phys.A 36 (2021) 13
Fan and Lin, PLB 823 (2021) 136778

Gluon PDFs from experimental data

PDFs: How much of the momentum of a fast-moving hadron is carried by its constituent gluons?



First proof-of-principle calculation of gluon PDF



Fan *et al*., PRL 121 (2018) 242001

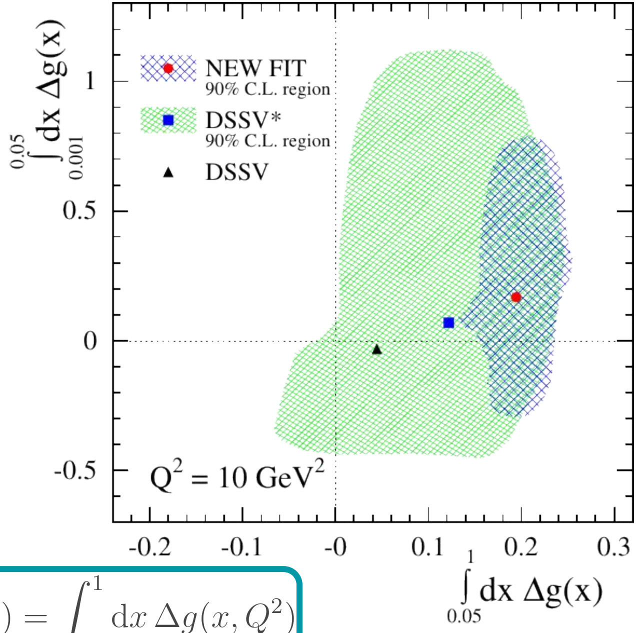
Aidala et al., RMP 85 (2013) 655
Deur et al., Rep. Prog. Phys. 82 (2019)
Ji et al., Nat. Rev. Phys. 3 (2021) 27

The origin of hadron spin

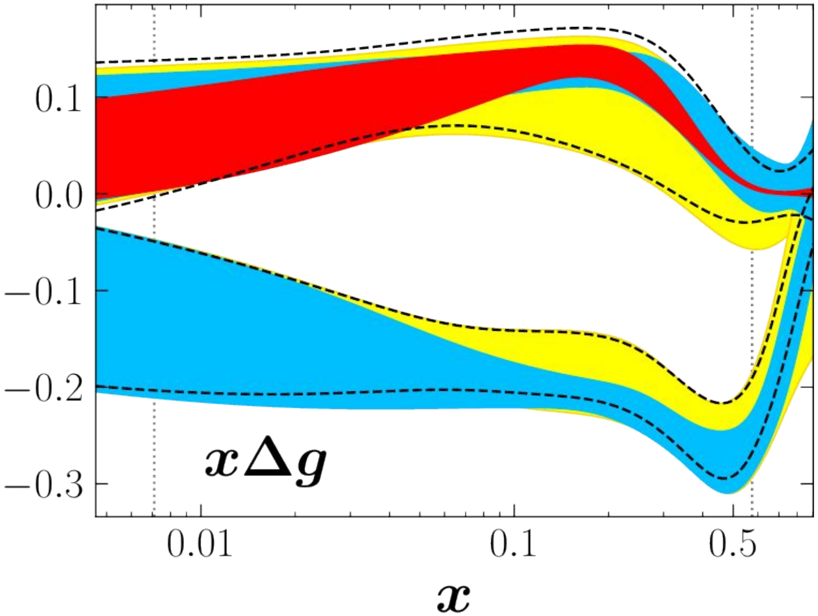
30 years after the EMC experiment precipitated the “proton spin crisis”, experimental picture still unclear

Quarks carry approximately 30% of the proton’s spin, gluon picture is much less clear

de Florian et al., PRL 113 (2014) 012001



Zhou et al., PRD 105 (2022) 074022



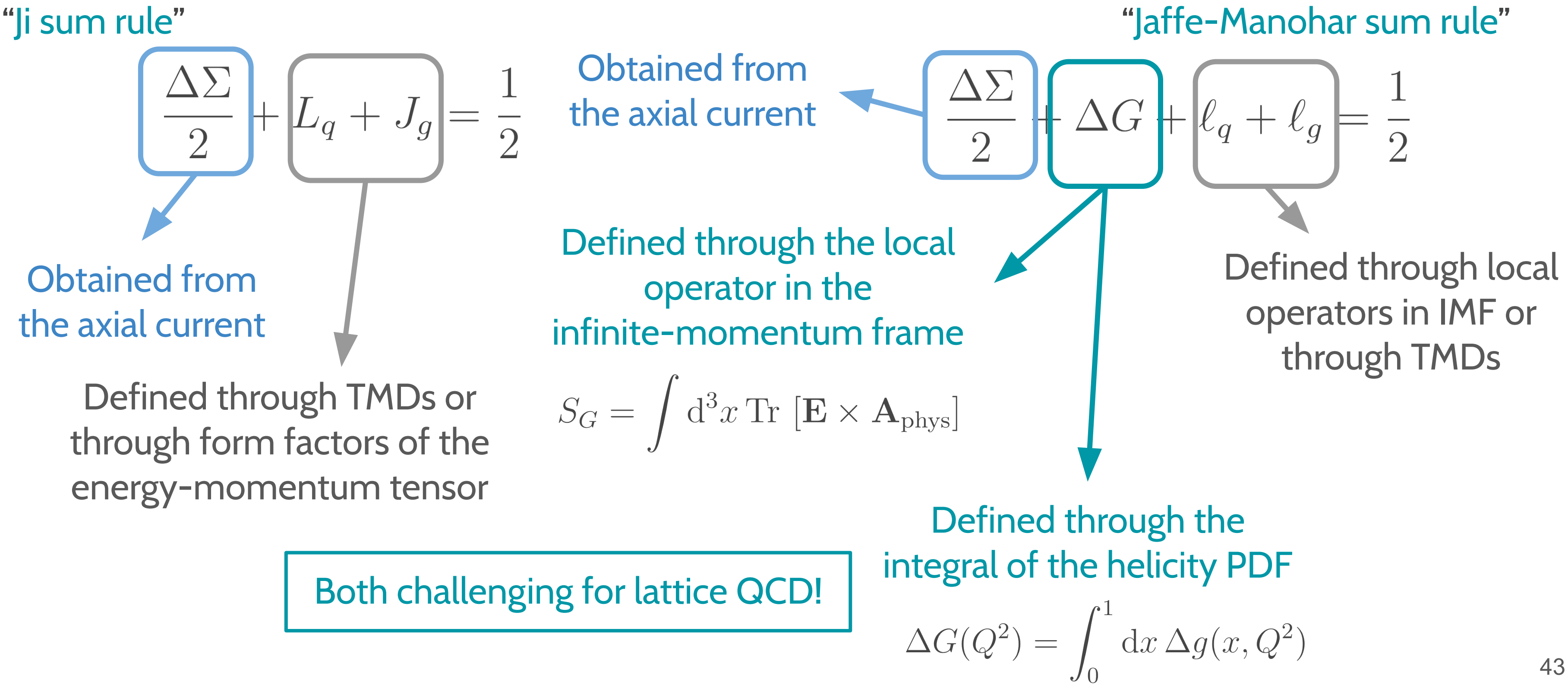
$$\boxed{\Delta G(Q^2) = \int_0^1 \mathrm{d}x\, \Delta g(x, Q^2)}$$

See positivity bound discussion: Collins et al., PRD 105 (2022) 076010

Liu, AAPPS Bull. (2022) 32
Ji et al., Nat. Rev. Phys. 3 (2021) 27

The origin of hadron spin

Broadly speaking - two approaches to first principles calculations of these contributions



“Ji sum rule”

$$\boxed{\frac{\Delta\Sigma}{2}} + \boxed{L_q + J_g} = \frac{1}{2}$$

Obtained from the axial current

Defined through TMDs or through form factors of the energy-momentum tensor

“Jaffe-Manohar sum rule”

$$\boxed{\frac{\Delta\Sigma}{2}} + \boxed{\Delta G} + \boxed{\ell_q + \ell_g} = \frac{1}{2}$$

Obtained from the axial current

Defined through the local operator in the infinite-momentum frame

$$S_G = \int \mathrm{d}^3x \,\mathrm{Tr}\,[\mathbf{E} \times \mathbf{A}_{\mathrm{phys}}]$$

Defined through local operators in IMF or through TMDs

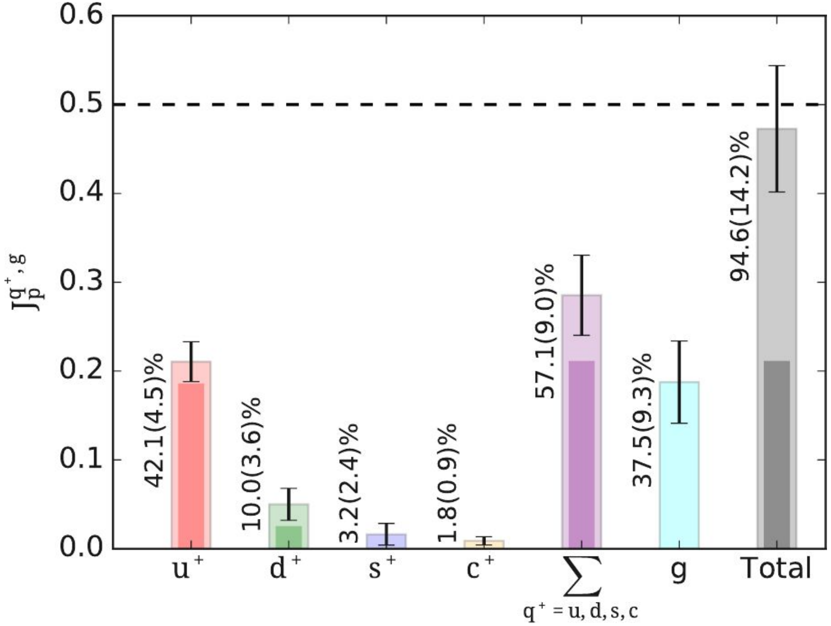
Defined through the integral of the helicity PDF

$$\Delta G(Q^2) = \int_0^1 \mathrm{d}x \, \Delta g(x, Q^2)$$

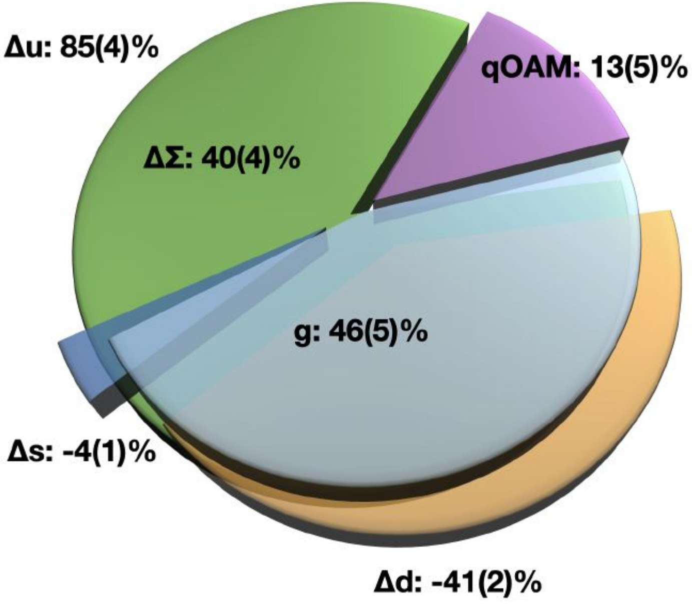
Both challenging for lattice QCD!

First calculations of the spin of the proton

Two state-of-the-art decompositions from lattice QCD



Alexandrou et al., PRD 101 (2020) 094513



Wang et al., PRD 106 (2022) 014512

Gluon PDFs: pseudo-distribution formalism

Starting point:

Balitsky, Morris and Radyushkin, PLB 808 (2020) 135621

$$M^{(0)}_{\mu\nu\rho\sigma;H}(P,n) = \langle H(P)|G_{\mu,\nu}(n^\alpha)W^{(A)}(n^\alpha,0)G_{\rho\sigma}|H(P)\rangle$$

$$W^{(A)}(n^\alpha,0) = \mathcal{P}\exp\left\{ig\int_0^n \mathrm{d}y^\mu A^{(A)}_\mu(y)\right\}$$

Gluon PDFs: pseudo-distribution formalism

Starting point:

Balitsky, Morris and Radyushkin, PLB 808 (2020) 135621

$$\begin{tikzcd}
M^{(0)}_{\mu\nu\rho\sigma;H}(P,n) = \langle H(P)|G_{\mu,\nu}(n^\alpha)W^{(A)}(n^\alpha,0)G_{\rho\sigma}|H(P)\rangle \arrow[dr, "n^2=0"'] & \\
& \mathcal{M}^{(0)}_{g/H}(\nu,0) = \frac{1}{2(P^+)^2}\left[M^{(0)}_H(P,z^-)\right]^{+\mu}{}_{+\mu} \arrow[d, "\xi^-", "\text{PDFs}"'] \\
& x\, f^{(0)}_{g/H}(x) = \int_{-1}^{1} \mathrm{d}\nu\, e^{ix\nu}\mathcal{M}^{(0)}_{g/H}(\nu,0)
\end{tikzcd}$$

Gluon PDFs: pseudo-distribution formalism

Starting point:

Balitsky, Morris and Radyushkin, PLB 808 (2020) 135621

$$\begin{tikzcd}
M^{(0)}_{\mu\nu\rho\sigma;H}(P,n) = \langle H(P)|G_{\mu,\nu}(n^\alpha)W^{(A)}(n^\alpha,0)G_{\rho\sigma}|H(P)\rangle \arrow[d, swap, "n^2=-z^2"] \arrow[dr, "n^2=0"] & \\
\mathcal{M}^{(0)}_{g/H}(\nu,z^2) = \frac{1}{2E_P^2}\left[M^{(0)}_{0ii0;H}(P,z) - M^{(0)}_{jiij;H}(P,z)\right] & \mathcal{M}^{(0)}_{g/H}(\nu,0) = \frac{1}{2(P^+)^2}\left[M^{(0)}_{H}(P,z^-)\right]^{+\mu}{}_{+\mu} \arrow[dd, "\xi^-", swap, "\text{PDFs}"] \\
\mathcal{M}^{(\mathrm{red.})}_{g/H}(\nu,z^2) = \left(\frac{\mathcal{M}^{(0)}_{g/H}(\nu,z^2)}{\mathcal{M}^{(0)}_{g/H}(\nu,0)|_{z=0}}\right)\Big/\left(\frac{\mathcal{M}^{(0)}_{g/H}(0,z^2)|_{p=0}}{\mathcal{M}^{(0)}_{g/H}(0,0)|_{p=0,z=0}}\right) \arrow[d, swap, "\zeta"] \arrow[d, "\text{pseudo PDFs}"] & \\
\mathcal{M}^{(\mathrm{red.})}_{g/H}(\nu,z^2) = \int_0^1 \frac{\mathrm{d}\xi\,\xi}{\langle\xi\rangle^2(\mu)}\left[c_{gg}(\xi\nu,\mu^2z^2)f_{g/H}(\xi,\mu^2) + \frac{P^z}{E_P}c_{gq}(\xi\nu,\mu^2z^2)f_{S/H}(\xi,\mu^2)\right] & x f^{(0)}_{g/H}(x) = \int_{-1}^{1}\mathrm{d}\nu\, e^{ix\nu}\mathcal{M}^{(0)}_{g/H}(\nu,0)
\end{tikzcd}$$

Gluon PDFs: pseudo-distribution formalism

Starting point:

Balitsky, Morris and Radyushkin, PLB 808 (2020) 135621

$$M^{(0)}_{\mu\nu\rho\sigma;H}(P,n) = \langle H(P)|G_{\mu,\nu}(n^\alpha)W^{(A)}(n^\alpha,0)G_{\rho\sigma}|H(P)\rangle$$

$n^2 = -z^2$ $\downarrow$

$$\mathcal{M}^{(0)}_{g/H}(\nu,z^2) = \frac{1}{2E_P^2}\left[M^{(0)}_{0ii0;H}(P,z) - M^{(0)}_{jiij;H}(P,z)\right]$$

$$\mathcal{M}^{(\mathrm{red.})}_{g/H}(\nu,z^2) = \left(\frac{\mathcal{M}^{(0)}_{g/H}(\nu,z^2)}{\mathcal{M}^{(0)}_{g/H}(\nu,0)|_{z=0}}\right) \Big/ \left(\frac{\mathcal{M}^{(0)}_{g/H}(0,z^2)|_{p=0}}{\mathcal{M}^{(0)}_{g/H}(0,0)|_{p=0,z=0}}\right)$$

ζ $\downarrow$ pseudo PDFs

$$\mathcal{M}^{(\mathrm{red.})}_{g/H}(\nu,z^2) = \int_0^1 \frac{\mathrm{d}\xi\,\xi}{\langle\xi\rangle^2(\mu)}\left[c_{gg}(\xi\nu,\boxed{\mu^2z^2)f_{g/H}(\xi,\mu^2)} + \frac{P^z}{E_P}c_{gq}\cancel{(\xi\nu,\mu^2z^2)f_{S/H}(\xi,\mu^2)}\right]$$

what we want

neglect mixing

$n^2 = 0$ $\searrow$

$$\mathcal{M}^{(0)}_{g/H}(\nu,0) = \frac{1}{2(P^+)^2}\left[M^{(0)}_H(P,z^-)\right]^{+\mu}{}_{+\mu}$$

PDFs $\downarrow$ ξ^-

$$x\,f^{(0)}_{g/H}(x) = \int_{-1}^{1}\mathrm{d}\nu\,e^{ix\nu}\mathcal{M}^{(0)}_{g/H}(\nu,0)$$

Balitsky, Morris and Radyushkin, PLB 808 (2020) 135621
Balitsky, Morris and Radyushkin, JHEP 02 (2022) 193

Gluon PDFs: pseudo-distribution formalism

Starting point:

$$\boxed{M^{(0)}_{\mu\nu\rho\sigma;H}(P,z) = \langle H(P)|G_{\mu\nu}(0,z,\mathbf{0}_\mathrm{T})W^{(A)}(z,0)\widetilde{G}_{\rho\sigma}(0)|H(P)\rangle}$$

$$M^{(0)}_{\mu\nu\rho\sigma;H}(P,n) = \langle H(P)|G_{\mu,\nu}(n^\alpha)W^{(A)}(n^\alpha,0)G_{\rho\sigma}|H(P)\rangle$$

$$n^2 = -z^2 \Big\downarrow$$

$$\boxed{M^{(0)}_{0i;0i}(P,z) + M^{(0)}_{ij;ij}(P,z) = -2P^z E_H \mathcal{M}_{\Delta g/H}(\nu,z^2) + 2E_H^3 z \mathcal{M}_{pp}(\nu,z^2)}$$

what we want

$$\mathcal{M}^{(0)}_{g/H}(\nu,z^2) = \frac{1}{2E_P^2}\left[M^{(0)}_{0ii0;H}(P,z) - M^{(0)}_{jiij;H}(P,z)\right]$$

$$\mathcal{M}^{(\mathrm{red.})}_{g/H}(\nu,z^2) = \left(\frac{\mathcal{M}^{(0)}_{g/H}(\nu,z^2)}{\mathcal{M}^{(0)}_{g/H}(\nu,0)|_{z=0}}\right) \Big/ \left(\frac{\mathcal{M}^{(0)}_{g/H}(0,z^2)|_{p=0}}{\mathcal{M}^{(0)}_{g/H}(0,0)|_{p=0,z=0}}\right)$$

$$\zeta \Big\downarrow$$ pseudo PDFs

$$\mathcal{M}^{(\mathrm{red.})}_{g/H}(\nu,z^2) = \int_0^1 \frac{\mathrm{d}\xi\,\xi}{\langle\xi\rangle^2(\mu)}\left[c_{gg}(\xi\nu,\mu^2z^2)f_{g/H}(\xi,\mu^2) + \frac{P^z}{E_P}c_{gq}(\xi\nu,\mu^2z^2)f_{S/H}(\xi,\mu^2)\right]$$

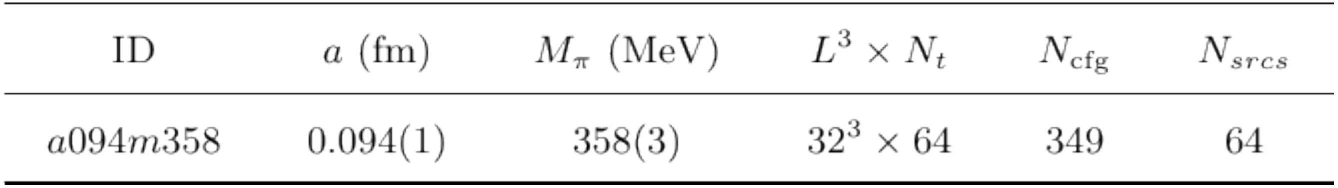
HadStruc lattice implementation

Gluons provide significant signal-to-noise challenges for lattice calculations, mitigated through:

1. Gradient flow smearing reduces ultraviolet fluctuations
2. Distillation and summed GEVP method improves operator overlap and reduces excited state contamination
3. Reduced Ioffe-time distribution reduces correlated uncertainties through ratio

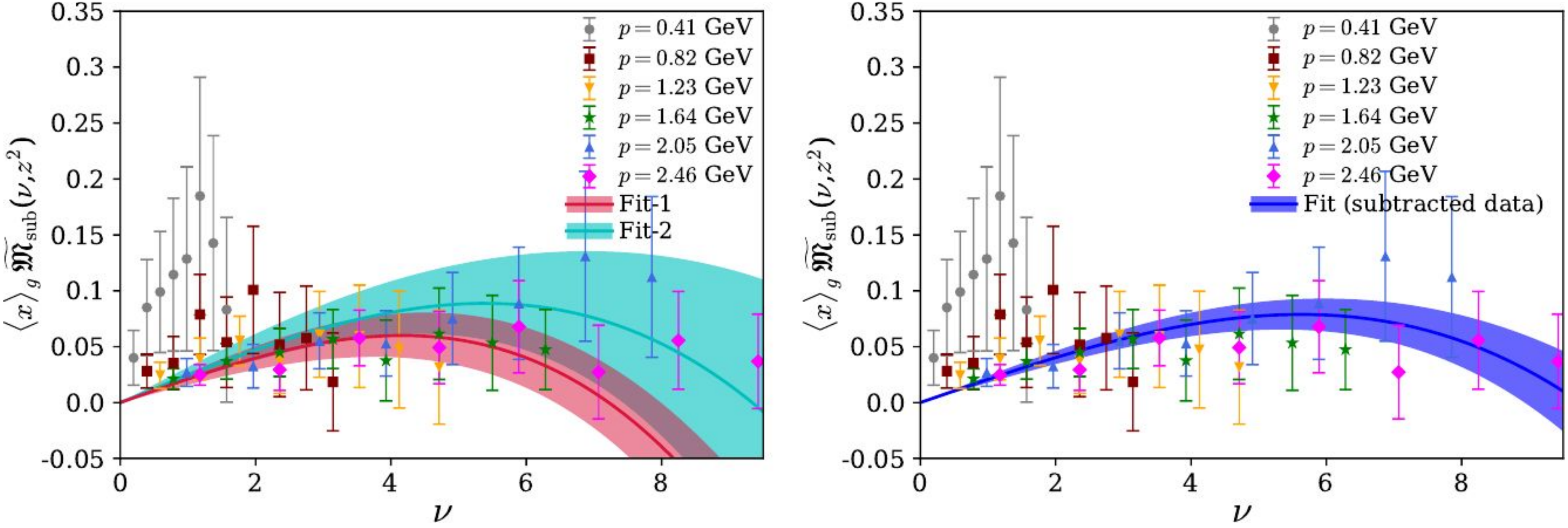
We neglect mixing with scalar quark distribution

Results calculated on a single lattice ensemble of 2+1 stout-smeared Wilson-improved clover fermions and tree-level tadpole-improved Symanzik gauge action, with Wilson flow, momentum-smeared nucleon interpolating operators and unimproved gauge field tensor



ID	a (fm)	M_π (MeV)	$L^3 \times N_t$	N_{cfg}	N_{srcs}
$a094m358$	0.094(1)	358(3)	$32^3 \times 64$	349	64

Polarised gluon pseudo-distribution



Egerer et al. (HadStruc), PRD 106 (2022) 094511