

# Global Fitting Paradigms

## Synergy Between Lattice And Phenomenology?

### QCD Real-Time Dynamics and Inverse Problems

**Date:** Monday, October 19, 2020 - 8:20am to Thursday, October 22, 2020 - 12:30pm



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Research: Partonic distributions  
from Lattice QCD



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Jefferson Lab Theory Center  
Research: QCD Global analysis  
(JAM) on hadron structure and  
hadronization

**Panel Discussion**

# What we are going to discuss?

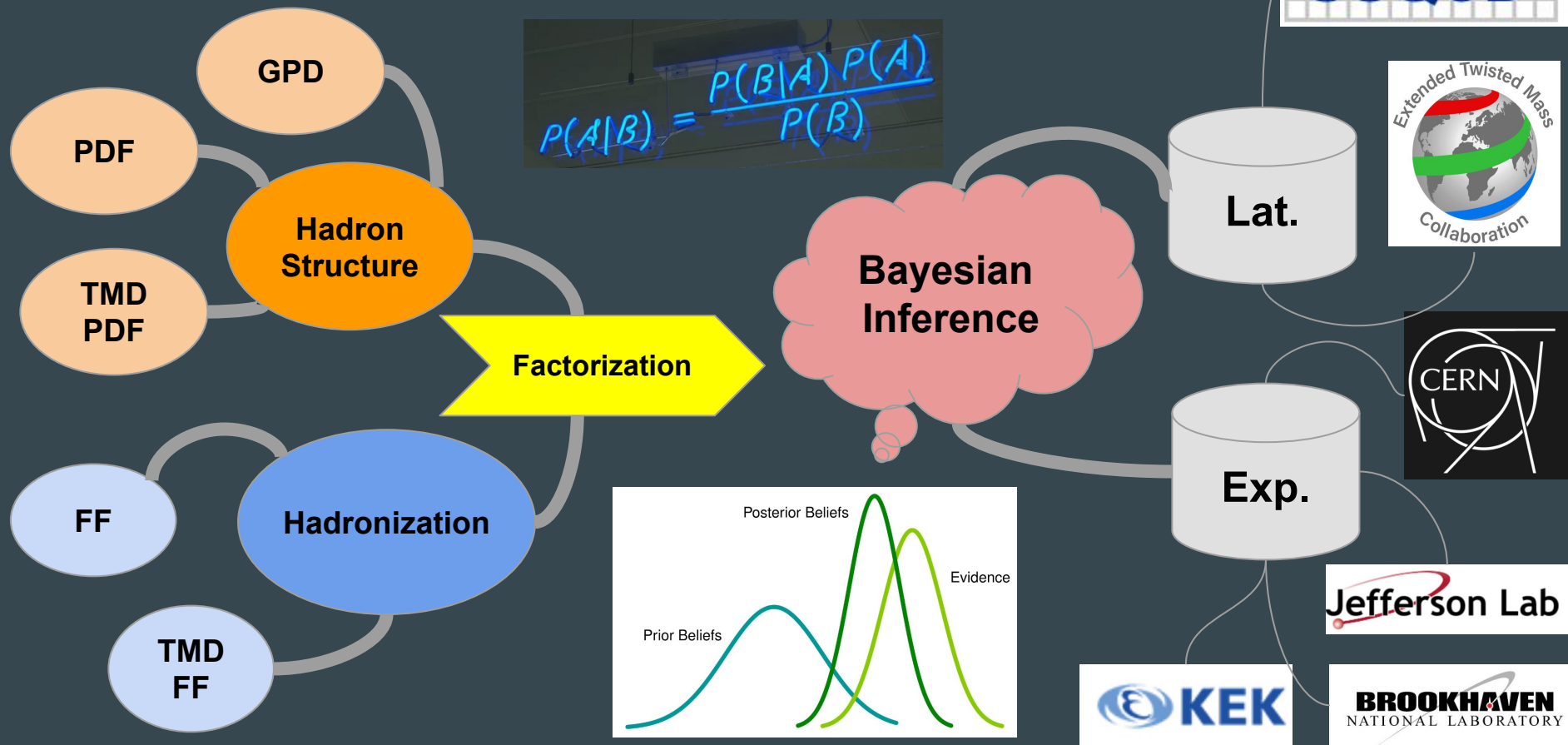
## Global analysis (NS)

- > What is a QCD global analysis?
- > The **Bayesian inference** in a nutshell
- > Why **lattice + experimental** data?

## Synergy between lattice and pheno (KC)

- > LQCD: **exploratory** vs. **precision** studies
- > **Lattice approaches** to partonic functions
- > Some state-of-the-art **lattice results**
- > **Synergy**: open problems/challenges

# What is ( the future of ) QCD global analysis?



# The Bayesian inference

$$f_i(\xi, \mu_0^2) = N_i \xi^{a_i} (1 - \xi)^{b_i} (1 + \dots)$$

$$d_i(\zeta, \mu_0^2) = N_i \zeta^{a_i} (1 - \zeta)^{b_i} (1 + \dots)$$

$$\mathbf{a} = (N_i, a_i, b_i, \dots)$$

$$d\sigma_{\text{DIS}} = \sum_i H_i^{\text{DIS}} \otimes f_i$$

$$d\sigma_{\text{DY}} = \sum_{ij} H_{ij}^{\text{DY}} \otimes f_i \otimes f_j$$

$$d\sigma_{\text{SIA}} = \sum_i H_i^{\text{SIA}} \otimes d_i$$

$$d\sigma_{\text{SIDIS}} = \sum_{ij} H_{ij}^{\text{SIDIS}} \otimes f_i \otimes d_j$$

Posterior  
distribution

Prior  
distribution

$$\rho(\mathbf{a}|\text{data}) \sim \mathcal{L}(\mathbf{a}, \text{data}) \pi(\mathbf{a})$$

Likelihood

$$E[f_i(\xi, \mu^2)] = \int d^n \mathbf{a} \rho(\mathbf{a}|\text{data}) f_i(\xi, \mu^2; \mathbf{a})$$

$$V[f_i(\xi, \mu^2)] = \int d^n \mathbf{a} \rho(\mathbf{a}|\text{data}) [f_i(\xi, \mu^2; \mathbf{a}) - E[f_i(\xi, \mu^2)]]^2$$

# The Bayesian inference

$$\mathcal{L}(\mathbf{a}, \text{data}) = \exp \left[ -\frac{1}{2} \chi^2(\mathbf{a}, \text{data}) \right]$$

$$\chi^2(\mathbf{a}, \text{data}) = \sum_{e,i} \left( \frac{d_{e,i} - \sum_k r_{e,k} \beta_{e,k,i} - t_{e,i}(\mathbf{a})/N_e}{\alpha_i} \right)^2$$

$$+ \sum_k r_{e,k}^2 + \left( \frac{1 - N_e}{\delta N_e} \right)^2$$

## Combined measurement and QCD analysis of the inclusive $e^\pm p$ scattering cross sections at HERA

Source	Data Samples
H1 $E'_e$	$\delta_1$ H1 NC [4] — $\delta_1$ H1 NC HY [5] — $\delta_1$ H1 NC [3] — $\delta_1$ H1 NC [5]
H1 $E_h$	$\delta_2$ H1 CC [3] — $\delta_2$ H1 CC [5] — $\delta_2$ H1 CC [4] — $\delta_3$ H1 NC [4]
	$\delta_3$ H1 NC HY [5] — $\delta_3$ H1 NC [3] — $\delta_3$ H1 NC [5]
H1 $\gamma p$ asymmetry	$\delta_6$ H1 NC HY [5] — $\delta_6$ H1 NC [5]
H1 $\gamma p$ background	$\delta_4$ H1 CC [3] — $\delta_4$ H1 CC [5] — $\delta_4$ H1 CC [4] — $\delta_5$ H1 NC [4]
	$\delta_5$ H1 NC HY [5] — $\delta_5$ H1 NC [3] — $\delta_5$ H1 NC [5]
H1 $\theta_e$	$\delta_2$ H1 NC [4] — $\delta_2$ H1 NC HY [5] — $\delta_2$ H1 NC [5]
H1 CC cuts	$\delta_1$ H1 CC [5] — $\delta_1$ H1 CC [4]
H1 LAr Noise	$\delta_3$ H1 CC [3] — $\delta_3$ H1 CC [5] — $\delta_3$ H1 CC [4] — $\delta_4$ H1 NC [4]
	$\delta_4$ H1 NC HY [5] — $\delta_4$ H1 NC [3] — $\delta_4$ H1 NC [5]
H1 Lumi 94 – 97	$\delta_5$ H1 CC [3] — $\delta_6$ H1 NC [3]
H1 Lumi 98 – 99	$\delta_5$ H1 CC [4] — $\delta_6$ H1 NC [4] — $\delta_7$ H1 NC HY [5]
H1 Lumi 99 – 00	$\delta_5$ H1 CC [5] — $\delta_7$ H1 NC [5]
ZEUS $E'_e$	$\delta_1$ ZEUS NC [11] — $\delta_1$ ZEUS NC [13]
ZEUS $E_h$ a	$\delta_1$ ZEUS CC [12] — $\delta_1$ ZEUS CC [14]
ZEUS $E_h$ b	$\delta_2$ ZEUS CC [12] — $\delta_2$ ZEUS CC [14]
ZEUS $E_h$ in BCAL	$\delta_2$ ZEUS CC [10] — $\delta_6$ ZEUS NC [9]
ZEUS $E_h$ in FCAL	$\delta_1$ ZEUS CC [10] — $\delta_5$ ZEUS NC [9]
ZEUS $\delta$ cut	$\delta_8$ ZEUS BPC [6] — $\delta_1$ ZEUS BPT [7]
ZEUS $\gamma p$ background	$\delta_2$ ZEUS NC [11] — $\delta_2$ ZEUS NC [13]
ZEUS $\gamma p$ background	$\delta_9$ ZEUS BPC [6] — $\delta_{14}$ ZEUS BPT [7] — $\delta_8$ ZEUS SVX [8]
ZEUS $y_h$ cut	$\delta_3$ ZEUS BPC [6] — $\delta_2$ ZEUS BPT [7]
ZEUS BPC linearity	$\delta_5$ ZEUS BPC [6] — $\delta_9$ ZEUS BPT [7]
ZEUS BPC shower	$\delta_4$ ZEUS BPC [6] — $\delta_3$ ZEUS BPT [7]
ZEUS CAL energy	$\delta_2$ ZEUS BPC [6] — $\delta_{12}$ ZEUS BPT [7] — $\delta_9$ ZEUS SVX [8]
ZEUS Cuts <sub>1</sub>	$\delta_3$ ZEUS NC [11] — $\delta_3$ ZEUS NC [13]
ZEUS Cuts <sub>2</sub>	$\delta_4$ ZEUS NC [11] — $\delta_4$ ZEUS NC [13]
ZEUS HFS model	$\delta_3$ ZEUS CC [10] — $\delta_3$ ZEUS CC [12] — $\delta_6$ ZEUS NC [11]
	$\delta_6$ ZEUS NC [13] — $\delta_3$ ZEUS CC [14]
ZEUS Lumi 94 – 97	$\delta_4$ ZEUS CC [10] — $\delta_{11}$ ZEUS NC [9]
ZEUS Lumi 98 – 99	$\delta_4$ ZEUS CC [12] — $\delta_7$ ZEUS NC [11]
ZEUS Lumi 99 – 00	$\delta_9$ ZEUS NC [13] — $\delta_4$ ZEUS CC [14]

**Table 5.** List of systematic sources that are correlated across the data samples. The type of the systematic uncertainty is given in the “source” column. The labels  $\delta_i$  denote the sources according to the sequential ordering in the list of the correlated systematic uncertainties of the corresponding publication. An overall 0.5% normalisation uncertainty, common to all data sets, is not included in this list.

# The Bayesian inference

$$\rho(\mathbf{a}|\text{data}) \sim \mathcal{L}(\mathbf{a}, \text{data})\pi(\mathbf{a})$$

$$\mathbb{E}[f_i(\xi, \mu^2)] = \int d^n \mathbf{a} \rho(\mathbf{a}|\text{data}) f_i(\xi, \mu^2; \mathbf{a})$$

$$\mathbb{V}[f_i(\xi, \mu^2)] = \int d^n \mathbf{a} \rho(\mathbf{a}|\text{data}) [f_i(\xi, \mu^2; \mathbf{a}) - \mathbb{E}[f_i(\xi, \mu^2)]]^2$$

## Maximum Likelihood

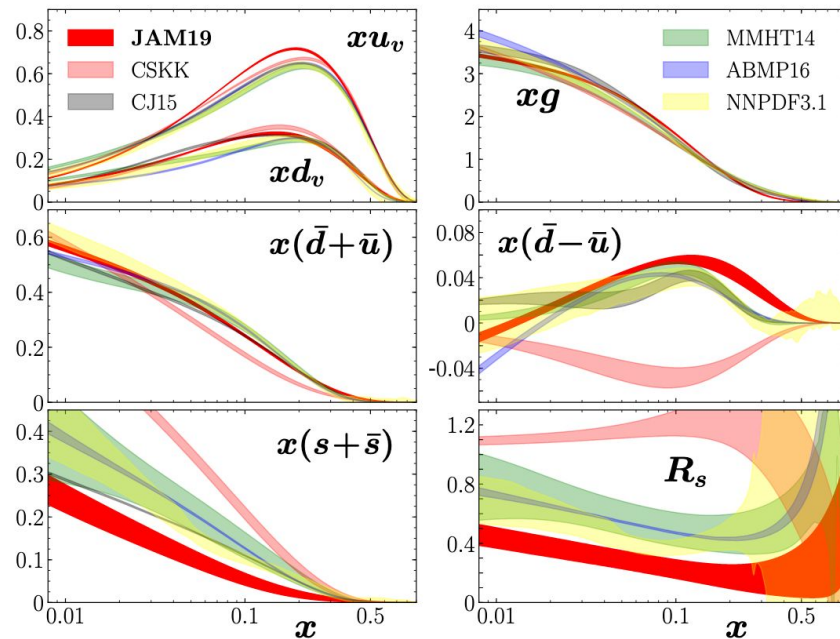
$$\mathbb{E}[f_i(\xi)] = f_i(\xi; \mathbf{a}_0)$$

$$\mathbb{V}[f_i(\xi)] = \text{Hessian, Lagrange}$$

## MC methods

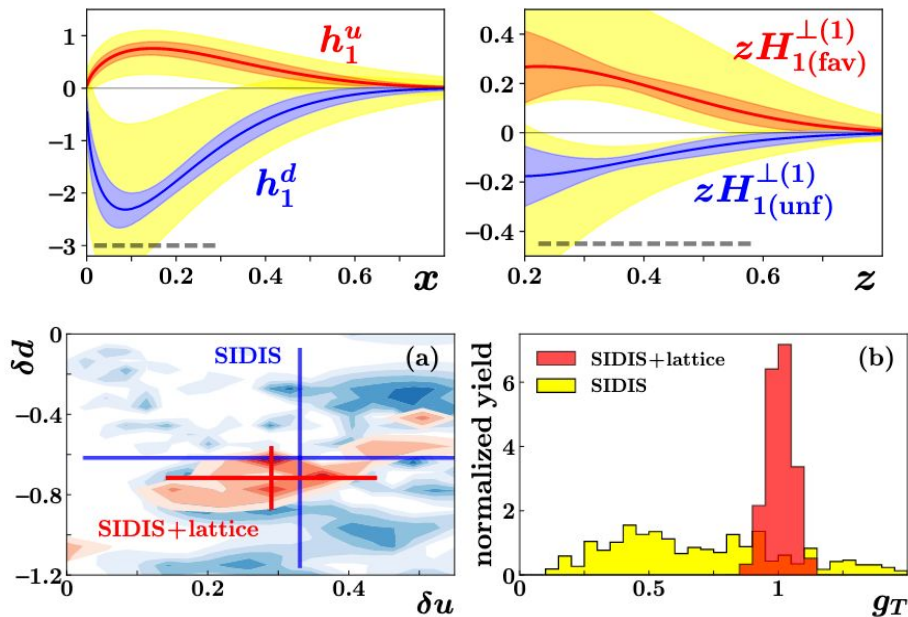
$$\mathbb{E}[f_i(\xi)] = \frac{1}{N} \sum_k f_i(\xi; \mathbf{a}_k)$$

$$\mathbb{V}[f_i(\xi)] = \frac{1}{N} \sum_k [f_i(\xi; \mathbf{a}_k) - \mathbb{E}[f_i(\xi)]]^2$$

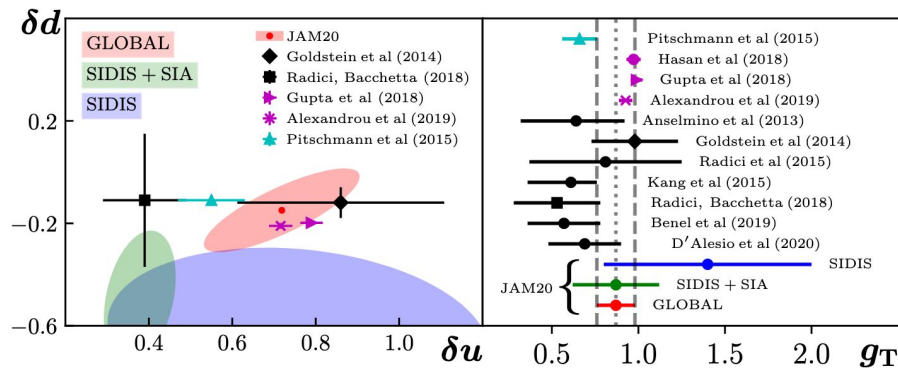




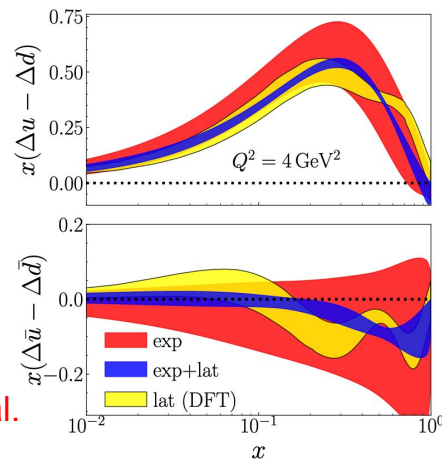
# Why lattice and experimental data?



Lin, et al.



Cammarota et al.



Bringewatt et al.

# Complementarity

## Experiment

- > Huge **amount of data** to access hadron structure and hadronization
- > Provides the **testing platform** for universality and QCD predictions
- > **Requires to separate** reaction dependent parts from intrinsic properties

## Lattice

- > Provides constraints on hadron structures **not accessible** experimentally
- > Universality of factorization **can be tested** within combined lattice observables and experimental data
- > **Direct access** to intrinsic properties of hadron structure that can be compared with infrared structures from experiments



# Challenges

## Uncertainty quantification

- > Modeling the **likelihood function** -> treatment of systematic uncertainties
- > Confidence levels in the presence of **incompatible data**
- > Bayesian posterior sampling on **large dimensional space**  $\sim O(100)$

## JLab 12 and EIC + (Lattice)

- > New era of high luminosity experiments -> **enormous amount of data**
- > New ideas emerging using **machine learning** models