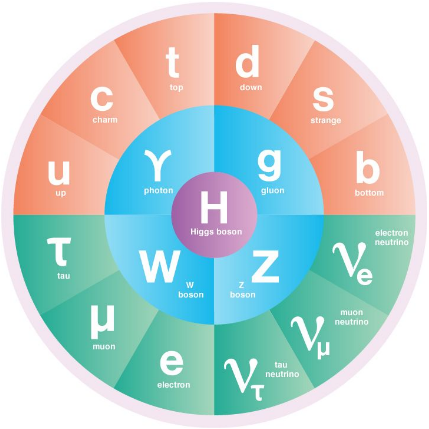
Nuclear **Femtography** in the era of Jefferson Lab 12 GeV program and the EIC

Nobuo Sato

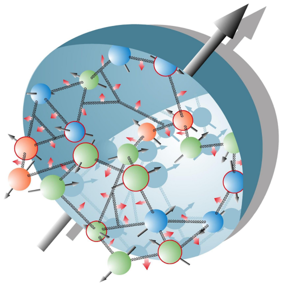
6th Colombian Meeting on High Energy Physics

Understanding the **emergent phenomena** of QCD

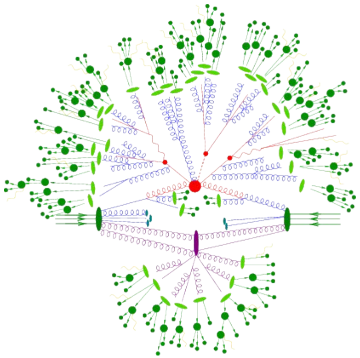


parts

"In philosophy, systems theory, science, and art, emergence occurs when an ***entity is observed*** *to have properties* ***its parts*** *do not have on their own, properties or behaviors which emerge only when the parts interact in a wider whole."* Wiki



Hadron Structure



Hadron formation

Observed entity

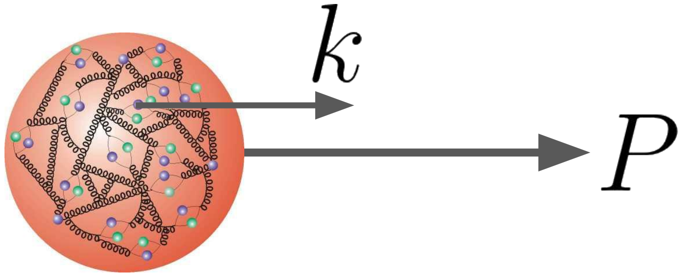
An example of hadron structure (1D)

parton distribution function (PDF)

$$\xi = \frac{k^+}{P^+}$$

Parton momentum fraction relative to parent hadron

$$f_i(\xi) = \int \frac{\mathrm{d}w^-}{4\pi} e^{-i\xi p^+ w^-} \boxed{\langle N|\bar{\psi}_i(0, w^-, \mathbf{0}_\mathrm{T})\gamma^+\psi_i(0)|N\rangle}$$

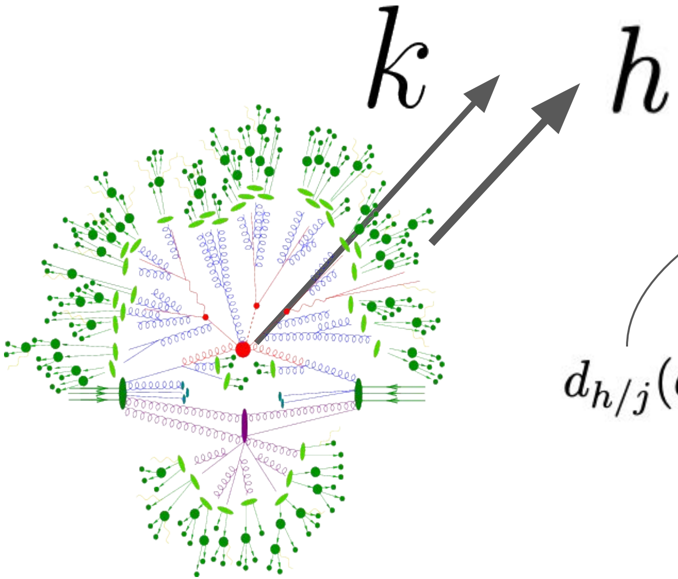


Interpretation in non-interacting QCD

$$\psi_i(x) = \sum_{k,\alpha} b_{k,\alpha}(x^+) u_{k,\alpha} e^{-ik^+x^- + ik_\mathrm{T}\cdot x_\mathrm{T}} + d^\dagger_{k,\alpha}(x^+) u_{k,-\alpha} e^{ik^+x^- - ik_\mathrm{T}\cdot x_\mathrm{T}}$$

$$f_i(\xi) \sim \sum_\alpha \int \mathrm{d}^2k_\mathrm{T} \, \langle N| \underbrace{b^\dagger_{k,\alpha} b_{k,\alpha}(\xi p^+, k_\mathrm{T}, \alpha)}_{\text{number operator}} |N\rangle$$

An example of **hadronization** (1D)



$$\zeta = \frac{p_h^+}{k^+}$$

hadron momentum fraction relative to **parent parton**

$$d_{h/j}(\zeta) \overset{!}{=} \frac{\mathrm{Tr}_{\text{color,Dirac}}}{4N_{c,j}} \sum_X \zeta \int \frac{dw^+}{2\pi} e^{i(p_h^-/\zeta)w^+}$$
$$\times \gamma^- \langle 0 | \bar{\psi}_j(0, w^+, \mathbf{0}_\mathrm{T}) \, | p_h, X \rangle \langle p_h, X | \, \psi_j(0) | 0 \rangle$$

Fragmentation functions (FFs)

X = all states except detected hadron ***h***

Hadron structure in interacting theory

Definition of PDFs in field theory requires renormalization

PDFs will depend on renormalization scale and its RGEs are the famous DGLAP equations

UV singularity when the field separation is zero

$$f_i(\xi) \stackrel{!}{=} \int \frac{\mathrm{d}w^-}{4\pi} e^{-i\xi p^+ w^-} \langle N|\bar{\psi}_i(0, w^-, \mathbf{0}_\mathrm{T})\gamma^+\psi_i(0)|N\rangle$$

Renormalization

$$f = Z_F \otimes f_{\mathrm{bare}}$$

$$f(\xi) \to f(\xi, \mu)$$

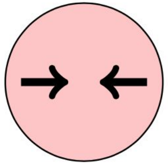
Dokshitzer–Gribov–Lipatov–Altarelli–Parisi

$$\frac{\mathrm{d} f_i(\xi, \mu^2)}{\mathrm{d} \ln \mu^2} = \sum_j \int_\xi^1 \frac{\mathrm{d}y}{y} P_{ij}(\xi, \mu^2)\, f_j\left(\frac{y}{\xi}, \mu^2\right)$$

aka **DGLAP**

Other examples of hadron structures: spin structures

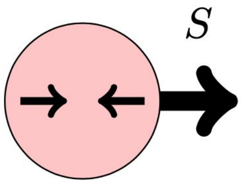
Unpolarized pdfs



$$f = f_\rightarrow + f_\leftarrow$$

$$\langle N|\bar{\psi}_i(0,w^-,\mathbf{0}_\mathrm{T})\gamma^+\psi_i(0)|N\rangle$$

Helicity distribution

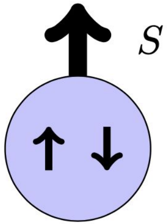


$$\Delta f = f_\rightarrow - f_\leftarrow$$

$$\langle N|\bar{\psi}_i(0,w^-,\mathbf{0}_\mathrm{T})\gamma^+\gamma_5\psi_i(0)|N\rangle$$

Spin crisis

Transversity

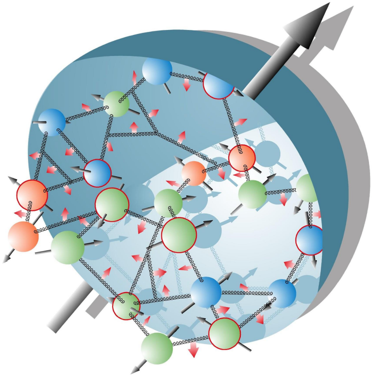


$$\delta_\mathrm{T} f = f_\uparrow - f_\downarrow$$

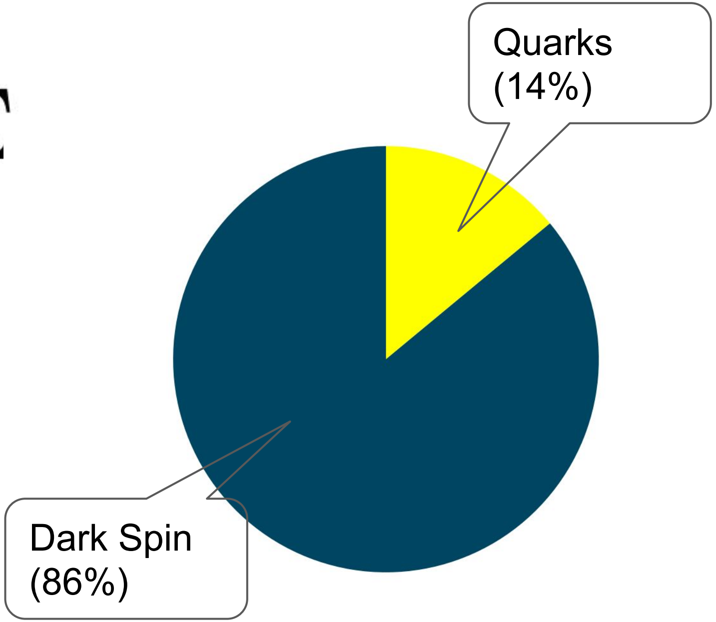
$$\langle N|\bar{\psi}_i(0,w^-,\mathbf{0}_\mathrm{T})\gamma^+\gamma_\perp\gamma_5\psi_i(0)|N\rangle$$

Spin crisis

$$\Delta\Sigma = \sum_{i\in\text{quarks}} \int_0^1 d\xi \Delta q_i(\xi)$$



$$\frac{1}{2} \stackrel{?}{=} \frac{1}{2}\Delta\Sigma$$



$$\Delta\Sigma \sim 0.28(4)$$

NS, Meltnitchouk, Kuhn, Ethier, Accardi ('15)

Today's understanding

$$\frac{1}{2} = J_q + J_g$$

Accessible via moments of generalized parton distributions

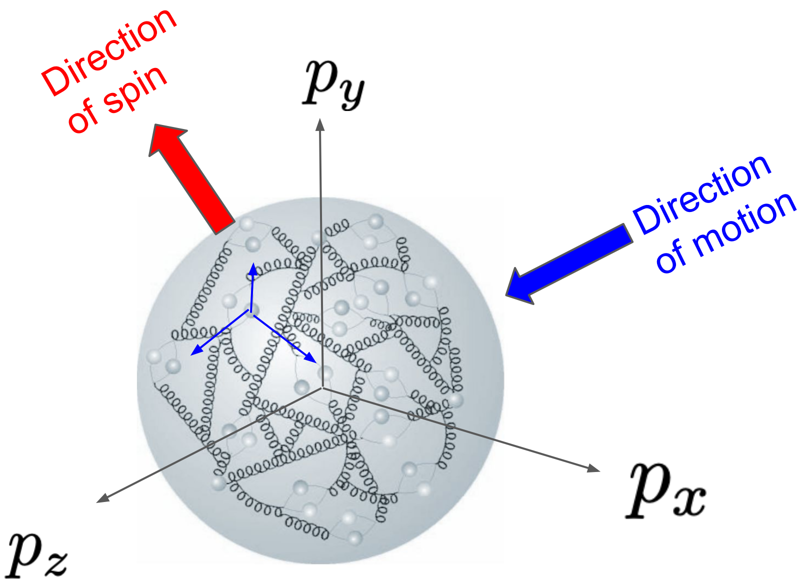
$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + \Delta g + L_g$$

Moments of helicity pdfs

Beyond 1D: Nuclear **femtography**

$$\xi = \frac{k^+}{P^+}$$

Parton distribution functions (PDFs)	$f(\xi)$
Transverse momentum distributions (TMDs)	$f(\xi, k_{\mathrm{T}})$
Generalized parton distributions (GPDs)	$f(\xi, b_{\mathrm{T}})$



3D structures

RGEs of TMDs are more complex -> Collins, Soper, Sterman (CSS)

TMDs

$$f(\xi, k_{\mathrm{T}}) = \int \frac{\mathrm{d}w^- \mathrm{d}^2 w_{\mathrm{T}}}{16\pi^3} e^{-i\xi p^+ w^- + ik_{\mathrm{T}} \cdot w_{\mathrm{T}}} \langle N| \bar{\psi}_i(0, w^-, w_{\mathrm{T}}) \gamma^+ \psi_i(0) |N\rangle$$

GPDs

$$\begin{aligned} F^q &= \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p'| \bar{q}(-\tfrac{1}{2}z) \gamma^+ q(\tfrac{1}{2}z) |p\rangle \Big|_{z^+=0,\, \mathbf{z}=0} \\ &= \frac{1}{2P^+} \left[H^q(x,\xi,t)\, \bar{u}(p')\gamma^+ u(p) + E^q(x,\xi,t)\, \bar{u}(p') \frac{i\sigma^{+\alpha}\Delta_\alpha}{2m} u(p) \right] \end{aligned}$$

F.T. $f(\xi, b_{\mathrm{T}})$

Ok, so how do we get these structures?

Route 1: Solve QCD on a supercomputer (lattice QCD)

eg.

$$\langle 0|T\phi(x_1)\ldots\phi(x_N)|0\rangle = \mathcal{N}\int[\mathrm{d}\phi]\mathrm{e}^{\mathrm{i}S[\phi]}\phi(x_1)\ldots\phi(x_N).$$

possible , but still in its infancy

Route 2: Use high energy experimental reactions and QCD factorization

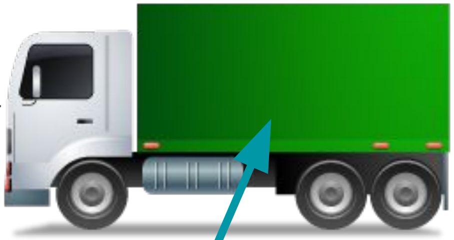
This talk

High energy scattering

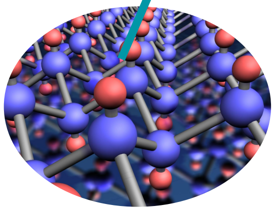
What part of this is the
“internal structure”?

Factorization

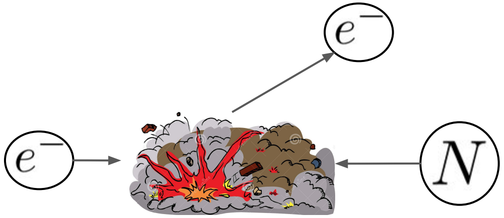
Want to see
internal structure



But we only see debris

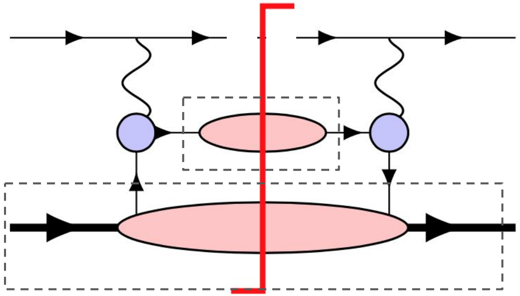


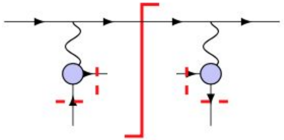
Example: Deep-inelastic scattering (DIS)



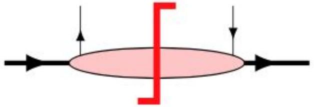
$$E' \frac{\mathrm{d}\sigma}{\mathrm{d}^3 l'}$$

Interpretation





Collision dependent factor



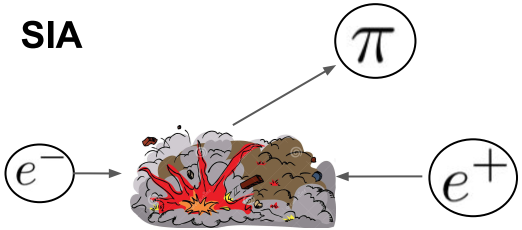
Internal structure

$$\sum_i \int_x^1 \frac{\mathrm{d}\xi}{\xi} H_i(\xi) f_i\left(\frac{x}{\xi}\right) + \mathcal{O}\left(\frac{m^2}{Q^2}\right)$$

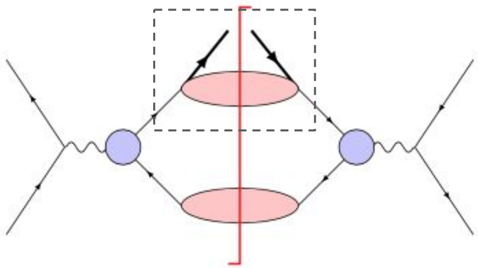
Error of approximations

Factorization in other reactions

SIA

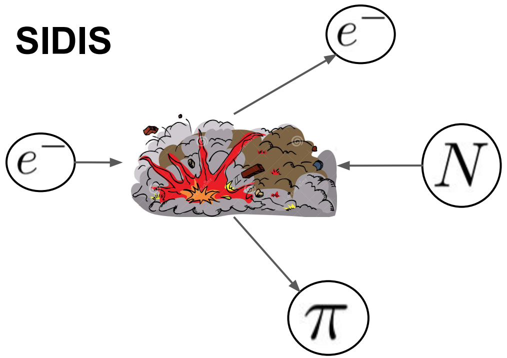


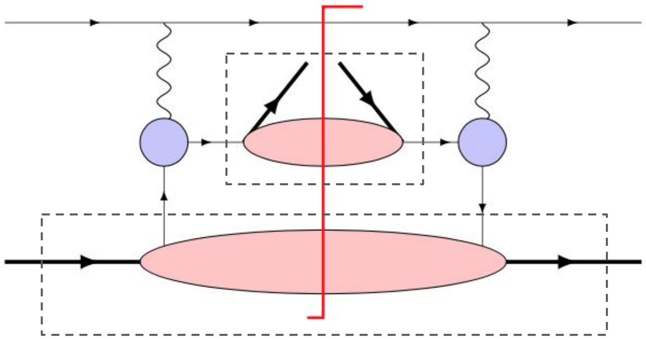
hadronization



$$d\sigma = \sum_i H_i^{\mathrm{SIA}} \otimes d_i$$

SIDIS



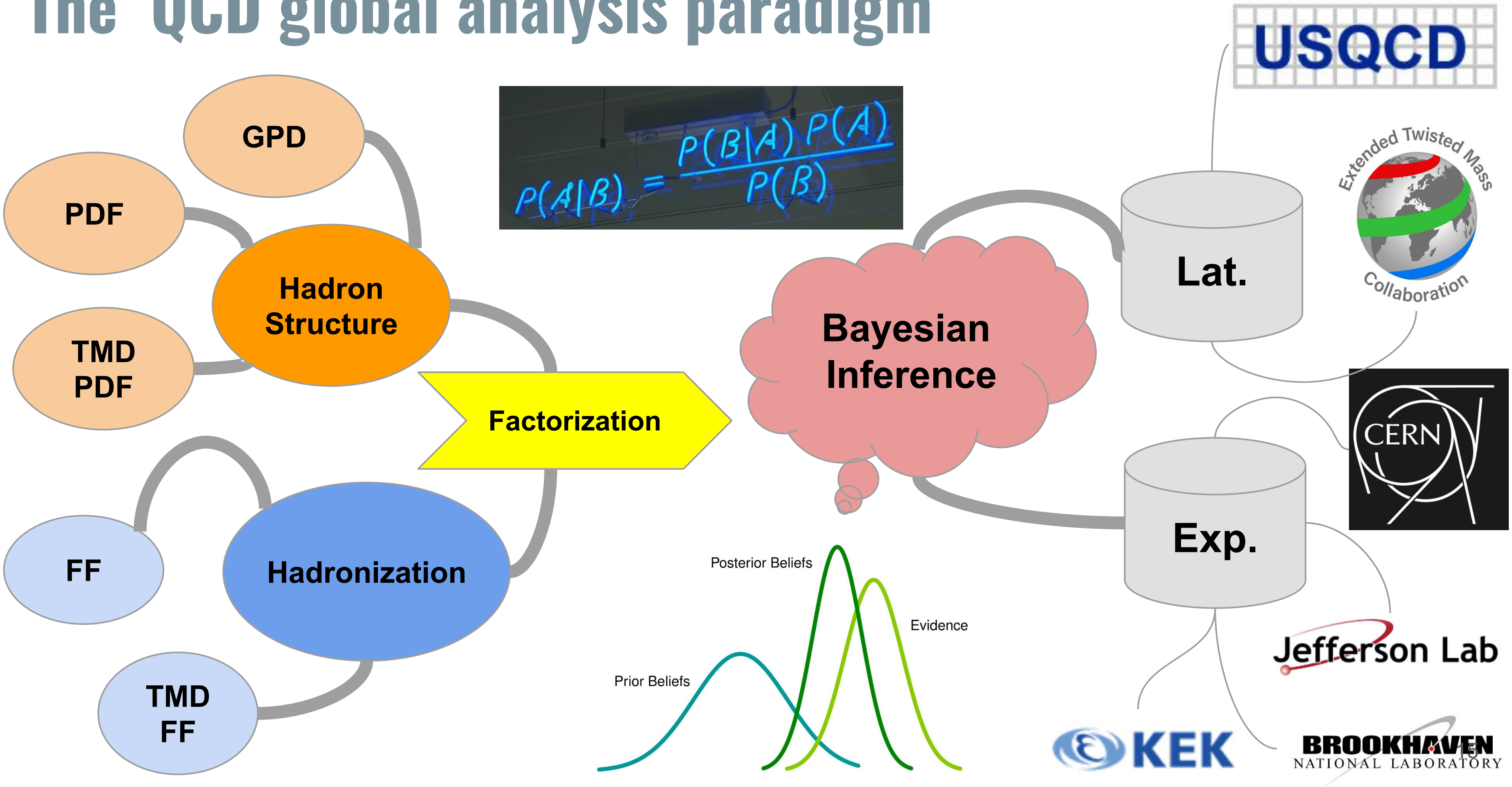


$$d\sigma = \sum_{ij} H_{ij}^{\mathrm{SIDIS}} \otimes f_i \otimes d_j$$

structure + hadronization

..and many more

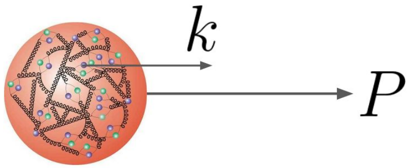
The QCD global analysis paradigm

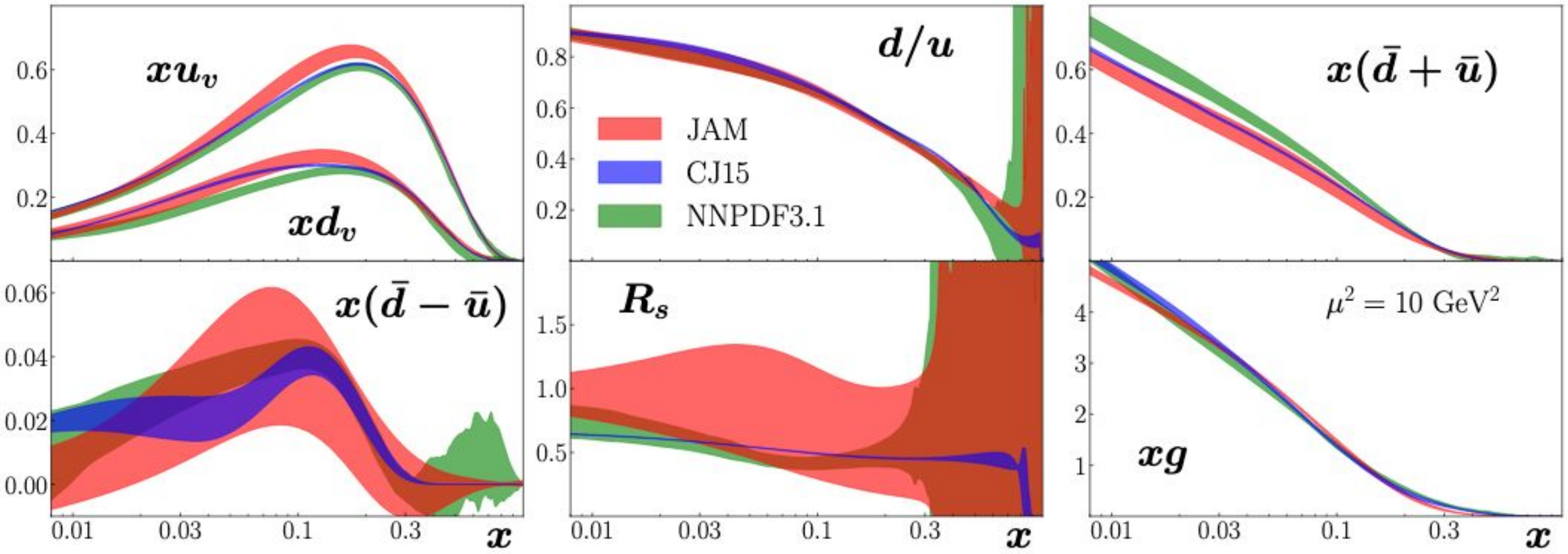


An example: JAM20-SIDIS

Moffat, Melnitchouk, Rogers, NS

PDFs





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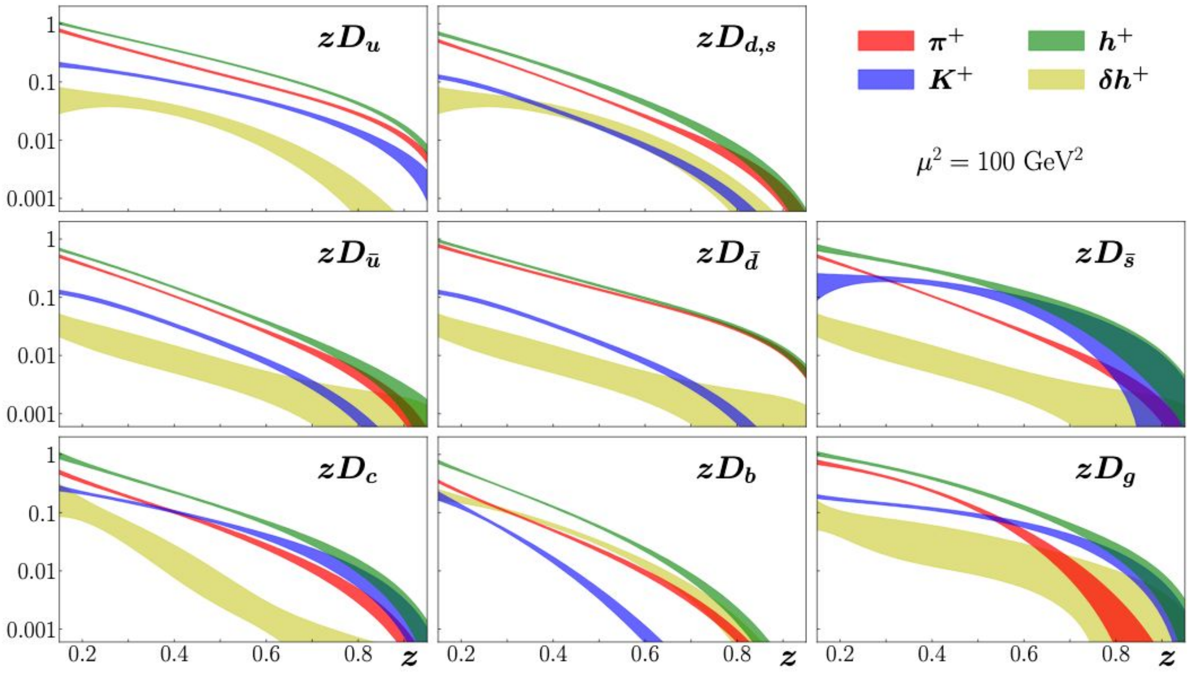
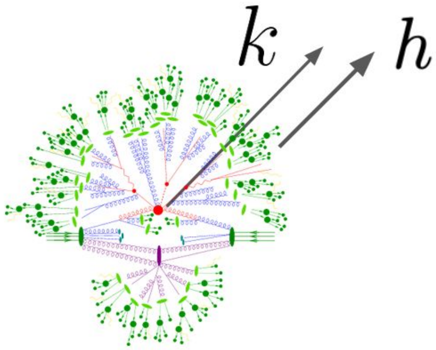
FFs





An example: JAM20-SIDIS

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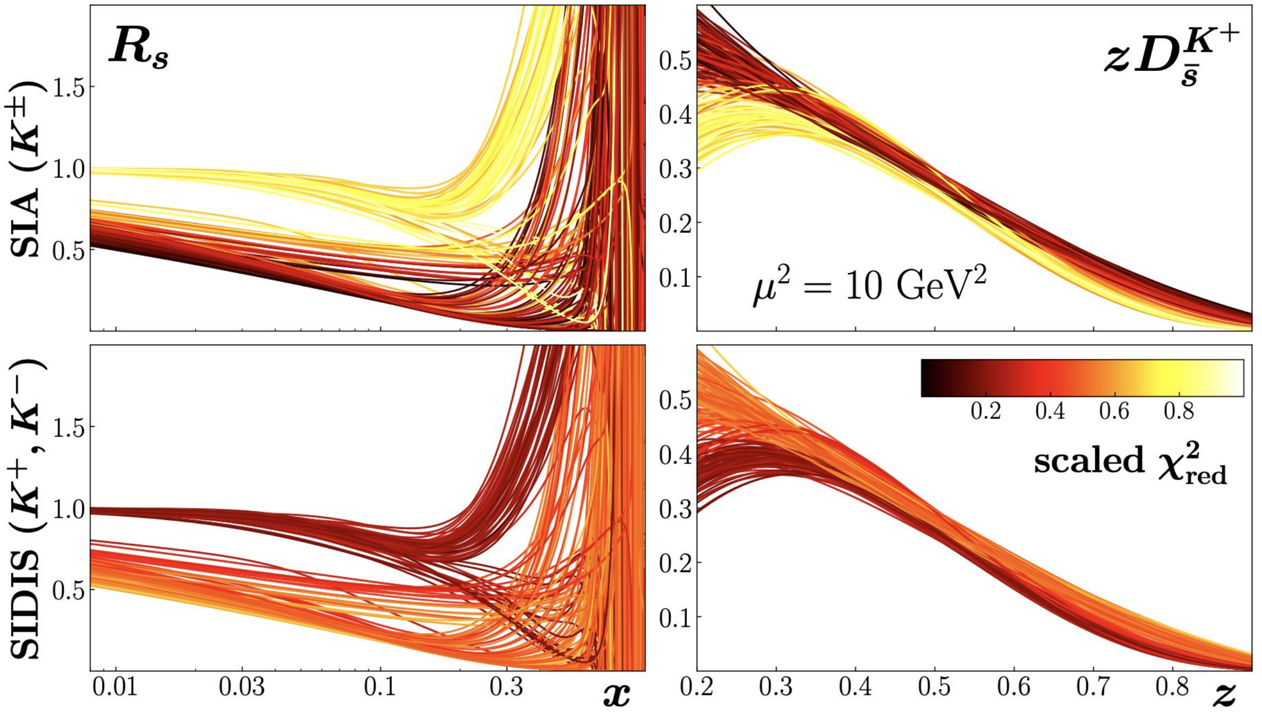
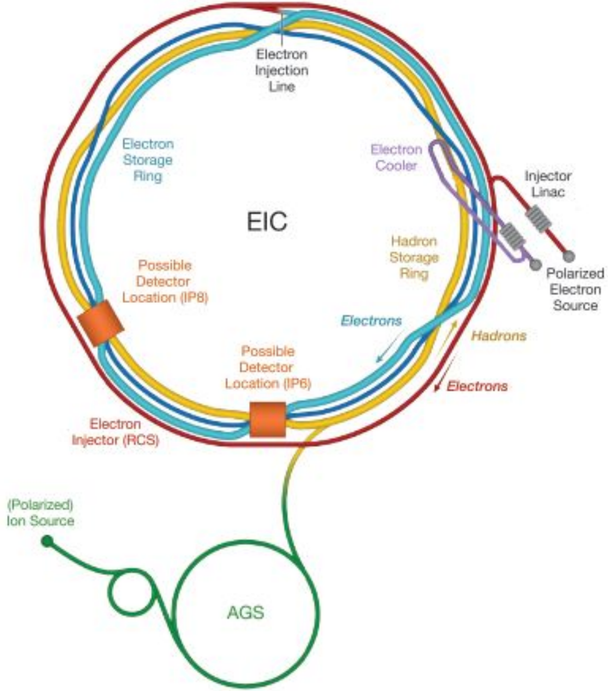
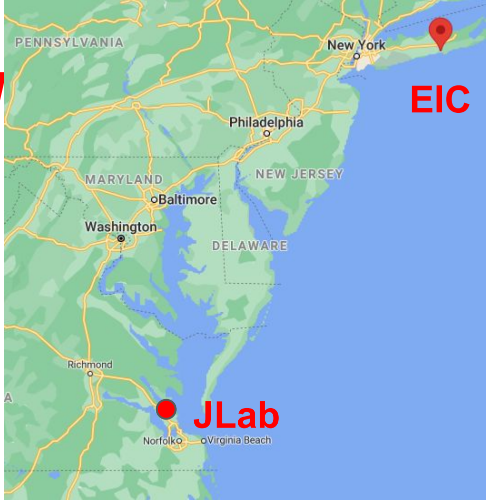


$$R_s = \frac{s + \bar{s}}{\bar{u} + \bar{d}}$$

The simultaneous fit of PDFs and FFs provides new insights on nucleon strangeness

Summary: Nuclear femtography a worldwide effort





Jefferson Lab

BROOKHAVEN NATIONAL LABORATORY

CERN

KEK

QCD global analysis

Experiments = theory + errors

$$d\sigma_{\mathrm{DIS}} = \sum_i H_i^{\mathrm{DIS}} \otimes f_i$$

$$d\sigma_{\mathrm{DY}} = \sum_{ij} H_{ij}^{\mathrm{DY}} \otimes f_i \otimes f_j$$

$$d\sigma_{\mathrm{SIA}} = \sum_i H_i^{\mathrm{SIA}} \otimes d_i$$

$$d\sigma_{\mathrm{SIDIS}} = \sum_{ij} H_{ij}^{\mathrm{SIDIS}} \otimes f_i \otimes d_j$$

Hadron Structure

Hadronization

RGE boundary conditions

$$f_i(\xi, \mu_0^2) = N_i \xi^{a_i}(1-\xi)^{b_i}(1+\ldots)$$

$$d_i(\zeta, \mu_0^2) = N_i \zeta^{a_i}(1-\zeta)^{b_i}(1+\ldots)$$

$$\mathbf{a} = (N_i, a_i, b_i, \ldots)$$

Posterior distribution

Prior distribution

$$\rho(\mathbf{a}|\mathrm{data}) \sim \mathcal{L}(\mathbf{a}, \mathrm{data})\pi(\mathbf{a})$$

Likelihood

$$\mathcal{L}(\boldsymbol{a}, \mathrm{data}) = \exp\left[-\frac{1}{2}\chi^2(\boldsymbol{a}, \mathrm{data})\right]$$

$$\mathrm{E}[f_i(\xi, \mu^2)] = \int d^n\mathbf{a}\, \rho(\mathbf{a}|\mathrm{data}) f_i(\xi, \mu^2; \mathbf{a})$$

$$\mathrm{V}[f_i(\xi, \mu^2)] = \int d^n\mathbf{a}\, \rho(\mathbf{a}|\mathrm{data}) \left[f_i(\xi, \mu^2; \mathbf{a}) - \mathrm{E}[f_i(\xi, \mu^2)]\right]^2$$