First global analysis of SSAs in SIDIS, Drell-Yan, $e^+e^-$ annihilation, and proton-proton collisions

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Outline

- SSAs within TMD and collinear twist-3 (CT3) factorization
- Global analysis – methodology and results
- Tensor charge of the nucleon
- EIC impact study on transversity and tensor charge
- Summary and outlook
Single Transverse-Spin Asymmetries (SSAs) within TMD and Collinear Twist-3 (CT3) Factorization
SIDIS (Sivers and Collins effects)

\[ h_1, f_{1T}^\perp, H_1^{\perp} \]

SIA (Collins effect)

DY (Sivers effect)
SIDIS (Sivers and Collins effects)

\[ h_1, f_{1T}^\perp, H_1^\perp \]

DY (Sivers effect)

SIA (Collins effect)

\[ H_1^\perp \]

pp collisions \((A_N)\)
\[ d\Delta \sigma(S_T) \sim H_{QS} \otimes f_1 \otimes F_{FT} \otimes D_1 + H_F \otimes f_1 \otimes h_1 \otimes \left( H_1^{\perp(1)}, \hat{H} \right) \]

- Qiu-Sterman term
- Fragmentation term
\[ H_{QS} \otimes f_1 \otimes F_{FT} \otimes D_1 \]

(Qiu and Sterman (1999), Kouvaris, et al. (2006))

\[ F_{FT} \sim \]

quark-gluon-quark correlator (soft-gluon pole)
\[ H_{QS} \otimes f_1 \otimes F_{FT} \otimes D_1 \]
(Qiu and Sterman (1999), Kouvaris, et al. (2006))

\[ F_{FT} \sim \text{quark-gluon-quark correlator (soft-gluon pole)} \]

**Connection to Sivers function**

\[
\tilde{f}_{1T}^{\perp(1)}(x, b_T; Q^2, \mu_Q) \sim F_{FT}(x, x; \mu_{b_*}) \exp \left[ -S_{pert}(b_*(b_T); \mu_{b_*}, Q, \mu_Q) - S_{NP}^{f_{1T}^\perp}(b_T, Q) \right]
\]

\[
g_{f_{1T}^\perp}(x, b_T) + g_K(b_T) \ln(Q/Q_0)
\]
(Aybat, et al. (2012); Echevarria, et al. (2014))

\[
\pi F_{FT}(x, x) = \int d^2 \vec{k}_T \frac{k_T^2}{2M^2} f_{1T}^{\perp}(x, k_T^2) \equiv f_{1T}^{\perp(1)}(x) \quad \text{(Boer, et al. (2003))}
\]

**NB:** TMD (\(\Lambda_{QCD} \sim q_T << Q\)) and CT3 factorization (\(\Lambda_{QCD} << q_T \sim Q\)) agree in their overlapping region of applicability (\(\Lambda_{QCD} << q_T << Q\))

(Ji, et al. (2006); Koike, et al. (2008); Zhou, et al. (2008, 2010); Yuan and Zhou (2009))
\[ H_F \otimes f_1 \otimes h_1 \otimes \left( H_1^{\perp(1)}, \tilde{H} \right) \]

(Metz & DP - PLB 723 (2013); Gamberg, Kang, DP, Prokudin, PLB 770 (2017))

\[ h_1 \sim \text{twist-2 collinear transversity PDF} \]

\[ H_1^{\perp(1)} \quad \tilde{H} \]

\[ \sim \int dz_1 \hat{H}_{FU}^{\perp} \sim \text{quark-gluon-quark fragmentation function} \]

(Kanazawa, Koike, Metz, DP, Schlegel, PRD 93 (2016))
\[ H_F \otimes f_1 \otimes h_1 \otimes \left( H_1^\perp(1), \tilde{H} \right) \]

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\[ h_1 \quad - \quad \text{twist-2} \quad \text{collinear} \quad \text{transversity PDF} \]

\[ H_1^\perp(1) \quad \tilde{H} \quad \sim \quad \int dz_1 \quad \hat{H}_FU^{\perp} \quad \sim \quad \text{quark-gluon-quark fragmentation function} \]

(Kanazawa, Koike, Metz, DP, Schlegel, PRD 93 (2016))

**Connection to Collins function**

\[ \tilde{H}_1^\perp(1)(z, b_T; Q^2, \mu_Q) \quad \sim \quad H_1^\perp(1)(z; \mu_{b_*}) \exp \left[ -S_{pert}(b_*(b_T); \mu_{b_*}, Q, \mu_Q) - S_{NP}^{H_1^\perp}(b_T, Q) \right] \]

\[ g_{H_1^\perp}(z, b_T) + g_K(b_T) \ln(Q/Q_0) \]

(Kang, et al. (2016))

\[ H_1^\perp(1)(z) \equiv z^2 \int d^2 p_\perp \frac{p_\perp^2}{2M_h^2} H_1^\perp(z, z^2 p_\perp^2) \]
\[ H_F \otimes f_1 \otimes h_1 \otimes \left( H_1^{\perp(1)}, \tilde{H} \right) \]

(Metz & DP - PLB 723 (2013), Gamberg, Kang, DP, Prokudin, PLB 770 (2017))

\[ h_1 \quad \text{twist-2 \textbf{collinear} transversity PDF} \]

\[ H_1^{\perp(1)} \quad \tilde{H} \quad \int dz_1 \hat{H}_{FU}^S \quad \text{quark-gluon-quark fragmentation function} \]

(Kanazawa, Koike, Metz, DP, Schlegel, PRD 93 (2016))

\textbf{NB:} \[ A_{UT}^{\sin \phi_S} \] in SIDIS integrated over \( P_T \) (Mulders, Tangerman (1996); Bacchetta, et al. (2007))

\[ F_{UT}^{\sin \phi_S} \sim \sum_a e_a^2 \frac{2M_h}{Q} h_1^a(x) \frac{\tilde{H}(z)}{z} \]
\[ H_F \otimes f_1 \otimes h_1 \otimes \left( H_{1}^{\perp(1)}, \tilde{H} \right) \]

(Metz & DP - PLB 723 (2013), Gamberg, Kang, DP, Prokudin, PLB 770 (2017))

**SSAs in** \( p^+p \rightarrow \pi X \) **are due to this fragmentation term!**

(Kanazawa, Koike, Metz, DP, PRD 89(RC) (2014))

(Gamberg, Kang, DP, Prokudin, PLB 770 (2017))

**This is a direct probe of transversity in collinear factorization!**
SIDIS (Sivers and Collins effects)

$S_\perp, P_h, \phi_h$

lepton plane

$h_1, F_{FT}, H_{1}^{\perp(1)}$

SIA (Collins effect)

$H_{1}^{\perp(1)}$

Thrust axis $\hat{n}$

DY (Sivers effect)

$F_{FT}$

pp collisions ($A_N$)

$h_1, F_{FT}, H_{1}^{\perp(1)}$
Do these SSAs indeed have a common origin (multi-parton correlators)?

Need a GLOBAL ANALYSIS of SSAs!
Global Analysis – Methodology and Results

(Cammarota, Gamberg, Kang, Miller, DP, Prokudin, Rogers, Sato, PRD 102 (2020))
We have performed the first QCD global analysis of SSAs in SIDIS, Drell-Yan, $e^+e^-$ annihilation, and proton-proton collisions and extracted a universal set of non-perturbative functions

$$h_1(x), F_{FT}(x, x), H_1^{\perp(1)}(z), \tilde{H}(z)$$

along with the relevant transverse momentum widths for the Sivers, transversity, and Collins functions: $\langle k_T^2 \rangle_{f_{1T}^+}, \langle k_T^2 \rangle_{h_1}, \langle p_{\perp}^2 \rangle_{H_1^{\perp}}, \langle p_{\perp}^2 \rangle_{H_1^{\perp}}$

We use a Gaussian ansatz: $F(x, k_T^2) \sim F(x) e^{-k_T^2/\langle k_T^2 \rangle}$ where

$$F^q(x) = \frac{N_q x^{a_q}(1-x)^{b_q}(1+\gamma_q x^{\alpha_q}(1-x)^{\beta_q})}{B[a_q+2, b_q+1] + \gamma_q B[a_q+\alpha_q+2, b_q+\beta_q+1]}$$

$NB$: $\{\gamma, \alpha, \beta\}$ only used for Collins function

DGLAP-type evolution for the collinear functions analogous to Duke & Owens (1984): double-log $Q^2$-dependent term explicitly added to the parameters
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We use a Gaussian ansatz: $F(x, k_T^2) \sim F(x) e^{-k_T^2/\langle k_T^2 \rangle}$ where

$$F^q(x) = \frac{N_q x^{a_q} (1 - x)^{b_q} (1 + \gamma_q x^{\alpha_q} (1 - x)^{\beta_q})}{B[a_q + 2, b_q + 1] + \gamma_q B[a_q + \alpha_q + 2, b_q + \beta_q + 1]}$$

$NB$: $\{\gamma, \alpha, \beta\}$ only used for Collins function

DGLAP-type evolution for the collinear functions analogous to Duke & Owens (1984): double-log $Q^2$-dependent term explicitly added to the parameters
18 observables and 6 non-perturbative functions (Sivers up/down; transversity up/down; Collins favored/unfavored)

- **Broad kinematical coverage:**
  
  **SIDIS:** \( x \lesssim 0.3 \), \( 0.2 \lesssim z \lesssim 0.6 \), \( 2 \lesssim Q^2 \lesssim 40 \text{ GeV}^2 \)

  **SIA:** \( 0.2 \lesssim z \lesssim 0.8 \), \( Q^2 \approx 13 \text{ GeV}^2 \) or \( 110 \text{ GeV}^2 \)

  **DY:** \( 0.1 \lesssim x \lesssim 0.35 \), \( Q^2 \approx 30 \text{ GeV}^2 \) or \( (80 \text{ GeV})^2 \)

  **A_N^h:** \( 0.2 \lesssim (x_{min}, z_{min}) \lesssim 0.7 \), \( 1 \lesssim Q^2 \lesssim 13 \text{ GeV}^2 \)
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<tr>
<th>Observable</th>
<th>Reactions</th>
<th>Non-Perturbative Function(s)</th>
<th>( \chi^2/N_{\text{pts.}} )</th>
</tr>
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<tbody>
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<td>( A_{\text{SIDIS}}^{Sivers} )</td>
<td>( e + (p, d)^\uparrow \rightarrow e + (\pi^+, \pi^-, \pi^0)^+ X )</td>
<td>( f_{1T}^\perp(x, k_T^2) )</td>
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<td>( A_{\text{SIDIS}}^{Collins} )</td>
<td>( e + (p, d)^\uparrow \rightarrow e + (\pi^+, \pi^-, \pi^0)^+ X )</td>
<td>( h_1(x, k_T^2), H_1^\perp(z, z^2 p_{1T}^2) )</td>
<td>111.3/126 = 0.88</td>
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<td>( e^+ + e^- \rightarrow \pi^+ \pi^-(UC,UL) + X )</td>
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<td>( A_{\text{DY}}^{Sivers} )</td>
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<td>( A_N^h )</td>
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<td>( h_1(x), F_{FT}(x, x) = \frac{1}{\pi} f_{1T}^{\perp(1)}(x), H_1^{\perp(1)}(z) )</td>
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Test of universality!
D. Pitonyak

### Predictions of $A_N$ using a fit of only TMD observables

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**Test of universality!**
- We use Bayesian inference in order to sample the posterior distribution for all parameters.

- Due to the large dimensionality of the parameter space we use a multi-step strategy to fit the data (Sato, et al. (2019)).

- Our results are based off ~1000 Monte Carlo samples.
\[
\chi^2 / N_{\text{pts.}} = 520 / 517 = 1.01
\]
Transversity

Sivers first moment (QS function)

Collins first moment
Tensor Charge of the Nucleon from JAM20
(Cammarota, Gamberg, Kang, Miller, DP, Prokudin, Rogers, Sato, PRD 102 (2020))
\[ \delta q \equiv \int_{0}^{1} dx \left[ h_{1}^{q}(x) - h_{1}^{\bar{q}}(x) \right] \quad g_{T} \equiv \delta u - \delta d \]
The tensor charge of the nucleon is one of its fundamental charges and is important for BSM studies (beta decays, EDM).

Lattice QCD has made very precise calculations at the physical point that are typically at odds with extractions from phenomenology – the central value for $\delta u$ from phenomenology tends to be smaller than lattice, which also causes $g_T$ to fall below lattice as well (“transverse spin puzzle”).

Up until now, the phenomenological extractions of the tensor charges have been based on either fits of SIDIS+SIA data or dihadron fragmentation data.

We have achieved the first phenomenological agreement with lattice for all the tensor charges ($\delta u, \delta d, g_T$), which highlights the importance of carrying out a global analysis (i.e., including $A_N$ data along with SIDIS+SIA).
This is the most precise phenomenological determination of $g_T$ to date.

Our tensor charges, especially $\delta u$, show excellent agreement with lattice:

$\delta u = 0.72(19), \delta d = -0.15(16)$

The inclusion of $A_N$ data is crucial in order to achieve agreement between phenomenology and lattice for the tensor charges!
EIC Impact Study on Transversity and Tensor Charge

(Gamberg, Kang, DP, Prokudin, Sato, Seidl, in preparation)
We generated SIDIS Collins EIC pseudo-data with PID+smearing systematics. The number of points listed is after the following cuts are made:

\[0.2 < z < 0.6, \quad Q^2 > 1.63 \text{ GeV}^2, \quad 0.2 < P_{hT} < 0.9 \text{ GeV}\]

### Data Points

<table>
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<tr>
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<tr>
<td>(e p \uparrow \to e \pi^+ X)</td>
<td>18x275 (756 pts.)</td>
</tr>
<tr>
<td>(e p \uparrow \to e \pi^- X)</td>
<td>18x275 (744 pts.)</td>
</tr>
<tr>
<td>(e n \uparrow \to e \pi^+ X)</td>
<td>18x100 (647 pts.)</td>
</tr>
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</tr>
<tr>
<td>10x100 (634 pts.)</td>
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</tr>
<tr>
<td>10x100 (622 pts.)</td>
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</tr>
<tr>
<td>5x100 (537 pts.)</td>
<td>5x100 (556 pts.)</td>
</tr>
<tr>
<td>5x41 (461 pts.)</td>
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</tr>
<tr>
<td>5x41 (464 pts.)</td>
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</tr>
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Total EIC points = 8223

JAM20 original fit (SSA points) = 517 (362 sensitive to Collins and/or transversity, i.e., SIDIS Collins, \(e^+e^-\), and \(A_N\))

We re-fit the 8223 + 517 = 8740 data points with ~200 replicas using priors from the JAM20 global analysis.
\[ x h_1(x) \]

\[ \delta d \]

\[ \delta u \]

JAM20
\[ \delta u = 0.72 \pm 0.19 \]
\[ \delta d = -0.15 \pm 0.16 \]
\[ g_T = 0.87 \pm 0.11 \]

JAM20 + EIC(ep)
\[ \delta u = 0.719 \pm 0.034 \]
\[ \delta d = -0.149 \pm 0.071 \]
\[ g_T = 0.869 \pm 0.044 \]

JAM20 + EIC(ep + eHe3)
\[ \delta u = 0.719 \pm 0.011 \]
\[ \delta d = -0.149 \pm 0.003 \]
\[ g_T = 0.869 \pm 0.012 \]
Summary and Outlook

- We have performed the first global analysis of SSAs in SIDIS, DY, $e^+e^-$ annihilation, and proton-proton collisions and extracted a universal set of non-perturbative functions.

- We have achieved the first phenomenological agreement with lattice QCD for all the tensor charges of the nucleon, which highlights the importance of carrying out a global analysis.

- The EIC impact on the tensor charge will be significant and make phenomenological extractions as precise as lattice calculations.

- Future studies will implement the proper TMD and CT3 evolution and include $A_{UT}^{\sin \phi_S}$ data to constrain $\tilde{H}$, as well as eventually other data (like hadron in a jet, dihadron, …).

Global analyses using data that cover a broad kinematical range are crucial if one is to claim we truly understand 3D hadronic structure!