Pion electro- and photoproduction on nucleons with explicit $\Delta(1232)$ in covariant Chiral Perturbation Theory

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JLab seminar

Outline

1 Motivation

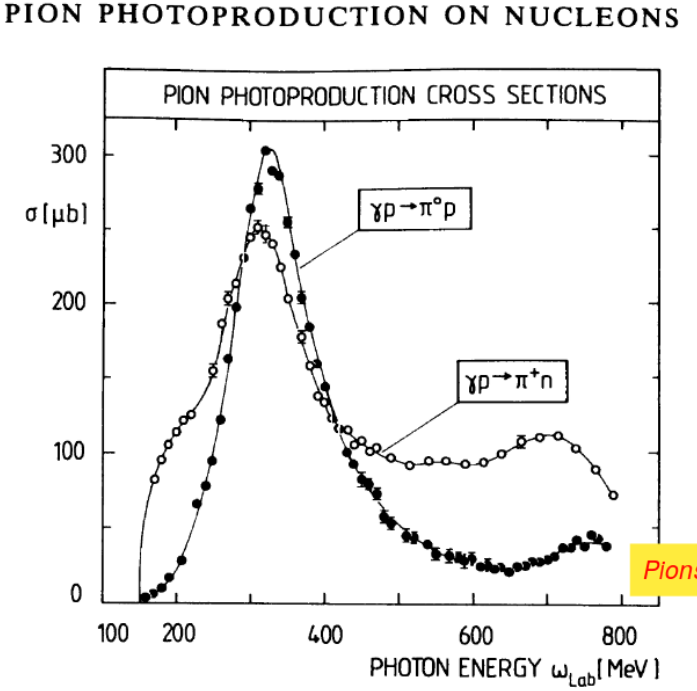
2 Theoretical approach

3 Results

4 Outlook

5 Summary

Motivation



Pions and Nuclei, Ericson & Weise

- We want to know how to deal properly at the lowest energy region in π-production processes
- In particular, at the peak the neutral channel has bigger cross section than the charged channel
- Near threshold there are huge cancellations for the neutral channel (not well described at low energy)
- For the charged channel there are not such cancellations
- How much contributes the resonances at low energies for neutral and charged channels ?

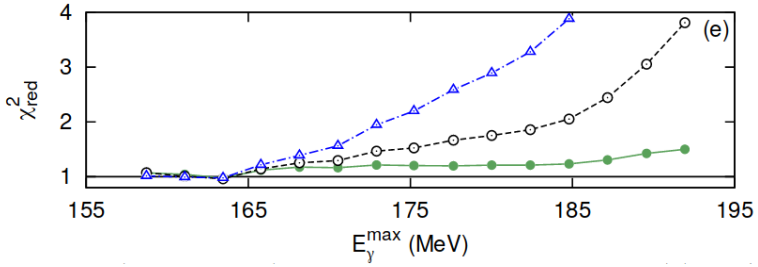
Comparing models with data

- We can compare different theoretical models (through its parameters) with experimental data
- Comparison can be made with chi-squared analysis

$$\chi^2 = \sum_i \frac{(y_i - theo_i)^2}{\delta y_i{}^2}$$

Previous works : $\gamma p \to \pi^0 p$ channel

Model with only N and π degrees of freedom



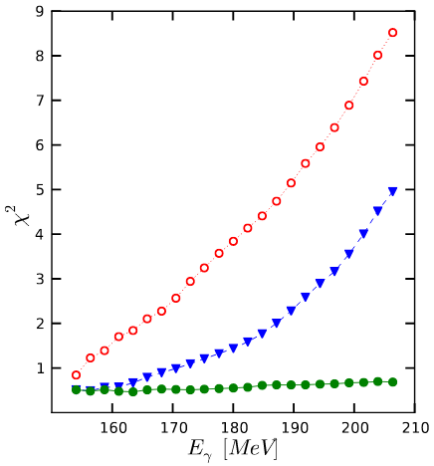
$O(p^4)$ relativistic ChPT
$O(p^4)$ HBChPT
Empirical fit

Hornidge et al. PRL(2013)

Also M. Hilt et al. PRC(2013)

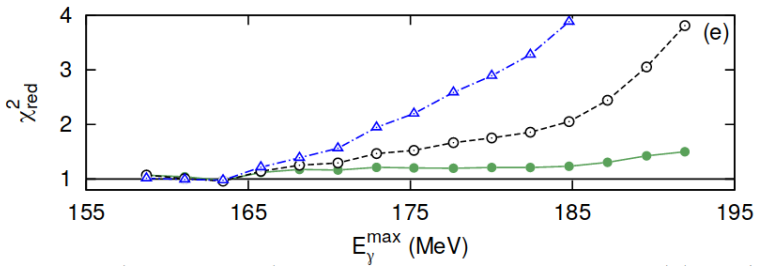
Starts failing at 20 MeV above π threshold

Model with $\Delta(1232)$ mechanism as explicit degree of freedom



- $O(p^3)$ relativistic [tree level] ChPT
- $O(p^3)$ relativistic [tree+loops] ChPT
- relativistic [tree+loops+Δ] H. Blin et al. PLB(2015)

Some problems : Theoretical models for $\gamma p \to \pi^0 p$



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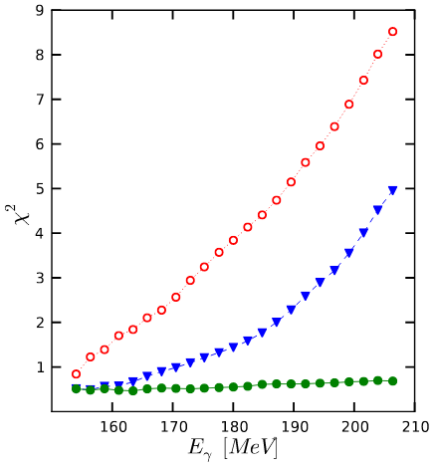
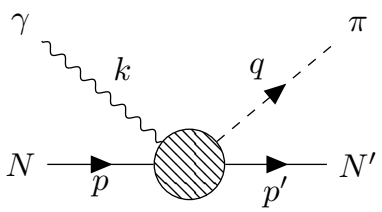
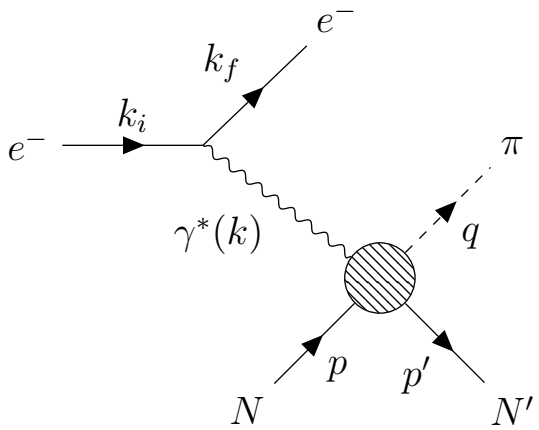


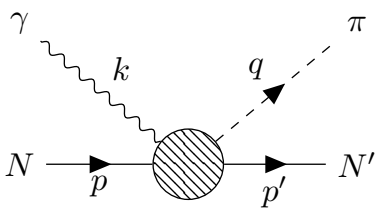
- Best model : relativistic ChPT with explicitly $\Delta(1232)$ inclusion

We want to study whats happen with charged and neutral π-EM production





Experimental data photo-production



$\sqrt{s} \sim 1074$ MeV $- 1133$ MeV

Measured observables :

$$\sigma \quad , \quad \frac{d\sigma}{d\Omega} \quad , \quad \Sigma = \frac{d\sigma_\perp - d\sigma_\parallel}{d\sigma_\perp + d\sigma_\parallel}, \quad T = \frac{d\sigma_+ - d\sigma_-}{d\sigma_+ + d\sigma_-}$$

$\gamma p \to \pi^0 p$

- S. Schumann et al. PLB (2015)
- Hornidge et al. PRL (2013)
- Schumann et al. EPJ A (2010)
- Blanpied et al. PRC (2001)
- Schmidt et al. PRL (2001)
- Others ...

1769 data points

$\gamma p \to \pi^+ n$

- Ahrens et al. EPJ A(2004)
- Blanpied et al. PRC (2001) $*$
- Korkmaz et al. PRL (1999)
- Fissum, et al. PRC (1996)
- Bergstrom et. al. PRC (1996)
- McPherson et. al. PRB (1964)
- Walker et al. PR (1963)

168 data points

$\gamma n \to \pi^- p$

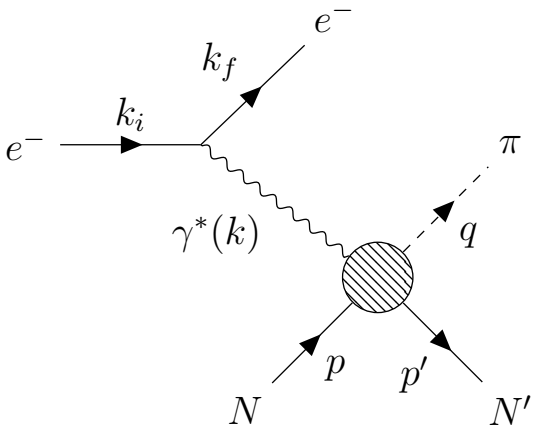
- W. Briscoe et. al. (2020) arXiv :2004.01742
- Liu, PhD thesis (1994)
- Wang, PhD thesis (1992)
- Bagheri, et al. PRC (1988)
- Salomon et al. NPA (1984)
- Rossi et al. NC A (1973)
- Benz et al. NPB (1973)
- White, et al. PR (1960)

100 data points

$\gamma n \to \pi^0 n$

Nothing yet
0 data points

Experimental data electro-production



$\sqrt{s} \sim 1074$ MeV $- 1133$ MeV
$Q^2 \sim 0.05$ GeV$^2 - 0.15$ GeV^2

Measured observables :
for an unpolarized target and without recoil polarization

$$\frac{d\sigma_v}{d\Omega_\pi} = \frac{d\sigma_T}{d\Omega_\pi} + \varepsilon \frac{d\sigma_L}{d\Omega_\pi} + \sqrt{2\varepsilon(1+\varepsilon)} \frac{d\sigma_{LT}}{d\Omega_\pi} \cos\phi_\pi + \varepsilon \frac{d\sigma_{TT}}{d\Omega_\pi} \cos 2\phi_\pi + h\sqrt{2\varepsilon(1-\varepsilon)} \frac{d\sigma_{LT'}}{d\Omega_\pi} \sin\phi_\pi$$

$\gamma^* p \to \pi^0 p$

- H. B. van den Brink et al., NPA 612, 391 (1997)
- M. O. Distler et al., PRL 80, 2294 (1998)
- M. Weis et al., EPJ A38, 27 (2008), 0705.3816.
- H. Merkel et al. (2011), 1109.5075.

751 data points
To be included from Hall A collaboration : Chirapatpimol et al. Phys.Rev.Lett. 114 (2015) 19, 192503

$\gamma^* p \to \pi^+ n$

- K. I. Blomqvist et al., ZPA 353, 415 (1996).
- A. Liesenfeld et al., PLB 468, 20 (1999), nucl-ex/9911003.
- D. Baumann, Ph.D. thesis, JGU, Mainz (2005).
- I. Friščić et al., PLB 766, 301 (2017), 1606.00970.

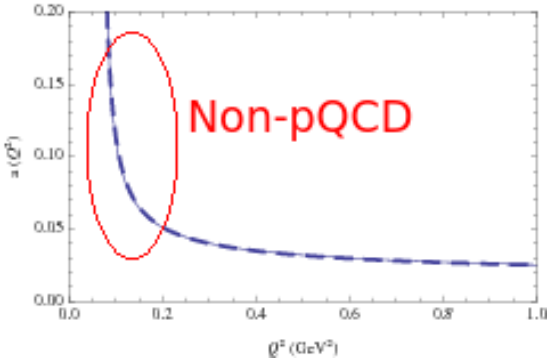
18 data points

$\gamma^* n \to \pi^- p$
Nothing yet
0 data points

$\gamma^* n \to \pi^0 n$

Nothing yet
0 data points

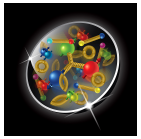
Approaching the experimental data



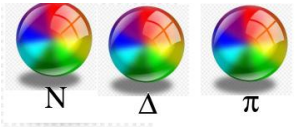
NON-PERTURBATIVE REGIME

- Low energy regime
- $\sqrt{s} \sim 1074$ MeV $- 1133$ MeV $\Longrightarrow \alpha_S >> 1$ perturbative QCD breakdown

  We need an Effective Theory approach $\Longrightarrow$ **Chiral Perturbation Theory**.
- At low energies : Relevant degrees of freedom



$\Longrightarrow$



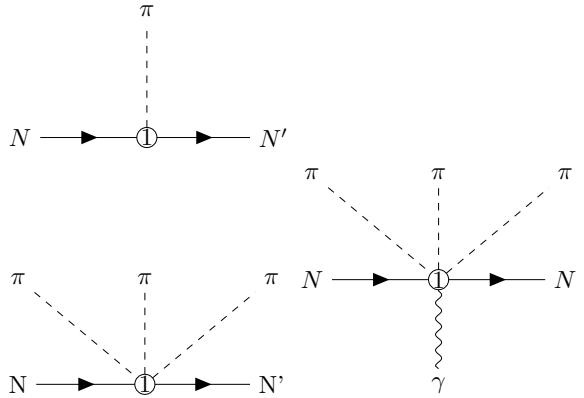
- We use as expansion parameters the relative $\frac{p}{\Lambda}$ and pion mass $\frac{M_\pi}{\Lambda}$

The nucleon Lagrangian

Each chiral order brings new LECs with it. For the process $\gamma(^*)N \to \pi N'$

$$\mathcal{L}_N^{(1)} = \bar{\Psi}\left(i\not{D} - m + \frac{g_0}{2}\not{u}\gamma_5\right)\Psi,$$

$O(p^1)$: m, g_0

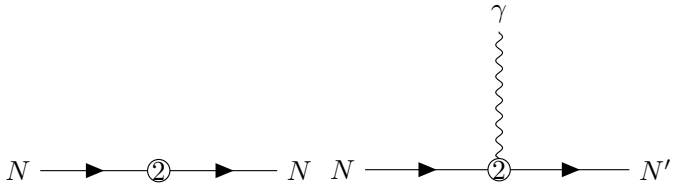


The nucleon Lagrangian

Each chiral order brings new LECs with it. For this process $\gamma N \to \pi N'$

$$\mathcal{L}_N^{(2)} = \bar{\Psi}\left(c_1 \mathrm{Tr}\left[\chi_+\right] \frac{c_6}{8m_N} F_{\mu\nu}^+ + \frac{c_7}{8m_N} \mathrm{Tr}\left[F_{\mu\nu}^+\right]\right) \sigma^{\mu\nu}\Psi + \cdots ,$$

$O(p^2) : c_1, c_6, c_7$

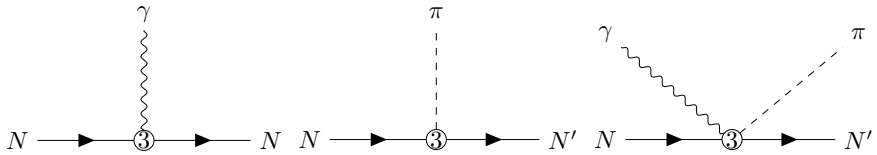


The nucleon Lagrangian

Each chiral order brings new LECs with it. For this process $\gamma(^*)N \to \pi N'$

$$\mathcal{L}_N^{(3)} = d_6 \overline{\Psi}\left(\frac{1}{2m_N} i[D^\mu, \widetilde{F}^+_{\mu\nu}]D^\nu + \text{H.c.}\right)\Psi + d_7 \overline{\Psi}\left(\frac{1}{2m_N} i[D^\mu, \text{Tr}\left[F^+_{\mu\nu}\right]]D^\nu + \text{H.c.}\right)\Psi$$
$$+ \overline{\Psi}\frac{i\epsilon^{\mu\nu\alpha\beta}}{2m}\left[d_8 \text{Tr}\left[\widetilde{F}^+_{\mu\nu}u_\alpha\right] + d_9 \text{Tr}\left[F^+_{\mu\nu}\right]u_\alpha + \text{H.c.}\right]D_\beta\Psi$$
$$+ \overline{\Psi}\frac{\gamma^\mu\gamma_5}{2}\left[d_{16}\text{Tr}\left[\chi_+\right]u_\mu + d_{18}\, i[D_\mu, \chi_-]\right]\Psi + d_{20}\overline{\Psi}\left[-i\frac{\gamma^\mu\gamma_5}{8m^2}[\widetilde{F}^+_{\mu\nu}, u_\lambda]D^{\lambda\nu} + \text{h.c.}\right]\Psi$$
$$+ d_{21}\overline{\Psi}\left[\frac{1}{2}i\gamma^\mu\gamma_5[\widetilde{F}^+_{\mu\nu}, u^\nu]\right]\Psi + d_{22}\overline{\Psi}\left[\frac{1}{2}\gamma^\mu\gamma_5[D^\nu, F^-_{\mu\nu}]\right]\Psi + \cdots$$

$O(p^3): d_6, d_7, d_8, d_9, d_{16}, d_{18}, d_{20}, d_{21}, d_{22}$



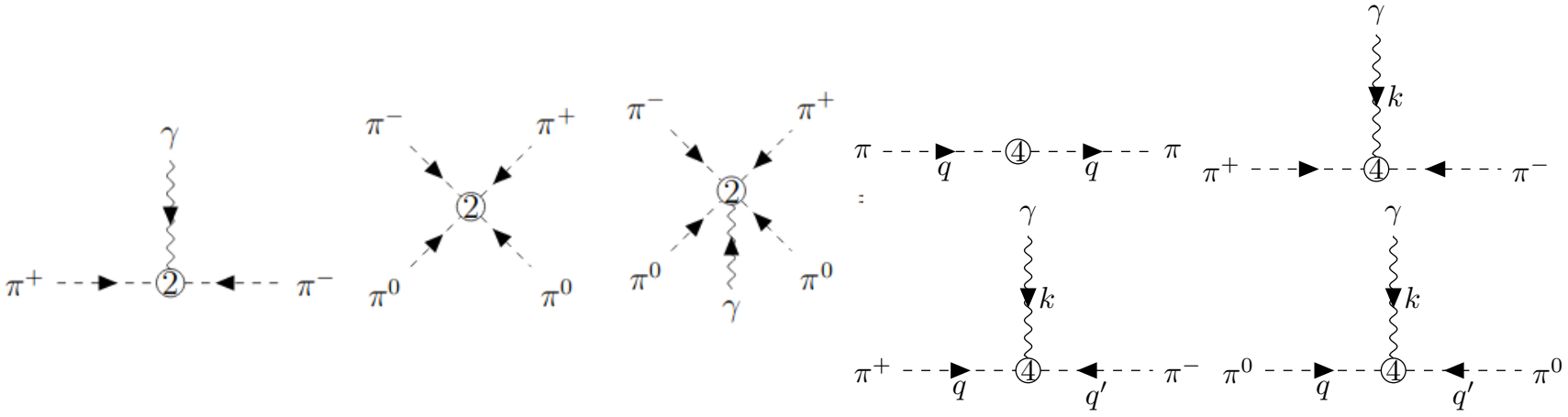
The pion Lagrangian

Up to $O(p^3)$ we do not need extra fitting Low-Energy-Constants

$$\mathcal{L}_{\pi\pi}^{(2)} = \frac{F_0^2}{4}\mathrm{Tr}\left[D^\mu U\left(D_\mu U\right)^\dagger + \chi U^\dagger + U\chi^\dagger\right].$$

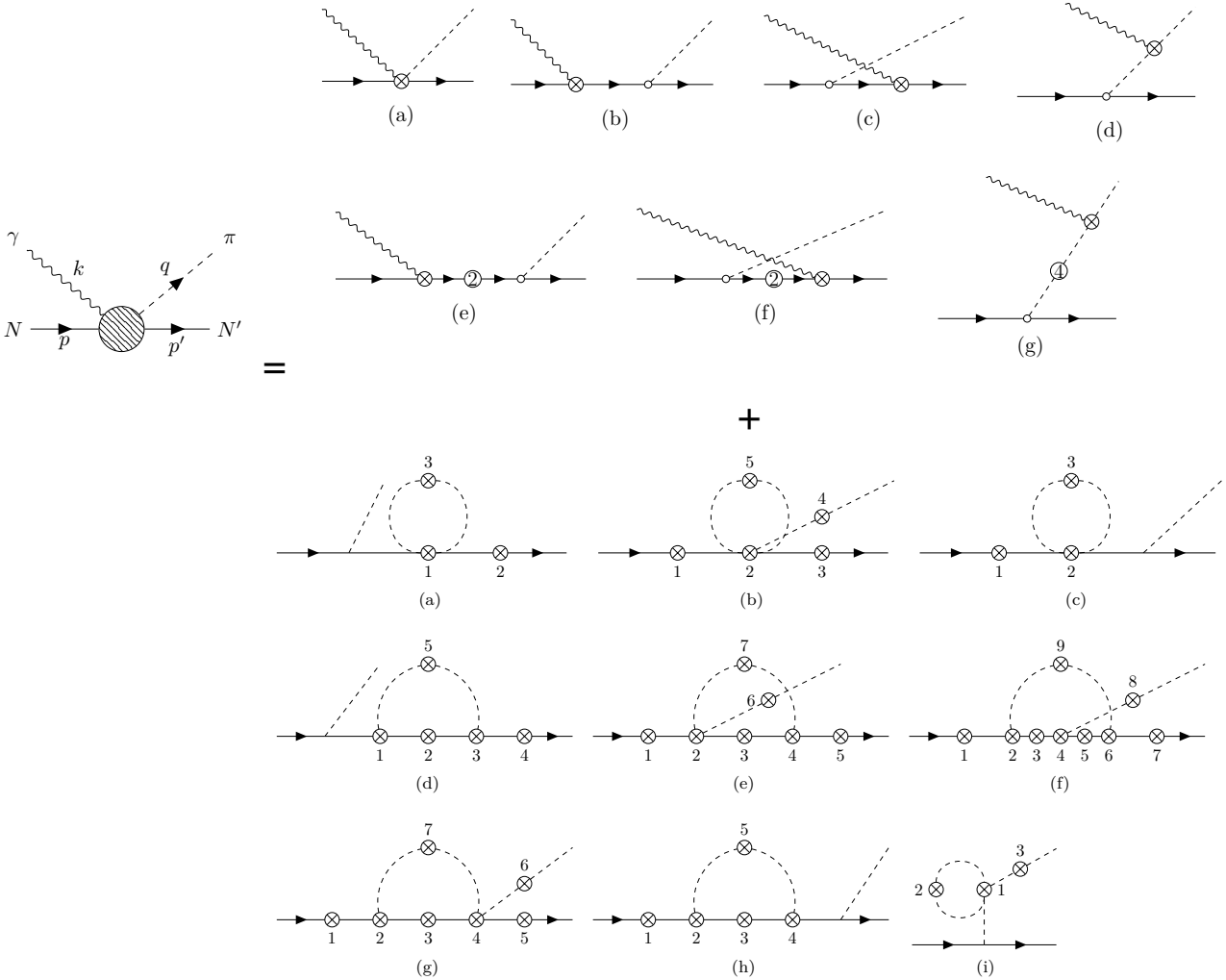
$$\mathcal{L}_{\pi\pi}^{GSS(4)} = \frac{l_3+l_4}{16}\mathrm{Tr}\left[\chi U^\dagger + U\chi^\dagger\right]^2 + \frac{l_4}{8}\mathrm{Tr}\left[\nabla_\mu U\left[\nabla^\mu U\right]^\dagger\right]\mathrm{Tr}\left[\chi U^\dagger + U\chi^\dagger\right]$$
$$+ i\frac{l_6}{2}\mathrm{Tr}\left[F_{R\mu\nu}\nabla^\mu U\left(\nabla^\nu U\right)^\dagger + F_{L\mu\nu}\left(\nabla^\mu U\right)^\dagger\nabla^\nu U\right] + \cdots,$$

All can be expanded in terms of no fitting parameters



Putting the pieces together

- Using all the ingredients, we calculate all possible diagrams up to $O(p^3)$ order



Power Counting Scheme

- For energies close to threshold, far from the $\Delta(1232)$ mass :

  Lensky & Pascalutsa EPJ (2010)

$$D = 4L + \sum_{k=1}^{\infty} kV^{(k)} - 2N_\pi - N_N \tag{1}$$

indicates the nominal chiral order for a given Feynman diagram graph

The Strategy

- We keep our expansion up to $O(p^3)$: Avoids inclusion of too many LECs at $O(p^4)$

Loop diagrams regularization

- Loop diagrams : UV divergences and power counting breaking terms

$\frac{1}{\epsilon_{UV}} = \frac{1}{4-dim}$ and terms $\propto p^1, p^2$, in amplitudes at order $O(p^3)$

- Renormalization $\widetilde{\text{MS}}$-EOMS :
  - $\widetilde{\text{MS}}$: Substracts multiples of $R = \gamma_E - 1/\epsilon_{UV} - \log(4\pi) - 1$,
  - EOMS : Substracts terms of lower order than the nominal one for loop diagrams

In short words

$$\mathcal{L}_{\text{ChPT}} = \mathcal{L}_N^{(1)}(m_0, g_0) + \mathcal{L}_N^{(2)}(c_1, c_6, c_7) + \mathcal{L}_N^{(3)}(d_6, d_7, d_8, d_9, d_{16}, d_{18}, d_{20}, d_{21}, d_{22}) + \mathcal{L}_{\pi\pi}^{(2)}(F_0, M_0) + \mathcal{L}_{\pi\pi}^{GSS(4)}(l_3, l_4, l_6) + \cdots$$

$\Downarrow$ After MS-1 UV and EOMS renormalization

$$\begin{aligned}\mathcal{M}^\mu = &\sqrt{\mathcal{Z}_\pi}(l_4)\mathcal{Z}_N \Big[\mathcal{M}_{tree}^{\mu(1)}(\widetilde{m}(m_N, \widetilde{c}_1), \widetilde{g}(d_{16}, g_A), F_0(l_4, F_\pi), M_0(l_3, M_\pi)) \\ &+ \mathcal{M}_{tree}^{\mu(2)}(\widetilde{c}_1, \widetilde{c}_6, \widetilde{c}_7) \\ &+ \mathcal{M}_{tree}^{\mu(3)}(d_6, d_7, d_8, d_9, d_{16}, d_{18}, d_{20}, d_{21}, d_{22}, l_3, l_4, l_6) \\ &+ \widetilde{\mathcal{M}}_{loop}^{\mu(3)} + \mathcal{O}(p^4)\Big]\end{aligned}$$

In short words : Which are the relevant LECs ?

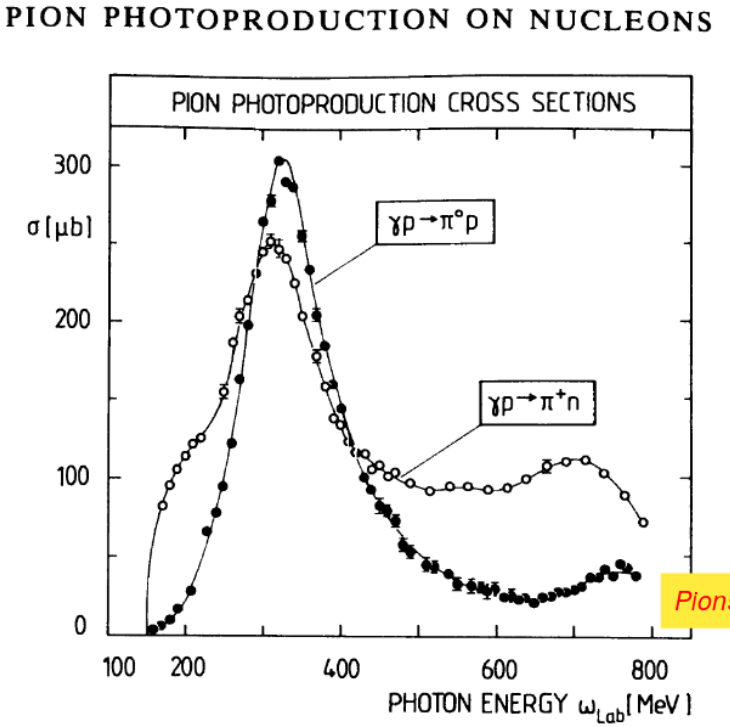
$$\mathcal{L}_{\text{ChPT}} = \mathcal{L}_N^{(1)}(m_0, g_0) + \mathcal{L}_N^{(2)}(c_1, c_6, c_7) + \mathcal{L}_N^{(3)}(d_6, d_7, d_8, d_9, d_{16}, d_{18}, d_{20}, d_{21}, d_{22}) + \mathcal{L}_{\pi\pi}^{(2)}(F_0, M_0) + \mathcal{L}_{\pi\pi}^{GSS(4)}(l_3, l_4, l_6) + \cdots$$

$\Downarrow$ After MS-1 UV and EOMS renormalization

$$\mathcal{M}^\mu = \sqrt{\mathcal{Z}_\pi}\mathcal{Z}_N \Big[\mathcal{M}_{tree}^{\mu(1)}(m_N, g_A, F_\pi, M_\pi) + \mathcal{M}_{tree}^{\mu(2)}(\widetilde{c}_6, \widetilde{c}_7) + \mathcal{M}_{tree}^{\mu(3)}(d_6, d_7, d_8, d_9, d_{18}, d_{20}, d_{21}, d_{22}, l_6) + \widetilde{\mathcal{M}}_{loop}^{\mu(3)} + \mathcal{O}(p^4)\Big]$$

- **Only d_8, d_9, d_{20}, d_{21}, d_{22} free parameters**

The Strategy to improve the approach



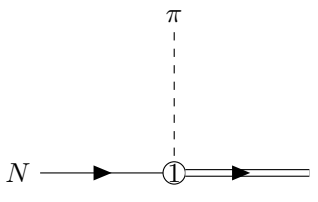
Pions and Nuclei, Ericson & Weise

- Explicit inclusion of the $\Delta(1232)$ spin $3/2$ resonance
- $\Delta(1232)$ becomes more relevant the closer we are to its mass
- $\Delta(1232)$ inclusion is important in charged and neutral π-photoproduction
- No more Low-Energy-Constants to be fitted (keep the model simple)

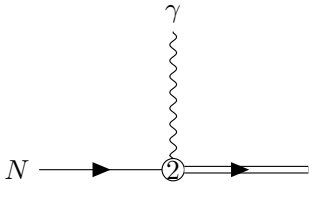
Contributions generated by the $\Delta(1232)$

$$\mathcal{L}^{(1)}_{\Delta\pi N} = \frac{ih_A}{2Fm_\Delta}\overline{\Psi}T^a\gamma^{\mu\nu\lambda}(\partial_\mu\Delta_\nu)\partial_\lambda\pi^a + \text{h.c.},$$

$$\mathcal{L}^{(2)}_{\Delta\gamma N} = \frac{3ieg_M}{2m(m+m_\Delta)}\overline{\Psi}T^3(\partial_\mu\Delta_\nu)\tilde{f}^{\mu\nu} + \text{h.c.},$$



h_A related to Γ^{strong}_Δ



g_M related to Γ^{EM}_Δ

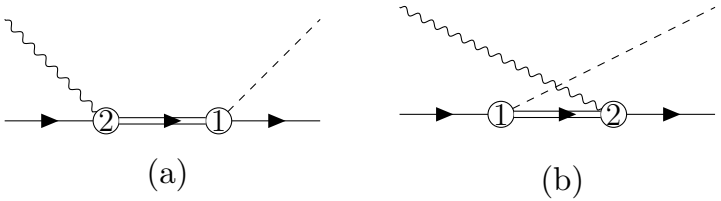
Only one constrained parameter added, $+\mathcal{M}^{\mu(5/2)}_\Delta(g_M h_A)$

δ-Power Counting Scheme

- For energies close to threshold, far from the $\Delta(1232)$ mass :
  Lensky & Pascalutsa EPJ (2010)

$$D = 4L + \sum_{k=1}^{\infty} kV^{(k)} - 2N_\pi - N_N - \frac{1}{2}N_\Delta .$$

- No loop diagrams with $\Delta(1232)$ up to $O(p^3)$.



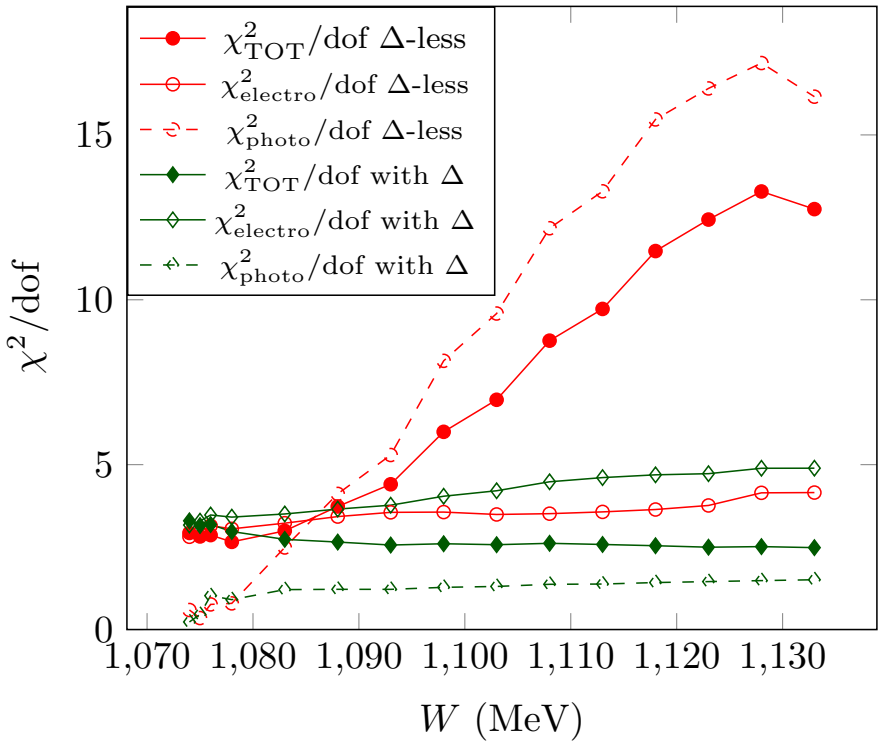
- $\Delta(1232)$ is only included at tree level $\longrightarrow O(p^{5/2})$ order

- WE HAVE OUR BEST MODEL ($\mathcal{O}(p^3)$ EOMS ChPT + Δ) IN TERMS OF LECs
- WE WANT TO DETERMINE THE LECs USING EXP. DATA

The first message ; The χ^2

Fitting LECs using data for all channels

Now we can see easily what can be obtained with the $\Delta(1232)$ inclusion, **without including extra fitting Low-Energy-Constants**



- ChPT up to $O(p^3)$ with loops
- ChPT up to $O(p^3)$ with loops & $\Delta(1232)$ inclusion

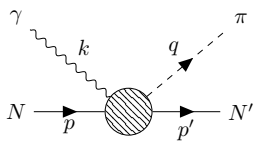
What about low-energy constants ?

Fitted LECs from other works using the same approach

TABLE – Values of the LECs determined from other processes.

	LEC	Value	Source
$\mathcal{L}_N^{(2)}$	$\widetilde{c}_6$	5.07 ± 0.15	μ_p and μ_n [Bauer :2012,Yao :2018,PDG]
	$\widetilde{c}_7$	-2.68 ± 0.08	μ_p and μ_n [Bauer :2012,Yao :2019,PDG]
$\mathcal{L}_N^{(3)}$	d_6	-0.70 GeV^{-2}	N EM Form factor [Fuchs :2003]
	d_7	-0.49 GeV^{-2}	N EM Form factor [Fuchs :2003]
	d_{18}	-0.02 ± 0.08 GeV^{-2}	πN scattering [Alarcon :2012]
$\mathcal{L}_{\pi\pi}^{(4)}$	l_6	$(-1.34 \pm 0.12) \times 10^{-2}$	$\langle r^2 \rangle_\pi$ [Yao :2018]
$\mathcal{L}_{\Delta N\pi}^{(1)}$	h_A	2.87 ± 0.03	$\Gamma_\Delta^{\text{strong}}$ [Bernard :2012]
$\mathcal{L}_{\Delta N\gamma}^{(2)}$	g_M	3.16 ± 0.16	$\Gamma_\Delta^{\text{EM}}$ [Blin :2015]

- We fit few LECs appearing in $\gamma(^*)N \to \pi N'$



$d_8, d_9, d_{20}, d_{21}, d_{22}$

Best fit results w/ Minuit : LECs

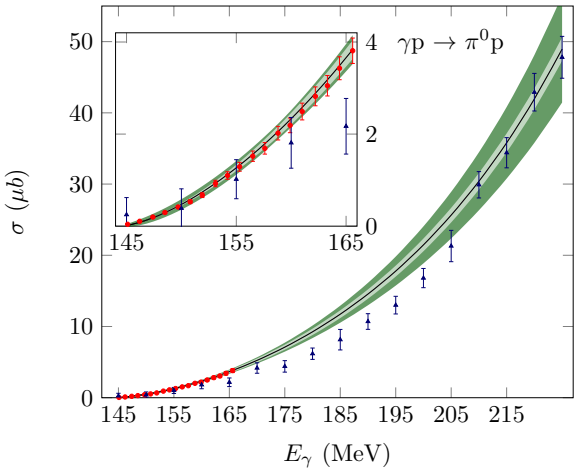
TABLE – The values of the LECs are dimensionless for g_M, in units of GeV^{-2} for d's. Fit I refers to the standard setting, Fit II removes Δ mechanisms

LECs	Fit-I	Fit -II Δ-less
$d_8 + d_9$	1.12 ± 0.01	3.44 ± 0.01
$d_8 - d_9$	0.63 ± 0.15	4.75 ± 0.18
d_{20}	-0.29 ± 0.09	-3.01 ± 0.09
d_{21}	1.64 ± 0.06	4.50 ± 0.06
d_{22}	0.95 ± 0.13	0.45 ± 0.12
g_M	2.90 ± 0.01	-
χ^2/dof	$\mathbf{2.7}$	$\mathbf{13.2}$
χ^2_γ/dof	1.7	16.8
χ^2_e/dof	5.1	4.4

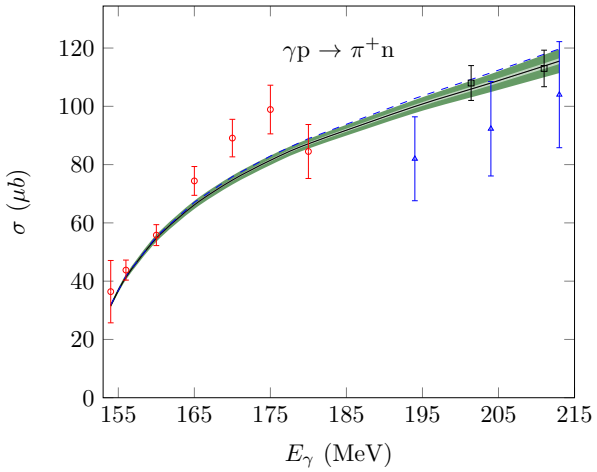
- c_6, c_7 LECS are fixed from nucleon form factors (μ_p,μ_n) (When they are included in the fit we get the same value, with or without Δ)
- $d's$ LECs are sensitive to the lower order corrections, i.e., to the Δ inclusion.
- h_A & g_M can slightly vary without change the values of the other LECs.

Comparing theoretical results vs experimental data

Cross sections



- A. Schmidt et al., PRL (2001)
- S. Schumann et al., EPJ A (2010)
- Fit I : $O(p^3)$ with Δ



- McPherson et al., PRB (1964)
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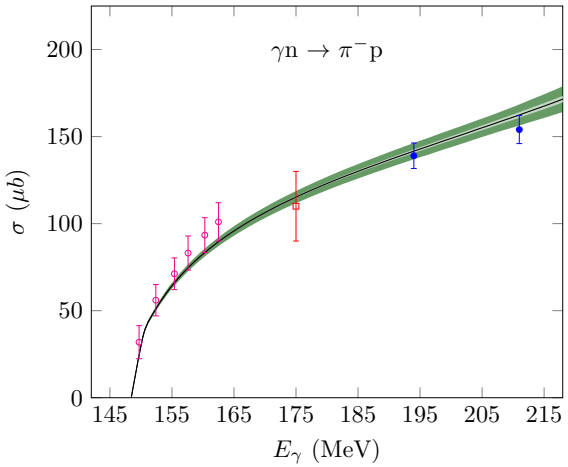

- W. Briscoe et. al. arxiv : 2004.01742 (2020)
- White, R. M. Schectman, and B. M. Chasan, Phys. Rev. 120 , 614 (1960)
- M. Wang, Ph. D. thesis, University of Kentucky (1992).
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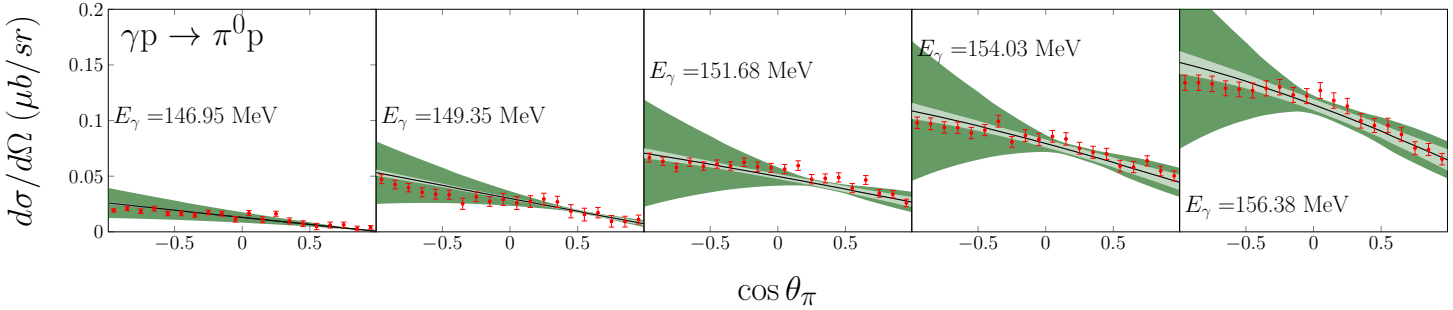
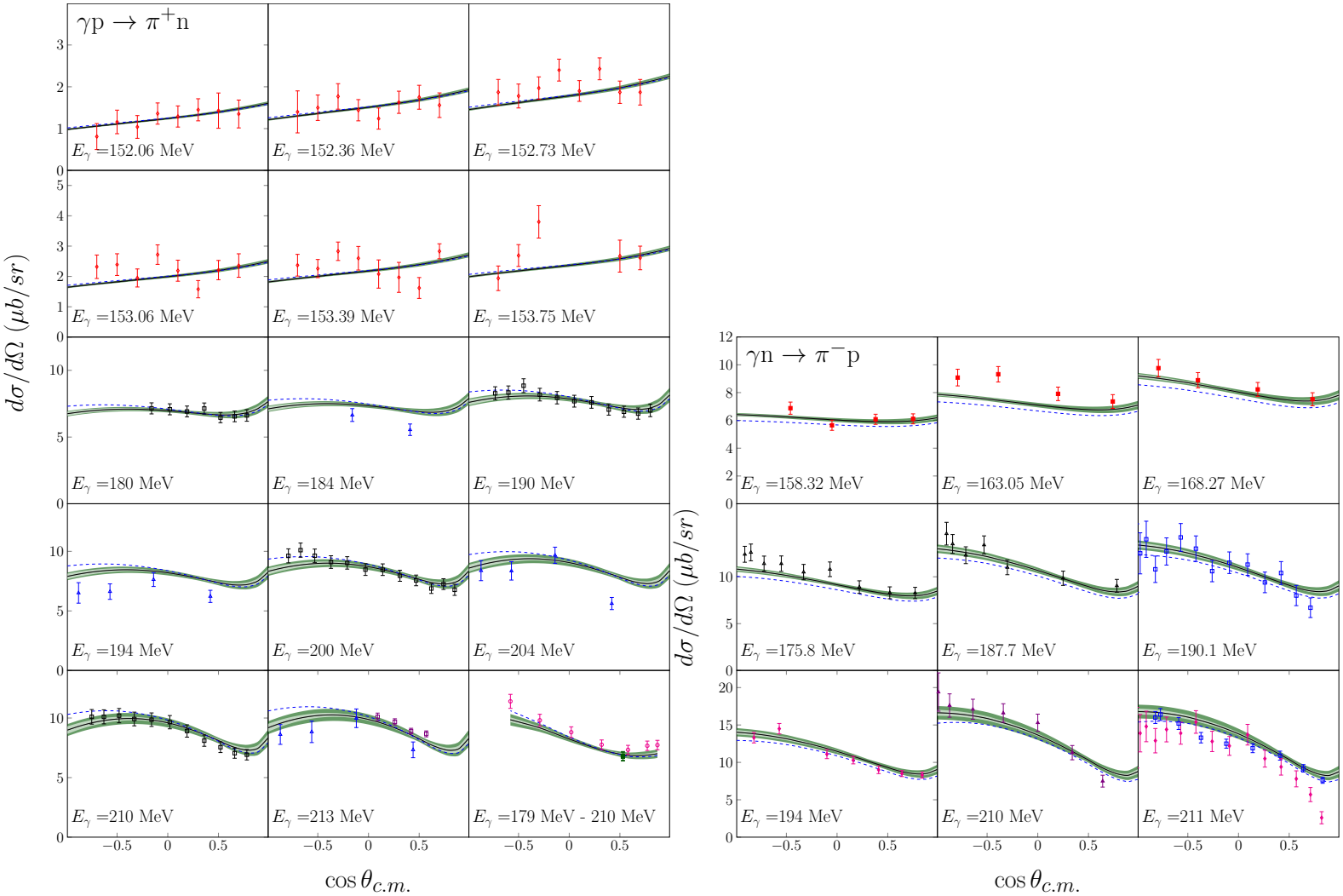
Differential cross sections



G. H. G. Navarro, M. J. Vicente Vacas, arXiv :2008.04244 (2020)[hep-ph]

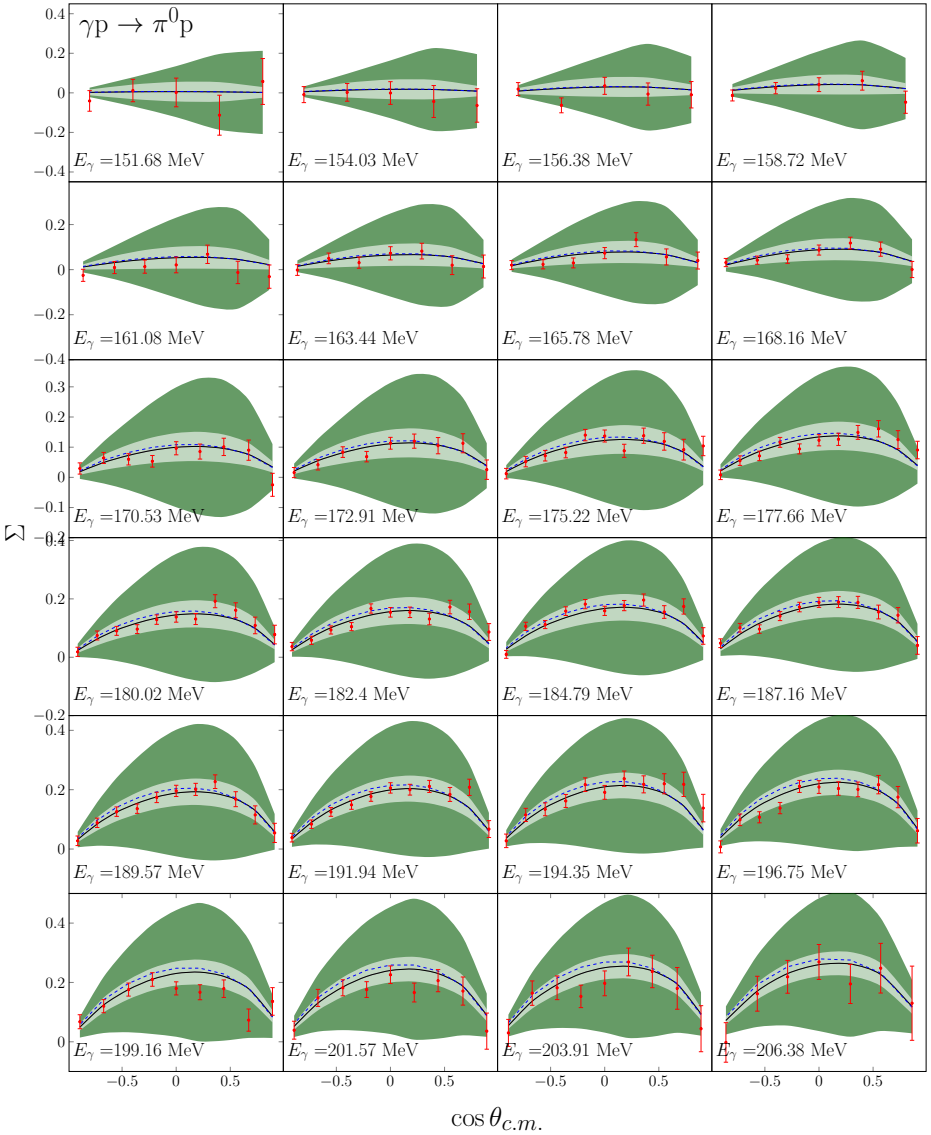
Differential cross sections

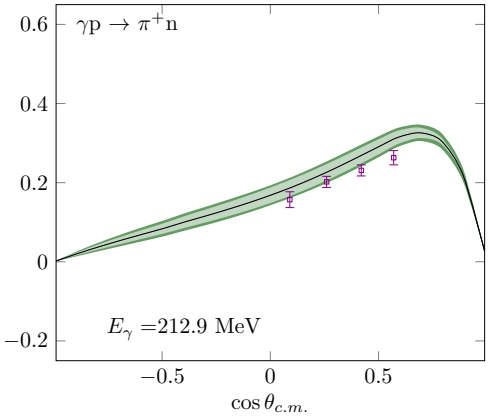


G. H. G. Navarro, A. N. H. Blin, M. J. Vicente Vacas, and D.-L. Yao, (2019), arXiv :1908.00890 [hep-ph]

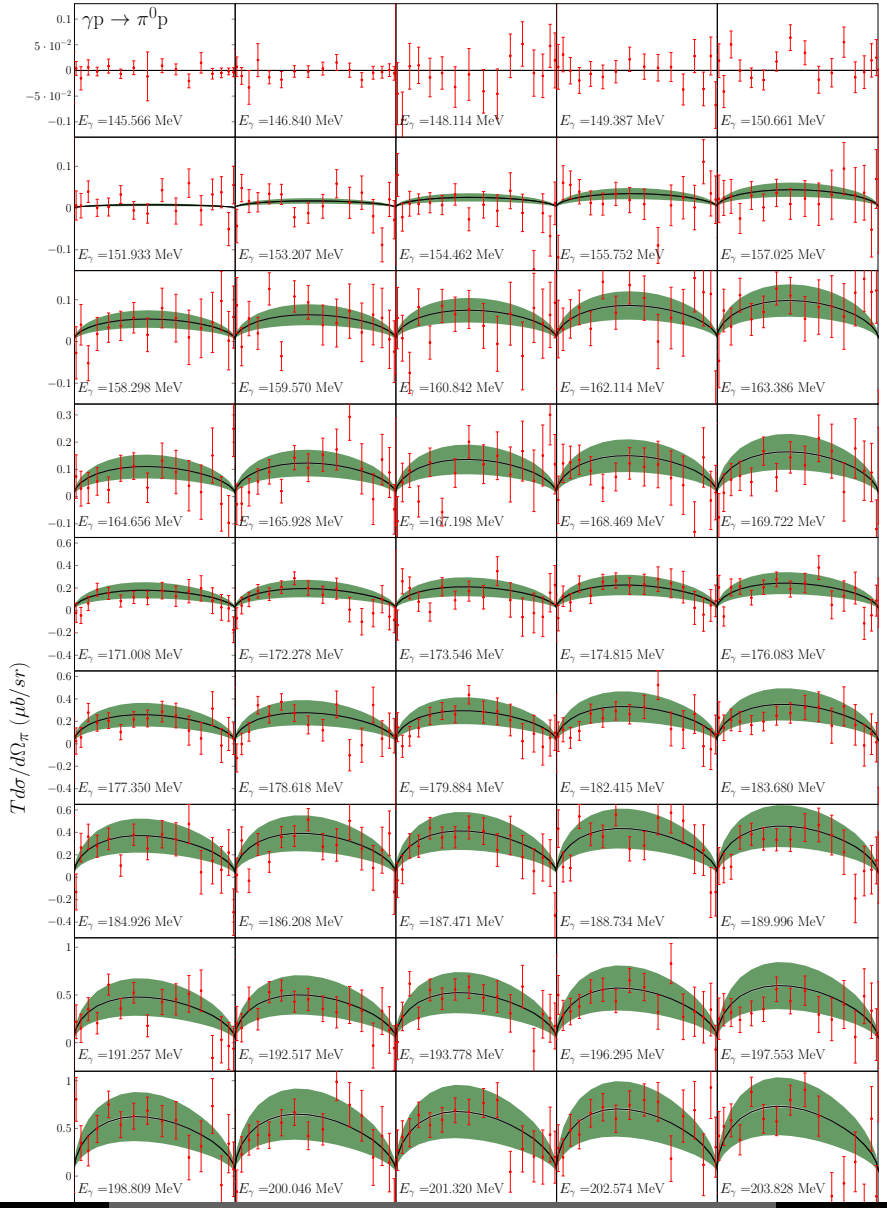
Beam asymmetries

arXiv :1908.00890 [hep-ph]

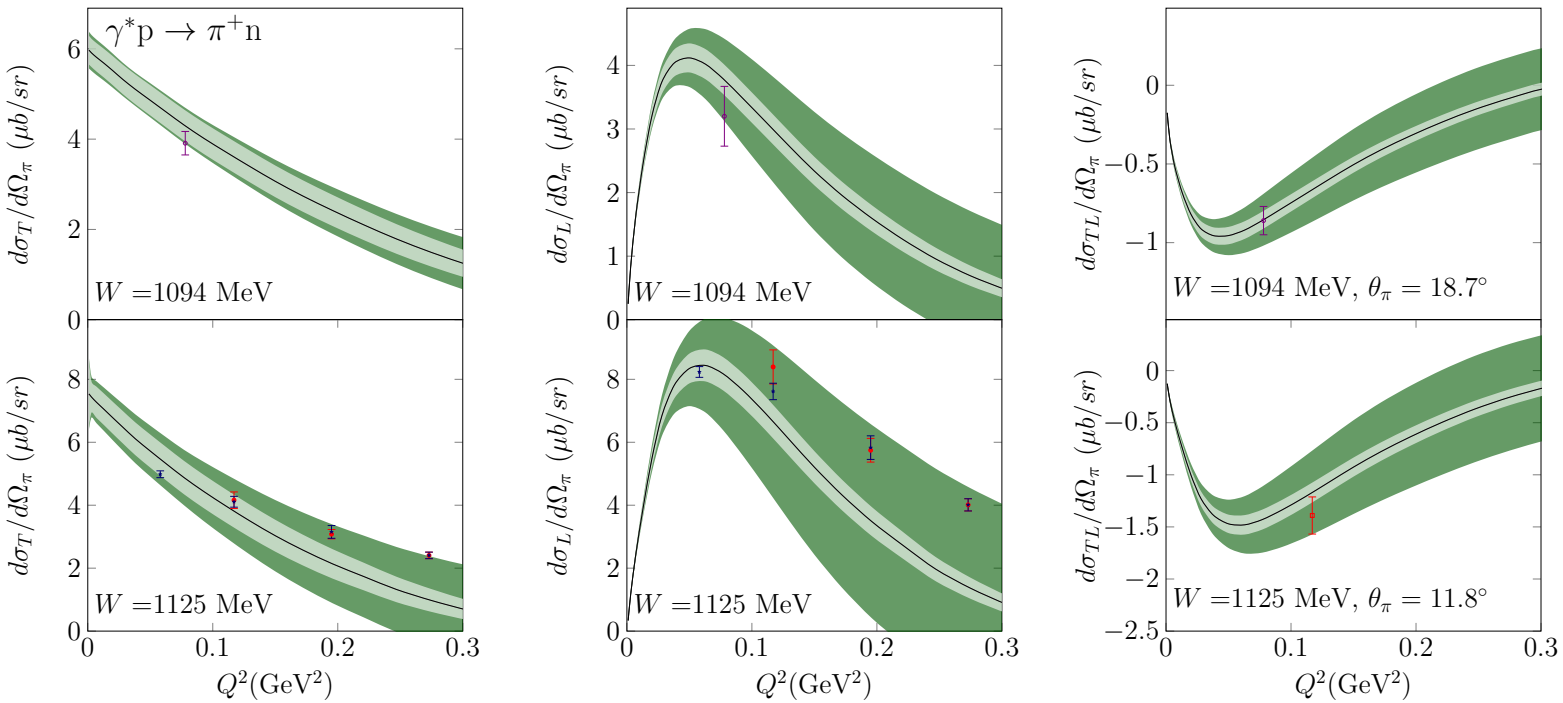




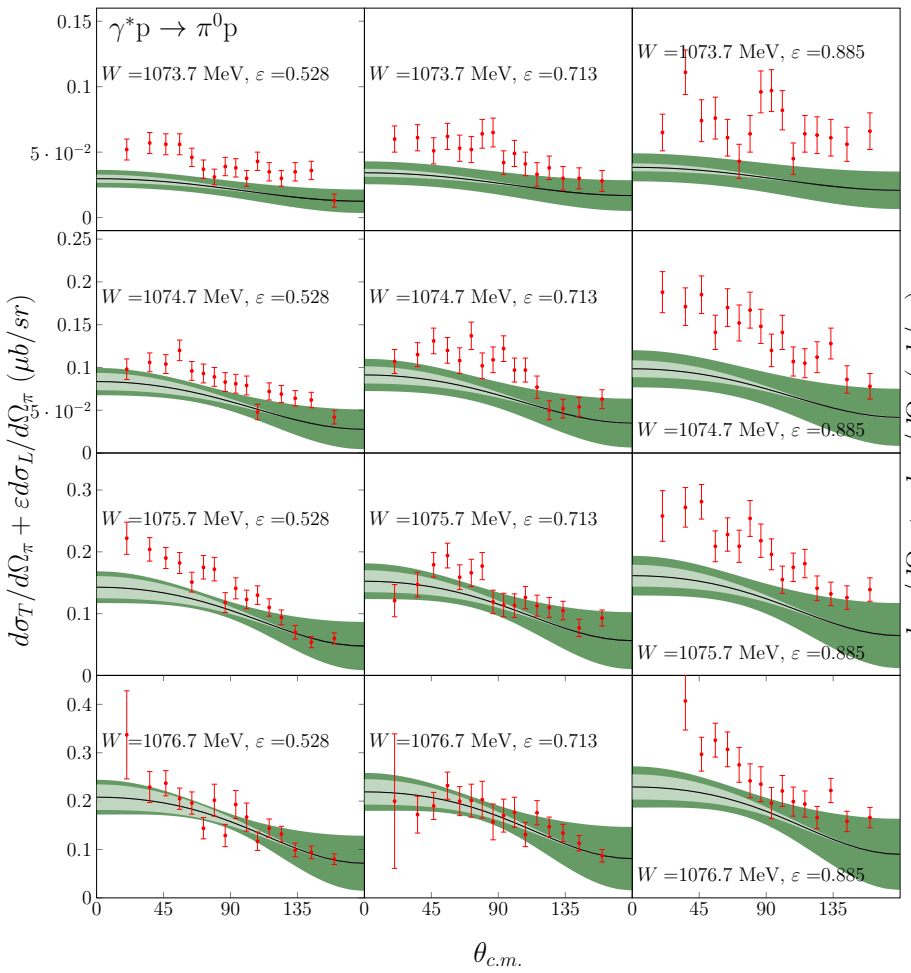
Target asymmetries

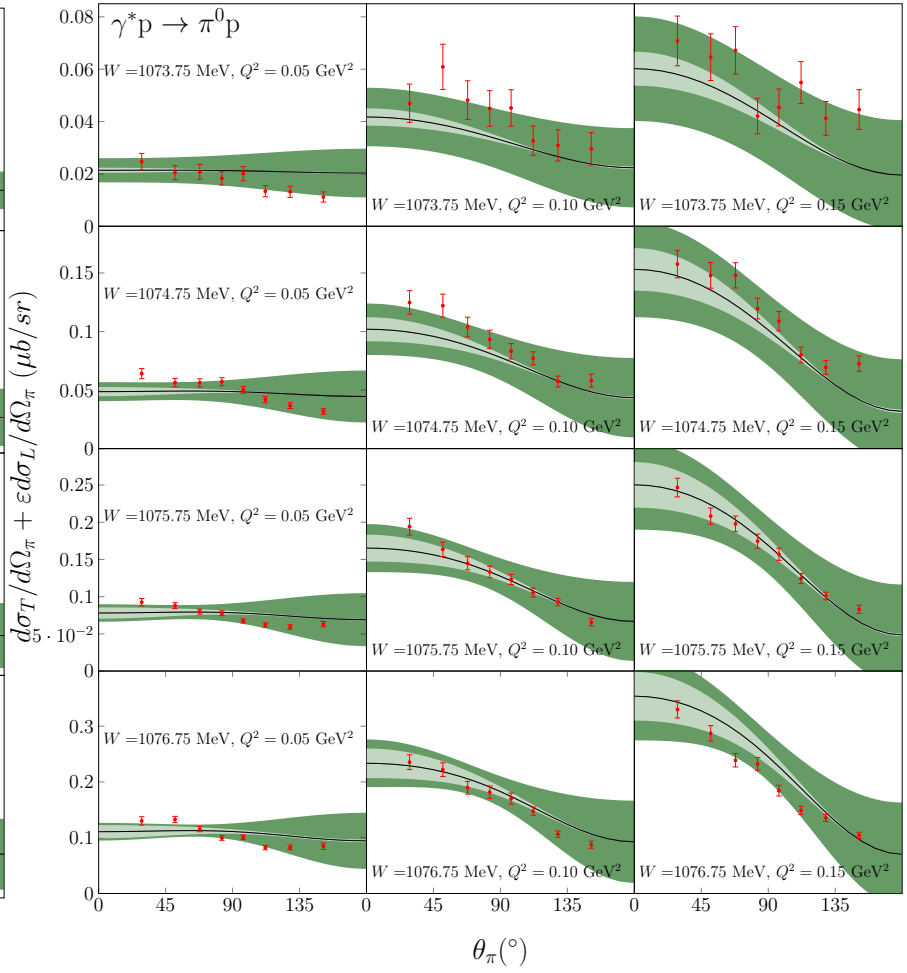


Observables electroproduction



Observables electroproduction





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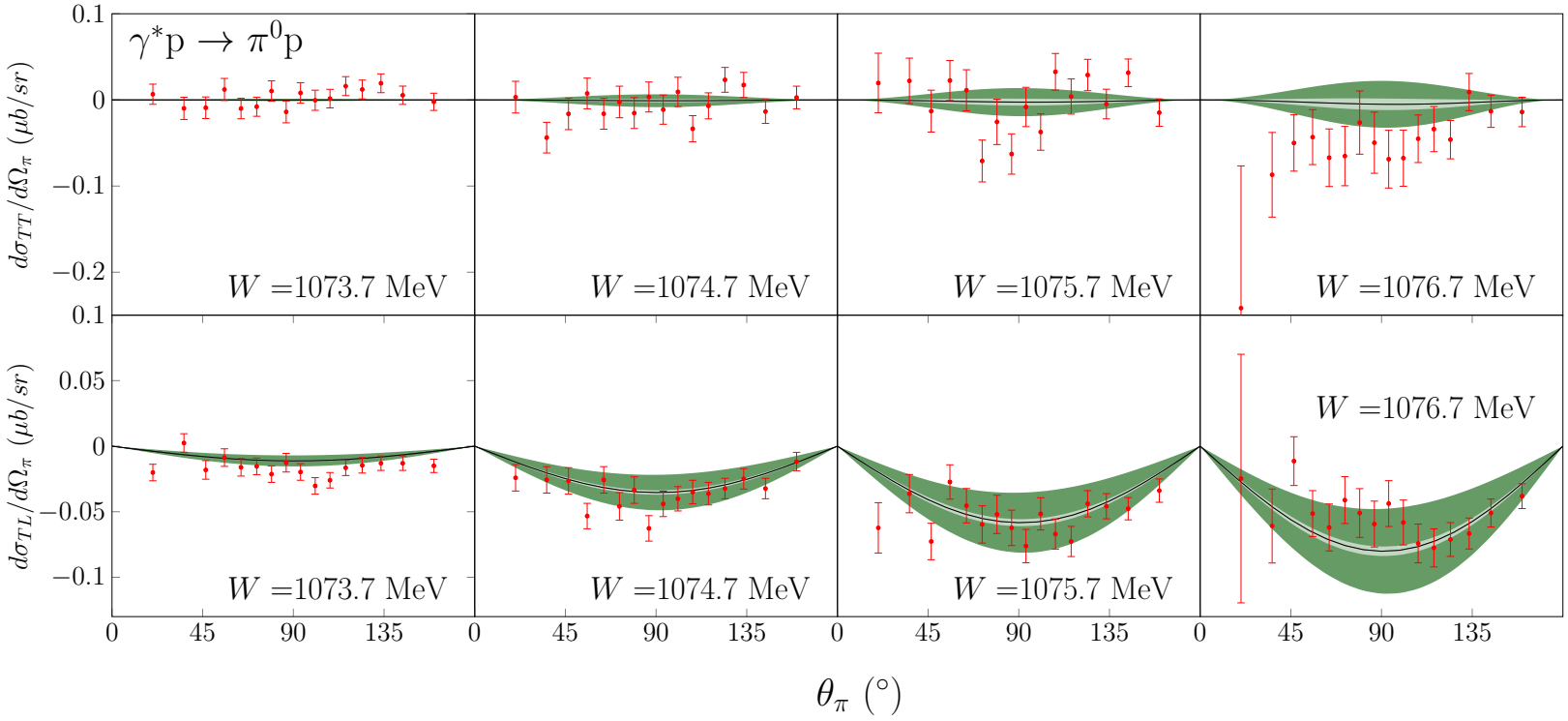

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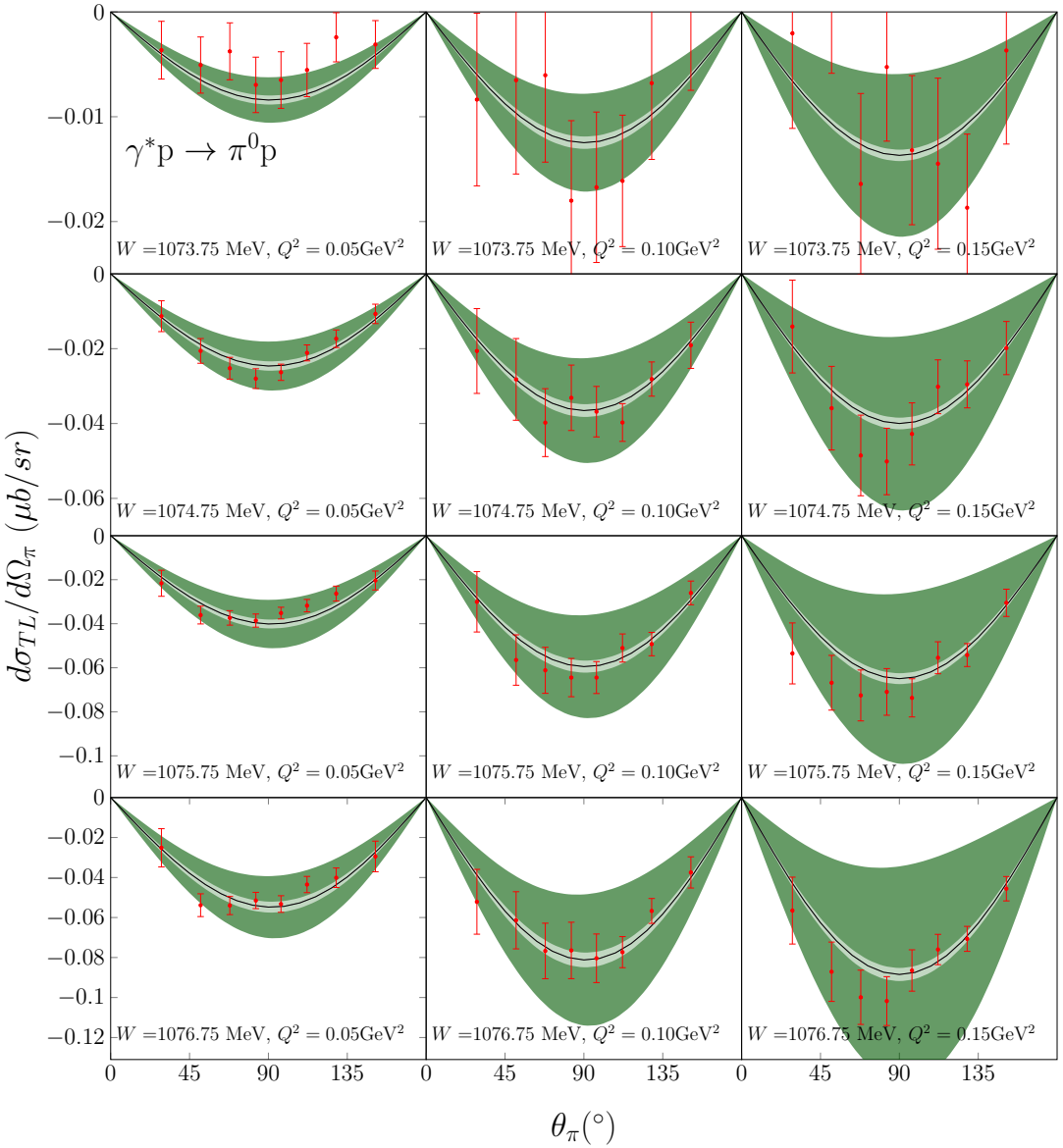

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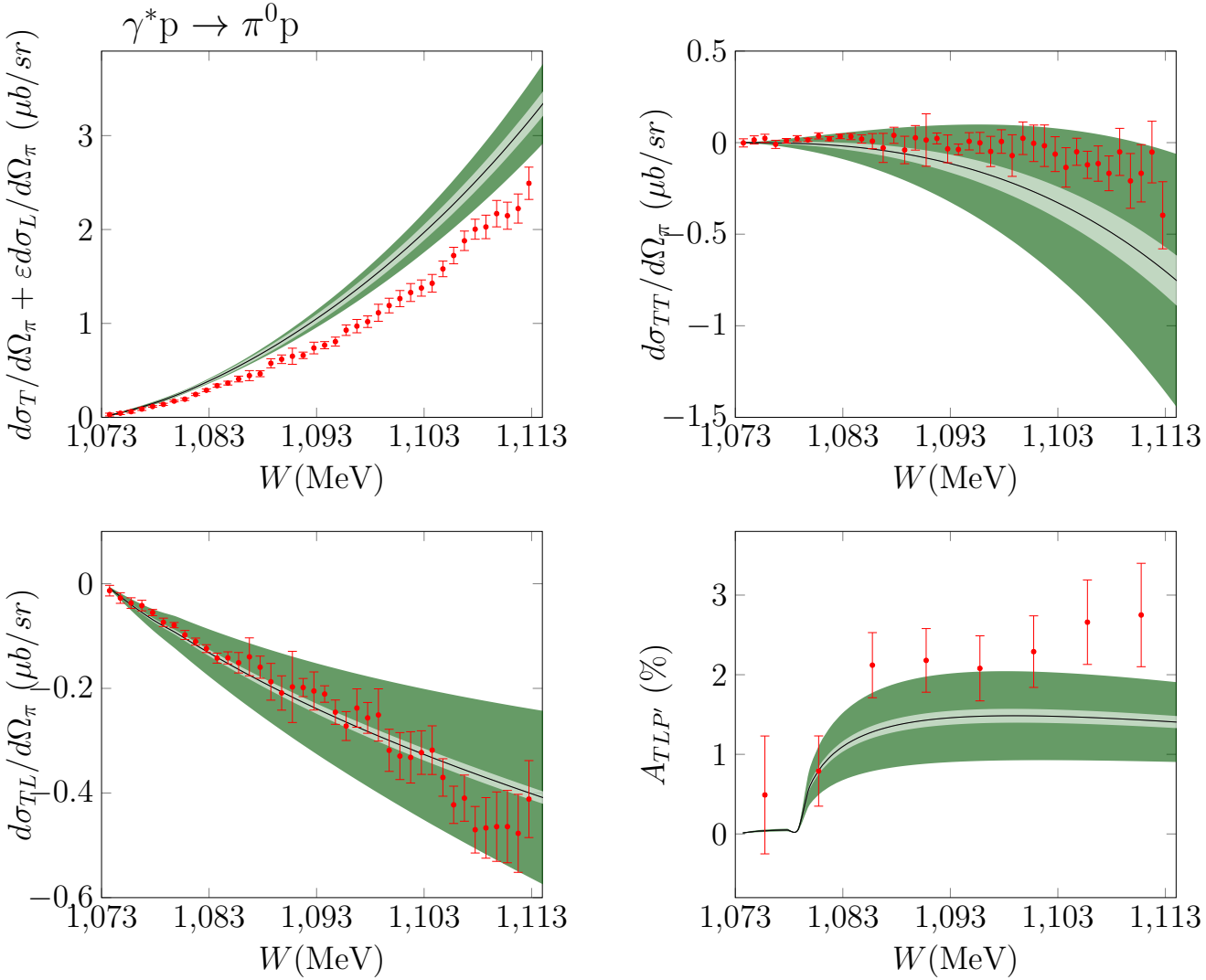

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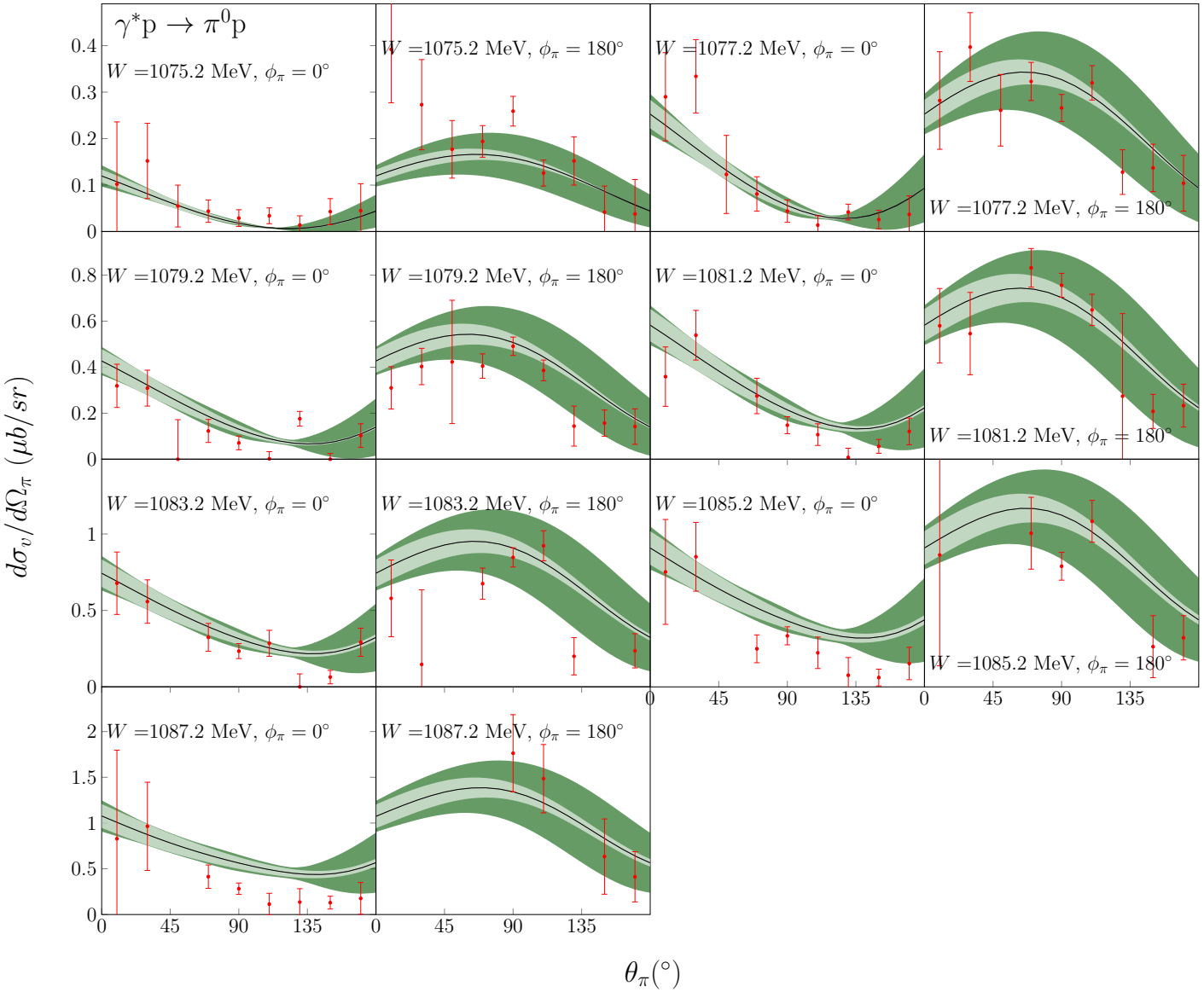
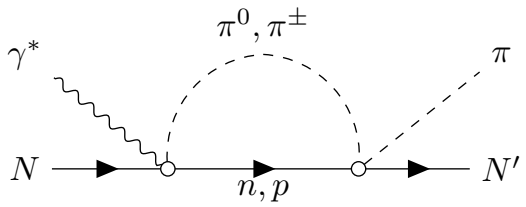
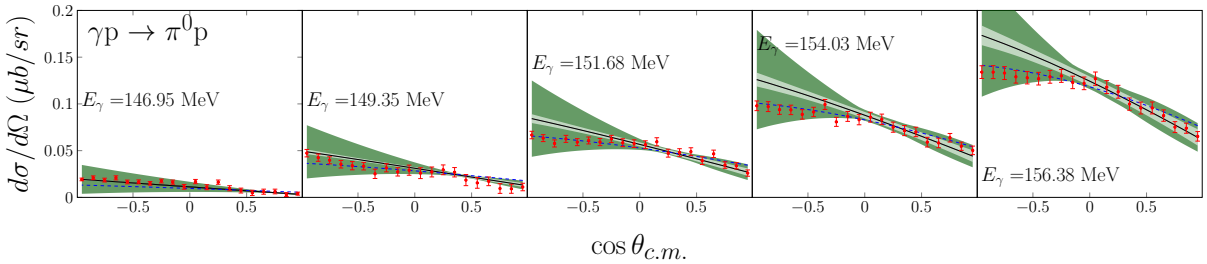


Note : Improvement in $d\sigma/d\Omega_\pi$



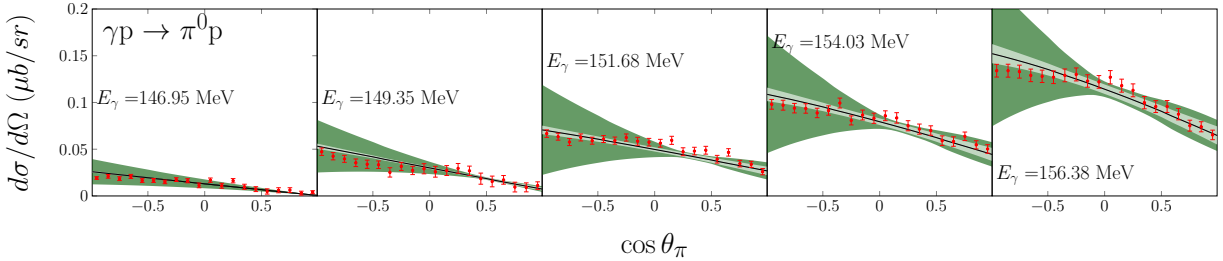
What's happen when we split explicitly $n, p, \pi^0, \pi^\pm$ masses at the one-loop amplitudes ?

Before



G. H. G. Navarro, A. N. H. Blin, M. J. Vicente Vacas, and D.-L. Yao, Phys. Rev. D 100, 094021 (2019)

Now



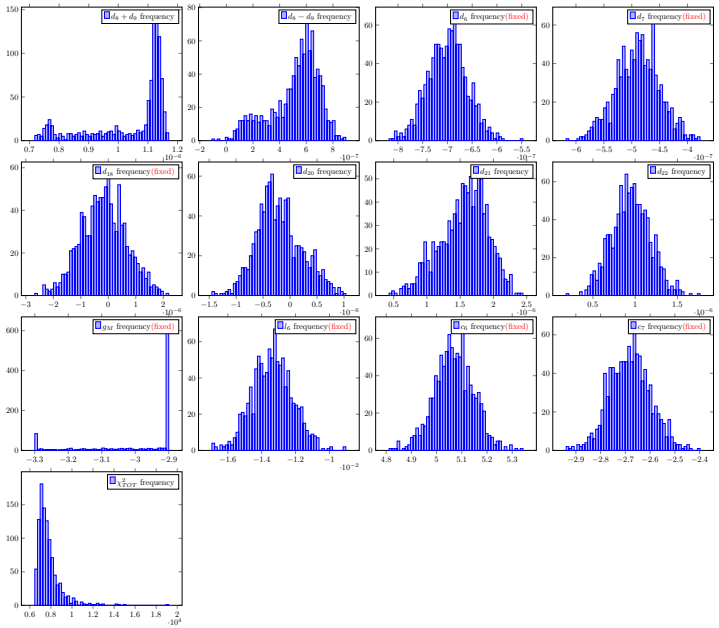
G. H. G. Navarro, M. J. Vicente Vacas (2019), arXiv :2008.04244 [hep-ph]

- d_{18} Goldelberg-Treiman relation, leads backward angles at lower energies together with one-loop imaginary part → **Best fit agreement !**

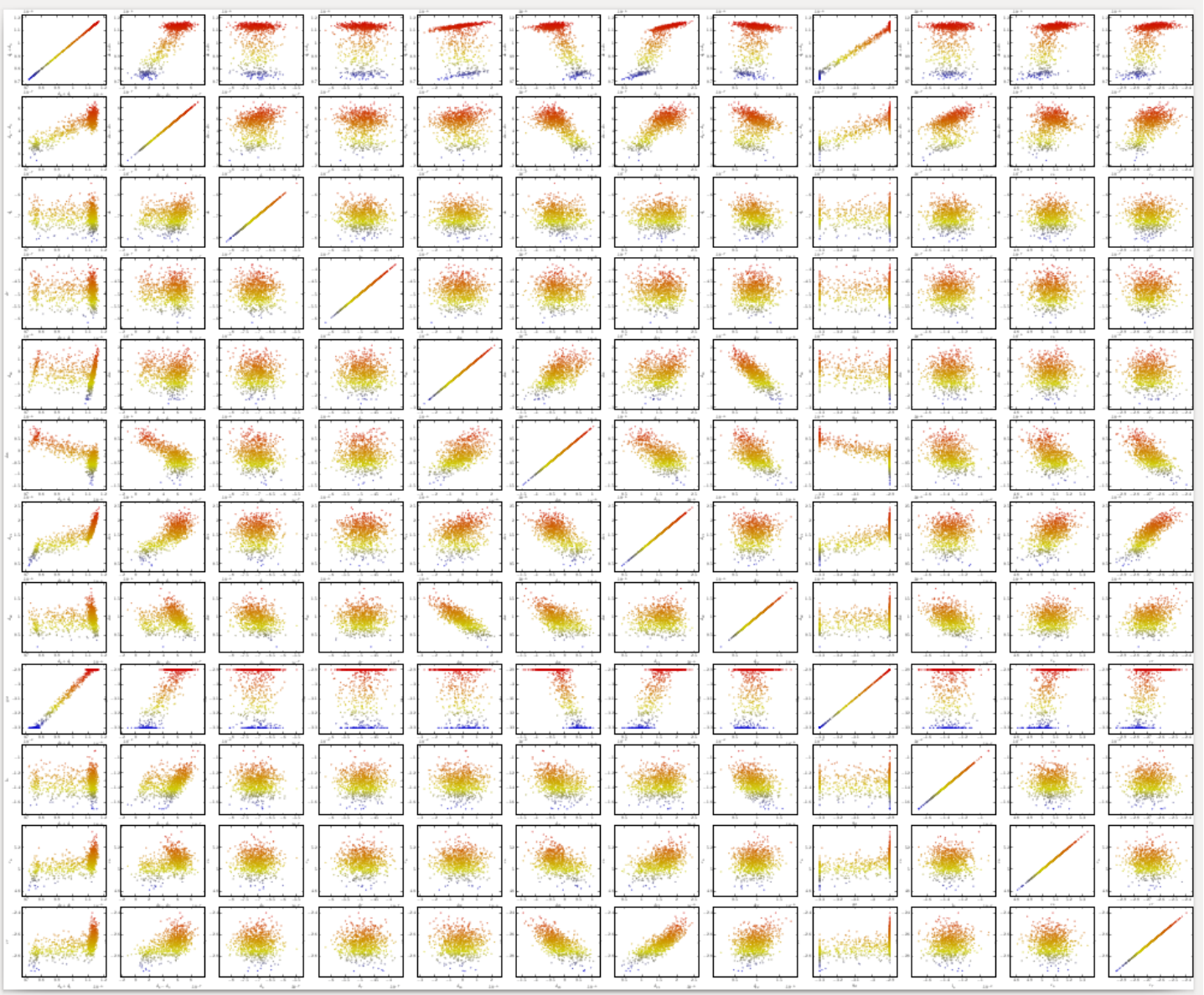
Error estimations

What are the mean sources of error ?

- Systematical error due to the truncation of chiral expansions ($\sim \mathcal{O}(p^4)$)
- Statistical errors due to the experimental data errors in the fitting procedure
- Statistical errors due to the fixed LECs from other processes
  $\hookrightarrow$ We introduce those errors through gaussian randomnization



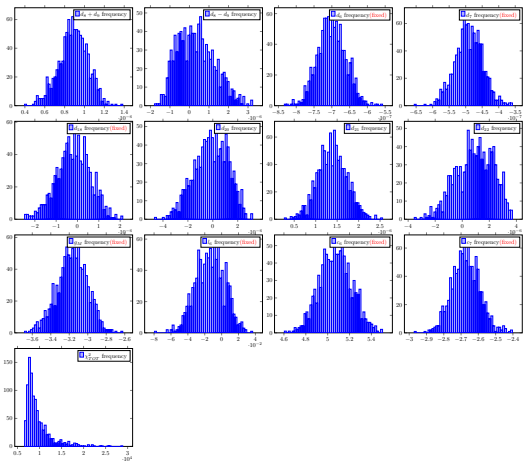
LEC correlations : $d_8 + d_9, d_8 - d_9, d_6, d_7, d_{18}, d_{20}, d_{21}, d_{22}, g_M, l_6, c_6, c_7$



Fits results : including LEC errors fromother processes

TABLE – The values of the LECs are dimensionless for g_M, in units of GeV^{-2} for d's. Fit I refers to the standard setting, Fit III includes non-fitting LEC errors

LECs	Fit-I	Fit -III
$d_8 + d_9$	1.12 ± 0.01	0.89 ± 0.14
$d_8 - d_9$	0.63 ± 0.15	0.26 ± 0.24
d_{20}	-0.29 ± 0.09	-0.03 ± 0.34
d_{21}	1.64 ± 0.06	1.38 ± 0.30
d_{22}	0.95 ± 0.13	0.99 ± 0.23
g_M	2.90 ± 0.01	$\mathbf{3.16 \pm 0.16}$
χ^2/dof	$\mathbf{2.7}$	$\mathbf{3.2 \pm 0.2}$



What have we learned ?

- When we include charged and neutral π EM-production reactions to the analysis, we uncorrelate many of the fitting-LECs ($d_{18}, g_0, d_{21}, d_{22}$)
- We get wider errors for the fitting LECs when including errors in fixed LECs from other processes
- Now we consider correlations among fitting and non-fitting LECs.
- Nevertheless, we get very similar errors for all the observables in both approaches, due to the correlations among fixed-fitted LECs
- $d_8 + d_9$ better determined than separated d_8, d_9
- In our case $d_8 + d_9$ is still correlated to g_M, we can choose the best fit

What is the next ?

Pion Photoproduction (Fixed & Fitting-LECs)

$$\gamma N \to \pi N'$$

Pion Electro-production (Fixed & Fitting-LECs)

$$eN \to e\pi N'$$

Weak Pion production–> Neutrino high precision processes (Predictions)

$$\nu N \to N'\pi\nu'$$

Pion photoproduction off nuclei(Predictions)

$$\gamma\ {}^{12}C \to {}^{12}C\pi^0, \quad \gamma\ {}^{12}C \to {}^{12}B\pi^+, \cdots$$

Summary

- It's particularly important the contribution of $\Delta(1232)$ at low energies for the neutral and charged channel.
- Globally the $\Delta(1232)$ inclusion improves much the agreement between data and ChPT models, No need to reach higher orders at this energy (without extra fitting constants).
- Wider agreement with data near Δ mass using $O(p^3)$ calculation with Δ inclusion than previous higher order $O(p^4)$ calculation without Δ.
- The inclusion of opening-channel effects at one-loop amplitudes improves the agreement near threshold (isospin mass splitting)
- We intend to increase the prediction capability through LECs and use its obtained accuracy for other processes.
  Fit LECs first $\rightarrow$ then make predictions for other processes

Thank you

Error estimations

$$\delta\mathcal{O}_{LEC} = \left(\sum_{i,j}[\text{Corr}(i,j)]\,\frac{\partial\mathcal{O}(\bar{x}_i)}{\partial x_i}\delta x_i\,\frac{\partial\mathcal{O}(\bar{x}_j)}{\partial x_j}\delta x_j\right)^{1/2},$$

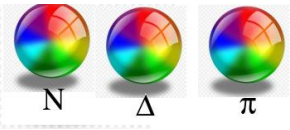
$$\delta\mathcal{O}_{th}^{(n)} = \max\left(\left|\mathcal{O}^{(n_{LO})}\right|Q^{n-n_{LO}+1},\left\{\left|\mathcal{O}^{(k)}-\mathcal{O}^{(j)}\right|Q^{n-j}\right\}\right),\quad n_{LO}\leq j\leq k\leq n$$

where $Q = m_\pi/\Lambda_b$, Λ_b is the breakdown scale of the chiral expansion. We have $\Lambda_b = 4\pi F_\pi \sim 1$ GeV.

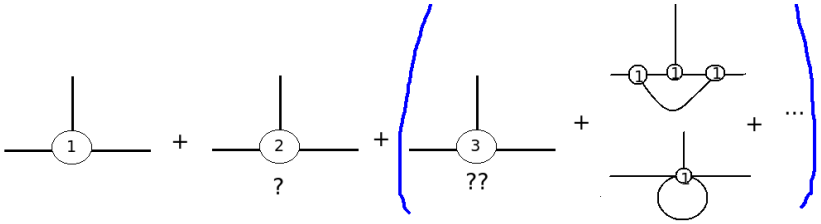
Chiral Perturbation Theory

χ PT

- At large distances (low energy) we are encouraged to use baryons and light mesons, instead of quarks and gluons, as degrees of freedom
- We use pion mass in the same spirit of the Chiral Symmetry Breaking in QCD with quark masses

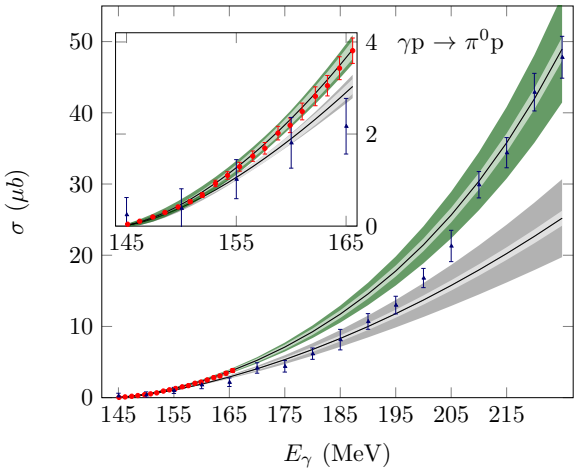


- We use as expansion parameters the relative $\frac{p}{\Lambda}$ and pion mass $\frac{m_\pi}{\Lambda}$
- We can construct Lagrangians that preserves Chiral Symmetry for each order in expansion.
- Appears Low Energy Constants (should be extracted from QCD, but in QCD cannot be solved)

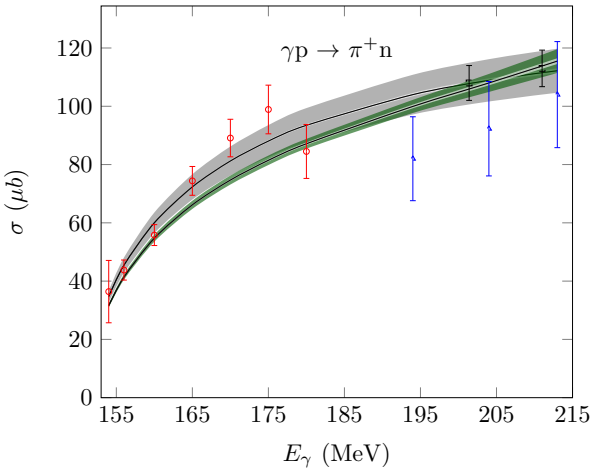


Comparing theoretical results vs experimental data

Cross sections



- A. Schmidt et al., PRL (2001)
- S. Schumann et al., EPJ A (2010)
- Fit I : $O(p^3)$ with Δ
- Fit II : $O(p^3)$ $\not\Delta$



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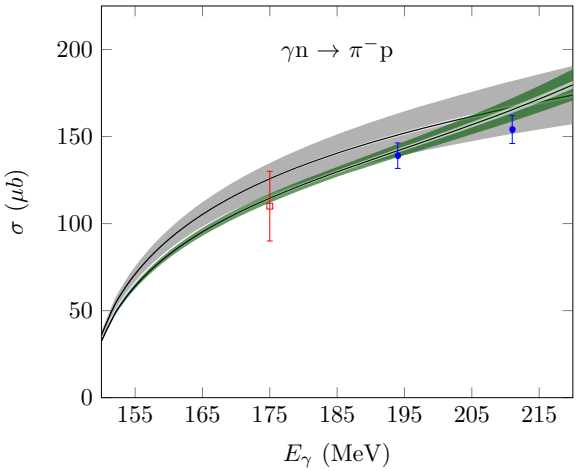

- White, R. M. Schectman, and B. M. Chasan, Phys. Rev. 120 , 614 (1960)
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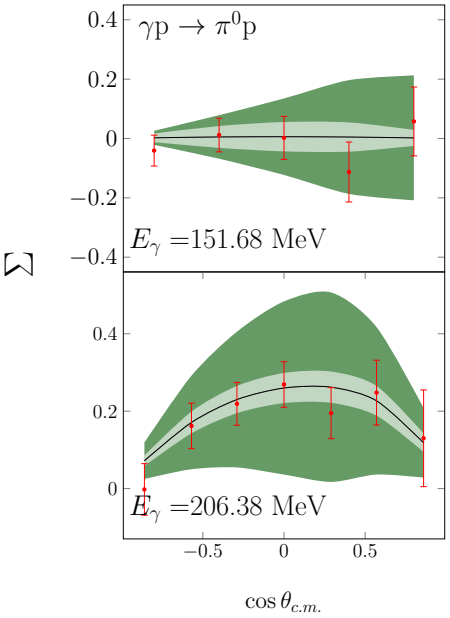
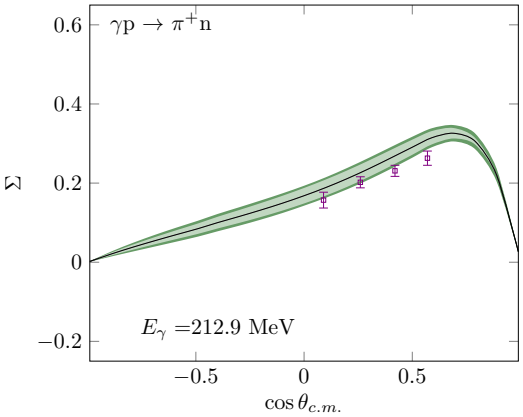


- Hornidge et al., (MAMI) PRL (2013)
- Fit I : $O(p^3)$ with Δ



- G. Blanpied et al., PRC (2001)
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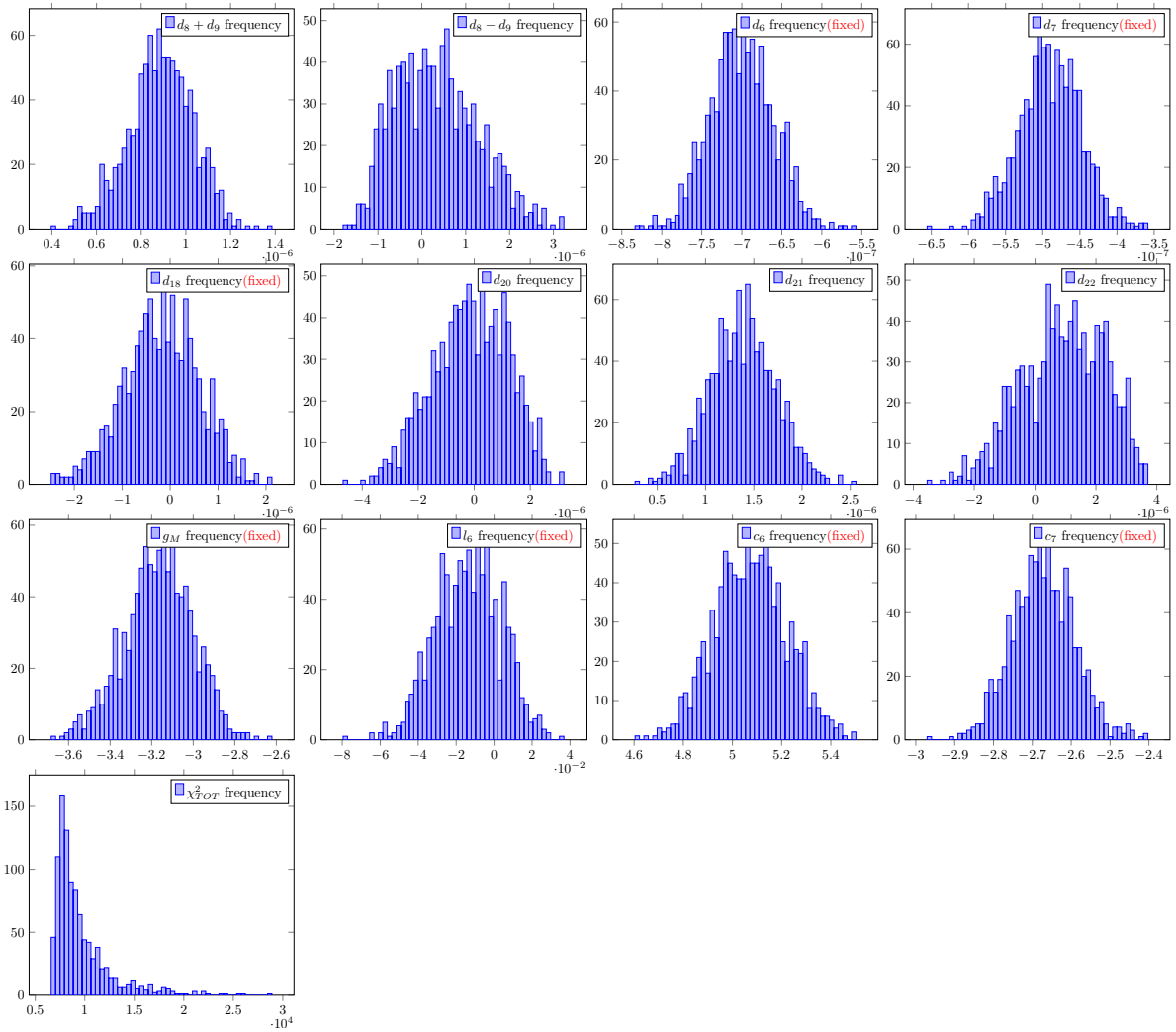
Frequency plots