Center for Frontiers in Nuclear Science

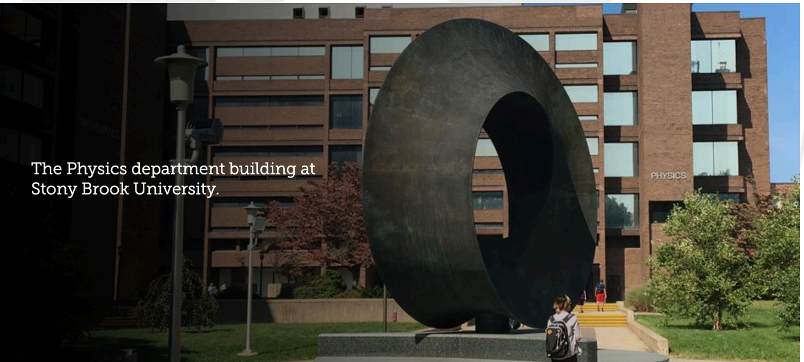
CFNS Ad-Hoc Meeting: Radiative Corrections

9 Jul 2020, 09:00 → 10 Jul 2020, 19:00 US/Eastern

Factorized Approach to Radiative Corrections for Inelastic Lepton-Hadron Collisions

Jianwei Qiu

Theory Center, Jefferson Lab



In collaboration with:
T. Liu, W. Melnitchouk and N. Sato

TMD Collaboration

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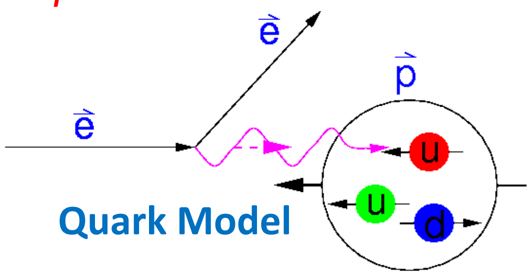
Inelastic Lepton-Hadron Scattering

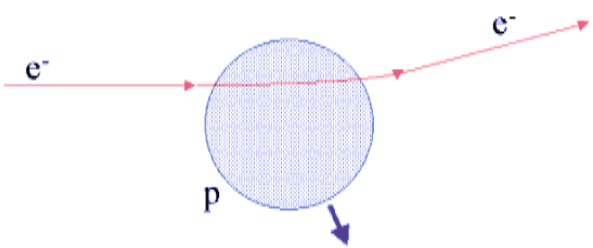
- A modern "Rutherford" experiment (over 50 years ago):

$e + p \rightarrow e + X$ **1968**

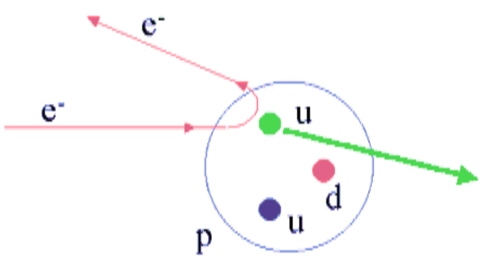


Prediction:

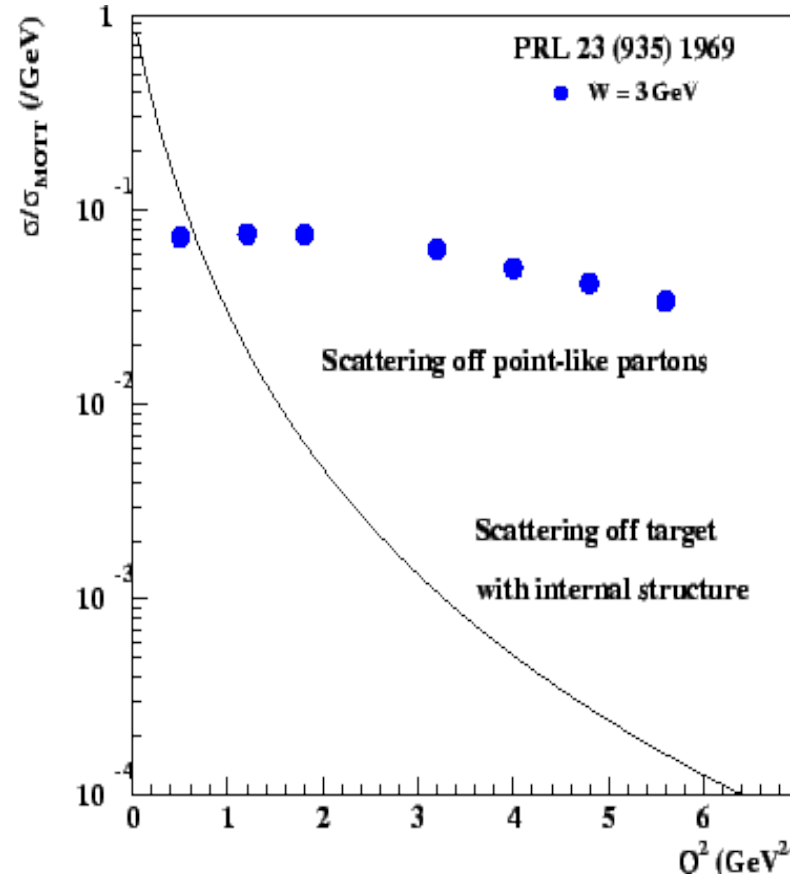
- If proton "charge cloud":



- If proton contains point charges, some of time see:



Discovery:



✧ **Point-like Partons/Quarks**

✧ **Probability Distributions**

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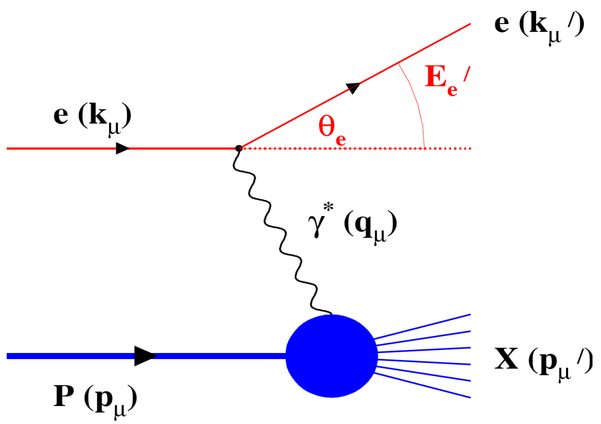
Inelastic Lepton-Hadron Scattering

❑ Approximation of one-photon exchange:



Q^2 = - (k-k')² → Measure of resolution

y = P.(k-k')/P.k → Measure of inelasticity

x_B = Q²/2P.(k-k')

→ Measure of momentum fraction of the struck quark in a proton

$Q^2 = S\, x_B\, y$

$$E'\frac{d\sigma}{d^3l'} = \frac{\alpha_{\rm EM}^2}{2\pi s}\int d^4q \sum_X \left| \langle k'|j_\mu|k\rangle \frac{1}{q^2}\langle X|J^\mu|P\rangle\right|^2 (2\pi)^4\delta^4(P+q-X)\,\delta^4(q-k+k')$$
$$= \frac{2\alpha_{\rm EM}^2}{Q^4 s}\, L^\mu(k,k';q)\, W_{\mu\nu}(q,P)$$

❑ Deep inelastic scattering (DIS) structure functions:

$$W_{\mu\nu}(q,P) = \frac{1}{4\pi}\sum_X (2\pi)^4\delta^4(P+q-X)\,\langle P|J_\mu(0)|X\rangle\,\langle X|J_\nu(0)|P\rangle + \text{spin}\ldots$$
$$= -\widetilde{g}_{\mu\nu}F_1(x_B,Q^2) + \frac{\widetilde{P}_\mu\widetilde{P}_\nu}{P\cdot q}F_2(x_B,Q^2) + \text{spin}\ldots$$
$$\widetilde{g}_{\mu\nu} = -g_{\mu\nu} + q_\mu q_\nu/q^2 \qquad\qquad \widetilde{P}_\mu = \widetilde{g}_{\mu\nu}P^\nu$$

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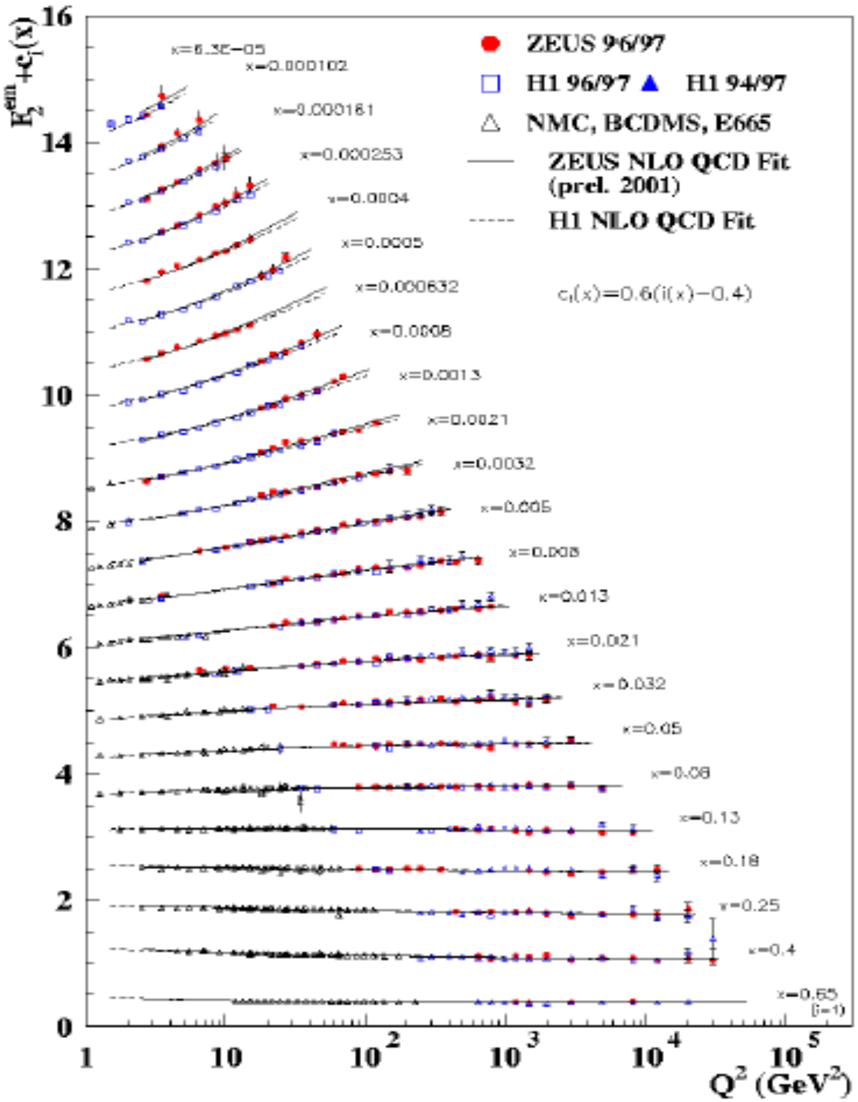
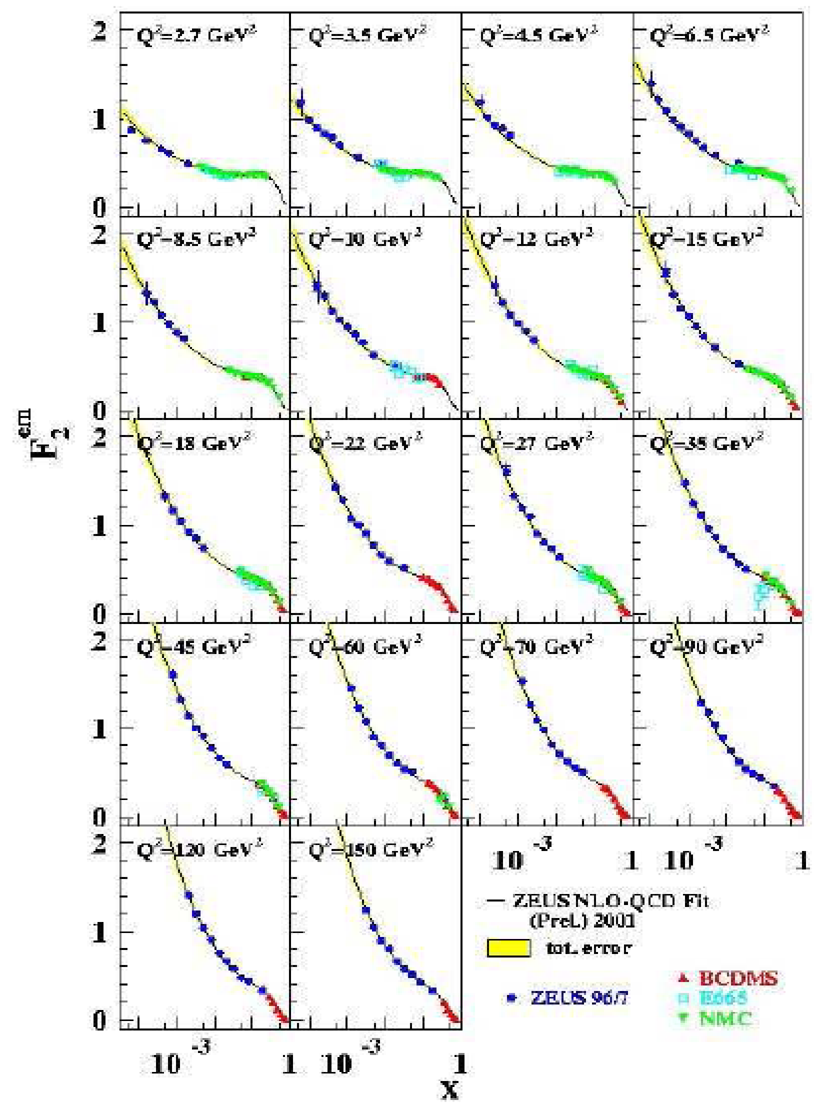
Inelastic Lepton-Hadron Scattering



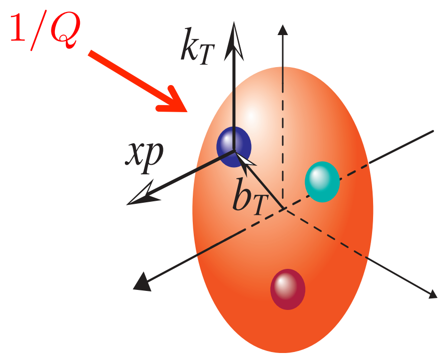


A very successful story of QCD, QCD Factorization, and QCD evolution!

Extraction of Parton Distribution Functions (PDFs) – hadron structure

New-Type Probes for 3D Hadron Structure

❑ Single scale hard probes is too "localized":



- It pins down the particle nature of quarks and gluons
- But, not very sensitive to the detailed structure of hadron ~ fm
- Transverse confined motion: k_T ~ 1/fm << Q
- Transverse spatial position: b_T ~ fm >> 1/Q

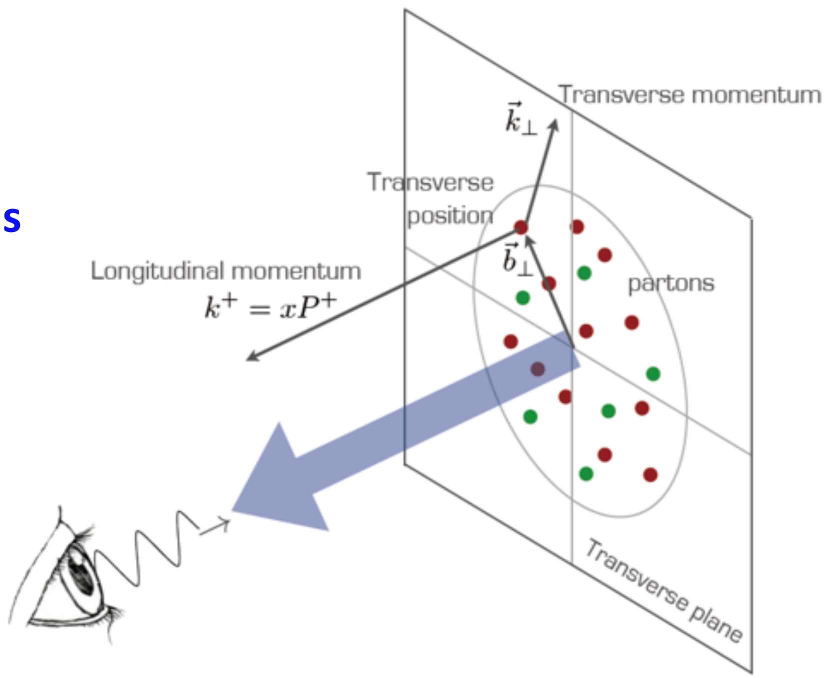
❑ Need new type of "Hard Probes" – Physical observables with TWO Scales:

$$Q_1 \gg Q_2 \sim 1/R \sim \Lambda_{\rm QCD}$$

Hard scale: Q_1 **To localize the probe**
particle nature of quarks/gluons

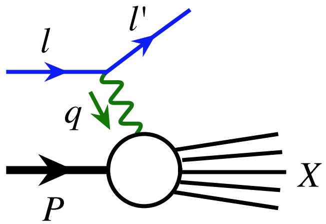
"Soft" scale: Q_2 **could be more sensitive to the**
hadron structure ~ 1/fm

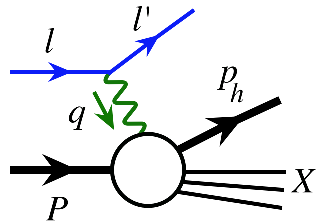
Hit the hadron "very hard" without breaking it,
clean information on the structure!

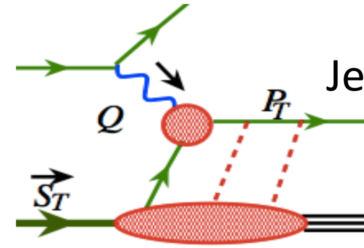


Inelastic Lepton-Hadron Scattering

❑ Semi-inclusive DIS:





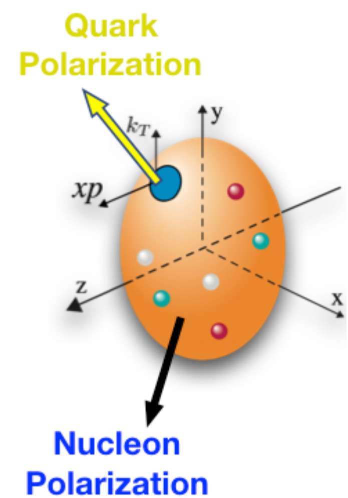


Jet, π, J/ψ, …

Scale: Q^2 $\quad$ $Q^2 \gg P_{h_T}^2$ $\quad$ $f(x, k_T, Q)$

In photon-hadron frame! $\quad$ **Parton's confined motion, …**

		Quark Polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1(x,k_T^2)$		$h_1^\perp(x,k_T^2)$ - Boer-Mulders
	L		$g_1(x,k_T^2)$ - Helicity	$h_{1L}^\perp(x,k_T^2)$ - Long-Transversity
	T	$f_1^\perp(x,k_T^2)$ - Sivers	$g_{1T}(x,k_T^2)$ - Trans-Helicity	$h_1(x,k_T^2)$ - Transversity $h_{1T}^\perp(x,k_T^2)$ - Pretzelosity



- Gluon: $f_q \to f_g$
- FFs
- Nuclei: $s \neq \frac{1}{2}$

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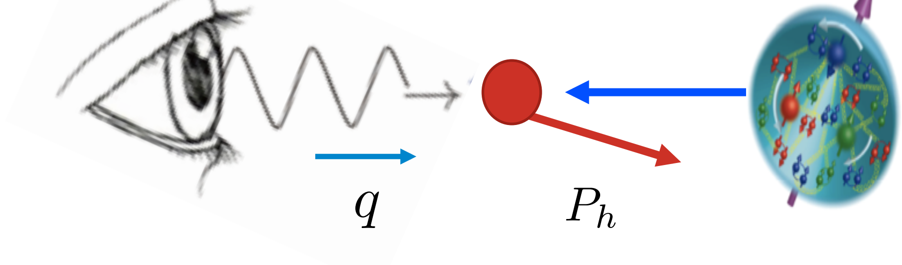
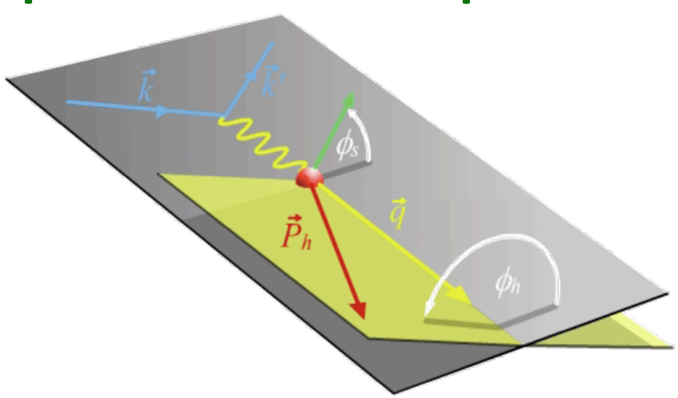
Inelastic Lepton-Hadron Scattering

- **Photon-hadron frame:**

e.g. Breit Frame



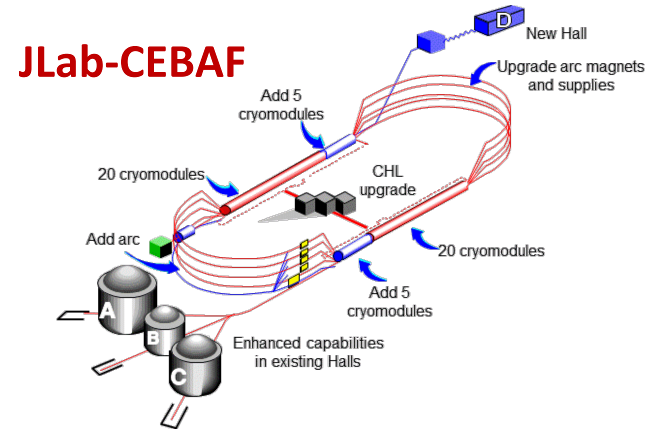
- **Leptonic + hadronic planes:**

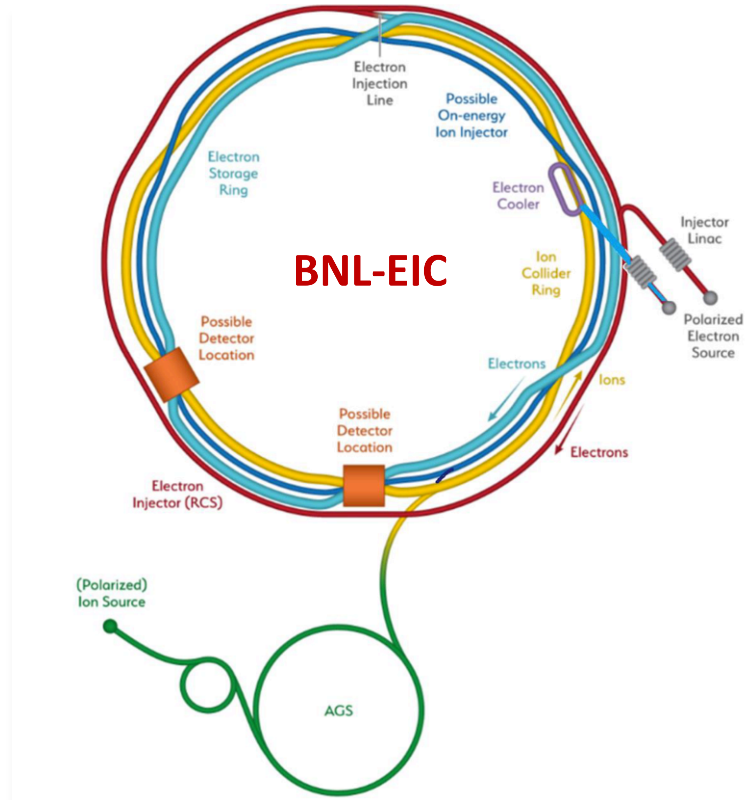


$$A_{UT}^{Collins} \propto \langle \sin(\phi_h + \phi_S) \rangle_{UT} \propto h_1 \otimes H_1^{\perp}$$

$$A_{UT}^{Sivers} \propto \langle \sin(\phi_h - \phi_S) \rangle_{UT} \propto f_{1T}^{\perp} \otimes D_1$$

$$A_{UT}^{Pretzelosity} \propto \langle \sin(3\phi_h - \phi_S) \rangle_{UT} \propto h_{1T}^{\perp} \otimes H_1^{\perp}$$





QED Radiative Corrections

- **Collision of charged particles triggers radiation!**
  - **Inelastic collision breaks the proton:**

    QCD radiation → QCD evolution, high order corrections, resummation, …

  - **QED radiation – emission of photon from lepton and quark, …**

    Well-studied topic – too many references to list here

    See all others talks at this workshop

    If not precisely observed, emission of real photon will

    - **change the inelastic cross section, …**
    - **change the kinematics – the meaning of x_B, Q^2, …**
    - **make the photon-hadron frame ill-defined – crucial for SIDIS, …**
    - **make the angular modulation between leptonic and hadronic planes inaccurate – critical for separating various TMDs, …**

    How big the effect is?

    How precisely we can account for this effect?

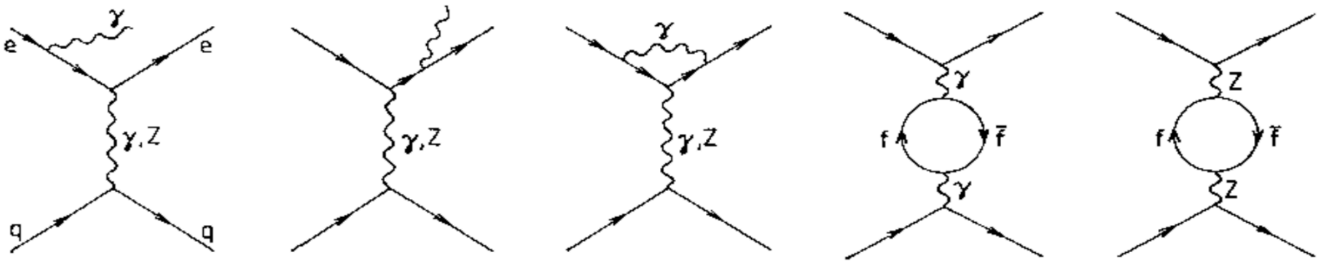
    What could be the impact for the future EIC?, or CEBAF in a near term, …

QED Radiative Corrections

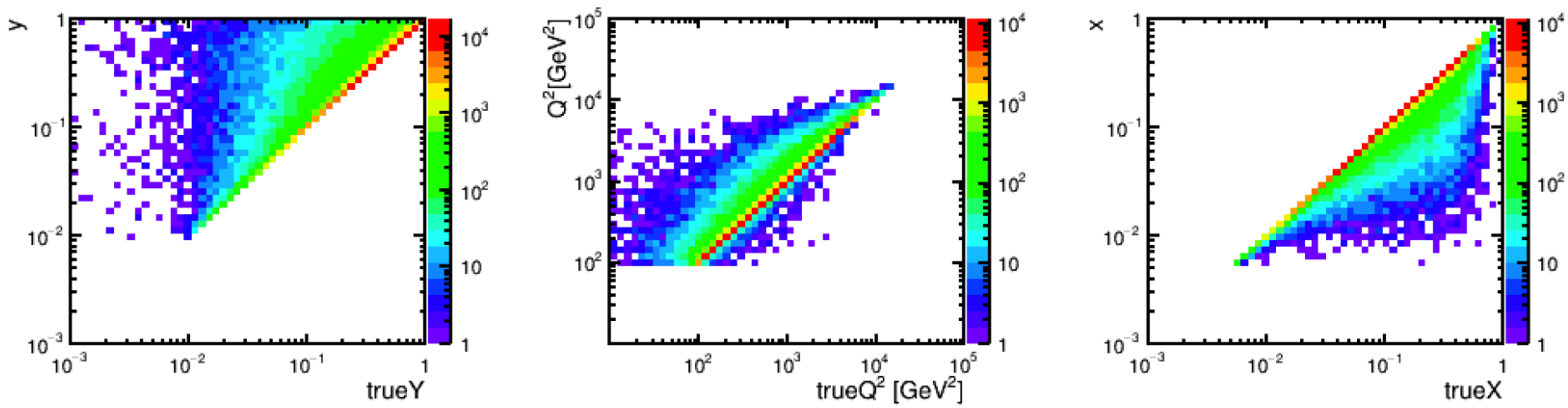
❑ Kinematics is smeared by radiative corrections:

See Xiaoxuan Chu @2nd EIC YR workshop

Data sample : Int L = 10 fb^{-1}, Kinematics settings: 0.01<y<0.95, 10^2 GeV^2 <Q^2<10^5 GeV^2



initial **final** **vacuum** **loops**



Instead of a straight line – linear correlation,

the kinematic variables, y, Q^2 , x_B, from the leptons are smeared so much to make them different from what the scattered "quark" experienced!

Trouble with the "photon-hadron" frame?!

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QED Radiative Corrections

❑ Radiative correction factor is too big to be comfort:

See B. Badelek et al.
Z Phys C 66 (1995) 591

$$\eta(x,y) = \frac{\sigma_{1\gamma}}{\sigma_{\text{meas}}}$$

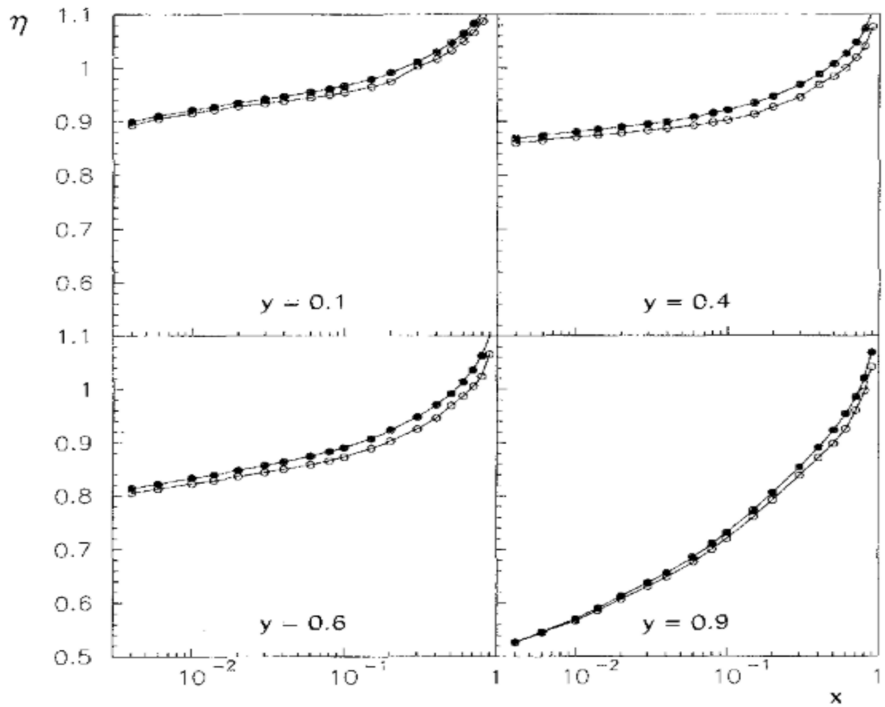

Fig. 5. Radiative correction factor η calculated in FERRAD35 (open symbols) and TERAD86 (closed symbols) for the muon – proton scattering at 280 GeV

Radiative corrections are very large, exceeding 50% at low x and high y region!

Recall: $y = \dfrac{2P\cdot q}{2P\cdot l}$ $\quad x_B = \dfrac{Q^2}{2P\cdot q}$

Larger momentum transfer, larger phase-space for shower, and larger radiative corrections!

Fits to EIC kinematics?!

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QED Radiative Corrections

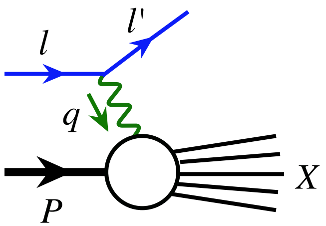
❑ Radiative corrections can cause trouble, …

Can mimic new/unexpected physics, if not handled correctly, *e.g.*

- **"HERMES effect", with apparent enhancement of nuclear $R=\sigma_L/\sigma_T$ ratio at $x_B < 0.03$ and $Q^2 < 2$ GeV^2**
  - **Original paper: Ackerstaff et al., PLB 475, 386 (2000)**
  - **Erratum: Airapetian et al., PLB 567, 339 (2003)**
  - **Interesting physics interpretations of original data [*e.g.*, Miller, Brodsky, Karliner, PLB 481, 245 (2000)]**
- **Nuclear EMC effect, with discrepancy between early EMC and BCDMS data at low x_B (enhancement rather than "shadowing")**
  - **Coulomb corrections have not always been consistently applied [*e.g*. Solvignon, Gaskell, Arrington, AIP Conf.Proc. 1160, 155 (2009)]**
- **…**

Questions

❑ Radiative corrections – Born kinematics:



$$\sigma_{\text{Measured}} = RC \otimes \sigma_{\text{No QED Radiation}}$$

- Uniquely determined "q" – a clean and controllable EM probe
- A well-defined hadronic tensor – DIS Structure Functions

$$W_{\mu\nu}(q,P) = \frac{1}{4\pi}\sum_X (2\pi)^4 \delta^4(P+q-X)\,\langle P|J_\mu(0)|X\rangle\,\langle X|J_\nu(0)|P\rangle + \text{spin}\ldots$$
$$= -\widetilde{g}_{\mu\nu} F_1(x_B,Q^2) + \frac{\widetilde{P}_\mu \widetilde{P}_\nu}{P\cdot q} F_2(x_B,Q^2) + \text{spin}\ldots$$

- OPE works – QCD factorization to all powers (or twists)

$$F_i(x_B,Q^2) = \sum_f C_i(x_B,x;Q^2,\mu^2)\otimes f(x,\mu^2) + \mathcal{O}(1/Q^2)$$

But,

- ○ Traditional approach of Mo and Tsai:
  - depends on unphysical parameter separating soft and hard regions of the phase space of radiated photon in order to cancel infrared divergences
  - is not easily transferrable to application to other processes, *e.g*. SIDIS
- ○ Differences between different schemes (*e.g*., Mo-Tsai and Bardin-Shumeiko) can be as large or larger than some systematic errors in the data analysis [Badelek et al., ZPC 66 (1995) 591]

Is the Born kinematics necessary for extracting PDFs?

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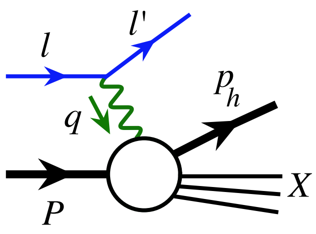
Questions

❑ Radiative corrections – Born kinematics:

$$\sigma_{\text{Measured}} = RC \otimes \sigma_{\text{No QED Radiation}}$$



- The "photon-hadron" frame is critical for SIDIS
- QCD factorization is proved to be valid for all P_T, as long as Q^2 is sufficiently large

✧ **Low P_{hT} (P_{hT} << Q) – TMD factorization:**

$$\sigma_{\text{SIDIS}}(Q, P_{h\perp}, x_B, z_h) = \hat{H}(Q) \otimes \Phi_f(x, k_\perp) \otimes \mathcal{D}_{f\to h}(z, p_\perp) \otimes \mathcal{S}(k_{s\perp}) + \mathcal{O}\left[\frac{P_{h\perp}}{Q}\right]$$

✧ **High P_{hT} (P_{hT} ~ Q) – Collinear factorization:**

$$\sigma_{\text{SIDIS}}(Q, P_{h\perp}, x_B, z_h) = \hat{H}(Q, P_{h\perp}, \alpha_s) \otimes \phi_f \otimes D_{f\to h} + \mathcal{O}\left(\frac{1}{P_{h\perp}}, \frac{1}{Q}\right)$$

✧ **P_{hT} Integrated - Collinear factorization:**

$$\sigma_{\text{SIDIS}}(Q, x_B, z_h) = \tilde{H}(Q, \alpha_s) \otimes \phi_f \otimes D_{f\to h} + \mathcal{O}\left(\frac{1}{Q}\right)$$

✧ **Very high P_{hT} >> Q – Collinear factorization:**

$$\sigma_{\text{SIDIS}}(Q, P_{h\perp}, x_B, z_h) = \sum_{abc} \hat{H}_{ab\to c} \otimes \phi_{\gamma\to a} \otimes \phi_b \otimes D_{c\to h} + \mathcal{O}\left(\frac{1}{Q}, \frac{Q}{P_{h\perp}}\right)$$

But, Radiative corrections can change the "direction" of the "q" and make the "real" Q^2 to be small

Is the Breit frame experimentally attainable?
Yes, but with errors, so precision QCD studies depends on how well such error is controlled e.g. SIDIS pT(Breit) spectrum?

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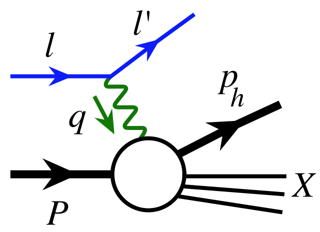
Questions

❑ Radiative corrections – Born kinematics:



$$\sigma_{\text{Measured}} = RC \otimes \sigma_{\text{No QED Radiation}}$$

- **The "photon-hadron" frame is critical for SIDIS**
- **QCD factorization is proved to be valid for all P_T, as long as Q^2 is sufficiently large**

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But, **Radiative corrections can change the "direction" of the "q" and make the "real" Q^2 to be small**

Can we achieve the factorization for extracting the same distributions without requiring the Born kinematics?

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Basic ideas for our new approach

- **Do not try to invent any new scheme to treat QED radiation to match to the Born kinematics – NO Radiative Correction!**

- **Develop a reliable formalism that can extract the PDFs, TMDs and parton correlation functions, systematically with controllable and consistent approximations**

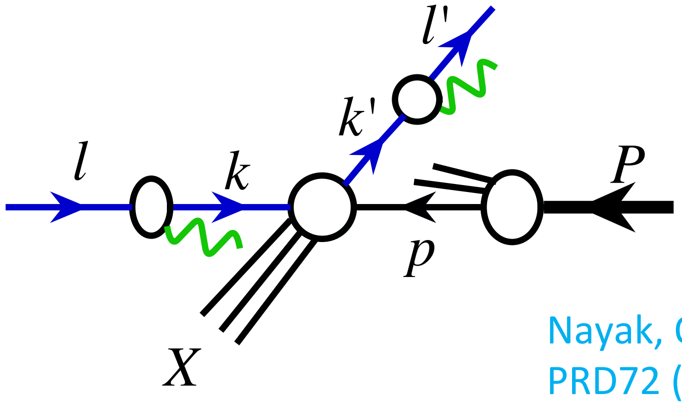
- **Generalize the QCD factorization to include Electroweak theory**
  - **QED radiation is a part of the production cross sections**
  - **QED radiation is treated in the same way as QCD radiation is treated**

Note: our new approach is more relevant for high-energy process where a large GeV scale exists.

Inclusive Deep Inelastic Scattering

- **Inclusive DIS**
  = Inclusive production of a high transverse momentum lepton in lepton-hadron collisions

$$e(l) + h(P) \to e'(l') + X$$



Nayak, Qiu, Sterman
PRD72 (2005) 114012

- **Factorization proof:**
  = Factorization proof of single hadron production hadronic collisions

$$E'\frac{d\sigma_{eh\to e'X}}{d^3l'} \approx \frac{1}{2s}\sum_{i,j,a}\int_{z_L}^{1}\frac{d\zeta}{\zeta^2}\int_{x_L}^{1}\frac{d\xi}{\xi}D_{e'/j}(\zeta)f_{i/e}(\xi)\int_{x_h}^{1}\frac{dx}{x}f_{a/h}(x)\,\hat{H}^{(m,n)}_{ia\to j}(\xi,\zeta,x;k')$$

$m:$ QED power

$n:$ QCD power

$$s = (P+l)^2 \approx 2P\cdot l$$

i, j, a include all QED and light flavor QCD particles

In the following discussion, we take valence approximation: $i = j = e$

$f_{i/e}(\xi), D_{e/j}(\zeta)$ Lepton PDFs and FFs include all collinear sensitivities as $m_e \to 0$

$\hat{H}_{ia\to j}$ Infrared safe, insensitive to $m_e \to 0$ $m_q \to 0$

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Inclusive Deep Inelastic Scattering

❑ Hard scale:

$$l'_T \gg \Lambda_{\rm QCD}$$

Physically measured, unlike the traditional Q^2, which could be very "small" due to photon radiation

❑ Lepton distribution:

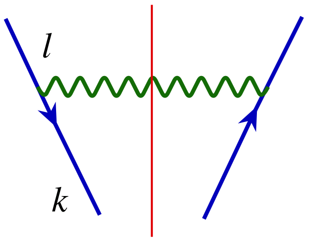
QED gauge link

$$f_{e/l}(\xi) = \int \frac{dy^-}{4\pi} e^{i\xi\, l^+ y^-} \langle l|\overline{\psi}(0)\gamma^+\Phi(0,y^-)\psi(y^-)|l\rangle$$

Obey DGLAP evolution with QED kernels

$$f^{(0)}_{e/l}(\xi) = \delta(\xi-1)\delta_{el}$$

$$f^{(1)}_{e/l}(\xi,\mu^2) = \frac{\alpha_{\rm em}}{2\pi}\left[\frac{1+\xi^2}{1-\xi}\ln\left(\frac{\mu^2}{(1-\xi)m_e^2}\right)\right]_+$$



❑ Lepton fragmentation function:

$$D_{l/e}(\zeta) = \int \frac{dy^-}{4\pi} e^{il^+y^-/\zeta}\,\frac{\zeta}{2}{\rm Tr}\left[\gamma^+\langle 0|\overline{\psi}(0)\Phi(0,\infty)|l,X\rangle\langle\psi(y^-)\Phi(y^-,\infty)|0\rangle\right]$$

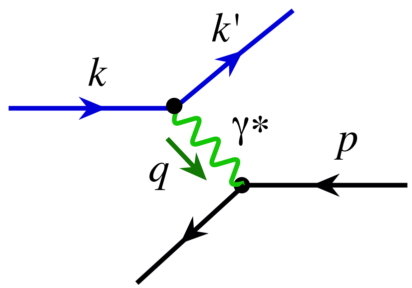
Obey DGLAP evolution with QED kernels

$$D^{(0)}_{l/e}(\zeta) = \delta(\zeta-1)\delta_{el}$$

$$D^{(1)}_{l/e}(\zeta) = \frac{\alpha_{\rm em}}{2\pi}\left[\frac{1+\zeta^2}{1-\zeta}\ln\left(\frac{\mu^2}{(1-\zeta)m_e^2+\Delta E^2}\right)\right]_+$$

Inclusive Deep Inelastic Scattering

❑ LO perturbatively calculable hard part:



$$\hat{H}^{(2,0)}_{eq\to e} = e_q^2 (4\alpha_{\rm EM}^2) \left[\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\right] \delta(\hat{s}+\hat{t}+\hat{u})$$

$$\hat{s} = x\,\xi\,s \qquad Q^2 = x_B\,y\,s$$

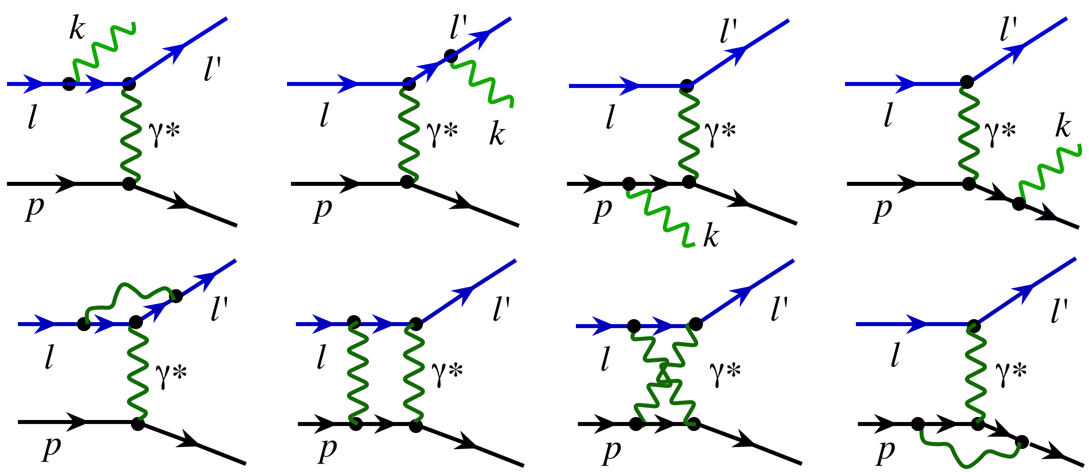
$$\hat{t} = (\xi/\zeta)t \qquad t = (l-l')^2 \equiv -Q^2$$

$$\hat{u} = (x/\zeta)u \qquad u = (P-l')^2 \equiv -(1-y)s$$

❑ NLO calculable hard part from QED:

Real:

Virtual:



Into QED Correction to QCD DGLAP kernels

$$\hat{H}^{(3,0)}_{eq\to e'} = \sigma^{(3,0)}_{eq\to e'} - D^{(1)}_{e'/e'} \otimes \hat{H}^{(2,0)}_{eq\to e'} - f^{(1)}_{e/e} \otimes \hat{H}^{(2,0)}_{eq\to e'} - f^{(1)}_{q/q} \otimes \hat{H}^{(2,0)}_{eq\to e'}$$

Completely IR safe, no dependence on lepton mass

Inclusive Deep Inelastic Scattering

❑ LO Factorized inclusive DIS cross section:

$$E' \frac{d\sigma^{(0)}_{eh\to e'X}}{d^3 l'} \approx \frac{2\alpha^2_{\rm EM}}{s} \sum_q \int_{z_L}^1 \frac{d\zeta}{\zeta^2} \int_{x_L}^1 \frac{d\xi}{\xi} D_{e/e}(\zeta) f_{e/e}(\xi) \int_{x_h}^1 \frac{dx}{x} e_q^2 \, f_{q/h}(x)\, \delta\!\left(x - \frac{-\xi t}{\xi\zeta s + u}\right) \times \left[\frac{(x\xi\zeta s)^2 + (xu)^2}{(\xi t)^2}\right] \left[\frac{\zeta}{(\xi\zeta s) + u}\right]$$

LO in hard part, but include all orders resummation into PDFs, FFs

$$\Rightarrow \frac{4\alpha^2_{\rm em}}{Q^2 s}\left[F_1(x_B, Q^2) + \frac{1-y}{x_B y^2} F_2(x_B, Q^2)\right]$$

With LO relation: $F_2(x_B) = 2x_B F_1(x_B) = \sum_q e_q^2\, x_B f_{q/h}(x_B)$

$$f_{e/e}(\xi) \approx f^{(0)}_{e/e}(\xi)$$

$$\xi = \zeta = 1$$

$$D_{e/e}(\zeta) \approx D^{(0)}_{e/e}(\zeta)$$

❑ NLO fixed order QED correction:

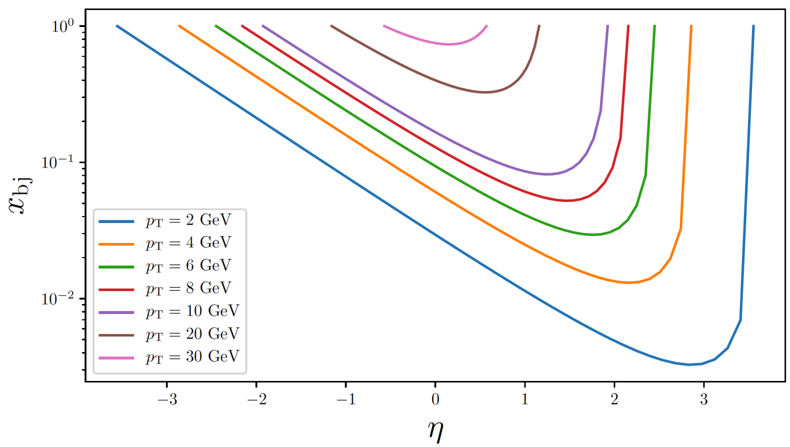
By taking: $f_{e/e}(\xi) \approx f^{(0)}_{e/e}(\xi) + f^{(1)}_{e/e}(\xi)$

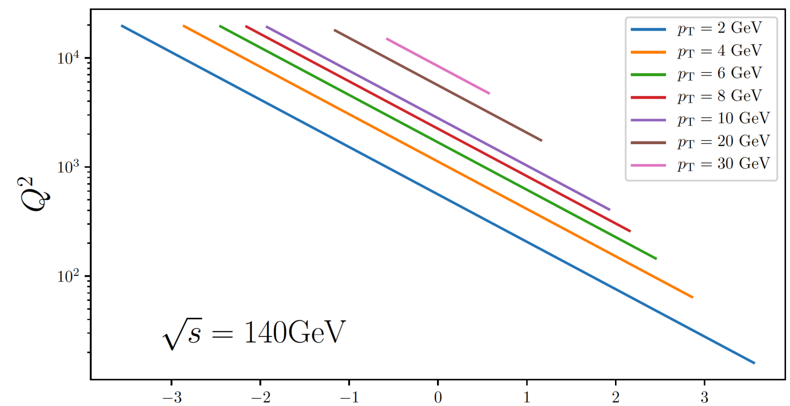
or: $D_{e/e}(\zeta) \approx D^{(0)}_{e/e}(\zeta) + D^{(1)}_{e/e}(\zeta)$

or: $\hat{H}^{(m,n)} \approx \hat{H}^{(2,0)} + \hat{H}^{(3,0)}$

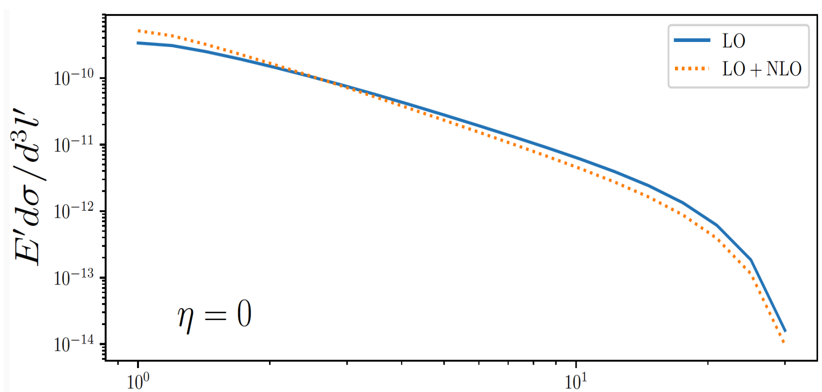
Inclusive Deep Inelastic Scattering

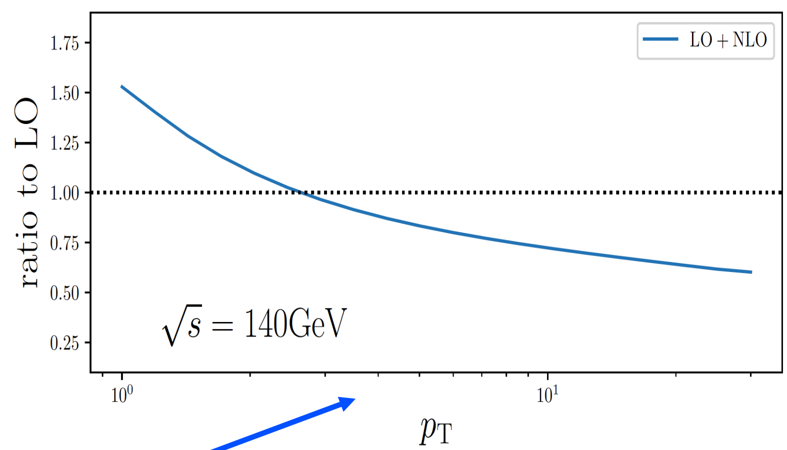
❑ Size of NLO fixed order QED corrections:





Preliminary





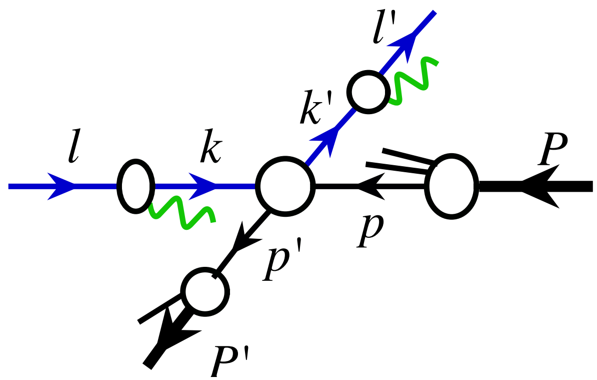
Factor of radiative corrections

Semi-inclusive DIS

❑ No photon-hadron frame!

Semi-inclusive DIS cross section

= inclusive electron + hadron (or jet) cross section in lepton-hadron collisions



Large hard scale: $P_T = \frac{1}{2}|\vec{l}'_T - \vec{P}'_T|$

"Soft" scale: $\Delta P_T = |\vec{l}'_T + \vec{P}'_T| \ll |l'_T| \sim |P'_T|$

Momentum imbalance between two particles (or jet(s))

❑ TMD factorization formula:

$$\frac{d\sigma}{dy\, dP_T^2\; d\Delta y\, d\Delta P_T^2} = \sum_{i,j,a,b} \widetilde{f}_{i/e} \otimes \widetilde{D}_{e/j} \otimes \widetilde{f}_{a/h} \otimes \widetilde{D}_{h'/b} \otimes \hat{H}^{(m,n)}_{ij\to ab}$$

TMDs

Only two hadrons are involved, the color flow is unique. NO factorization breaking effect identified by Collins & Qiu (2007) and Mulders & Rogers (2010)

Summary and outlook

- **Radiative corrections are very important for lepton-hadron scattering**
  - **Especially difficult for a consistent treatment beyond the inclusive DIS**
  - **No well-defined photon-hadron frame, if we cannot recover all QED radiation**
  - **Radiative corrections are more important for events with high momentum transfers and large phase space to shower – such as those at the EIC**
- **We proposed a factorization based treatment of QED radiation**
  - **QED radiation is a part of production cross sections, treated in the same way as radiation from quarks and gluons**
  - **No artificial and/or process dependent scale(s) introduced other than the standard factorization scale**
  - **All perturbatively calculable hard parts are IR safe for both QCD and QED**
  - **All lepton mass or resolution sensitivity are included into "Universal" lepton distribution and fragmentation functions (or jet functions)**

Thank you!

Special thanks to JLab experimental colleagues for helpful discussions!

Jefferson Lab