

Factorized Approach to Radiative Corrections for Inelastic Lepton-Hadron Collisions

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arXiv:2008.02895, in collaboration with T. Liu, W. Melnitchouk and N. Sato

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TMD Collaboration

U.S. DEPARTMENT OF ENERGY | Office of Science

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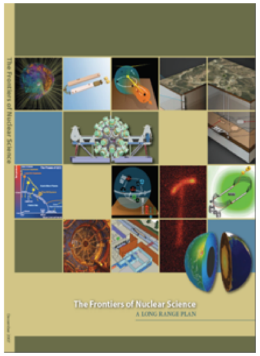
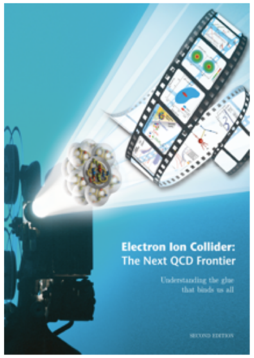
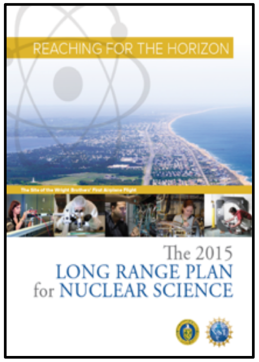
U.S. - based Electron-Ion Collider

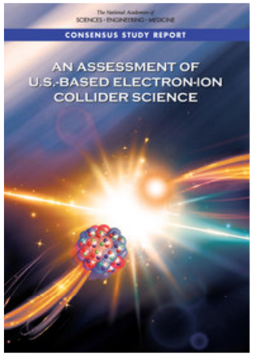
- A long journey, a joint effort of the full community:

…









…

"… answer science questions that are compelling, fundamental, and timely, and help maintain U.S. scientific leadership in nuclear physics."

… three profound questions:

How does the mass of the nucleon arise?
How does the spin of the nucleon arise?
What are the emergent properties of dense systems of gluons?

- On January 9, 2020:

The U.S. DOE announced the selection of BNL as the site for the Electron-Ion Collider

A new era to explore the emergent phenomena of QCD!

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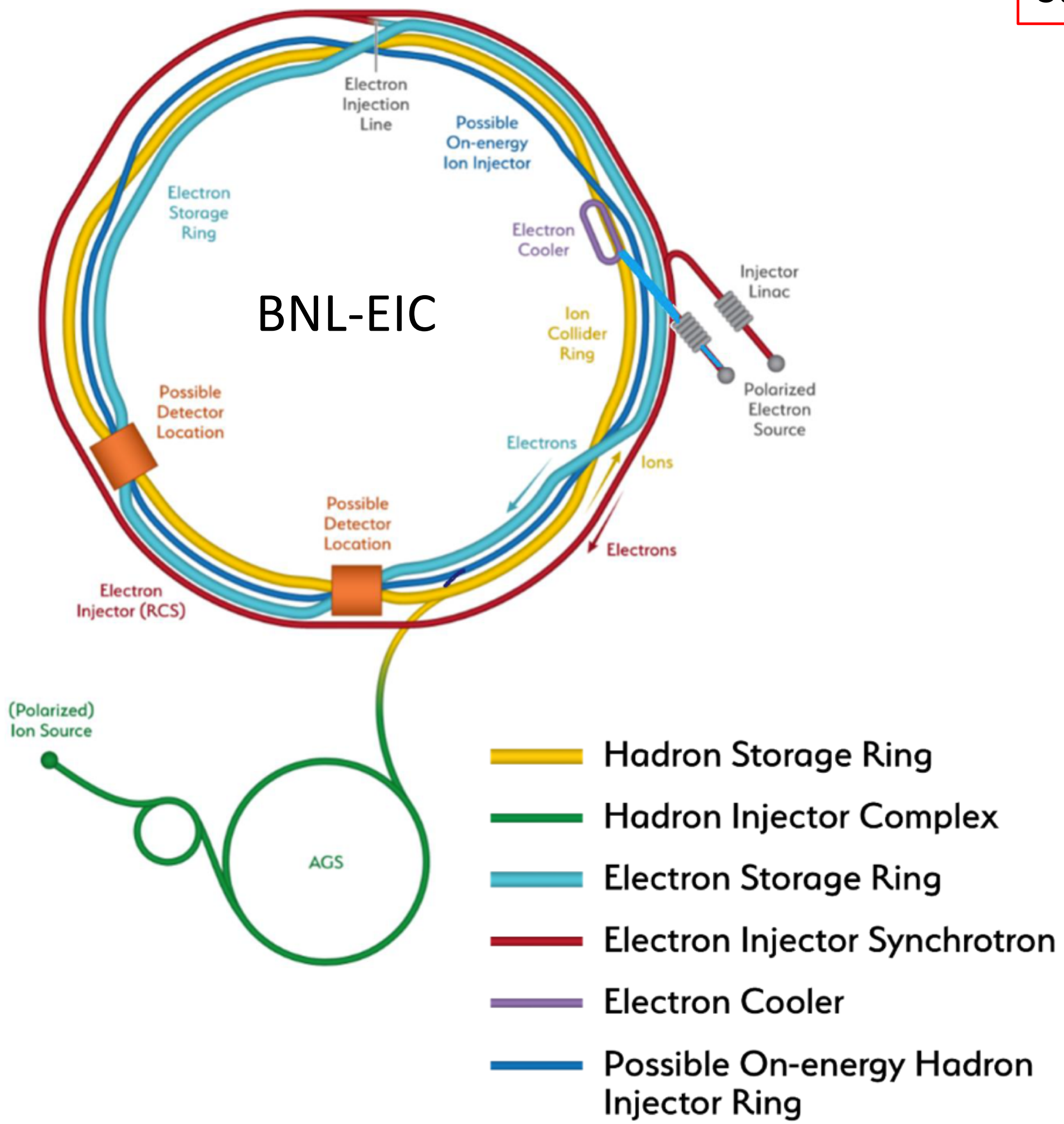
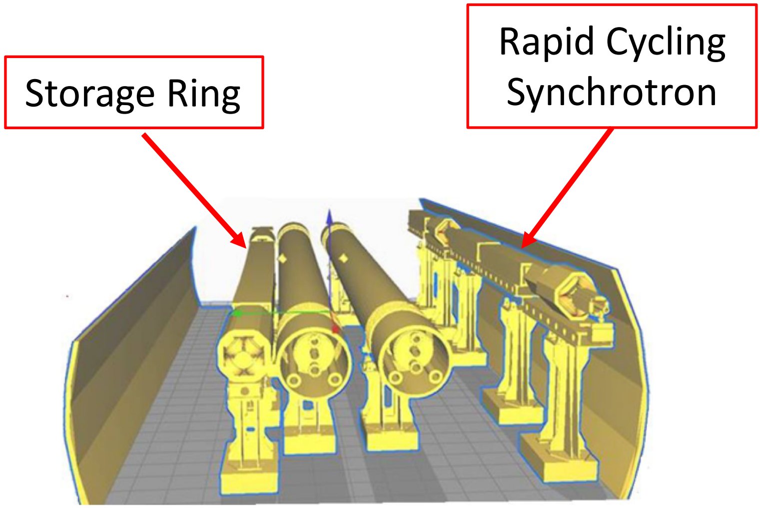
U.S. - based Electron-Ion Collider

- **The winning design - BNL:**

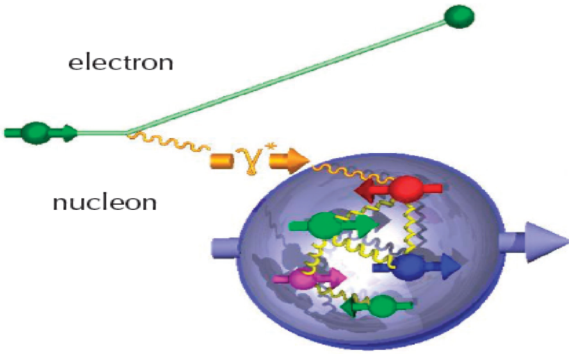




- **Center of Mass Energies:**
  20 GeV – 141 GeV
- **Required Luminosity:**
  10^{33} - 10^{34} $cm^{-2}s^{-1}$
- **Hadron Beam Polarization:**
  80%
- **Electron Beam Polarization:**
  80%
- **Ion Species Range:**
  p to Uranium
- **Number of interaction regions:**
  up to 2

Many Complementary Probes at One Facility

- **A new generation of the "Rutherford" experiment:**



- **A controlled "probe" – virtual photon**
- **Can either break or not break the hadron**

One facility covers all!

- **Inclusive events:** e+p/A → e'+X

  Detect only the scattered lepton in the detector

  (Modern Rutherford experiment!)

- **Semi-Inclusive events:** e+p/A → e'+h(p,K,p,jet)+X

  Detect the scattered lepton in coincidence with identified hadrons/jets

  (Initial hadron is broken – confined motion! – cleaner than h-h collisions)

- **Exclusive events:** e+p/A → e'+ p'/A'+ h(p,K,p,jet)

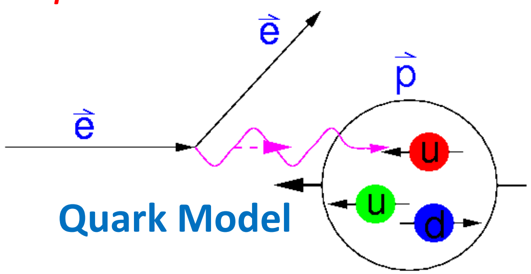
  Detect every things including scattered proton/nucleus (or its fragments)

  (Initial hadron is NOT broken – tomography! – almost impossible for h-h collisions)

Inelastic Lepton-Hadron Scattering

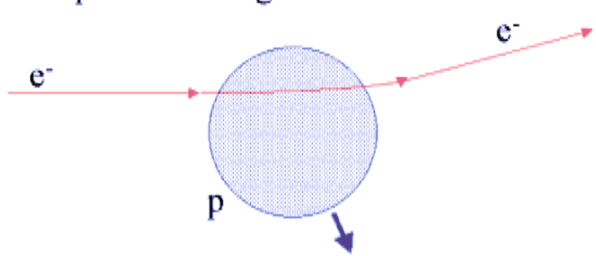
- A modern "Rutherford" experiment (over 50 years ago):

$e + p \rightarrow e + X$ **1968**

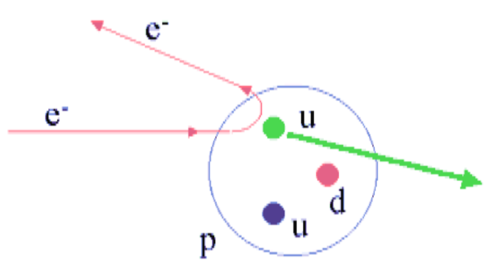


Prediction:

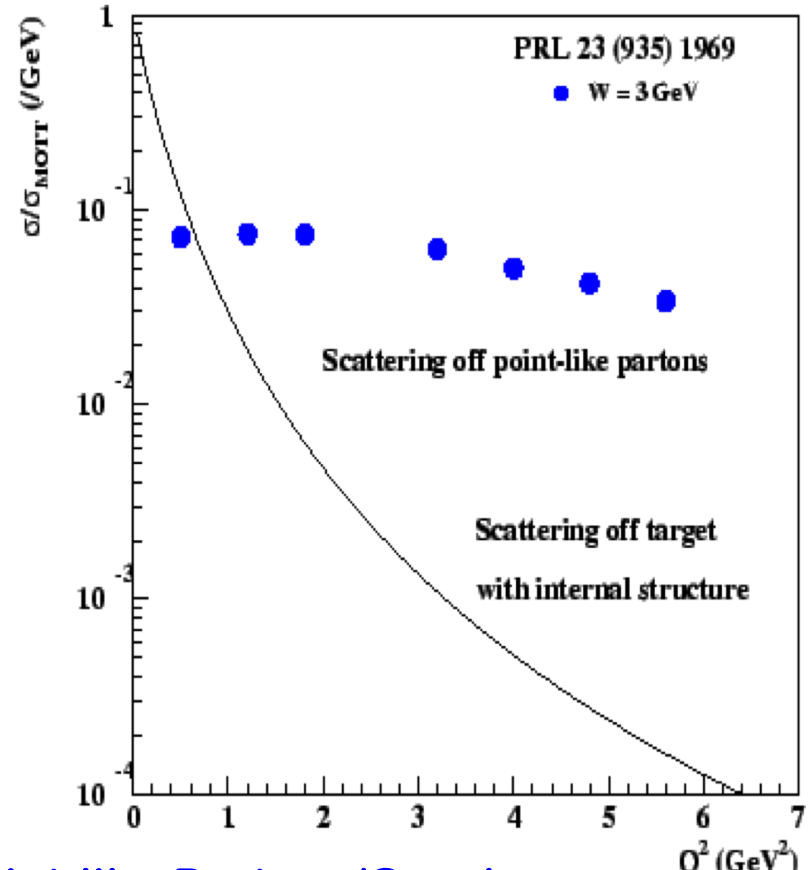
- If proton "charge cloud":



- If proton contains point charges, some of time see:



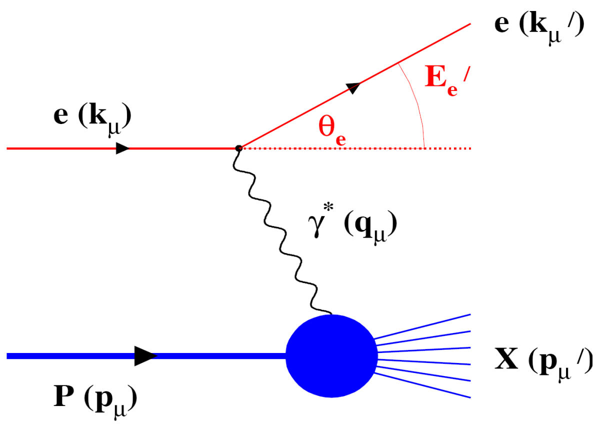
Discovery:



- Point-like Partons/Quarks
- Probability Distributions

Inelastic Lepton-Hadron Scattering

❑ Approximation of one-photon exchange:



$Q^2 = -(k-k')^2$ → Measure of the resolution

$y = P.(k-k')/P.k$ → Measure of inelasticity

$x_B = Q^2/2P.(k-k')$

→ Measure of momentum fraction of the struck quark in a proton

$Q^2 = S\, x_B\, y$

$$E' \frac{d\sigma}{d^3 l'} = \frac{\alpha_{\rm EM}^2}{2\pi s} \int d^4 q \sum_X \left| \langle k' | j_\mu | k \rangle \frac{1}{q^2} \langle X | J^\mu | P \rangle \right|^2 (2\pi)^4 \delta^4(P + q - X)\, \delta^4(q - k + k')$$

$$= \frac{2\alpha_{\rm EM}^2}{Q^4 s} L^\mu(k, k'; q)\, W_{\mu\nu}(q, P)$$

❑ Deep inelastic scattering (DIS) structure functions:

$$W_{\mu\nu}(q, P) = \frac{1}{4\pi} \sum_X (2\pi)^4 \delta^4(P + q - X)\, \langle P | J_\mu(0) | X \rangle\, \langle X | J_\nu(0) | P \rangle + \text{spin...}$$

$$= -\widetilde{g}_{\mu\nu} F_1(x_B, Q^2) + \frac{\widetilde{P}_\mu \widetilde{P}_\nu}{P \cdot q} F_2(x_B, Q^2) + \text{spin...}$$

$$\widetilde{g}_{\mu\nu} = -g_{\mu\nu} + q_\mu q_\nu / q^2 \qquad \widetilde{P}_\mu = \widetilde{g}_{\mu\nu} P^\nu$$

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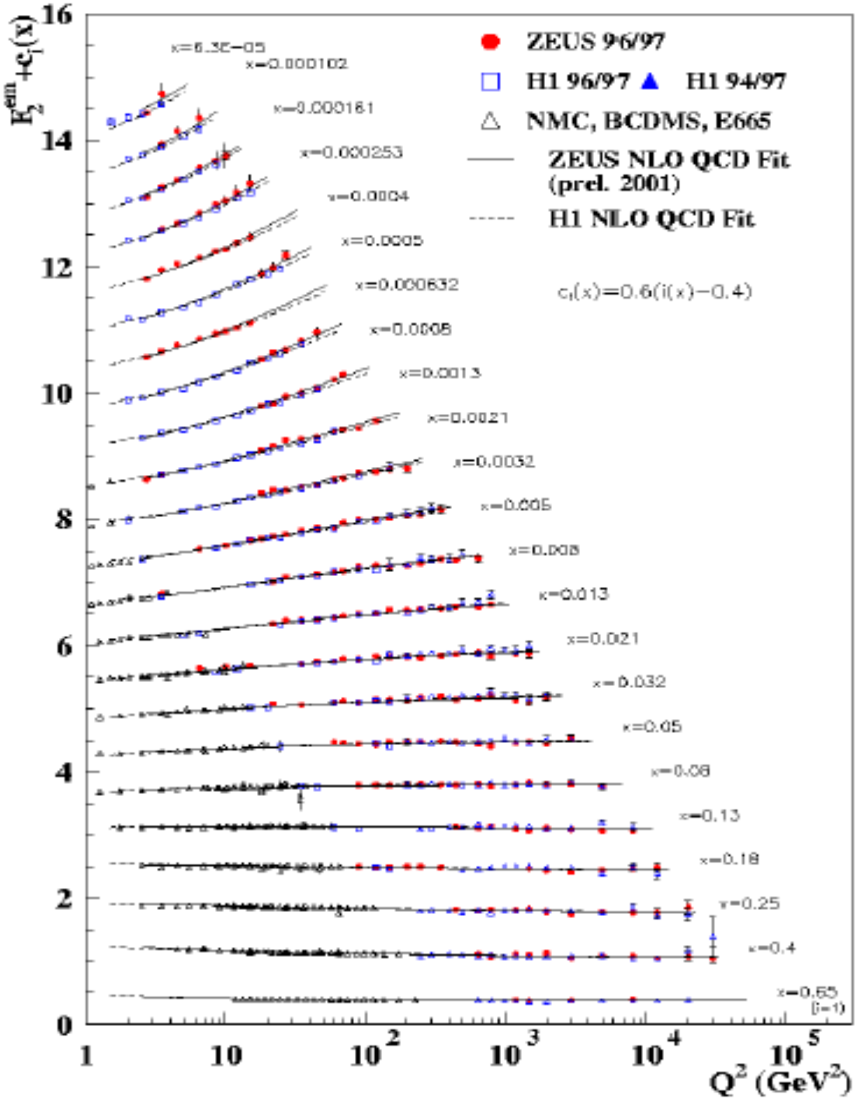
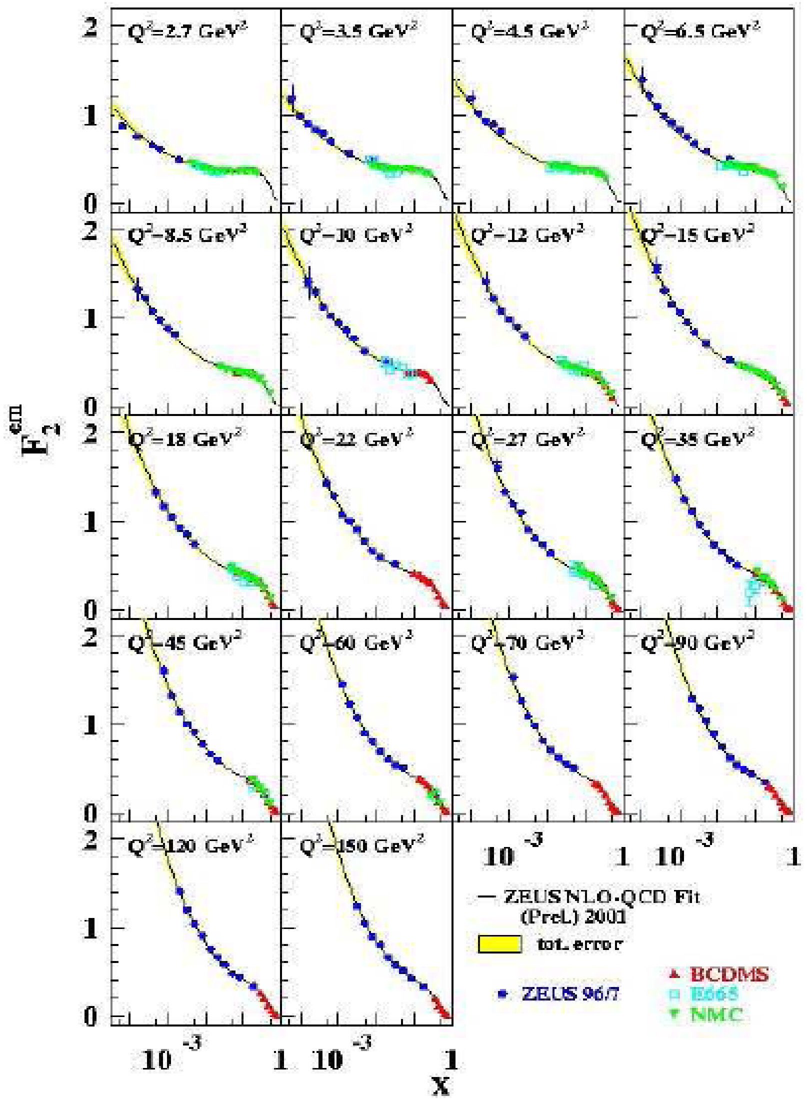
Inelastic Lepton-Hadron Scattering





A very successful story of QCD, QCD Factorization, and QCD evolution!

Extraction of Parton Distribution Functions (PDFs) – hadron structure

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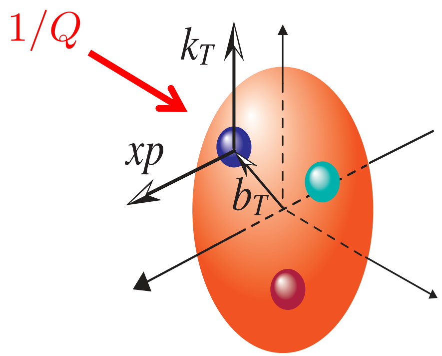
New-Type Probes for 3D Hadron Structure

❑ Single scale hard probes is too "localized":



- It pins down the particle nature of quarks and gluons
- But, not very sensitive to the detailed structure of hadron ~ fm
- Transverse confined motion: k_T ~ 1/fm << Q
- Transverse spatial position: b_T ~ fm >> 1/Q

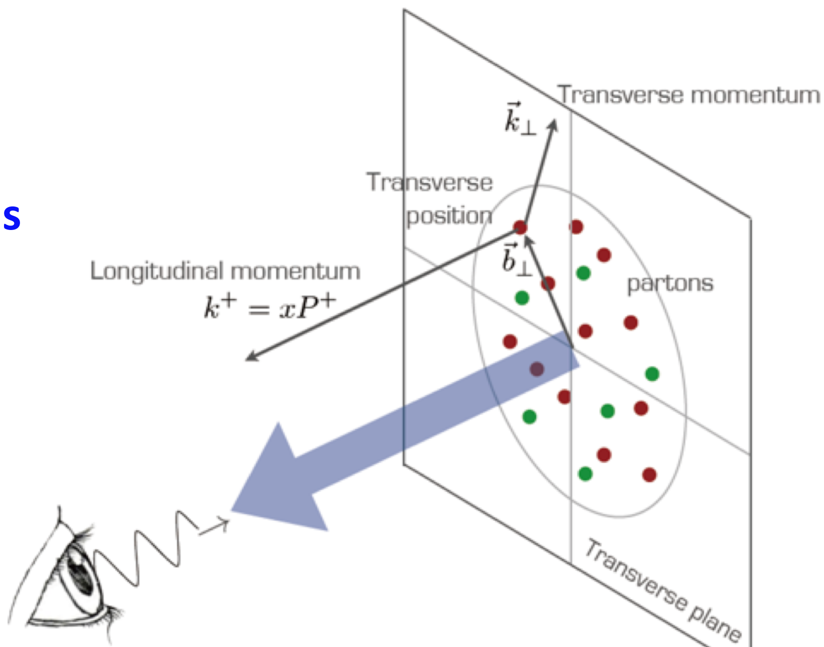
❑ Need new type of "Hard Probes" – Physical observables with TWO Scales:

$$Q_1 \gg Q_2 \sim 1/R \sim \Lambda_{\rm QCD}$$

Hard scale: Q_1 To localize the probe
particle nature of quarks/gluons

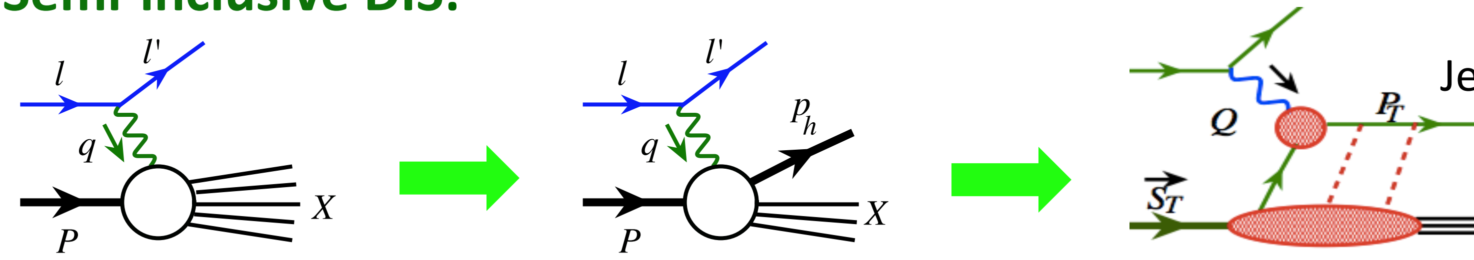
"Soft" scale: Q_2 could be more sensitive to the
hadron structure ~ 1/fm

**Hit the hadron "very hard" without breaking it,
clean information on the structure!**



Inelastic Lepton-Hadron Scattering

❑ Semi-inclusive DIS:

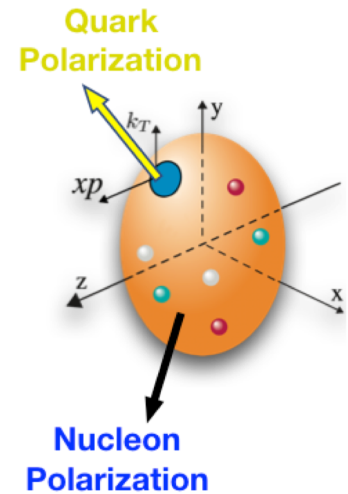


Jet, π, J/ψ, …

Scale: Q^2 $\qquad Q^2 \gg P_{h_T}^2$ $\qquad f(x, k_T, Q)$ **- TMDs**

In photon-hadron frame! **Parton's confined motion, …**

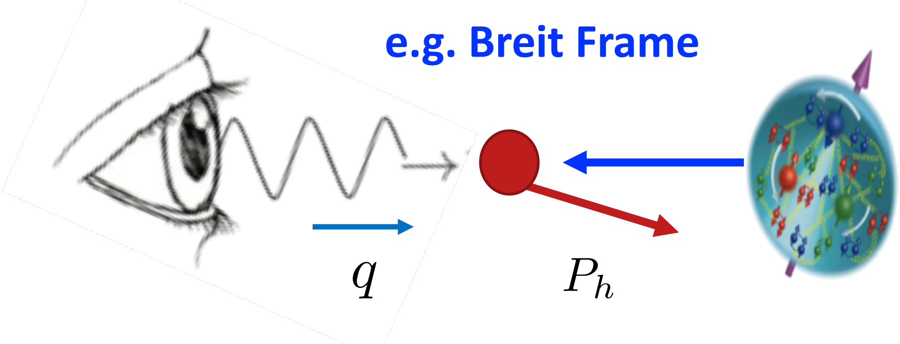
		Quark Polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1(x,k_T^2)$		$h_1^\perp(x,k_T^2)$ - Boer-Mulders
	L		$g_1(x,k_T^2)$ - Helicity	$h_{1L}^\perp(x,k_T^2)$ - Long-Transversity
	T	$f_{1T}^\perp(x,k_T^2)$ - Sivers	$g_{1T}(x,k_T^2)$ - Trans-Helicity	$h_1(x,k_T^2)$ - Transversity $h_{1T}^\perp(x,k_T^2)$ - Pretzelosity

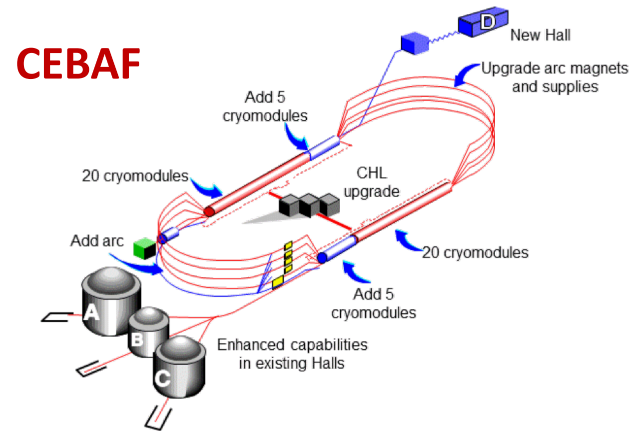


- Gluon: $f_q \to f_g$
- FFs
- Nuclei: $s \neq \frac{1}{2}$

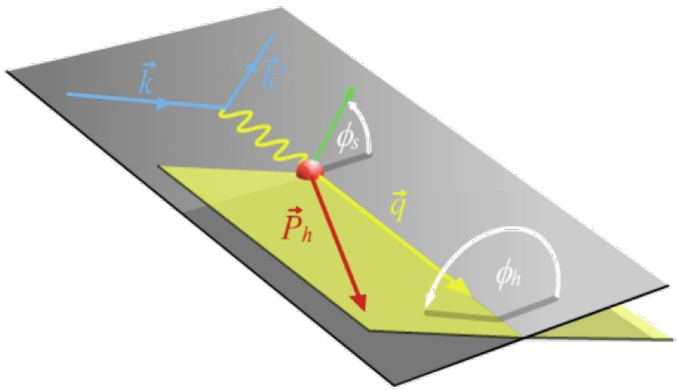
Inelastic Lepton-Hadron Scattering

❑ Photon-hadron frame:





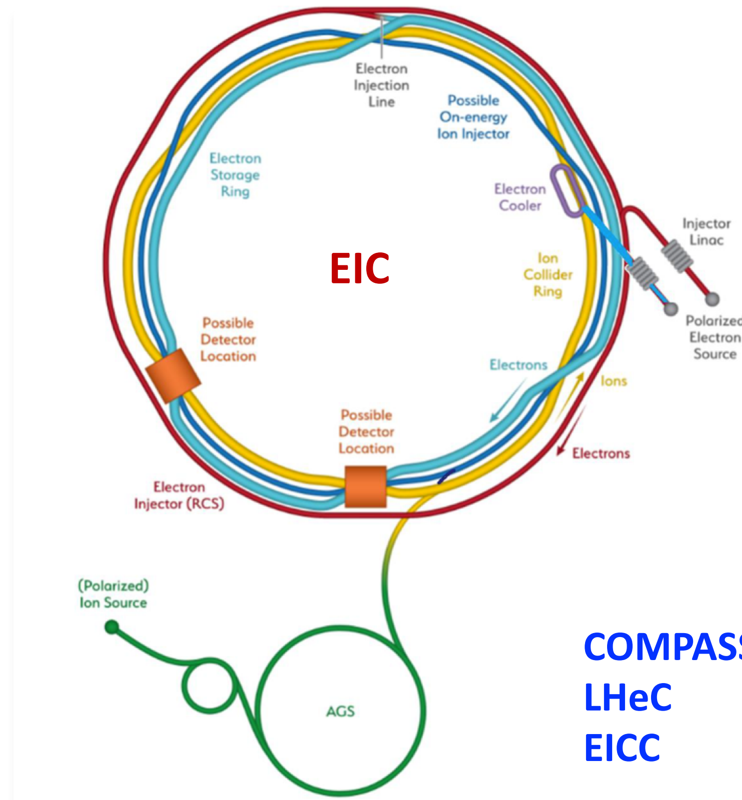
❑ Leptonic + hadronic planes:



$$A_{UT}^{Collins} \propto \langle \sin(\phi_h + \phi_S) \rangle_{UT} \propto h_1 \otimes H_1^{\perp}$$

$$A_{UT}^{Sivers} \propto \langle \sin(\phi_h - \phi_S) \rangle_{UT} \propto f_{1T}^{\perp} \otimes D_1$$

$$A_{UT}^{Pretzelosity} \propto \langle \sin(3\phi_h - \phi_S) \rangle_{UT} \propto h_{1T}^{\perp} \otimes H_1^{\perp}$$



COMPASS
LHeC
EICC

QED Radiative Corrections

- **Collision of charged particles triggers radiation!**
  - **Inelastic collision breaks the proton:**

    QCD radiation → **QCD evolution, high order corrections, resummation, …**

  - **QED radiation – emission of photon from lepton and quark, …**

    Well-studied topic – too many references to list here

    L.W. Mo and Y.S. Tsai, Rev. Mod. Phys. 41 (1969) 205

    D.Y. Bardin, et al. Z. Phys. C 42 (1989) 679

    If not precisely observed, emission of real photon will

    - **change the inelastic cross section, …**
    - **change the kinematics – the meaning of x_B, Q^2, …**
    - **make the photon-hadron frame ill-defined – crucial for SIDIS, …**
    - **make the angular modulation between leptonic and hadronic planes inaccurate – critical for separating various TMDs, …**

    How big the effect is?

    How precisely we can account for this effect?

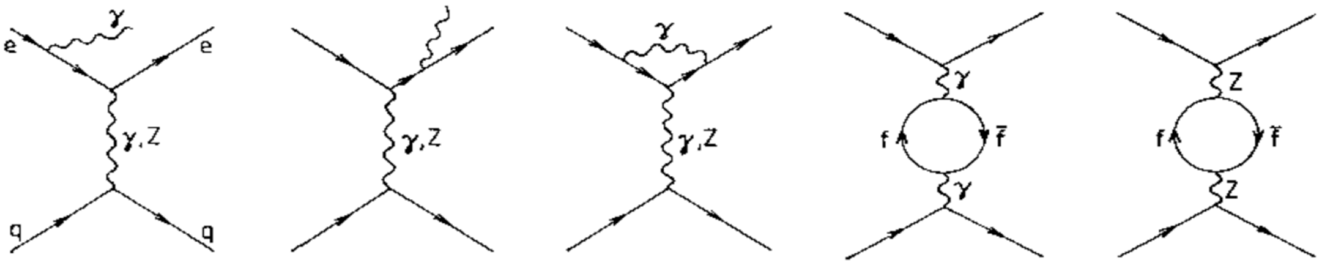
    What could be the impact for the future EIC?, or CEBAF in a near term, …

QED Radiative Corrections

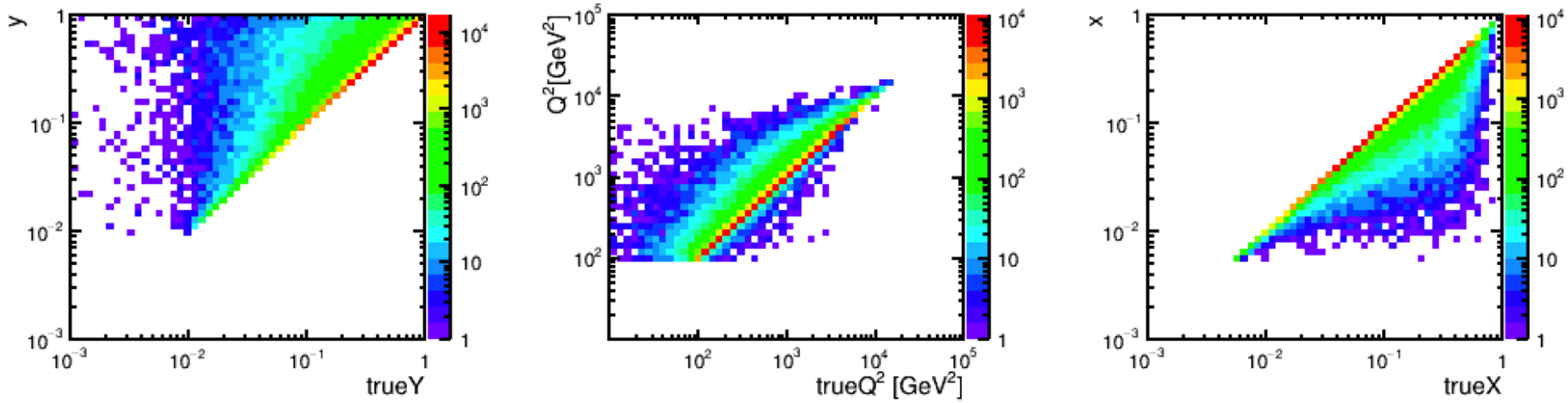
❑ Kinematics is smeared by radiative corrections:

See Xiaoxuan Chu @2nd EIC YR workshop

Data sample : Int L = 10 fb^{-1}, Kinematics settings: $0.01<y<0.95$, 10^2 GeV^2 $<Q^2<10^5$ GeV^2



initial **final** **vacuum** **loops**



Instead of a straight line – linear correlation,

the kinematic variables, y, Q^2 , x_B, from the leptons are smeared so much to make them different from what the scattered "quark" experienced!

Trouble with the "photon-hadron" frame?!

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QED Radiative Corrections

See B. Badelek et al.
Z Phys C 66 (1995) 591

❑ Radiative correction factor is too big to be comfort:

$$\eta(x,y) = \frac{\sigma_{1\gamma}}{\sigma_{\text{meas}}}$$

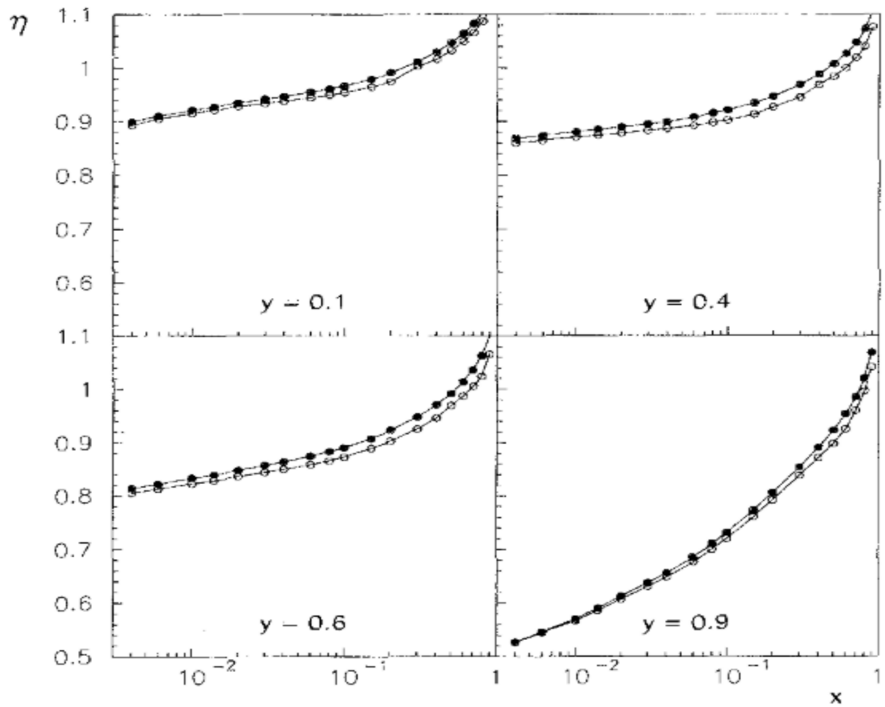

Fig. 5. Radiative correction factor η calculated in FERRAD35 (open symbols) and TERAD86 (closed symbols) for the muon – proton scattering at 280 GeV

Radiative corrections are very large, exceeding 50% at low x and high y region!

Recall: $y = \dfrac{2P\cdot q}{2P\cdot l}$ $\quad x_B = \dfrac{Q^2}{2P\cdot q}$ $\quad Q^2 = x_B\, y\, S$

larger phase-space for shower (smaller x_B), Larger momentum transfer (larger y) larger radiative corrections!

Fits to EIC kinematics?!

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QED Radiative Corrections

❑ Radiative corrections can cause trouble, …

Can mimic new/unexpected physics, if not handled correctly, *e.g.*

- **"HERMES effect", with apparent enhancement of nuclear $R=\sigma_L/\sigma_T$ ratio at $x_B < 0.03$ and $Q^2 < 2$ GeV^2**
    - **Original paper: Ackerstaff et al., PLB 475, 386 (2000)**
    - **Erratum: Airapetian et al., PLB 567, 339 (2003)**
    - **Interesting physics interpretations of original data [*e.g.*, Miller, Brodsky, Karliner, PLB 481, 245 (2000)]**
- **Nuclear EMC effect, with discrepancy between early EMC and BCDMS data at low x_B (enhancement rather than "shadowing")**
    - **Coulomb corrections have not always been consistently applied [*e.g.* Solvignon, Gaskell, Arrington, AIP Conf. Proc. 1160, 155 (2009)]**
- **…**

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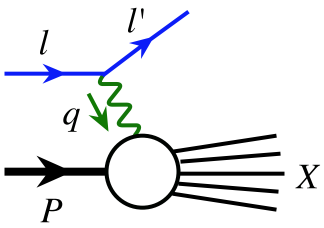
Questions

❑ Radiative corrections – Born kinematics:



$$\sigma_{\text{Measured}} = RC \otimes \sigma_{\text{No QED Radiation}}$$

- **Uniquely determined "q" – a clean and controllable EM probe**
- **A well-defined hadronic tensor – DIS Structure Functions**

$$W_{\mu\nu}(q,P) = \frac{1}{4\pi}\sum_X (2\pi)^4 \delta^4(P+q-X) \langle P|J_\mu(0)|X\rangle \langle X|J_\nu(0)|P\rangle + \text{spin...}$$
$$= -\tilde{g}_{\mu\nu} F_1(x_B,Q^2) + \frac{\tilde{P}_\mu \tilde{P}_\nu}{P\cdot q} F_2(x_B,Q^2) + \text{spin...}$$

- **OPE works – QCD factorization to all powers (or twists)**

$$F_i(x_B,Q^2) = \sum_f C_i(x_B,x;Q^2,\mu^2) \otimes f(x,\mu^2) + \mathcal{O}(1/Q^2)$$

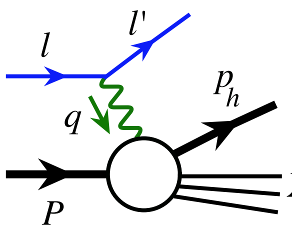
But,

- **Traditional approach of Mo and Tsai:**
  - **depends on unphysical parameter separating soft and hard regions of the phase space of radiated photon in order to cancel infrared divergences**
  - **is not easily transferrable to application to other processes, *e.g*. SIDIS**
- **Differences between different schemes (*e.g.*, Mo-Tsai and Bardin-Shumeiko) can be as large or larger than some systematic errors in the data analysis [Badelek et al., Z. Phys. C66 (1995) 591]**

Is the Born kinematics necessary for extracting PDFs?

Questions

❑ Radiative corrections – Born kinematics:



$$\sigma_{\text{Measured}} = RC \otimes \sigma_{\text{No QED Radiation}}$$

- The "photon-hadron" frame is critical for SIDIS
- QCD factorization is proved to be valid for all P_T, as long as Q^2 is sufficiently large

✧ **Low P_{hT} ($P_{hT} << Q$) – TMD factorization:**

$$\sigma_{\text{SIDIS}}(Q, P_{h\perp}, x_B, z_h) = \hat{H}(Q) \otimes \Phi_f(x, k_\perp) \otimes \mathcal{D}_{f\to h}(z, p_\perp) \otimes \mathcal{S}(k_{s\perp}) + \mathcal{O}\left[\frac{P_{h\perp}}{Q}\right]$$

✧ **High P_{hT} ($P_{hT} \sim Q$) – Collinear factorization:**

$$\sigma_{\text{SIDIS}}(Q, P_{h\perp}, x_B, z_h) = \hat{H}(Q, P_{h\perp}, \alpha_s) \otimes \phi_f \otimes D_{f\to h} + \mathcal{O}\left(\frac{1}{P_{h\perp}}, \frac{1}{Q}\right)$$

✧ **P_{hT} Integrated - Collinear factorization:**

$$\sigma_{\text{SIDIS}}(Q, x_B, z_h) = \tilde{H}(Q, \alpha_s) \otimes \phi_f \otimes D_{f\to h} + \mathcal{O}\left(\frac{1}{Q}\right)$$

✧ **Very high $P_{hT} >> Q$ – Collinear factorization:**

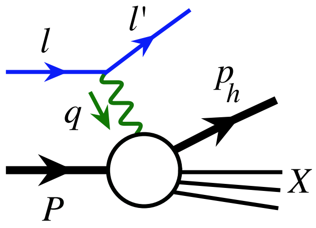
$$\sigma_{\text{SIDIS}}(Q, P_{h\perp}, x_B, z_h) = \sum_{abc} \hat{H}_{ab\to c} \otimes \phi_{\gamma\to a} \otimes \phi_b \otimes D_{c\to h} + \mathcal{O}\left(\frac{1}{Q}, \frac{Q}{P_{h\perp}}\right)$$

But, Radiative corrections can change the "direction" and "value" of the "q" and make the "real" Q^2 to be small (and very small !)

Is the Breit frame experimentally attainable?
Yes, but with errors, so precision QCD studies depends on how well such error is controlled e.g. SIDIS pT(Breit) spectrum?

Questions

❑ Radiative corrections – Born kinematics:



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But, Radiative corrections can change the "direction" and "value" of the "q" and make the "real" Q^2 to be small (and very small !)

Can we achieve the factorization for extracting the same distributions without requiring the Born kinematics?

Basic ideas for our new approach

- Do not try to invent any new scheme to treat QED radiation to match to the Born kinematics – NO Radiative Correction !

- Develop a reliable formalism that can extract the PDFs, TMDs and parton correlation functions, systematically with controllable and consistent approximations, without requiring the "one-photon" approximation and the "photon-hadron" frame !

- Generalize the QCD factorization to include Electroweak theory
  - QED radiation is a part of the production cross sections
  - QED radiation is treated in the same way as QCD radiation is treated

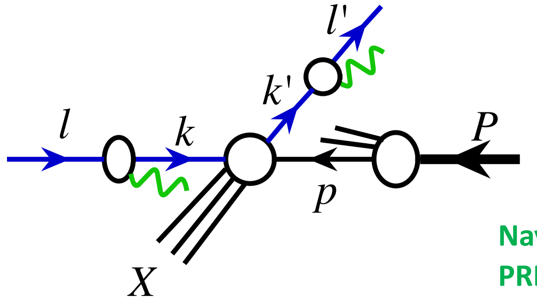
Note: our new approach is more relevant for high-energy process
where a large GeV scale exists and
collinear logarithms caused by radiation are important

Inclusive Deep Inelastic Scattering

- **Inclusive DIS**

  = **Inclusive production of a high transverse momentum lepton in lepton-hadron collision frame**

$$e(l) + h(P) \to e'(l') + X$$



Nayak, Qiu, Sterman
PRD72 (2005) 114012

- **Factorization proof:**

  = **Factorization proof of single hadron production hadronic collisions**

$$E' \frac{d\sigma_{eh\to e'X}}{d^3l'} \approx \frac{1}{2s} \sum_{i,j,a} \int_{z_L}^{1} \frac{d\zeta}{\zeta^2} \int_{x_L}^{1} \frac{d\xi}{\xi} D_{e'/j}(\zeta) f_{i/e}(\xi) \int_{x_h}^{1} \frac{dx}{x} f_{a/h}(x)\, \hat{H}^{(m,n)}_{ia\to j}(\xi,\zeta,x;l')$$

m : QED power

n : QCD power

$$s = (P+l)^2 \approx 2P\cdot l$$

i, j, a include all QED and light flavor QCD particles

In the following discussion, we take valence approximation: $i = j = e$

$f_{i/e}(\xi)$ **Lepton distribution functions (LDFs)**

$D_{e/j}(\zeta)$ **Lepton fragmentation functions (LFFs)**

$\Big\}$ **include all collinear sensitivities as** $m_e \to 0$

$\hat{H}_{ia\to j}$ **Infrared safe, insensitive to** $m_e \to 0 \quad m_q \to 0$

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Inclusive Deep Inelastic Scattering

- **Physically measured "hard" scales (in lepton-hadron frame):**

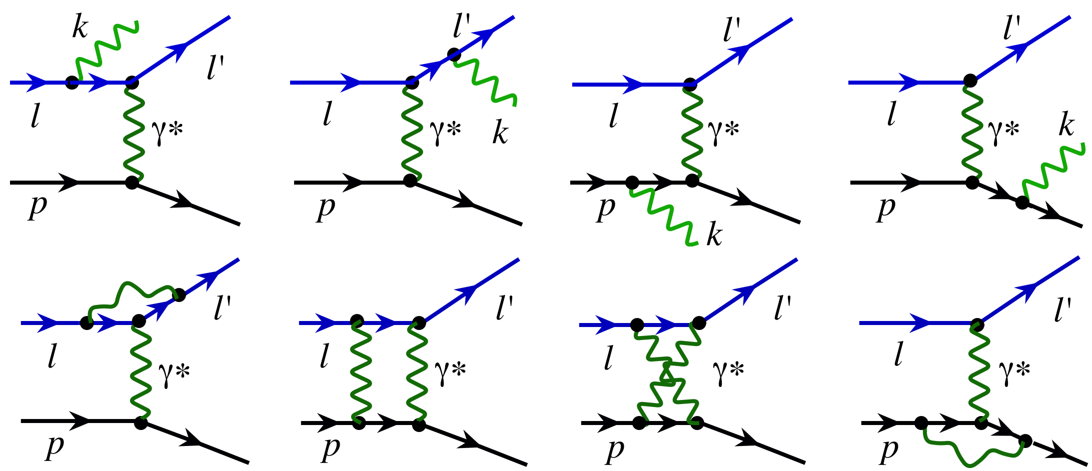
$$l'_T \gg \Lambda_{\rm QCD} \qquad\qquad Q^2 = -(l - l')^2 \gg \Lambda_{\rm QCD}^2$$

Invariant mass of the "virtual photon" could be very "small" due to photon radiation

- **Factorization approach to organize QED contribution:**

Real:

Virtual:



- **Collinear sensitive to incoming lepton** ➡ **lepton distribution function (LDF)**
- **Collinear sensitive to observed lepton** ➡ **lepton fragmentation function (LFF)**
- **Collinear sensitive to incoming quark** ➡ **evolution of PDFs**
- **The rest** ➡ **Infrared safe perturbative hard parts**

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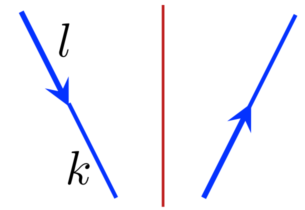
❑ Lepton distribution:

QED gauge link

$$f_{e/l}(\xi) = \int \frac{dy^-}{4\pi} e^{i\xi\, l^+ y^-} \langle l | \overline{\psi}(0)\gamma^+ \Phi(0,y^-)\psi(y^-) | l \rangle$$

Similar to the definition of quark PDFs

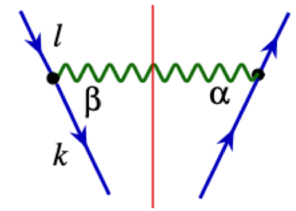
- **Leading order:**



$$f^{(0)}_{e/l}(\xi) = \frac{1}{4l\cdot n}\mathrm{Tr}\left[\gamma\cdot n\gamma\cdot l\right]\delta(\xi - \frac{k\cdot n}{l\cdot n})d^4k\delta^4(k-l)\delta_{el}$$

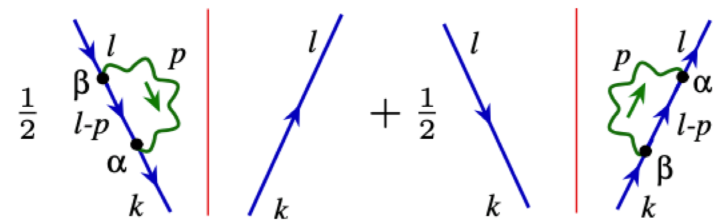
$$= \delta(\xi - 1)\delta_{el}$$

- **Next-to-Leading order (MSbar scheme):**



$$f^{\mathrm{Real}(1)}_{e/e}(\xi,\mu^2) = \frac{\alpha_{\mathrm{em}}}{2\pi}\left[\frac{1+\xi^2}{1-\xi}\ln\left(\frac{\mu^2}{(1-\xi)m_e}\right)\right]$$

$$f^{(1)}_{e/e}(\xi,\mu^2) = \frac{\alpha_{\mathrm{em}}}{2\pi}\left[\frac{1+\xi^2}{1-\xi}\ln\left(\frac{\mu^2}{(1-\xi)m_e}\right)\right]_+$$



❑ Resummation:

$$\mu^2\frac{d}{d\mu^2}f_{e/e}(\xi,\mu^2) = \int_\xi^1 \frac{d\xi'}{\xi'}P_{ee}(\xi/\xi',\alpha)f_{e/e}(\xi',\mu^2) \qquad P^{(1)}_{e/e}(z,\alpha) = \frac{\alpha_{\mathrm{em}}}{2\pi}\left[\frac{1+z^2}{1-z}\right]_+$$

QED DGLAP evolution with QED kernels

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- **Lepton fragmentation function:**

QED gauge link

$$D_{l/e}(\zeta) = \int \frac{dy^-}{4\pi} e^{il^+ y^-/\zeta} \frac{\zeta}{2} \mathrm{Tr}\left[\gamma^+ \langle 0|\overline{\psi}(0)\Phi(0,\infty)|l,X\rangle\langle\psi(y^-)\Phi(y^-,\infty)|0\rangle\right]$$

Similar to definition of quark FFs

- **Leading order:**

$$D^{(0)}_{l/e}(\zeta) = \delta(\zeta - 1)\delta_{el}$$

- **Next-to-Leading order (MSbar scheme):**

$$D^{(1)}_{l/e}(\zeta) = \frac{\alpha_{\rm em}}{2\pi}\left[\frac{1+\zeta^2}{1-\zeta}\ln\left(\frac{(1-\zeta)\mu^2}{\Delta E^2}\right)\right]_+$$

ΔE^2

invariant mass resolution of the radiated photon

$\Delta E^2 = 0.01\ \mathrm{GeV}^2$

Used in numerical calculations

- **Resummation:**

LFFs obey the same QED DGLAP evolution of the LPFs

- **Input distributions at the input scale $\mu_0^2 \sim m_e^2$:**

Unlike input distributions for PDFs, which are non-perturbative, input distribution for LDFs and LFFs are perturbatively calculable, …

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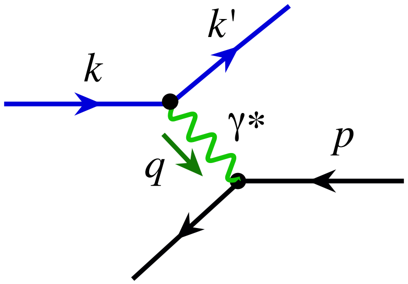
❑ LO perturbatively calculable hard part:



$$\hat{H}^{(2,0)}_{eq\to e} = e_q^2(4\alpha_{\rm EM}^2)\left[\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\right]\delta(\hat{s}+\hat{t}+\hat{u})$$

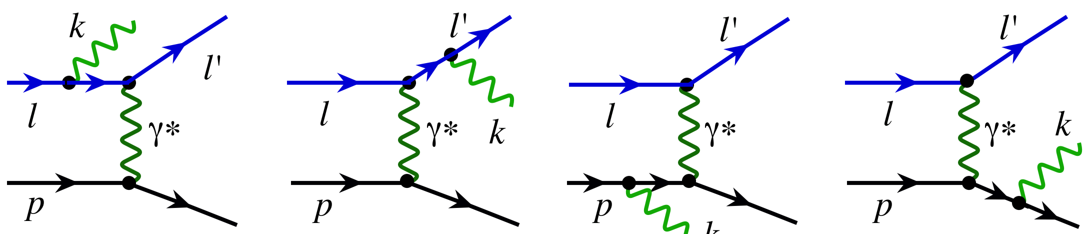
$$\hat{s} = x\,\xi\,s \qquad Q^2 = x_B\,y\,s$$

$$\hat{t} = (\xi/\zeta)t \qquad t = (l-l')^2 \equiv -Q^2$$

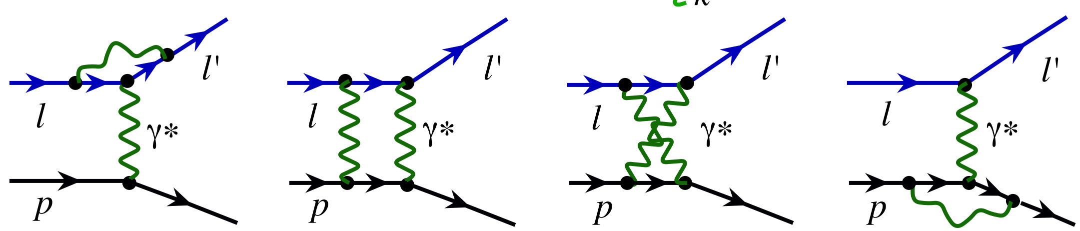
$$\hat{u} = (x/\zeta)u \qquad u = (P-l')^2 \equiv -(1-y)s$$

❑ NLO calculable hard part from QED:

Real:



Virtual:



Into QED Correction to QCD DGLAP kernels

$$\hat{H}^{(3,0)}_{eq\to e'} = \sigma^{(3,0)}_{eq\to e'} - D^{(1)}_{e'/e'}\otimes\hat{H}^{(2,0)}_{eq\to e'} - f^{(1)}_{e/e}\otimes\hat{H}^{(2,0)}_{eq\to e'} - f^{(1)}_{q/q}\otimes\hat{H}^{(2,0)}_{eq\to e'}$$

Completely IR safe, no dependence on lepton mass

Inclusive Deep Inelastic Scattering

❑ LO Factorized inclusive DIS cross section:

$$E'\frac{d\sigma^{(0)}_{eh\to e'X}}{d^3l'} \approx \frac{2\alpha^2_{\rm EM}}{s}\sum_q \int_{z_L}^1 \frac{d\zeta}{\zeta^2}\int_{x_L}^1\frac{d\xi}{\xi} D_{e/e}(\zeta) f_{e/e}(\xi)\int_{x_h}^1 \frac{dx}{x} e_q^2\, f_{q/h}(x)\,\delta\!\left(x - \frac{-\xi t}{\xi\zeta s + u}\right)$$
$$\times\left[\frac{(x\xi\zeta s)^2 + (xu)^2}{(\xi t)^2}\right]\left[\frac{\zeta}{(\xi\zeta s)+u}\right]$$

LO in hard part, but include all orders resummation into PDFs, FFs

$$\Rightarrow \frac{4\alpha^2_{\rm em}}{Q^2 s}\left[F_1(x_B,Q^2) + \frac{1-y}{x_B y^2}F_2(x_B,Q^2)\right]$$

With LO relation: $F_2(x_B) = 2x_B F_1(x_B) = \sum_q e_q^2\, x_B f_{q/h}(x_B)$

$$f_{e/e}(\xi) \approx f^{(0)}_{e/e}(\xi) \qquad \xi = \zeta = 1$$
$$D_{e/e}(\zeta) \approx D^{(0)}_{e/e}(\zeta)$$

❑ NLO fixed order QED correction:

By taking: $f_{e/e}(\xi) \approx f^{(0)}_{e/e}(\xi) + f^{(1)}_{e/e}(\xi)$ **or**: $\hat{H}^{(m,n)} \approx \hat{H}^{(2,0)} + \hat{H}^{(3,0)}$

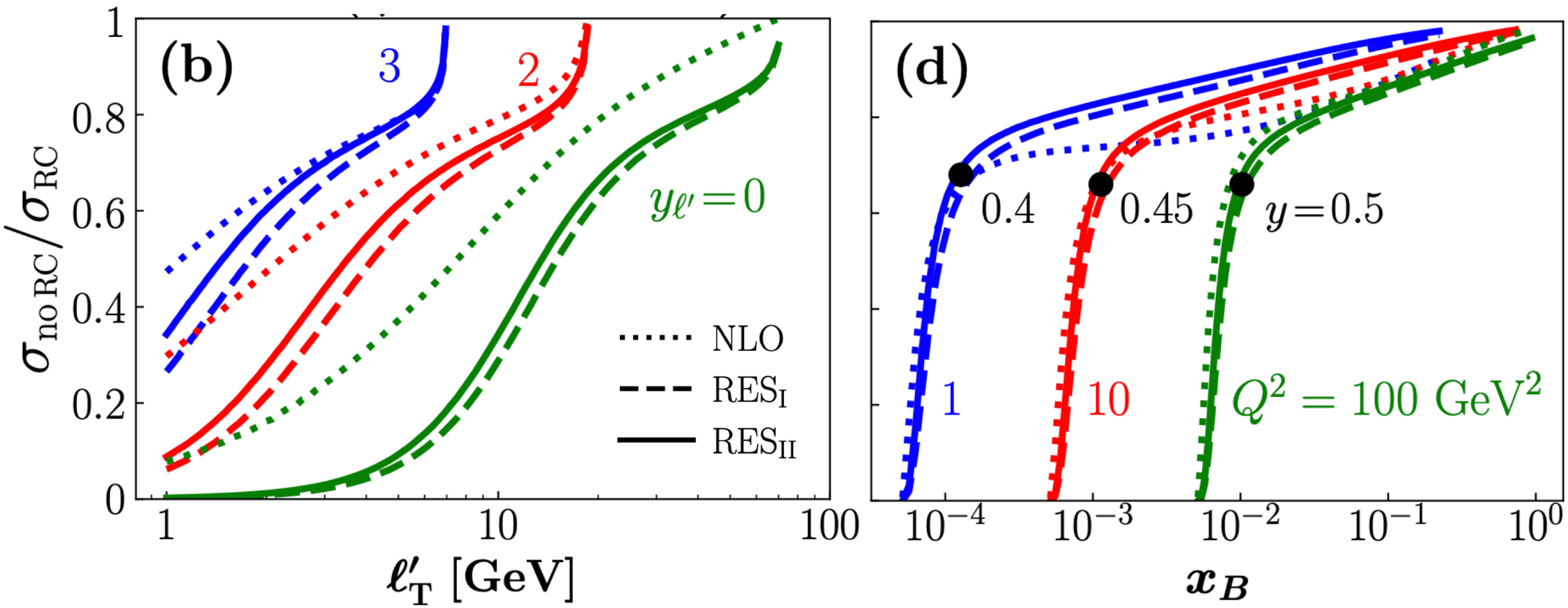
or: $D_{e/e}(\zeta) \approx D^{(0)}_{e/e}(\zeta) + D^{(1)}_{e/e}(\zeta)$

Inclusive Deep Inelastic Scattering

- **Numerical impact of QED contribution at EIC ($\sqrt{S} = 140$ GeV):**

$$\frac{\sigma_{\rm noRC}}{\sigma_{\rm RC}} \leftrightarrow \frac{\sigma_{1\gamma}}{\sigma_{\rm measured}} = \eta(x_B, y)$$

B. Badelek et al.
Z Phys C 66 (1995) 591



$\sigma_{\rm noRC} = E'\frac{d\sigma}{d^3l'}$ With $f_{e/e}(\xi) \approx f^{(0)}_{e/e} = \delta(\xi - 1)$ and $D_{e/e}(\zeta) \approx D^{(0)}_{e/e} = \delta(\zeta - 1)$

NLO : With $f_{e/e}(\xi,\mu) \approx f^{(0)}_{e/e}(\xi) + f^{(1)}_{e/e}(\xi,\mu^2)$ and $D_{e/e}(\zeta,\mu) \approx D^{(0)}_{e/e}(\zeta) + D^{(1)}_{e/e}(\zeta,\mu^2)$

$\mathrm{RES_I}$: DGLAP evolved with 0th input $f_{e/e} \approx f^{(0)}_{e/e}$ and $D_{e/e} \approx D^{(0)}_{e/e}$ at $\mu_0^2 = m_e^2$

$\mathrm{RES_{II}}$: DGLAP evolved with input $f_{e/e} \approx f^{(0)}_{e/e} + f^{(1)}_{e/e}$, $D_{e/e} \approx D^{(0)}_{e/e} + D^{(1)}_{e/e}$

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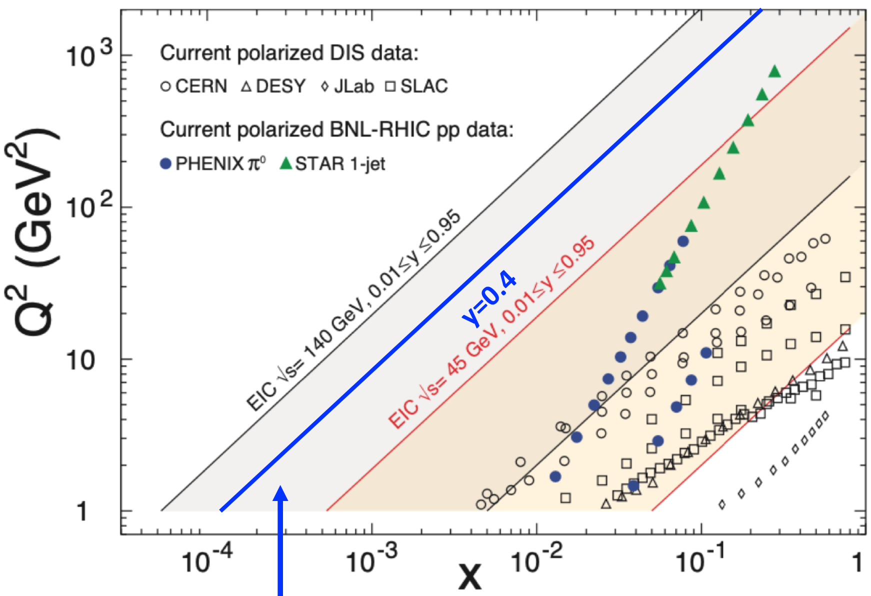
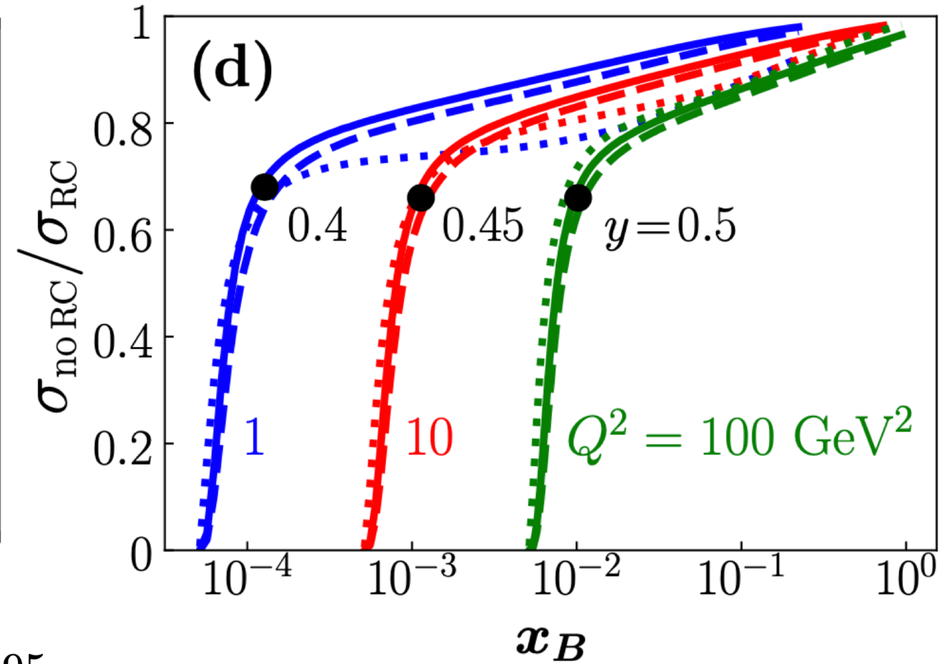
Inclusive Deep Inelastic Scattering

- Numerical impact of QED contribution at EIC ($\sqrt{S} = 140$ GeV):





Almost the same as $\sqrt{S} = 90$ GeV at $y = 0.95$

- Renormalization and factorization scale choice:

$$\mu^2 = \max\left[m_c^2, w_1\, l'^2_T + w_2\, Q^2\right]$$

with: $(w_1\,, w_2) = (1,0),\ (1/2, 1/2)$

Weak scale dependence on the ratios

Both JAM and CTEQ hadron PDFs were used, not visible difference in the ratios

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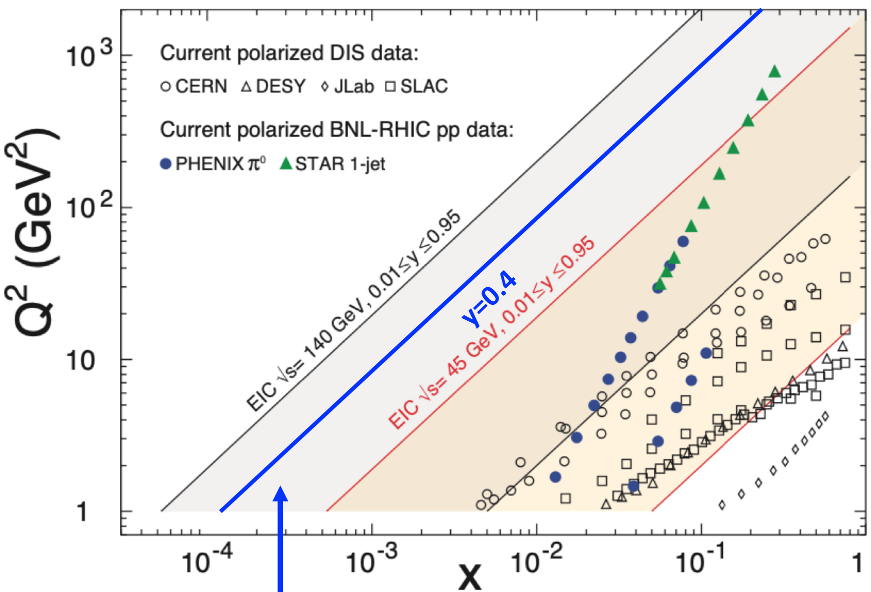
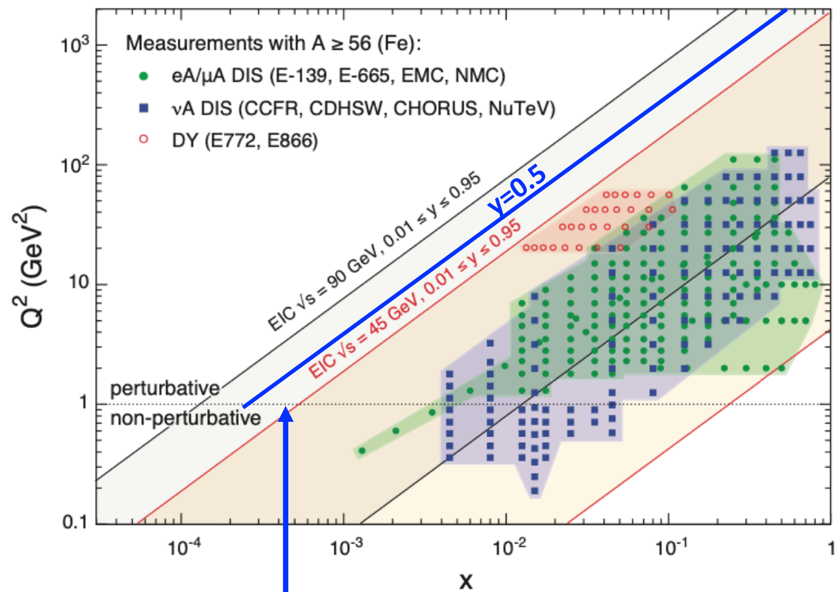
Inclusive Deep Inelastic Scattering

❑ Numerical impact of QED contribution at EIC ($\sqrt{S} = 140\ \mathrm{GeV}$):





Almost the same as $\sqrt{S} = 90\ \mathrm{GeV}$ at $y = 0.95$

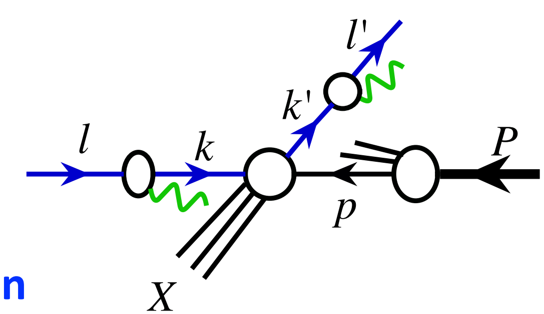
Almost the same as $\sqrt{S} = 65\ \mathrm{GeV}$ at $y = 0.95$

❑ Why the impact of RC is so large?

$\alpha_{\rm em} \ln(\mu^2/m_e^2)$ **is not a large number!**

It is the shift in kinematics caused by the RC that matters!

- $\delta(\xi - 1)\ \Rightarrow\ f_{e/e}(\xi, \mu^2)$ **More evolution, more small ξ lepton**
- **Smaller** $\widetilde{Q}^2 \equiv -(k - k')^2$**, much larger hard cross section!**



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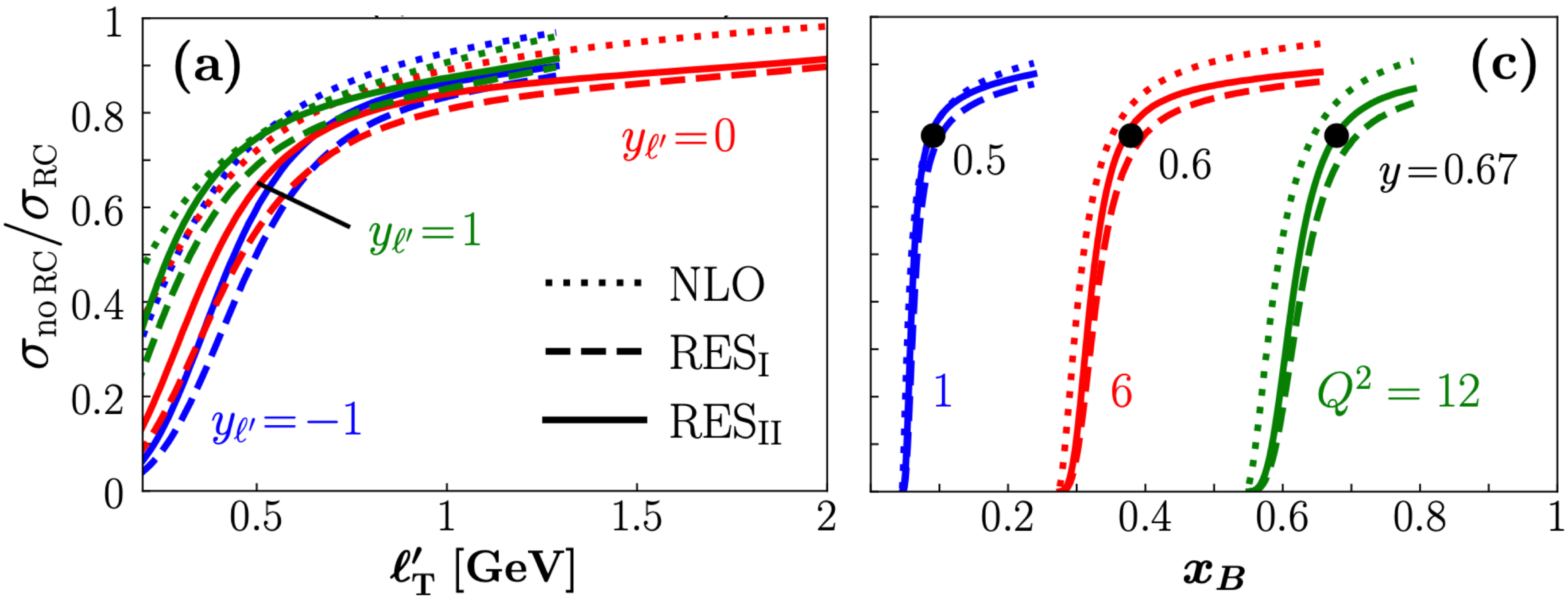
Inclusive Deep Inelastic Scattering

❑ Numerical impact of QED contribution at CEBAF ($\sqrt{S} = 4.7\ \mathrm{GeV}$):

$$\frac{\sigma_{\mathrm{noRC}}}{\sigma_{\mathrm{RC}}} \leftrightarrow \frac{\sigma_{1\gamma}}{\sigma_{\mathrm{measured}}} = \eta(x_B, y)$$

B. Badelek et al.
Z Phys C 66 (1995) 591



$$\sigma_{\mathrm{noRC}} = E' \frac{d\sigma}{d^3 l'}$$

With $f_{e/e}(\xi) \approx f_{e/e}^{(0)} = \delta(\xi - 1)$ and $D_{e/e}(\zeta) \approx D_{e/e}^{(0)} = \delta(\zeta - 1)$

- **Final-state hadronic mass (W > 2 GeV) was imposed**
- **Difference between curves is small, not much room for shower**
- **Transverse momentum of scattered electron is a good "hard" scale**

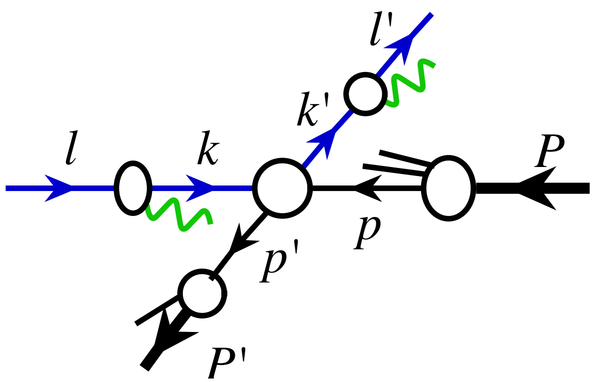
Semi-inclusive DIS

❑ No photon-hadron frame!

Semi-inclusive DIS cross section

= inclusive electron + hadron (or jet) cross section in lepton-hadron collisions

(lepton or lepton jet)



(hadron or jet)

- **Transverse plane to the colliding axis:**

$$\overline{\mathbf{P}}_T \equiv \frac{1}{2}(\boldsymbol{\ell}'_T - \boldsymbol{P}_{hT})$$

$$\overline{\mathbf{p}}_T \equiv \boldsymbol{\ell}'_T + \boldsymbol{P}_{hT}$$

- **Collinear factorization:**

$$|\overline{\mathbf{P}}_T| \sim |\overline{\mathbf{p}}_T|$$

- **TMD factorization:**

$$|\overline{\mathbf{P}}_T| \gg |\overline{\mathbf{p}}_T|$$

"Hard" scale = $\overline{\mathbf{P}}_T$

"Soft" scale = $\overline{\mathbf{p}}_T$

Momentum imbalance between two particles (or jet(s))

❑ The "Parton frame":

Boost along the collision axis, such that $y_{\ell'} + y_h = 0$

$y_{\ell'}$ Rapidity of the observed lepton (or jet) in the lab frame

y_h Rapidity of the observed hadron (or jet) in the lab frame

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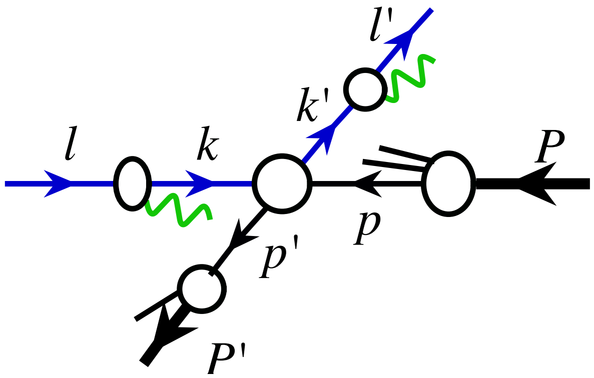
Semi-inclusive DIS

❑ Collinear factorization:



Effectively the same as two-jet (or hadron) cross section at a hadron collider

CO-Functions

$$\frac{d\sigma_{\text{SIDIS}}}{dy_{\ell'}\,dy_h\,d^2\overline{\mathbf{P}}_T\,d^2\overline{\mathbf{p}}_T} = \frac{1}{2s}\sum_{i,j,a,b} f_{i/e}\otimes D_{e/j}\otimes f_{a/h}\otimes D_{h'/b}\otimes H^{(m,n)}_{i+a\to j+b+X} + \mathcal{O}(1/|\overline{\mathbf{P}}|, 1/|\overline{\mathbf{p}}|)$$

❑ TMD factorization:

For final-state jet(s): $D \to J$
or fully calculated H_J without D's

TMDs

$$\frac{d\sigma_{\text{SIDIS}}}{dy_{\ell'}\,dy_h\,d^2\overline{\mathbf{P}}_T\,d^2\overline{\mathbf{p}}_T} = \frac{1}{2s}\sum_{i,j,a,b} \tilde{f}_{i/e}\otimes \tilde{D}_{e/j}\otimes \tilde{f}_{a/h}\otimes \tilde{D}_{h'/b}\otimes \tilde{H}^{(m,n)}_{i+a\to j+b+X} + \mathcal{O}(|\overline{\mathbf{p}}|/|\overline{\mathbf{P}}|)$$

The "+"-direction of the final-state TMDs are defined in the "parton frame" where the observed lepton (or jet) and hadron (or jet) are about back-to-back

Only two hadrons (or jet) are observed. Factorization breaking effect, identified by Collins & Qiu (2007) and Mulders & Rogers (2010), is not relevant here

Summary and outlook

- Radiative corrections are very important for lepton-hadron scattering
  - Especially difficult for a consistent treatment beyond the inclusive DIS
  - No well-defined photon-hadron frame, if we cannot recover all QED radiation
  - Radiative corrections are more important for events with high momentum transfers and large phase space to shower – such as those at the EIC
- We proposed a factorization based treatment of QED radiation
  - QED radiation is a part of production cross sections, treated in the same way as radiation from quarks and gluons
  - No artificial and/or process dependent scale(s) introduced other than the standard factorization scale
  - All perturbatively calculable hard parts are IR safe for both QCD and QED
  - All lepton mass or resolution sensitivity are included into "Universal" lepton distribution and fragmentation functions (or jet functions)

Thank you!

Special thanks to JLab experimental colleagues for helpful discussions!

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