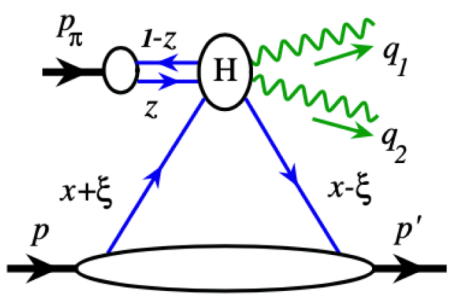
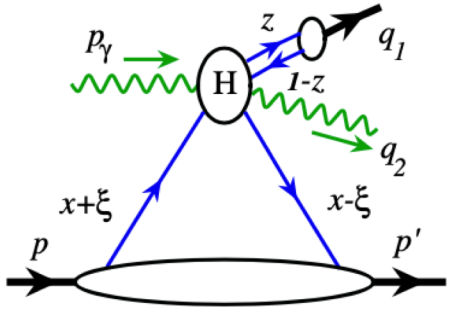
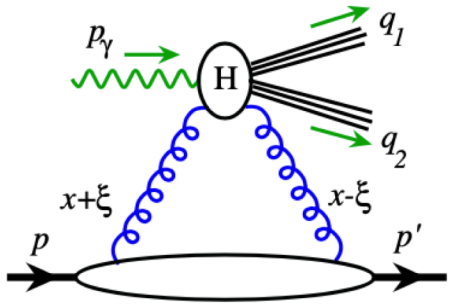
DNP 2021
Fall Meeting of the Division of Nuclear Physics
of the American Physical Society
October 11-14, 2021

A new class of exclusive processes to better measure the x-dependence of DAs and GPDs

Exclusive massive pair production (EMPP):







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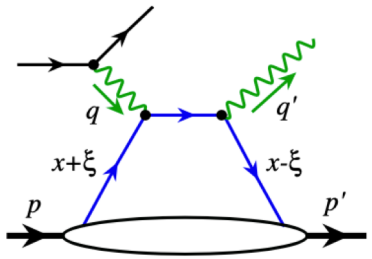
Jian-Wei Qiu
Jefferson Lab, Theory Center

In collaboration with: Zhite Yu (Michigan State University)

Jefferson Lab
TMD Collaboration
U.S. DEPARTMENT OF ENERGY | Office of Science
JSA

QCD Tomography

- **Imagining spatial distribution of quarks and gluons – *two-scale observables*:**



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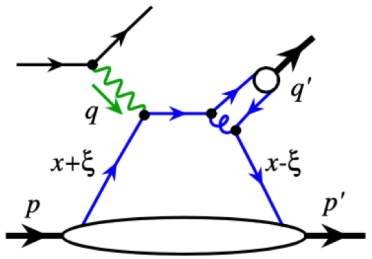

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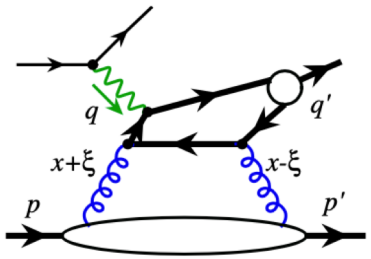

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DVCS: $Q^2 \gg |t|$ **DVMP** **DVHQ**

$$\frac{d\sigma}{dt} \qquad t = (p - p')^2$$

Factorization $Q^2 \gg |t|$

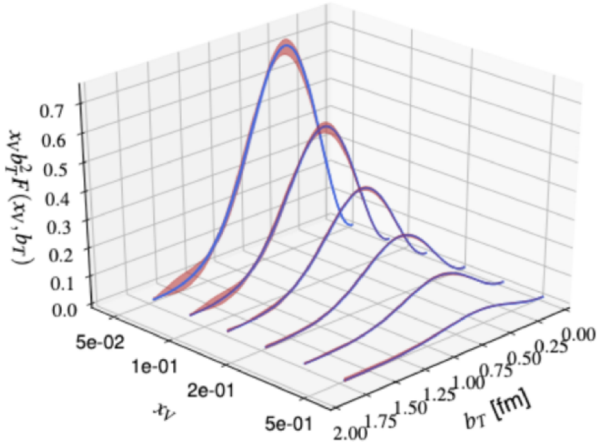
GPDs: $f_{i/h}(x, \xi, t; \mu)$

F.T. t_T **to** b_T **at** $\xi \propto (p - p')^+ \to 0$

- **Proton radii of quark and gluon spatial distribution, $r_q(x)$ & $r_g(x)$**

  Should $r_q(x)$ > $r_g(x)$, or vice versa? Could $r_g(x)$ saturate as $x \to 0$?

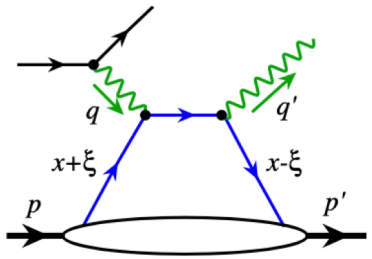
  …



Jefferson Lab

QCD Tomography

- **Imagining spatial distribution of quarks and gluons – *two-scale observables*:**



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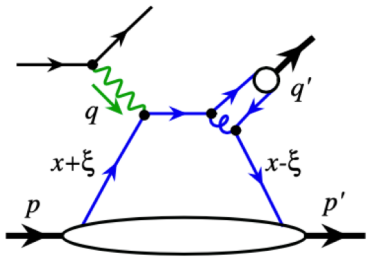

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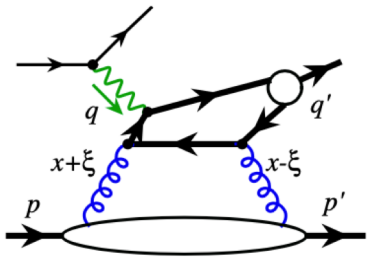

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$$\frac{d\sigma}{dt} \qquad t = (p - p')^2$$

Factorization $Q^2 \gg |t|$

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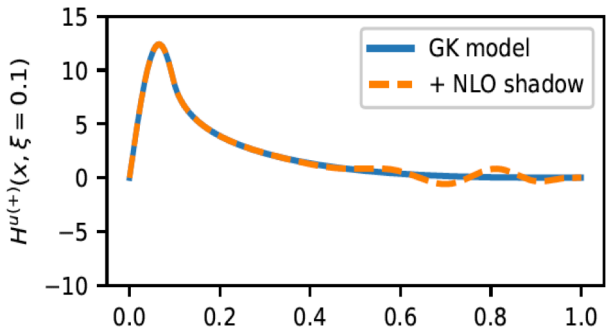
- **Proton radii of quark and gluon spatial distribution, $r_q(x)$ & $r_g(x)$**

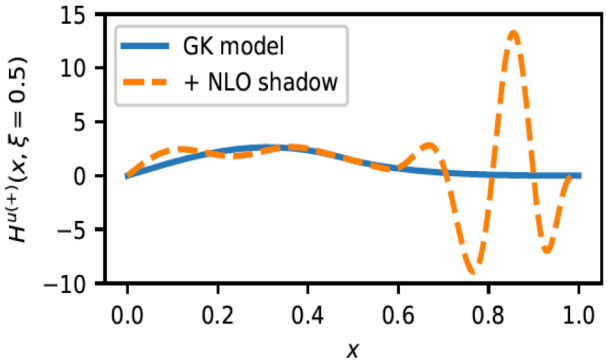
  Should $r_q(x)$ > $r_g(x)$, or vice versa? Could $r_g(x)$ saturate as $x \to 0$?

  …

- **But, all these observables are not sensitive to the *x*-dependence!**

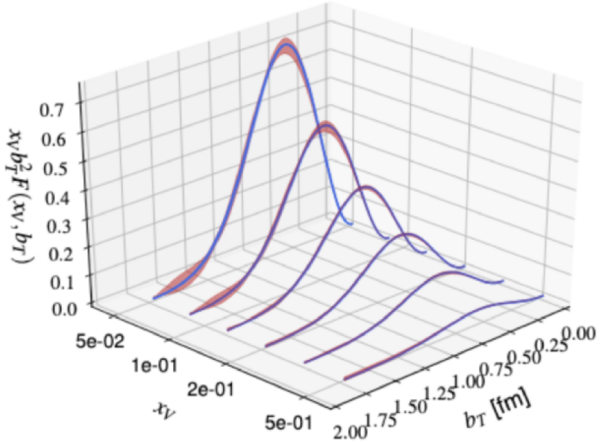
  Sensitive to the total momentum of the pair, not the relative momentum





Blue and dashed
Fit the same CFFs !

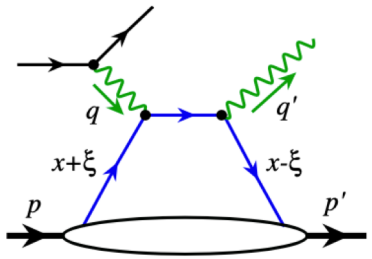
Phys.Rev. D103 (2021) 114019



Jefferson Lab

Exclusive Massive Pair Production (EMPP)

- **Imagining spatial distribution of quarks and gluons – *two-scale observables*:**



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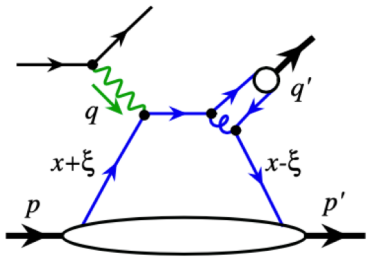

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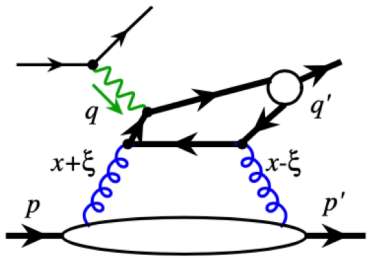
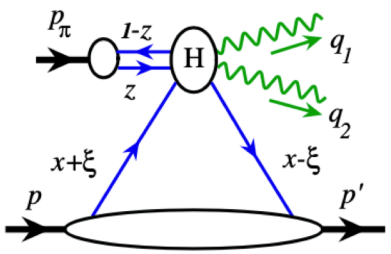
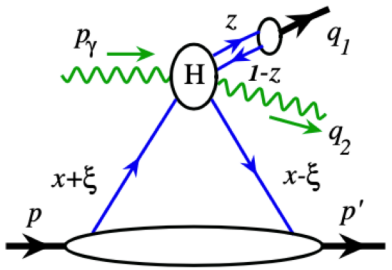


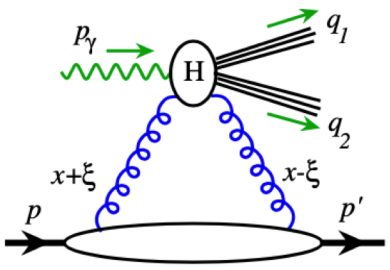
+…

$$Q^2 \equiv -q^2$$
$$t = (p - p')^2$$
$$Q^2 \gg |t|$$

DVCS: Q^2 >> |t| **DVMP** **DVHQ**



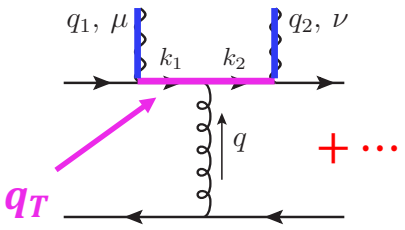




+ …

$$Q^2 \equiv (q_1 + q_2)^2$$
$$t = (p - p')^2$$
$$Q^2 \gg |t|$$
$$q_T \gg \Lambda_{\rm QCD}$$

- Both are $2 \to 3$ exclusive processes
- Key difference is the source of the hard scale
- Allow x-dependence to flow through the production of the "pair"
- Additional sensitivity from angular distribution of q_1 or q_T in the pair's rest frame



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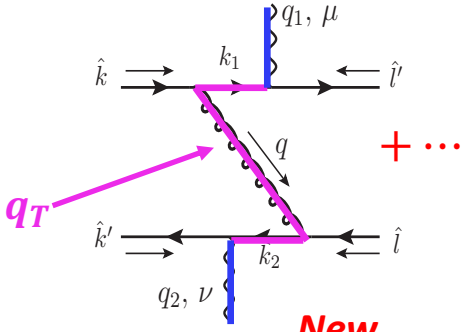
Like "time-like" form factor



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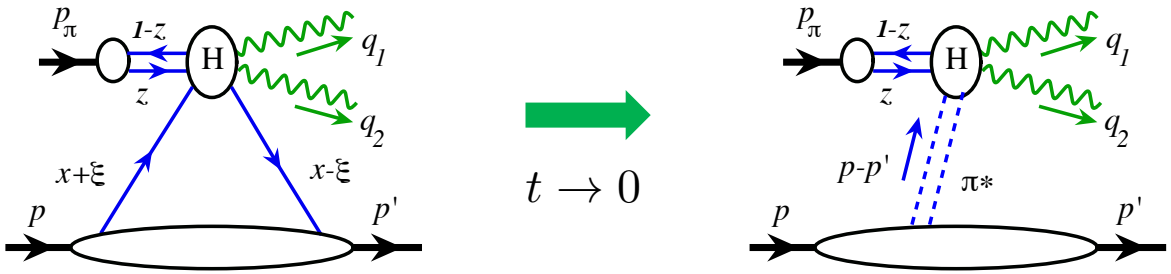
New

Jefferson Lab

Exclusive Massive Two-Photon Production

- **Massive photon pair:** $\pi^-(p_\pi) + P(p) \to \gamma(q_1) + \gamma(q_2) + N(p')$



$$\xi = \frac{(p-p')^+}{(p+p')^+} \qquad P^+ = \frac{(p+p')^+}{2}$$

Momentum transfer: $\Delta \equiv p - p'$

Leading power: $\Delta^+ = 2\xi P^+ = (p-p')^+$

Hard scale: $Q^2 = \hat{s} \equiv (q_1 + q_2)^2$

Soft scale: $t = (p-p')^2$

- **Factorization – *necessary conditions*:**

$\hat{s} \gg |t| \gtrsim \Lambda^2_{\rm QCD}$ — **Requires the time of the hard collision** $\sim 1/\sqrt{\hat{s}}$

$\Delta^+ = (p-p')^+ \gg \sqrt{|t|}$ — **to be much shorter than the lifetime of the exchanged** $q\bar{q}$ **(or** gg **) state**

$q_T^2 \gg \Lambda^2_{\rm QCD}$ — q_T **could be as small as 0 while** $\sqrt{\hat{s}}$ **is the hard scale**

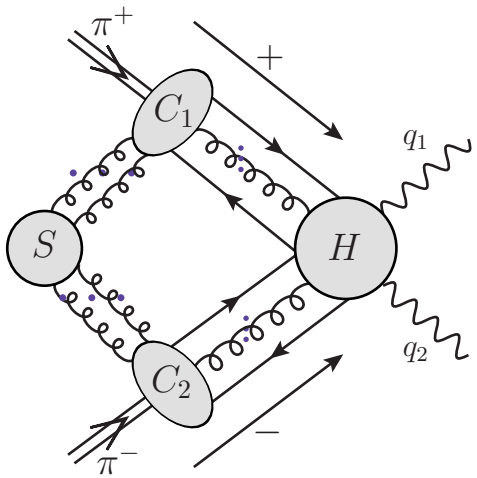
In this short talk, show factorization proof for a simplified process:

$$\pi(p_1) + \pi(p_2) \to \gamma(q_1) + \gamma(q_2)$$

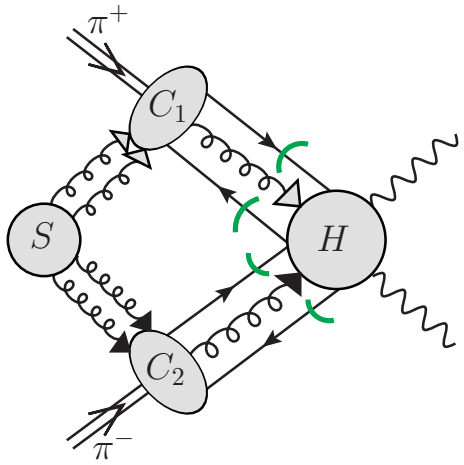
Exclusive Massive Two-Photon Production

- **Factorization:** $\pi(p_1) + \pi(p_2) \to \gamma(q_1) + \gamma(q_2)$

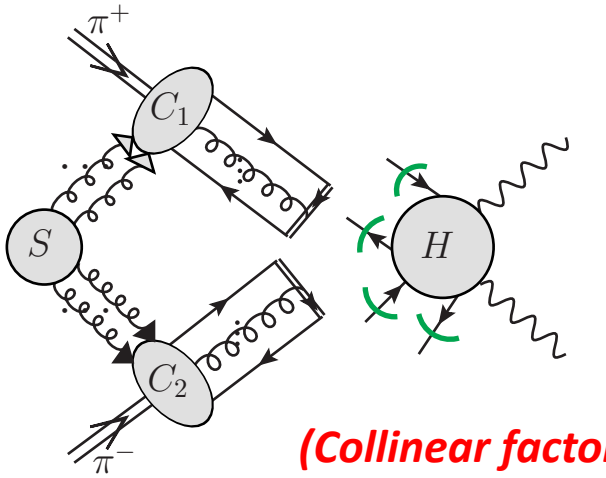
  - **Leading region**



  - **Approximations**

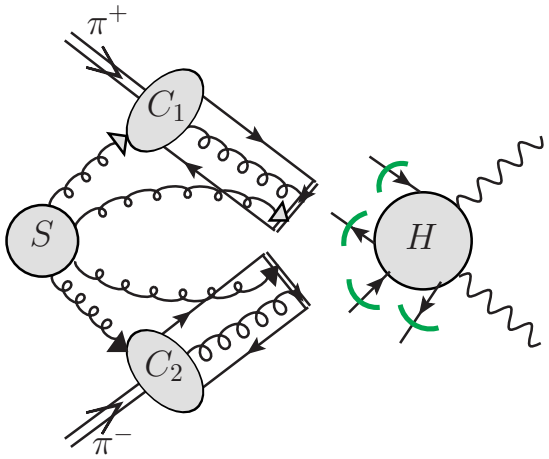


  - **Ward identity for collinear gluons**



(Collinear factorization)

  - **Ward identity for soft gluons**
    - **Soft gluons are as if attached to a "*closed fermion loop*"**
    - **Sum over diagrams**
      $\Longrightarrow \boldsymbol{S = 0}$

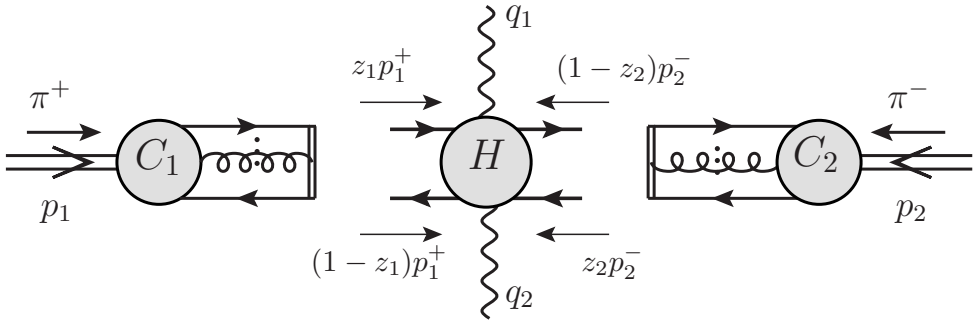


Soft gluons cancel because collinear parton lines are in color singlet states.

Jefferson Lab

Exclusive Massive Two-Photon Production

- **Factorization:** $\pi(p_1) + \pi(p_2) \to \gamma(q_1) + \gamma(q_2)$



$$\mathcal{M} = \frac{s}{2}\int_0^1 \mathrm{d}z_1 \mathrm{d}z_2\, \phi_{\pi^+}(z_1)\phi_{\pi^-}(z_2)\cdot \mathrm{Tr}\left[\frac{\gamma_5\gamma^-}{2} H(\hat{k}_1,\hat{k}_2;q_1,q_2;\mu)\frac{\gamma_5\gamma^+}{2}\right] + \mathcal{O}\left(\frac{m_\pi}{q_T}\right) \longrightarrow \frac{\mathrm{d}\sigma}{\mathrm{d}q_T} \propto |\mathcal{M}|^2$$

- **Hadron functions: distribution amplitudes (DA):**

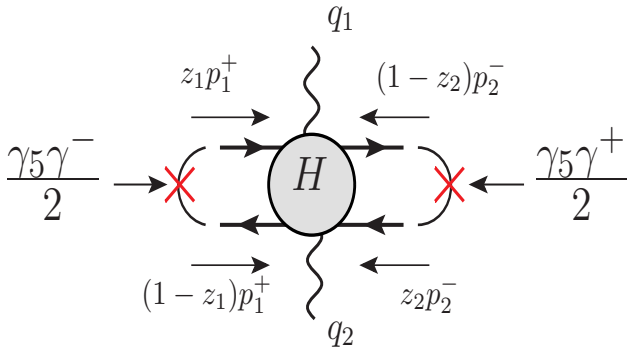
$$\phi_{\pi^+}(z_1) = \int \frac{dx^-}{4\pi} e^{i\, z_1\, p_1^+ x^-} \langle 0|\, \bar{d}(0)\, \gamma^+\gamma_5\, W(0,x^-)\, u(x^-)\, |\pi^+(p_1)\rangle$$

$$\phi_{\pi^-}(z_2) = \int \frac{dx^+}{4\pi} e^{i\, z_2\, p_2^- x^+} \langle 0|\, \bar{u}(0)\, \gamma^-\gamma_5\, W(0,x^+)\, d(x^+)\, |\pi^-(p_2)\rangle$$

$\boldsymbol{\phi_{\pi^+}(z) = \phi_{\pi^-}(z) = \phi(z)}$
are universal DAs

- **Hard coefficient**

$$C\left(z_1, z_2; \frac{q_T^2}{s}; \frac{q_T^2}{\mu^2}\right) =$$



Projections (for $\pi^\mp$):

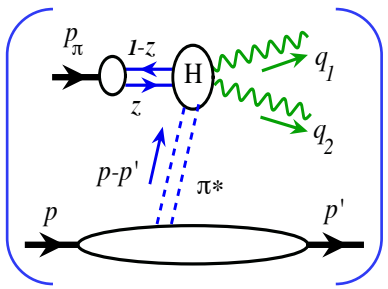
1. **Spin-0** $\gamma_5\gamma^\pm$
2. **P-odd**

Jefferson Lab

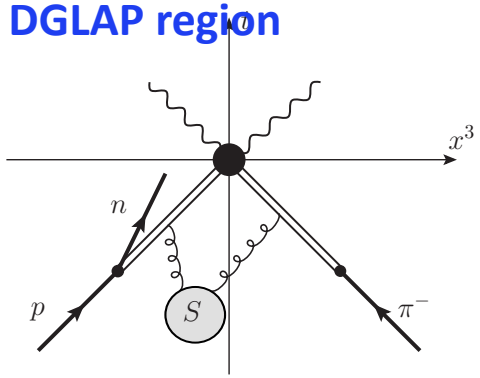
Exclusive Massive Two-Photon Production

❑ Factorization: $\pi^-(p_\pi) + P(p) \to \gamma(q_1) + \gamma(q_2) + N(p')$

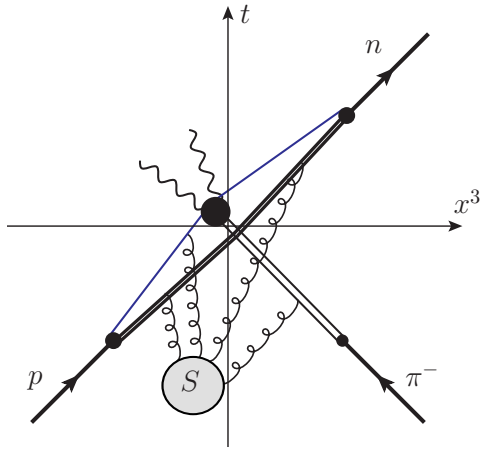
- **Additional region: DGLAP region**



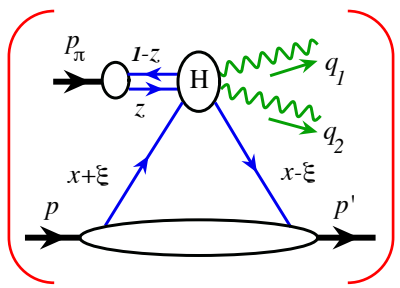
(Efremov, Radyushkin, Brodsky, Lepage)



ERBL region



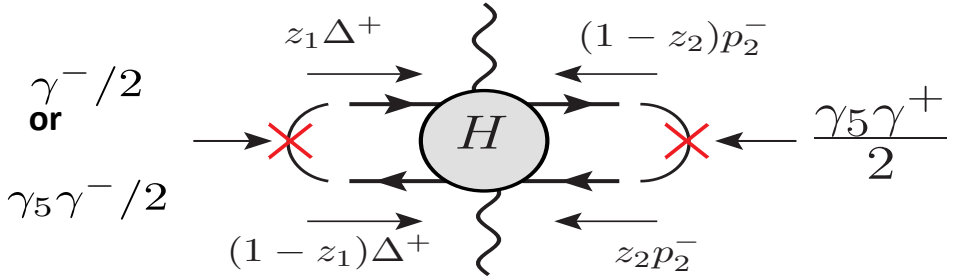
DGLAP region



- Different soft structures
- Factorization proof needs to be modified

$$\mathcal{M} = \mathrm{GPD} \otimes \mathrm{DA} \otimes \mathrm{Hard}$$

❑ Additional channels – more GPDs:



$\gamma^-/2$ corresponds to $F^u_{pn} \supset (H, E)$

$\gamma_5\gamma^-/2$ corresponds to $\widetilde{F}^u_{pn} \supset (\widetilde{H}, \widetilde{E})$

❑ Factorization formula:

$$\mathcal{M} = \frac{\hat{s}}{2} \int \mathrm{d}z_1\, \mathrm{d}z_2 \left[F^u_{pn}(z_1, \xi, t)\, \phi(z_2)\, \widetilde{C}(z_1, z_2) + \widetilde{F}^u_{pn}(z_1, \xi, t)\, \phi(z_2)\, C(z_1, z_2) \right] + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{q_T}, \frac{|t|}{\hat{s}} \right)$$

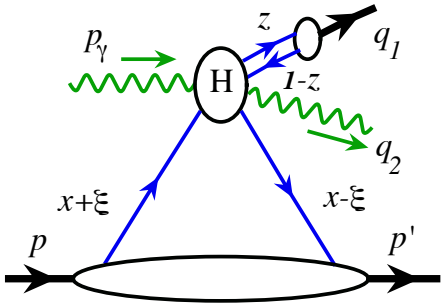
$$\frac{\mathrm{d}\sigma}{\mathrm{d}t\, \mathrm{d}\xi\, \mathrm{d}q_T^2} \propto |\mathcal{M}|^2$$

Jefferson Lab

Exclusive $\pi^0\gamma$ Pair Production

- **Factorization:** $\gamma(p_\gamma) + h(p) \to \pi^0(q_1) + \gamma(q_2) + h(p')$

  Factorization is proved similarly



Hall D at JLab

- **Hard scales:**

  Invariant mass of $\pi\gamma$ pair

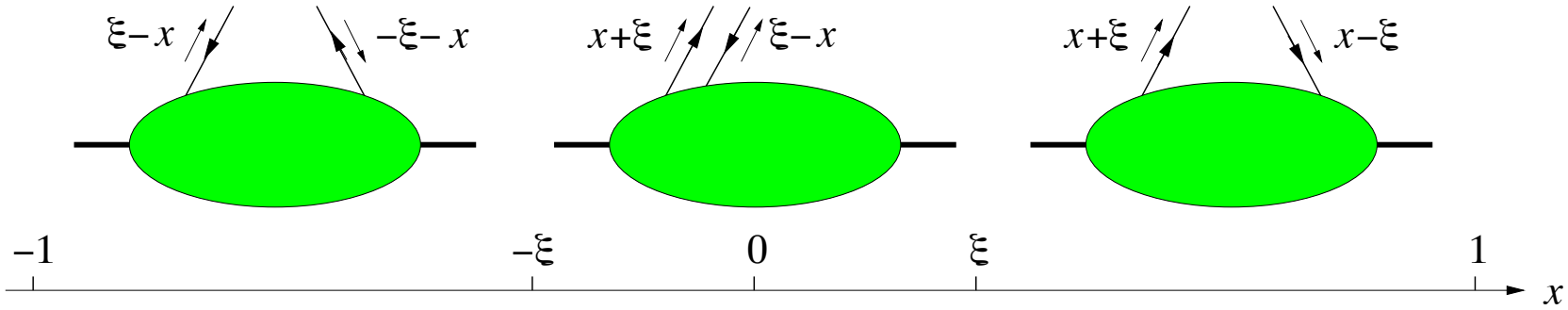
  q_T of π (or γ) in pair's rest frame – convert to $\cos\theta_\pi$

$\frac{d\sigma}{d\cos\theta}$ is sensitive to x-dependence of GPDs

- **Cancellation of unwanted propagators & $\cos\theta$ dependence:**

$$\mathrm{Re}\, O_{++} = (e_1 - e_2)^2 \left[\frac{1-\cos\theta}{1+\cos\theta} \cdot P \frac{x+z-2xz}{2\,x\,z\,(1-x)\,(1-z)}\right] + (e_1^2 - e_2^2)\left[\frac{2}{1-\cos\theta} \cdot P \frac{x-z}{x\,z\,(1-x)\,(1-z)}\right]$$
$$- e_1 e_2 P \frac{1-\cos\theta}{x\,z\,(1-x)\,(1-z)} \cdot \frac{(xz + (1-x)(1-z))\,(x(1-x) + z(1-z))}{(2(1-x)(1-z) - (1+\cos\theta)xz)\,(2xz - (1+\cos\theta)(1-x)(1-z))}$$

- **Sensitive to ERBL region (complementary)**



Also sensitive to DA in the bulk region.

Jefferson Lab

Exclusive $\pi^0\gamma$ Pair Production

❑ Phenomenology:

$$\frac{d\sigma}{d|t|\, d\xi\, d\cos\theta_\pi\, d\phi_\pi} = \frac{|\mathcal{A}|^2}{32\, s\, (2\pi)^4\, (1+\xi)^2}$$

$$\frac{1}{2}\overline{|\mathcal{A}|^2} = \left(\frac{2\pi\alpha_s}{s} f_\pi\right)^2 \left(\frac{C_F}{N_c}\right)^2 \left(\frac{1+\xi}{\xi}\right)^2 (1-\xi^2)$$
$$\times \left[|O_{++}^{[\widetilde{H}]}|^2 + |O_{+-}^{[\widetilde{H}]}|^2 + |\widetilde{O}_{++}^{[H]}|^2 + |\widetilde{O}_{+-}^{[H]}|^2\right]$$

- **Factorized helicity amplitude:**

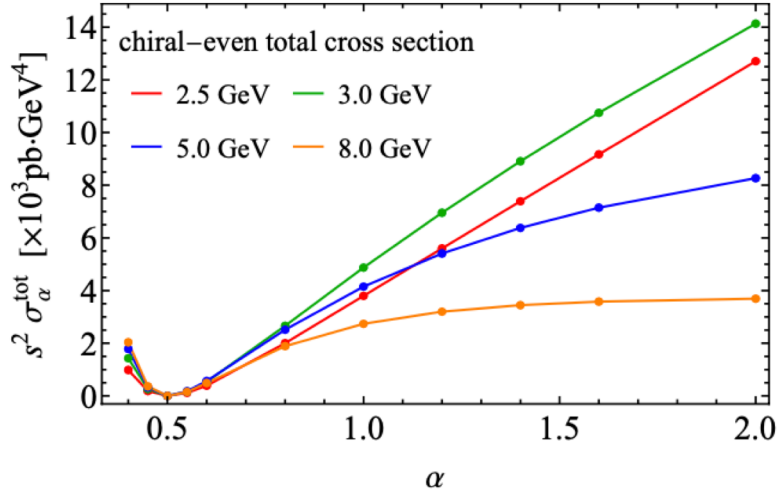
$$O_{\lambda\lambda'}^{[\widetilde{H}]} = \sum_q \int_{x_L}^{x_R} dx \int_0^1 dz\, \widetilde{H}^q(x,\xi,t)\, \phi_\pi^q(z)\, O_{\lambda\lambda'}^q(x,z)$$

- **Pion distribution amplitude:**

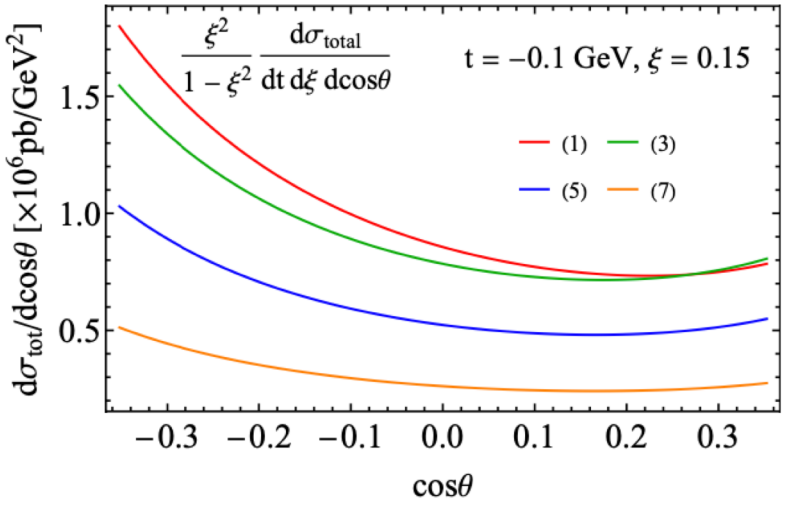
$$\phi_{\pi^0}^d(z) = \phi_{\pi^0}^u(z) = \frac{1}{\sqrt{2}} \frac{z^\alpha (1-z)^\alpha}{B(1+\alpha, 1+\alpha)}, \quad (\alpha > 0)$$

- **Model GPDs = simplified GK model:**
  - **Taking** $n_i = 0$
  - **Parametrizing the forward limit as** $x^a(1-x)^b$
  - **Neglecting the D-term**

❑ Sensitivity on DAs (total – q_T > 1 GeV):



❑ Sensitivity on GPDs (α = 0.63):



Jefferson Lab

Exclusive $\pi^0\gamma$ Pair Production

❑ Phenomenology:

$$\frac{d\sigma}{d|t|\, d\xi\, d\cos\theta_\pi\, d\phi_\pi} = \frac{|\mathcal{A}|^2}{32\, s\, (2\pi)^4\, (1+\xi)^2}$$

$$\frac{1}{2}\overline{|\mathcal{A}|^2} = \left(\frac{2\pi\alpha_s}{s} f_\pi\right)^2 \left(\frac{C_F}{N_c}\right)^2 \left(\frac{1+\xi}{\xi}\right)^2 (1-\xi^2)$$
$$\times \left[|O_{++}^{[\widetilde{H}]}|^2 + |O_{+-}^{[\widetilde{H}]}|^2 + |\widetilde{O}_{++}^{[H]}|^2 + |\widetilde{O}_{+-}^{[H]}|^2\right]$$

- **Factorized helicity amplitude:**

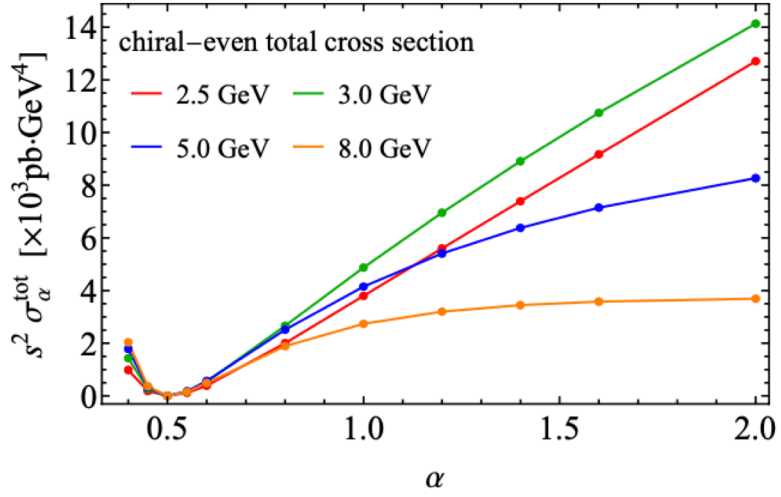
$$O_{\lambda\lambda'}^{[\widetilde{H}]} = \sum_q \int_{x_L}^{x_R} dx \int_0^1 dz\, \widetilde{H}^q(x,\xi,t)\, \phi_\pi^q(z)\, O_{\lambda\lambda'}^q(x,z)$$

- **Pion distribution amplitude:**

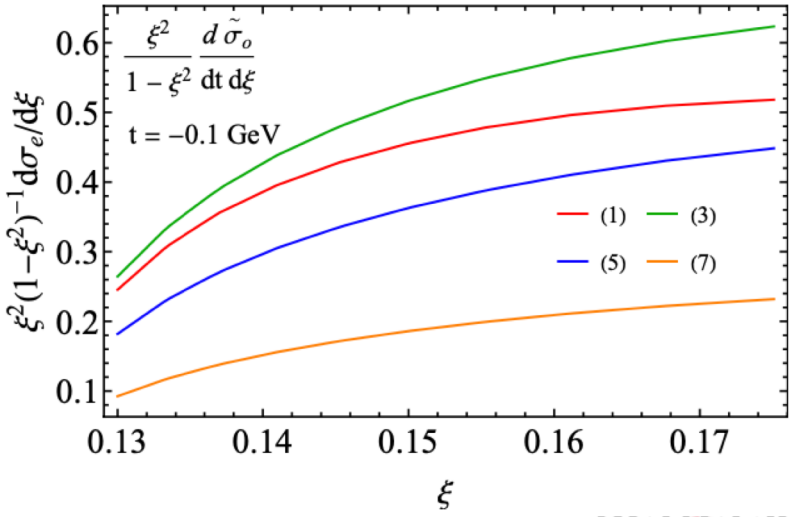
$$\phi_{\pi^0}^d(z) = \phi_{\pi^0}^u(z) = \frac{1}{\sqrt{2}} \frac{z^\alpha (1-z)^\alpha}{\mathrm{B}(1+\alpha, 1+\alpha)}, \quad (\alpha > 0)$$

- **Model GPDs = simplified GK model:**
  - Taking $n_i = 0$
  - Parametrizing the forward limit as $x^a(1-x)^b$
  - Neglecting the D-term

❑ Sensitivity on DAs (total – q_T > 1 GeV):



❑ Sensitivity on GPDs (α = 0.63):



Summary and Outlook

- **Proposed a new type of exclusive two-scale observables – exclusive massive pair production (EMPP):**
  - These processes are factorizable and sensitive to pion DAs and hadron GPDs
  - Complementary to exclusive deep virtual lepton-hadron scattering processes, such as DVCS, DVMP, …
  - Hard scale is given by the invariant mass of the produced pair, not by a single virtual photon
  - More sensitive to the x-dependence of GPDs

- **This new type of exclusive two-scale observables could be measured at JLab, COMPASS, EIC, …**
  - More realistic numerical predictions are underway
  - More observables could be explored – hard part (the probe) should be sensitive to the momentum difference of the two partons from the diffracted hadron

Thank you!