5th Workshop on the
QCD Structure of the Nucleon
Alcalá de Henares, Madrid, SPAIN
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Universidad de Alcalá
QCD Structure of the Nucleon (QCD-N2021)

A new approach to inclusive lepton-hadron scattering with both QED & QCD factorization

- Lepton-hadron scattering, QED and QCD radiation
- Radiative correction vs. Radiative contribution
- Factorization and approximation
- Impact on Semi-Inclusive DIS
- Summary and outlook

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TMD Collaboration

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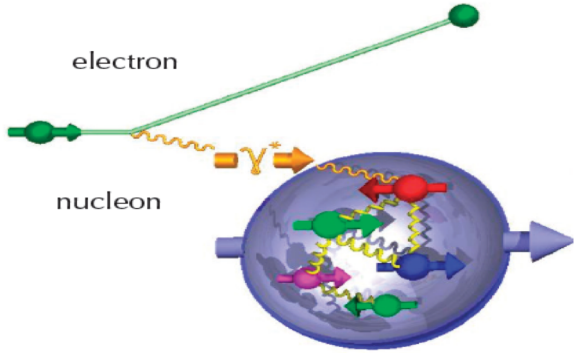
High-Energy Lepton-Hadron Scattering

- The new generation of "Rutherford" experiments for probing hadron structure:



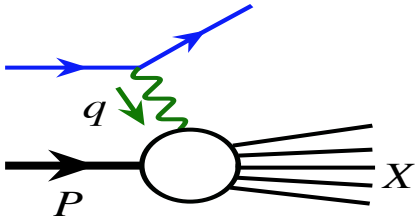
- A controlled "probe" – virtual photon
- Can either break or not break the hadron

High-energy lepton-hadron facilities have been built, or to be built, at SLAC, CERN, FNAL, DESY, JLab, BNL, …

- **Inclusive events**: e+p/A → e'+X

  Detect only the scattered lepton in the detector

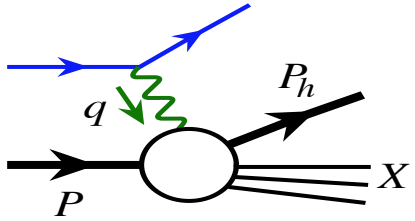
  (Modern Rutherford experiment!)



- **Semi-Inclusive events**: e+p/A → e'+h(p,K,p,jet)+X

  Detect the scattered lepton in coincidence with identified hadrons/jets

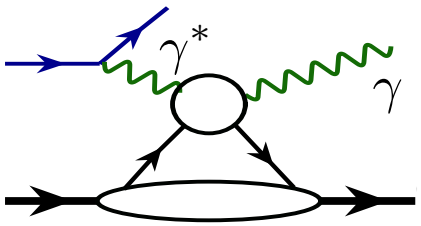
  (Initial hadron is broken – confined motion! – cleaner than h-h collisions)



- **Exclusive events:** e+p/A → e'+ p'/A'+ h(p,K,p,jet)

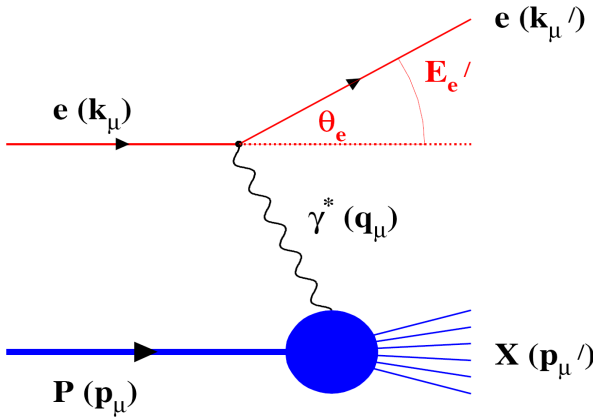
  Detect every things including scattered proton/nucleus (or its fragments)

  (Initial hadron is NOT broken – tomography! – almost impossible for h-h collisions)



Lepton-Hadron Inclusive Deep Inelastic Scattering

❑ Approximation of one-photon exchange:



$Q^2 = -(k-k')^2$ → Measure of the resolution

$y = P.(k-k')/P.k$ → Measure of inelasticity

$x_B = Q^2/2P.(k-k')$ → Measure of momentum fraction of the struck quark in a proton

$Q^2 = S\, x_B\, y$

$$E' \frac{d\sigma}{d^3k'} = \frac{2\alpha_{\rm EM}^2}{s} \frac{1}{Q^4} L^{\mu\nu}(k,k;q) W_{\mu\nu}(q,P)$$

$$L^{\mu\nu}(k,k;q) = 2(k^\mu k'^\nu + k^\nu k'^\mu - k\cdot k' g^{\mu\nu}) + \text{spin...}$$

❑ Deep inelastic scattering (DIS) structure functions:

$$W_{\mu\nu}(q,P) = \frac{1}{4\pi}\sum_X (2\pi)^4 \delta^4(P+q-X)\,\langle P|J_\mu(0)|X\rangle\,\langle X|J_\nu(0)|P\rangle + \text{spin...}$$

$$= -\widetilde{g}_{\mu\nu} F_1(x_B,Q^2) + \frac{\widetilde{P}_\mu \widetilde{P}_\nu}{P\cdot q} F_2(x_B,Q^2) + \text{spin...}$$

$$\widetilde{g}_{\mu\nu} = -g_{\mu\nu} + q_\mu q_\nu/q^2 \qquad \widetilde{P}_\mu = \widetilde{g}_{\mu\nu} P^\nu$$

Factorization

$$F_i(x_B,Q^2) \approx \sum_f C_{if}(x_B,Q^2;x,\mu^2) \otimes f(x,\mu^2) + \mathcal{O}(1/Q^2)$$

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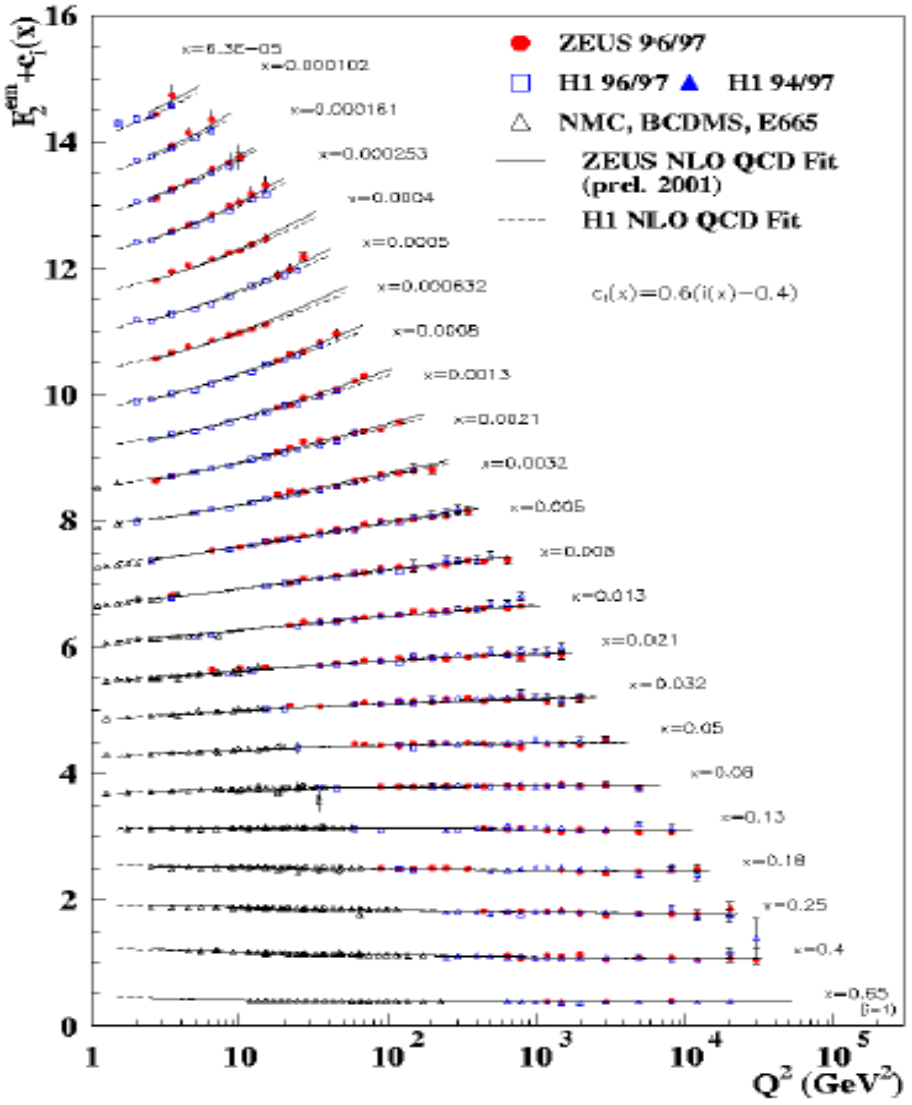
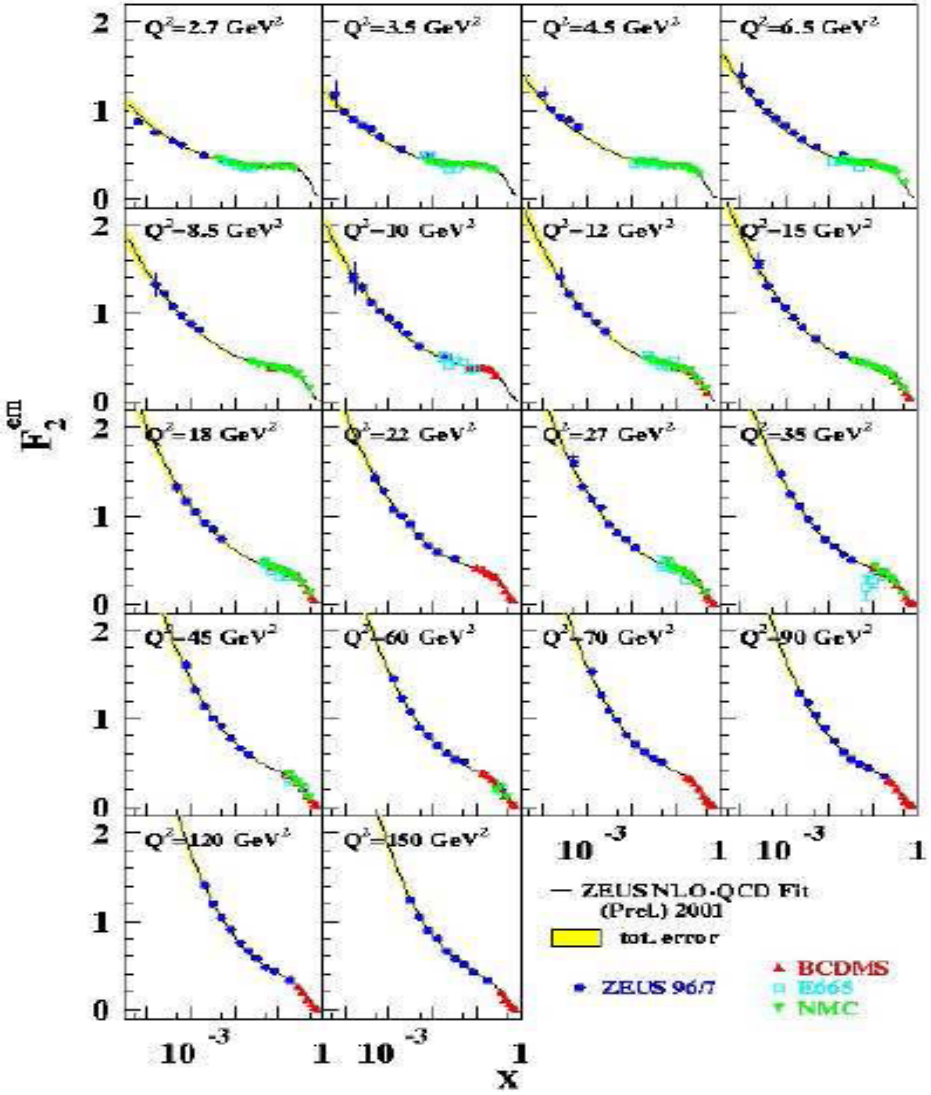
Lepton-Hadron Inclusive Deep Inelastic Scattering





A very successful story of QCD, QCD Factorization, and QCD evolution!

Extraction of Parton Distribution Functions (PDFs) – 1D hadron structure

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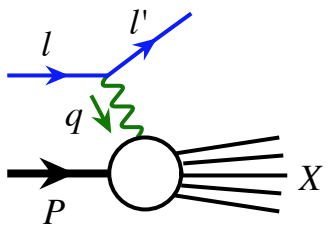
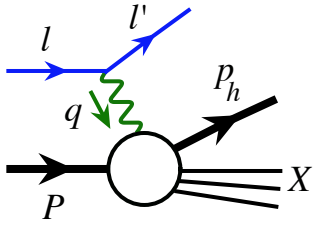
Lepton-Hadron Semi-Inclusive Deep Inelastic Scattering

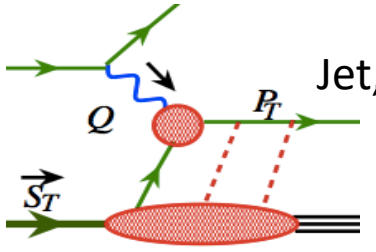


Scale: Q^2 **- PDFs**



$Q^2 \gg P_{h_T}^2$

In photon-hadron frame!

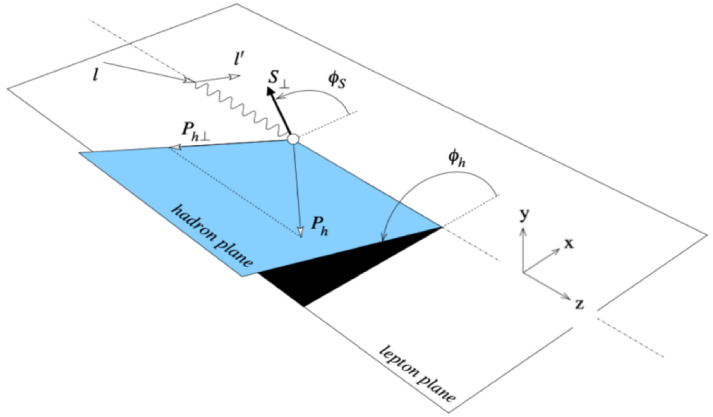


Jet, π, J/ψ, …

$f(x, k_T, Q)$ **- TMDs**

Parton's confined motion, …

$$
\begin{aligned}
\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} = \\
\frac{\alpha^2}{xyQ^2}\,\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Bigg\{ & F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)}\cos\phi_h\, F_{UU}^{\cos\phi_h} \\
& + \varepsilon\cos(2\phi_h)\, F_{UU}^{\cos 2\phi_h} + \lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\sin\phi_h\, F_{LU}^{\sin\phi_h} \\
& + S_\parallel\left[\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_h\, F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\, F_{UL}^{\sin 2\phi_h}\right] \\
& + S_\parallel\lambda_e\left[\sqrt{1-\varepsilon^2}\, F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_h\, F_{LL}^{\cos\phi_h}\right] \\
& + |\boldsymbol{S}_\perp|\Bigg[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) \\
& \quad + \varepsilon\sin(\phi_h+\phi_S)\, F_{UT}^{\sin(\phi_h+\phi_S)} + \varepsilon\sin(3\phi_h-\phi_S)\, F_{UT}^{\sin(3\phi_h-\phi_S)} \\
& \quad + \sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_S\, F_{UT}^{\sin\phi_S} + \sqrt{2\varepsilon(1+\varepsilon)}\sin(2\phi_h-\phi_S)\, F_{UT}^{\sin(2\phi_h-\phi_S)}\Bigg] \\
& + |\boldsymbol{S}_\perp|\lambda_e\Bigg[\sqrt{1-\varepsilon^2}\cos(\phi_h-\phi_S)\, F_{LT}^{\cos(\phi_h-\phi_S)} + \sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_S\, F_{LT}^{\cos\phi_S} \\
& \quad + \sqrt{2\varepsilon(1-\varepsilon)}\cos(2\phi_h-\phi_S)\, F_{LT}^{\cos(2\phi_h-\phi_S)}\Bigg]\Bigg\}
\end{aligned}
$$



18 SIDIS
Structure Functions

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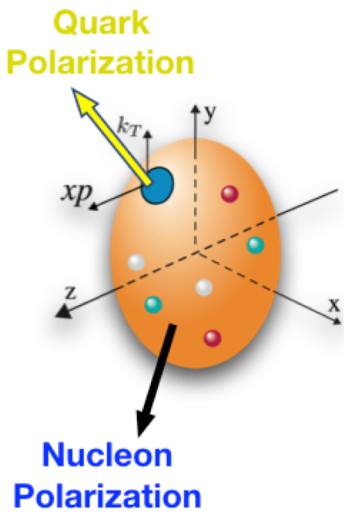
Transverse Momentum Dependent PDFs (TMDs)

❑ Quark TMDs with polarization:

Nucleon Polarization	Quark Polarization: Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
U	$f_1(x,k_T^2)$		$h_1^\perp(x,k_T^2)$ Boer-Mulders
L		$g_1(x,k_T^2)$ Helicity	$h_{1L}^\perp(x,k_T^2)$ Long-Transversity
T	$f_1^\perp(x,k_T^2)$ Sivers	$g_{1T}(x,k_T^2)$ Trans-Helicity	$h_1(x,k_T^2)$ Transversity; $h_{1T}^\perp(x,k_T^2)$ Pretzelosity

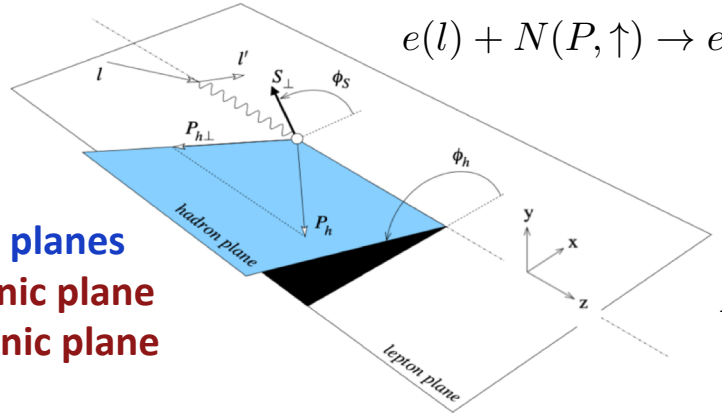


Analogous tables for:

- **Gluons** $f_1 \to f_1^g$ etc
- **Fragmentation functions**
- **Nuclear targets** $S \neq \frac{1}{2}$

❑ Polarized SIDIS:

$$e(l) + N(P,\uparrow) \to e(l') + h(P_h) + X$$



Two planes
Leptonic plane
Hadronic plane

Single Transverse-Spin Asymmetry

$$A_{UT} = \frac{1}{P}\frac{\sigma_{lN(\uparrow)} - \sigma_{lN(\downarrow)}}{\sigma_{lN(\uparrow)} + \sigma_{lN(\downarrow)}}$$

In photon-hadron frame:

$$A_{UT}^{Collins} \propto \langle \sin(\phi_h + \phi_S) \rangle_{UT} \propto h_1 \otimes H_1^\perp$$

$$A_{UT}^{Sivers} \propto \langle \sin(\phi_h - \phi_S) \rangle_{UT} \propto f_{1T}^\perp \otimes D_1$$

$$A_{UT}^{Pretzelosity} \propto \langle \sin(3\phi_h - \phi_S) \rangle_{UT} \propto h_{1T}^\perp \otimes H_1^\perp$$

Angular modulation provides the best way to separate TMDs

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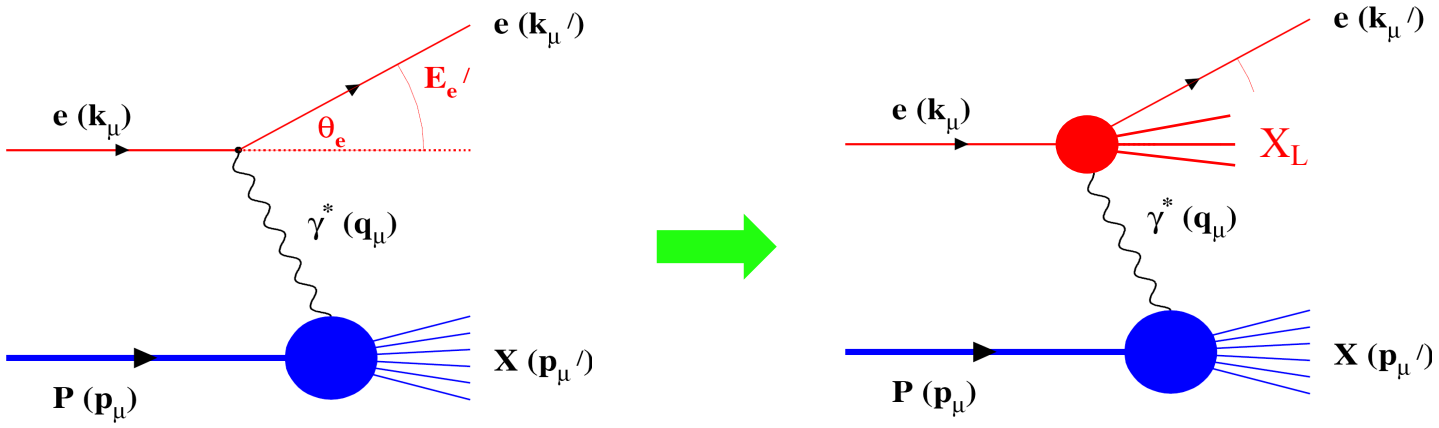
Collision Induced QED Radiation

- **Uncertainty in determining the photon-hadron frame:**



$$q_\mu \rightarrow \hat{q}_\mu$$

$$Q^2 = -q^2 \rightarrow \widehat{Q}^2 = -\hat{q}^2$$

$$x_B = \frac{Q^2}{2P\cdot q} \rightarrow \hat{x}_B = \frac{\widehat{Q}^2}{2P\cdot \hat{q}}$$

- **Under the "one-photon" approximation:**

$$E'\frac{d\sigma}{d^3k'} = \frac{2\alpha_{\rm EM}^2}{s}\frac{1}{Q^4}L^{\mu\nu}(k,k;q)W_{\mu\nu}(q,P) \rightarrow E'\frac{d\sigma}{d^3k'} = \frac{2\alpha_{\rm EM}^2}{s}\int d^4\hat{q}\left(\frac{1}{\hat{q}^2}\right)^2 \widetilde{L}^{\mu\nu}(k,k;\hat{q})W_{\mu\nu}(\hat{q},P)$$

$$\widetilde{L}^{\mu\nu}(k,k;\hat{q}) = \sum_{X_L}\int\prod_{i\in X_L}\frac{d^3k_i}{(2\pi)^3 2E_i}\,\delta^{(4)}\Big(k-k'-\hat{q}-\sum_{i\in X_L}k_i\Big)\,\langle k|j^\mu(0)|k'X_L\rangle\langle k'X_L|j^\nu(0)|k\rangle$$

$$= -\widetilde{g}^{\mu\nu}(\hat{q})\,L_1 + \frac{\widetilde{k}^\mu\widetilde{k}^\nu}{k\cdot k'}L_2 + \frac{\widetilde{k'}^\mu\widetilde{k'}^\nu}{k\cdot k'}L_3 + \frac{\widetilde{k}^\mu\widetilde{k'}^\nu+\widetilde{k'}^\mu\widetilde{k}^\nu}{2k\cdot k'}L_4$$

$$\rightarrow 2\big(k^\mu k'^\nu + k'^\mu k^\nu - k\cdot k' g^{\mu\nu}\big)\,\delta^{(4)}(k-k'-\hat{q})$$

$$\widetilde{g}^{\mu\nu}(\hat{q}) = g^{\mu\nu} - \frac{\hat{q}^\mu\hat{q}^\nu}{\hat{q}^2}$$

$$\widetilde{k}^\mu = \widetilde{g}^{\mu\nu}(\hat{q})k_\nu$$

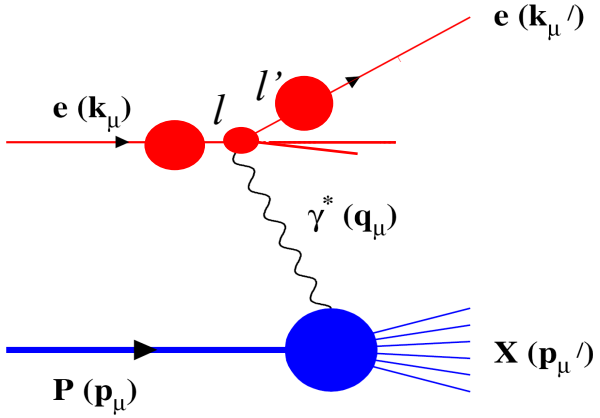
$$\widetilde{k'}^\mu = \widetilde{g}^{\mu\nu}(\hat{q})k'_\nu$$

4 Lepton structure functions! Small α leads to $\hat{q}_T^2 \ll \widehat{Q}^2$ in lepton back-to-back frame!

Collinear Factorization for QED Radiative Conntribution

- **Collinear factorization with the "one-photon" approximation:**

Liu, Melnitchouk, Qiu, Sato
2008.02895, 2108.13371



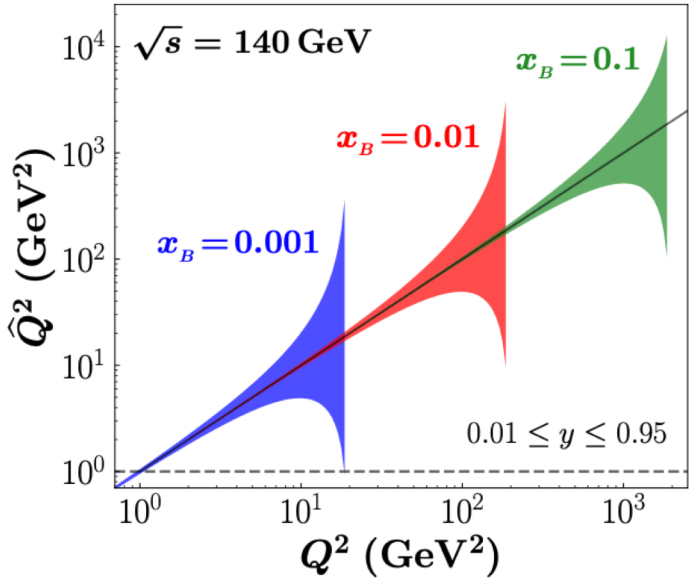
$$E_{k'}\frac{d^3\sigma_{kP\to k'X}}{d^3k'} \approx \int_{\zeta_{\min}}^{1}\frac{d\zeta}{\zeta^2}D_{e/e}(\zeta,\mu^2)\int_{\xi_{\min}}^{1}d\xi\, f_{e/e}(\xi,\mu^2)\left[E_{l'}\frac{d^3\hat{\sigma}_{lP\to l'X}}{d^3l'}\right]_{l=\xi k,\, l'=k'/\zeta}$$

$$\approx \int_{\zeta_{\min}}^{1}\frac{d\zeta}{\zeta^2}D_{e/e}(\zeta,\mu^2)\int_{\xi_{\min}}^{1}d\xi\, f_{e/e}(\xi,\mu^2)$$

$$\times\frac{\alpha^2}{\hat{y}\,\widehat{Q}^4}\left[\hat{x}_B\hat{y}^2\,F_1(\hat{x}_B,\widehat{Q}^2)+\left(1-\hat{y}-\frac{1}{4}\hat{y}^2\hat{\gamma}^2\right)F_2(\hat{x}_B,\widehat{Q}^2)\right]$$

- QED radiation prevents a well-defined "photon-hadron" frame
- Radiation is IR sensitive as $m_e/Q\to 0$, into LDFs & LFFs
- Hadron is probed by $(x_B,Q^2)\to(\hat{x}_B,\widehat{Q}^2)$

$$x_B\to\hat{x}_B\in[x_B,1] \qquad \widehat{Q}^2_{\min}=Q^2\frac{(1-y)}{(1-x_B\,y)} \qquad \widehat{Q}^2_{\max}=Q^2\frac{1}{(1-y+x_B\,y)}$$



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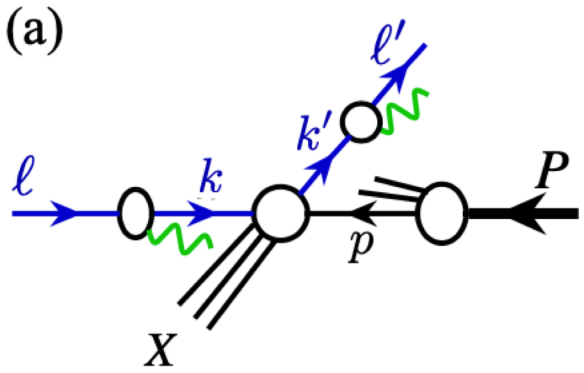
Collinear Factorization for QED Radiative Conntribution

- **Without the "one-photon" approximation:**

Liu, Melnitchouk, Qiu, Sato
2008.02895, 2108.13371

~ Inclusive single lepton production at high transverse momentum

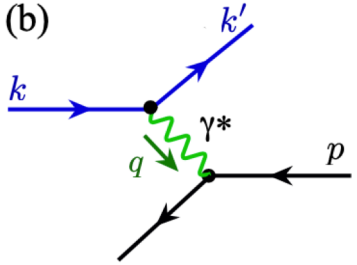


$$E_{k'}\frac{d\sigma_{kP\to k'X}}{d^3k'} = \frac{1}{2s}\sum_{i,j,a}\int_{\zeta_{\min}}^{1}\frac{d\zeta}{\zeta^2}\int_{\xi_{\min}}^{1}\frac{d\xi}{\xi}\,D_{e/j}(\zeta,\mu^2)\,f_{i/e}(\xi,\mu^2)$$
$$\times\int_{x_{\min}}^{1}\frac{dx}{x}\,f_{a/N}(x,\mu^2)\,\widehat{H}_{ia\to jX}(\xi k, xP, k'/\zeta,\mu^2)\;+\;\cdots$$

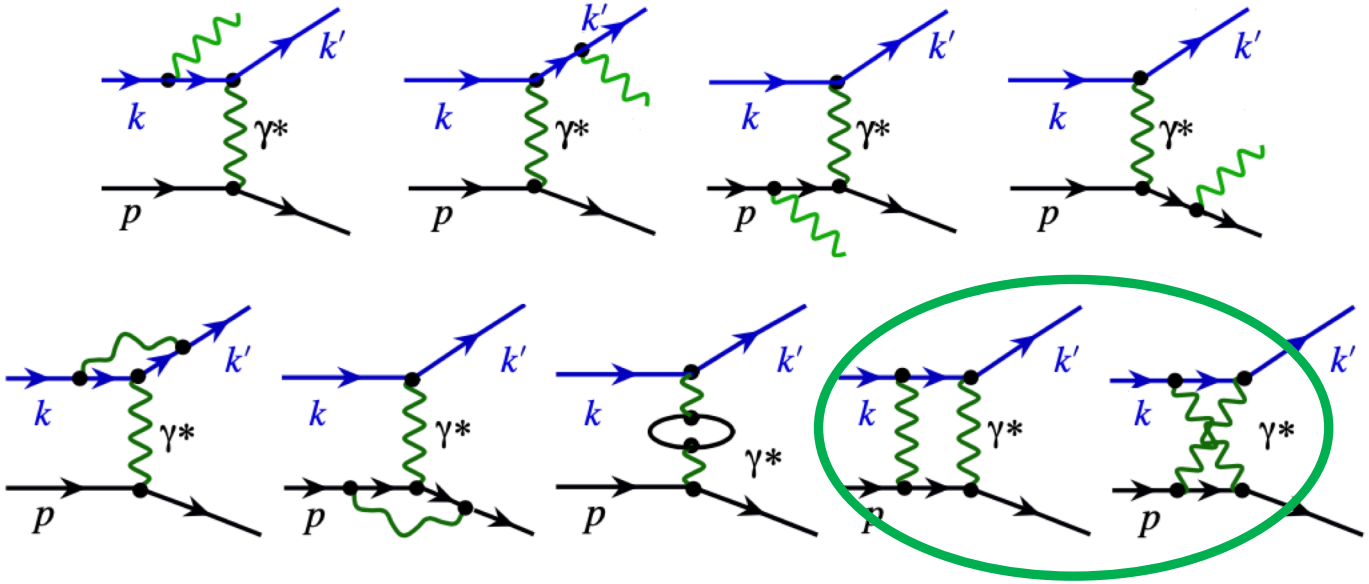
No structure functions, but have PDFs, LDFs, LFFs, …

- **Calculated hard parts in power of** $\alpha^m\alpha_s^n$ **:**

LO



NLO:



Beyond one-photon exchange

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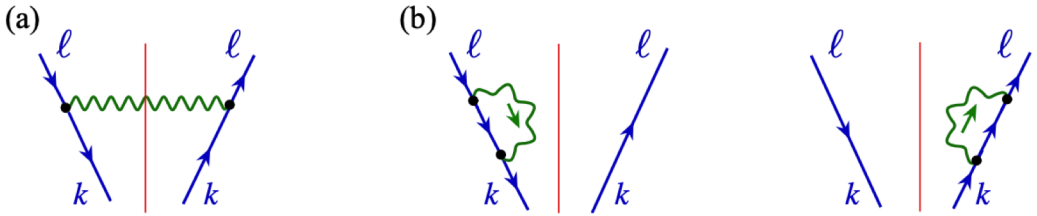
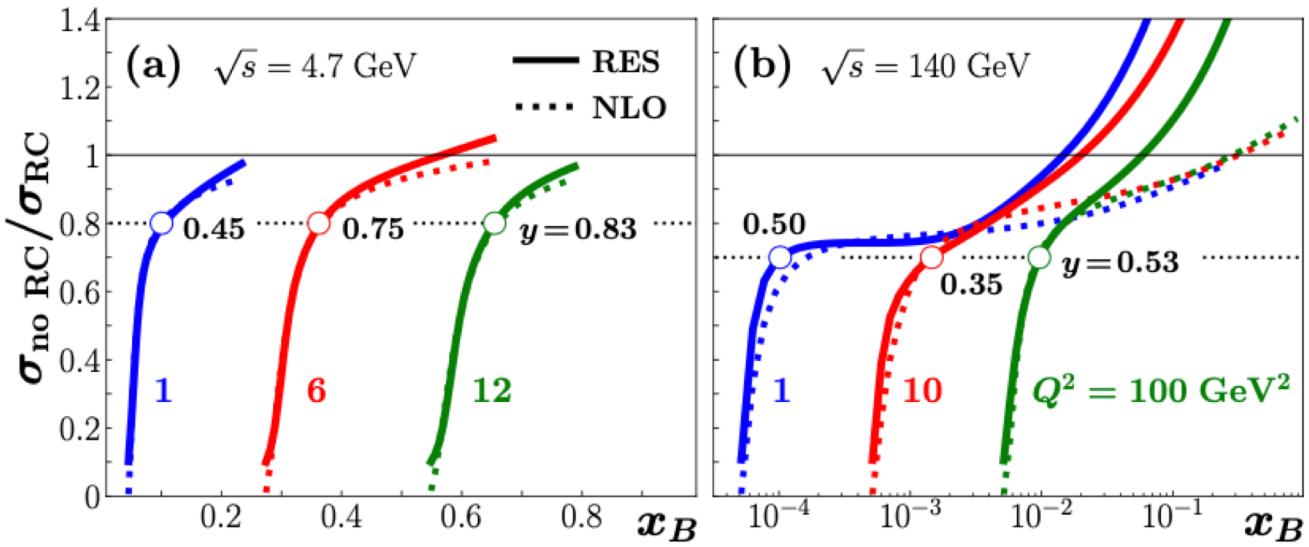
QED Radiative Corrections vs Radiative Contributions

- **Lepton distribution function:**



$$f^{(1)}_{e/e}(\xi,\mu^2) = \frac{\alpha}{2\pi}\left[\frac{1+\xi^2}{1-\xi}\ln\frac{\mu^2}{(1-\xi)^2 m_e^2}\right]_+$$

+ nonperturbative contributions …



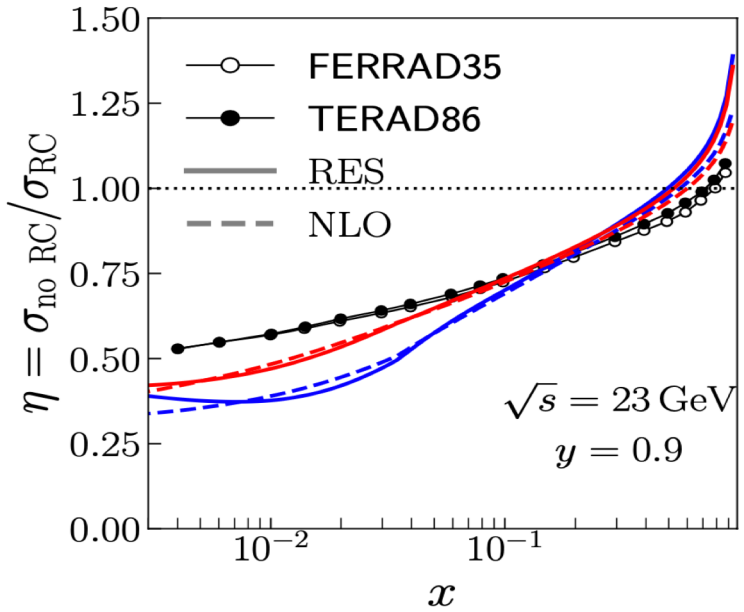
- **Lepton evolution – e.g., valence:**

$$\mu^2\frac{\mathrm{d}}{\mathrm{d}\mu^2}f_{e/e}(\xi,\mu^2) = \int_\xi^1 \frac{\mathrm{d}\xi'}{\xi'}P_{ee}\left(\frac{\xi}{\xi'},\alpha\right)f_{e/e}(\xi',\mu^2)$$

- **Lepton fragmentation function:**

$$D^{(1)}_{e/e}(\zeta,\mu) = \frac{\alpha}{2\pi}\left[\frac{1+\zeta^2}{1-\zeta}\ln\frac{\zeta^2\mu^2}{(1-\zeta)^2 m_e^2}\right]_+$$

+ nonperturbative contributions …



QED Radiative Corrections vs Radiative Contributions

Liu, Melnitchouk, Qiu, Sato
2008.02895, 2108.13371

❑ QED radiative corrections:

$$\sigma_{\rm obs}(x_B, Q^2) \neq R_{\rm QED}(x_B, Q^2; x_{B,\rm true}, Q^2_{\rm true}) \times \sigma_{\rm Born}(x_{B,\rm true}, Q^2_{\rm true}) + \sigma_X(x_B, Q^2)$$

- The correction factors R_{QED} and σ_x should not depend on the hadron structure that we wish to extract, and they can be systematically calculated in QED to high precision;
- The effective scale Q^2_{true} for the Born cross section σ_{Born} should be large enough to keep the "true" scattering within the DIS regime.
- Extraction of $\sigma_{\rm Born}$ is an inverse problem

❑ QED radiative contributions:

$$\sigma_{\rm obs}(x_B, Q^2) = \sigma^{\rm univ}_{\rm lep}(\mu^2; m_e^2) \otimes \sigma^{\rm univ}_{\rm had}(\mu^2; \Lambda^2_{\rm QCD}) \otimes \widehat{\sigma}_{\text{IR-safe}}(\hat{x}_B, \widehat{Q}^2, \mu^2) + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{Q^2}, \frac{m_e^2}{Q^2}\right)$$

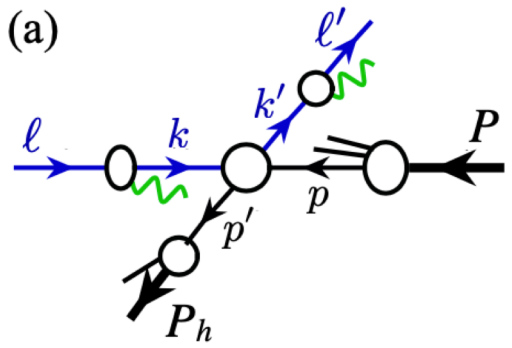
- Infrared sensitive QED contributions – divergent as $m_e/Q \to 0$, are absorbed to universal LDFs and LFFs
- Infrared safe QED contributions – finite as $m_e/Q \to 0$, are calculated order-by-order in power of α
- Power suppressed contributions as $m_e/Q \to 0$, are neglected

Predictive power: Universality of LDFs and LFFs, their evolution, calculable hard parts
Neglect power corrections

Lepton-Hadron Semi-Inclusive Deep Inelastic Scattering

- **Inclusive production of a lepton and a hadron:**

Liu, Melnitchouk, Qiu, Sato
2008.02895, 2108.13371



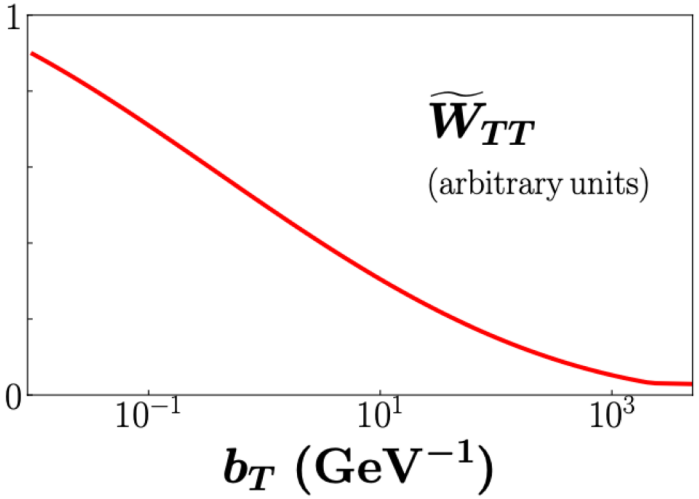
$$e(\ell) + N(P) \to e(\ell') + h(P_h) + X$$

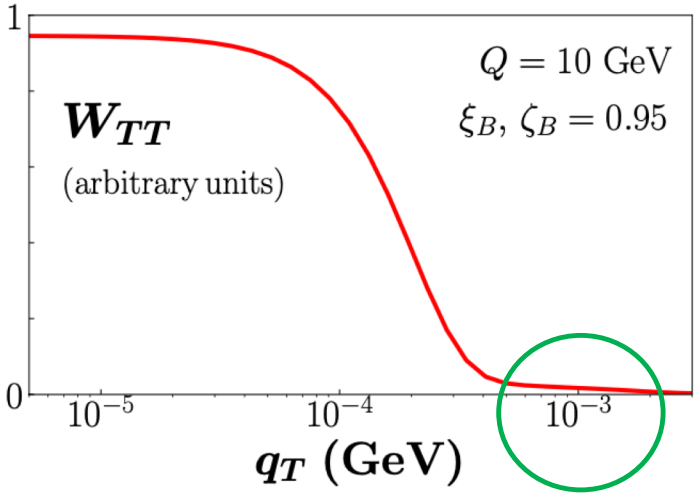
Momentum imbalance between the lepton and the hadron could be sensitive to both parton TMDs and lepton TMDs

Typical parton transverse momentum: $k_T^2 \sim \Lambda_{\rm QCD}^2 + \langle k_T^2 \rangle_{\text{generated by QCD shower}}$

- **Estimate of lepton transverse momentum generated by QED shower:**

Resummation to lepton TMD





QED broadening for lepton is so much smaller than typical parton kT!

Collinear factorization for high order QED contributions

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Lepton-Hadron Semi-Inclusive Deep Inelastic Scattering

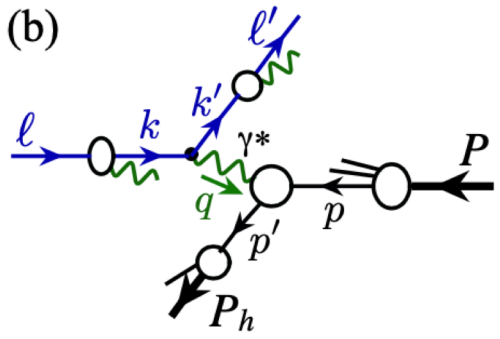
- **QED factorization of collision induced radiation – collinear:**

Liu, Melnitchouk, Qiu, Sato
2008.02895, 2108.13371

$$E_{\ell'}E_{P_h}\frac{\mathrm{d}^6\sigma_{\ell(\lambda_\ell)P(S)\to\ell' P_h X}}{\mathrm{d}^3\ell'\,\mathrm{d}^3P_h} \approx \sum_{ij\lambda_k}\int_{\zeta_{\min}}^1 \frac{\mathrm{d}\zeta}{\zeta^2}D_{e/j}(\zeta)\int_{\xi_{\min}}^1 \mathrm{d}\xi\, f_{i(\lambda_k)/e(\lambda_\ell)}(\xi)\left[E_{k'}E_{P_h}\frac{\mathrm{d}^6\hat\sigma_{k(\lambda_k)P(S)\to k' P_h X}}{\mathrm{d}^3k'\,\mathrm{d}^3P_h}\right]_{k=\xi\ell,k'=\ell'/\zeta} + \mathcal{O}\left(\frac{m_e^n}{Q^n}\right)$$

- Leading power IR sensitive contribution is universal, as $m_e/Q\to 0$, factorized into LDFs and LFFs
- IR safe contributions are calculated order-by-order in powers of α
- Neglect m_e/Q power suppressed contributions
- Collinear QED factorization for both inclusive DIS and SIDIS, or e^+e^-, … [global fits of LDFs, LFFs]

- **"One photon"-approximation:**



$$\frac{\mathrm{d}^6\sigma_{\ell(\lambda_\ell)P(S)\to\ell' P_h X}}{\mathrm{d}x_B\mathrm{d}y\,\mathrm{d}\psi\,\mathrm{d}z_h\,\mathrm{d}\phi_h\mathrm{d}P_{hT}^2} = \sum_{ij\lambda_k}\int_{\zeta_{\min}}^1\frac{\mathrm{d}\zeta}{\zeta^2}\int_{\xi_{\min}}^1\frac{\mathrm{d}\xi}{\xi}f_{i(\lambda_k)/e(\lambda_\ell)}(\xi)\,D_{e/j}(\zeta)$$

$$\times\frac{\hat x_B}{x_B\xi\zeta}\left[\frac{\alpha^2}{\hat x_B\,\hat y\,\widehat{Q}^2}\frac{\hat y^2}{2(1-\hat\varepsilon)}\left(1+\frac{\hat\gamma^2}{2\hat x_B}\right)\sum_n \hat w_n F_n^h(\hat x_B,\widehat{Q}^2,\hat z_h,\widehat{P}_{hT}^2)\right]$$

Evaluated in a "virtual photon-hadron" frame

Apply a (ξ,ζ)-dependent Lorentz transformation:

$$\{\hat q, P, \widehat{P}_h\} \xrightarrow{(\xi,\zeta)} \{q, P, P_h\}$$

In a frame to compare with exp. measurements

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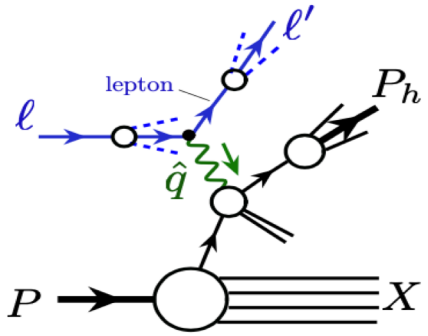
❑ Two-step approach to SIDIS:



One-photon approximation

1) In "virtual-photon" frame, defined by $\hat{q}(\xi,\zeta) - p$

- TMD factorization when $\widehat{P}_T^2 \ll \widehat{Q}^2$
- CO factorization when $\widehat{P}_T^2 \sim \widehat{Q}^2$
- Matching to get the $\widehat{P}_T$-distribution

2) Lorentz transformation from the "virtual-photon" frame to any experimentally defined frame – lepton-hadron Lab frame, Breit frame (x_B,Q^2), …

QED contribution (not correction) can be systematically improved order-by-order in power α!

❑ Case study F_{UU}:

$$
\begin{aligned}
\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} = \\
&\frac{\alpha^2}{xyQ^2}\,\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Bigg\{\boxed{F_{UU,T}} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} \\
&+ \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h} + \lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} \\
&+ S_\parallel\left[\sqrt{2\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right] \\
&+ S_\parallel\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right] \\
&+ |\boldsymbol{S}_\perp|\Bigg[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) \\
&\quad + \varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)} + \varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} \\
&\quad + \sqrt{2\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S} + \sqrt{2\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Bigg] \\
&+ |\boldsymbol{S}_\perp|\lambda_e\Bigg[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)} + \sqrt{2\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} \\
&\quad + \sqrt{2\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Bigg]\Bigg\}
\end{aligned}
$$

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- **Case study F_{UU}:**

$$\frac{d\sigma^h_{\rm SIDIS}}{dx_B dy\, dz\, dP^2_{hT}} = \int_{\zeta_{\min}}^{1} d\zeta \int_{\xi_{\min(\zeta)}}^{1} d\xi\, D_{e/e}(\zeta)\, f_{e/e}(\xi) \times \underbrace{\left[\frac{\hat{x}_B}{x_B\,\xi\zeta}\right]\left[\frac{(2\pi)^2\,\alpha}{\hat{x}_B\hat{y}\,\widehat{Q}^2}\frac{\hat{y}^2}{2(1-\hat{\varepsilon})}F^h_{UU}(\hat{x}_B,\widehat{Q}^2,\hat{z},\widehat{P}_{hT})\right]}_{\text{Evaluated in a “virtual photon-hadron” frame}}$$

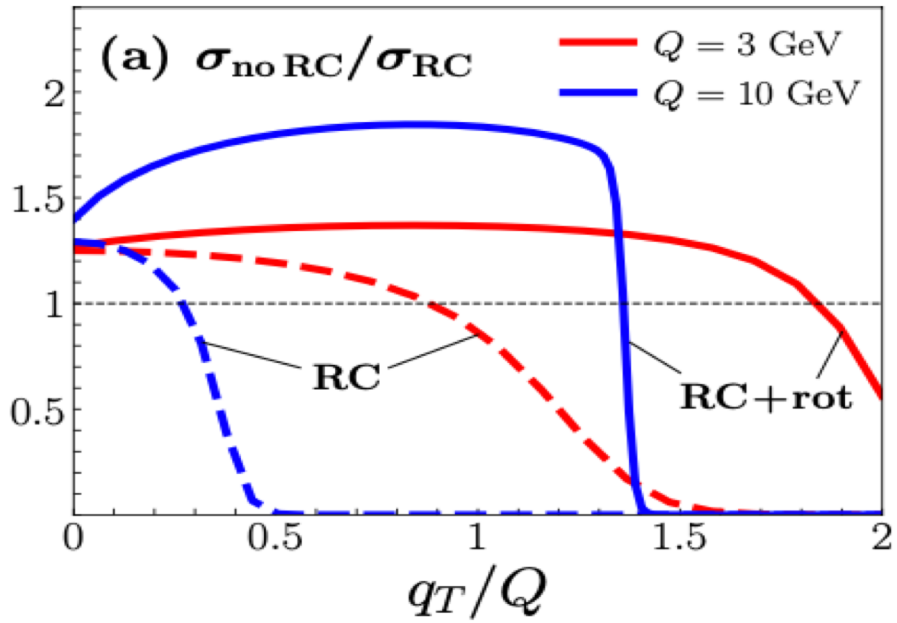
Unpolarized structure function:

$$F^h_{UU} = x_B \sum_q e_q^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, \delta^{(2)}(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{q}_T) \times f_{q/N}(x_B, \boldsymbol{p}_T^2)\, D_{h/q}(z, \boldsymbol{k}_T^2) \qquad \boldsymbol{q}_T = \boldsymbol{P}_{hT}/z$$

(ξ, ζ) **- Dependent Lorentz transformation**

Effectively, a rotation in hadron-rest frame

Solid – with rotation
Dashed – without rotation



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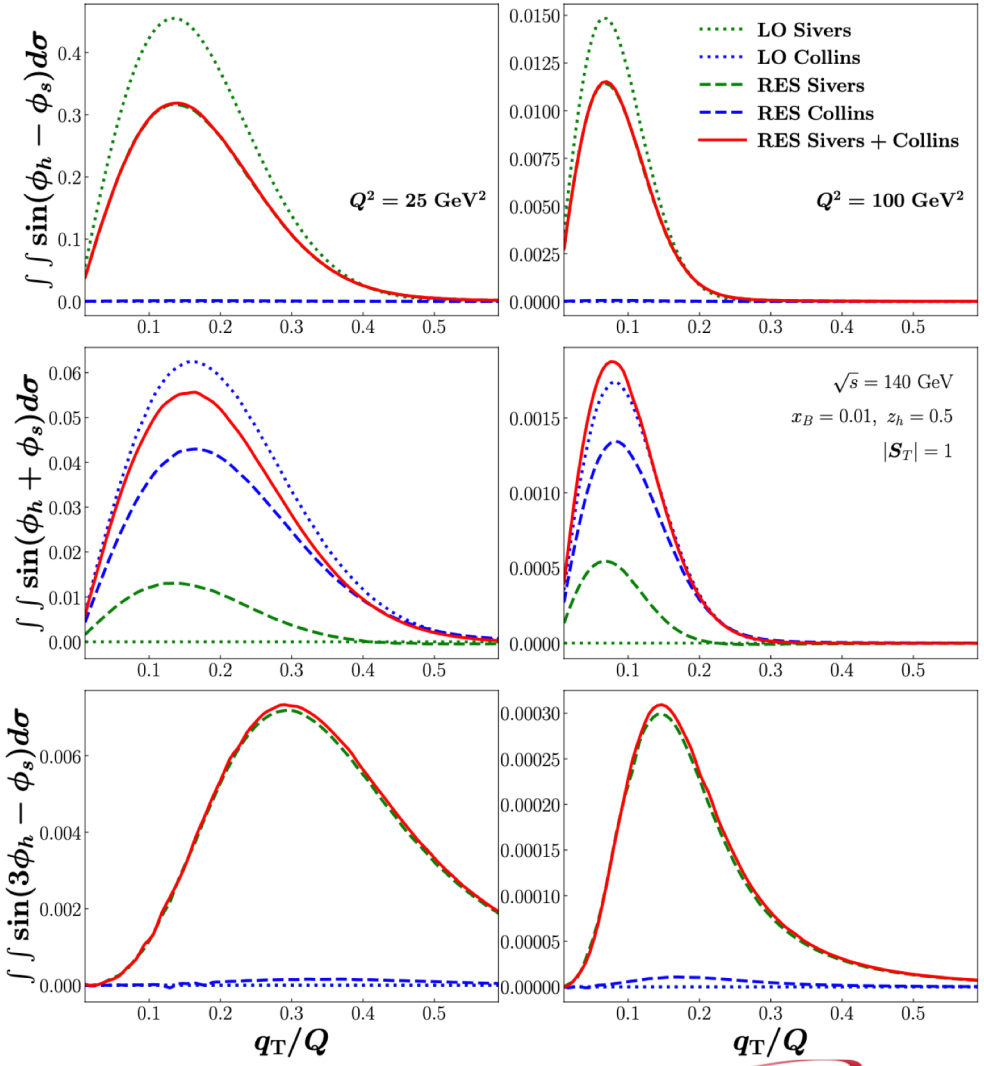
- **Case study – single transverse spin asymmetry:**

Liu, Melnitchouk, Qiu, Sato
2008.02895, 2108.13371

$$
\begin{aligned}
\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} = \\
&\frac{\alpha^2}{xyQ^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Bigg\{F_{UU,T}+\varepsilon\,F_{UU,L}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} \\
&+\varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}+\lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} \\
&+S_\parallel\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h}+\varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right] \\
&+S_\parallel\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right] \\
&+|\boldsymbol{S}_\perp|\Bigg[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}+\varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) \\
&\quad+\varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}+\varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} \\
&\quad+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Bigg] \\
&+|\boldsymbol{S}_\perp|\lambda_e\Bigg[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} \\
&\quad+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Bigg]\Bigg\}
\end{aligned}
$$



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Lepton-Hadron Semi-Inclusive Deep Inelastic Scattering

- **QED radiative corrections:**

Radiative Effects in the Processes of Hadron Electroproduction

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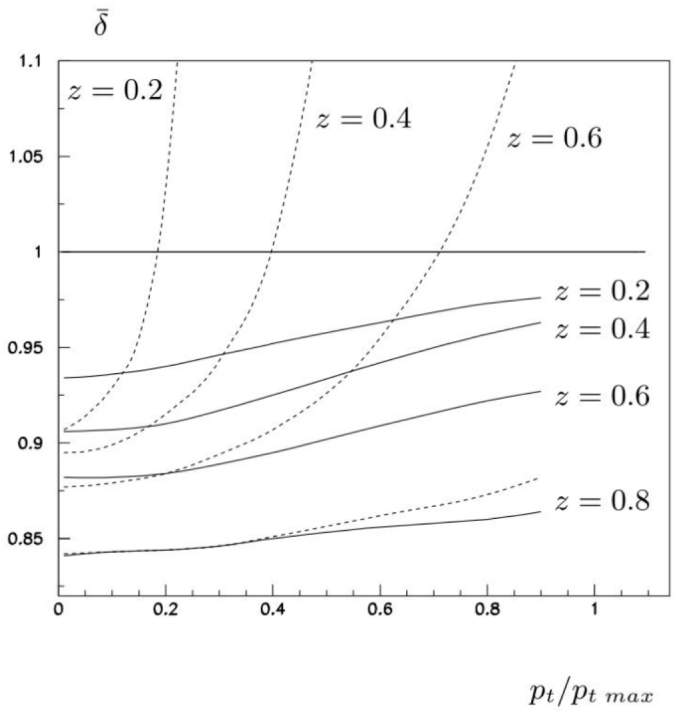
Received: date / Revised version: date

Abstract. An approach to calculate radiative corrections to unpolarized cross section of semi-inclusive electroproduction is developed. An explicit formulae for the lowest order QED radiative correction are presented. Detailed numerical analysis is performed for the kinematics of experiments at the fixed targets.



Similar trends. Eg. RCs depends on hadronic input

- **Our formalism is different. It does not depend on hadronic input, which is what we want to probe!**
- **Our formalism is organized in terms of IR safe quantities and universal functions – advantage of factorization**

Summary and Outlook

- **Radiative corrections are very important for lepton-hadron scattering**
  - Especially difficult for a consistent treatment beyond the inclusive DIS
  - No well-defined photon-hadron frame, if we cannot recover all QED radiation
  - Radiative corrections are more important for events with high momentum transfers and large phase space to shower – such as those at the EIC
- **We proposed a factorization based treatment of QED radiation, along with QCD factorization**
  - QED radiation is a part of production cross sections, treated in the same way as QCD radiation from quarks and gluons
  - No artificial and/or process dependent scale(s) introduced for treating QED radiation, other than the standard factorization scale
  - All perturbatively calculable hard parts are IR safe for both QCD and QED
  - All lepton mass or resolution sensitivity are included into "Universal" lepton distribution and fragmentation functions (or jet functions)

Thank you!

Special thanks to experimental colleagues for helpful discussions!