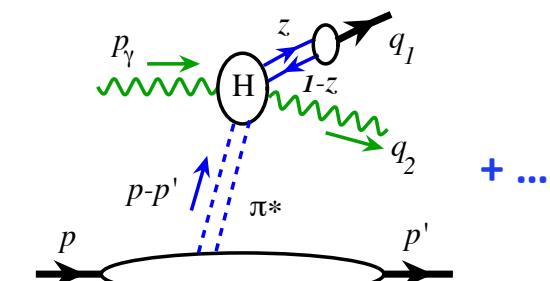
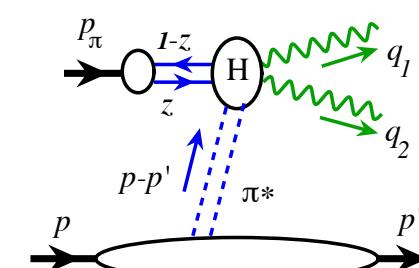
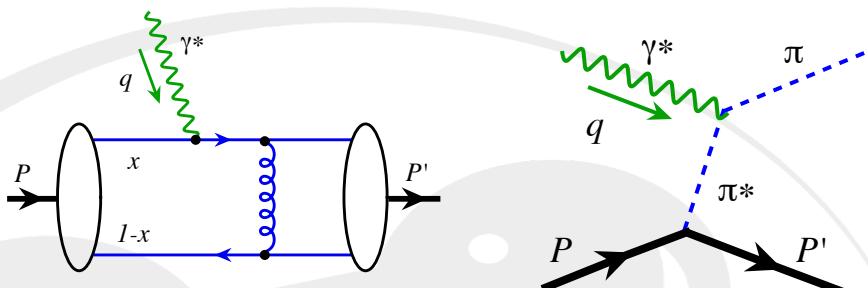


New Exclusive Processes to Better Measure the x-Dependence of Pion DAs

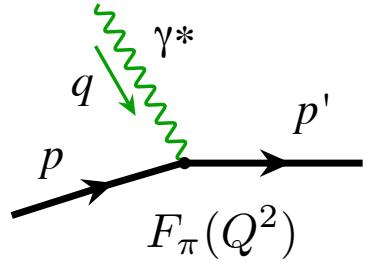


Jian-Wei Qiu
Jefferson Lab, Theory Center

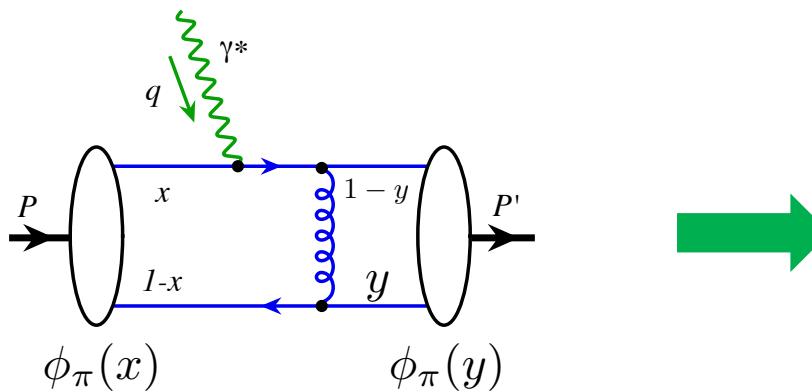
In collaboration with: Zhite Yu (Michigan State University)

Pion Form Factor – Pion Distribution Amplitude

□ Pion form factor: $F_\pi(Q^2)$



$$Q^2 = -q^2 \approx$$

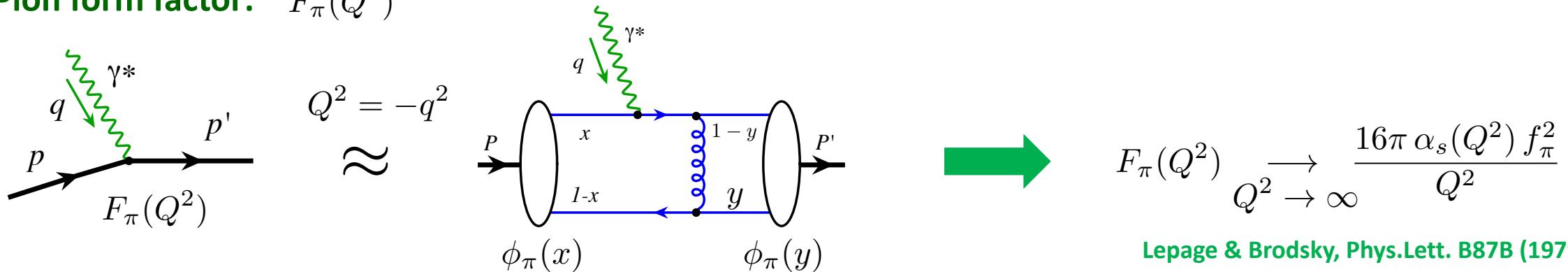


$$F_\pi(Q^2) \xrightarrow[Q^2 \rightarrow \infty]{\frac{16\pi \alpha_s(Q^2) f_\pi^2}{Q^2}}$$

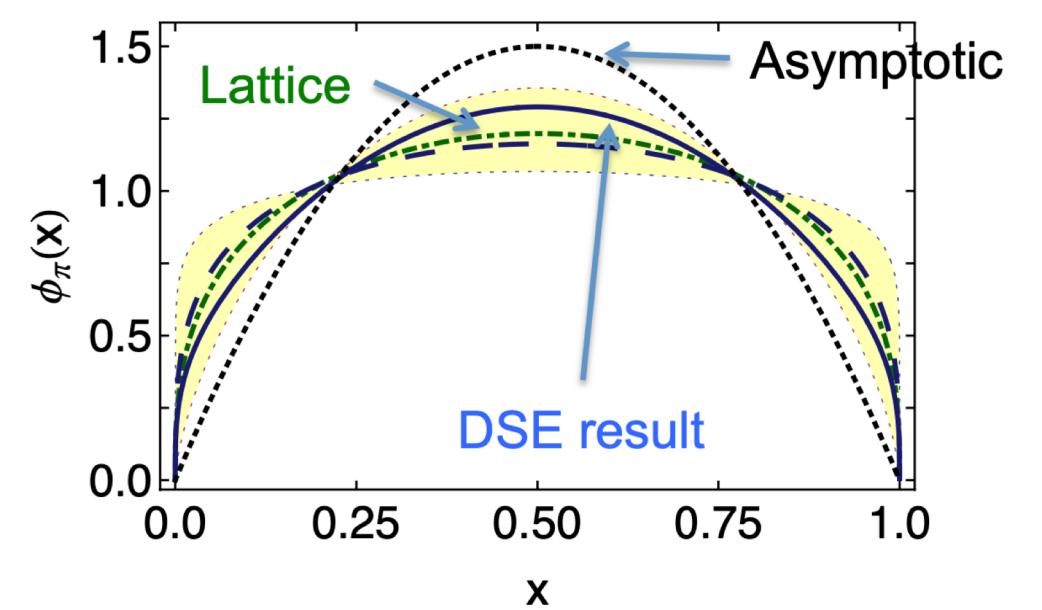
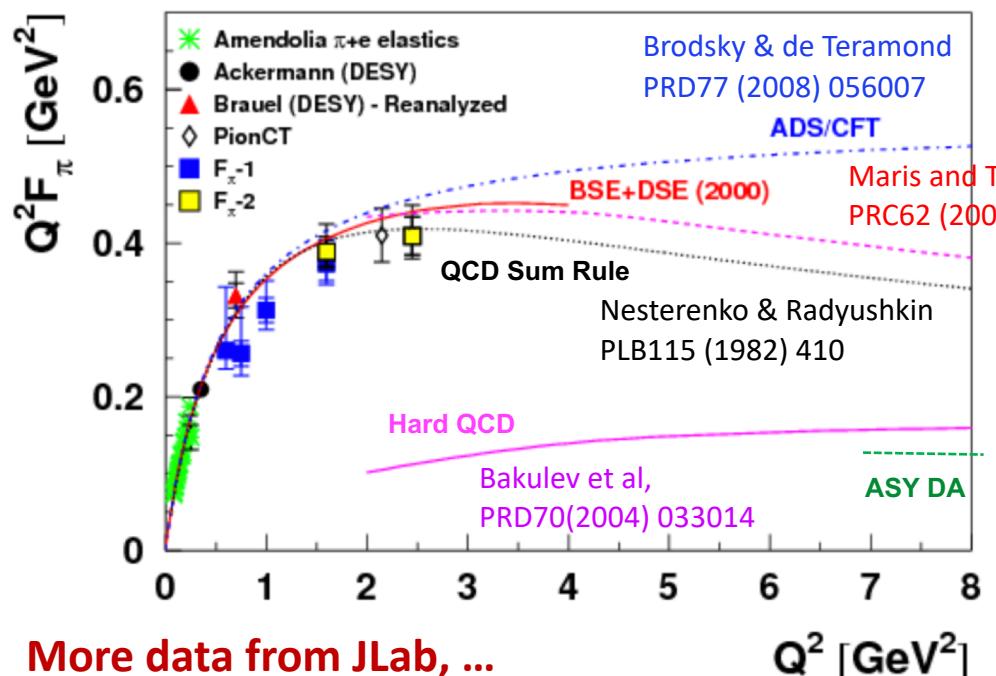
Lepage & Brodsky, Phys.Lett. B87B (1979)359

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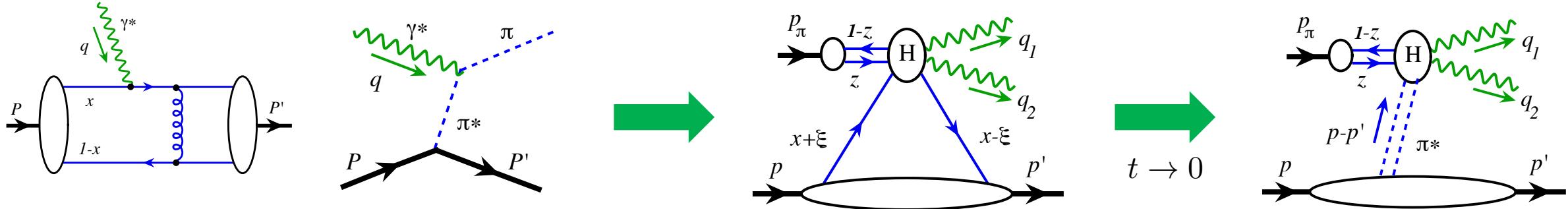
□ Data and models:



FFs not sensitive to DA's x -dependence!

Exclusive Massive Two-Photon Production

Massive photon pair: $\pi^-(p_\pi) + P(p) \rightarrow \gamma(q_1) + \gamma(q_2) + N(p')$



Hard scale: $Q^2 = \hat{s} \equiv (q_1 + q_2)^2$

Soft scale: $t = (p - p')^2$

$$\xi = \frac{(p - p')^+}{(p + p')^+} \quad P^+ = \frac{(p + p')^+}{2}$$

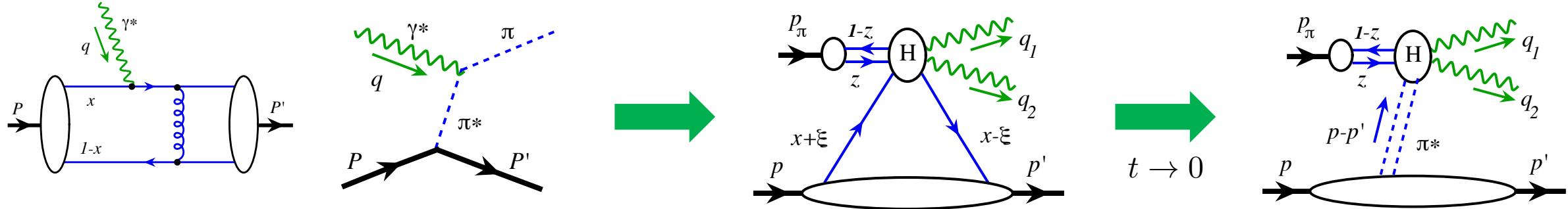
Momentum transfer: $\Delta \equiv p - p'$

Leading power:

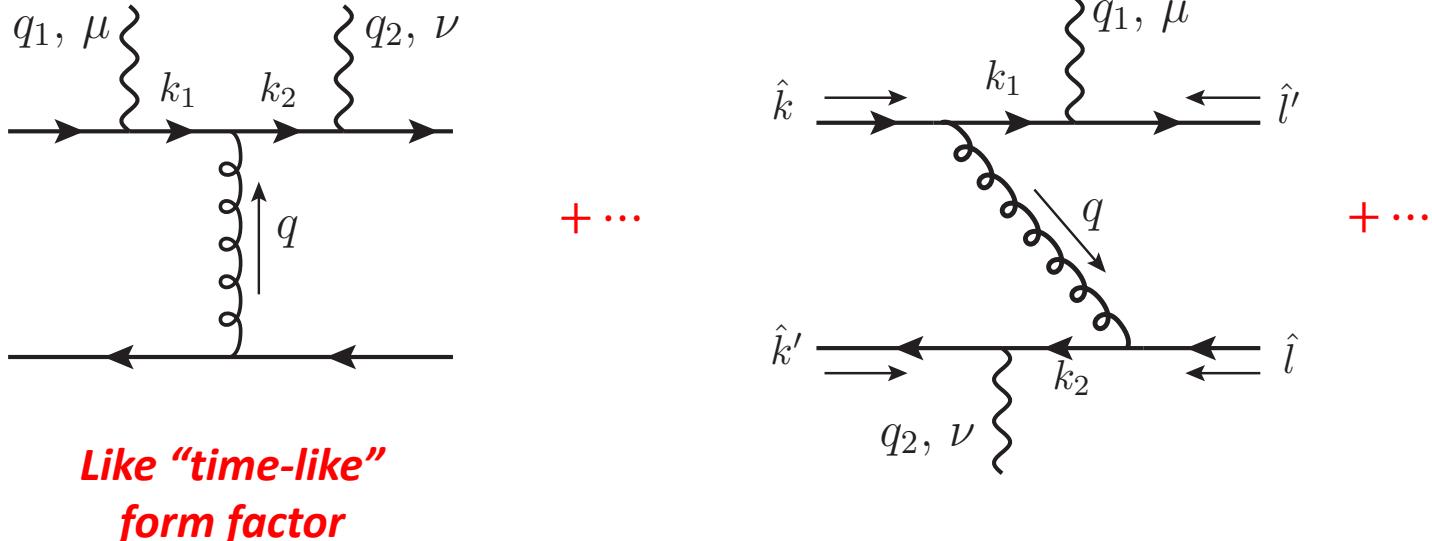
$$\Delta^+ = 2\xi P^+ = (p - p')^+$$

Exclusive Massive Two-Photon Production

- **Massive photon pair:** $\pi^-(p_\pi) + P(p) \rightarrow \gamma(q_1) + \gamma(q_2) + N(p')$



- Much more sensitive to the x -dependence of DAs:



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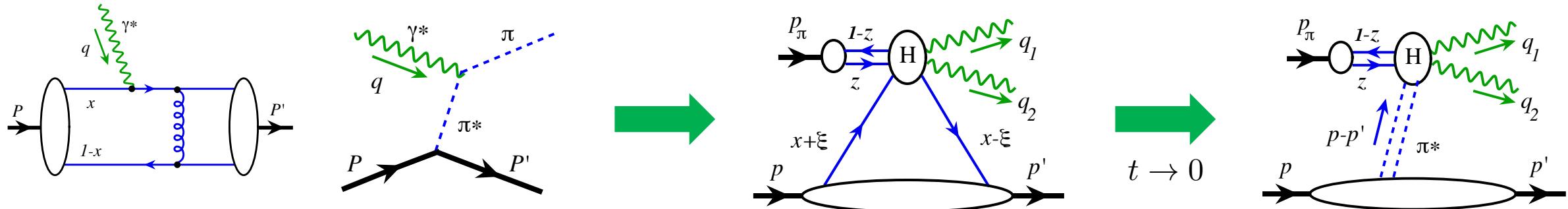
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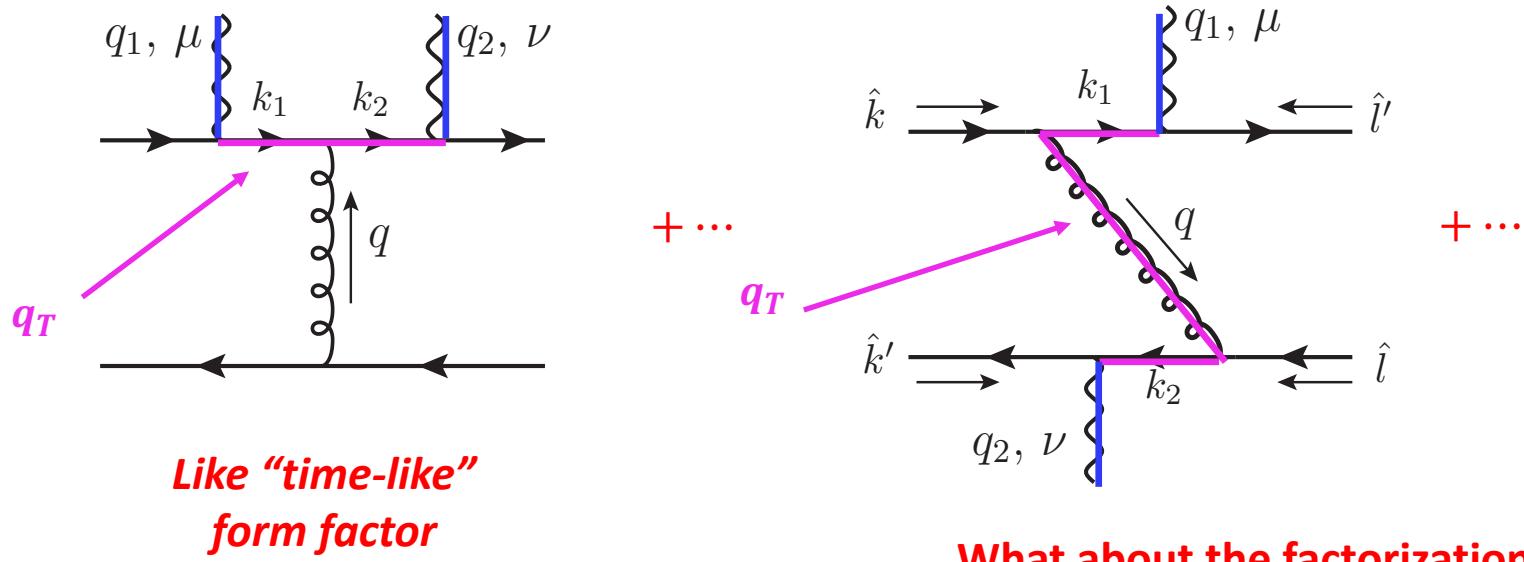
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Exclusive Massive Two-Photon Production

□ **Massive photon pair:**

$$\pi^-(p_\pi) + P(p) \rightarrow \gamma(q_1) + \gamma(q_2) + N(p')$$

□ **The sensitive observable: Q, q_T**

(in $\gamma\gamma$ CM, with π^- in $-\hat{z}$ direction)

$$\gamma(q_T) + \gamma(-q_T)$$

□ **Factorization – necessary conditions:**

$$\hat{s} \gg |t| \gtrsim \Lambda_{\text{QCD}}^2$$

Requires the time of the hard collision $\sim 1/\sqrt{\hat{s}}$

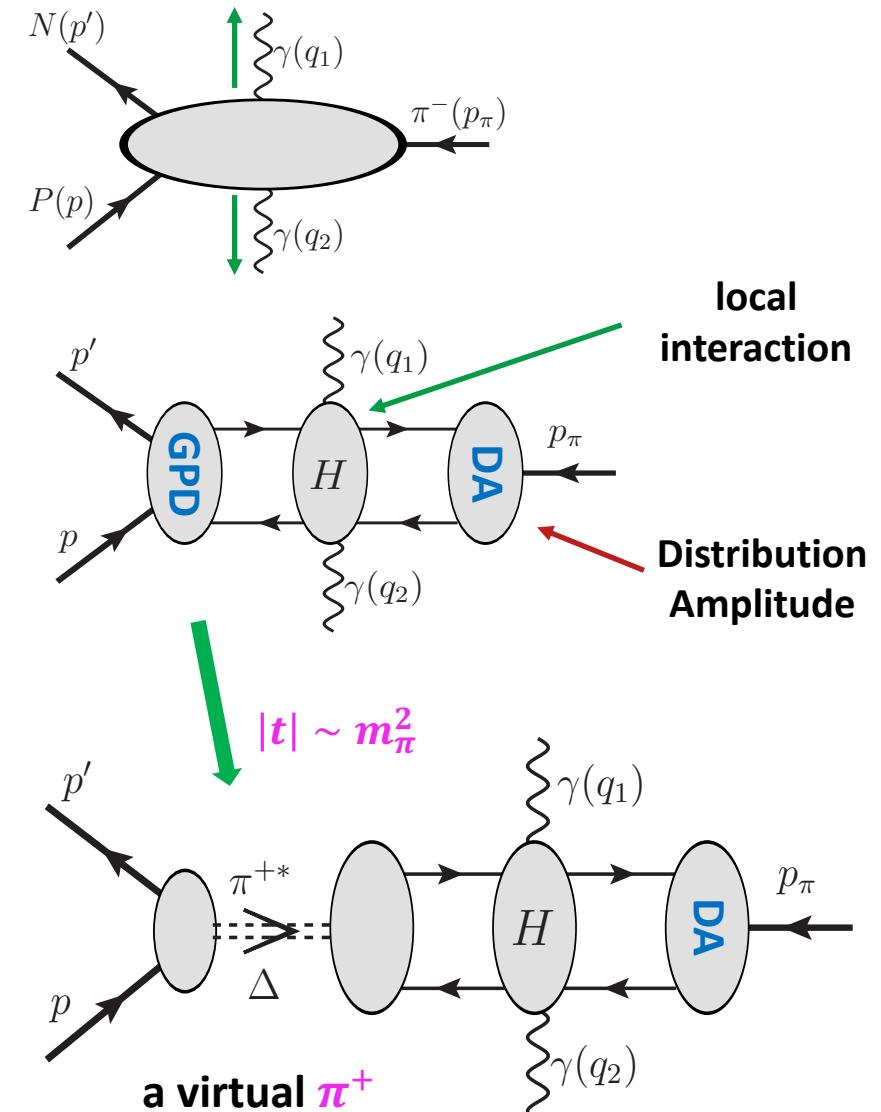
$$\Delta^+ = (p - p')^+ \gg \sqrt{|t|}$$

to be much shorter than the lifetime of the exchanged

$q\bar{q}$ (or gg) state

$$q_T^2 \gg \Lambda_{\text{QCD}}^2$$

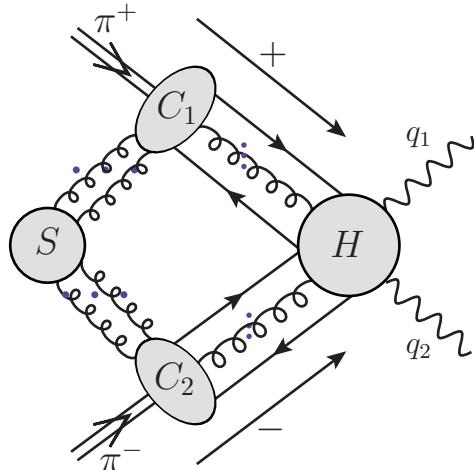
q_T could be as small as 0 while $\sqrt{\hat{s}}$ is the hard scale



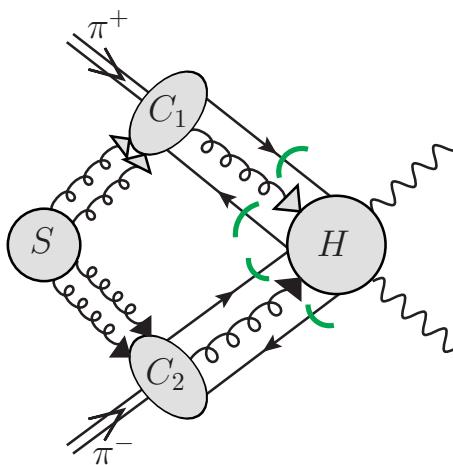
Exclusive Massive Two-Photon Production

□ Factorization: $\pi(p_1) + \pi(p_2) \rightarrow \gamma(q_1) + \gamma(q_2)$

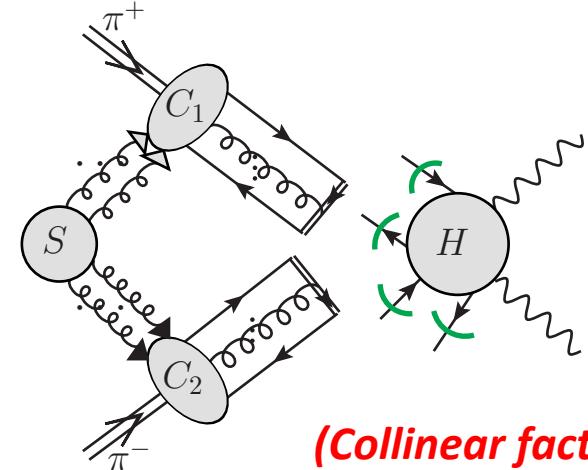
■ Leading region



■ Approximations



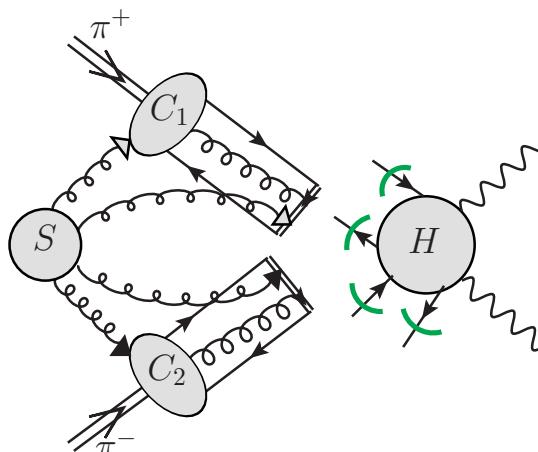
■ Ward identity for collinear gluons



(Collinear factorization)

■ Ward identity for soft gluons

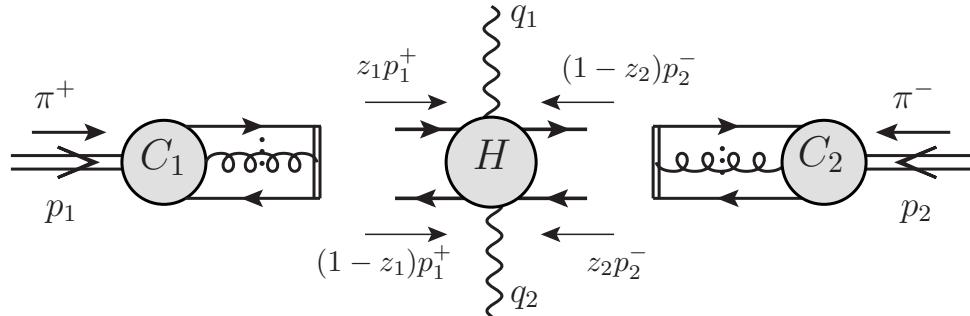
- Soft gluons are as if attached to a “closed fermion loop”
- Sum over diagrams
 $\Rightarrow S = 0$



Soft gluons cancel because collinear parton lines are in color singlet states.

Exclusive Massive Two-Photon Production

□ Factorization: $\pi(p_1) + \pi(p_2) \rightarrow \gamma(q_1) + \gamma(q_2)$



$$\mathcal{M} = \frac{s}{2} \int_0^1 dz_1 dz_2 \phi_{\pi^+}(z_1) \phi_{\pi^-}(z_2) \cdot \text{Tr} \left[\frac{\gamma_5 \gamma^-}{2} H(\hat{k}_1, \hat{k}_2; q_1, q_2; \mu) \frac{\gamma_5 \gamma^+}{2} \right] + \mathcal{O}\left(\frac{m_\pi}{q_T}\right) \quad \longrightarrow \quad \frac{d\sigma}{dq_T} \propto |\mathcal{M}|^2$$

□ Hadron functions: distribution amplitudes (DA):

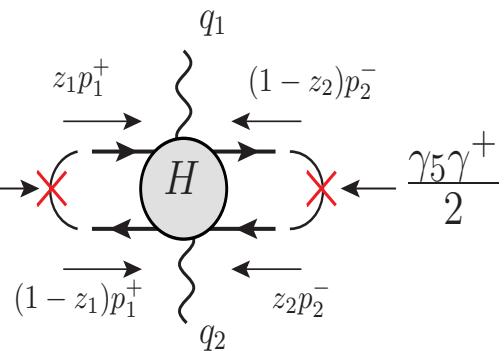
$$\phi_{\pi^+}(z_1) = \int \frac{dx^-}{4\pi} e^{i z_1 p_1^+ x^-} \langle 0 | \bar{d}(0) \gamma^+ \gamma_5 W(0, x^-) u(x^-) | \pi^+(p_1) \rangle$$

$$\phi_{\pi^-}(z_2) = \int \frac{dx^+}{4\pi} e^{i z_2 p_2^- x^+} \langle 0 | \bar{u}(0) \gamma^- \gamma_5 W(0, x^+) d(x^+) | \pi^-(p_2) \rangle$$

$\phi_{\pi^+}(z) = \phi_{\pi^-}(z) = \phi(z)$
are universal DAs

□ Hard coefficient

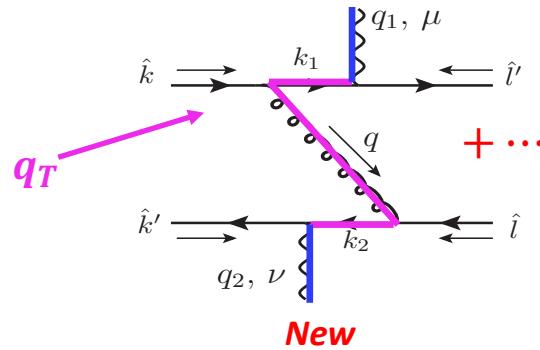
$$C \left(z_1, z_2; \frac{q_T^2}{s}; \frac{q_T^2}{\mu^2} \right) = \frac{\gamma_5 \gamma^-}{2} \rightarrow \cancel{x} \quad \cancel{x} \quad \frac{\gamma_5 \gamma^+}{2}$$



Projections (for π^\mp):
 1. Spin-0 $\gamma_5 \gamma^\pm$
 2. P-odd

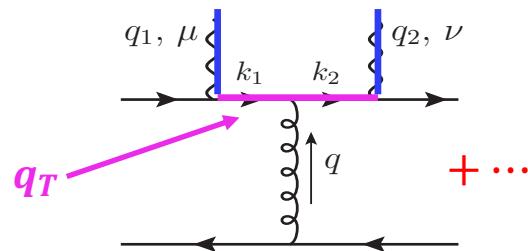
Exclusive Massive Two-Photon Production

□ Hard part for A-type:



- **Gluon propagator** $q^2 = -\frac{\hat{s}}{4} [(2z_1 - 1 - \sqrt{1 - \kappa})(2z_2 - 1 - \sqrt{1 - \kappa}) + \kappa]$
- ➡ $\mathcal{M} \propto \int_0^1 dz_1 dz_2 \frac{\phi(z_1)\phi(z_2)}{(1-z_1)(1-z_2) [(2z_1 - 1 - \sqrt{1 - \kappa})(2z_2 - 1 - \sqrt{1 - \kappa}) + \kappa]}$
- Change q_T changes the z_1 - z_2 integral.
 - $d\sigma/dq_T^2$ provides sensitivity to the DA's functional form of z .

□ Hard part for B-type:



Like “time-like”
form factor

- **Gluon propagator** $q^2 = z_2(1 - z_1)\hat{s}$
- ➡ $\mathcal{M} \propto \int_0^1 dz_1 dz_1 \frac{\phi(z_1)\phi(z_2)}{z_1(1-z_1)z_2(1-z_2)} \sim \left[\int_0^1 dz \frac{\phi(z)}{z(1-z)} \right]^2$
- Not sensitive to DA functional form.
 - Relies on $\phi(z) = 0$ at end points.
 - Sudakov resummation could suppress the end-point sensitivity.

Li, Sterman, 1992

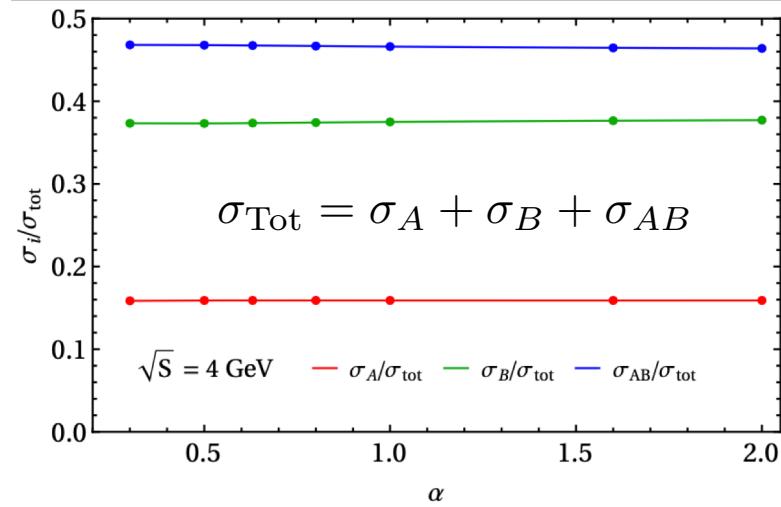
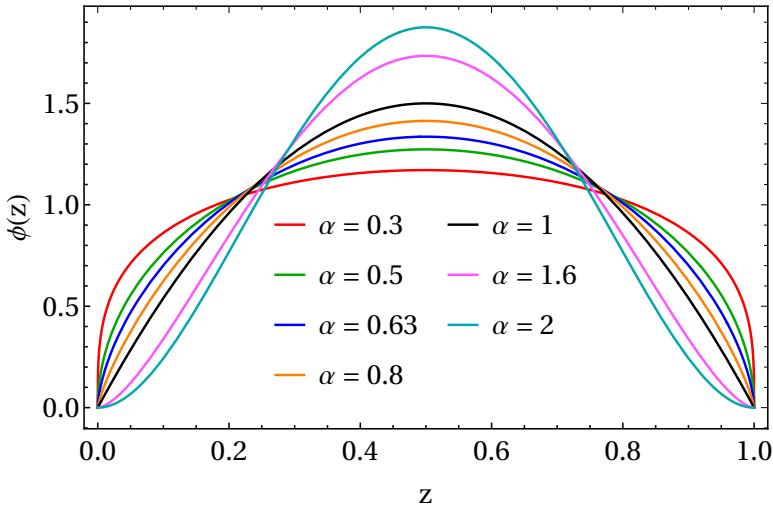
Exclusive Massive Two-Photon Production

□ DA parametrization:

$$\phi_\alpha(z) = \frac{i f_\pi}{2} \cdot \left[\frac{z^\alpha (1-z)^\alpha}{B(1+\alpha, 1+\alpha)} \right]$$

Change α

⇒ Change z dependence



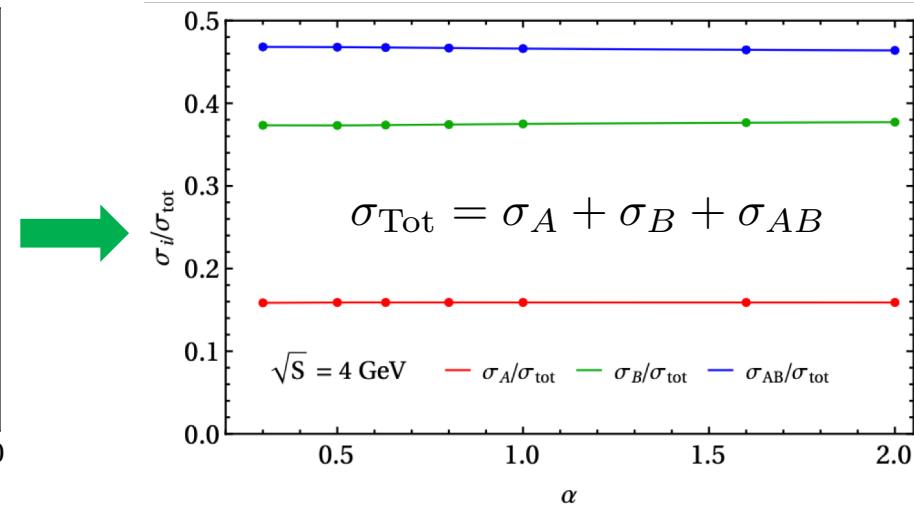
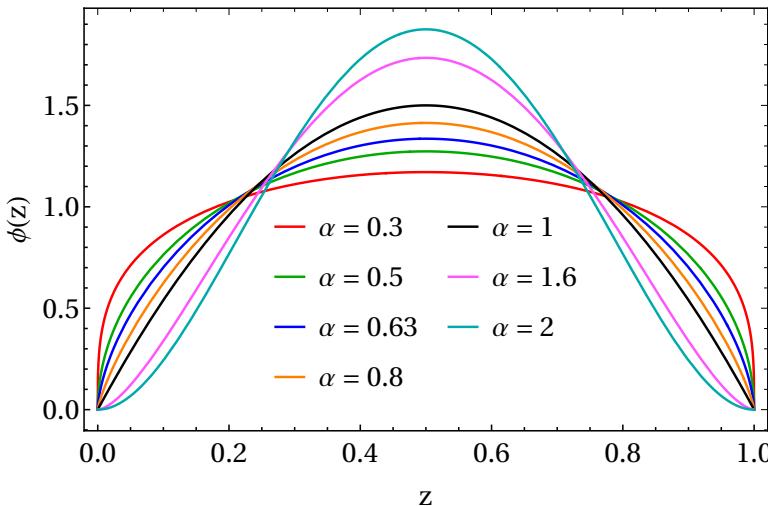
Exclusive Massive Two-Photon Production

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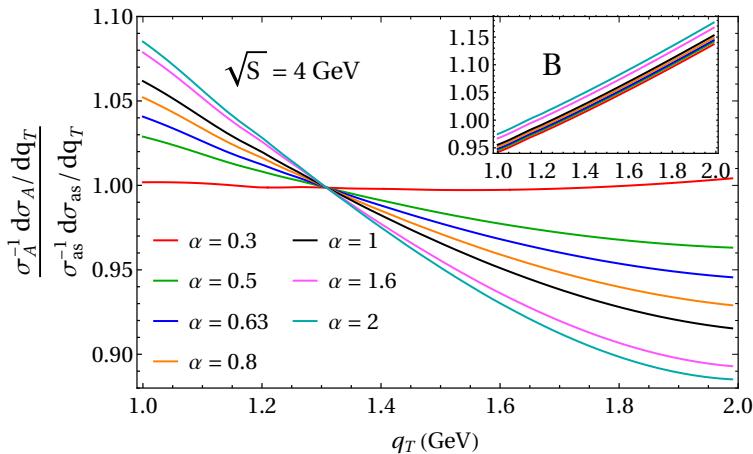
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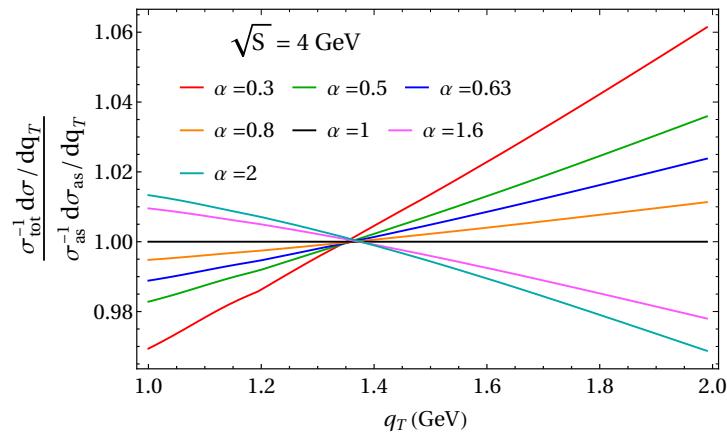


□ q_T distribution:

$$\frac{d\sigma}{dq_T} \sim |\phi(\textcolor{red}{z})|^2$$

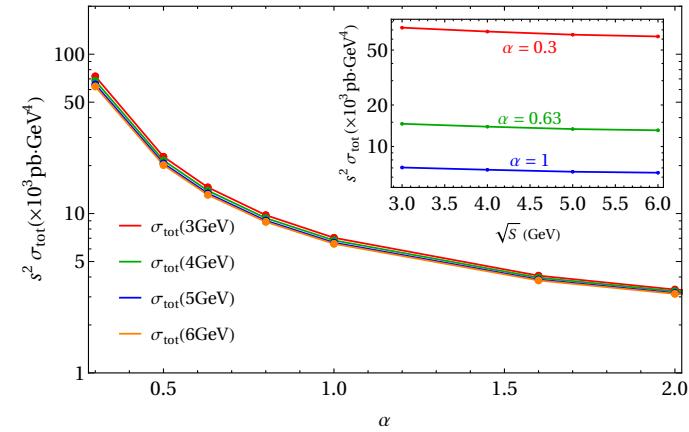


A: photons from two quark lines
B: photons from one quark line



Total:

$$\sigma_{\text{Tot}} = \sigma_A + \sigma_B + \sigma_{AB}$$

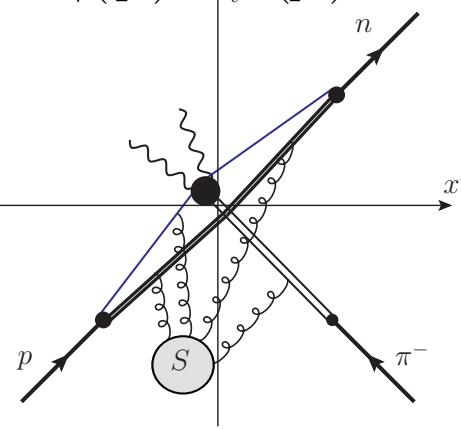
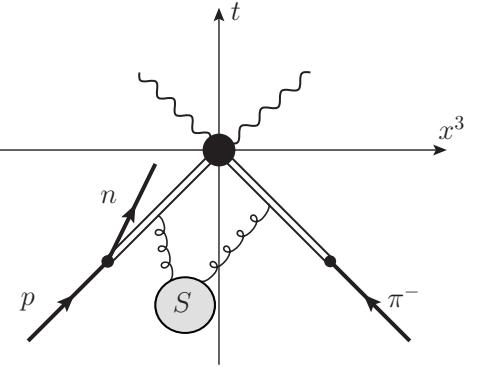
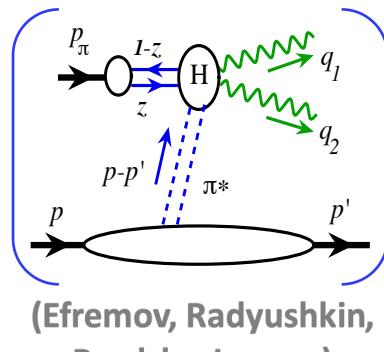


$$\sigma_{\text{tot}} = \int_{1 \text{ GeV}}^{\sqrt{s}/2} dq_T \frac{d\sigma}{dq_T}$$

Exclusive Massive Two-Photon Production

□ Factorization:

$$\pi^-(p_\pi) + P(p) \rightarrow \gamma(q_1) + \gamma(q_2) + N(p')$$



Additional region: DGLAP region!

- Different soft structures
- Factorization proof needs to be modified

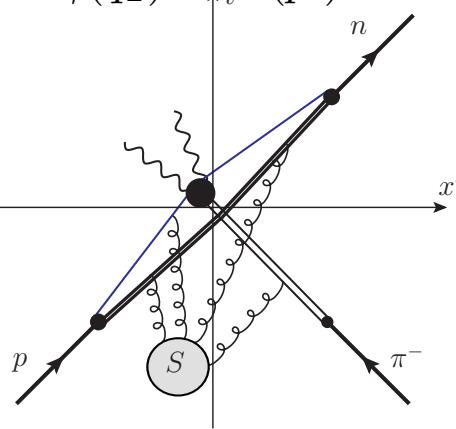
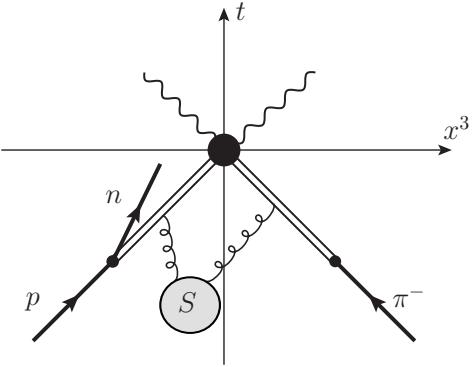
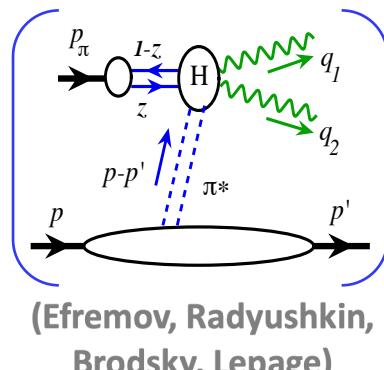


$$\mathcal{M} = \text{GPD} \otimes \text{DA} \otimes \text{Hard}$$

Exclusive Massive Two-Photon Production

☐ Factorization:

$$\pi^-(p_\pi) + P(p) \rightarrow \gamma(q_1) + \gamma(q_2) + N(p')$$



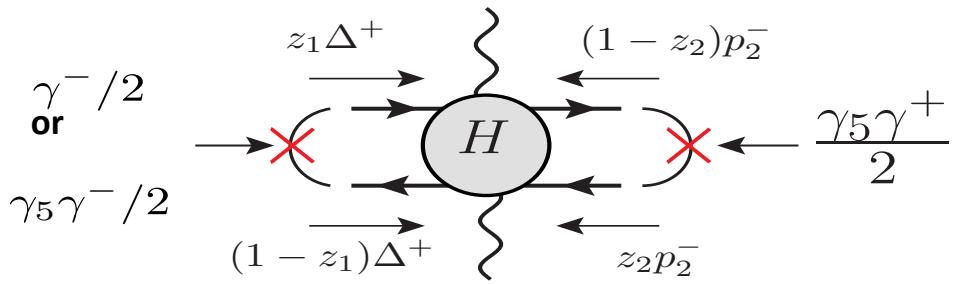
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$$\mathcal{M} = \text{GPD} \otimes \text{DA} \otimes \text{Hard}$$

☐ Additional channels – more GPDs:



$\gamma^-/2$ corresponds to $F_{pn}^u \supset (H, E)$
 $\gamma_5\gamma^-/2$ corresponds to $\tilde{F}_{pn}^u \supset (\tilde{H}, \tilde{E})$

☐ Factorization formula:

$$N = -2ig^2(C_F/N_c)(1/\hat{s})$$

$$x_L = \frac{\xi - 1}{2\xi}, \quad x_R = \frac{\xi + 1}{2\xi}$$

$$\begin{aligned} \mathcal{M}_{\lambda\lambda'} &= N \int_{x_L}^{x_R} dx \int_0^1 dz \tilde{H}(x, \xi, t) D(z) O_{\lambda\lambda'}(x, z) \\ \widetilde{\mathcal{M}}_{\lambda\lambda'} &= N \int_{x_L}^{x_R} dx \int_0^1 dz H(x, \xi, t) D(z) \widetilde{O}_{\lambda\lambda'}(x, z) \end{aligned}$$

$$\frac{d\sigma}{dt d\xi dq_T^2} \propto |\mathcal{M}|^2$$



Exclusive Massive Two-Photon Production

□ GPD models – simplified GK model:

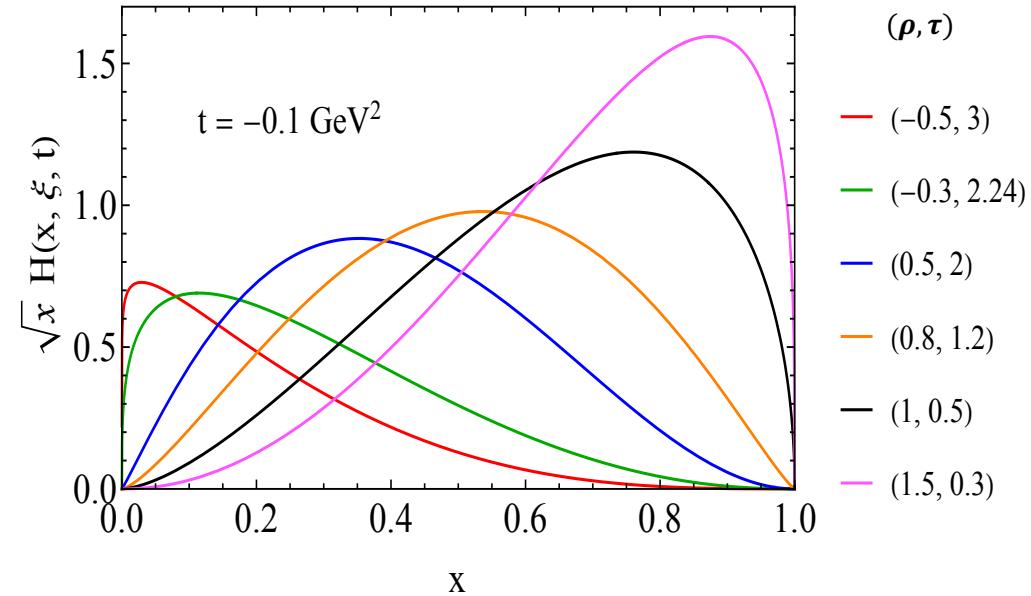
$$H_{pn}(x, \xi, t) = \theta(x) x^{-0.9(t/\text{GeV}^2)} \frac{x^\rho(1-x)^\tau}{B(1+\rho, 1+\tau)}$$

$$\tilde{H}_{pn}(x, \xi, t) = \theta(x) x^{-0.45(t/\text{GeV}^2)} \frac{1.267 x^\rho(1-x)^\tau}{B(1+\rho, 1+\tau)}$$

- Neglect E, \tilde{E} . Neglect evolution effect.
- Tune (ρ, τ) to control x shape.
- Fix DA: $D(z) = N z^{0.63} (1-z)^{0.63}$



Goloskokov, Kroll
[hep-ph/0501242](#)
[arXiv: 0708.3569](#)
[arXiv: 0906.0460](#)



Exclusive Massive Two-Photon Production

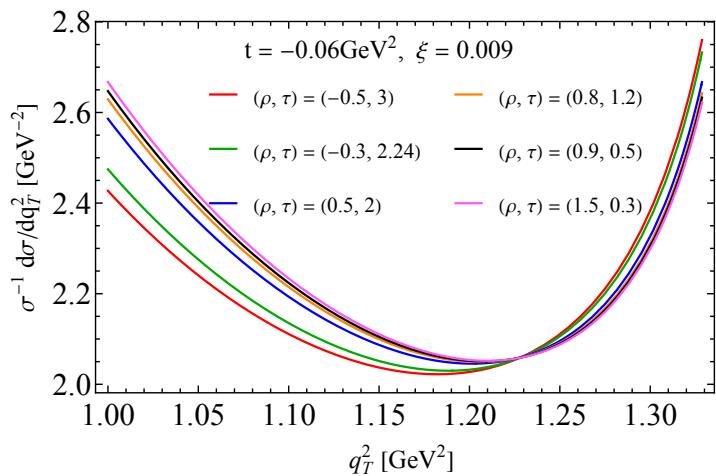
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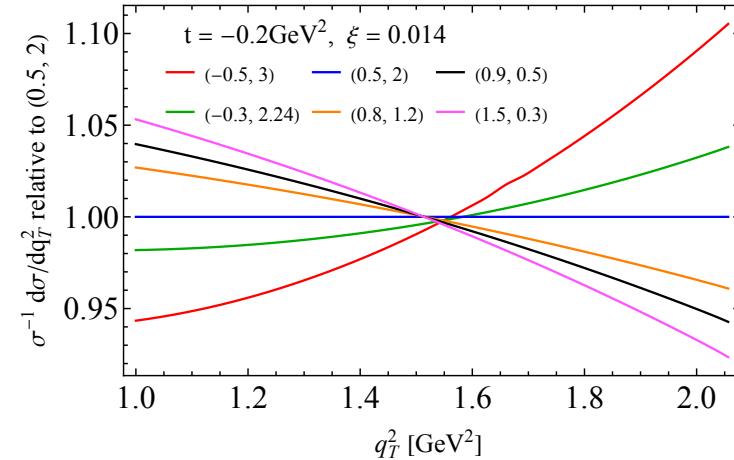
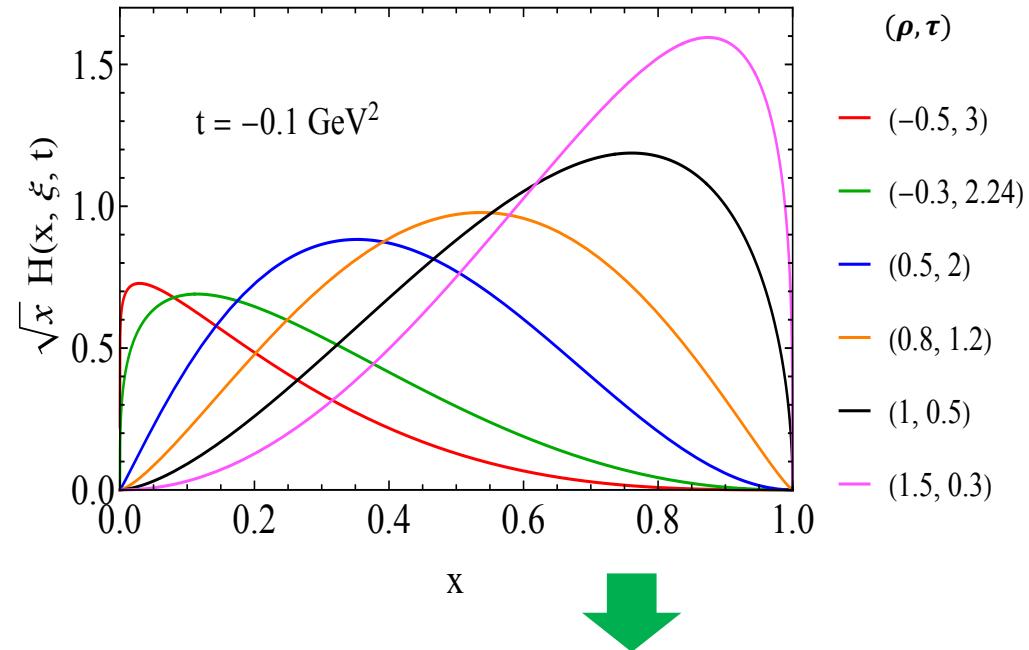
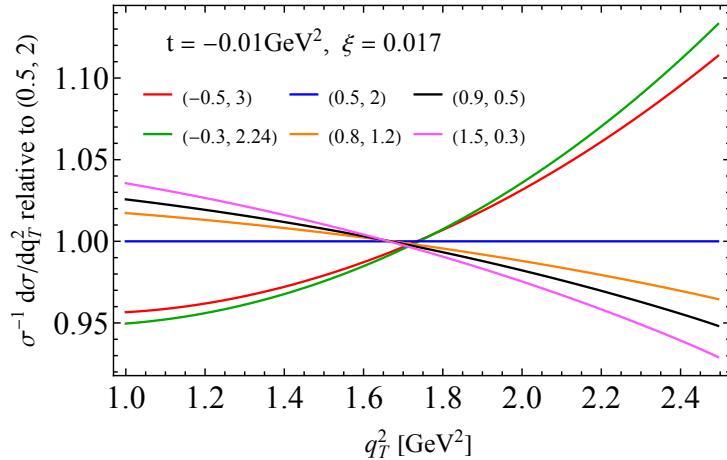
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□ Normalized q_T distribution:



$$\frac{d\sigma}{dt d\xi dq_T^2} \sim |H(x, \xi, t)|^2$$

Goloskokov, Kroll
[hep-ph/0501242](#)
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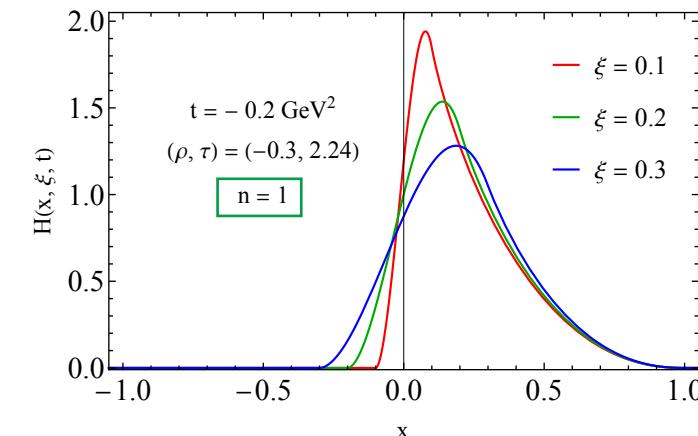
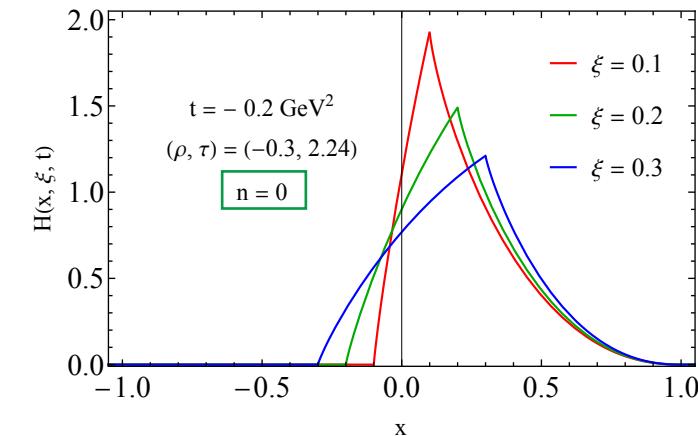
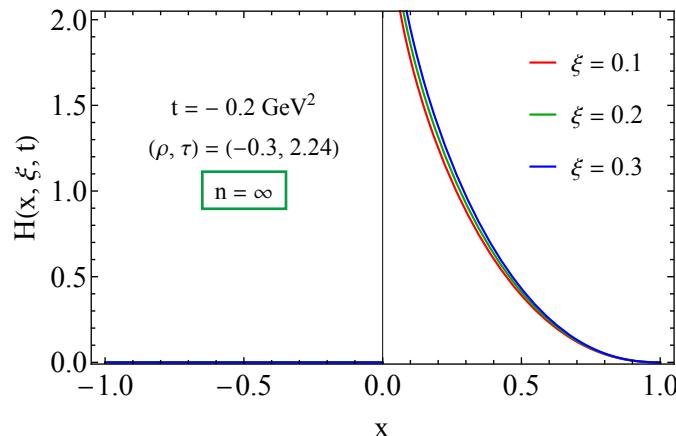
$E_\pi = 150 \text{ GeV}$

Exclusive Massive Two-Photon Production

□ GPD models – modified GK model:

$$H(x, \xi, t) = \int d\beta d\alpha \delta(x - \beta - \xi\alpha) f(\beta, \alpha, t)$$
$$f(\beta, \alpha, t) = e^{(b + \alpha' \ln |\beta|^{-1})t} \cdot h(\beta) \cdot w(\beta, \alpha)$$
$$w(\beta, \alpha) = \frac{\Gamma(2n+2)}{2^{2n+1}\Gamma^2(n+1)} \left[(1 - |\beta|)^2 - \alpha^2 \right]^n$$

- Change n to change ξ dependence
- Choose $n = 0, 1, \infty$



Exclusive Massive Two-Photon Production

□ GPD models – modified GK model:

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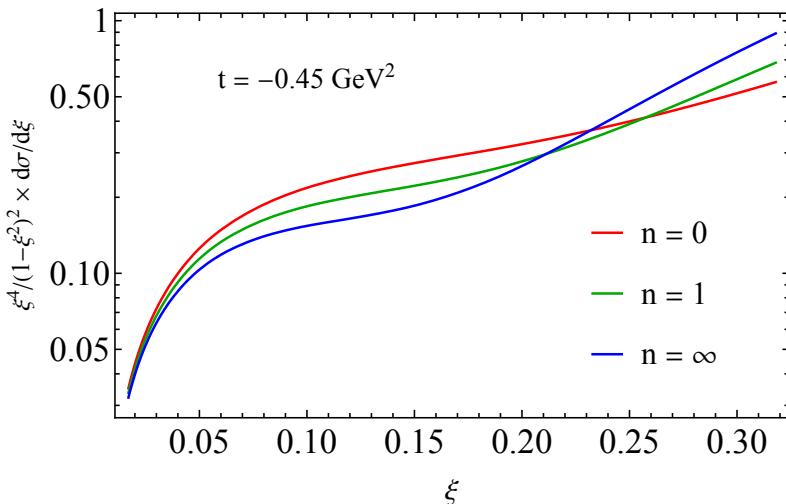
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$$w(\beta, \alpha) = \frac{\Gamma(2n+2)}{2^{2n+1}\Gamma^2(n+1)} \frac{[(1-|\beta|)^2 - \alpha^2]^n}{(1-|\beta|)^{2n+1}}$$

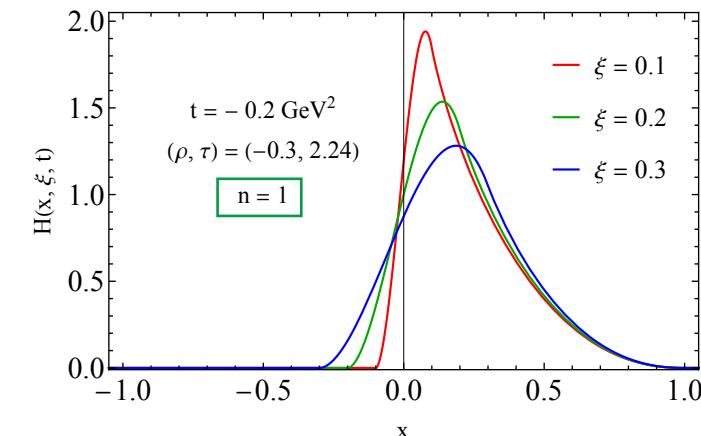
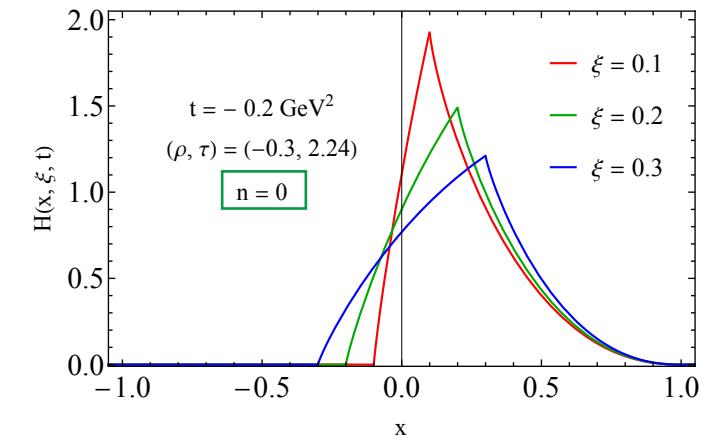
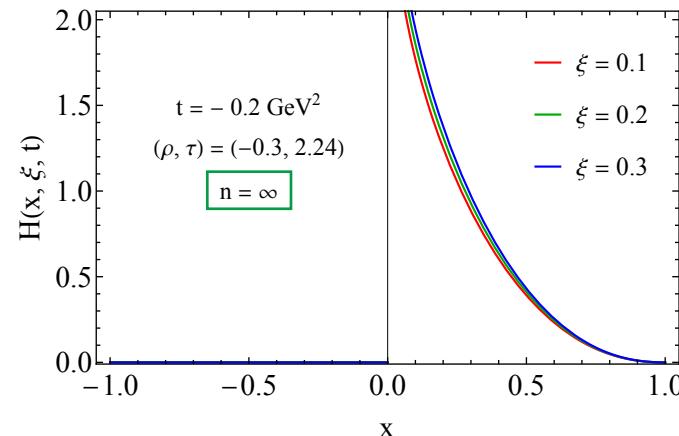
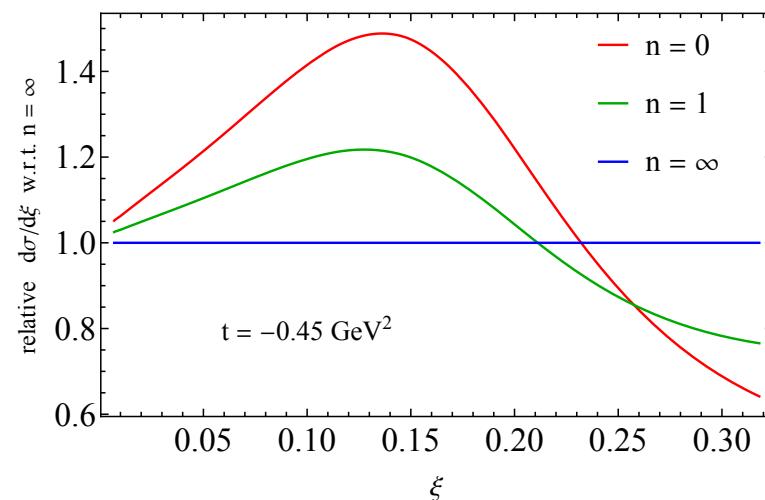
- Change n to change ξ dependence
- Choose $n = 0, 1, \infty$

□ ξ distribution (integrate out q_T):

$$q_T \geq 1 \text{ GeV}$$



$$\frac{d\sigma}{dt d\xi dq_T^2} \sim |H(x, \xi, t)|^2$$



Exclusive $\pi^0\gamma$ Pair Production

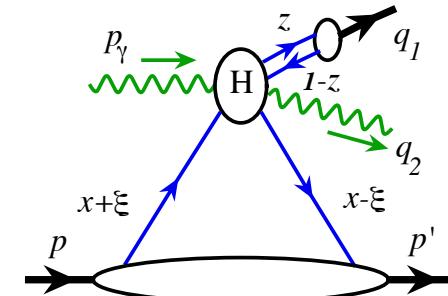
□ Factorization: $\gamma(p_\gamma) + h(p) \rightarrow \pi^0(q_1) + \gamma(q_2) + h(p')$

Factorization is proved similarly ($q_T >> 1/R!$)

□ Hard scales:

Invariant mass of $\pi\gamma$ pair

q_T of π (or γ) in pair's rest frame – convert to $\cos\theta_\pi$



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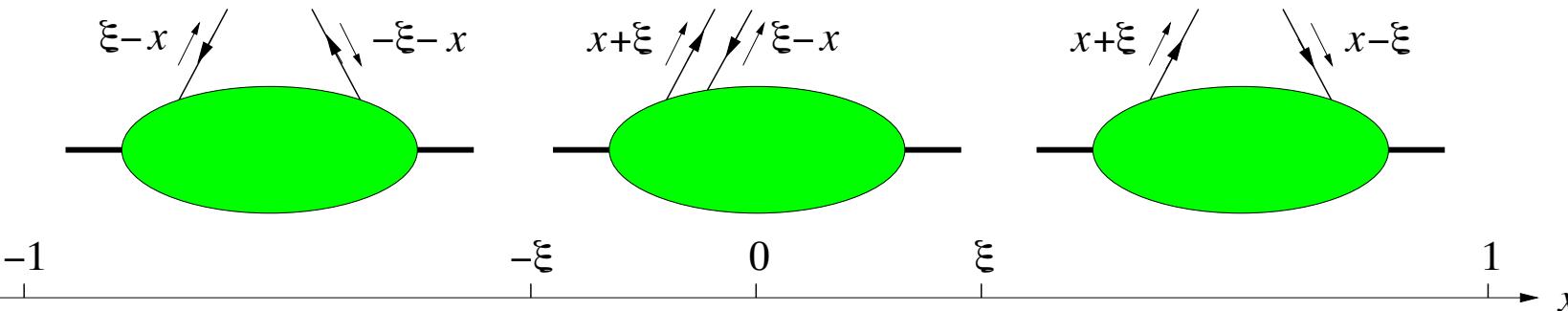
Hall D at JLab?

$\frac{d\sigma}{dcos\theta}$ is sensitive to x -dependence of GPDs

□ Cancellation of unwanted propagators & $\cos\theta$ dependence:

$$\begin{aligned} \text{Re } O_{++} = & (e_1 - e_2)^2 \left[\frac{1 - \cos\theta}{1 + \cos\theta} \cdot P \frac{x + z - 2xz}{2xz(1-x)(1-z)} \right] + (e_1^2 - e_2^2) \left[\frac{2}{1 - \cos\theta} \cdot P \frac{x - z}{xz(1-x)(1-z)} \right] \\ & - e_1 e_2 P \frac{1 - \cos\theta}{xz(1-x)(1-z)} \cdot \frac{(xz + (1-x)(1-z))(x(1-x) + z(1-z))}{(2(1-x)(1-z) - (1 + \cos\theta)xz)(2xz - (1 + \cos\theta)(1-x)(1-z))} \end{aligned}$$

□ Sensitive to ERBL region (complementary)



Also sensitive to DA
in the bulk region.

Exclusive $\pi^0\gamma$ Pair Production

□ Phenomenology:

$$\frac{d\sigma}{d|t| d\xi d\cos\theta_\pi d\phi_\pi} = \frac{|\mathcal{A}|^2}{32 s (2\pi)^4 (1 + \xi)^2}$$

$$\begin{aligned} \frac{1}{2} \overline{|\mathcal{A}|^2} &= \left(\frac{2\pi\alpha_s}{s} f_\pi \right)^2 \left(\frac{C_F}{N_c} \right)^2 \left(\frac{1+\xi}{\xi} \right)^2 (1 - \xi^2) \\ &\times \left[|\tilde{O}_{++}^{[\tilde{H}]}|^2 + |\tilde{O}_{+-}^{[\tilde{H}]}|^2 + |\tilde{O}_{++}^{[H]}|^2 + |\tilde{O}_{+-}^{[H]}|^2 \right] \end{aligned}$$

■ Factorized helicity amplitude:

$$O_{\lambda\lambda'}^{[\tilde{H}]} = \sum_q \int_{x_L}^{x_R} dx \int_0^1 dz \tilde{H}^q(x, \xi, t) \phi_\pi^q(z) O_{\lambda\lambda'}^q(x, z)$$

■ Pion distribution amplitude:

$$\phi_{\pi^0}^d(z) = \phi_{\pi^0}^u(z) = \frac{1}{\sqrt{2}} \frac{z^\alpha (1-z)^\alpha}{B(1+\alpha, 1+\alpha)}, \quad (\alpha > 0)$$

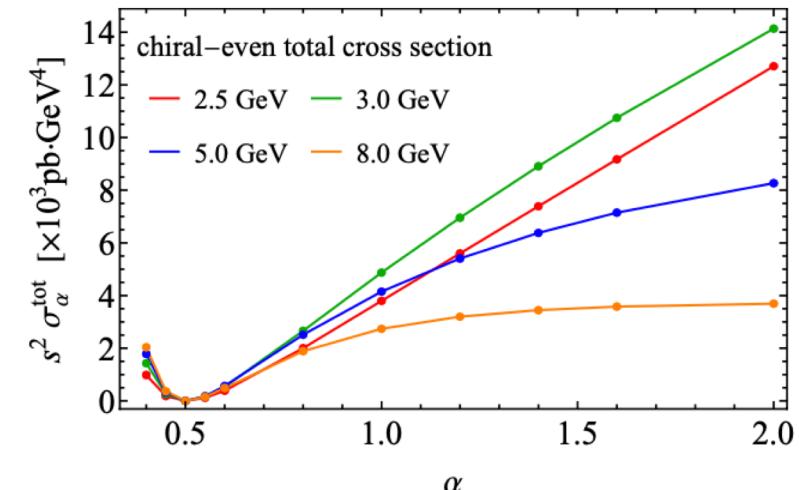
■ Model GPDs = simplified GK model:

- Taking $n_i = 0$

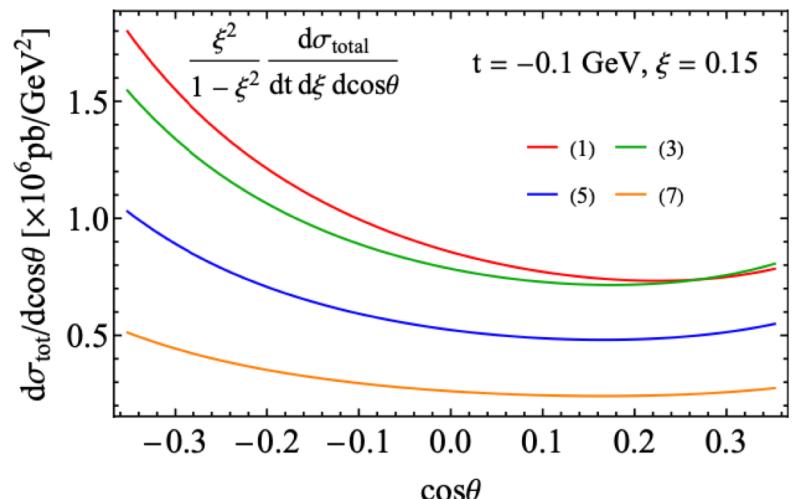
- Parametrizing the forward limit as $x^a(1-x)^b$

- Neglecting the D-term

□ Sensitivity on DAs (total – $q_T > 1$ GeV):



□ Sensitivity on GPDs ($\alpha = 0.63$):



Exclusive $\pi^0\gamma$ Pair Production

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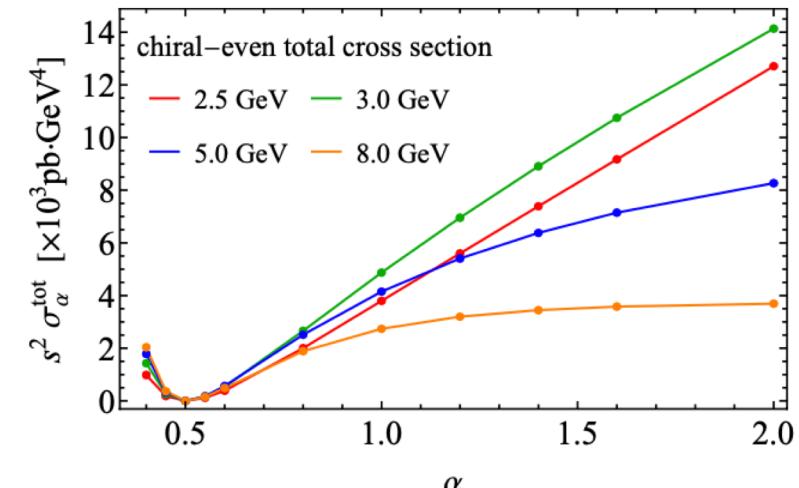
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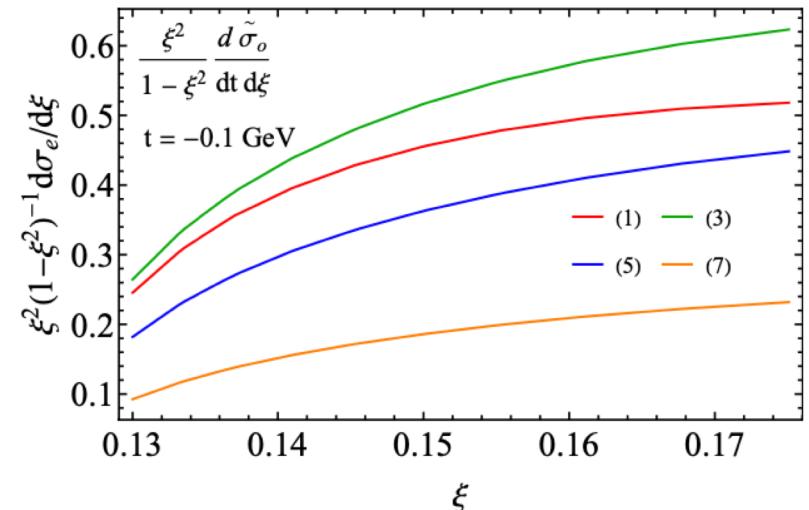
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Summary and Outlook

- Proposed new type of exclusive two-scale observable –
exclusive massive photon-pair production:
 - This process is factorizable and sensitive to pion DAs and hadron GPDs
 - Complementary to the pion form factor, and exclusive deep virtual lepton-hadron scattering processes, such as DVCS, DVMP, ..., and measurable at Amber, ...
 - Hard scale is given by the invariant mass of the produced pair, not by a single virtual photon
 - More sensitive to the x-dependence of pion DA and hadron GPDs
- This process can be generated to other exclusive two-scale observables that could be measured at JLab, Amber, EIC, ElcC, ...
 - More realistic numerical predictions are underway
 - More observables could be explored – hard part (the probe) should be sensitive to the momentum difference of the two active partons from the pion or the diffracted hadron

Thank you!