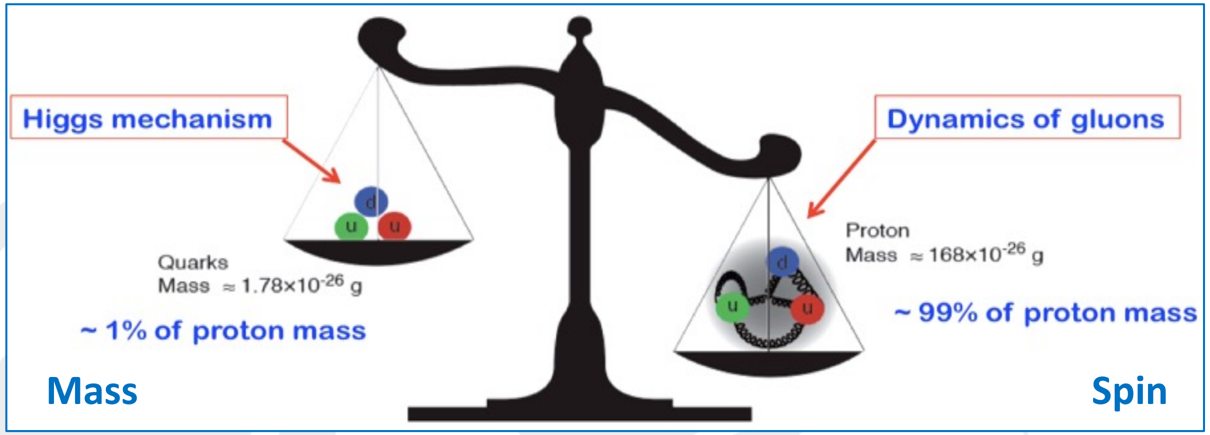
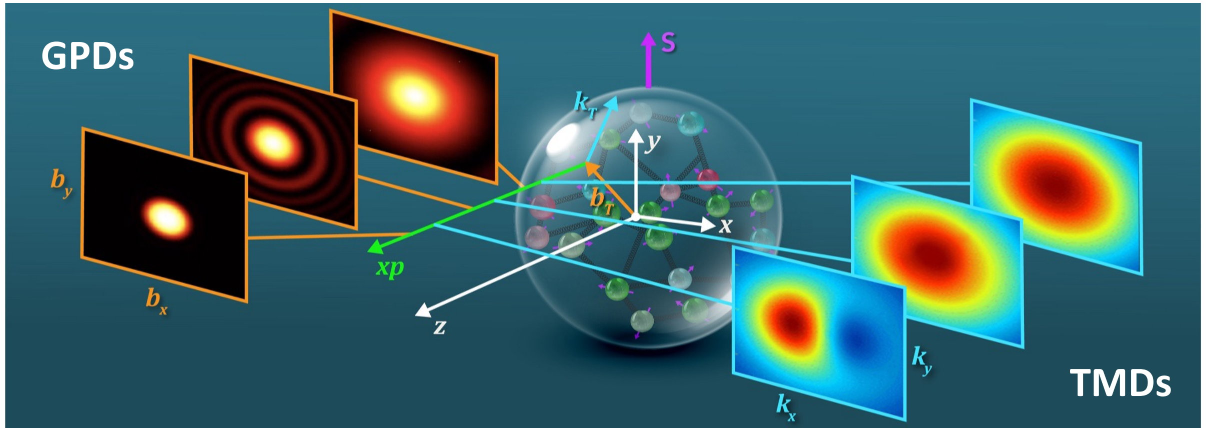
Exclusive hard processes for extracting Generalized Parton Distributions





Emergent Properties

Dynamical Structure

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JHEP 08 (2022) 103, PRD 107 (2023) 1
and in preparation

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TMD Collaboration
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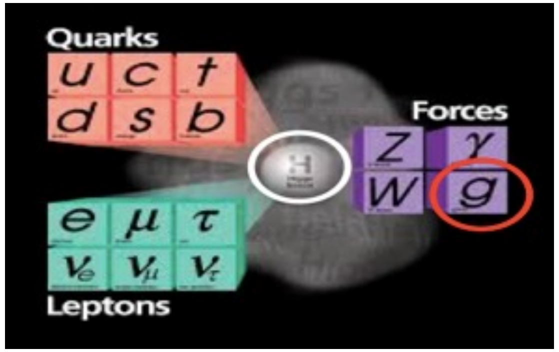
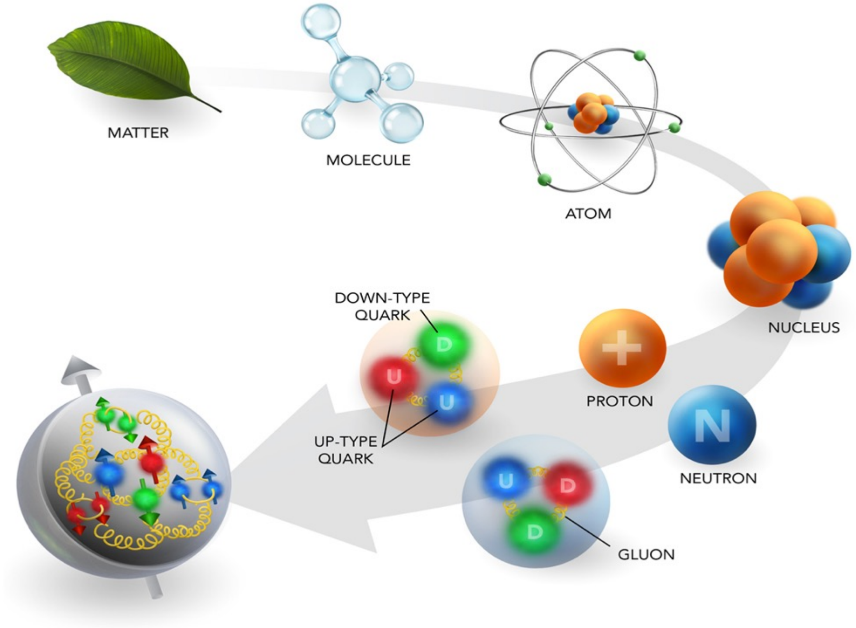
Nucleon: Fundamental Building Block of All Atomic Nuclei

❑ Nucleon is not elementary:

Nucleon is a strongly interacting, relativistic bound state of quarks and gluons of QCD
Our understanding of the nucleon has been evolving, and will continue to evolve, …





- **Nucleon Mass:**

  = Energy of the nucleon when it is at Rest!

$$M_n = \left.\frac{\langle P|H_{\rm QCD}(\psi,A)|P\rangle}{\langle P|P\rangle}\right|_{\rm at\ \ rest}$$

- **Nucleon Spin:**

  = Angular momentum of the nucleon when it is at Rest!

$$S = \sum_f \langle P, S_z = 1/2|\, \hat{J}^z_f \,|P, S_z = 1/2\rangle = \frac{1}{2}$$

$$J^i_{\rm QCD} = \frac{1}{2}\,\epsilon^{ijk}\int d^3x\; M^{0jk}_{\rm QCD} \;\Longleftarrow\; M^{\alpha\mu\nu}_{\rm QCD} = T^{\alpha\nu}_{\rm QCD}\,x^\mu - T^{\alpha\mu}_{\rm QCD}\,x^\nu$$

Understanding it fully is still beyond the best mind that we have!

$H_{\rm QCD}(\psi, A)$ **is known, but not** $|P\rangle = ?$

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QCD Landscape of Nucleons and Nuclei

Color Confinement — **Asymptotic freedom** — $Q\ \mathrm{(GeV)}$

20 MeV (10 fm) — 200 MeV (1 fm) — 2 GeV (1/10 fm)

Probing scale

QCD at the Fermi Scale: Femto-science (0.1-10 fm)

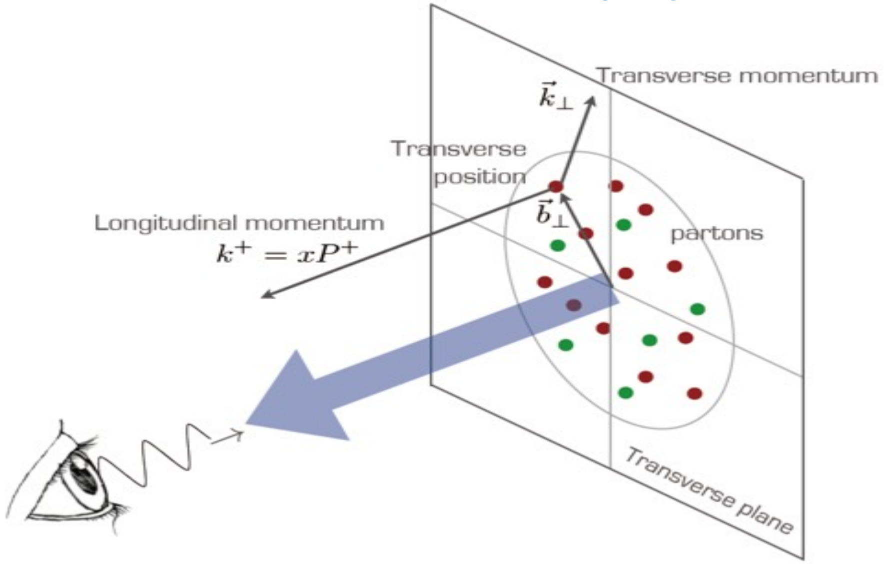
- *The most interesting, rich, and complex regime of the theory!*
- *All emergent phenomena depend on the scale at which we probe them!*

Asymptotic regime
PQCD works beautifully!

❑ **Need new observables with two distinctive: scales:**

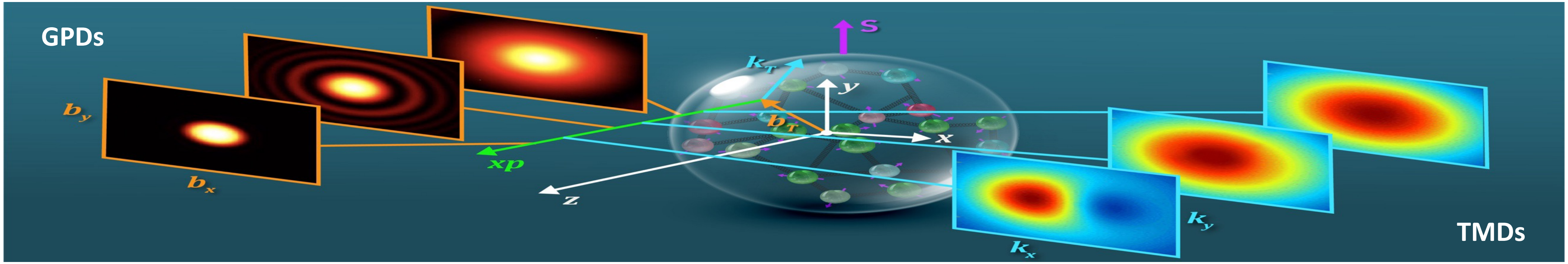
$$Q_1 \gg Q_2 \sim 1/R \sim \Lambda_{\mathrm{QCD}}$$

- **Hard scale:** Q_1 **to localize the probe to see the particle nature of quarks/gluons**
- **"Soft" scale:** Q_2 **to be more sensitive to the emergent regime of hadron structure ~ 1/fm**



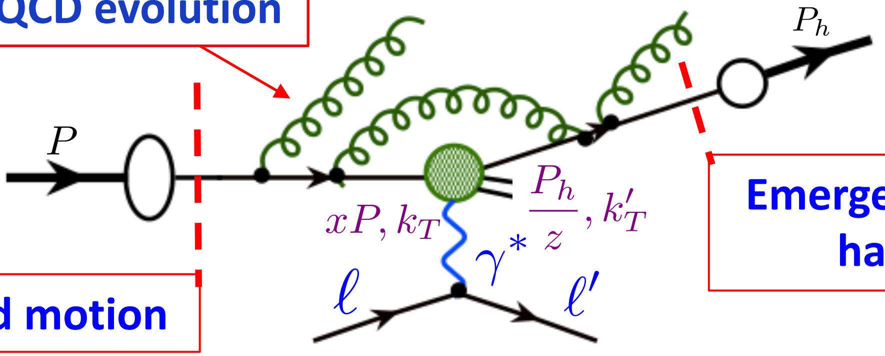
"See" Internal Structure of Hadron without seeing quarks/gluons?

❑ 3D hadron structure:



NO quarks and gluons can be seen in isolation!

❑ If the nucleon is broken, e.g., in SIDIS, …



Transverse momentum broadening:

$$\Delta k_T^2 \propto \Lambda_{\rm QCD}^2 \times \alpha_s(C_F, C_A) \times \log(Q^2/\Lambda_{\rm QCD}^2) \times \log(s/Q^2) \quad \} \gtrsim 1$$

Structure information is diluted by the collision induced shower!

- ***Measured k_T is NOT the same as k_T of the confined motion!***
- ***Too larger Q^2 could weaken our precision to probe the true hadron structure!***

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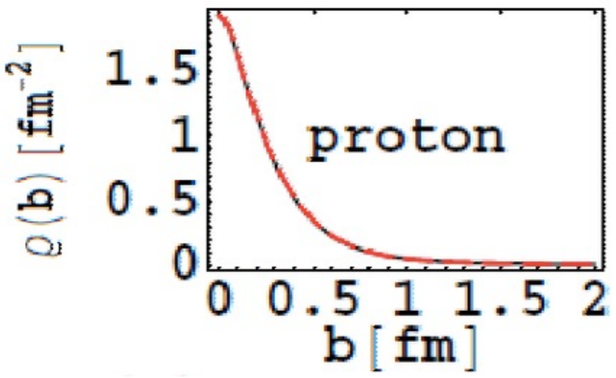
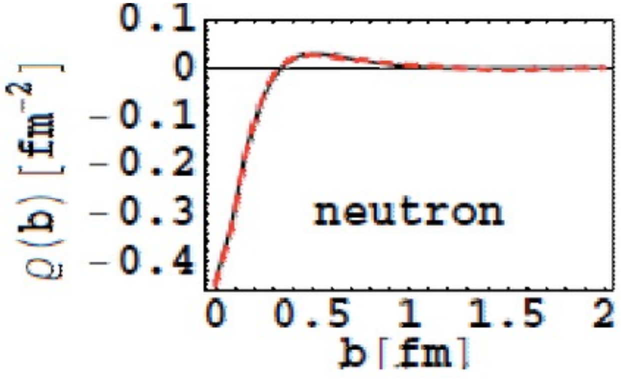
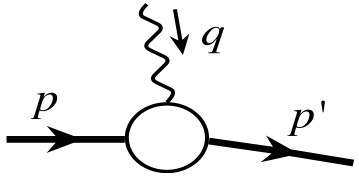
How to Explore Internal Structure of Hadron without Breaking it?

- **Form factors:** Elastic electric form factor ➡ Charge distributions

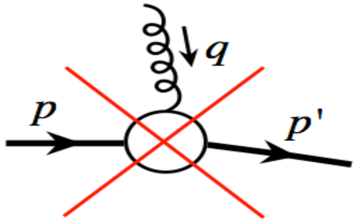






Proton "Radius"
in EM charge distribution

- **But, there is NO elastic "color" form factor!**



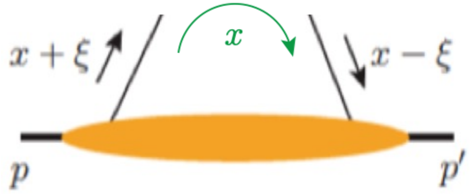
No Proton "Radius"
in color charge distribution!

- **3D hadron tomography:**

Generalized "form factor" for quark and gluon "density" distribution

Generalized PDFs (GPDs) – without breaking the proton

$$F_{q/h}(x,\xi,t) \qquad \text{skewness} \quad \xi = \frac{(p-p')^+}{(p+p')^+} \quad t=(p-p')^2$$



$t=(p-p')^2$

Spatial distribution of quark/gluon density, quark/gluon correlations, …

Generalized Parton Distribution (GPD)

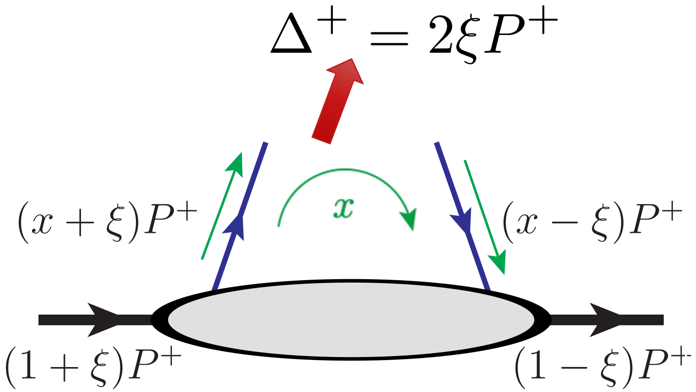
❑ Definition:

$$F^q(x,\xi,t) = \int \frac{dz^-}{4\pi} e^{-ixP^+z^-} \langle p'|\bar{q}(z^-/2)\gamma^+ q(-z^-/2)|p\rangle$$

$$= \frac{1}{2P^+}\left[H^q(x,\xi,t)\,\bar{u}(p')\gamma^+ u(p) - E^q(x,\xi,t)\,\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}u(p)\right],$$

$$\widetilde{F}^q(x,\xi,t) = \int \frac{dz^-}{4\pi} e^{-ixP^+z^-} \langle p'|\bar{q}(z^-/2)\gamma^+\gamma_5 q(-z^-/2)|p\rangle$$

$$= \frac{1}{2P^+}\left[\widetilde{H}^q(x,\xi,t)\,\bar{u}(p')\gamma^+\gamma_5 u(p) - \widetilde{E}^q(x,\xi,t)\,\bar{u}(p')\frac{\gamma_5\Delta^+}{2m}u(p)\right].$$



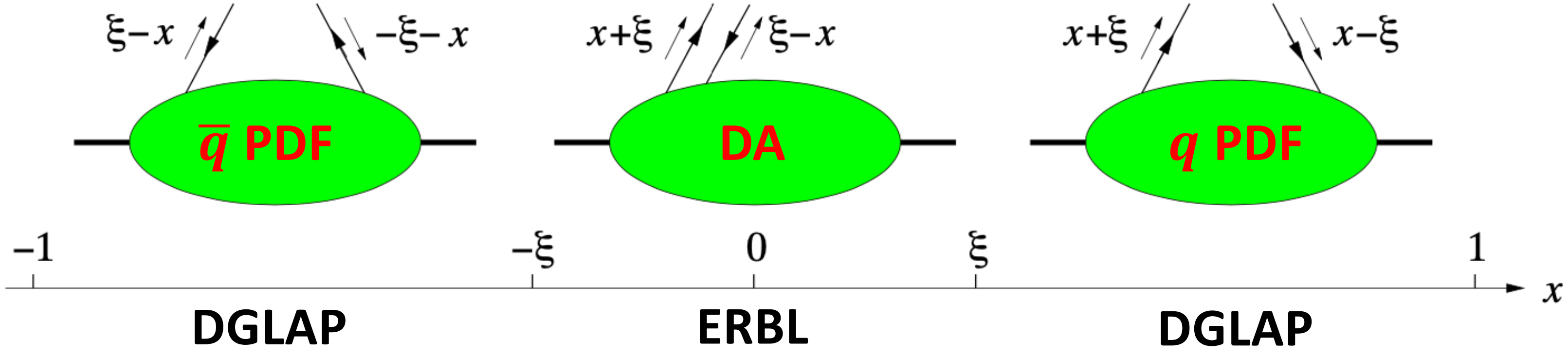
$$P^+ = \frac{p^+ + p'^+}{2}$$

$$\Delta = p - p' \qquad t = \Delta^2$$

Similar definition for gluon GPDs

❑ Combine *PDF* and *Distribution Amplitude (DA):*

Forward limit $\xi = t = 0$ **:** $\quad H^q(x,0,0) = q(x), \quad \tilde{H}^q(x,0,0) = \Delta q(x)$



Properties of GPDs

❑ "Mass" – QCD energy-momentum tensor:

Ji, PRL78, 1997

$$T^{\mu\nu} = \sum_{i=q,g} T_i^{\mu\nu} \quad \textbf{with} \quad T_q^{\mu\nu} = \bar{\psi}_q \, i\gamma^{(\mu} \overleftrightarrow{D}^{\nu)} \, \psi_q - g^{\mu\nu} \bar{\psi}_q \left(i\gamma \cdot \overleftrightarrow{D} - m_q \right) \psi_q \quad \textbf{and} \quad T_g^{\mu\nu} = F^{a,\mu\eta} F^{a,}{}_{\eta}{}^{\nu} + \frac{1}{4} g^{\mu\nu} \left(F^a_{\rho\eta} \right)^2$$

❑ Gravitational form factors:

$$\langle p' \,| T_i^{\mu\nu} |\, p \rangle = \bar{u}(p') \left[A_i(t) \frac{P^\mu P^\nu}{m} + J_i(t) \frac{i P^{(\mu} \sigma^{\nu)\Delta}}{2m} + D_i(t) \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{4m} + m \, \bar{c}_i(t) \, g^{\mu\nu} \right] u(p)$$

$$C_i(t) \leftrightarrow D_i(t)/4$$

Related to pressure & stress force inside h

Polyakov, schweitzer, Inntt. J. Mod. Phys. A33, 1830025 (2018)
Burkert, Elouadrhiri , Girod Nature 557, 396 (2018)

❑ Connection to GPD moments:

$$\int_{-1}^{1} dx \, x \, F_i(x, \xi, t) \propto \langle p' | T_i^{++} | p \rangle \quad \propto \quad \bar{u}(p') \left[\underbrace{\left(A_i + \xi^2 D_i \right)}_{\int_{-1}^{1} dx \, x \, H_i(x,\xi,t)} \gamma^+ + \underbrace{\left(B_i - \xi^2 D_i \right)}_{\int_{-1}^{1} dx \, x \, E_i(x,\xi,t)} \frac{i\sigma^{+\Delta}}{2m} \right] u(p)$$

❑ "Spin" – Angular momentum sum rule:

$$J_i = \lim_{t \to 0} \int_{-1}^{1} dx \, x \left[H_i(x, \xi, t) + E_i(x, \xi, t) \right] \qquad i = q, g$$

➡ **3D tomography**
Relation to GFF
Angular Momentum

➡ ***x*-dependence of GPDs!**

Need to know the x-dependence of GPDs to construct the proper moments!

Properties of GPDs

M. Burkdart, PRD 2000

❑ Impact parameter dependent parton density distribution:

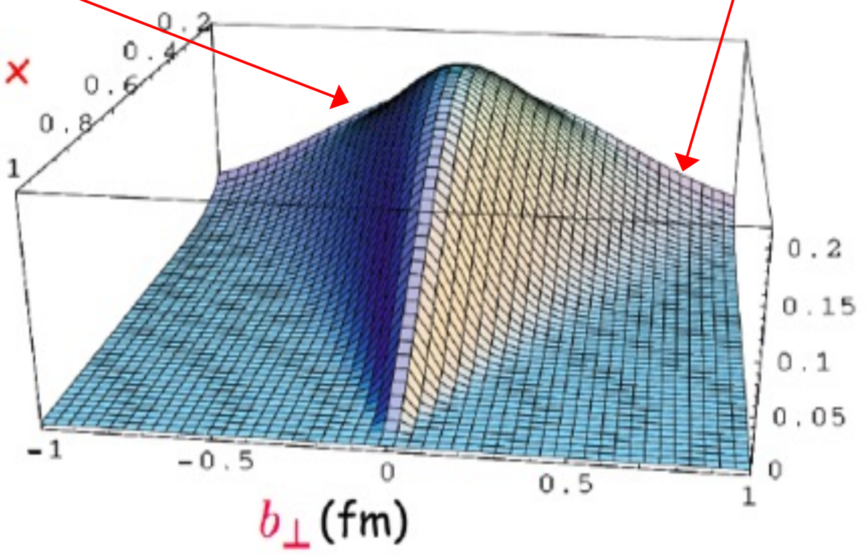
$$q(x, b_\perp, Q) = \int d^2\Delta_\perp e^{-i\Delta_\perp \cdot b_\perp} H_q(x, \xi = 0, t = -\Delta_\perp^2, Q)$$

Quark density in $\mathrm{d}x\,\mathrm{d}^2 \boldsymbol{b}_T$

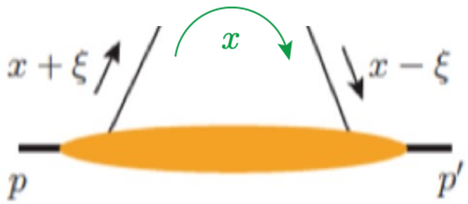
How fast does glue density fall?

3D image

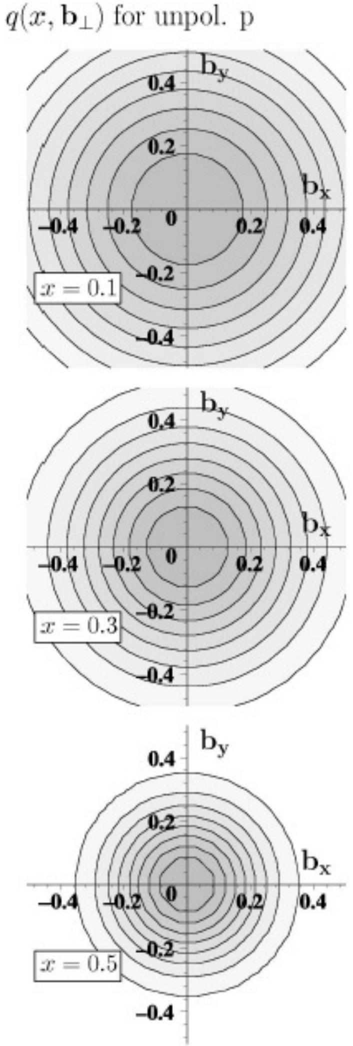
How far does glue density spread?



Unpolarized proton



- x = momentum flow between the pair
- $b_\perp$ = conjugate to the diffracted momentum
- Small x: large "meson cloud"
- Large x: compact "valence core"
- $b_\perp \to 0$ = $x \to 1$ narrow distribution



➡ Proton radii of quark and gluon spatial density distribution, $r_q(x)$ & $r_g(x)$

Properties of GPDs

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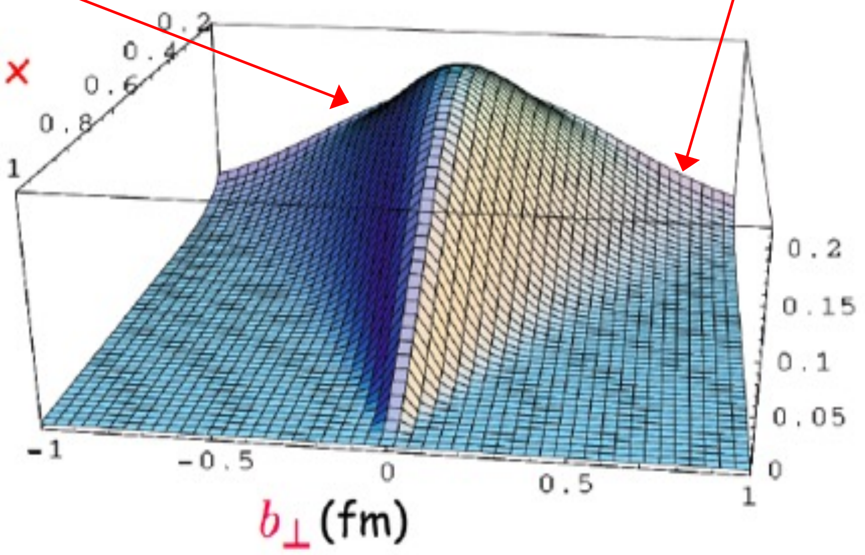

Proton radii of quark and gluon spatial density distribution, $r_q(x)$ & $r_g(x)$

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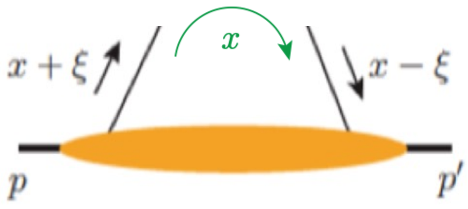
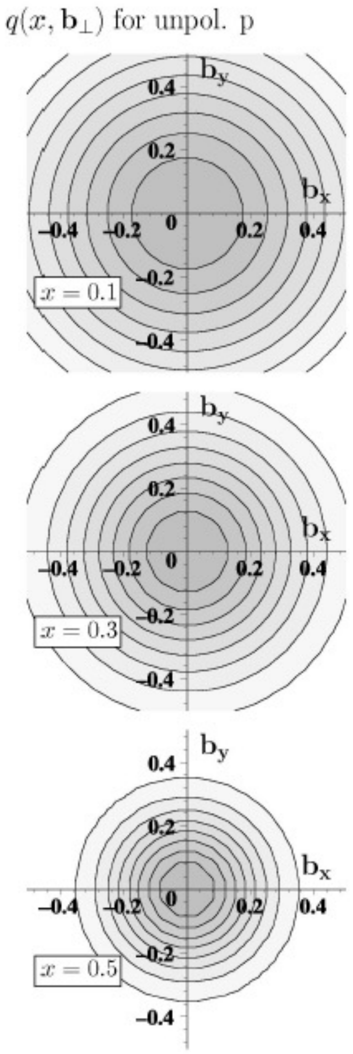
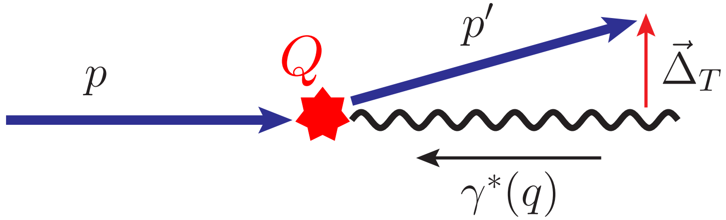


- Should $r_q(x)$ > $r_g(x)$, or vice versa?
- Could $r_g(x)$ saturates as $x \to 0$
- How do they compare with known radius (EM charge radius, mass radius, …)?
- Tomographic images in slides of different x value!



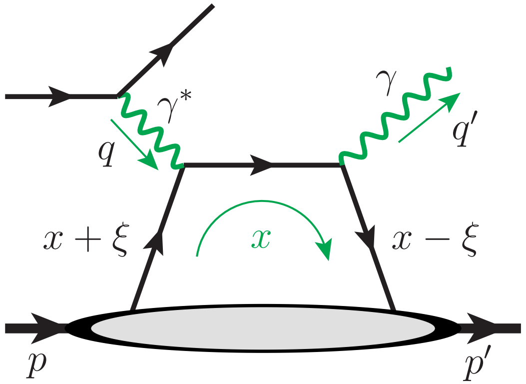
Exclusive Diffractive Process for Extracting GPDs

- Hit the proton hard without breaking it $\Longrightarrow$ Diffractive scattering to keep proton intact

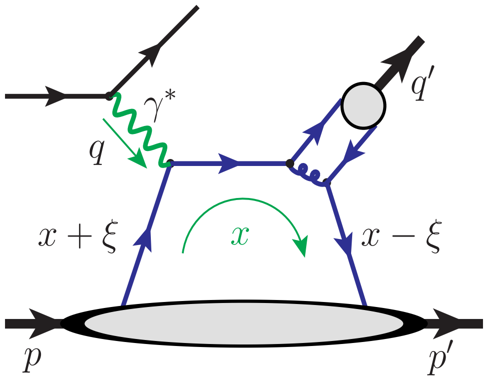


HERA discovery:
~15% of HERA events with the Proton stayed intact

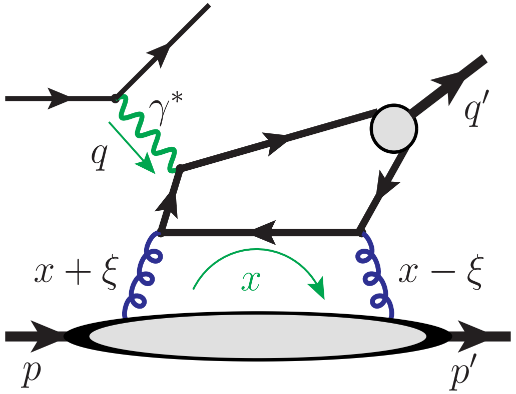
- Known exclusive processes for extracting GPDs:



DVCS: $Q^2 >> |t|$



DVMP



DVQP

+ DDVCS, …

Feature: Two-scale observables $Q^2 \gg |t|$ $\quad t = (p - p')^2$

- Hard scale Q: allows pQCD, factorization
- Low scale t: probes non-pert. hadron structure

Factorization

GPDs: $f_{i/h}(x, \xi, t; \mu)$

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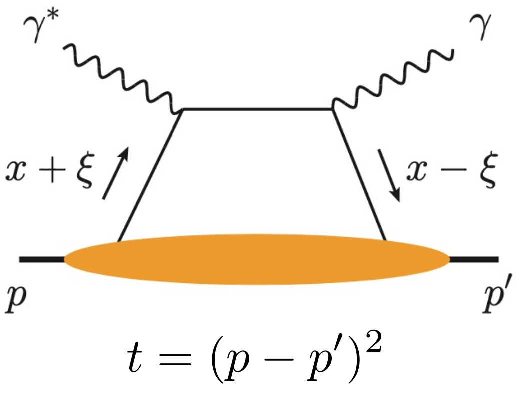
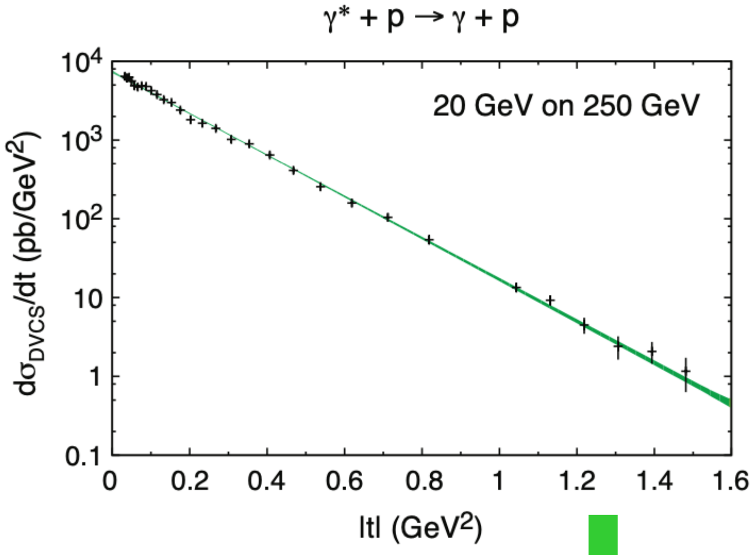
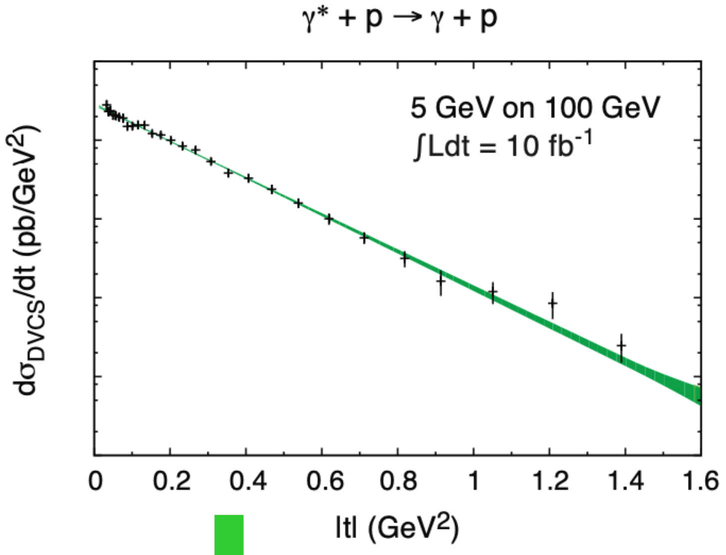
DVCS at a Future EIC (White Paper)

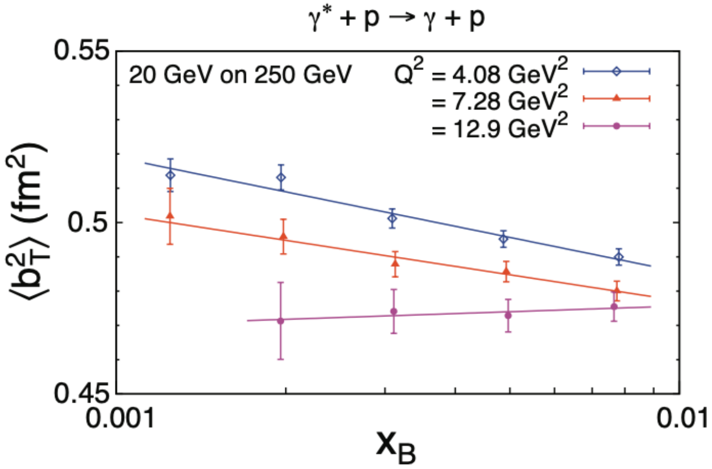
❑ Cross Sections:

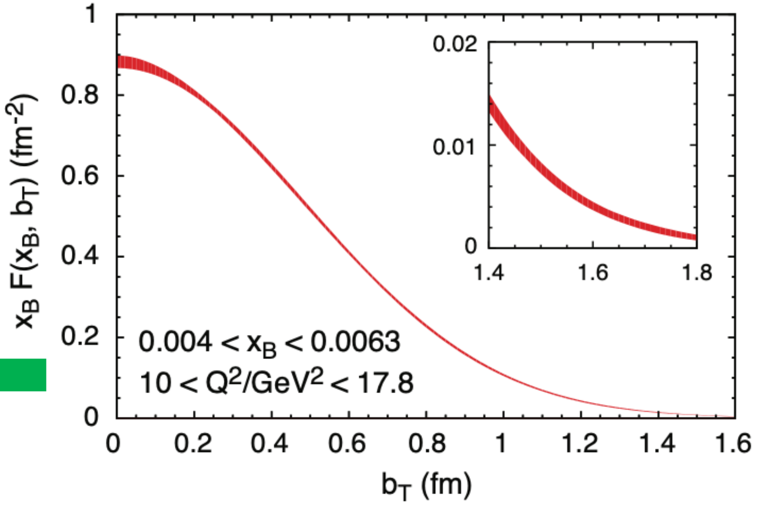


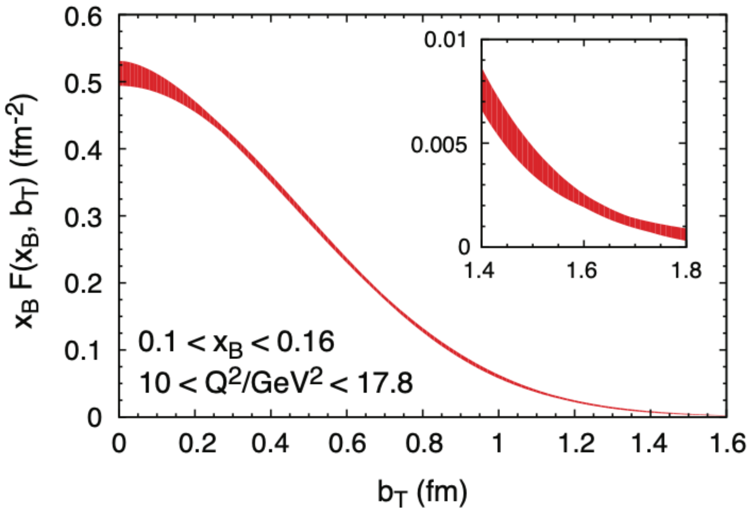




❑ Spatial distributions:



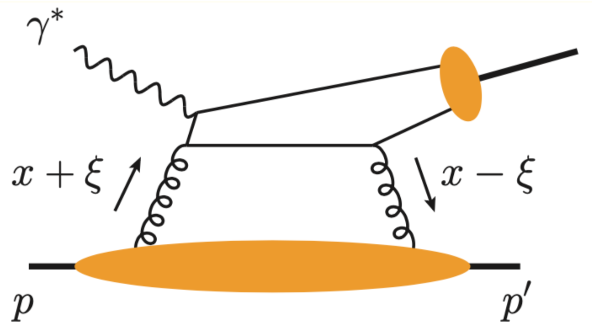




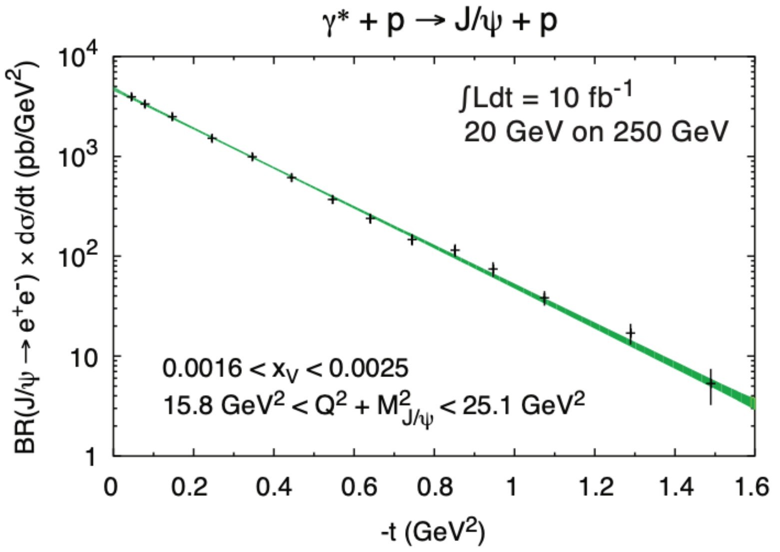
Effective "proton radius" in terms of quarks as a function of x_B

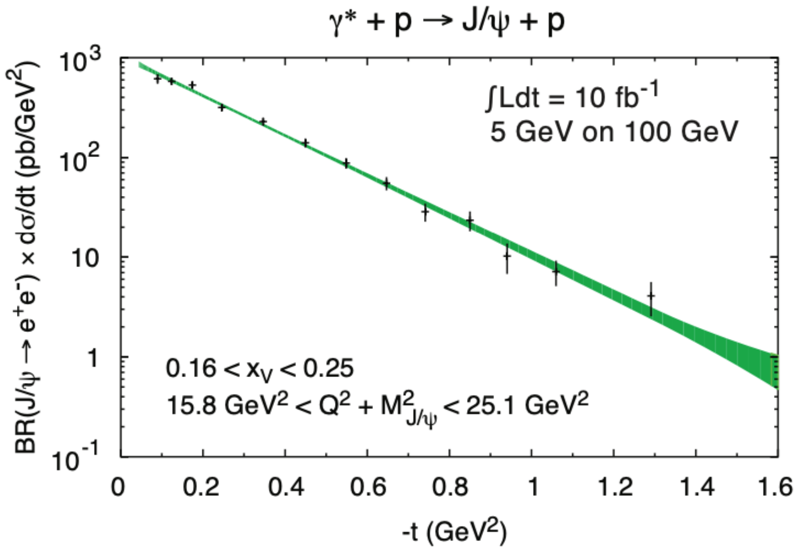
Imaging the Gluon at the EIC (White Paper)

❑ Exclusive vector meson production:

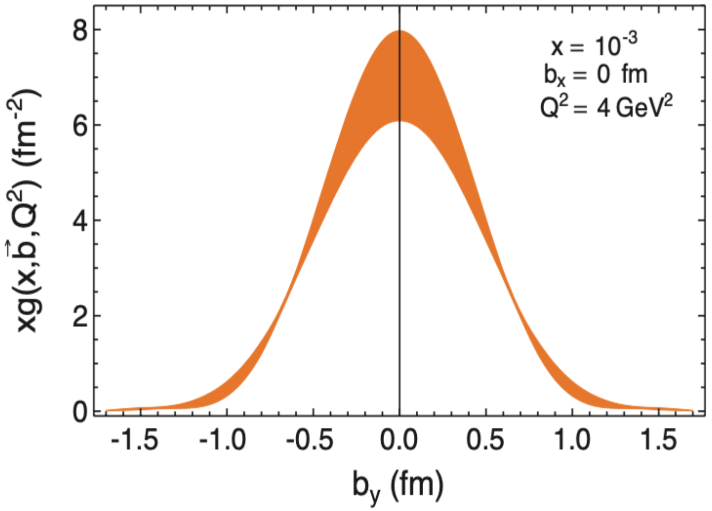


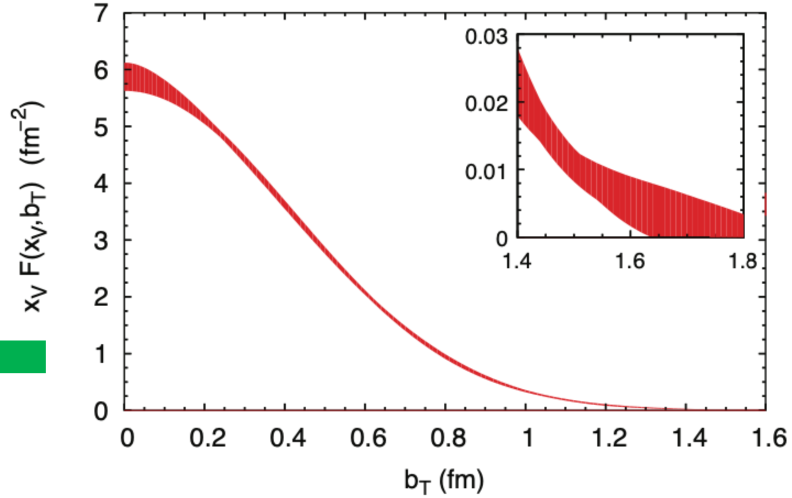
$$t = (p - p')^2$$

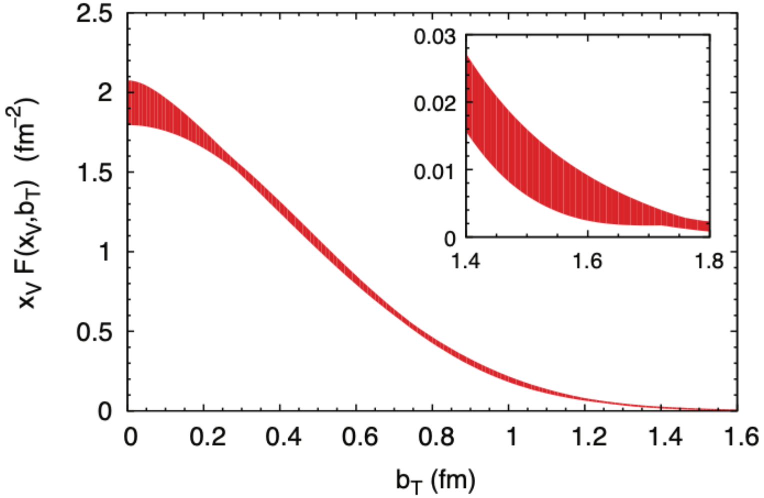




❑ Spatial distributions:







The b_T space density for gluons

Effective "proton radius" in terms of gluons

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It is Difficult to Extract the x-dependence of GPD – Why?

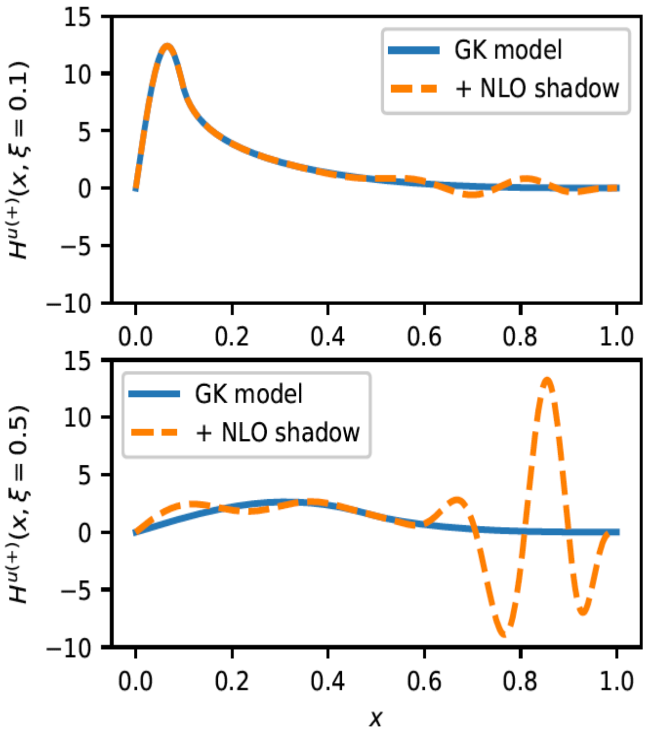
❑ Amplitude nature: exclusive processes

$x \sim$ loop momentum

$$\mathcal{M} \sim \int_{-1}^{1} \mathrm{d}x F(x,\xi,t) \cdot C(x,\xi;Q/\mu)$$

never pin down to some x

❑ "Shadow GPDs" $F(x,\xi,t) \to F(x,\xi,t) + S(x,\xi,t)$



with

$$\int_{-1}^{1} \mathrm{d}x \frac{S(x,\xi,t)}{x-\xi+i\varepsilon} = 0$$

Blue and dashed Fit the same CFFs !

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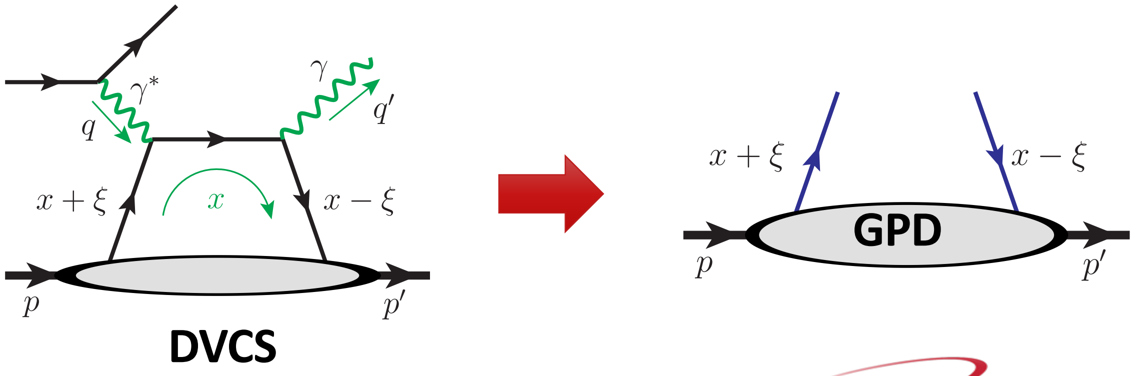
❑ Sensitivity to x comes from $C(x,\xi;Q/\mu)$

At LO, DVCS hard coefficient factorizes

$$C(x,\xi;Q/\mu) = C_Q(Q/\mu) \cdot C_x(x,\xi) \propto \frac{1}{x-\xi+i\varepsilon} \cdots$$

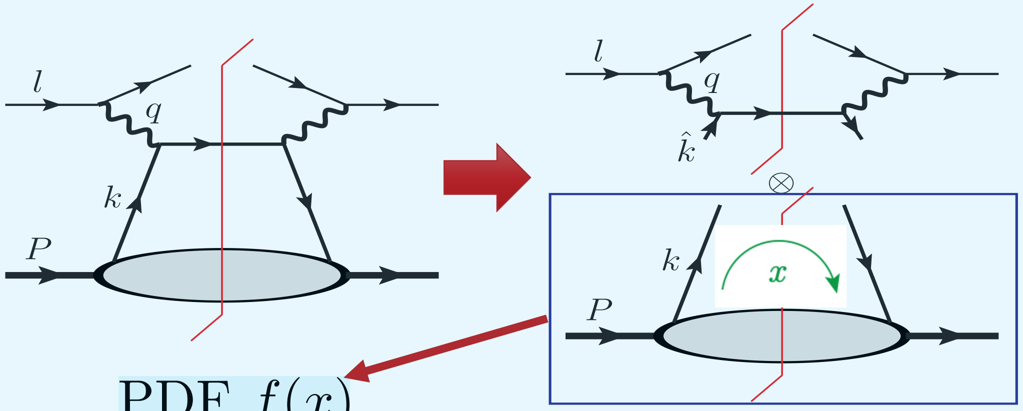
$$\Rightarrow \quad i\mathcal{M} \propto \int_{-1}^{1} \mathrm{d}x \frac{F(x,\xi,t)}{x-\xi+i\varepsilon} \equiv \text{“}F_0(\xi,t)\text{”}$$

- also true for most other processes
- x-dependence is only constrained by a "moment"
- easy to fit to the data



Inclusive Process vs. Exclusive Process

❑ Deeply Inelastic Scattering (DIS):



PDF $f(x)$

$x = k^+/P^+$

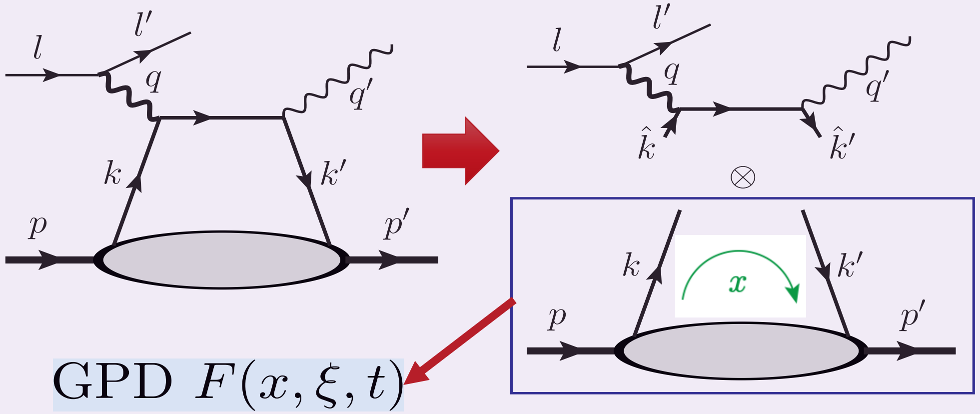
Cross section: Cut diagrams

$$\sigma_{\rm DIS} \simeq \int_{x_B}^{1} \mathrm{d}x \, f(x) \, \hat{\sigma}(x/x_B)$$

- PDF ~ probability
- At LO: $x = x_B$
- Beyond LO: $x \in [x_B, 1]$

x-dependence: Part of measurement

❑ Deeply Virtual Compton Scattering (DVCS)



GPD $F(x, \xi, t)$

$$x = \frac{(k+k')^+}{(p+p')^+}$$

$$\xi = \frac{(p-p')^+}{(p+p')^+}$$

$$t = (p-p')^2$$

Amplitude: Uncut diagrams

$$\mathcal{M}_{\rm DVCS}(\xi, t) \simeq \int_{-1}^{1} \mathrm{d}x \, F(x, \xi, t) \, \hat{\mathcal{M}}(x, \xi)$$

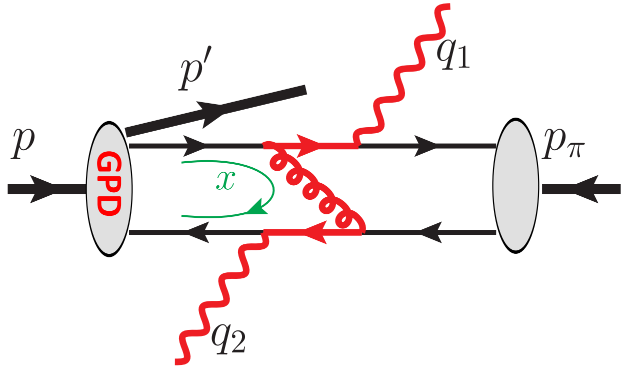
- GPD ~ amplitude
- $k^+ = (x + \xi) \, P^+$ is loop momentum
- At any order: $x \in [-1, 1]$

x-dependence: Hard to measure

What kind of process/observable could be sensitive to the x-dependence?

- **Create an entanglement between the internal x and an externally measured variable?**
  - **Production of two back-to-back high pT particles (say, two photons):**

$$\pi^-(p_\pi) + P(p) \to \gamma(q_1) + \gamma(q_2) + N(p')$$



  - **Kinematical observables:**
    - $t = (p - p')^2$
    - $\xi = (p^+ - p'^+)/(p^+ + p'^+)$

Hard scale: $q_T \gg \Lambda_{QCD}$

Soft scale: $t \sim \Lambda^2_{QCD}$

  - **Factorization:**

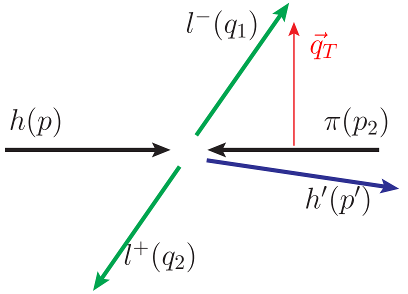
[*suppressing pion DA factor*]

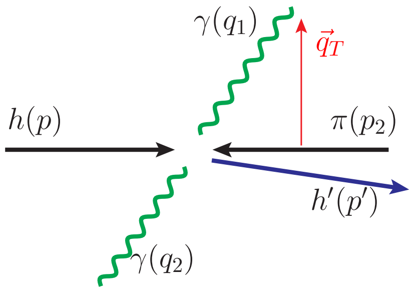
$$\mathcal{M}(t, \xi, q_T) = \int_{-1}^{1} \mathrm{d}x\, F(x, \xi, t; \mu) \cdot C(x, \xi; q_T/\mu) + \mathcal{O}(\Lambda_{QCD}/q_T)$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t\ \mathrm{d}\xi\ \mathrm{d}q_T} \sim |\mathcal{M}(t, \xi, q_T)|^2$$

$x \leftrightarrow q_T$

q_T distribution is "conjugate" to x distribution

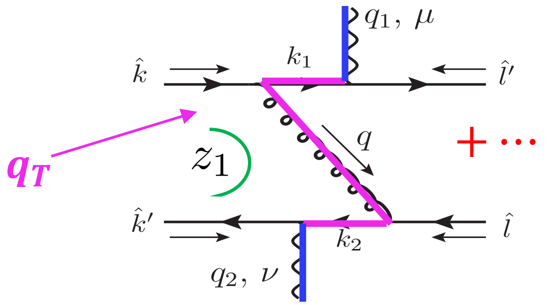




\+ photon-meson pair, meson-meson pair

What kind of process/observable could be sensitive to the x-dependence?

❑ Hard part for A-type:



New

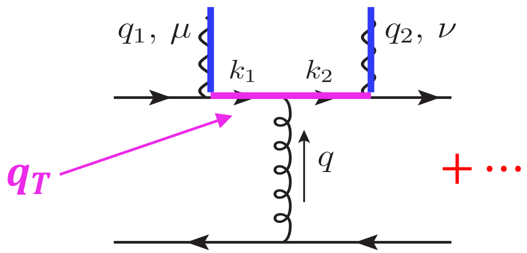
$\kappa = 4\, q_T^2/\hat{s}$

- **Gluon propagator** $q^2 = -\frac{\hat{s}}{4}\left[\left(2z_1 - 1 - \sqrt{1-\kappa}\right)\left(2z_2 - 1 - \sqrt{1-\kappa}\right) + \kappa\right]$

➡ $$\mathcal{M} \propto \int_0^1 \mathrm{d}z_1\, \mathrm{d}z_2\, \frac{\phi(z_1)\phi(z_2)}{(1-z_1)\,(1-z_2)\left[\left(2z_1 - 1 - \sqrt{1-\kappa}\right)\left(2z_2 - 1 - \sqrt{1-\kappa}\right) + \kappa\right]}$$

- **Change q_T changes the z_1-z_2 integral.**
- **$\mathrm{d}\sigma/\mathrm{d}q_T^2$ provides sensitivity to the DA's functional form of z.**

❑ Hard part for B-type:



Like "time-like" form factor

- **Gluon propagator** $q^2 = z_2(1-z_1)\hat{s}$

➡ $$\mathcal{M} \propto \int_0^1 \mathrm{d}z_1\, \mathrm{d}z_1\, \frac{\phi(z_1)\phi(z_2)}{z_1\,(1-z_1)\,z_2\,(1-z_2)} \sim \left[\int_0^1 \mathrm{d}z\, \frac{\phi(z)}{z\,(1-z)}\right]^2$$

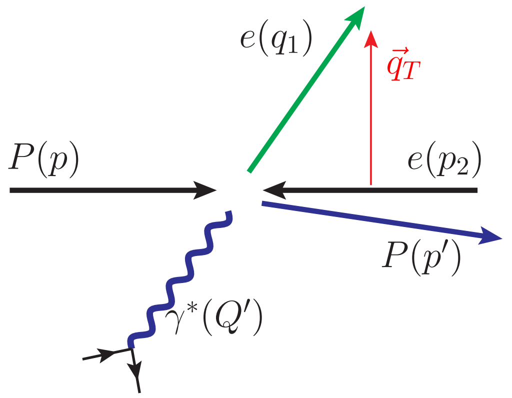
- ***Not* sensitive to DA functional form.**
- **Relies on $\phi(z) = 0$ at end points.**
- **Sudakov resummation could suppress the end-point sensitivity.**

Li, Sterman, 1992

What kind of process/observable could be sensitive to the x-dependence?

- **Create an entanglement between the internal x and an externally measured variable?**
  - **Double DVCS – invariant mass of a lepton-pair:**

$$e(p_2) + P(p) \to e(q_1) + \gamma^*(q_2)[\to \ell^+\ell^-] + P(p')$$



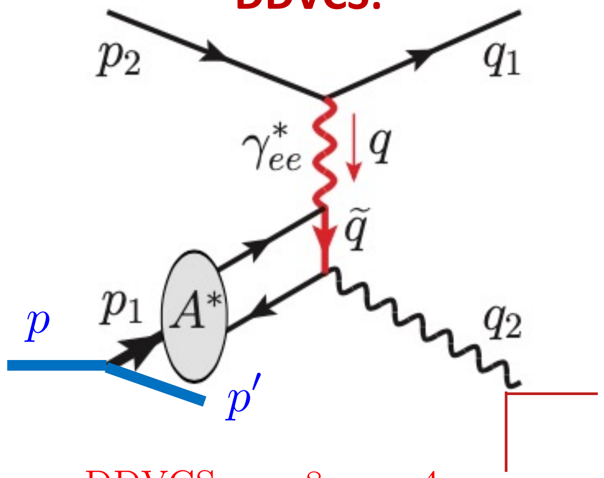
Factorization: Can be factorized in the same way as DVCS!

  - **The x-dependence on GPDs:**

The DVCS and DVMP type of processes only sensitive to:

$$\int_{-1}^{1} dx \frac{F(x,\xi,t)}{x-\xi \pm i\varepsilon}$$

DDVCS:



$$\sigma^{\mathrm{DDVCS}} \propto e^8 \propto \alpha_{\mathrm{EM}}^4$$

Requires a very high luminosity!

Transverse momentum flows from the final-state lepton to the virtual photon, and the quark line of momentum, $\widetilde{q}$, whose invariant mass is sensitive to the invariant mass of the lepton pair:

$$q_2^2 = (2\xi P + q)^2 = (2\xi)2P\cdot q - Q^2 + \mathcal{O}(|t|)$$

$$\widetilde{q}^{\,2} = ((x+\xi)P + q)^2$$
$$= \frac{Q^2 + q_2^2}{2\xi}\left[x - \xi\left(\frac{1 - q_2^2/Q^2}{1 + q_2^2/Q^2}\right)\right] \to x - \xi \quad \text{as} \quad q_2^2 \to 0 \quad \textbf{DVCS}$$

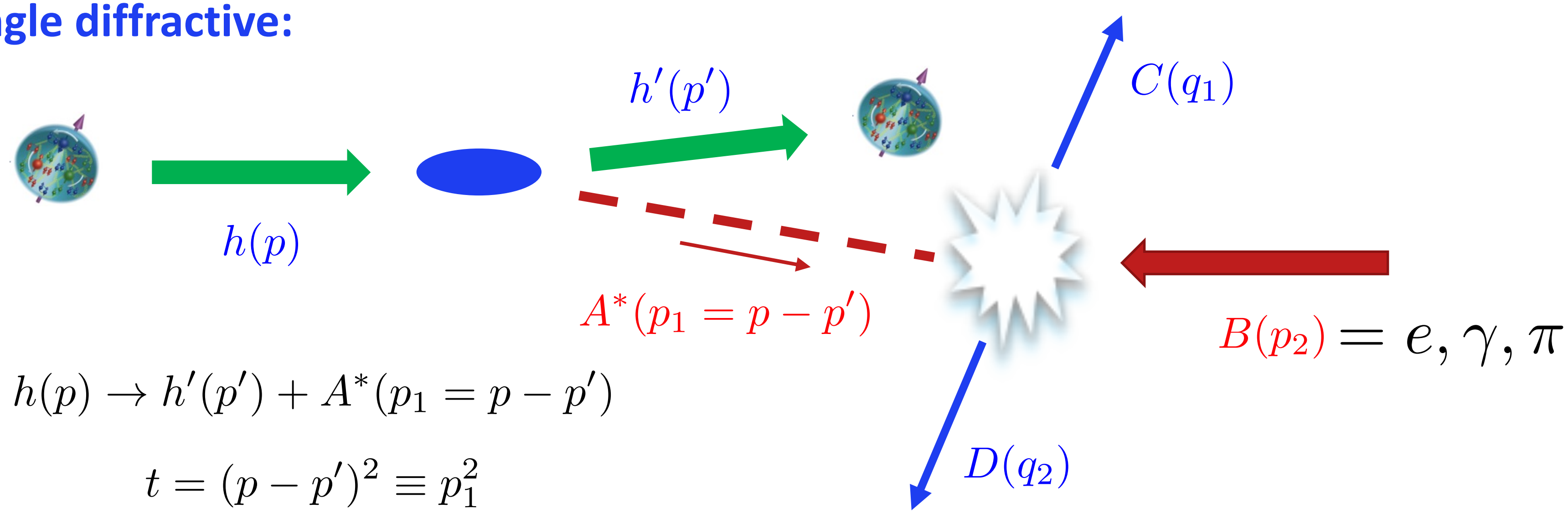
Direct sensitive to external variable, q_2^2 , directly sensitive to q_T

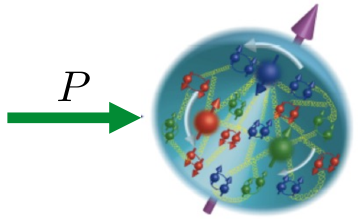
Single-Diffractive Hard Exclusive Processes (SDHEP)

Qiu & Yu, JHEP 08 (2022) 103, PRD 107 (2023) 1, in preparation

❑ Two-stage diffractive $2 \to 3$ hard exclusive processes:

- **Single diffractive:**





Probing its structure without breaking it!

$$h(p) \to h'(p') + A^*(p_1 = p - p')$$

$$t = (p - p')^2 \equiv p_1^2$$

- **Hard probe:** $2 \to 2$ **high** q_T **exclusive process**

$$A^*(p_1) + B(p_2) \to C(q_1) + D(q_2)$$

$$(p - p') \cdot n \gg \sqrt{|t|} \quad \Longleftrightarrow \quad |q_{1_T}| = |q_{2_T}| \gg \sqrt{-t}$$

- **The single diffractive $2 \to 3$ exclusive hard processes:**

$$h(p) + B(p_2) \to h'(p') + C(q_1) + D(q_2)$$

- **Necessary condition for QCD factorization:**

$$|q_{1_T}| = |q_{2_T}| \gg \sqrt{-t}$$

The state $A^*(p_1)$ lives much longer than $2 \to 2$ hard exclusive collision!

Not necessarily sufficient!

Single-Diffractive Hard Exclusive Processes (SDHEP)

- **Two-stage diffractive $2 \to 3$ hard exclusive processes:**

Qiu & Yu, JHEP 08 (2022) 103, PRD 107 (2023) 1, in preparation

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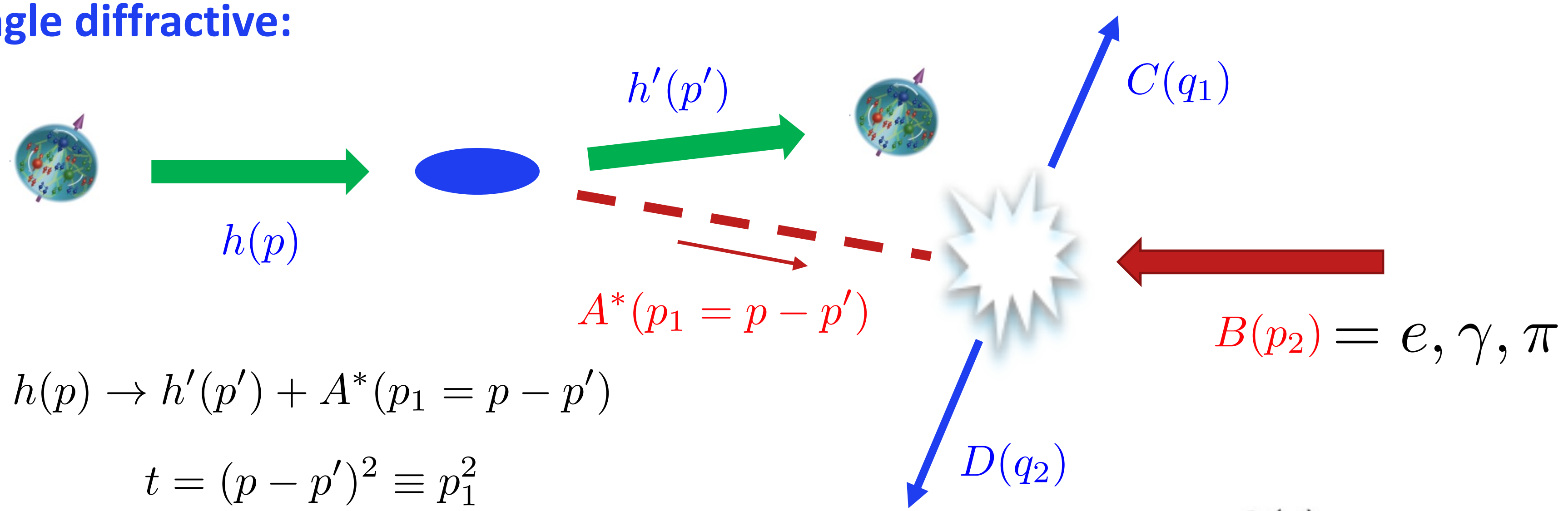

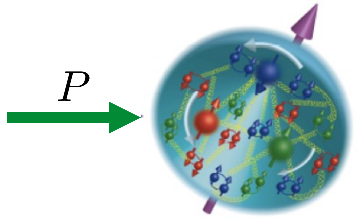

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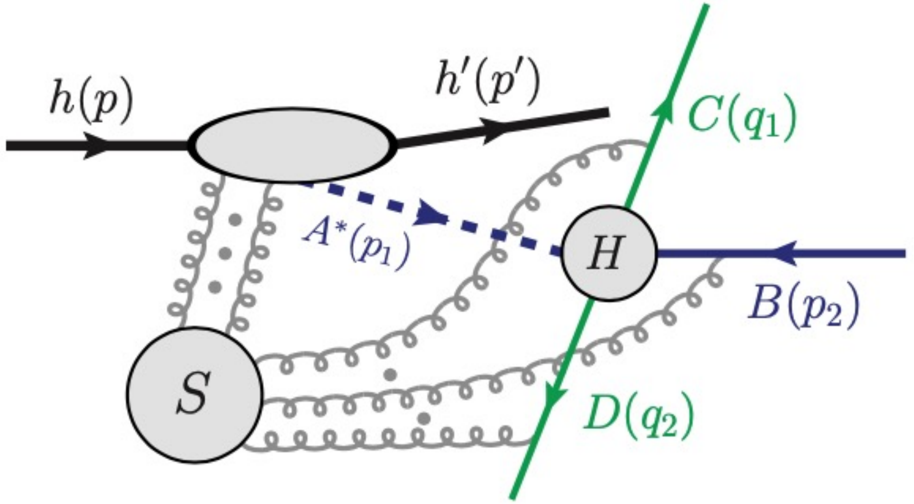

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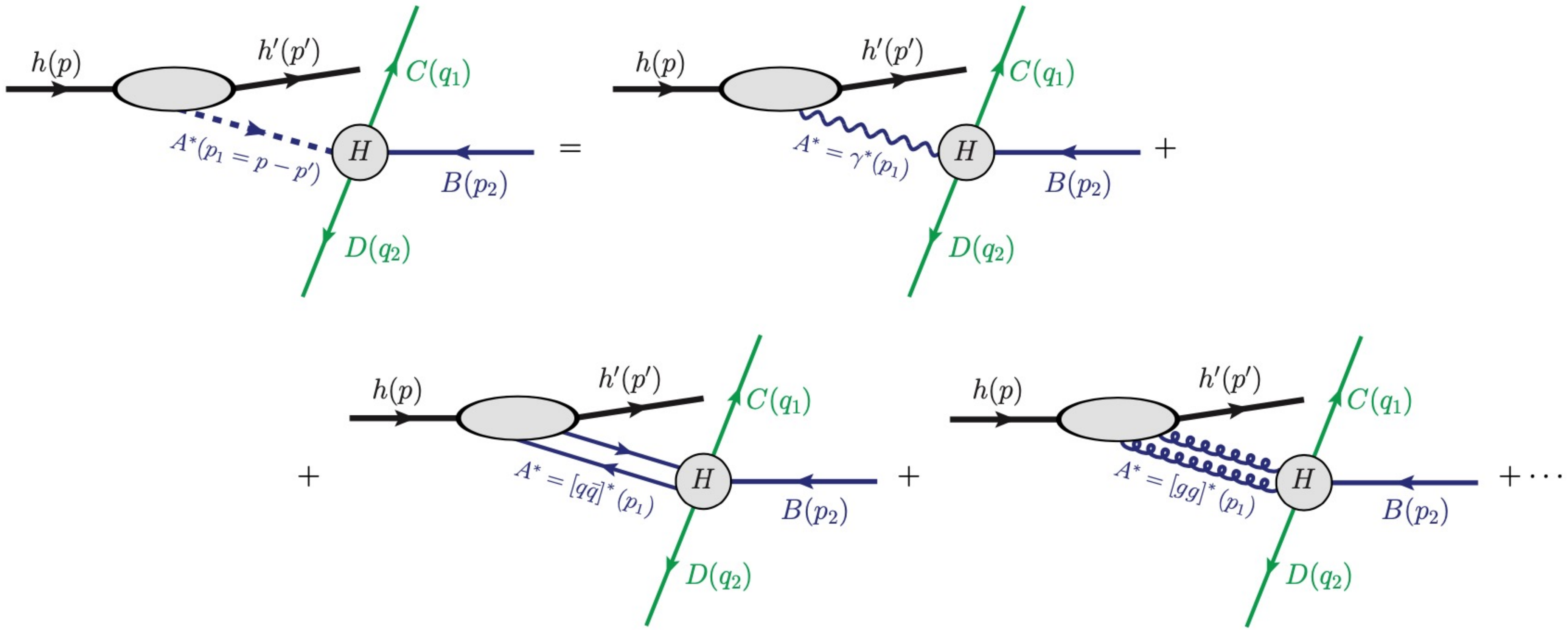
Single-Diffractive Hard Exclusive Processes (SDHEP)

- **Two-stage diffractive $2 \to 3$ hard exclusive processes:**

Qiu & Yu, JHEP 08 (2022) 103, PRD 107 (2023) 1, in preparation



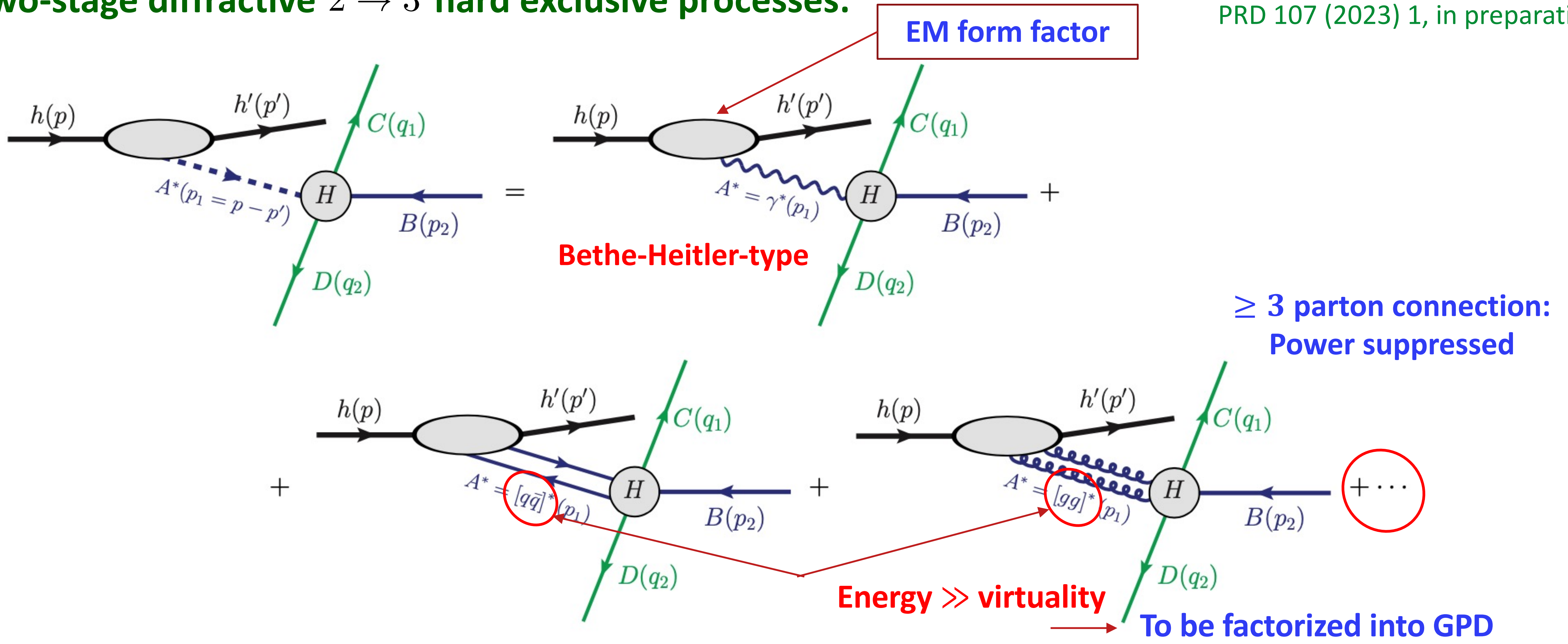
The exchanged state $A^*(p\text{-}p')$ is a sum of all possible partonic states, $\sum_{n=1,2,..}$, allowed by

- Quantum numbers of *h(p) – h'(p')*
- Symmetry of producing non-vanishing *H*

Single-Diffractive Hard Exclusive Processes (SDHEP)

Qiu & Yu, JHEP 08 (2022) 103, PRD 107 (2023) 1, in preparation

- **Two-stage diffractive $2 \to 3$ hard exclusive processes:**



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- Symmetry of producing non-vanishing H

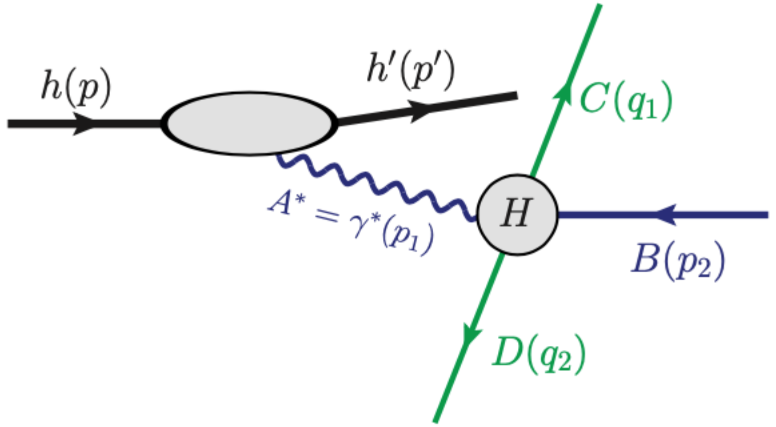
General Discussion on n=1 state: γ^*

❑ Exchange of a virtual photon:

Qiu & Yu, PRD 107 (2023) 1

$$\mathcal{M}^{(1)} = \frac{ie^2}{t}\langle h'(p')|J^\mu(0)|h(p)\rangle\,\langle C(q_1)D(q_2)|J_\mu(0)|B(p_2)\rangle$$
$$\equiv \frac{ie^2}{t}F^\mu(p,p')\,\mathcal{H}_\mu(p_1,p_2,q_1,q_2)$$

$$J^\mu = \sum_{i\in q} Q_i \bar{\psi}_i \gamma^\mu \psi_i$$



$$F^\mu(p,p') = \langle h'(p')|J^\mu(0)|h(p)\rangle$$
$$= F_1^h(t)\,\bar{u}(p')\gamma^\mu u(p) + F_2^h(t)\,\bar{u}(p')\frac{i\sigma^{\mu\nu}p_{1\nu}}{2m_h}u(p)$$

Forbidden for $p \to n$ (or $n \to p$) transition GPDs
Or not allowed by H

Has a leading component , $F^+ \propto \mathcal{O}(Q)$, as h-h' fast along "+"

$$F^+\mathcal{H}^- = \frac{1}{p_1^+}F^+\left(p_1^+\mathcal{H}^-\right) = \frac{1}{p_1^+}F^+\left(p_1\cdot\mathcal{H} + \boldsymbol{p}_{1\perp}\cdot\boldsymbol{\mathcal{H}}_\perp - p_1^-\mathcal{H}^+\right) \sim \mathcal{O}(\sqrt{|t|})$$

Leading power of $F\cdot\mathcal{H}$

➡ $\mathcal{M}^{(1)} \sim \mathcal{O}(1/\sqrt{|t|})$ Higher power than n=2 contribution, but, higher power in power of α_{EM}

$\mathcal{M}^{(2)} \sim \mathcal{O}(1/Q)$ ➡ $\mathcal{M}^{(1)}/\mathcal{M}^{(2)} \sim \mathcal{O}(Q/\sqrt{|t|})$

If we neglect contribution from $n \geq 3$, $\mathcal{M}^{(1+2)}_{\text{SDHEP}} \sim$ is up to corrections at $\mathcal{O}(\sqrt{|t|}/Q^2)$

Jefferson Lab

Exclusive Massive Photon-Pair Production in Meson-Hadron Collision

- **Challenge for QCD factorization:** $\pi^-(p_\pi) + P(p) \to \gamma(q_1) + \gamma(q_2) + N(p')$ $\quad \lambda \sim m_\pi/Q, \quad Q \sim q_T$

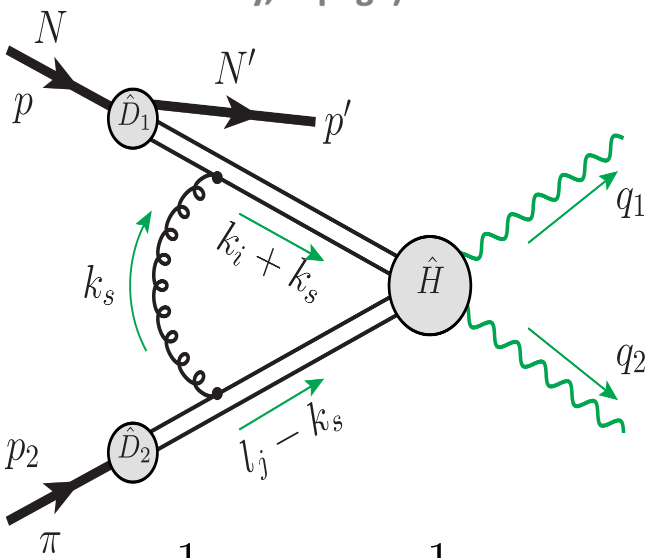
Gluons in the Glauber region: $k_s = (\lambda^2, \lambda^2, \lambda)\, Q$ ***Transverse component contribute to the leading region!***

$k_i = (1, \lambda^2, \lambda)\, Q$

$l_j = (\lambda^2, 1, \lambda)\, Q$

ERBL region

(Efremov, Radyushkin, Brodsky, Lepage)



$$\frac{1}{k_s^2 + i\varepsilon} \to \frac{1}{-\mathbf{k}_s^2 + i\varepsilon}$$

$$\frac{1}{(k_i + k_s)^2 + i\varepsilon} \to \frac{1}{k_s^- + i\varepsilon}$$

$$\frac{1}{(l_j - k_s)^2 + i\varepsilon} \to \frac{1}{-k_s^+ + i\varepsilon}$$

No pinch!

$k_s^+ \to k_s^+ + i\,\mathcal{O}(\lambda Q), \quad k_s^- \to k_s^- - i\,\mathcal{O}(\lambda Q)$

after deformation

$k_s \sim (\lambda, \lambda, \lambda)\, Q$

Keep only "+" or "-" component in collinear function

Exclusive Massive Photon-Pair Production in Meson-Hadron Collision

- **Challenge for QCD factorization:** $\pi^-(p_\pi) + P(p) \to \gamma(q_1) + \gamma(q_2) + N(p')$ $\lambda \sim m_\pi/Q,$ $Q \sim q_T$

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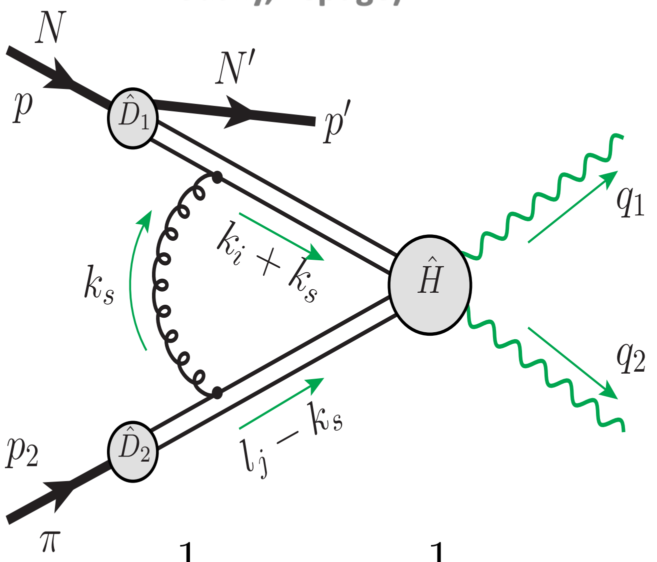


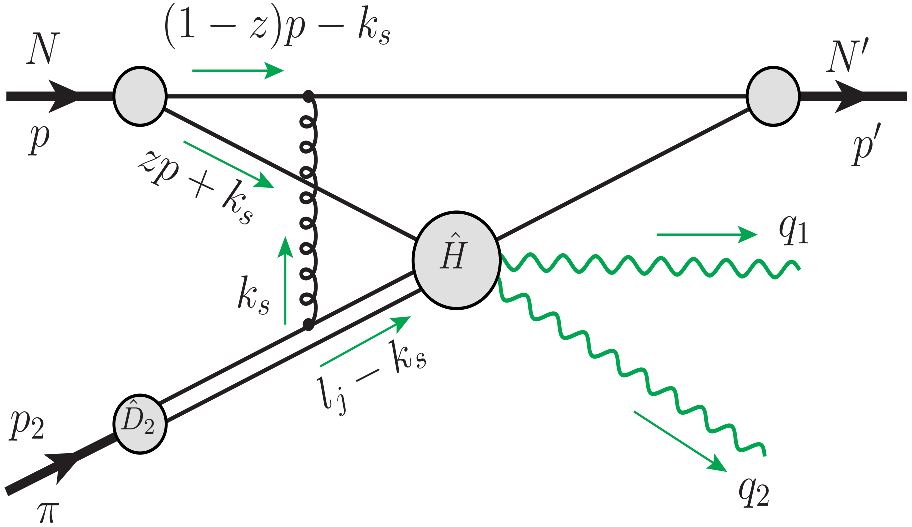
$$\frac{1}{k_s^2 + i\varepsilon} \to \frac{1}{-\mathbf{k}_s^2 + i\varepsilon}$$

$$\frac{1}{(k_i + k_s)^2 + i\varepsilon} \to \frac{1}{k_s^- + i\varepsilon}$$

$$\frac{1}{(l_j - k_s)^2 + i\varepsilon} \to \frac{1}{-k_s^+ + i\varepsilon}$$

No pinch!

DGLAP region



$$\frac{1}{((1-z)p - k_s)^2 + i\varepsilon} \to \frac{1}{k_s^- - i\varepsilon}$$

$$\frac{1}{(zp + k_s)^2 + i\varepsilon} \to \frac{1}{k_s^- + i\varepsilon}$$

Pinched!

Same conclusion if k_s flows Through N'!

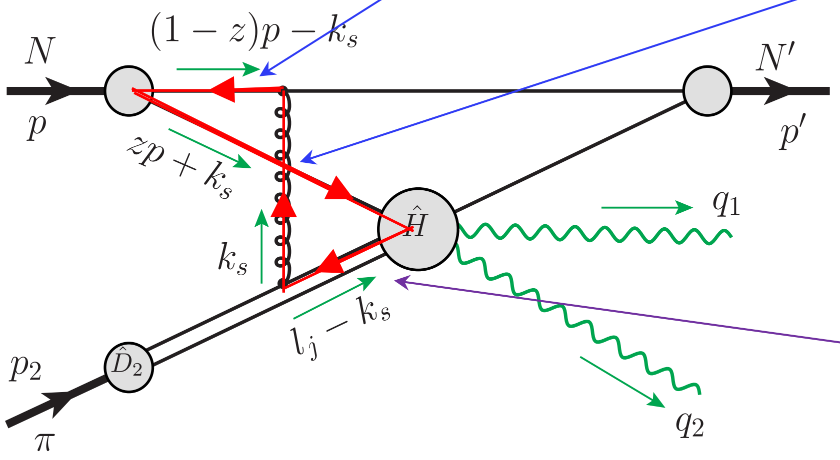
Exclusive Massive Photon-Pair Production in Meson-Hadron Collision

- **Factorization:** $\pi^-(p_\pi) + P(p) \to \gamma(q_1) + \gamma(q_2) + N(p')$

Qiu & Yu, JHEP 08 (2022) 103

$$\frac{1}{-k_s^- + i\varepsilon}$$

$$\frac{1}{+k_s^- + i\varepsilon}$$



k_s^- **is pinched around 0**

$$\frac{1}{-k_s^+ + i\varepsilon}$$

Only pole for k_s^+

DGLAP region: pinched

Deformation out of the Glauber region:

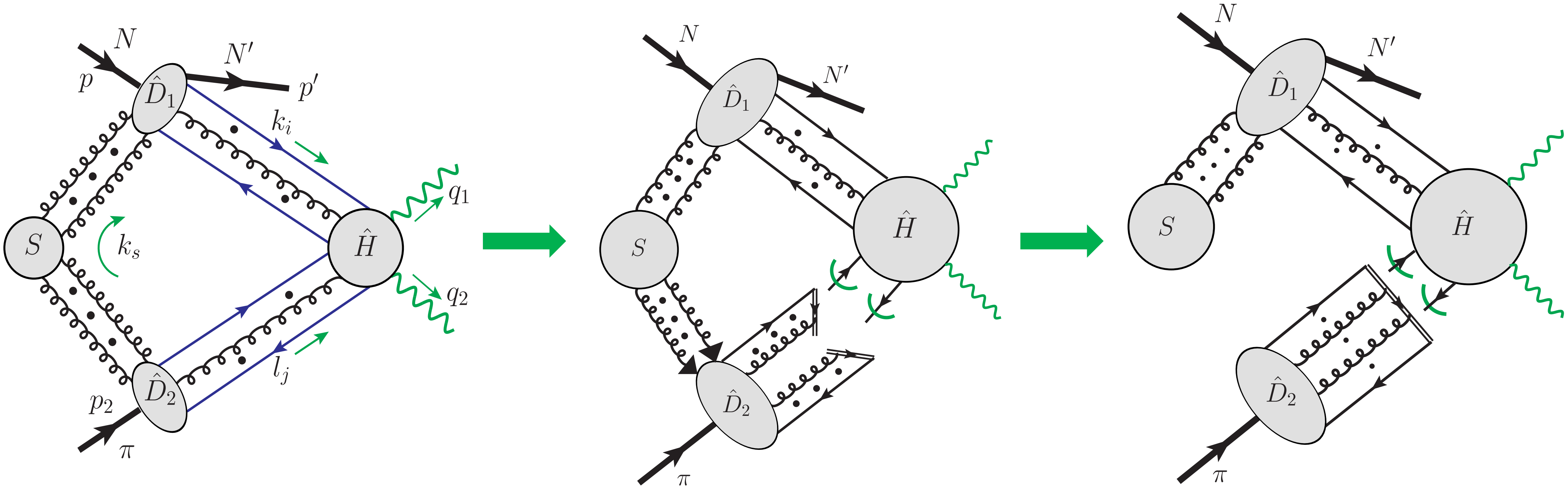
$k_s^+ \to k_s^+ - i\mathcal{O}(Q)$ ➡ $k_s \sim (1, \lambda^2, \lambda)Q$ **Collinear region**

Works for both ERBL and DGLAP regions!

Exclusive Massive Photon-Pair Production in Meson-Hadron Collision

❑ **Factorization:** $\pi^-(p_\pi) + P(p) \to \gamma(q_1) + \gamma(q_2) + N(p')$

Qiu & Yu, JHEP 08 (2022) 103



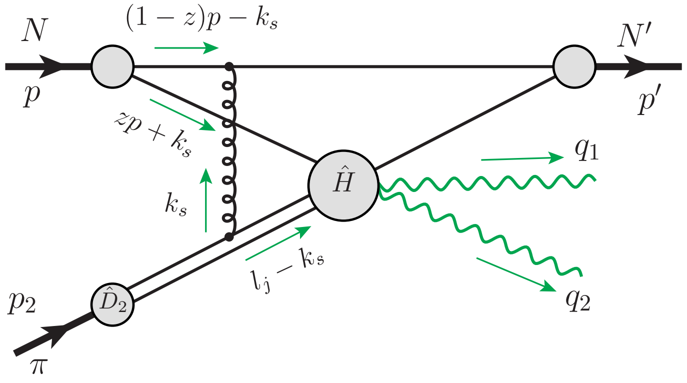
- **Same strategy for proving the factorization of first sub-leading power for inclusive processes** — Qiu, Sterman, 1991
- **No QCD factorization for double diffractive hadronic scattering!**

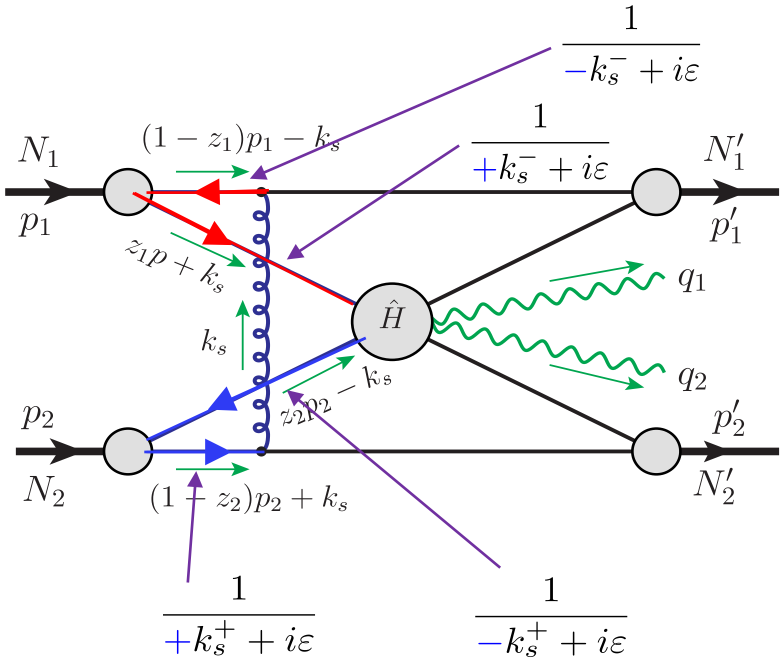
$$p+n \to p+n+\gamma+\gamma, \quad p+n \to p+n+\text{jet}+\text{jet}, \quad p+p \to p+p+\text{jet}+\text{jet}, \quad p+\bar{p} \to p+\bar{p}+\text{jet}+\text{jet}, \ldots$$

Why *single* diffractive?

- **Double diffractive process**

Glauber pinch for diffractive scattering



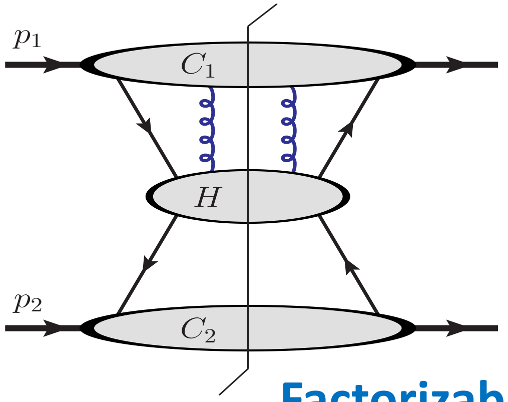


Both k_s^+ and k_s^- are pinched in Glauber region!

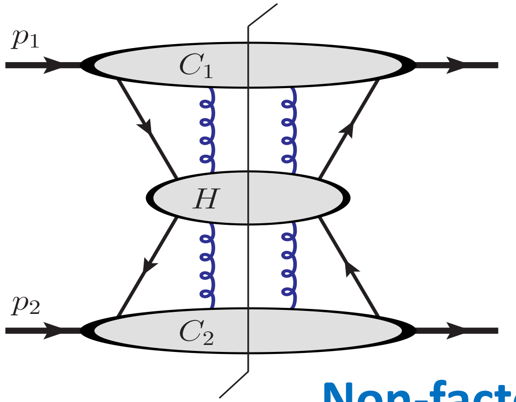
Break of factorization

Factorizable if all pion momentum flows into hard part

- **Compare: Drell-Yan process at high twist:**



Factorizable



Non-factorizable

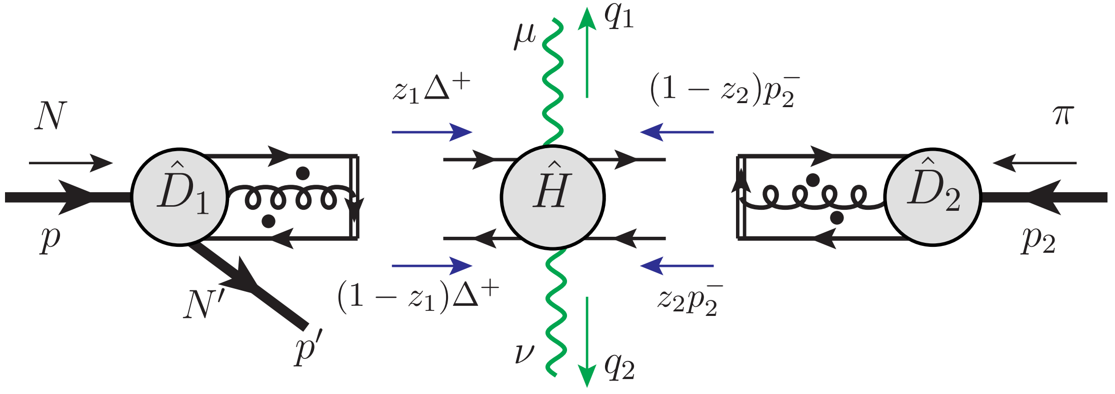
Only the 1st sub-leading twist is factorizable!

Qiu & Sterman, NPB, 1991

Exclusive Massive Photon-Pair Production in Meson-Hadron Collision

- **Factorization formula:** $\pi^-(p_\pi) + P(p) \to \gamma(q_1) + \gamma(q_2) + N(p')$

$$\mathcal{M}^{\mu\nu} = \int \mathrm{d}z_1 \mathrm{d}z_2 \left[\widetilde{\mathcal{F}}^{ud}_{NN'}(z_1,\xi,t) D(z_2) C^{\mu\nu}(z_1,z_2) + \mathcal{F}^{ud}_{NN'}(z_1,\xi,t) D(z_2) \widetilde{C}^{\mu\nu}(z_1,z_2) \right] + \mathcal{O}(\Lambda_{\mathrm{QCD}}/q_T)$$



$$\mathcal{F}^{ud}_{NN'}(z_1,\xi,t) = \int \frac{\mathrm{d}y^-}{4\pi} e^{iz_1\Delta^+ y^-} \langle N'(p')|\bar{d}(0)\gamma^+ \Phi(0,y^-;w_2)\, u(y^-)|N(p)\rangle$$

$$= \frac{1}{2P^+}\left[H^{ud}_{NN'}(z_1,\xi,t)\, \bar{u}(p')\gamma^+ u(p) - E^{ud}_{NN'}(z_1,\xi,t)\, \bar{u}(p') \frac{i\sigma^{+\alpha}\Delta_\alpha}{2m_p} u(p) \right],$$

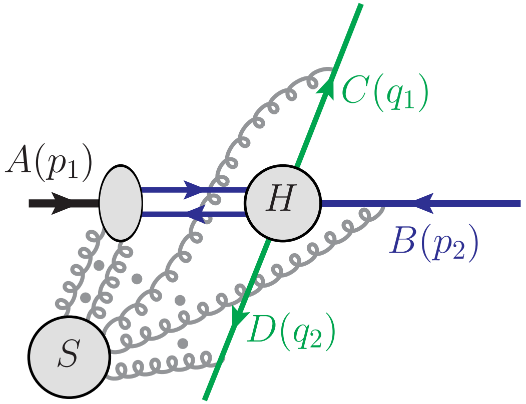
$$\widetilde{\mathcal{F}}^{ud}_{NN'}(z_1,\xi,t) = \int \frac{\mathrm{d}y^-}{4\pi} e^{iz_1\Delta^+ y^-} \langle N'(p')|\bar{d}(y^-)\gamma^+\gamma_5 \Phi(0,y^-;w_2)\, u(0)|N(p)\rangle$$

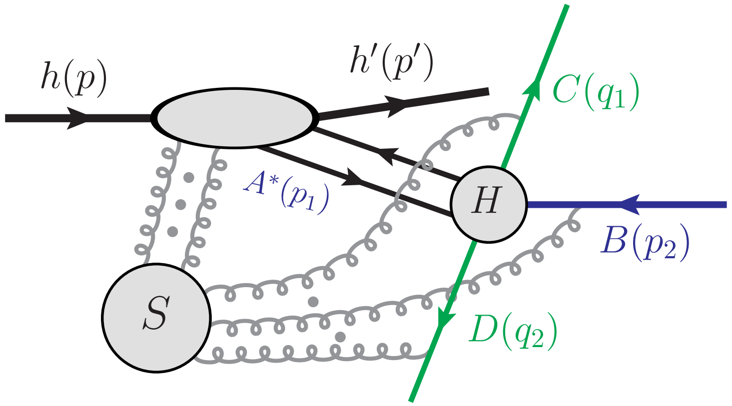
$$= \frac{1}{2P^+}\left[\widetilde{H}^{ud}_{NN'}(z_1,\xi,t)\, \bar{u}(p')\gamma^+\gamma_5 u(p) - \widetilde{E}^{ud}_{NN'}(z_1,\xi,t)\, \bar{u}(p') \frac{i\gamma_5\sigma^{+\alpha}\Delta_\alpha}{2m_p} u(p) \right]$$

Factorization for SDHEP in the Two-stage Paradigm

Qiu & Yu, JHEP 08 (2022) 103,
PRD 107 (2023) 1

❑ Factorization for 2-parton channel factorization:





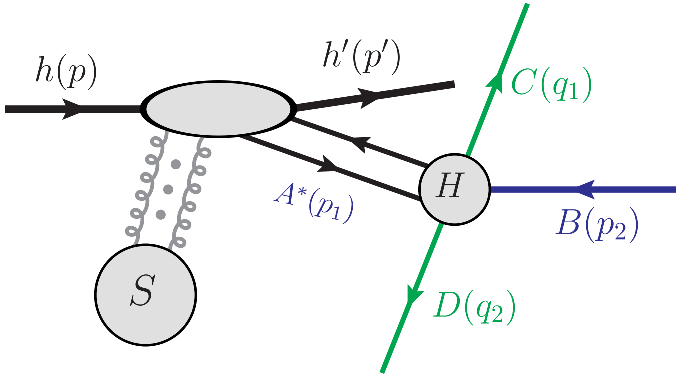
Only complication:

k_s^- is pinched in Glauber region for DGLAP region.

$$k_s^+ \mapsto k_s^+ \pm i\mathcal{O}(Q)$$

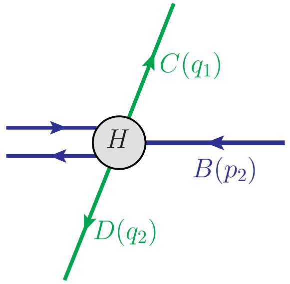
Glauber → *h*-collinear region

❑ Soft gluons cancel for the meson-initialized process if *C* and *D* are mesons:

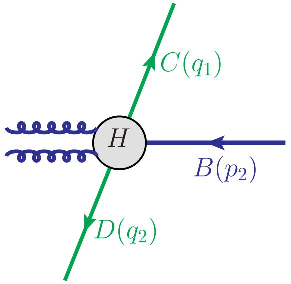


Soft gluons are no longer pinched and can be deformed into *h*-collinear region

GPD $\otimes$



or

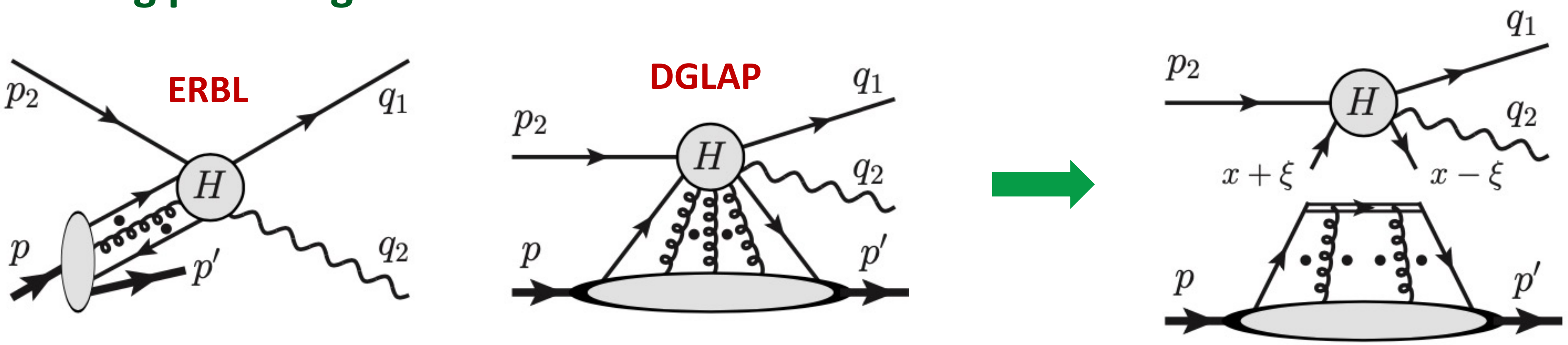


SDHEP with a Lepton Beam – JLab, EIC

❑ DVCS:

$$h(p) = \text{Proton}(p),\ h'(p') = \text{Proton}(p'),\ B(p_2) = \text{electron}(p_2),\ C(q_1) = \text{electron}(q_1),\ D(q_2) = \text{photon}(q_2)$$

❑ Leading pinch region:



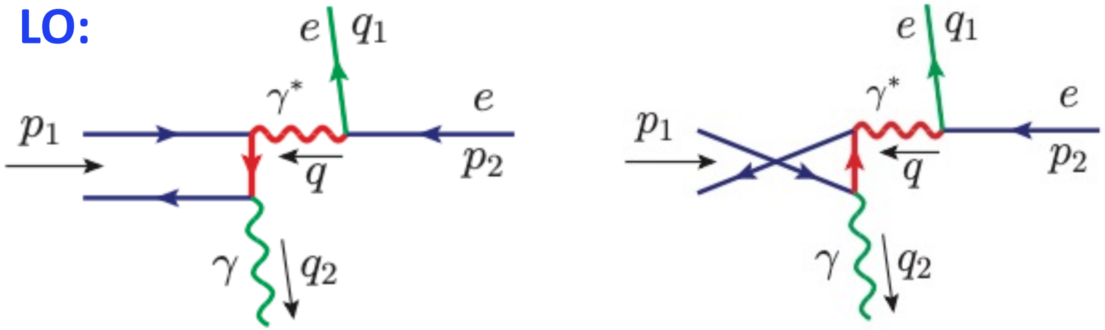
$$x = \frac{(k+k')^+}{(p+p')^+} \qquad \xi = \frac{(p-p')^+}{(p+p')^+}$$

❑ Factorization formula:

$$\mathcal{M}^{(2)}_{he\to h'e\gamma} = \sum_i \int_{-1}^{1} \mathrm{d}x\, F_i^h(x,\xi,t)\, C_{ie\to e\gamma}(x,\xi,q_T),$$

$$\Rightarrow \quad C^{(0)} \propto \frac{1}{x-\xi+i\varepsilon} - \frac{1}{x+\xi-i\varepsilon}$$

LO:



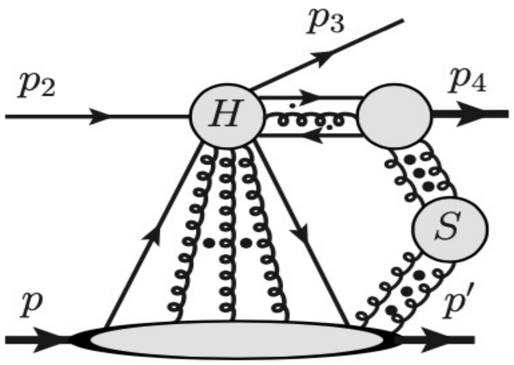
“moment-type” x-dependence

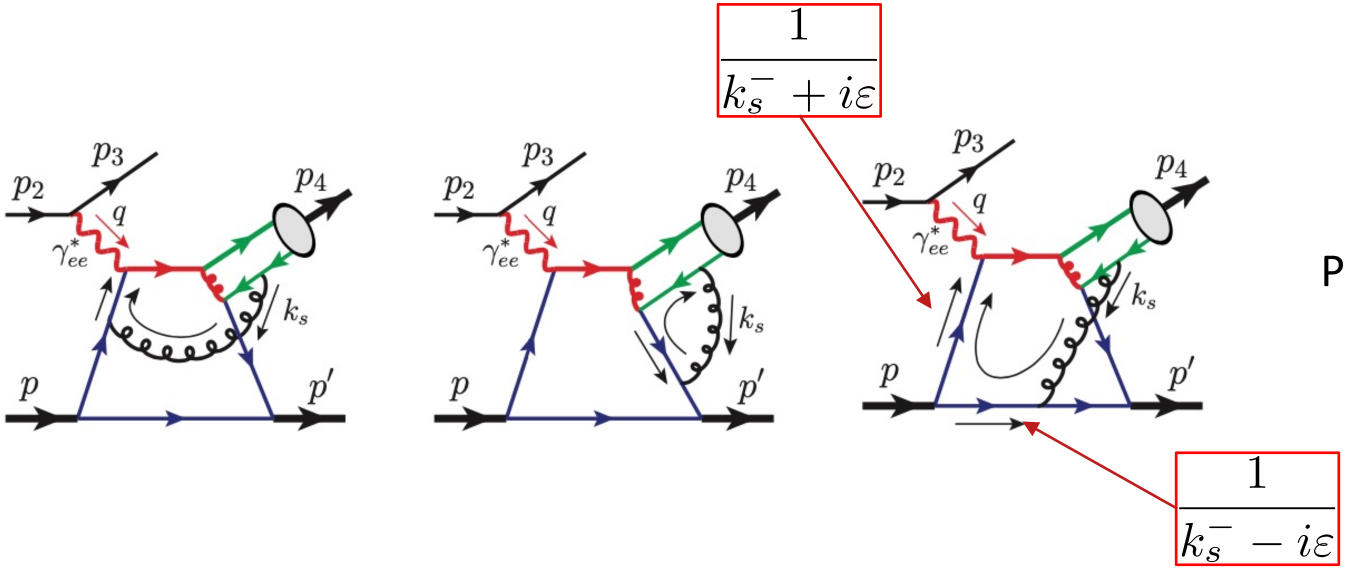
SDHEP with a Lepton Beam – JLab, EIC

❑ DVMP:

$$h(p) = \text{Proton}(p),\ \ h'(p') = \text{Proton}(p'),\ \ B(p_2) = \text{electron}(p_2),\ \ C(q_1) = \text{electron}(p_3),\ \ D(q_2) = \text{Meson}(p_4)$$

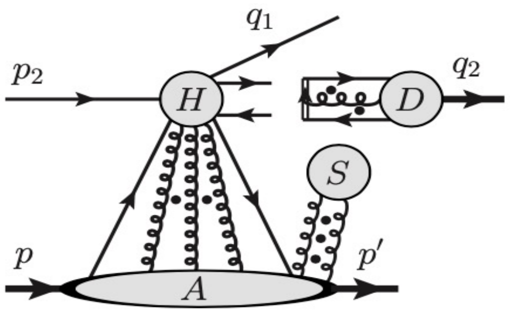
❑ Leading pinch region:

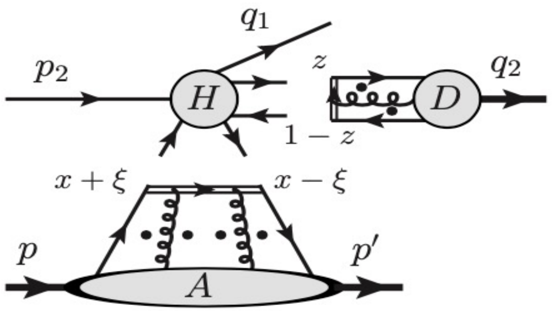




Pinched in k_s^-

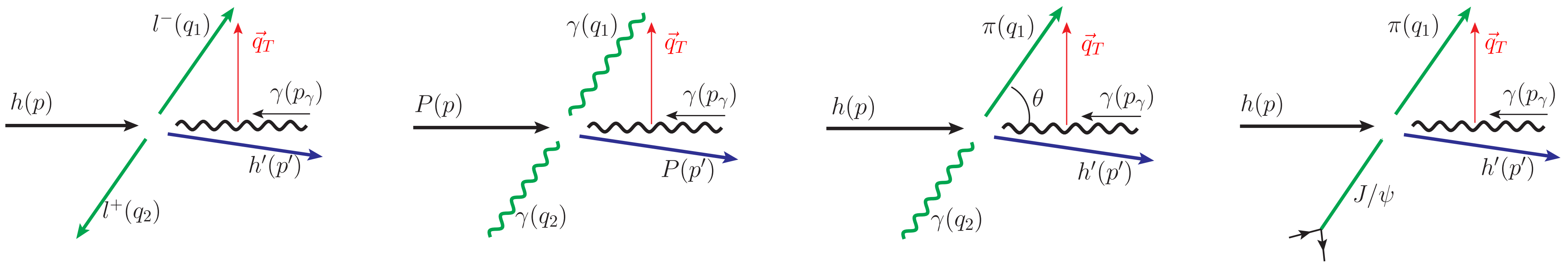
❑ Factorization:





$$\mathcal{M}^{(2)}_{he\to h'eM_D} = \sum_{i,j}\int_{-1}^{1} dx \int_0^1 dz_D \times F_i^{hh'}(x,\xi,t)\, C_{ie\to ej}(x,\xi;z_D;q_T)\,\phi_{j/D}(z_D)$$

SDHEP with a Photon Beam – JLab, EIC



(JLab Hall-D, …)

❑ Dilepton & Diphoton production:

A. Pedrak, et al. Phys.Rev.D96 (2017)074008, …

Both n=1 and n=2 should contribution, and factorizable

❑ Real photon + meson pair production:

G. Duplancic, et al. JHEP 11 (2018) 179, …

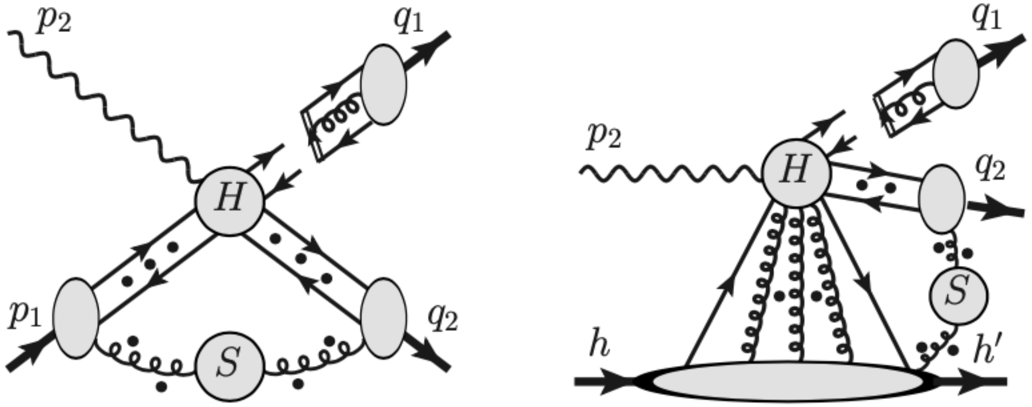
The n=1 channel is forbidden for a charge meson: $\pi^\pm$, or transversely polarized vector meson, ρ_T,

but, allowed for the production of a longitudinally polarized vector meson like ρ_L.

Factorization arguments are the same as that for DVMP.

❑ Light meson pair production – New:

$$\mathcal{M}_{h\gamma\to h' M_C M_D} = \sum_{i,j,k} \int_{-1}^{1} \mathrm{d}x \int_0^1 \mathrm{d}z_C\, \mathrm{d}z_D\, F_i^{hh'}(x,\xi,t) \times C_{i\gamma\to jk}(x,\xi;z_C,z_D;q_T)\, D_{j/C}(z_C)\, D_{k/D}(z_D)$$



Numerical results

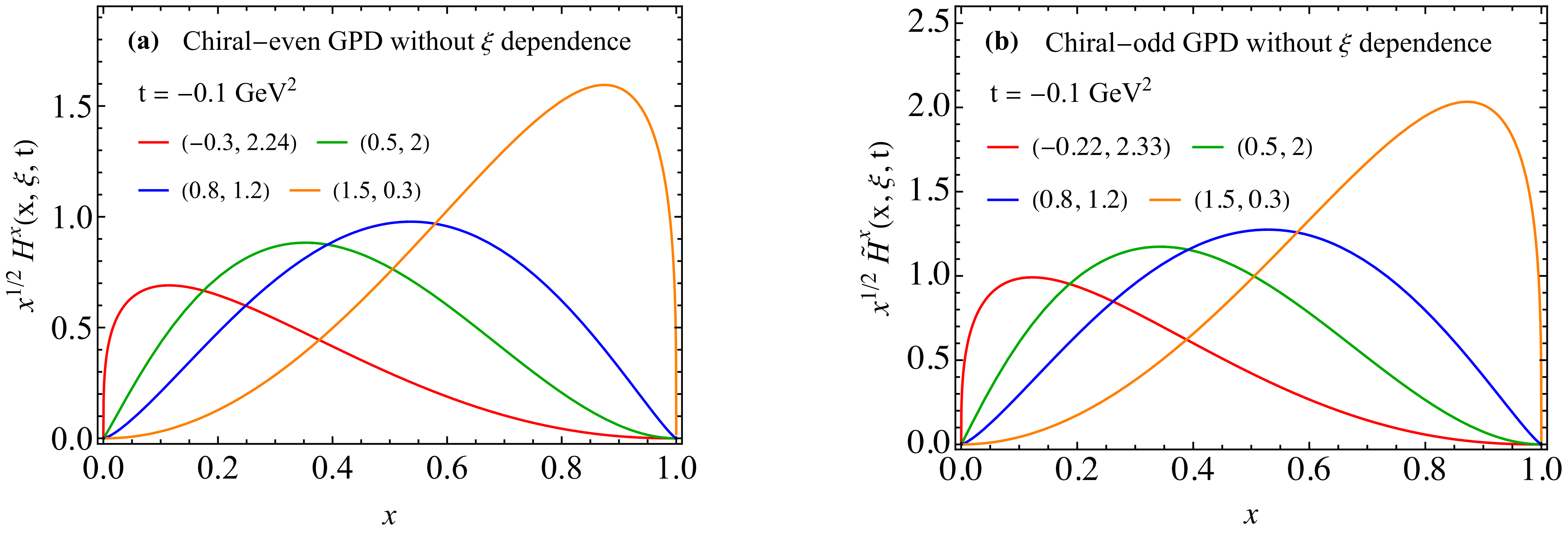
❑ GPD models – simplified GK model:

Goloskokov, Kroll
hep-ph/0501242
arXiv: 0708.3569
arXiv: 0906.0460

$$H_{pn}(x,\xi,t) = \theta(x)\, x^{-0.9\,(t/\mathrm{GeV}^2)}\, \frac{x^{\rho}(1-x)^{\tau}}{B(1+\rho,1+\tau)}$$

$$\widetilde{H}_{pn}(x,\xi,t) = \theta(x)\, x^{-0.45\,(t/\mathrm{GeV}^2)}\, \frac{1.267\, x^{\rho}(1-x)^{\tau}}{B(1+\rho,1+\tau)}$$

- **Neglect $E, \widetilde{E}$. Neglect evolution effect.**
- **Tune (ρ, τ) to control x shape.**
- **Fix DA: $D(z) = N\, z^{0.63}(1-z)^{0.63}$**



Jefferson Lab

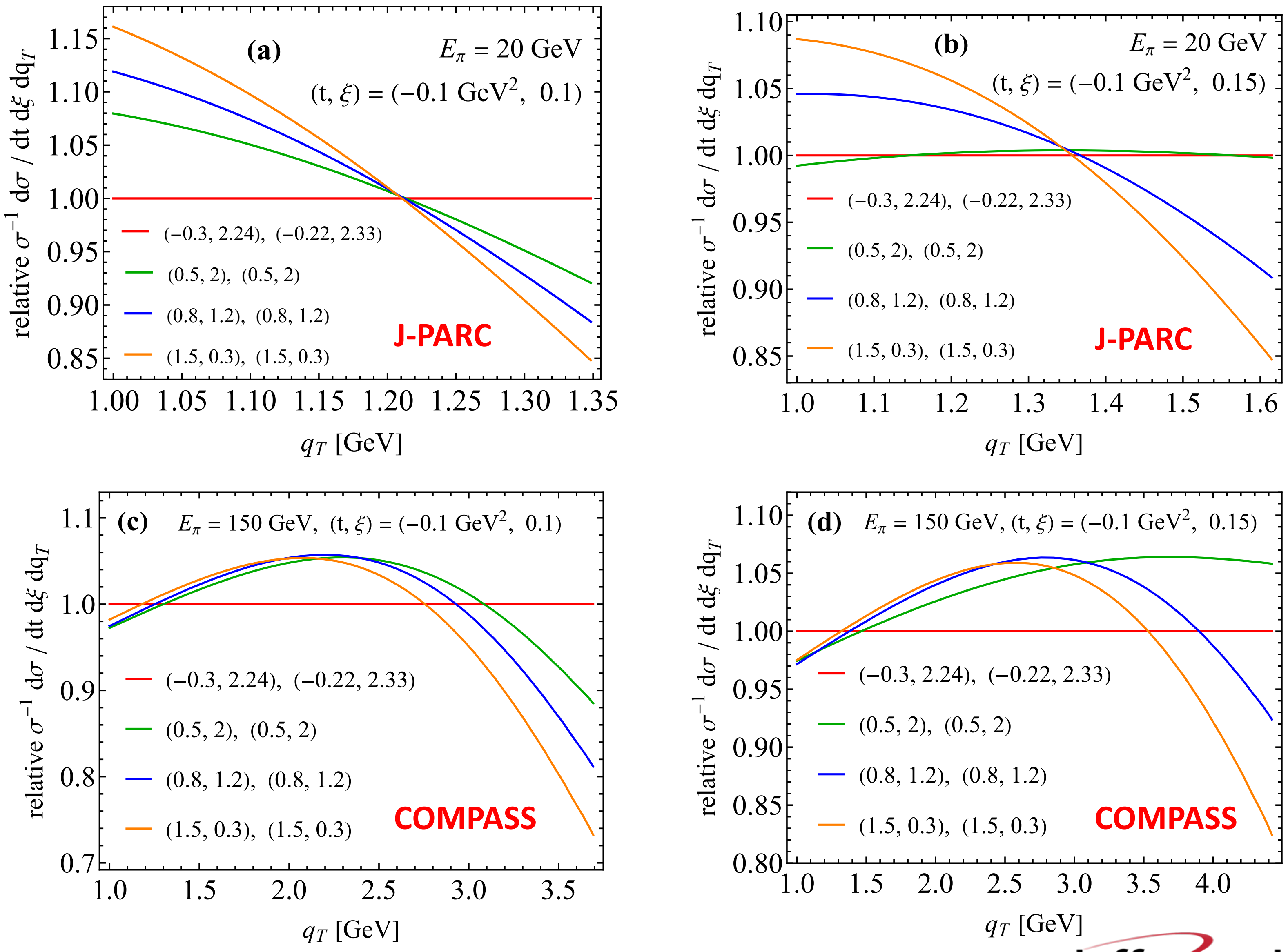
Numerical results

$$\frac{d\sigma}{dt\, d\xi\ dq_T} \sim |H(x, \xi, t)|^2$$

Relative q_T shape

$$\frac{\sigma_{\rm tot}^{-1} d\sigma / dq_T}{\text{some shape func}}$$

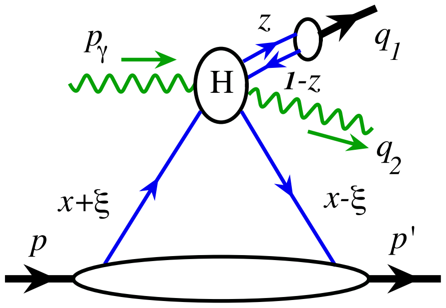
$$\sigma_{\rm tot} = \int_{1\ \rm GeV}^{\sqrt{\hat{s}}/2} dq_T \frac{d\sigma}{dt d\xi dq_T}$$



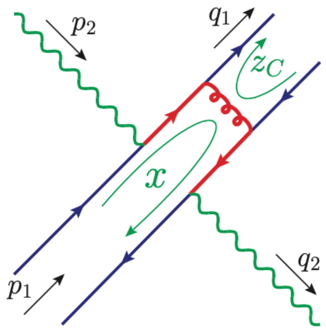
Exclusive Photo-Production of a $\pi\gamma$ Pair – Hall D at JLab

- **Process:** $\gamma(p_\gamma) + h(p) \to \pi^{\pm}(q_1) + \gamma(q_2) + h'(p')$

  First introduced by G. Duplancic et al. [JHEP 11 (2018) 179],
  No contribution from gluon GPDs



$$\frac{d\sigma}{dt\,d\xi\,dq_T^2}$$ or $$\frac{d\sigma}{dt\,d\xi\,d\cos\theta}$$

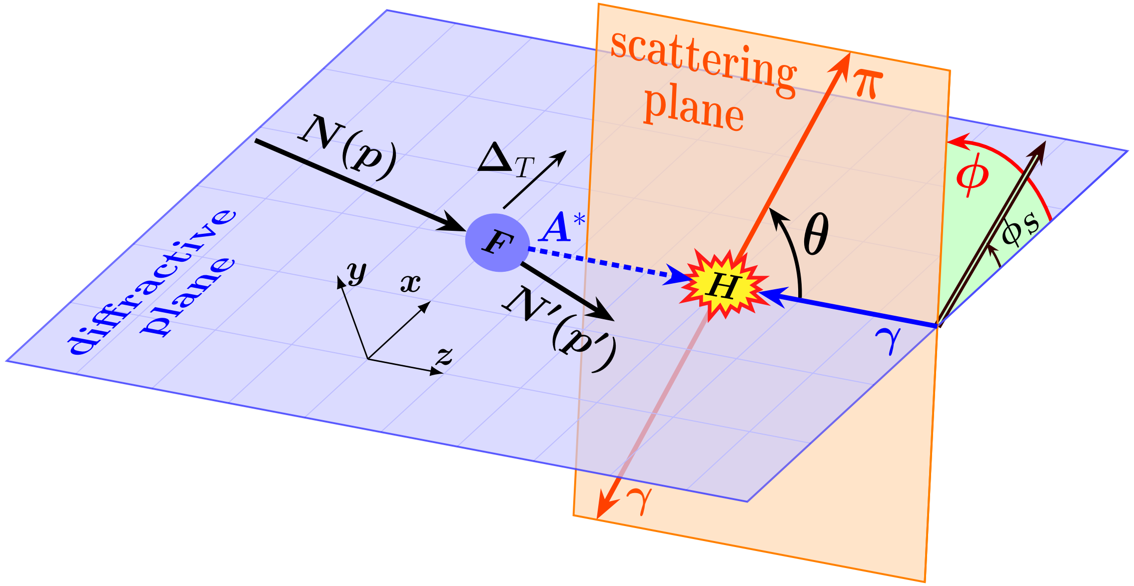


- **Factorization:**

  Proved to be valid when $q_T \gg \sqrt{|t|} \gtrsim \Lambda_{\rm QCD}$

- **Polarization of photon and hadron:**

$$\frac{d\sigma}{d|t|\,d\xi\,d\cos\theta\,d\phi} = \frac{1}{2\pi}\frac{d\sigma}{d|t|\,d\xi\,d\cos\theta}\cdot\Big[1+\lambda_N\lambda_\gamma\,A_{LL} + \zeta\,A_{UT}\cos 2(\phi-\phi_S) + \lambda_N\,\zeta A_{LT}\sin 2(\phi-\phi_S)\Big]$$



Unpolarized cross section:

$$\frac{d\sigma}{d|t|\,d\xi\,d\cos\theta} = \frac{N^2\,(1-\xi^2)}{32\,s\,(2\pi)^3\,(1+\xi)^2}\Sigma_{UU}$$

$$\Sigma_{UU} = |\widetilde{C}_+^{[H]}|^2 + |\widetilde{C}_-^{[H]}|^2 + |C_+^{[\widetilde{H}]}|^2 + |C_-^{[\widetilde{H}]}|^2$$

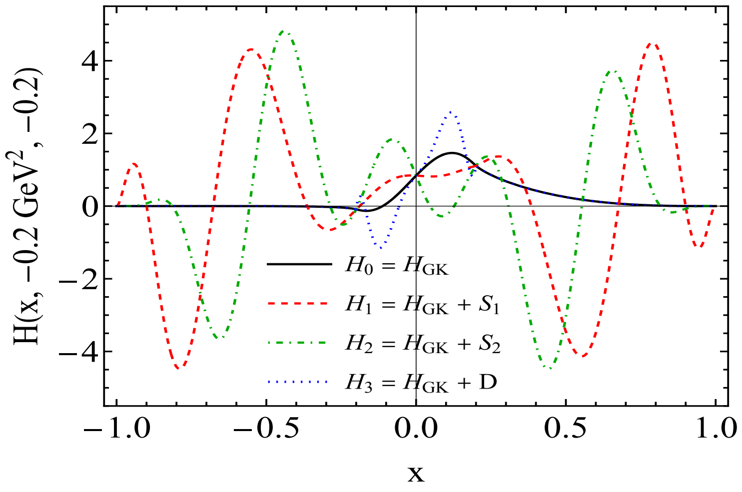
Exclusive $\pi^0\gamma$ Pair Production – Phenomenology

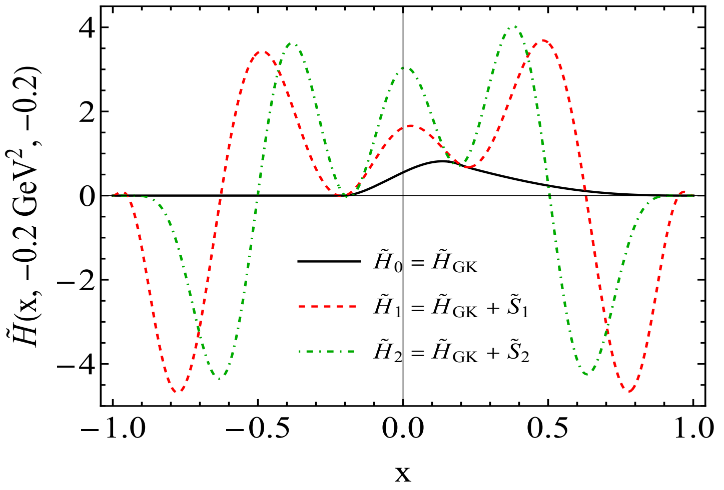
❑ Impact of shadow GPDs:

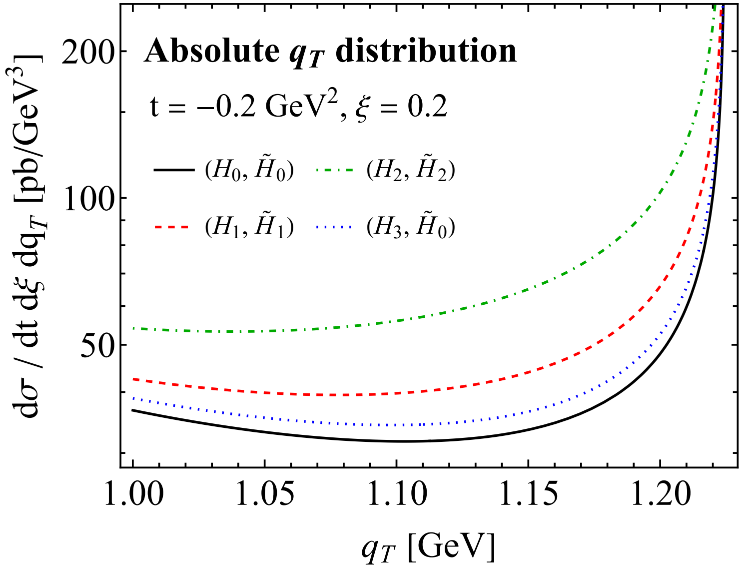
$$F(x,\xi,t) \to F(x,\xi,t) + S(x,\xi,t)$$

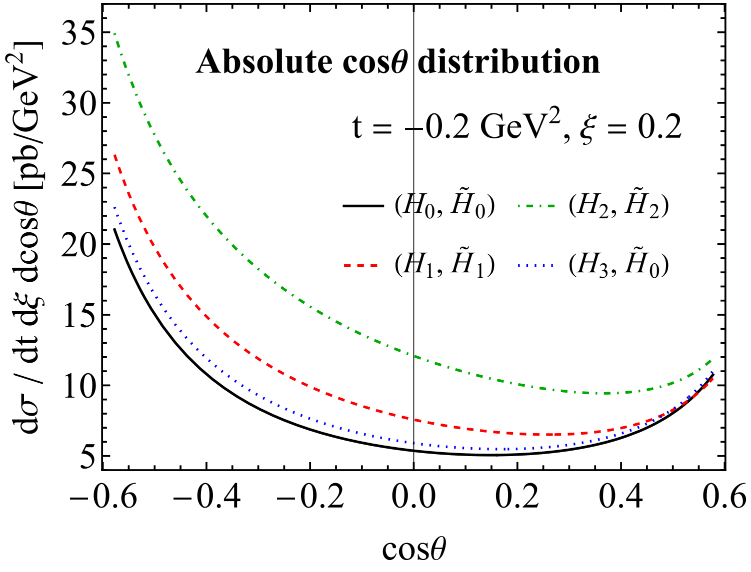
with

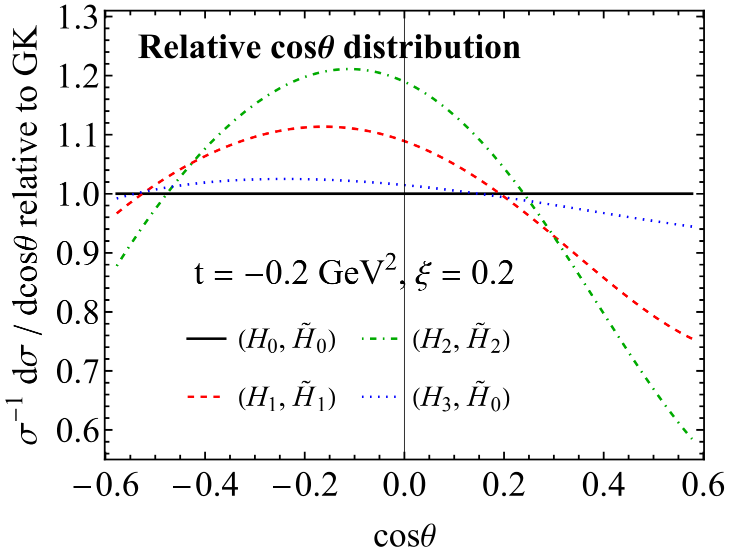
$$\int_{-1}^{1} \mathrm{d}x \, \frac{S(x,\xi,t)}{x-\xi+i\varepsilon} = 0$$





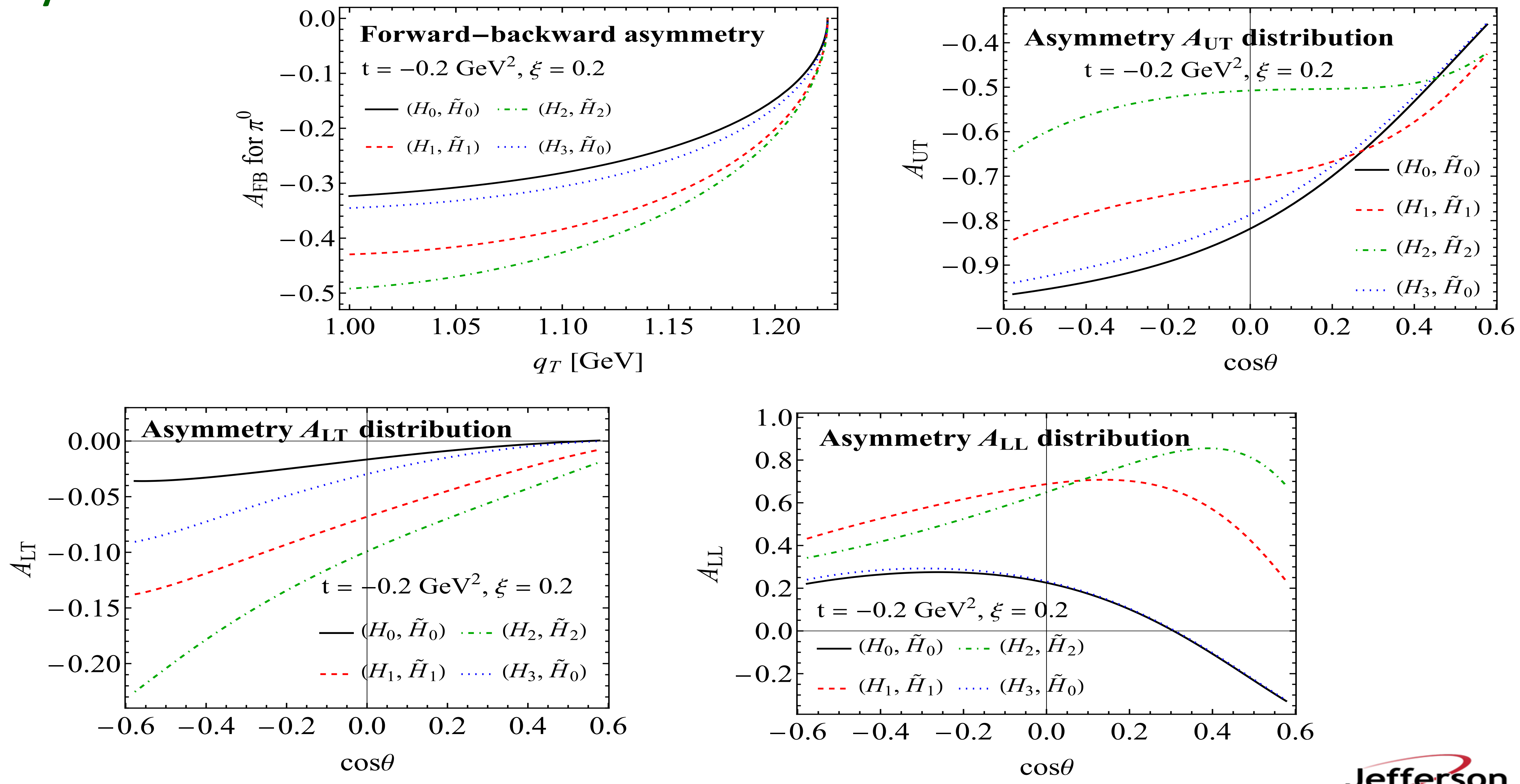






Exclusive $\pi^0\gamma$ Pair Production – Phenomenology

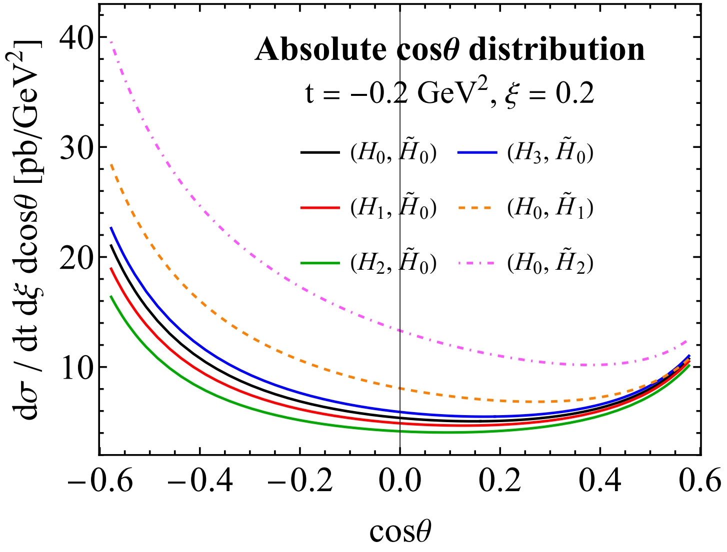
- **Asymmetries:**

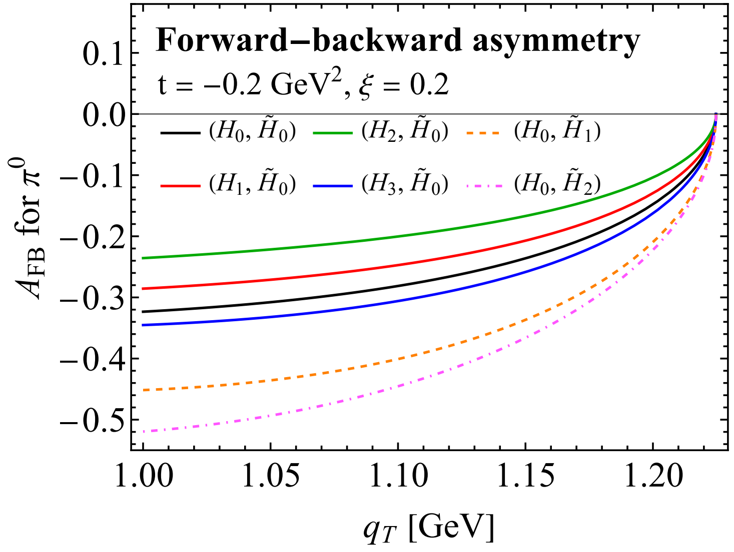


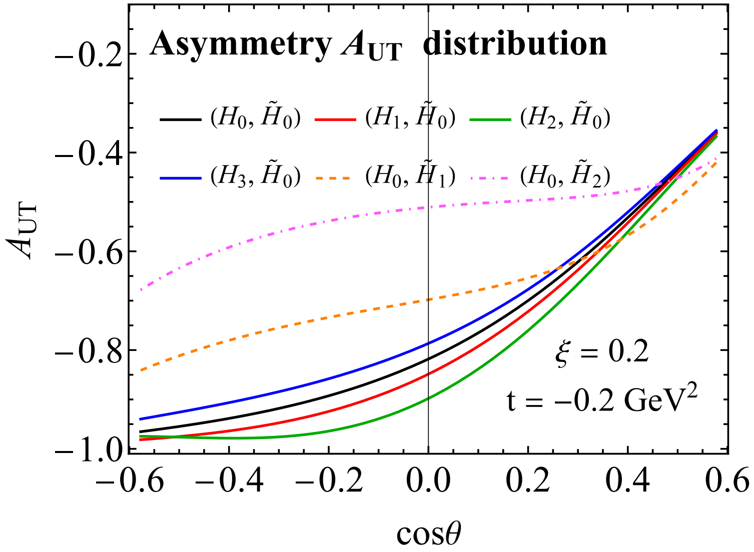
Exclusive $\pi^0\gamma$ Pair Production – Phenomenology

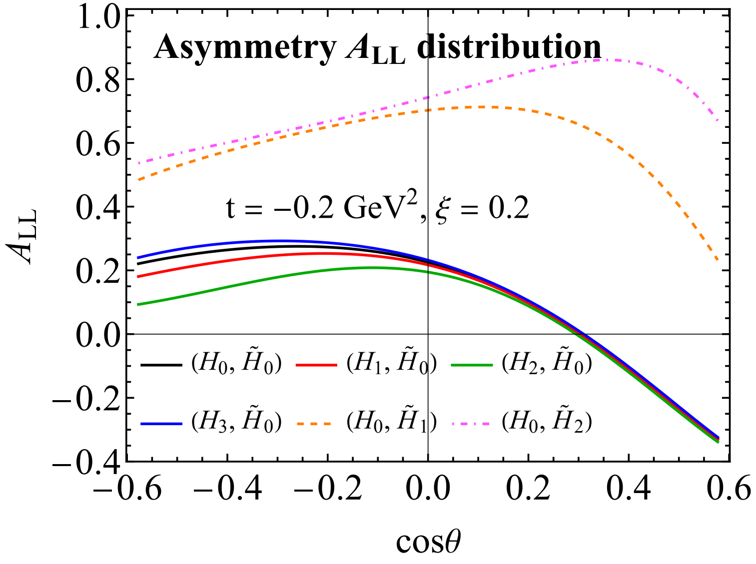
❑ Sensitivities on GPDs:

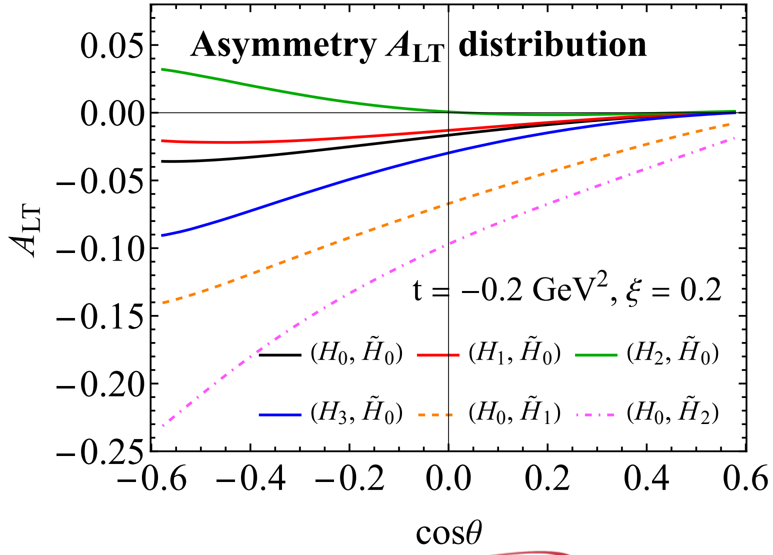
GPD models
= simplified GK model











Summary and Outlook

- GPDs are fundamental parton correlation functions:
  - Carry rich information on emergent hadron properties (mass, spin, …) from QCD/parton dynamics
  - Are responsible for the tomographic images of confined quarks and gluons inside a bound hadron
  - Provide the much needed hints on how confined quarks/gluons respond to the probing scale, …

Extracting their x-dependence from experimental observable(s) is non-trivial, but, full of opportunities, …

- Introduced the single diffractive $2 \to 3$ hard exclusive processes (SDHEP) for extracting GPDs, …
  - Explored both necessary and sufficient conditions for the leading power QCD factorization
  - Covered all existing/known processes for extracting GPDs, plus ideas for new observables, …
  - Introduced a path forward to identify new SDHEPs that could be sensitive to x-dependence of GPDs
  - Angular modulation between diffractive plane and hard scattering plane could provide unique opportunity to separate various GPDs

Following the pioneering work on GPDs over 26 years ago, we now have renewed opportunities for exploring the physics of GPDs and the confined phenomena of QCD.

Thanks!