Explore Proton's Quark/Gluon Structure without Breaking it

- **Challenges:**

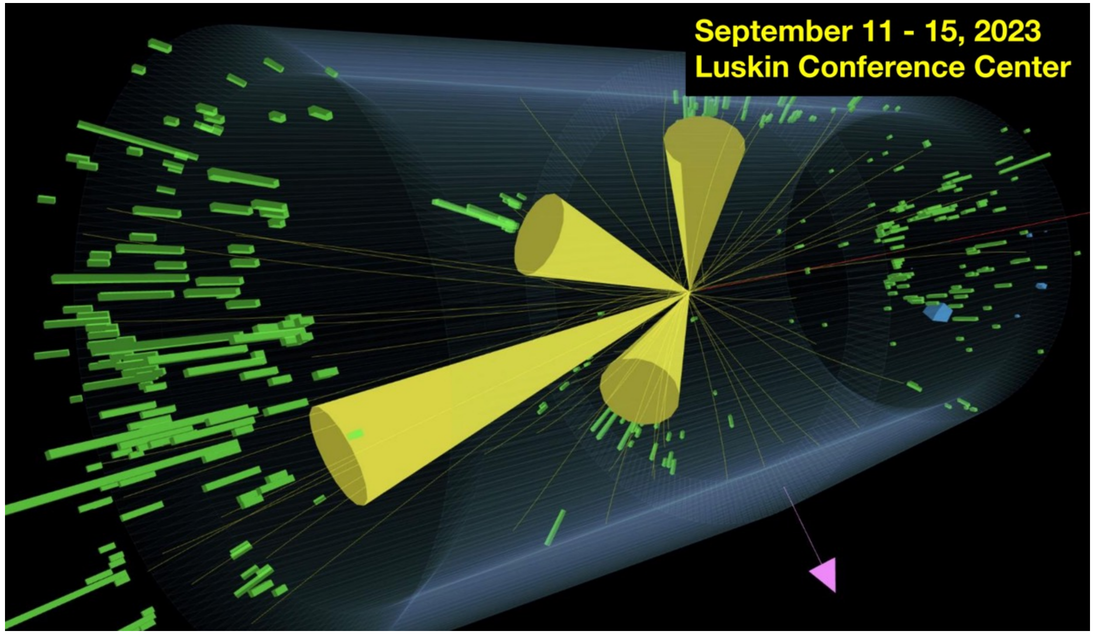
  Seeing quarks and gluons without breaking the hadron

- **Factorization:**

  Imaging the spatial distributions of quarks and gluons inside a bound hadron with controllable approximations

QuantOm Collaboration

Pixelating the hadron in terms of probabilities to find quarks and gluons in slices of the momentum fraction x



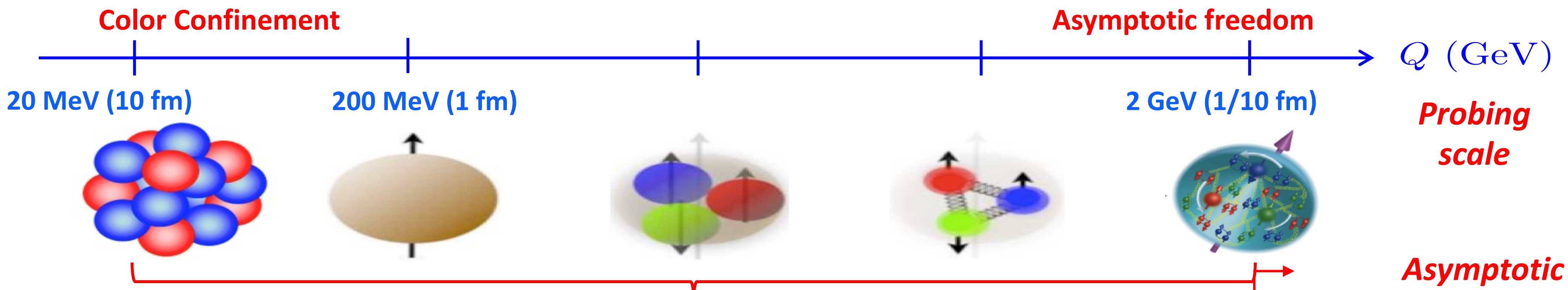
Jianwei Qiu
Jefferson Lab, Theory Center

Jefferson Lab

U.S. DEPARTMENT OF ENERGY | Office of Science

JSA

QCD Landscape of Nucleons and Nuclei

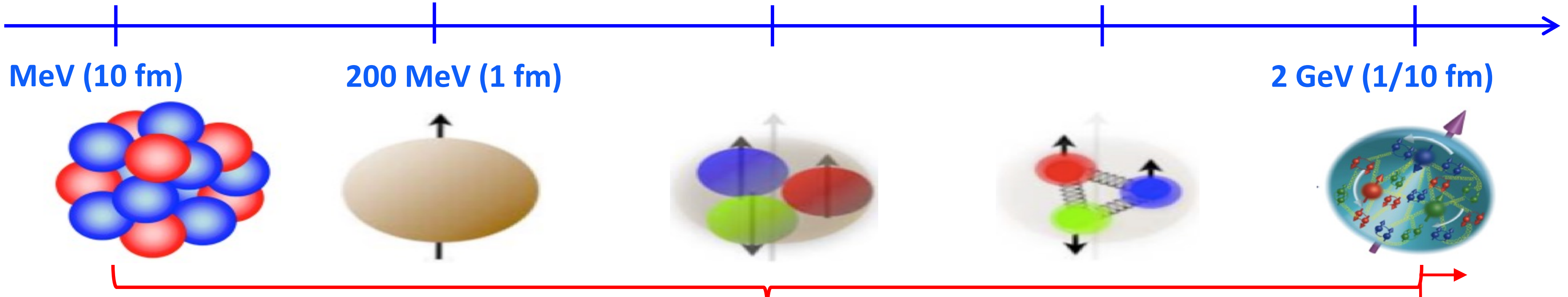


QCD at the Fermi Scale: Femto-science (0.1-10 fm)

- ***The most interesting, rich, and complex regime of the theory!***
- ***All emergent phenomena depend on the scale at which we probe them!***

Asymptotic regime PQCD works beautifully!

QCD Landscape of Nucleons and Nuclei



Probing scale

QCD at the Fermi Scale: Femto-science (0.1-10 fm)

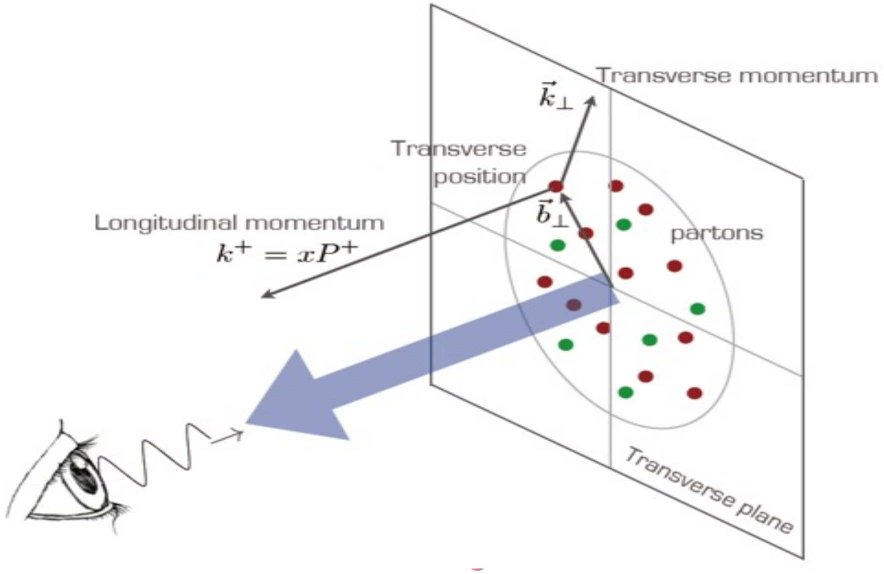
Asymptotic regime PQCD works beautifully!

- *The most interesting, rich, and complex regime of the theory!*
- *All emergent phenomena depend on the scale at which we probe them!*

❑ Need new observables with two distinctive scales:

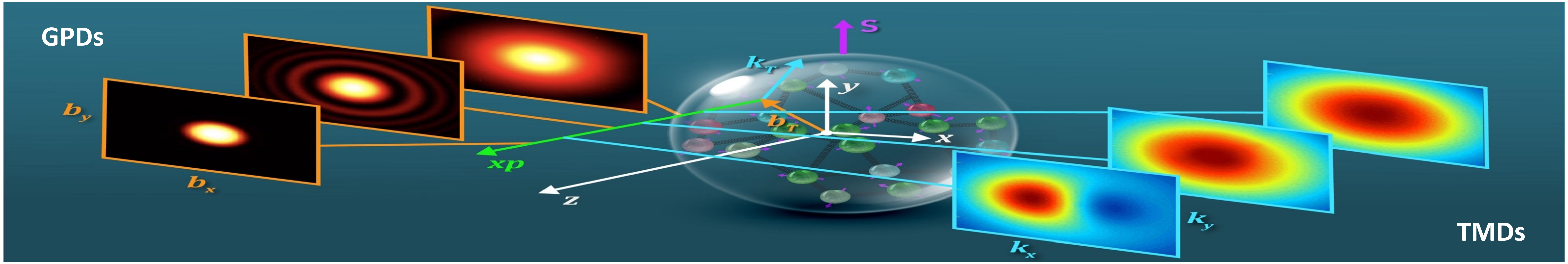
$$Q_1 \gg Q_2 \sim 1/R \sim \Lambda_{\rm QCD}$$

- **Hard scale:** Q_1 **to localize the probe to see the particle nature of quarks/gluons**
- **"Soft" scale:** Q_2 **to be more sensitive to the emergent regime of hadron structure ~ 1/fm**



"See" Internal Structure of Hadron without seeing quarks/gluons?

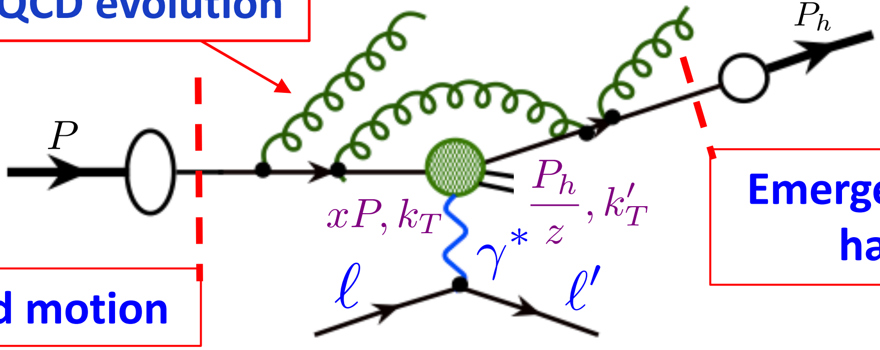
❑ 3D hadron structure:



NO quarks and gluons can be seen in isolation!

❑ If the nucleon is broken, e.g., in SIDIS, …

Gluon shower – QCD evolution



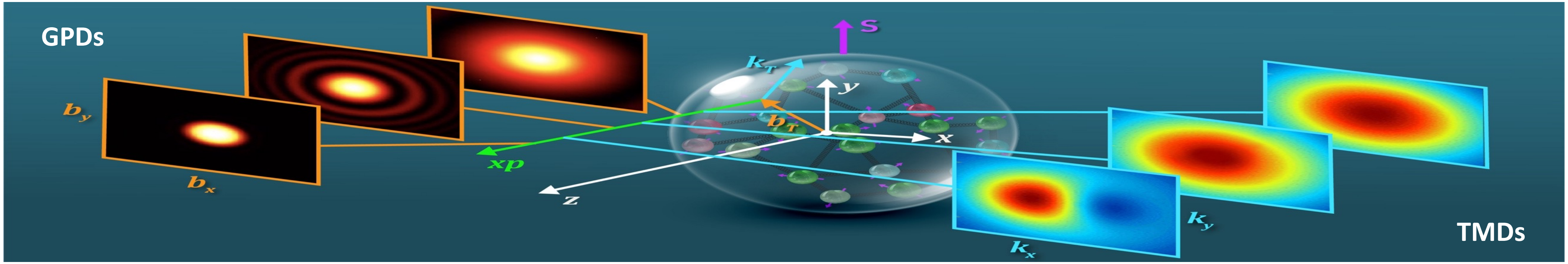
Emergence of a hadron
hadronization

Confined motion

- *Measured k_T is NOT the same as k_T of the confined motion!*
- *Too larger Q^2 could weaken our precision to probe the true hadron structure!*

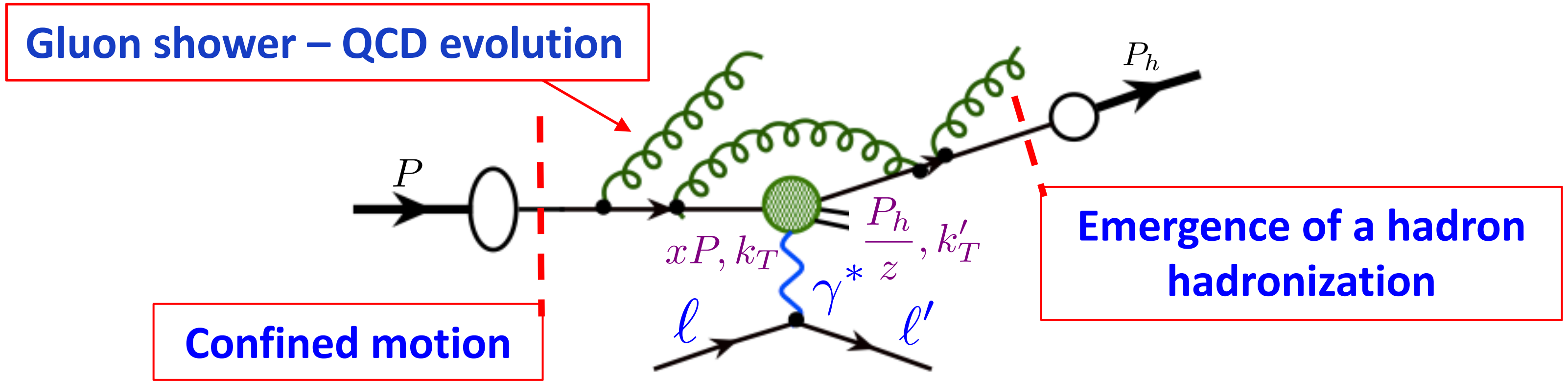
"See" Internal Structure of Hadron without seeing quarks/gluons?

❑ 3D hadron structure:



NO quarks and gluons can be seen in isolation!

❑ If the nucleon is broken, e.g., in SIDIS, …



Transverse momentum Broadening from the shower:

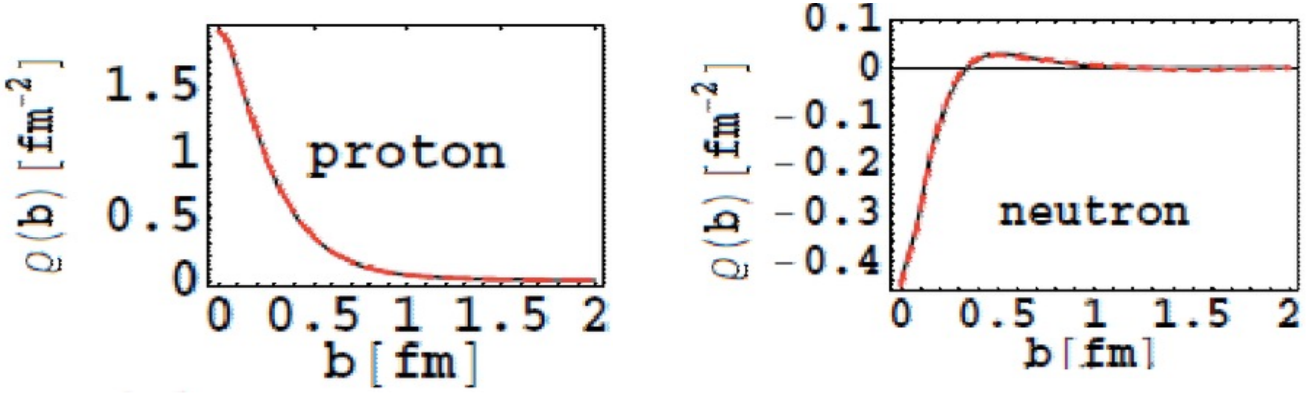
$$\Delta k_T^2 \propto \Lambda_{\rm QCD}^2 \times \alpha_s(C_F, C_A) \left.\begin{matrix} \times \log(Q^2/\Lambda_{\rm QCD}^2) \\ \times \log(s/Q^2) \end{matrix}\right\} \gtrsim 1$$

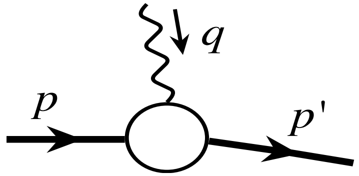
Structure information can be diluted by the collision induced shower!

- *Measured k_T is NOT the same as k_T of the confined motion!*
- *Too larger Q^2 could weaken our precision to probe the true hadron structure!*

Challenges for Exploring Internal Structure of Hadron without Breaking it

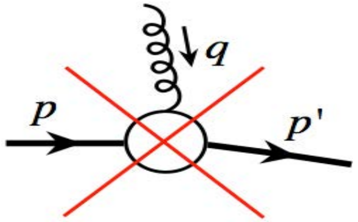
- ❑ **Form factors:** Elastic electric form factor ➡ Charge distributions





➡ Proton "Radius" in EM charge distribution

- ❑ **But, there is NO elastic "color" form factor!**

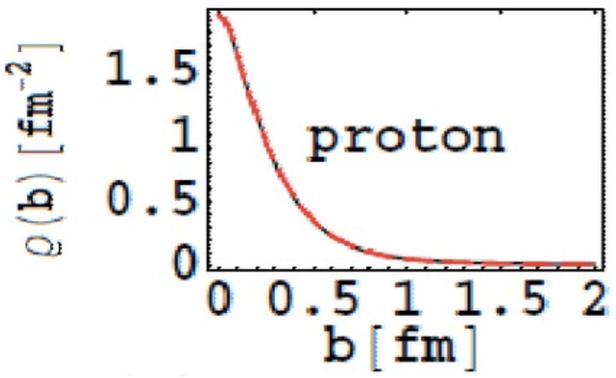


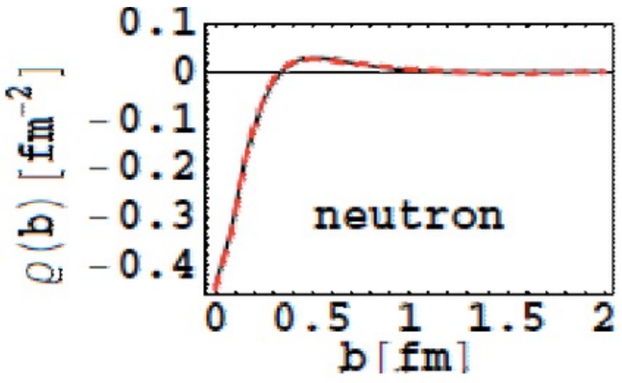
➡ No Proton "Radius" in color charge distribution!

Jefferson Lab

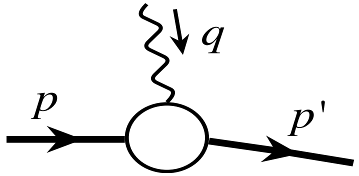
Challenges for Exploring Internal Structure of Hadron without Breaking it

- **Form factors:** Elastic electric form factor → Charge distributions



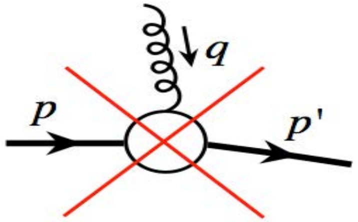


→



Proton "Radius"
in EM charge distribution

- **But, there is NO elastic "color" form factor!** →



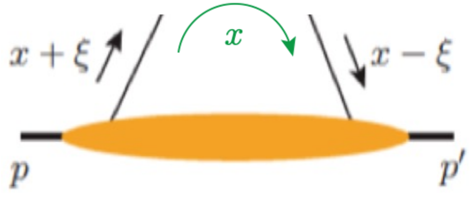
No Proton "Radius"
in color charge distribution!

- **3D hadron tomography:**

Generalized "form factor" for quark and gluon "density" distribution

Generalized PDFs (GPDs) – without breaking the proton

$F_{q/h}(x,\xi,t)$ skewness $\xi = \frac{(p-p')^+}{(p+p')^+}$ $t=(p-p')^2$



$t=(p-p')^2$

F.T. to get spatial distribution of quark/gluon density, quark/gluon correlations, …

Jefferson Lab

Generalized Parton Distributions (GPDs)

- **Definition:**

D. Müller, D. Robaschik, B. Geyer, F.-M. Dittes, J. Hořejši, *Fortsch. Phys.* 42 (1994) 101

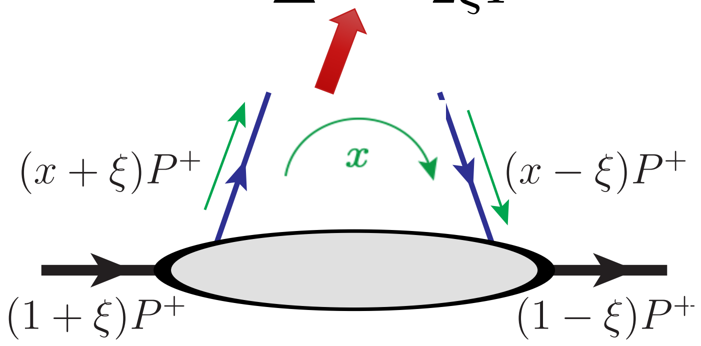
$$F^q(x,\xi,t) = \int \frac{dz^-}{4\pi} e^{-ixP^+z^-} \langle p'|\bar{q}(z^-/2)\gamma^+ q(-z^-/2)|p\rangle$$

$$= \frac{1}{2P^+}\left[H^q(x,\xi,t)\,\bar{u}(p')\,\gamma^+ u(p) - E^q(x,\xi,t)\,\bar{u}(p')\,\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}u(p)\right],$$

$$\widetilde{F}^q(x,\xi,t) = \int \frac{dz^-}{4\pi} e^{-ixP^+z^-} \langle p'|\bar{q}(z^-/2)\gamma^+\gamma_5 q(-z^-/2)|p\rangle$$

$$= \frac{1}{2P^+}\left[\widetilde{H}^q(x,\xi,t)\,\bar{u}(p')\,\gamma^+\gamma_5 u(p) - \widetilde{E}^q(x,\xi,t)\,\bar{u}(p')\,\frac{\gamma_5\Delta^+}{2m}u(p)\right].$$

$$\Delta^+ = 2\xi P^+$$



$$P^+ = \frac{p^+ + p'^+}{2}$$

$$\Delta = p - p' \qquad t = \Delta^2$$

Similar definition for gluon GPDs

Jefferson Lab

Generalized Parton Distributions (GPDs)

D. Müller, D. Robaschik, B. Geyer, F.-M. Dittes, J. Hořejši, *Fortsch. Phys.* 42 (1994) 101

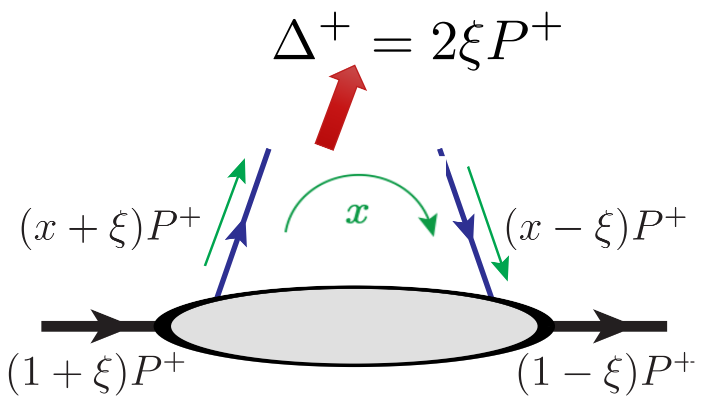
❑ Definition:

$$F^q(x,\xi,t) = \int \frac{dz^-}{4\pi} e^{-ixP^+z^-} \langle p'|\bar{q}(z^-/2)\gamma^+ q(-z^-/2)|p\rangle$$

$$= \frac{1}{2P^+}\left[H^q(x,\xi,t)\,\bar{u}(p')\,\gamma^+ u(p) - E^q(x,\xi,t)\,\bar{u}(p')\,\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}u(p)\right],$$

$$\widetilde{F}^q(x,\xi,t) = \int \frac{dz^-}{4\pi} e^{-ixP^+z^-} \langle p'|\bar{q}(z^-/2)\gamma^+\gamma_5 q(-z^-/2)|p\rangle$$

$$= \frac{1}{2P^+}\left[\widetilde{H}^q(x,\xi,t)\,\bar{u}(p')\,\gamma^+\gamma_5 u(p) - \widetilde{E}^q(x,\xi,t)\,\bar{u}(p')\,\frac{\gamma_5\Delta^+}{2m}u(p)\right].$$



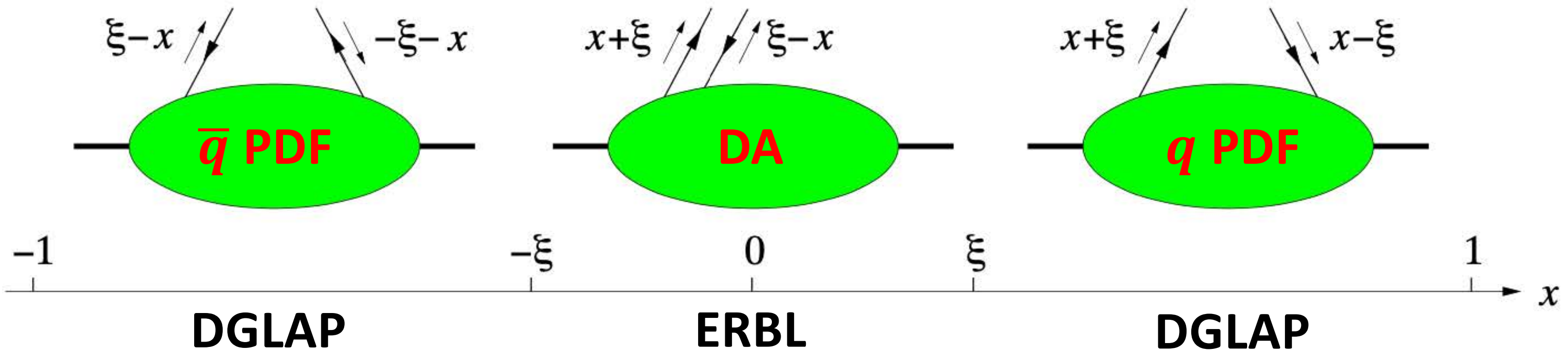
$$P^+ = \frac{p^+ + p'^+}{2}$$

$$\Delta = p - p' \qquad t = \Delta^2$$

Similar definition for gluon GPDs

❑ Combine *PDF* and *Distribution Amplitude (DA):*

Forward limit $\xi = t = 0$ **:** $\quad H^q(x,0,0) = q(x), \quad \tilde{H}^q(x,0,0) = \Delta q(x)$

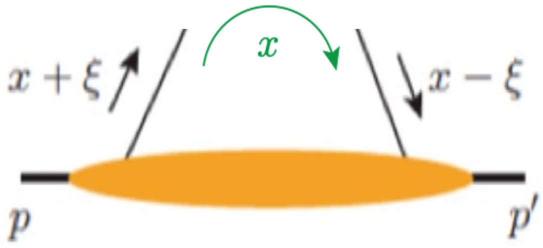


Properties of GPDs - I

- **Impact parameter dependent parton density distribution:**

$$q(x, b_\perp, Q) = \int d^2\Delta_\perp e^{-i\Delta_\perp \cdot b_\perp} H_q(x, \xi = 0, t = -\Delta_\perp^2, Q)$$

➡ Quark density in $\mathrm{d}x\,\mathrm{d}^2\boldsymbol{b}_T$



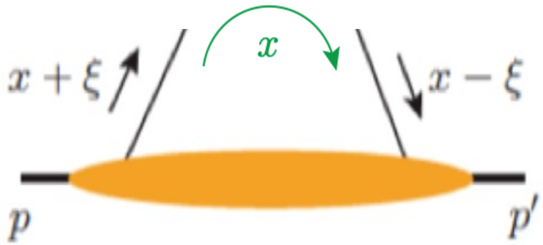
Measurement of p' fixes (t, ξ)

x = momentum flow between the pair

Properties of GPDs - I

❑ **Impact parameter dependent parton density distribution:**

$$q(x, b_\perp, Q) = \int d^2\Delta_\perp e^{-i\Delta_\perp \cdot b_\perp} H_q(x, \xi = 0, t = -\Delta_\perp^2, Q)$$



Measurement of p' fixes (t, ξ)

x = momentum flow between the pair

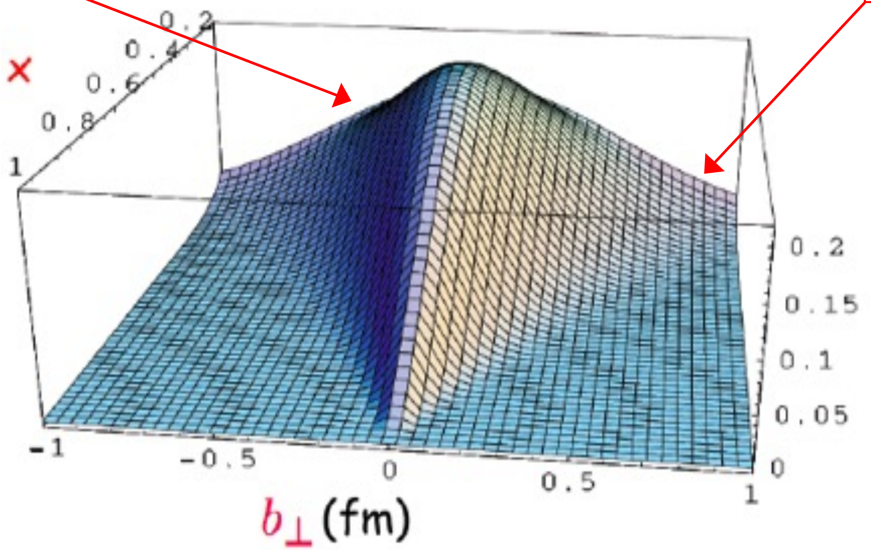
➡ **Quark density in** $\mathrm{d}x\,\mathrm{d}^2\boldsymbol{b}_T$

How fast does glue density fall?

Tomographic image of hadron in slice of x

How far does glue density spread?

➡



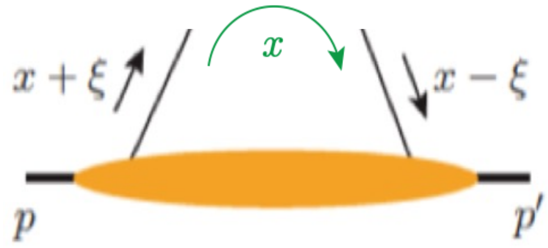
Modeled by M. Burkdart, PRD 2000

➡ **Proton radii from quark and gluon spatial density distribution,** $r_q(x)$ **&** $r_g(x)$

Properties of GPDs - I

- **Impact parameter dependent parton density distribution:**

$$q(x, b_\perp, Q) = \int d^2\Delta_\perp e^{-i\Delta_\perp \cdot b_\perp} H_q(x, \xi = 0, t = -\Delta_\perp^2, Q)$$



Measurement of p' fixes (t, ξ)

x = momentum flow between the pair

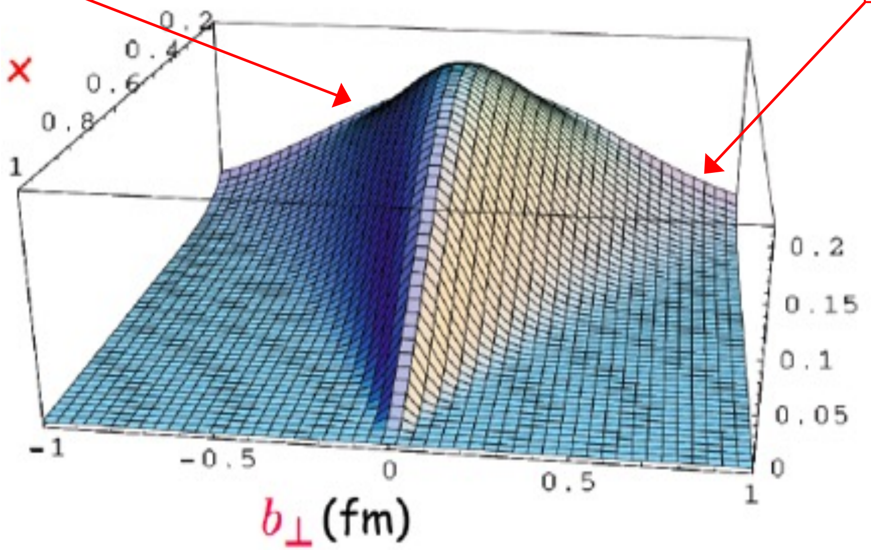
→ **Quark density in** $\mathrm{d}x\,\mathrm{d}^2\boldsymbol{b}_T$

How fast does glue density fall?

Tomographic image of hadron in slice of x

How far does glue density spread?

→



Modeled by M. Burkdart, PRD 2000

→ **Proton radii from quark and gluon spatial density distribution,** $r_q(x)$ **&** $r_g(x)$

- **Should** $r_q(x)$ **>** $r_g(x)$**, or vice versa?**
- **Could** $r_g(x)$ **saturates as** $x \to 0$
- **How do they compare with known radius (EM charge radius, mass radius, …), & why?**
- **How the image correlate to hadron spin, … ?**
- …

Properties of GPDs - II

❑ QCD energy-momentum tensor:

Ji, PRL78, 1997

$$T^{\mu\nu} = \sum_{i=q,g} T_i^{\mu\nu} \quad \text{with} \quad T_q^{\mu\nu} = \bar{\psi}_q \, i\gamma^{(\mu} \overleftrightarrow{D}^{\nu)} \, \psi_q - g^{\mu\nu} \bar{\psi}_q \left(i\gamma \cdot \overleftrightarrow{D} - m_q \right) \psi_q \quad \text{and} \quad T_g^{\mu\nu} = F^{a,\mu\eta} F^{a,}{}_{\eta}{}^{\nu} + \frac{1}{4} g^{\mu\nu} \left(F^a_{\rho\eta}\right)^2$$

❑ "Gravitational" form factors:

$$\langle p' \, | T_i^{\mu\nu} | \, p \rangle = \bar{u}(p') \left[A_i(t) \frac{P^\mu P^\nu}{m} + J_i(t) \frac{i P^{(\mu} \sigma^{\nu)\Delta}}{2m} + D_i(t) \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{4m} + m \, \bar{c}_i(t) \, g^{\mu\nu} \right] u(p)$$

❑ Connection to GPD moments:

$$\int_{-1}^{1} dx \, x \, F_i(x,\xi,t) \propto \langle p' | T_i^{++} | p \rangle \quad \propto \quad \bar{u}(p') \left[\underbrace{(A_i + \xi^2 D_i)}_{\int_{-1}^{1} dx\, x\, H_i(x,\xi,t)} \gamma^+ + \underbrace{(B_i - \xi^2 D_i)}_{\int_{-1}^{1} dx\, x\, E_i(x,\xi,t)} \frac{i\sigma^{+\Delta}}{2m} \right] u(p)$$

$C_i(t) \leftrightarrow D_i(t)/4$

Related to pressure & stress force inside h

Polyakov, schweitzer, Inntt. J. Mod. Phys. A33, 1830025 (2018)

Burkert, Elouadrhiri , Girod Nature 557, 396 (2018)

❑ Angular momentum sum rule:

$$J_i = \lim_{t\to 0} \int_{-1}^{1} dx \, x \left[H_i(x,\xi,t) + E_i(x,\xi,t) \right]$$

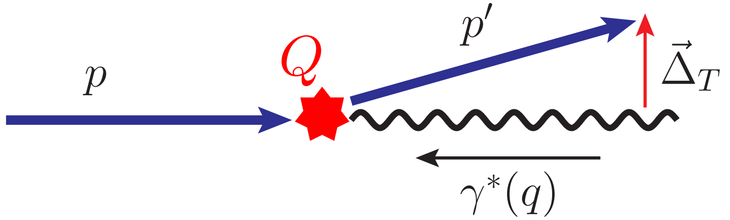
$$i = q, g$$

➡ **3D tomography**
Relation to GFF
Angular Momentum ➡ ***x*-dependence of GPDs!**

Need to know the x-dependence of GPDs to construct the proper moments!

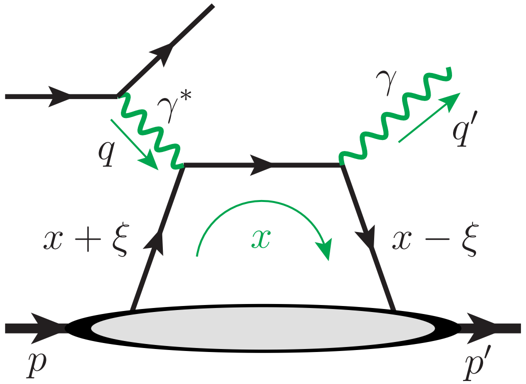
Exclusive Diffractive Processes for Extracting GPDs

- ❑ **Hit the proton hard without breaking it** $\Longrightarrow$ **Diffractive scattering to keep proton intact**

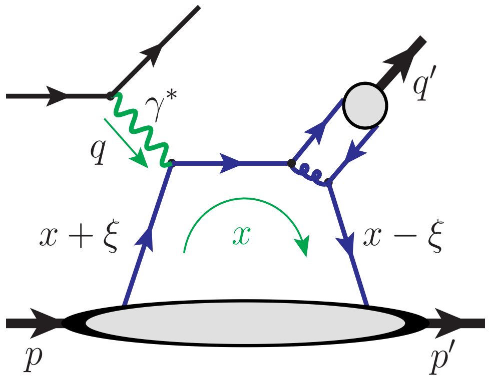


HERA discovery:
~ 10-15% of HERA events with the Proton stayed intact

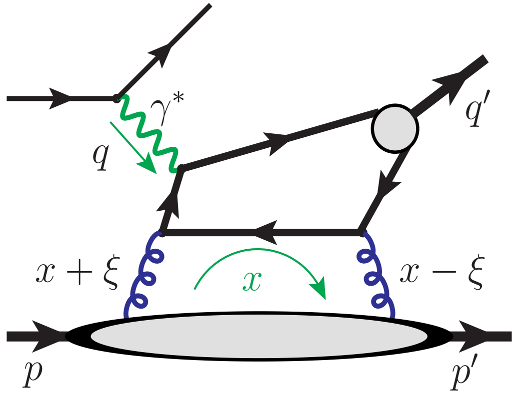
- ❑ **Known exclusive processes for extracting GPDs:**



DVCS: Q^2 >> |t|



DVMP



DVQP

+ DDVCS, …

Feature: Two-scale observables $\quad Q^2 \gg |t| \qquad t=(p-p')^2$

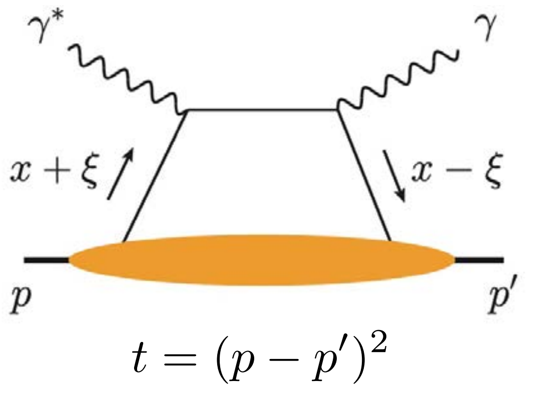
- **Hard scale Q: allows pQCD, factorization**
- **Low scale t: probes non-pert. hadron structure**

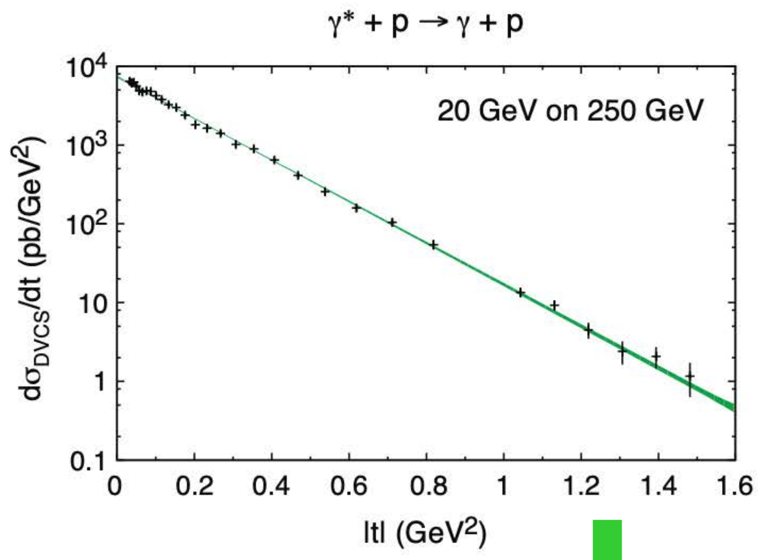
Factorization

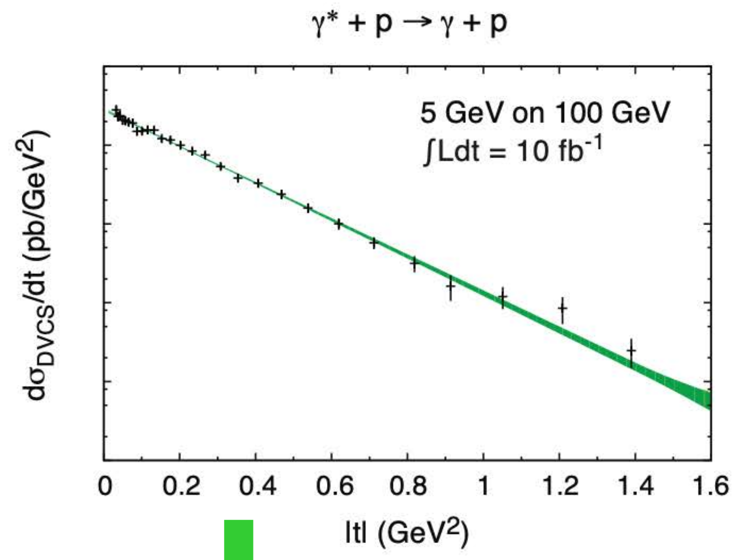
GPDs: $f_{i/h}(x,\xi,t;\mu)$

Imaging the quarks at a Future EIC (White Paper)

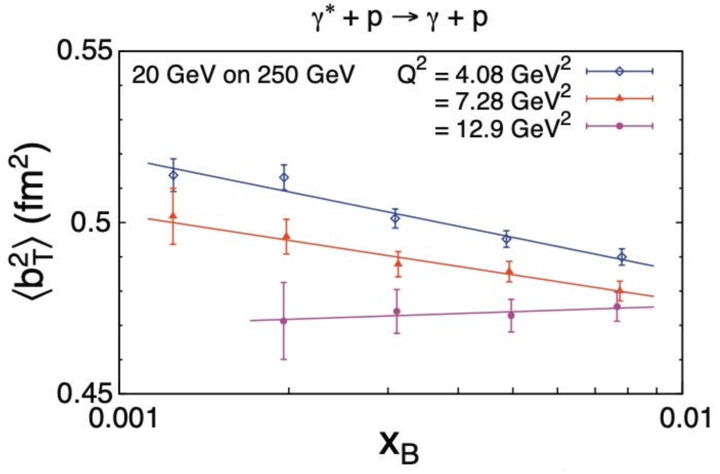
❑ DVCS Cross Sections:

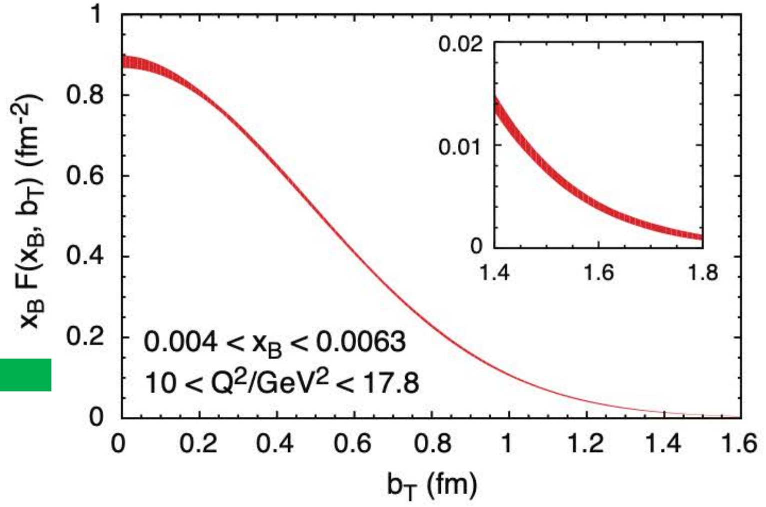


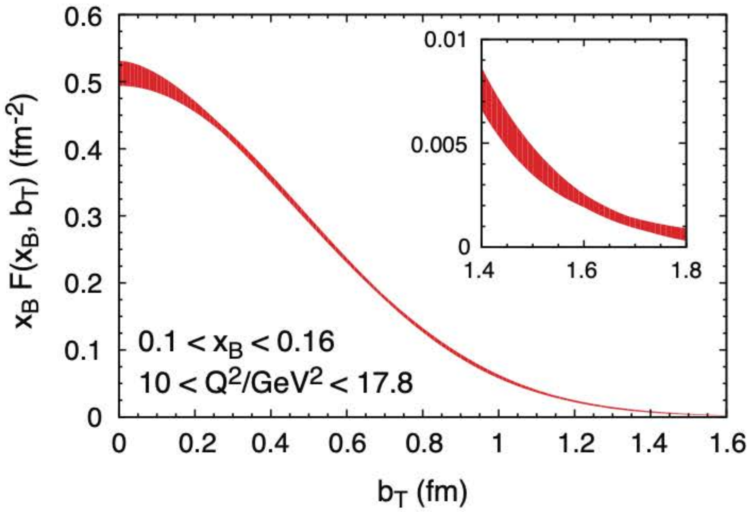




❑ Spatial distributions:



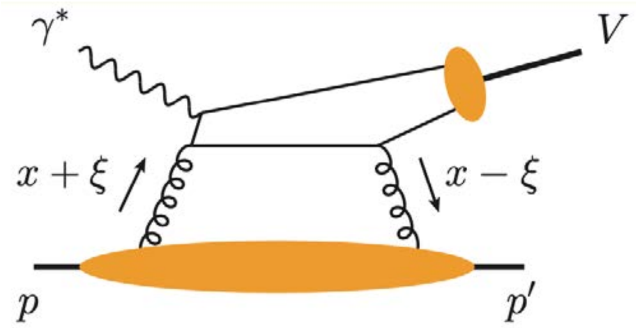




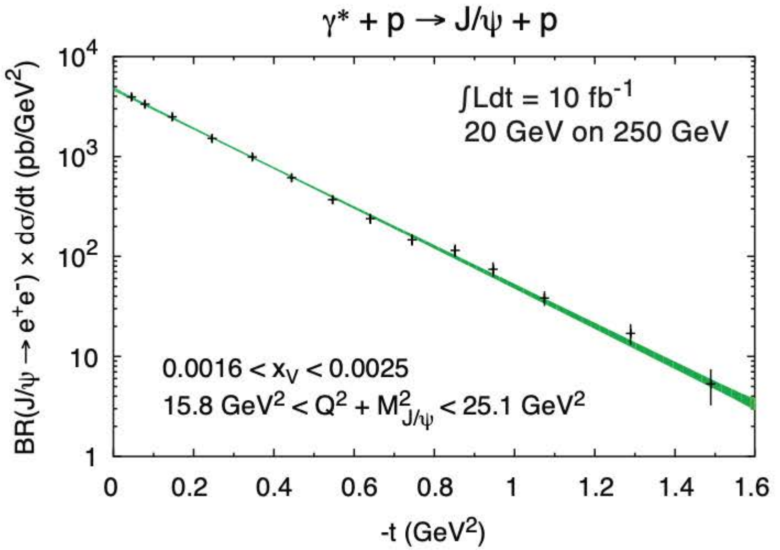
Effective "proton radius" in terms of quark distributions as a function of x_B

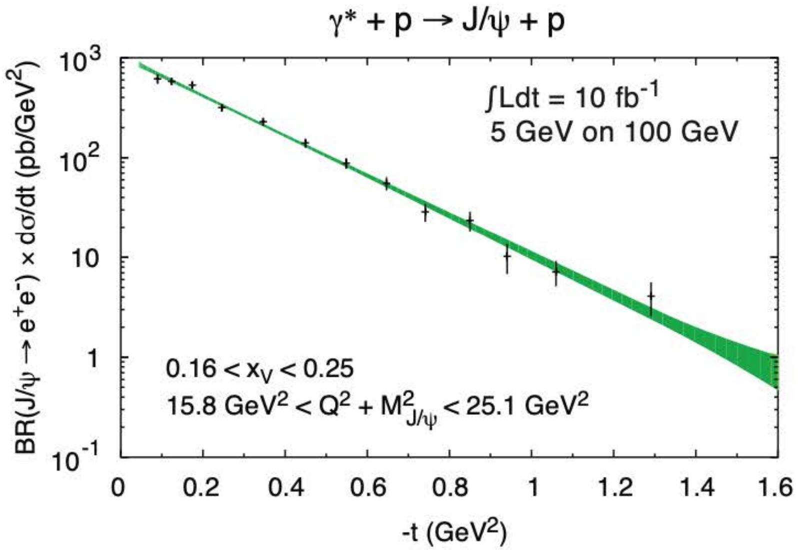
Imaging the gluons at the EIC (White Paper)

- **Exclusive vector meson production:**

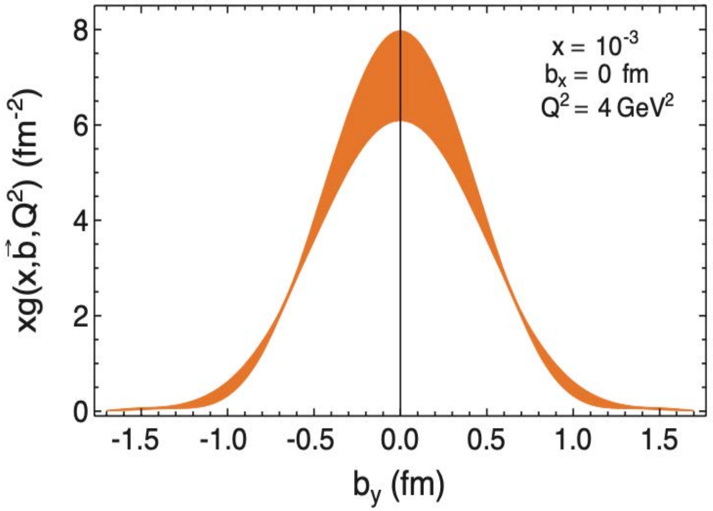


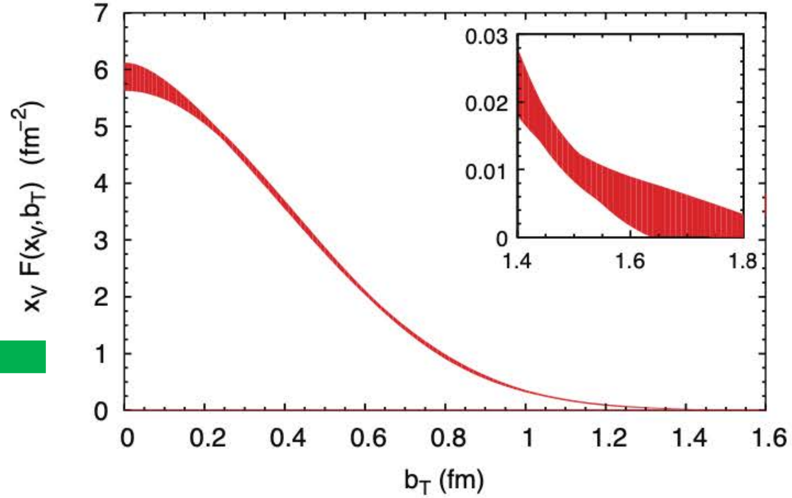
$t = (p - p')^2$

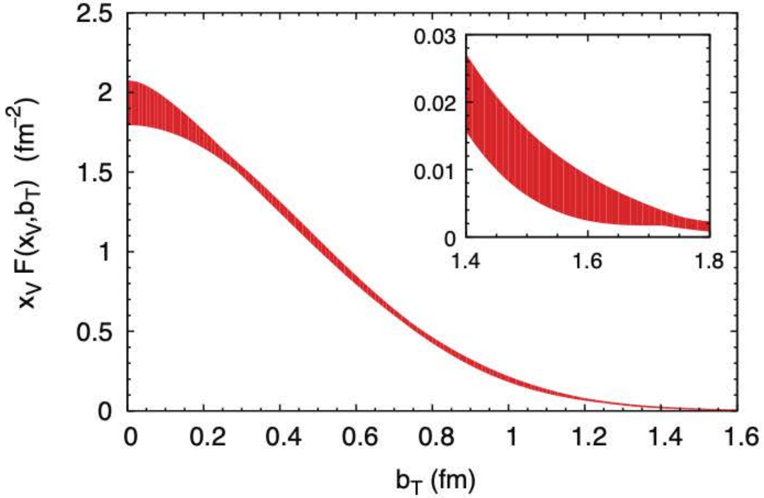




- **Spatial distributions:**





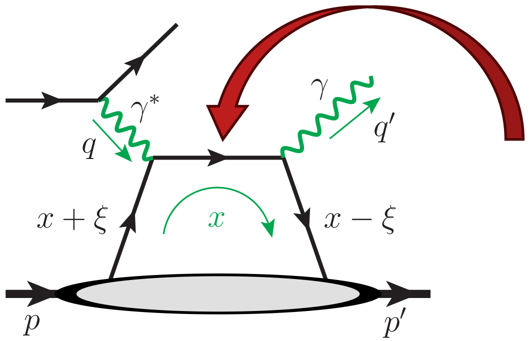


The b_T space density for gluons → "proton radius" in terms of distribution of gluons

Jefferson Lab

Difficult to Extract the x-dependence of GPDs?

- **Amplitude nature: $x \sim$ loop momentum**



DVCS

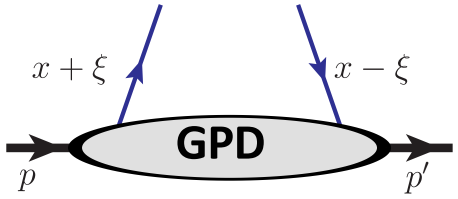
Smaller propagator
= bigger amplitude

$$\propto \frac{1}{x-\xi+i\varepsilon}$$

PRD56 (1997) 5524
PRD58 (1998) 094018
PRD59 (1999) 074009

$$\Rightarrow \quad i\mathcal{M} \propto \int_{-1}^{1} \mathrm{d}x \, \frac{F(x,\xi,t)}{x-\xi+i\varepsilon} \equiv \text{“}F_0(\xi,t)\text{”}$$

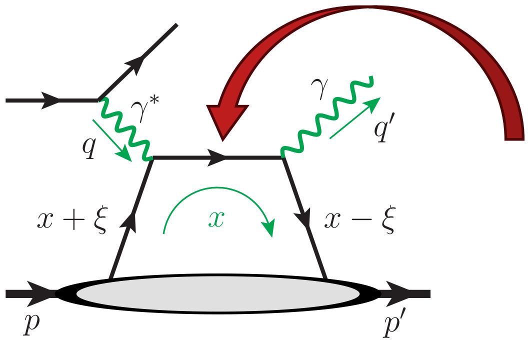
- **also true for most other processes**
- **x-dependence is only constrained by a “moment”**
- **x-integration decouples from external Q^2**

$\Rightarrow$ 

NO full x-dependence for given t and ξ

Difficult to Extract the x-dependence of GPDs?

❑ Amplitude nature: $x \sim$ loop momentum



DVCS

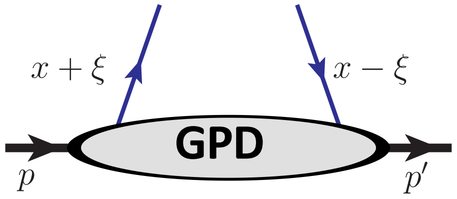
Smaller propagator = bigger amplitude

$$\propto \frac{1}{x - \xi + i\varepsilon}$$

PRD56 (1997) 5524
PRD58 (1998) 094018
PRD59 (1999) 074009

$$\Rightarrow \quad i\mathcal{M} \propto \int_{-1}^{1} \mathrm{d}x \, \frac{F(x,\xi,t)}{x - \xi + i\varepsilon} \equiv \text{"}F_0(\xi,t)\text{"}$$

- also true for most other processes
- x-dependence is only constrained by a "moment"
- x-integration decouples from external Q^2



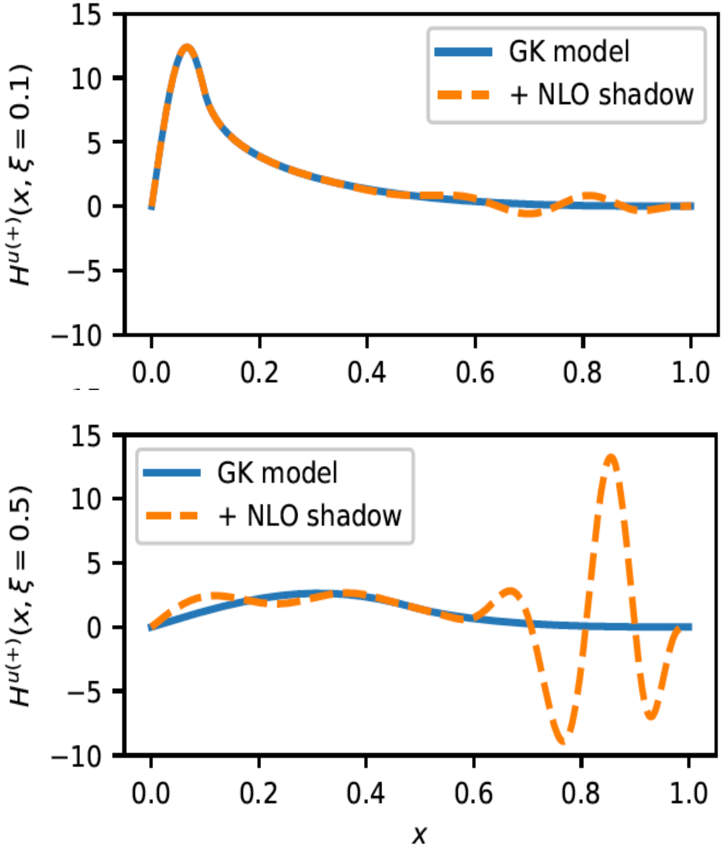
NO full x-dependence for given t and ξ

❑ "Shadow GPDs"

PRD103 (2021) 114019

$$F(x,\xi,t) \to F(x,\xi,t) + S(x,\xi,t)$$

with $$\int_{-1}^{1} \mathrm{d}x \, \frac{S(x,\xi,t)}{x - \xi + i\varepsilon} = 0$$



Blue and dashed Fit the same CFFs !

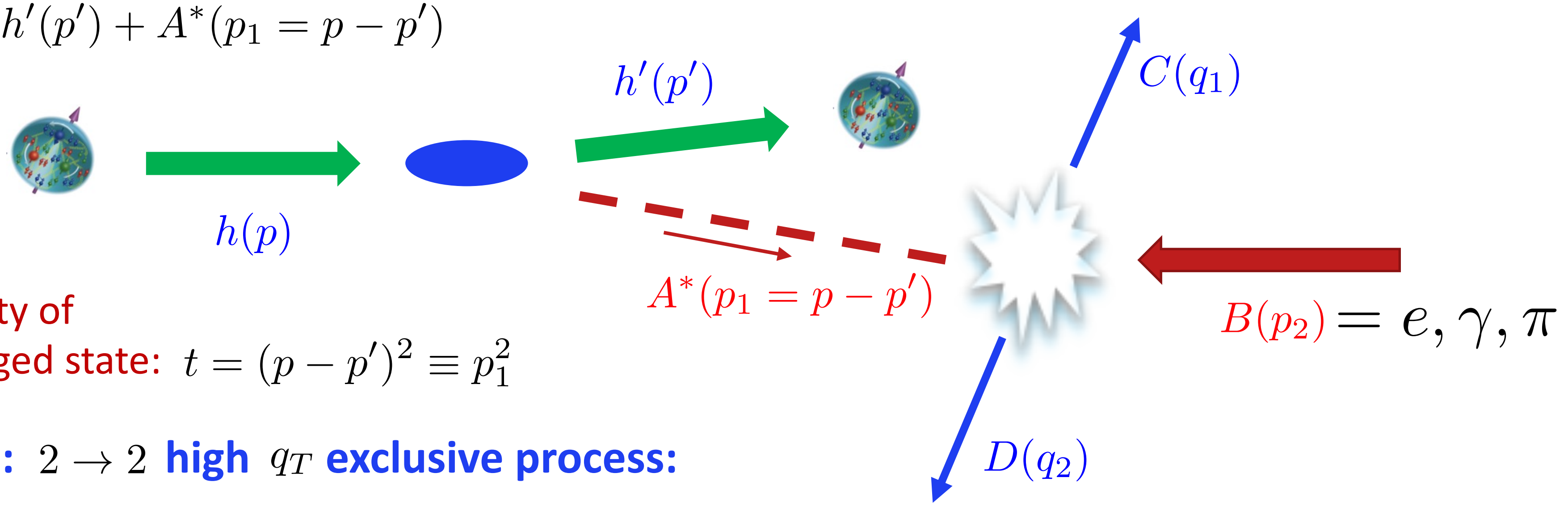
Single-Diffractive Hard Exclusive Processes (SDHEP)

- **Diffractive $2 \to 3$ hard exclusive processes:**

Qiu & Yu, JHEP 08 (2022) 103,
PRD 107 (2023) 1
2305.15397 (PRL in press)

  - **Single diffractive – keep the hadron intact:**

$$h(p) \to h'(p') + A^*(p_1 = p - p')$$



Virtuality of
exchanged state: $t = (p - p')^2 \equiv p_1^2$

  - **Hard probe: $2 \to 2$ high q_T exclusive process:**

$$A^*(p_1) + B(p_2) \to C(q_1) + D(q_2)$$

Probing time: $\sim 1/|q_{1T}| \approx 1/|q_{2T}|$

Jefferson Lab

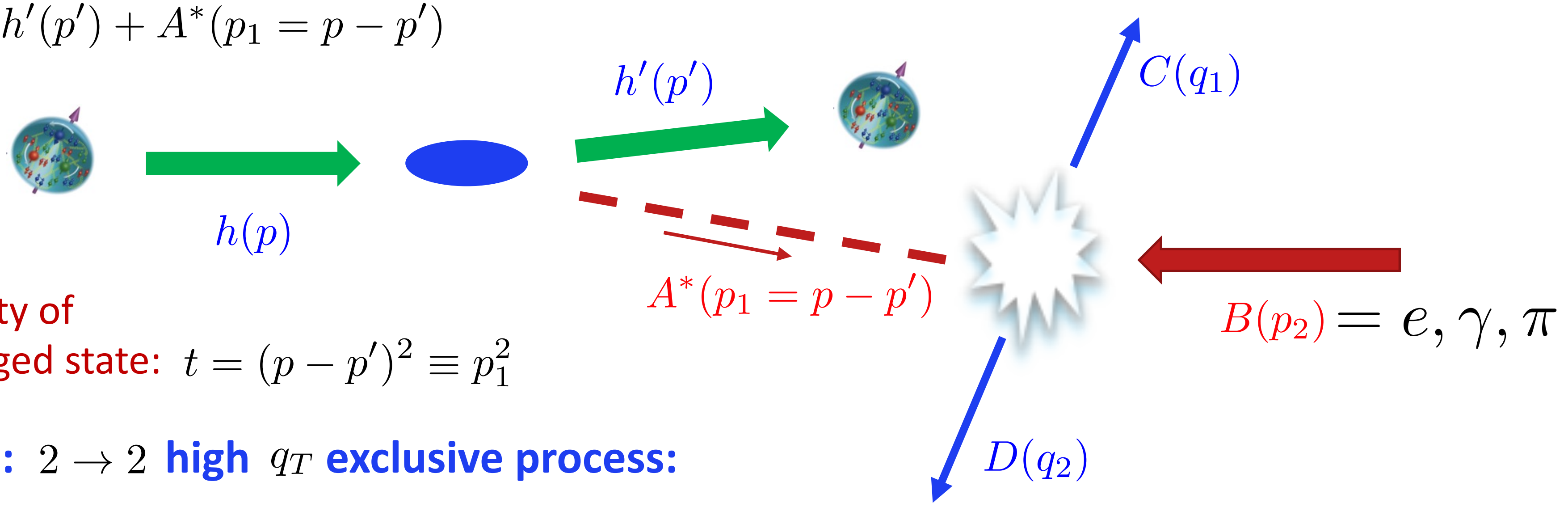
Single-Diffractive Hard Exclusive Processes (SDHEP)

Qiu & Yu, JHEP 08 (2022) 103,
PRD 107 (2023) 1
2305.15397 (PRL in press)

❑ Diffractive $2 \to 3$ hard exclusive processes:

- **Single diffractive – keep the hadron intact:**

$$h(p) \to h'(p') + A^*(p_1 = p - p')$$



Virtuality of
exchanged state: $t = (p - p')^2 \equiv p_1^2$

- **Hard probe: $2 \to 2$ high q_T exclusive process:**

$$A^*(p_1) + B(p_2) \to C(q_1) + D(q_2)$$

Probing time: $\sim 1/|q_{1T}| \approx 1/|q_{2T}|$

The single diffractive $2 \to 3$ exclusive hard processes (SDHEP):

$$h(p) + B(p_2) \to h'(p') + C(q_1) + D(q_2)$$

A 2-scale observable!

- **Necessary condition for QCD factorization:**

Lifetime of $A^*(p_1)$ is much longer than collision time of the probe!

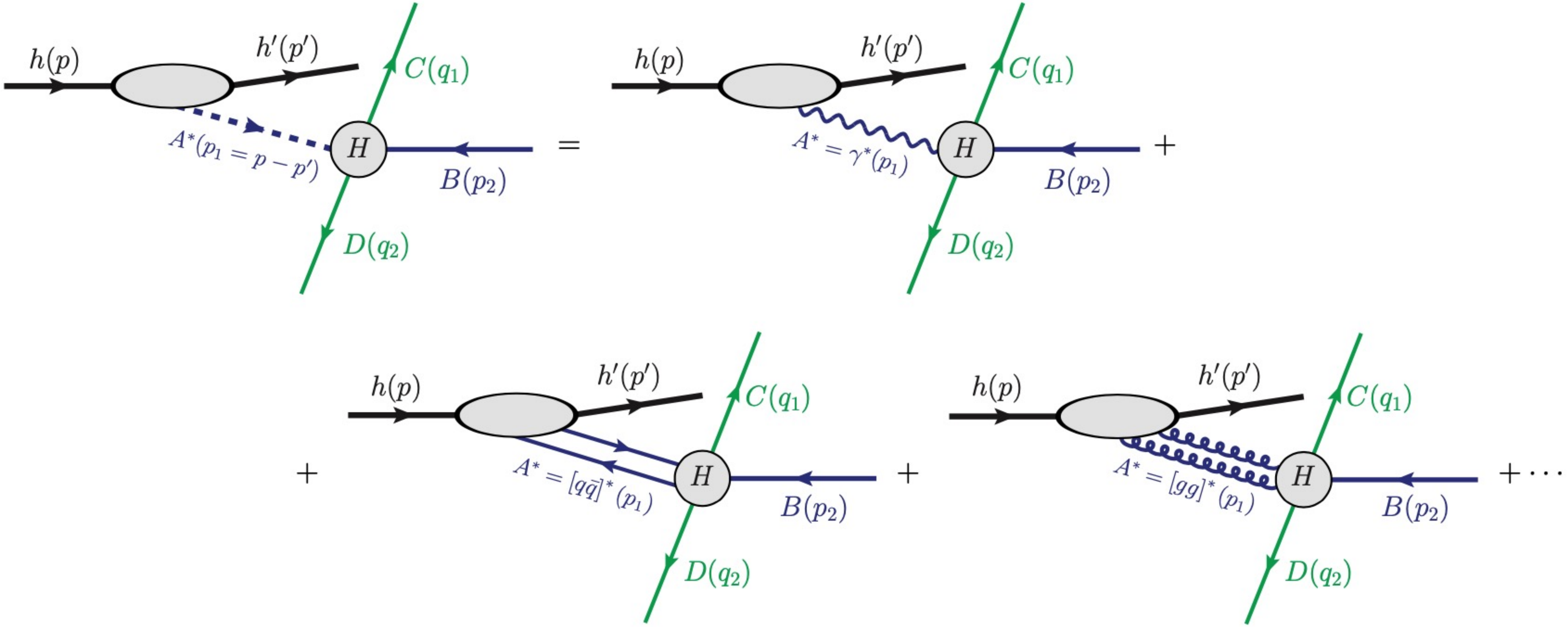
$\Rightarrow$ $|q_{1_T}| = |q_{2_T}| \gg \sqrt{-t}$

Not necessarily sufficient!

Single-Diffractive Hard Exclusive Processes (SDHEP)

Qiu & Yu, JHEP 08 (2022) 103,
PRD 107 (2023) 1
2305.15397 (PRL in press)

❑ Two-stage diffractive $2 \to 3$ hard exclusive processes:



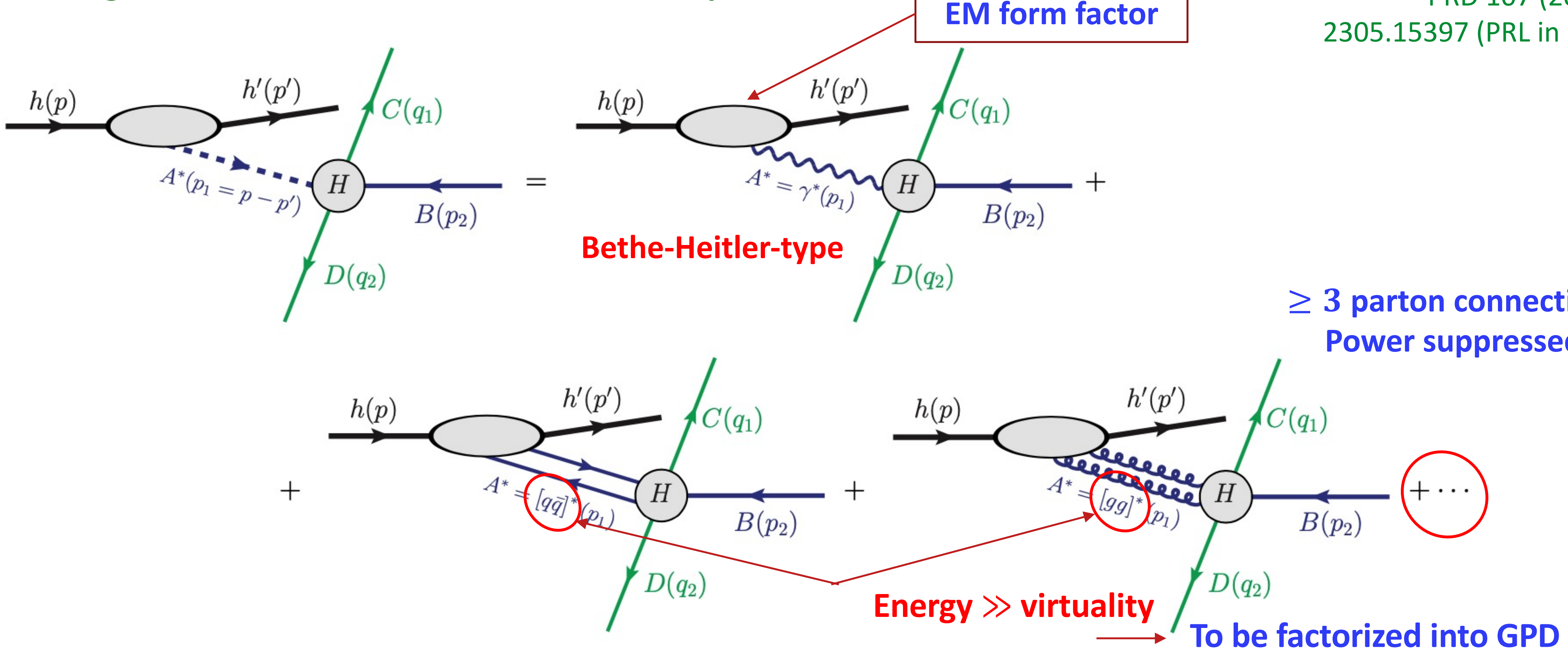
The exchanged state *A*(p-p')* is a sum of all possible partonic states, $\sum_{n=1,2,..}$, allowed by

- **Quantum numbers of *h(p) – h'(p')***
- **Symmetry of producing non-vanishing *H***

Single-Diffractive Hard Exclusive Processes (SDHEP)

❑ **Two-stage diffractive $2 \to 3$ hard exclusive processes:**

Qiu & Yu, JHEP 08 (2022) 103,
PRD 107 (2023) 1
2305.15397 (PRL in press)



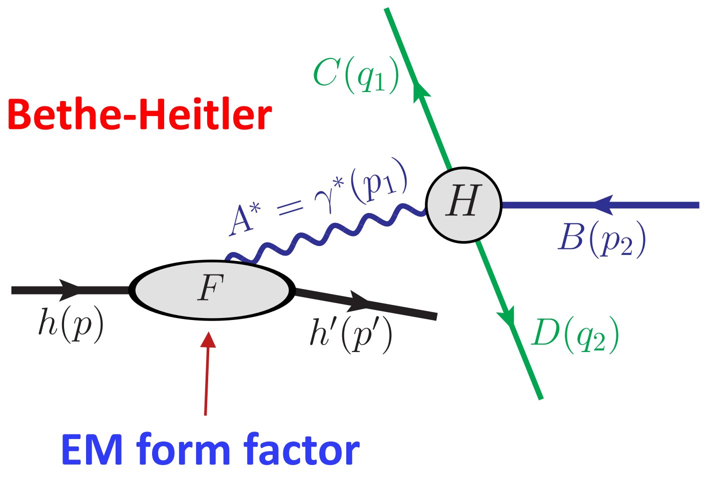
The exchanged state $A^*(p-p')$ is a sum of all possible partonic states, $\sum_{n=1,2,..}$, allowed by

- **Quantum numbers of $h(p) - h'(p')$**
- **Symmetry of producing non-vanishing H**

General Discussion on n=1 state: γ^*

Qiu & Yu, PRD 107 (2023) 1

❑ Exchange of a virtual photon – "GPD background":



$$\mathcal{M}^{(1)} = \frac{ie^2}{t}\langle h'(p')|J^\mu(0)|h(p)\rangle\langle C(q_1)D(q_2)|J_\mu(0)|B(p_2)\rangle$$

$$\equiv \frac{ie^2}{t}F^\mu(p,p')\mathcal{H}_\mu(p_1,p_2,q_1,q_2)$$

Leading component

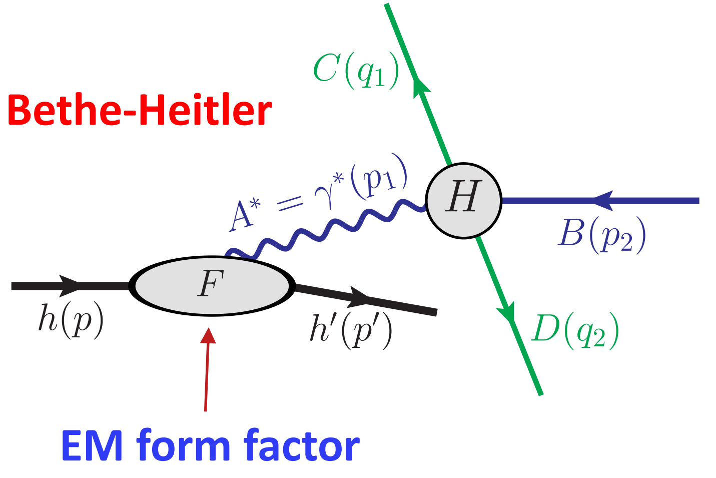
$$F^+\mathcal{H}^- = \frac{1}{p_1^+}F^+\left(p_1^+\mathcal{H}^-\right) = \frac{1}{p_1^+}F^+\left(p_1\cdot\mathcal{H} + \boldsymbol{p}_{1\perp}\cdot\mathcal{H}_\perp - p_1^-\mathcal{H}^+\right) \sim \mathcal{O}(\sqrt{|t|})$$

$$\mathcal{M}^{(1)} \sim \mathcal{O}(1/\sqrt{|t|})$$

General Discussion on n=1 state: γ^*

Qiu & Yu, PRD 107 (2023) 1

❑ Exchange of a virtual photon – "GPD background":



$$\mathcal{M}^{(1)} = \frac{ie^2}{t}\langle h'(p')|J^\mu(0)|h(p)\rangle\langle C(q_1)D(q_2)|J_\mu(0)|B(p_2)\rangle$$

$$\equiv \frac{ie^2}{t}F^\mu(p,p')\mathcal{H}_\mu(p_1,p_2,q_1,q_2)$$

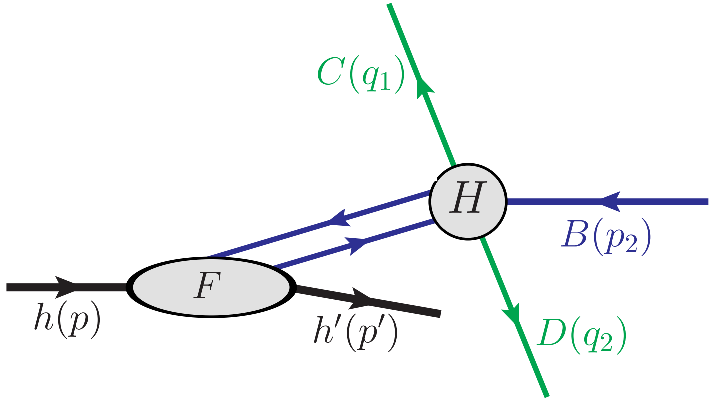
Leading component

$$F^+\mathcal{H}^- = \frac{1}{p_1^+}F^+\left(p_1^+\mathcal{H}^-\right) = \frac{1}{p_1^+}F^+\left(p_1\cdot\mathcal{H} + \boldsymbol{p}_{1\perp}\cdot\mathcal{H}_\perp - p_1^-\mathcal{H}^+\right) \sim \mathcal{O}(\sqrt{|t|})$$

$$\mathcal{M}^{(1)} \sim \mathcal{O}(1/\sqrt{|t|})$$

$$\mathcal{M}^{(2)} \sim \mathcal{O}(1/Q)$$

$\Rightarrow$ $\mathcal{M}^{(1)}/\mathcal{M}^{(2)} \sim \mathcal{O}(Q/\sqrt{|t|})$



γ^* channel is of a ***more leading power*** than GPD contribution, but higher power in α_{EM}

Generally allowed, except

(1) flavor changing ($p \to n$, $n \to p$, etc.)

(2) forbidden by symmetry in the hard part

Jefferson Lab

Extract GPDs from SDHEP with controllable approximation - Factorization

50 years of QCD
2212.11107

- **QCD Facts:**
  - **Any scattering cross section with identified hadron(s) cannot be calculated fully in QCD perturbation theory**

Jefferson Lab

Extract GPDs from SDHEP with controllable approximation - Factorization

QCD Facts:

50 years of QCD
2212.11107

- **Any scattering cross section with identified hadron(s) cannot be calculated fully in QCD perturbation theory**
- **QCD factorization is a controllable approximation with following 3 key features:**
  - All process-dependent nonperturbative contributions to factorizable cross sections are suppressed by powers of 1/(RQ), which could be neglected if the hard scale Q is sufficiently large;
  - All factorizable nonperturbative contributions are process independent, representing the characteristics of identified hadron(s); and
  - Process dependence of factorizable contributions is perturbatively calculable from partonic scattering at the short-distance.
- **Predictions follow** when cross sections with different hard scatterings but the same nonperturbative long-distance effect of identified hadron are compared

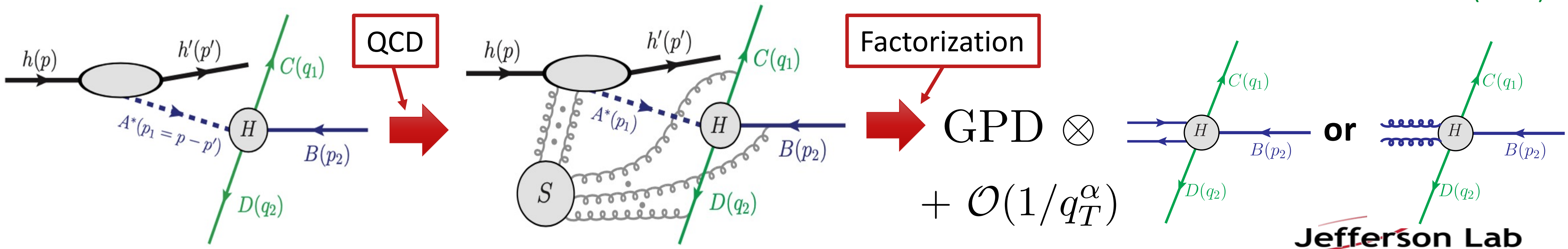
Extract GPDs from SDHEP with controllable approximation - Factorization

50 years of QCD
2212.11107

- **QCD Facts:**
  - **Any scattering cross section with identified hadron(s) cannot be calculated fully in QCD perturbation theory**
  - **QCD factorization is a controllable approximation with following 3 key features:**
    - All process-dependent nonperturbative contributions to factorizable cross sections are suppressed by powers of 1/(RQ), which could be neglected if the hard scale Q is sufficiently large;
    - All factorizable nonperturbative contributions are process independent, representing the characteristics of identified hadron(s); and
    - Process dependence of factorizable contributions is perturbatively calculable from partonic scattering at the short-distance.
  - **Predictions follow** when cross sections with different hard scatterings but the same nonperturbative long-distance effect of identified hadron are compared

- **Factorization for 2-parton channels – Very nontrivial:**

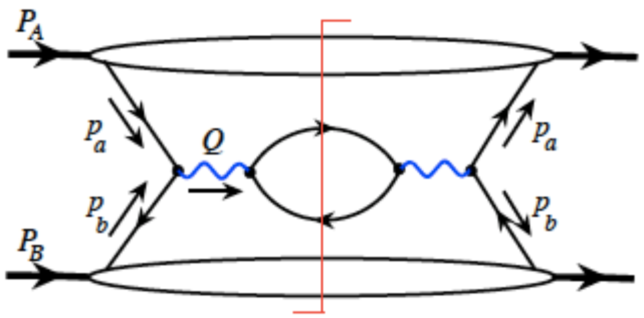
Qiu & Yu, JHEP 08 (2022) 103,
PRD 107 (2023) 1

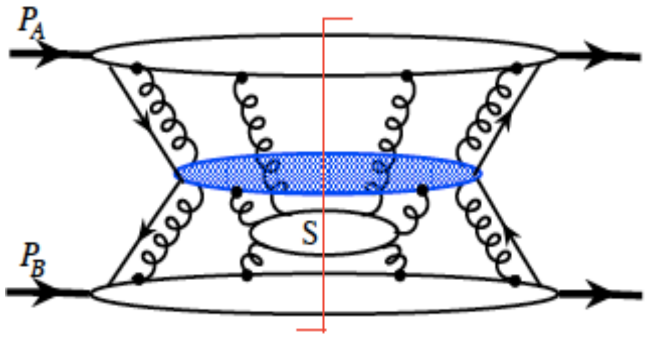


Extract GPDs from SDHEP with controllable approximation - Factorization

- **Lessons learned from QCD factorization for hadronic collisions (e.g., Drell-Yan):**

Collins, Soper, Sterman 1989





Leading pinch surface

Hard: all lines off-shell by Q

Collinear:

- lines collinear to A and B
- One "physical parton" per hadron

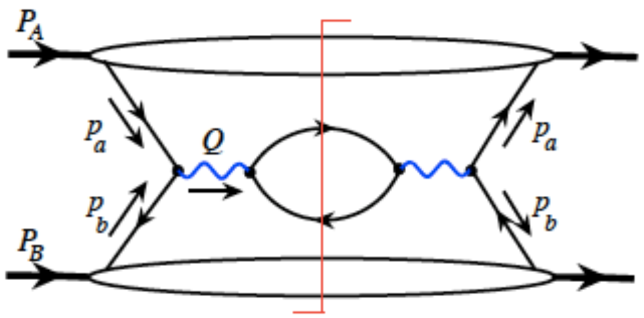
Soft: all components are soft

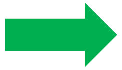
Jefferson Lab

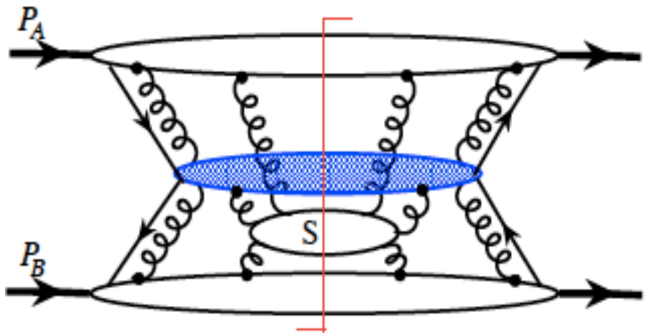
Extract GPDs from SDHEP with controllable approximation - Factorization

Collins, Soper, Sterman 1989

- **Lessons learned from QCD factorization for hadronic collisions (e.g., Drell-Yan):**







Leading pinch surface

Hard: all lines off-shell by Q

Collinear:

✧ lines collinear to A and B

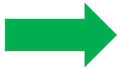
✧ One "physical parton" per hadron

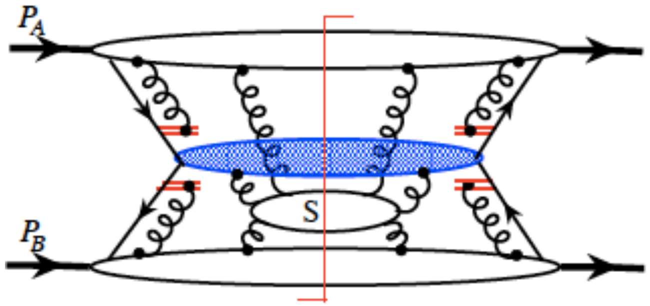
Soft: all components are soft

- **Collinear and longitudinally polarized gluons:**

  Easy to factorize:

  - Apply Ward Identity to decouple them from the hard part
  - Reconnect them the gauge links

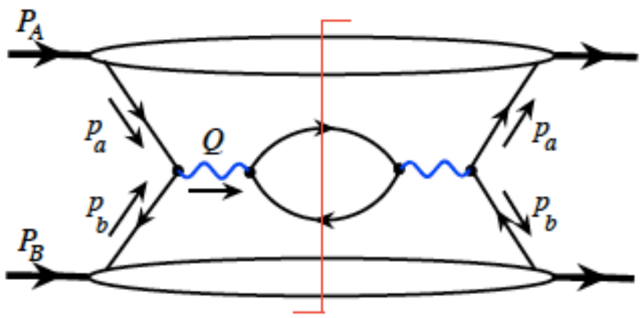


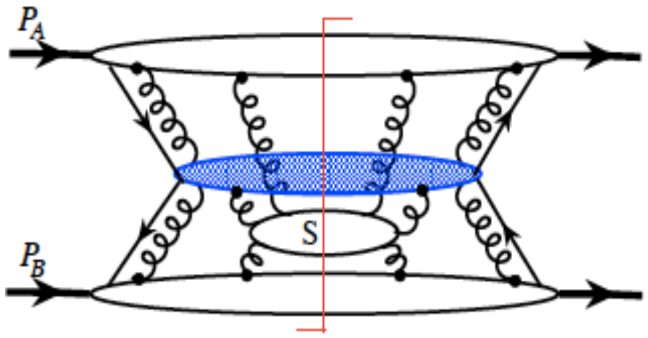


Extract GPDs from SDHEP with controllable approximation - Factorization

❑ Lessons learned from QCD factorization for hadronic collisions (e.g., Drell-Yan):

Collins, Soper, Sterman 1989





Leading pinch surface

Hard: all lines off-shell by Q

Collinear:

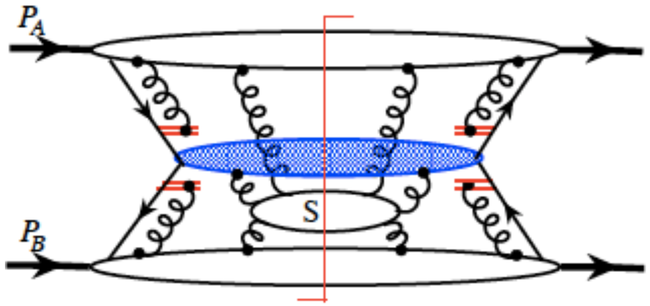
- ✧ lines collinear to A and B
- ✧ One "physical parton" per hadron

Soft: all components are soft

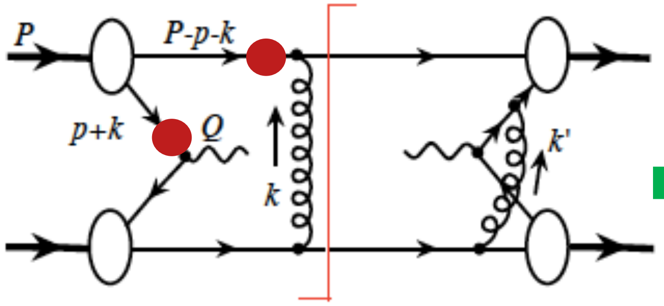
❑ Collinear and longitudinally polarized gluons:

Easy to factorize:

- Apply Ward Identity to decouple them from the hard part
- Reconnect them the gauge links



❑ Trouble with the soft gluons:



$$(xp+k)^2 + i\epsilon \propto k^- + i\epsilon$$

$$((1-x)p-k)^2 + i\epsilon \propto k^- - i\epsilon$$

$$k \to (\lambda^2, \lambda^2, \lambda) \quad \lambda \sim \frac{\Lambda_{\rm QCD}}{Q}$$

Pinched in Glauber regime

Solution:

- Sum over all final states,
- Cancelation of all poles in one-half plane (remove pinches)

Difficulty for exclusive processes:

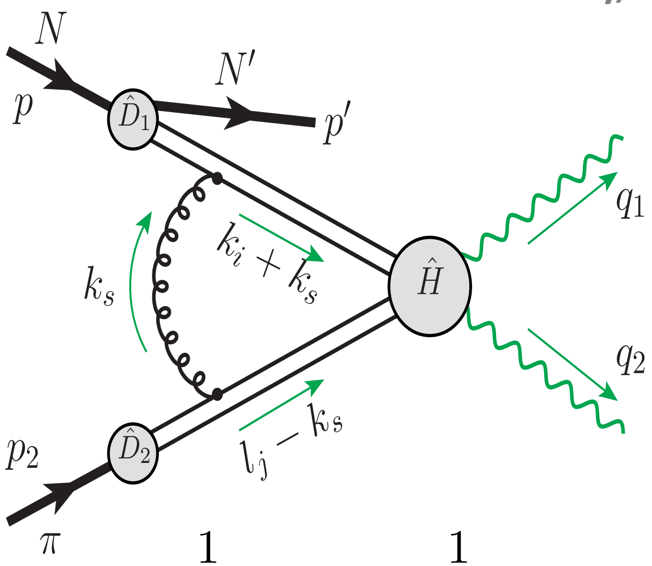
No final-states to sum!

Extract GPDs from SDHEP with controllable approximation - Factorization

- **Glauber pinch for SDHEP, e.g.** $\pi^-(p_\pi) + P(p) \to \gamma(q_1) + \gamma(q_2) + N(p')$

ERBL region

(Efremov, Radyushkin, Brodsky, Lepage)



$$\frac{1}{k_s^2 + i\varepsilon} \to \frac{1}{-\mathbf{k}_s^2 + i\varepsilon}$$

$$\frac{1}{(k_i + k_s)^2 + i\varepsilon} \to \frac{1}{k_s^- + i\varepsilon}$$

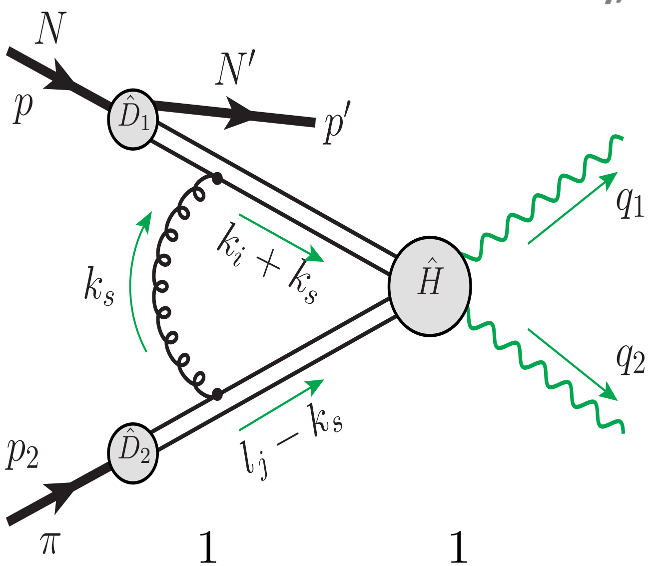
$$\frac{1}{(l_j - k_s)^2 + i\varepsilon} \to \frac{1}{-k_s^+ + i\varepsilon}$$

No pinch!

Extract GPDs from SDHEP with controllable approximation - Factorization

- **Glauber pinch for SDHEP, e.g.** $\pi^-(p_\pi) + P(p) \to \gamma(q_1) + \gamma(q_2) + N(p')$ $\lambda \sim m_\pi/Q, \quad Q \sim q_T$

ERBL region (Efremov, Radyushkin, Brodsky, Lepage)



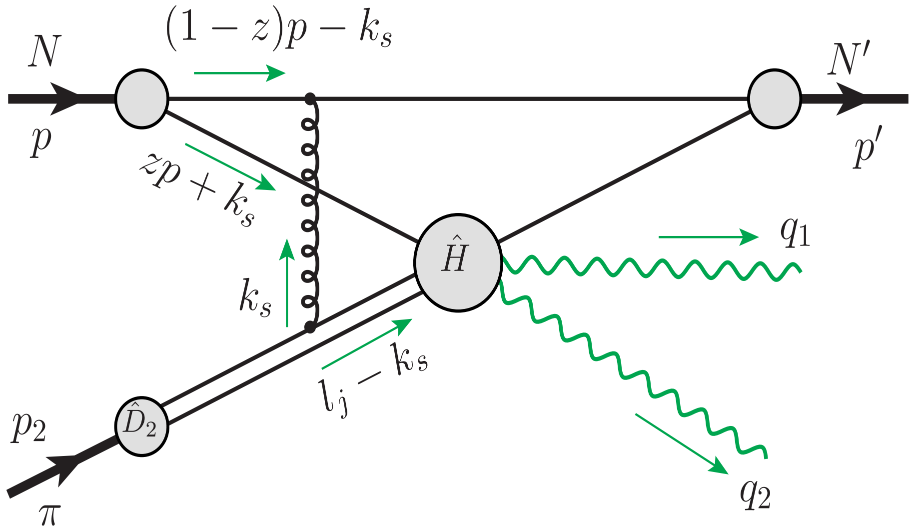
$$\frac{1}{k_s^2 + i\varepsilon} \to \frac{1}{-\mathbf{k}_s^2 + i\varepsilon}$$

$$\frac{1}{(k_i + k_s)^2 + i\varepsilon} \to \frac{1}{k_s^- + i\varepsilon}$$

$$\frac{1}{(l_j - k_s)^2 + i\varepsilon} \to \frac{1}{-k_s^+ + i\varepsilon}$$

No pinch!

DGLAP region



$$\frac{1}{((1-z)p - k_s)^2 + i\varepsilon} \to \frac{1}{k_s^- - i\varepsilon}$$

$$\frac{1}{(zp + k_s)^2 + i\varepsilon} \to \frac{1}{k_s^- + i\varepsilon}$$

Pinched!

Same conclusion if k_s flows through N'!

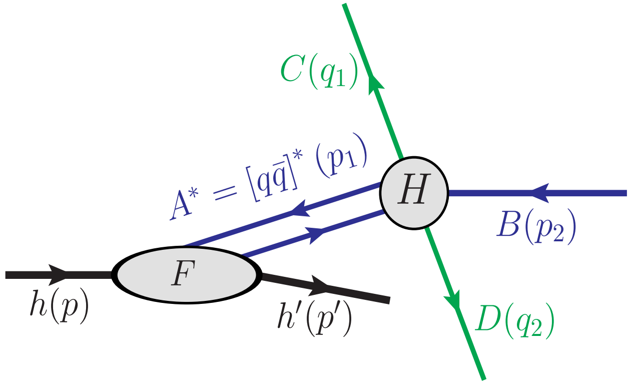
➡ Gluons pinched in the Glauber region: $k_s = (\lambda^2, \lambda^2, \lambda)\, Q$

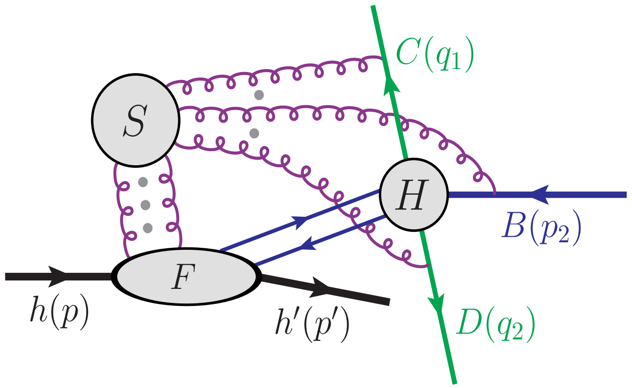
➡ ***Transverse component contribute to the leading region!***

Factorization for SDHEP in the Two-stage Paradigm

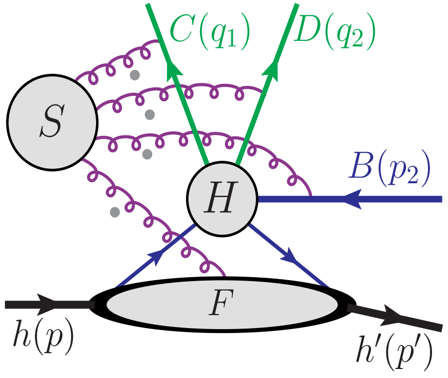
Qiu & Yu, JHEP 08 (2022) 103, PRD 107 (2023) 1

❑ Factorization for 2-parton channels (CO gluons are easy to factorize):



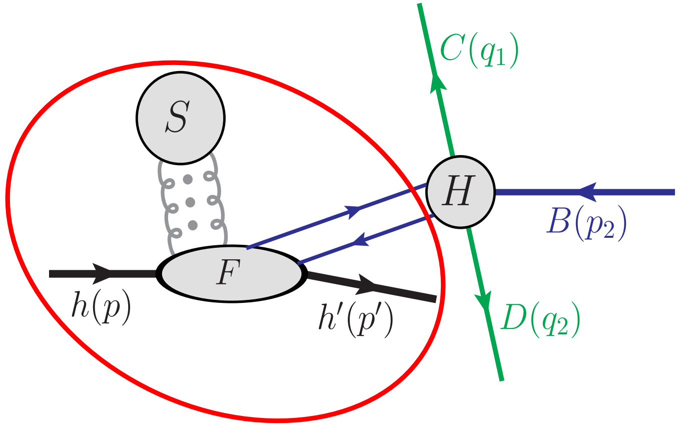


+



ERBL region: $[q\bar{q}']$ ~ meson **DGLAP region: Glauber pinch**

❑ Soft gluons cancel when coupling to color neutral hadrons:

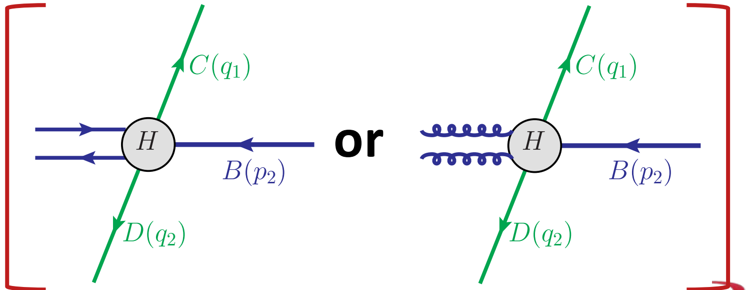


Glauber gluons of SDHEP (*only* k_s^- is pinched in Glauber region):

$$k_s^+ \mapsto k_s^+ \pm i\mathcal{O}(Q) \quad \Rightarrow \quad k_s = (\lambda^2, \lambda^2, \lambda) \;\to\; (1, \lambda^2, \lambda)$$

CO gluons

$\mathrm{GPD} \otimes$



Hard probes

No factorization for double diffractive scattering! Soper, 1997

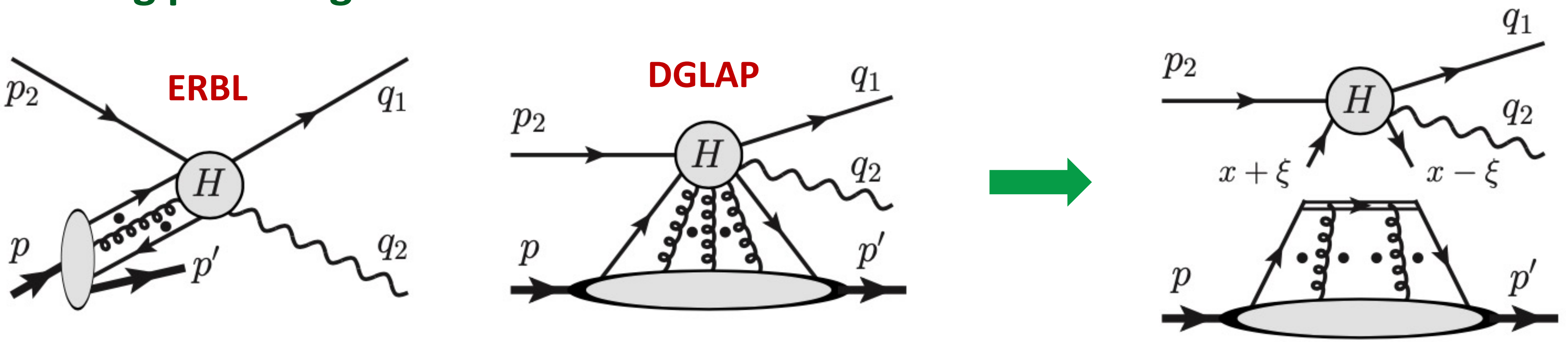
SDHEP with a Lepton Beam – JLab, EIC

PRD56 (1997) 5524; PRD58 (1998) 094018; PRD59 (1999) 074009

❑ DVCS:

$$h(p) = \text{Proton}(p),\ \ h'(p') = \text{Proton}(p'),\ \ B(p_2) = \text{electron}(p_2),\ \ C(q_1) = \text{electron}(q_1),\ \ D(q_2) = \text{photon}(q_2)$$

❑ Leading pinch region:



$$x = \frac{(k+k')^+}{(p+p')^+}$$

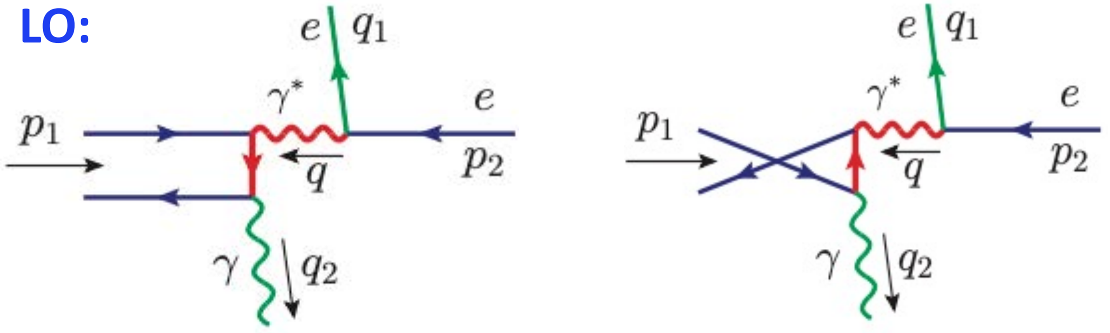
$$\xi = \frac{(p-p')^+}{(p+p')^+}$$

❑ Factorization formula:

$$\mathcal{M}^{(2)}_{he\to h'e\gamma} = \sum_i \int_{-1}^{1} \mathrm{d}x\, F_i^h(x,\xi,t)\, C_{ie\to e\gamma}(x,\xi,q_T),$$

$$C^{(0)} \propto \frac{1}{x-\xi+i\varepsilon} - \frac{1}{x+\xi-i\varepsilon}$$

LO:



The x-integration is NOT sensitive to externally measured hard scale, q_T or Q^2!

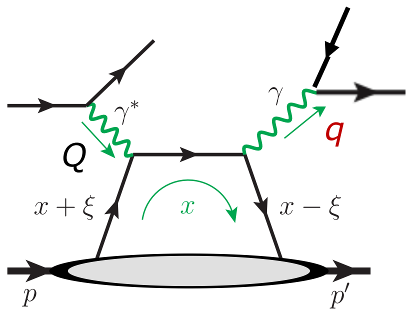
Jefferson Lab

What kind of process/observable could be sensitive to the x-dependence?

- **Create an entanglement between the internal x and an externally measured variable?**

$$i\mathcal{M} \propto \int_{-1}^{1} \mathrm{d}x \frac{F(x, \xi, t)}{x - x_p(\xi, q) + i\varepsilon}$$

Change external q to sample different part of x.



- **Double DVCS (two scales):**

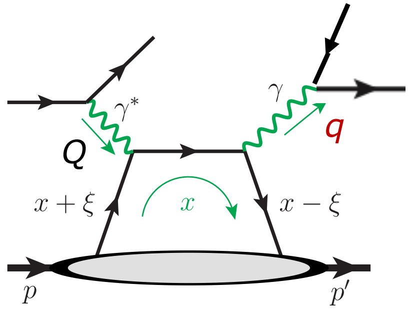
$$x_p(\xi, q) = \xi \left(\frac{1 - q^2/Q^2}{1 + q^2/Q^2} \right) \to \xi \quad \text{same as} \quad \text{DVCS} \quad \text{if } q \to 0$$

What kind of process/observable could be sensitive to the x-dependence?

❑ **Create an entanglement between the internal x and an externally measured variable?**

$$i\mathcal{M} \propto \int_{-1}^{1} \mathrm{d}x \frac{F(x,\xi,t)}{x - x_p(\xi,q) + i\varepsilon}$$

Change external *q* to sample different part of x.



- **Double DVCS (two scales):**

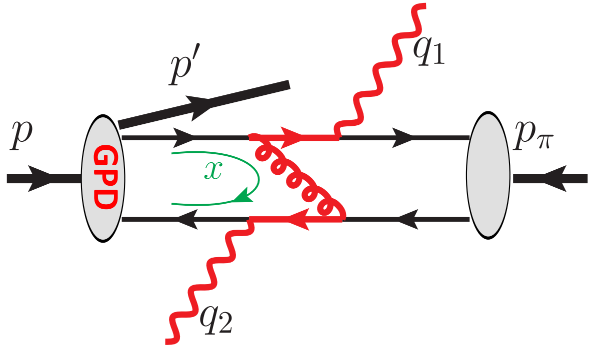
$$x_p(\xi,q) = \xi\left(\frac{1-q^2/Q^2}{1+q^2/Q^2}\right) \to \xi \quad \text{same as} \quad \text{DVCS} \quad \text{if } q \to 0$$

- **Production of two back-to-back high pT particles (say, two photons):**

$$\pi^-(p_\pi) + P(p) \to \gamma(q_1) + \gamma(q_2) + N(p')$$

Hard scale: $q_T \gg \Lambda_{\mathrm{QCD}}$ **Soft scale:** $t \sim \Lambda^2_{\mathrm{QCD}}$

Qiu & Yu
JHEP 08 (2022) 103



- **Factorization:**

[*suppressing pion DA factor*]

$$\mathcal{M}(t,\xi,q_T) = \int_{-1}^{1} \mathrm{d}x\, F(x,\xi,t;\mu)\cdot C(x,\xi;q_T/\mu) + \mathcal{O}(\Lambda_{\mathrm{QCD}}/q_T)$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t\,\mathrm{d}\xi\,\mathrm{d}q_T} \sim |\mathcal{M}(t,\xi,q_T)|^2$$

q_T distribution is "conjugate" to x distribution

$$x \leftrightarrow q_T$$

GPD Models for Testing the x-dependence

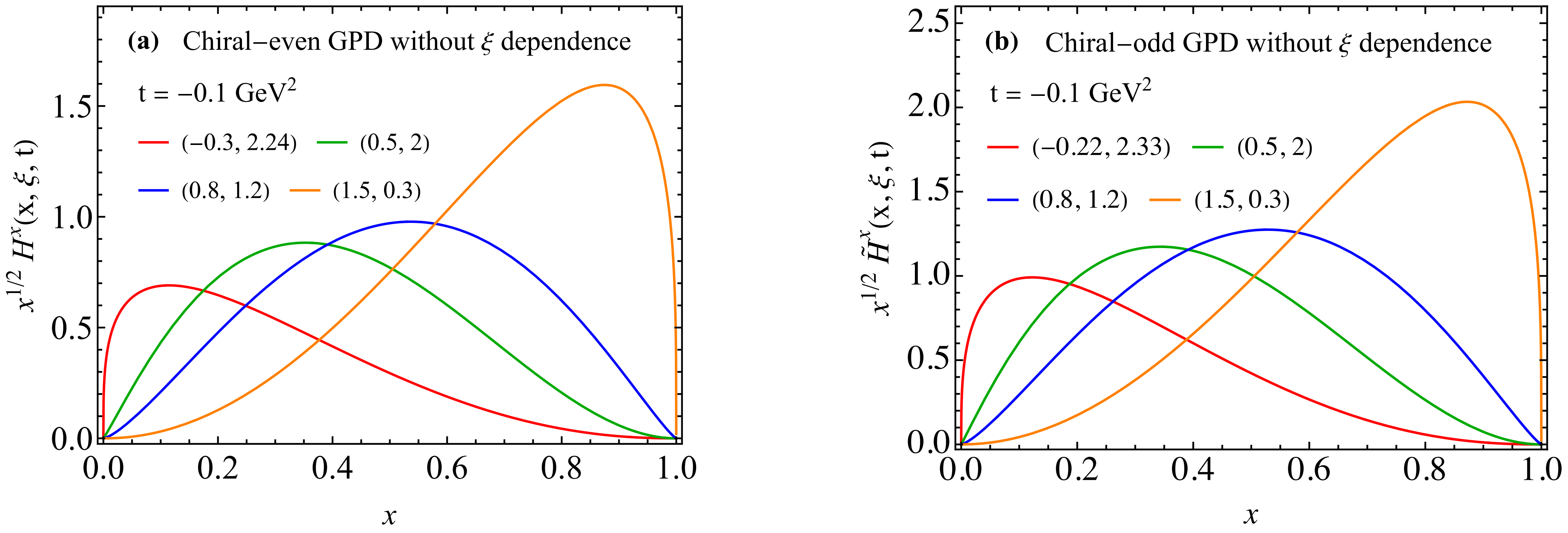
❑ Simplified GK models:

Goloskokov, Kroll
hep-ph/0501242
arXiv: 0708.3569
arXiv: 0906.0460
Qiu & Yu,
arXiv:2305.15397

$$H_{pn}(x,\xi,t) = \theta(x)\, x^{-0.9\,(t/\mathrm{GeV}^2)}\, \frac{x^{\rho}(1-x)^{\tau}}{B(1+\rho,1+\tau)}$$

$$\widetilde{H}_{pn}(x,\xi,t) = \theta(x)\, x^{-0.45\,(t/\mathrm{GeV}^2)}\, \frac{1.267\, x^{\rho}(1-x)^{\tau}}{B(1+\rho,1+\tau)}$$

- Neglect $E, \widetilde{E}$. Neglect evolution effect.
- Tune (ρ, τ) to control x shape.
- Fix DA: $D(z) = N\, z^{0.63}(1-z)^{0.63}$

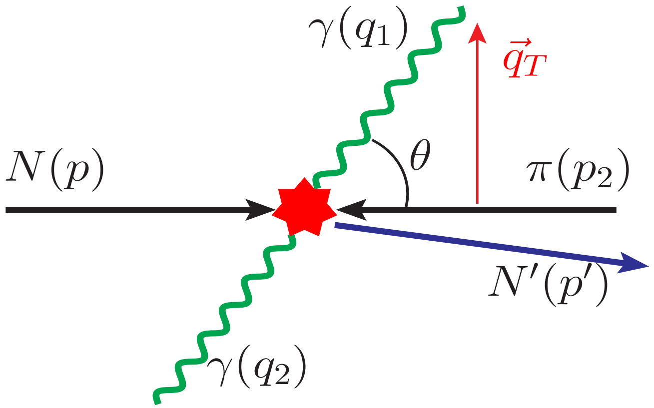


Jefferson Lab

Enhanced Sensitivity on x-dependence of GPDs

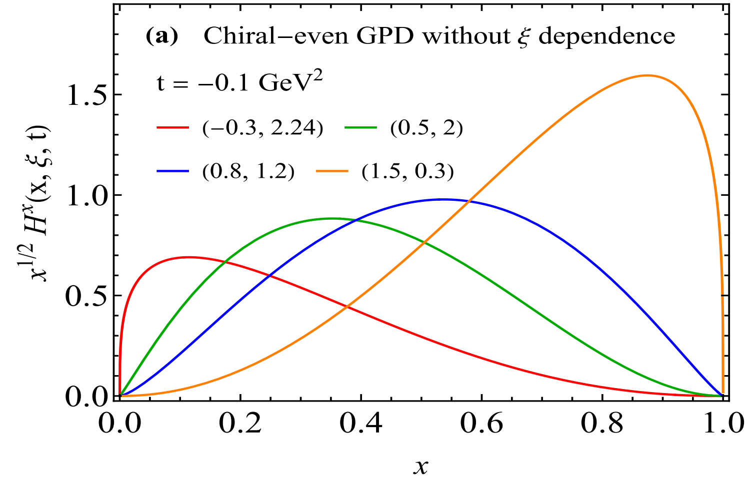
- **Two-photon production:** $\pi^-(p_\pi) + P(p) \to \gamma(q_1) + \gamma(q_2) + N(p')$ **J-PARC, COMPASS**

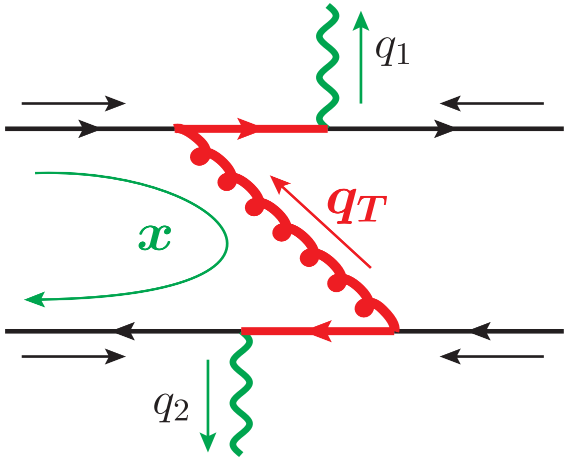
Qiu & Yu, JHEP 08 (2022) 103

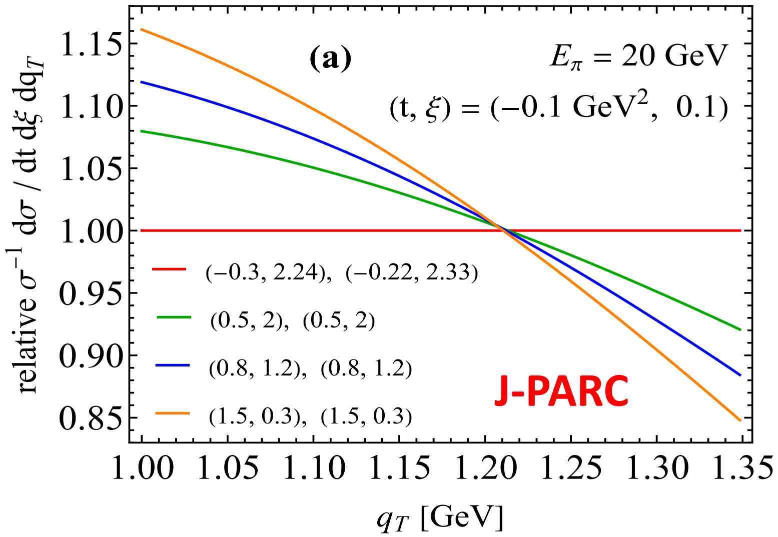


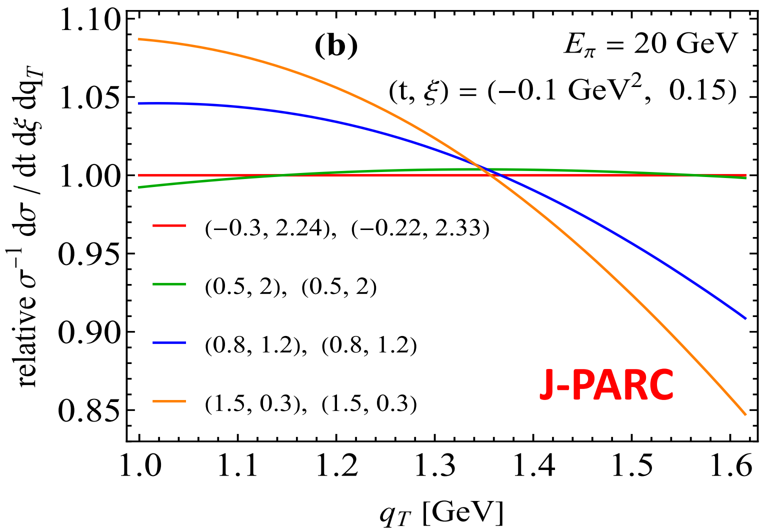
Vary GPD *x* shapes

Different q_T shapes





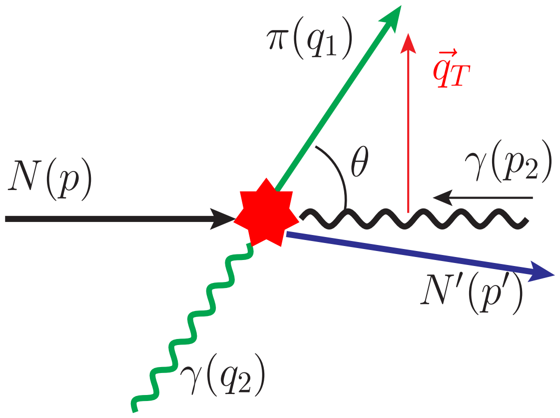




Enhanced Sensitivity on x-dependence of GPDs

- **Pion-photon production:** $\gamma(p_\gamma) + h(p) \to \pi^{\pm}(q_1) + \gamma(q_2) + h'(p')$

JLab-Hall D, other Halls & EIC with a quasi-photon beam

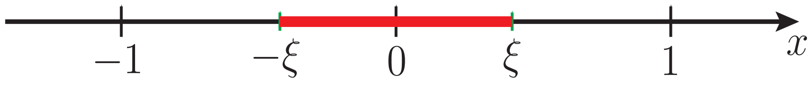


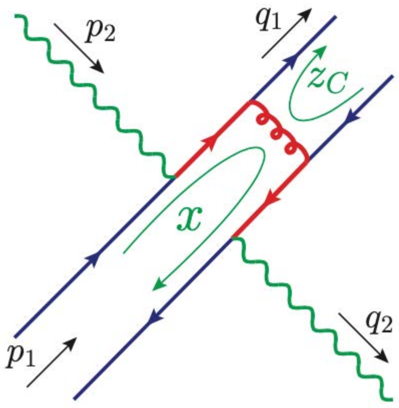
$i\mathcal{M}$ **contains the entanglement between x and qT**

Qiu & Yu, arXiv:2305.15397

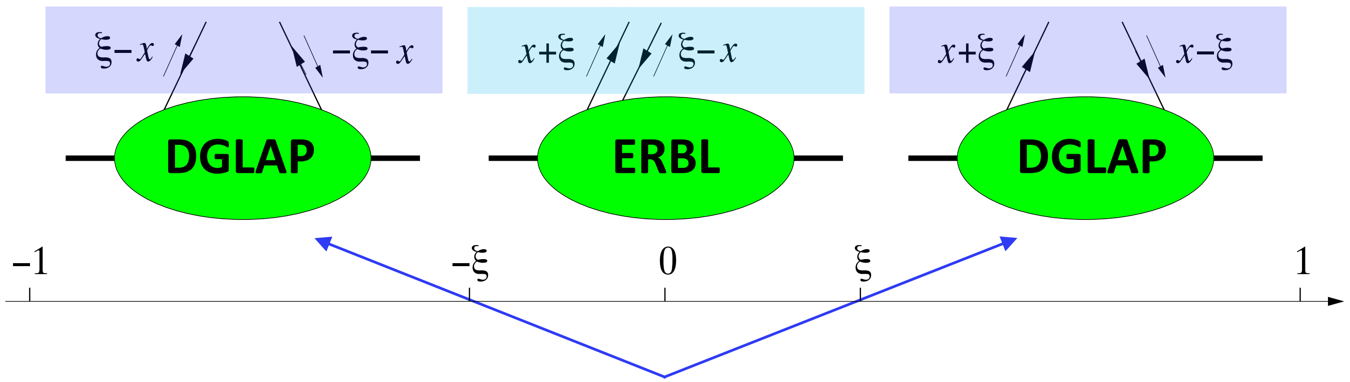
$$I'(t,\xi;z,\theta) = \int_{-1}^{1} \frac{dx\, F(x,\xi,t)}{x - \rho'(z;\theta) + i\epsilon}$$

$$\rho'(z;\theta) = \xi \cdot \left[\frac{\cos^2(\theta/2)\,(1-z) - z}{\cos^2(\theta/2)\,(1-z) + z}\right] \in [-\xi, \xi]$$





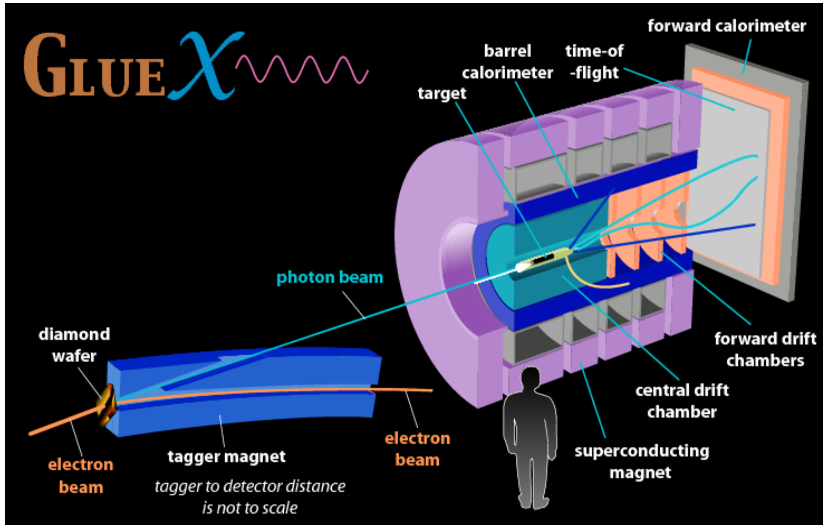
Complementary sensitivity:

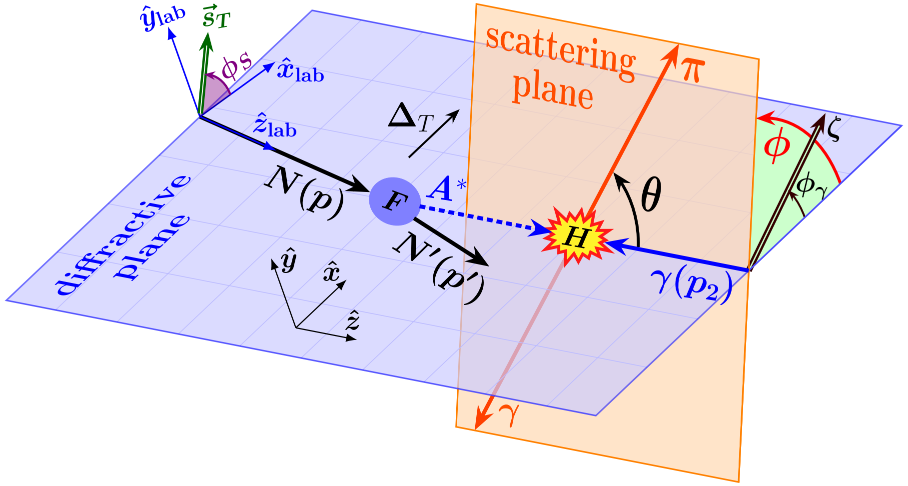


$N\,\pi \to N'\,\gamma\,\gamma$

Exclusive Photo-Production of a π γ Pair – Hall D at JLab

Qiu & Yu, arXiv:2305.15397
PRL (in press)





❑ Polarization asymmetries

$$\frac{d\sigma}{d|t|\,d\xi\,d\cos\theta\,d\phi} = \frac{1}{2\pi}\,\frac{d\sigma}{d|t|d\xi d\cos\theta}\cdot\left[1+\lambda_N\lambda_\gamma\,A_{LL} + \zeta\,A_{UT}\cos 2\left(\phi-\phi_\gamma\right) + \lambda_N\zeta\,A_{LT}\sin 2\left(\phi-\phi_\gamma\right)\right]$$

$$\frac{d\sigma}{d|t|\,d\xi\,d\cos\theta} = \pi\left(\alpha_e\alpha_s\right)^2\left(\frac{C_F}{N_c}\right)^2\frac{1-\xi^2}{\xi^2 s^3}\,\Sigma_{UU}$$

$$\Sigma_{UU} = |\mathcal{M}_+^{[\tilde{H}]}|^2 + |\mathcal{M}_-^{[\tilde{H}]}|^2 + |\widetilde{\mathcal{M}}_+^{[H]}|^2 + |\widetilde{\mathcal{M}}_-^{[H]}|^2,$$

$$A_{LL} = 2\,\Sigma_{UU}^{-1}\,\mathrm{Re}\left[\mathcal{M}_+^{[\tilde{H}]}\,\widetilde{\mathcal{M}}_+^{[H]*} + \mathcal{M}_-^{[\tilde{H}]}\,\widetilde{\mathcal{M}}_-^{[H]*}\right],$$

$$A_{UT} = 2\,\Sigma_{UU}^{-1}\,\mathrm{Re}\left[\widetilde{\mathcal{M}}_+^{[H]}\,\widetilde{\mathcal{M}}_-^{[H]*} - \mathcal{M}_+^{[\tilde{H}]}\,\mathcal{M}_-^{[\tilde{H}]*}\right],$$

$$A_{LT} = 2\,\Sigma_{UU}^{-1}\,\mathrm{Im}\left[\mathcal{M}_+^{[\tilde{H}]}\,\widetilde{\mathcal{M}}_-^{[H]*} + \mathcal{M}_-^{[\tilde{H}]}\,\widetilde{\mathcal{M}}_+^{[H]*}\right].$$

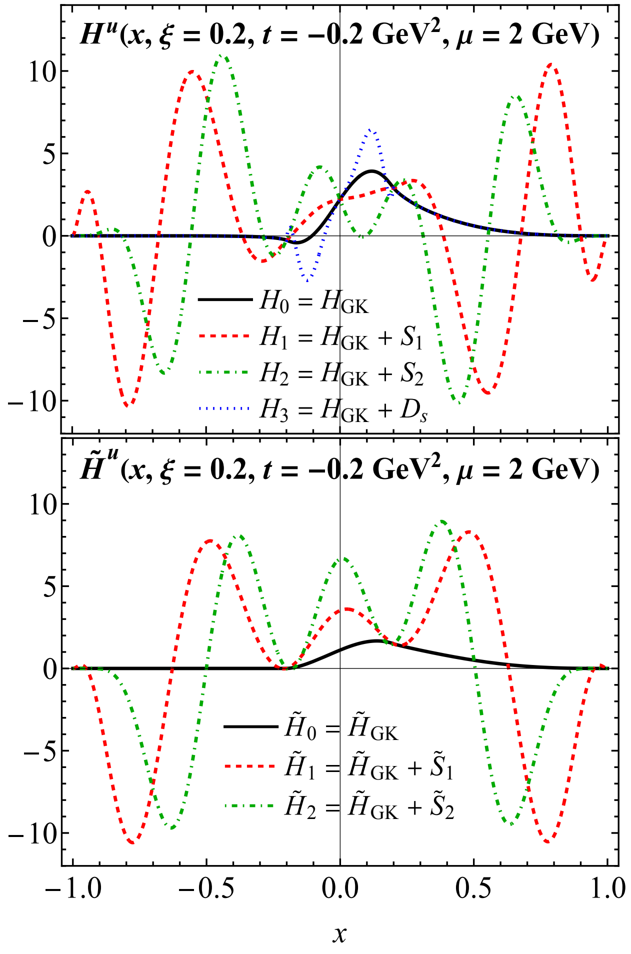
Exclusive Photo-Production of a π γ Pair – Hall D at JLab

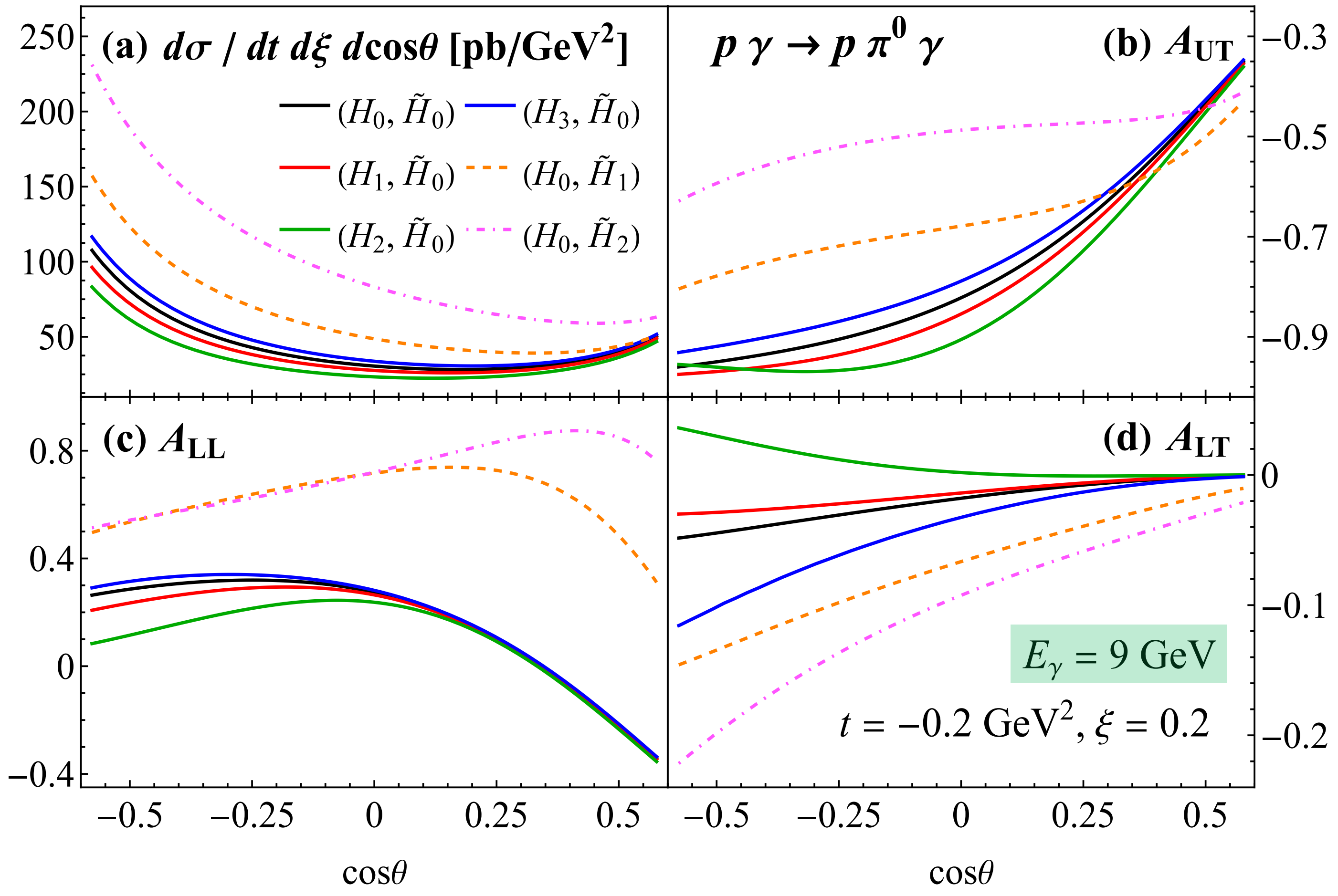
❑ GPD Models:

= GK model + shadow GPDs

$$\int_{-1}^{1} \frac{dx\, S(x,\xi)}{x-\xi \pm i\epsilon} = 0$$

Goloskokov, Kroll, `05, `07, `09
Bertone et al. `21
Moffat et al. `23
Qiu & Yu, arXiv:2305.15397





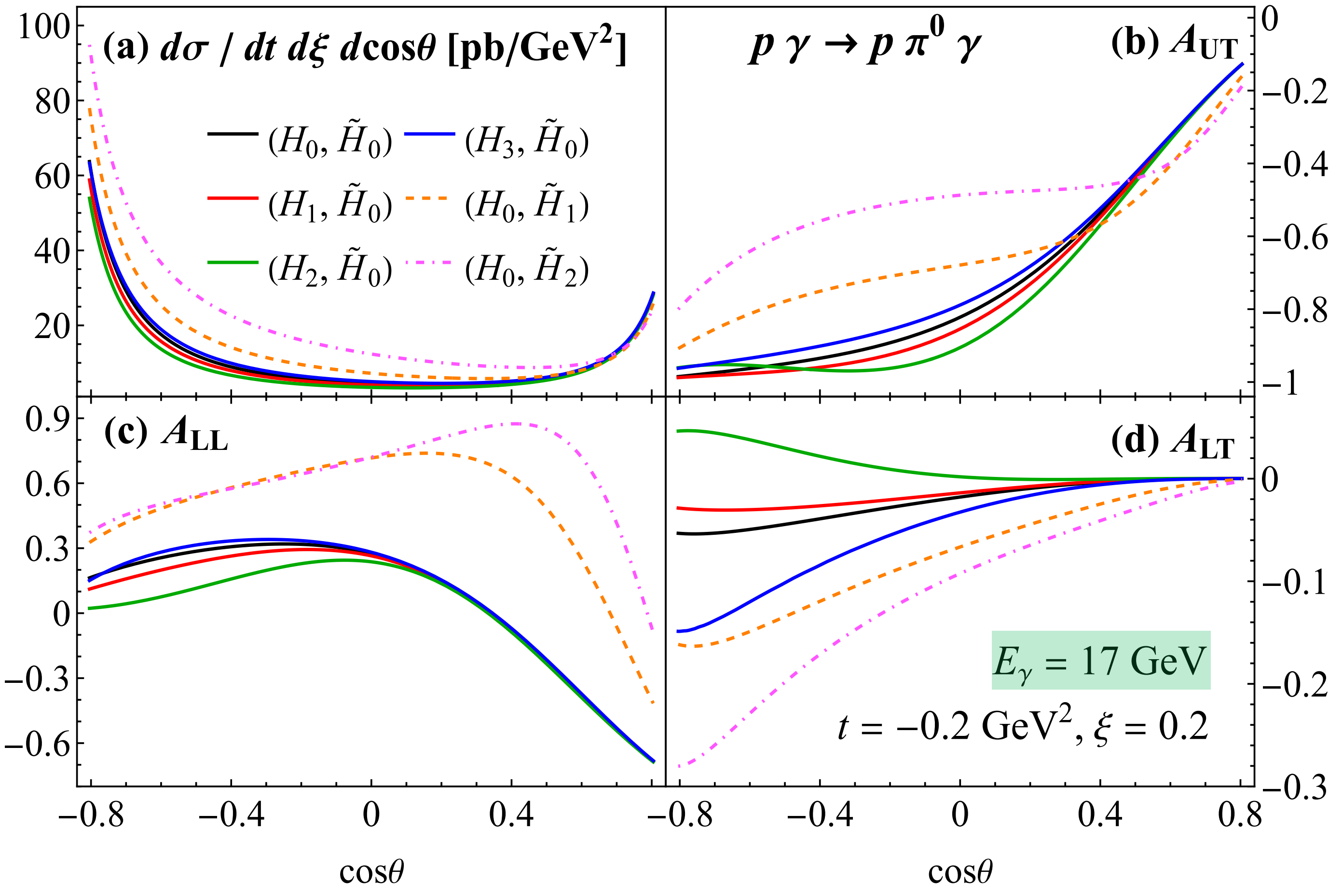
Exclusive Photo-Production of a π γ Pair – Hall D at JLab

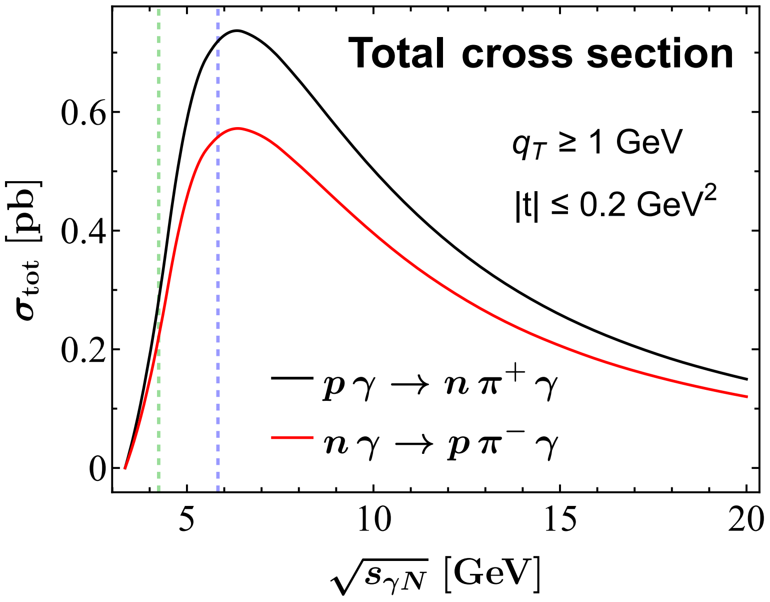
❑ GPD Models:

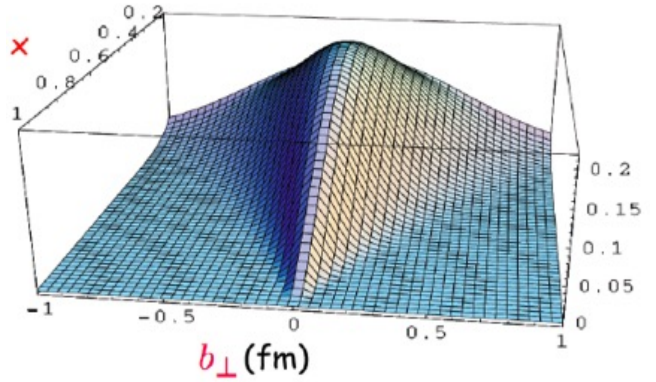
= GK model + shadow GPDs

$$\int_{-1}^{1} \frac{dx\, S(x,\xi)}{x-\xi \pm i\epsilon} = 0$$

Goloskokov, Kroll, \`05, \`07, \`09
Bertone et al. \`21
Moffat et al. \`23
Qiu & Yu, arXiv:2305.15397





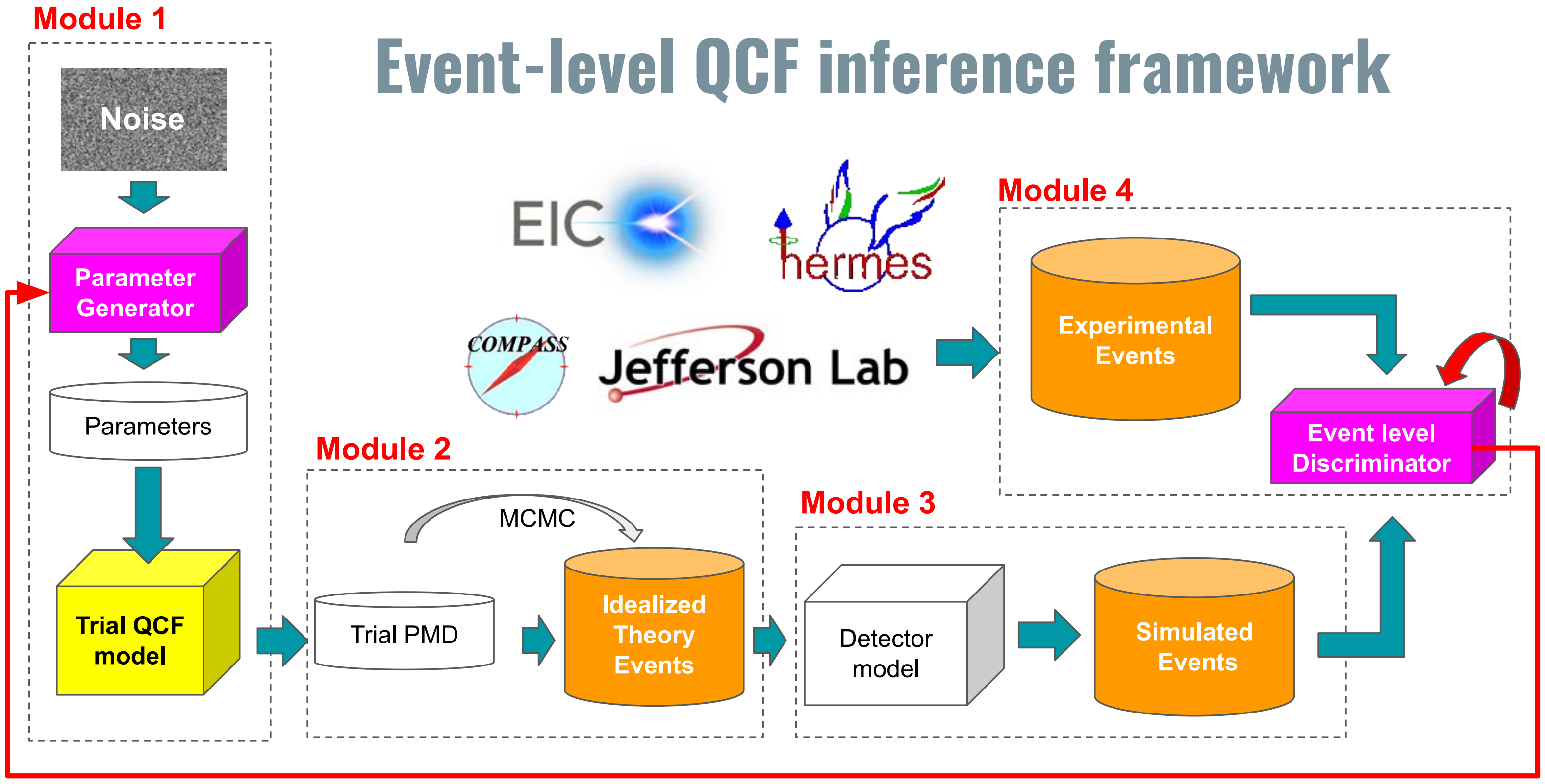


Extracting GPDs is a challenging inverse problem! $\sigma \propto [\mathcal{F} \otimes C]^2$

QuantOm Collaboration – a 5-year SciDAC project

- **Femtoscale Imaging of Nuclei using Exascale Platforms:**

  Pixelating hadron in terms of probabilities to find quarks and gluons in slices of the momentum fraction x



PMD: Particle Momentum Distribution - Observables

QCF: Quantum Correlation Functions: PDFs, TMDs, GPDs, …

QuantOm Collaboration

NP: ANL(Lead), JLab, VT

ASCR: FASTMath, RAPIDs

Exp Events (PMD):

- **DIS:**
  1 particle inclusive
- **SIDIS:**
  2 particle inclusive
- SDHEP:
  3 particle exclusive

Generated Events:

Many templates from trial QCFs & trusted theory

Inference:

Optimized QCFs or pixelated images in trusted phase space

New regimes:

Go beyond the trusted phase space

Summary and Outlook

- SDHEP provides a reliable way to explore tomography of nuclei without breaking them:
  - GPDs are fundamental functions carrying the pixelated images of a bound hadron/nucleus,
  - Carry rich information on emergent hadron properties (mass, spin, …) from QCD dynamics,
  - Provide the much needed hints on how confined quarks/gluons respond to the hard probes, …

  Extracting their x-dependence from experimental observable(s) is non-trivial, but, full of opportunities, …

- 50 years of QCD established it as the right theory of strong interactions:
  - Many challenges and open questions remain, including confinement, emergent phenomena, …
  - QCD at the femto-scale (0.1 – 10 fm) is the most interesting, rich, and complex regime of the theory

Summary and Outlook

- SDHEP provides a reliable way to explore tomography of nuclei without breaking them:
  - GPDs are fundamental functions carrying the pixelated images of a bound hadron/nucleus,
  - Carry rich information on emergent hadron properties (mass, spin, …) from QCD dynamics,
  - Provide the much needed hints on how confined quarks/gluons respond to the hard probes, …

  Extracting their x-dependence from experimental observable(s) is non-trivial, but, full of opportunities, …

- 50 years of QCD established it as the right theory of strong interactions:
  - Many challenges and open questions remain, including confinement, emergent phenomena, …
  - QCD at the femto-scale (0.1 – 10 fm) is the most interesting, rich, and complex regime of the theory

  I would like to thank all pioneers who discovered the QCD and methods allowing us to explore the QCD!

  I would like to thank Prof. Al Mueller who introduced me to the QCD and its excitements, and Prof. George Sterman who introduced me to the factorization and the predictive power of perturbative QCD!

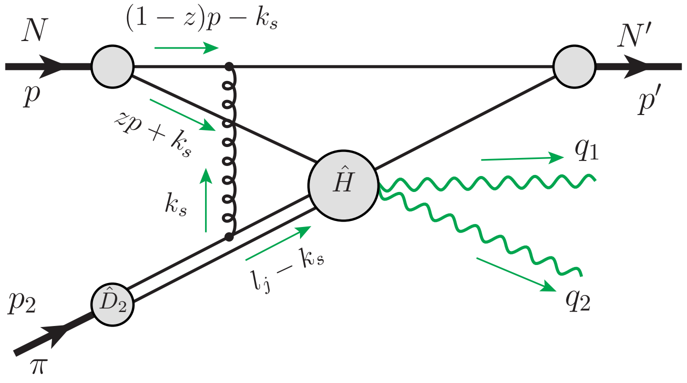
  I would like to thank the organizers for hosting such a nice and historic meeting, and the opportunity to speak and to celebrate the 50 years of QCD with all of you!

Thanks!

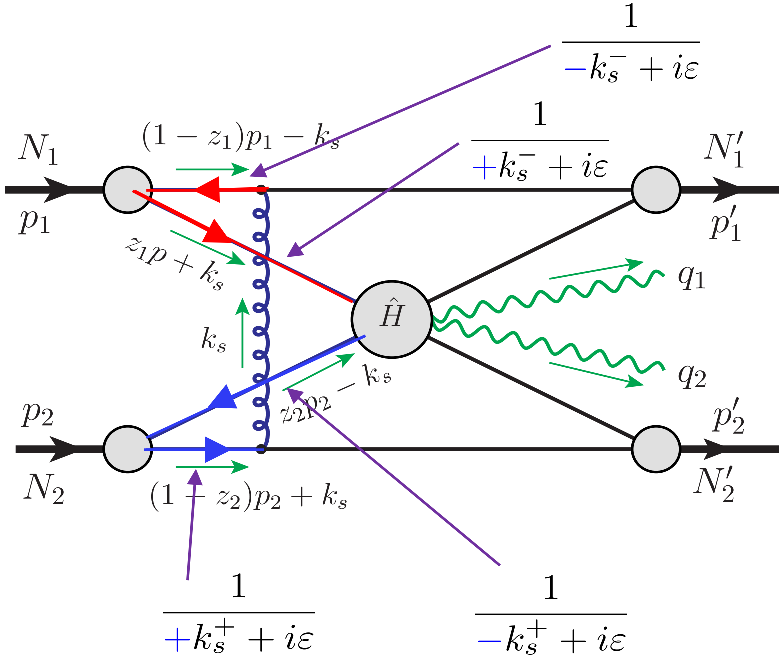
Why *single* diffractive?

- **Double diffractive process**

Glauber pinch for diffractive scattering



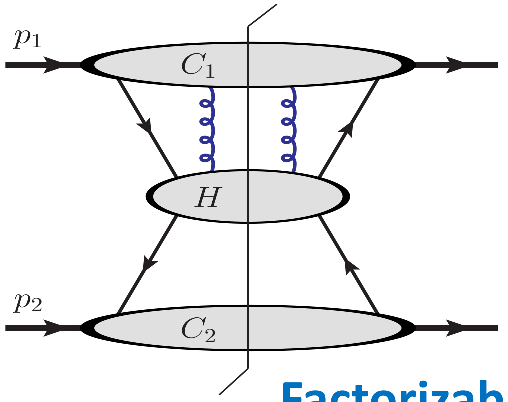
Factorizable if all pion momentum flows into hard part



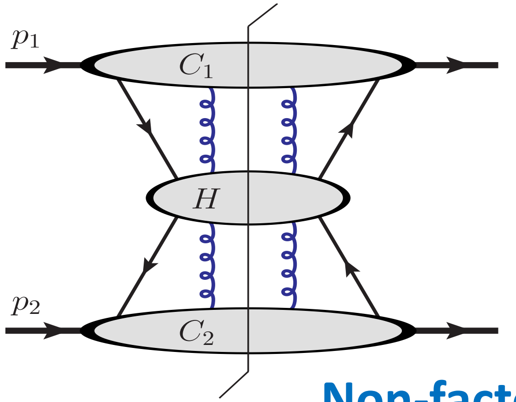
Both k_s^+ and k_s^- are pinched in Glauber region!

Break of factorization

- **Compare: Drell-Yan process at high twist:**



Factorizable



Non-factorizable

Only the 1st sub-leading twist is factorizable!

Qiu & Sterman, NPB, 1991