# Extraction of pseudo-PDFs
# *or why Ioffe-Time Distributions are our friend*

David Richards

*Jefferson Lab*

# Outline

- LaMET, quasi-PDFs, pseudo-PDFs and Good Lattice Cross Sections
- Pion as a theatre for PDFs - pPDFs and GLCS
- pPDFs in Nucleon
- Summary

# Introduction

- First Challenge:
  - Euclidean lattice precludes calculation of light-cone/time-separated correlation functions

$$q(x,\mu) = \int \frac{d\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P \mid \bar{\psi}(\xi^-)\gamma^+ e^{-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)} \psi(0) \mid P \rangle$$

So…. …Use *Operator-Product-Expansion to formulate in terms of Mellin Moments* with respect to Bjorken x.

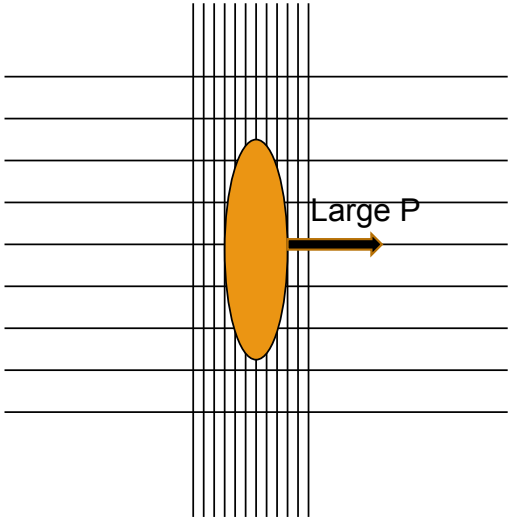
$$\longrightarrow \quad \langle P \mid \bar{\psi}\gamma_{\mu_1}(\gamma_5) D_{\mu_2} \ldots D_{\mu_n} \psi \mid P \rangle \to P_{\mu_1} \ldots P_{\mu_n} a^{(n)}$$

- Second Challenge:
  - Discretised lattice: power-divergent mixing for higher moments

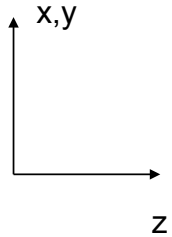
Moment Methods

- Extended operators: Z.Davoudi and M. Savage, PRD 86,054505 (2012)
- Valence heavy quark: W.Detmold and W.Lin, PRD73, 014501 (2006)

# Solution….




Large-Momentum Effective Theory (LaMET)




**"Equal time" correlator**

**X. Ji, Phys. Rev. Lett. 110, 262002 (2013).**
**X. Ji, J. Zhang, and Y. Zhao, Phys. Rev. Lett. 111, 112002 (2013).**
**J. W. Qiu and Y. Q. Ma, arXiv:1404.686.**

$$q(x,\mu^2,P^z) = \int \frac{dz}{4\pi} e^{izk^z} \langle P \mid \bar{\psi}(z)\gamma^z e^{-ig\int_0^z dz'\, A^z(z')} \psi(0) \mid P > + \mathcal{O}((\Lambda^2/(P^z)^2), M^2/(P^z)^2))$$

$$q(x,\mu^2,P^z) = \int_x^1 \frac{dy}{y} Z\left(\frac{x}{y}, \frac{\mu}{P^z}\right) q(y,\mu^2) + \mathcal{O}(\Lambda^2/(P^z)^2, M^2/(P^z)^2)$$

# Pseudo-PDFs

- Pseudo-PDF (pPDF) recognizing generalization of PDFs in terms of *Ioffe Time.* $\nu = p \cdot z$

B.Ioffe, PL39B, 123 (1969); V.Braun *et al*, PRD51, 6036 (1995)

A.Radyushkin, Phys. Rev. D 96, 034025 (2017)

$$M^\alpha(p,z) = \langle p \mid \bar{\psi}\gamma^\alpha U(z;0)\psi(0) \mid p\rangle$$

$p = (p^+, m^2/2p^+, 0_T)$ $\quad$ $z = (0, z_-, 0_T)$

Ioffe-Time Distribution

$$\Downarrow$$

$$M^\alpha(z,p) = 2p^\alpha \mathcal{M}(\nu, z^2) + 2z^\alpha \mathcal{N}(\nu, z^2)$$

Ioffe-time pseudo-Distribution (pseudo-ITD) generalization to *space-like* z

Lattice "building blocks" that of quasi-PDF approach.

$\Downarrow$ *Lorentz covariant*

$$\mathcal{M}(\nu, z^2) = \int_{-1}^{1} dx\, e^{i\nu x} \mathcal{P}(x, z^2)$$

$\longleftarrow$ *pseudo-PDF*

$$\Downarrow$$

$$f(x) = \mathcal{P}(x,0) \underset{z_3^2 \to 0}{=} \frac{1}{2\pi} \int_{-\infty}^{\infty} d\nu e^{-i\nu x} \mathcal{M}(\nu, -z_3^2)$$

# pPDFs - II

To deal with UV divergences, introduce reduced distribution $\mathfrak{M} = \frac{\mathcal{M}(\nu, z^2)}{\mathcal{M}(0,z^2)}$

$$\mathfrak{M}(\nu, z^2) = \int_0^1 du\, K(u, z^2\mu^2, \alpha_s) Q(u\nu, \mu^2)$$

Computed on lattice

Perturbatively calculable

Ioffe-time Distribution

$$Q(\nu,\mu) = \mathfrak{M}(\nu, z^2) - \frac{\alpha_s C_F}{2\pi} \int_0^1 du \left[ \ln\left( z^2\mu^2 \frac{e^{2\gamma_E+1}}{4} \right) B(u) + L(u) \right] \mathfrak{M}(u\nu, z^2).$$

K. Orginos et al., PRD96 (2017), 094503

Match data at different *z*

$\Downarrow$

$$Q(\nu) = \int_{-1}^{1} dx\, q(x) e^{i\nu x}$$

$$q(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\nu\, e^{-i\nu x} Q(\nu)$$

Need data for all v, *or additional physics input*

Inverse problem

# Moments

J Karpie, K Orginos, S Zafeiropoulos, arXiv:1807.10933

- Can obviate need for inverse through computation of moments

$$\mathfrak{M}(\nu, z^2) = \sum_{n=0}^{\infty} i^n \frac{\nu^n}{n!} a_n(\mu^2) K_n(\mu^2 z^2) + \mathcal{O}(z^2)$$

Mellin moments of PDF

Matching coefficient

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First “dedicated” lattice calculation of pseudo-PDFs - K. Orginos et al., PRD96 (2017), 094503

c.f. TMD calculation in Ioffe time with straight link - B. Musch et al., PRD83:094507 (2011)

See Michael Englehardt, Wednesday PM

# "Good Lattice Cross Sections"

$$\sigma_n(\nu, \xi^2, P^2) = \langle P \mid T\{\mathcal{O}_n(\xi)\} \mid P \rangle$$

Ma and Qiu, Phys. Rev. Lett. 120 022003

*Expressed in coordinate space*

Short distance scale

where

$$\sigma_n(\nu, \xi^2, P^2) = \sum_a \int_{-1}^{1} \frac{dx}{x} f_a(x, \mu^2) K_n^a(x\nu, \xi^2, x^2 P^2, \mu^2) + \mathcal{O}(\xi^2 \Lambda_{\mathrm{QCD}}^2)$$

*Calculated in LQCD*

Parton Distribution function

*Calculated in perturbation theory ("process dependent")*

$$\mathcal{O}(\xi) = \bar{\psi}(0)\Gamma W(0, 0+\xi)\psi(\xi)$$

Encompasses qPDF/pPDF

Gauge-Invariant Currents

$$\begin{aligned} \mathcal{O}_S(\xi) &= \xi^4 Z_S^2 [\bar{\psi}_q \psi_q](\xi)[\bar{\psi}_q \psi](0) \\ \mathcal{O}_{V'}(\xi) &= \xi^2 Z_{V'}^2 [\bar{\psi}_q \xi \cdot \gamma \psi_{q'}](\xi)[\bar{\psi}_{q'} \xi \cdot \gamma \psi](0) \end{aligned}$$

Flavor-changing

+ analogous gluon operators

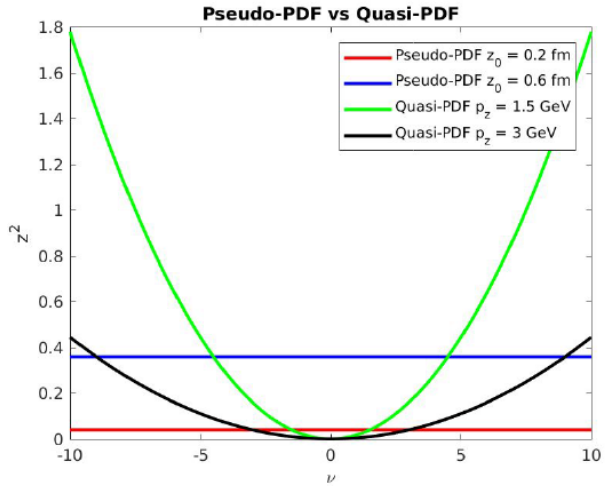
# Quasi- vs Pseudo- vs GLCS

- All integrals of Ioffe-Time Distribution Function
- Should yield same PDF after matching and systematic controls

*Quasi-PDF*

$$\mathcal{Q}(x, p_3^2) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\nu \, e^{-i\nu x} \mathcal{M}(\nu, -\nu^2/p_3^2)$$

$$\mathcal{P}(x, -z_3^2) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\nu \, e^{-i\nu x} \mathcal{M}(\nu, -z_3^2)$$




*Pseudo-PDF and GLCS*

For pPDF + GLCS, z sets short-distance scale.

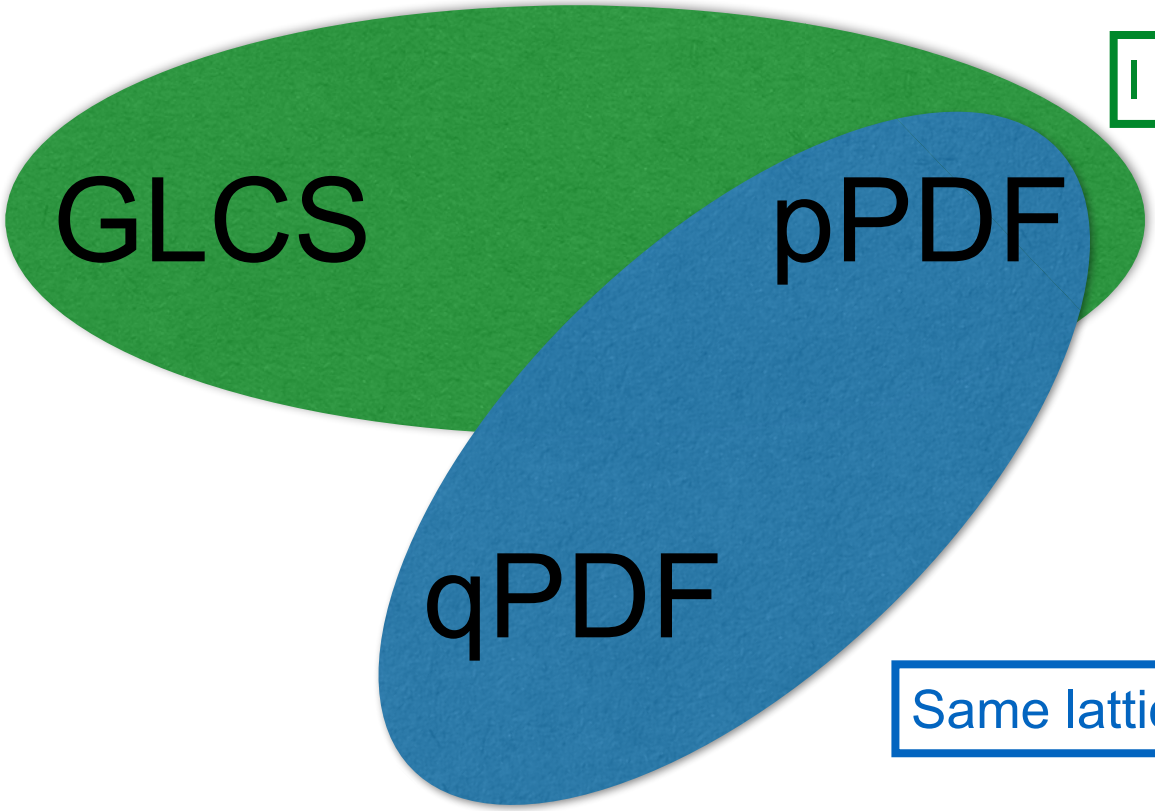
$$z << \frac{1}{\Lambda_{\mathrm{QCD}}}$$

*N.B.* All approaches require large momentum - but for pPDF and GLCS to ensure range in Ioffe time to solve *inverse problem*.

$P \longrightarrow \sqrt{s}$ *Collision energy*

$z \longrightarrow \frac{1}{Q}$ *Hard Probe*

Analogous matching to light-cone PDFs

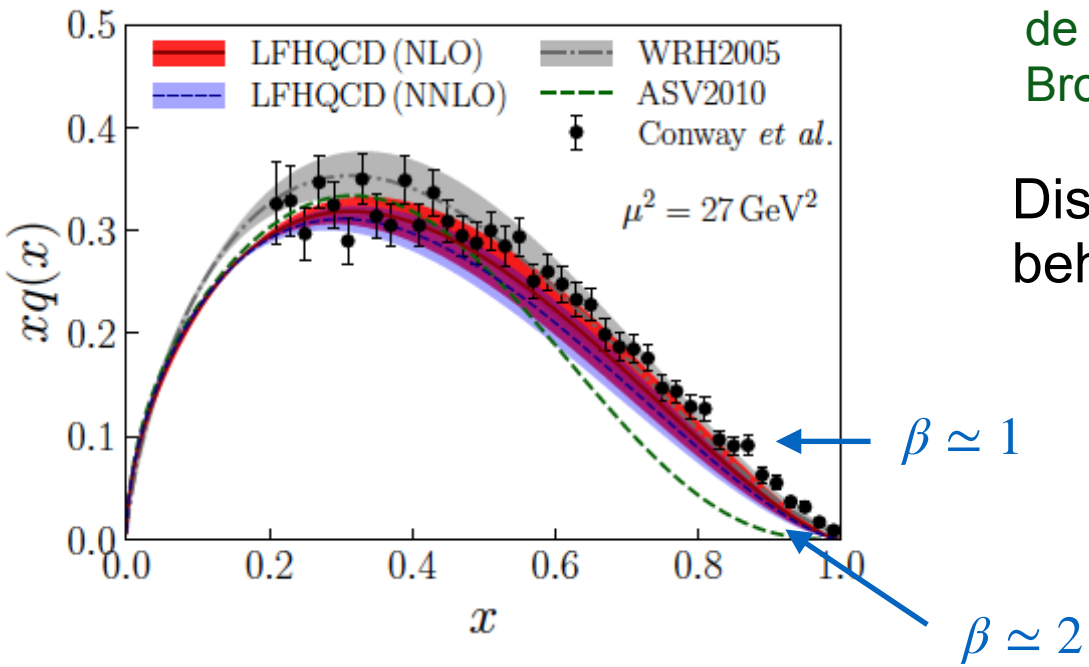



I will discuss these

Same lattice building blocks

# Pion Valence PDF

- u distribution of FNAL E615 to leading order
- C12-15-006 at Hall A will look at structure of pion
- C12-15-006A at Hall A will look at structure of Kaon
- Pion structure: DY at COMPASS++/AMBER
- No free pion target




de Teramond, liu, Sufian, Dosch, Brodsky, Deur, PRL (2018)

Discrepancy in large-x behavior of pion distribution

# Why the Pion?

- Pion less computationally demanding that nucleon?
  - *Larger signal-to-noise ratio*

$$C(t,\vec{p}) \equiv \sum_{\vec{x}} \langle 0 \mid \mathcal{O}(t,\vec{x})\mathcal{O}^{\dagger}(0,0) \mid 0 \rangle e^{-i\vec{p}\cdot\vec{x}} \rightarrow e^{-E(\vec{p})t}$$

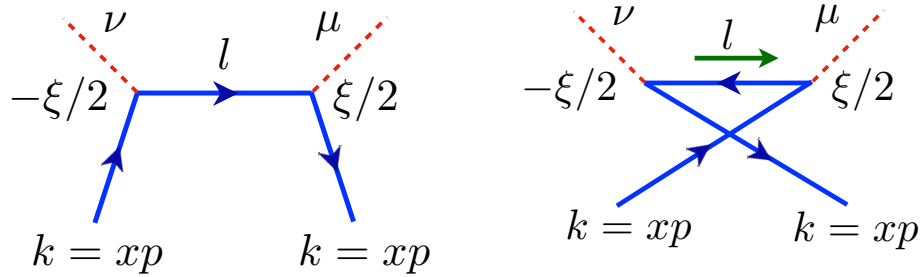
$$C_{\sqrt{\sigma^2}}(t,\vec{p}) \rightarrow \begin{cases} e^{-m_\pi t} & \text{Mesons} \\ e^{-(3m_\pi/2)t} & \text{Baryons} \end{cases}$$

- Important constraint on systematic uncertainty is understanding operator renormalization
  - *Operator renormalization "independent" of external states*
- Several different calculations using the different approaches
  - *Lattice cross-section approach straightforward for mesons, challenging for baryons*

# Good Lattice Cross Section

$$\sigma_{ij}^{\mu\nu}(\xi,p) = \langle \pi(p) \mid \mathcal{O}_{ij}^{\mu\nu}(\xi) \mid \pi(p) \rangle$$
$$= \xi^4 \langle \pi(p) \mid \mathcal{J}_i^{\mu}(\xi/2)\mathcal{J}^{\nu} + j(-\xi/2) \mid \pi(p) \rangle$$

Calculate *K* at tree-level between quark states




Process, i.e. current, dependent

$$\frac{1}{2}[\sigma_{V,A}^{\mu\nu}(\xi,p) + \sigma_{A,V}^{\mu\nu}(\xi,p)]$$

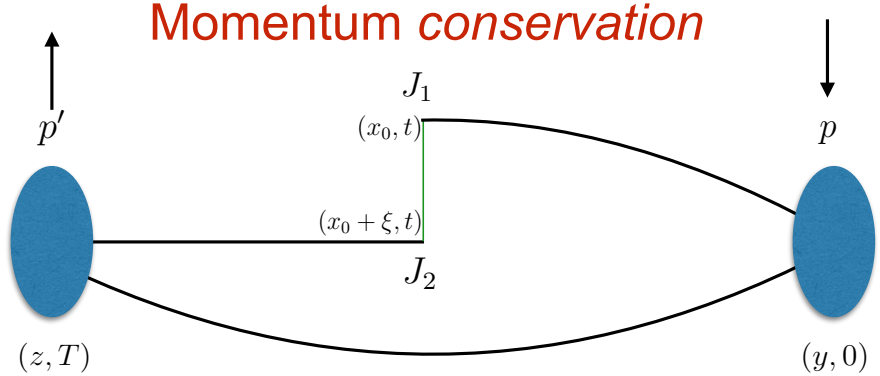
$$\equiv \epsilon^{\mu\nu\alpha\beta}\xi_\alpha p_\beta T_1(\nu,\xi^2) + (p^\mu\xi^\nu - \xi^\mu p^\nu)T_2(\nu,\xi^2)$$

Sequential-Source Approach

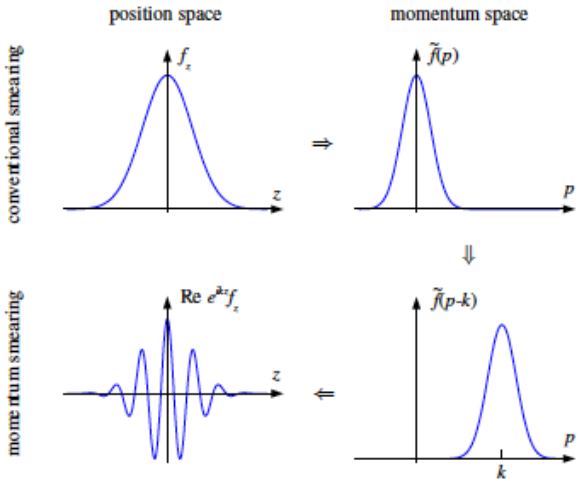
Momentum *conservation*

Momentum projection

Momentum projection

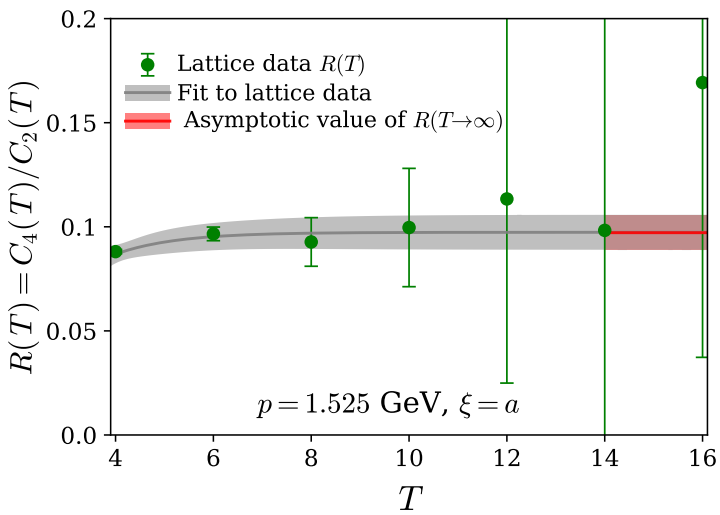


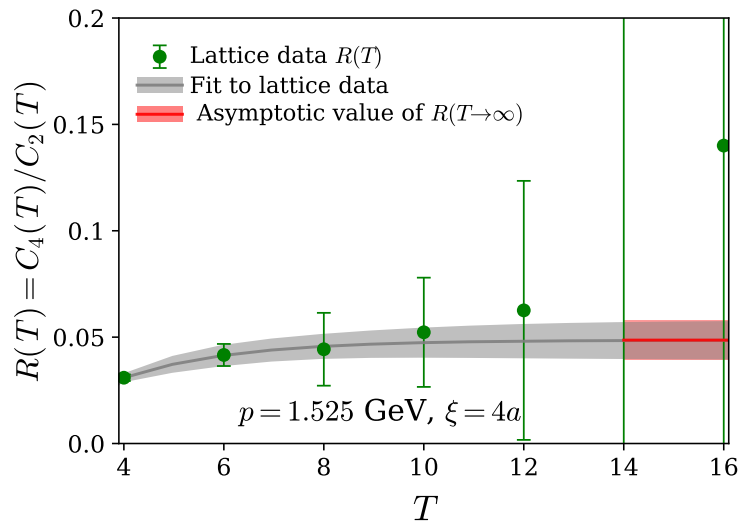
# Challenges of Higher Momenta




**Boosted interpolating operators**

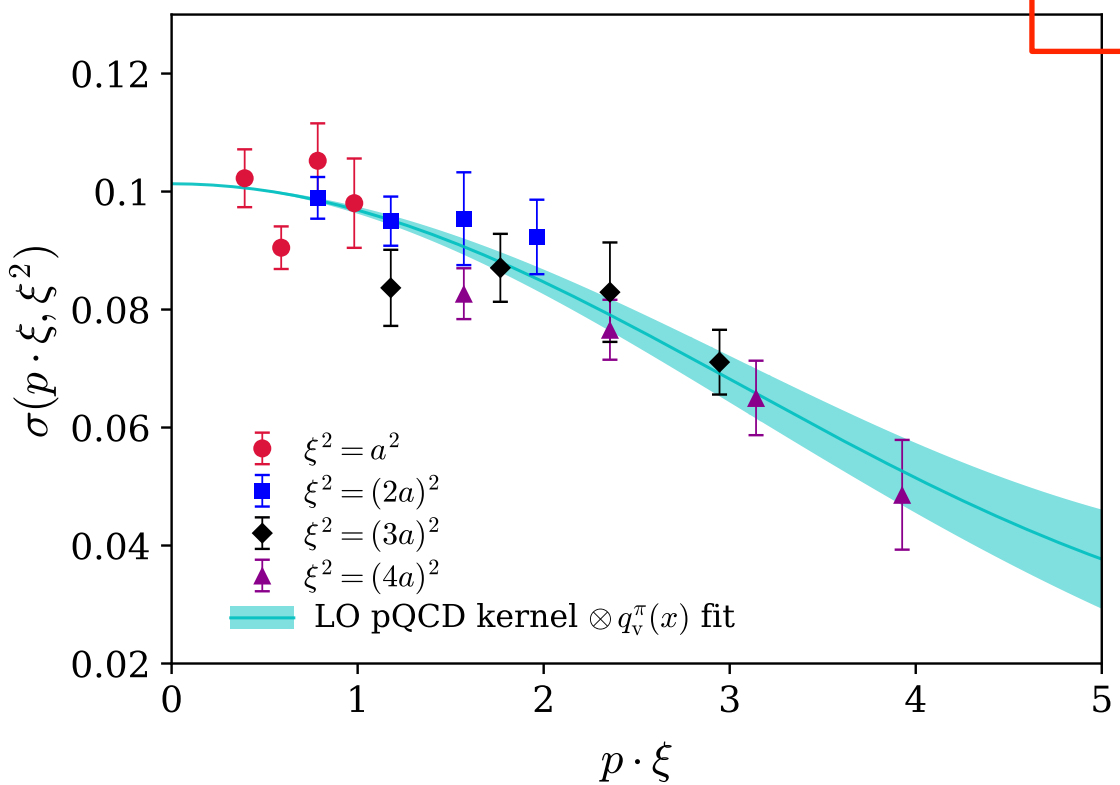
**Bali *et al.*, Phys. Rev. D 93, 094515 (2016)**



# Good Lattice Cross Section

2 + 1 Clover-fermion action
- Lattice spacing a ~ 0.127 fm
- Pion mass 415 MeV
- $32^3$ x 96 lattice, 490 Configs



# Inverse problem: extract PDF

"Inverse Problem" - ill-posed inverse Fourier transform.

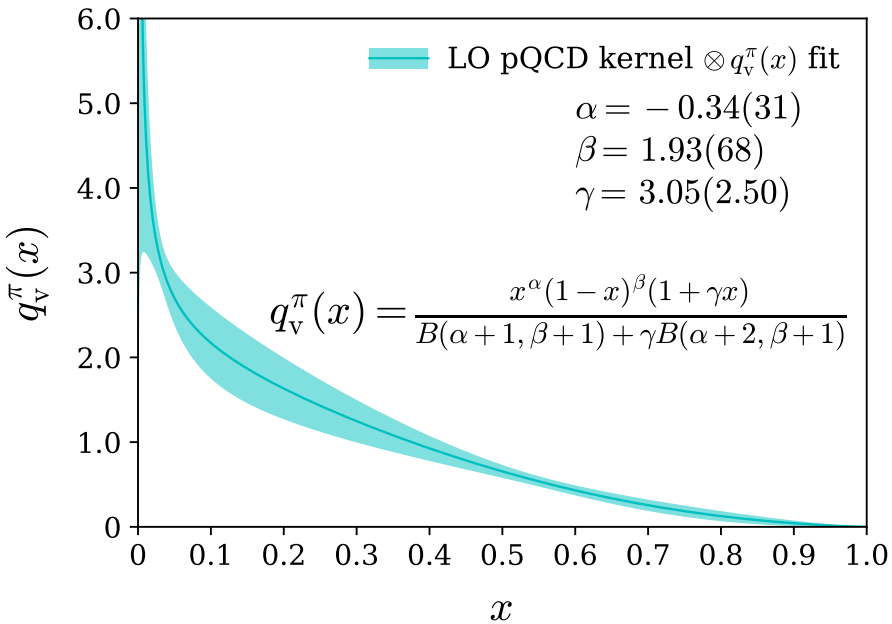
$$\sigma_n(\nu,\xi^2,P^2) = \sum_a \int_{-1}^{1} \frac{dx}{x} f_a(x,\mu^2) K_n^a(x\nu,\xi^2,x^2P^2,\mu^2) + \mathcal{O}(\xi^2\Lambda_{\rm QCD}^2)$$

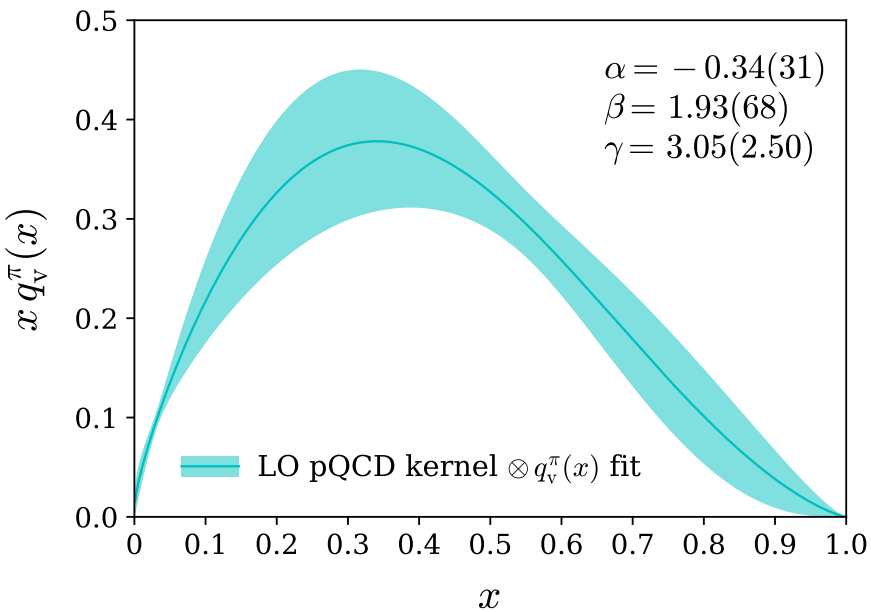
Calculate on Lattice

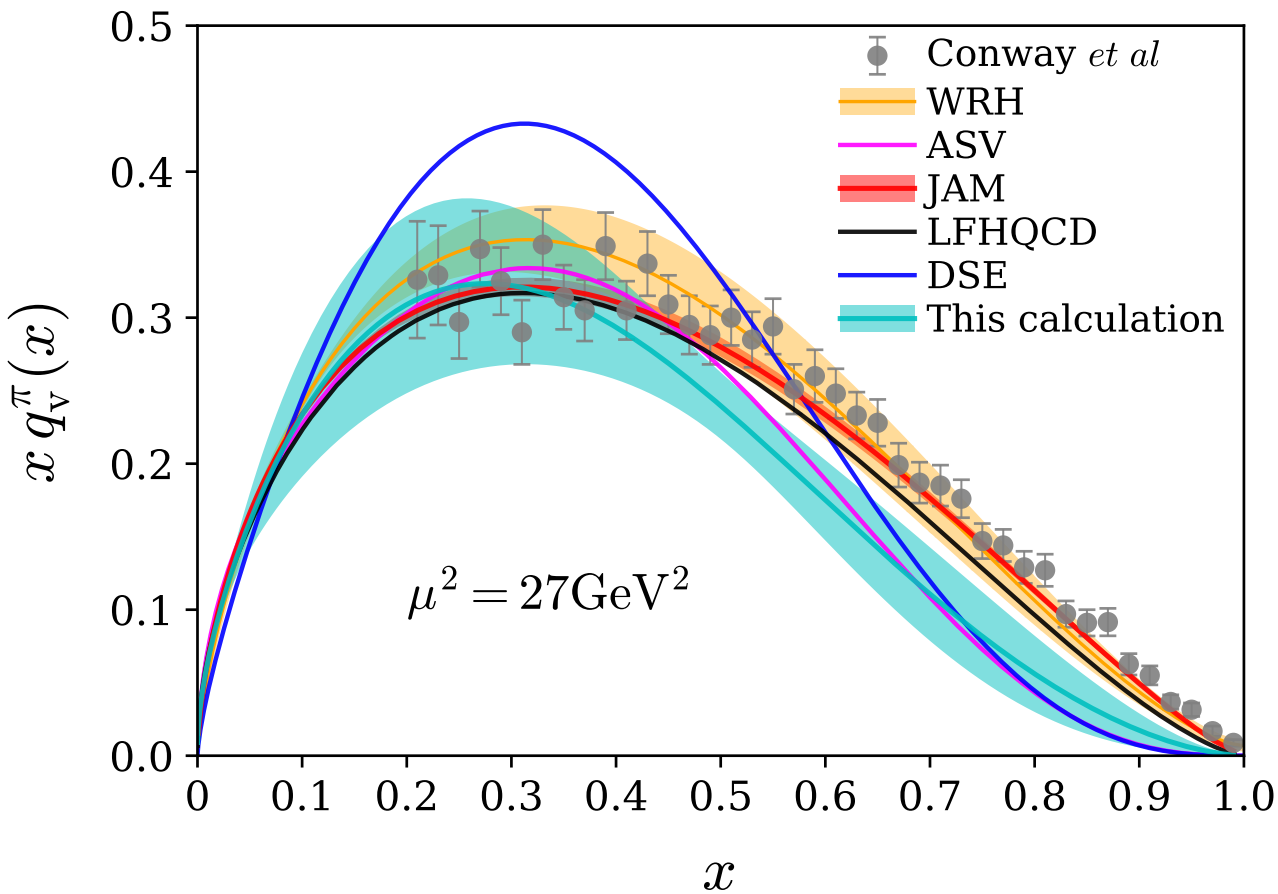
Calculate in PQCD

Extract PDF?

Similar challenge to global fitting community!






LO Calculation: “guess” scale

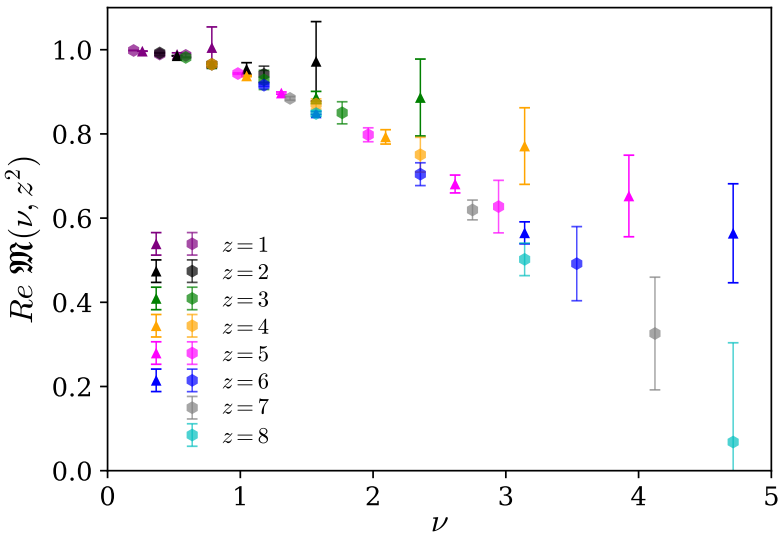
Systematic study at final lattice spacing NLO kernel: coming soon

# Pseudo-PDF Approach

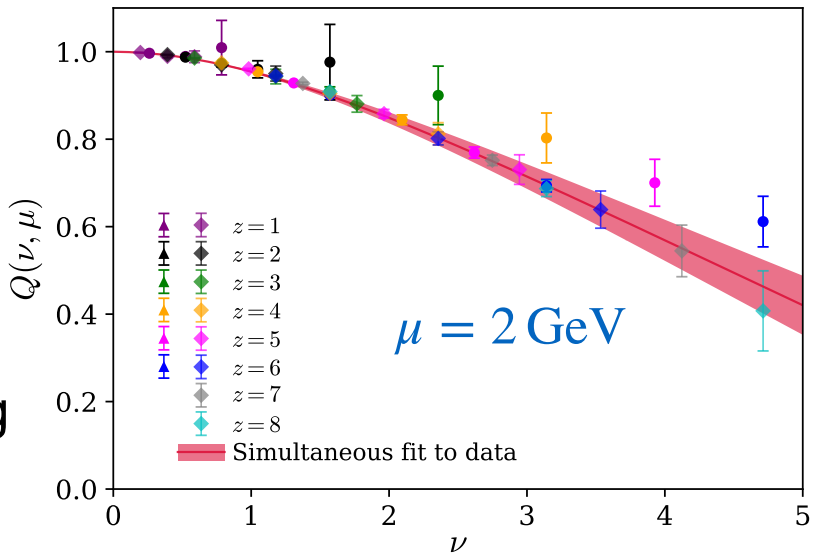
| ID | $a$ (fm) | $m_\pi$ (MeV) | $\beta$ | $am_l$ | $am_s$ | $L^3 \times T$ | $N_{cfg}$ |
|---|---|---|---|---|---|---|---|
| a127m415 | 0.127(2) | 415(23) | 6.1 | -0.280 | -0.245 | $24^3 \times 64$ | 2147 |
| a127m415$L$ | 0.127(2) | 415(23) | 6.1 | -0.280 | -0.245 | $32^3 \times 96$ | 2560 |

See Savvas Zeiferopoulos, Thursday PM

Same ensemble as GLCS

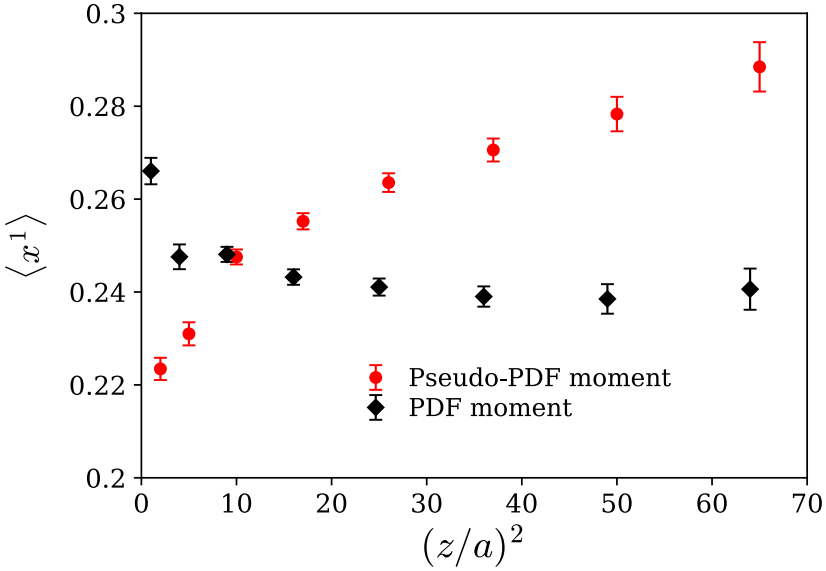


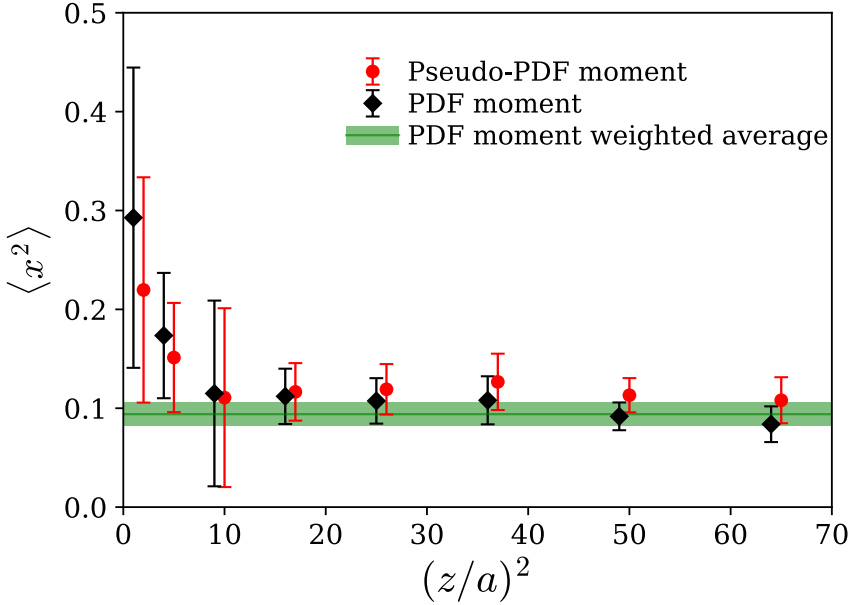

After Matching



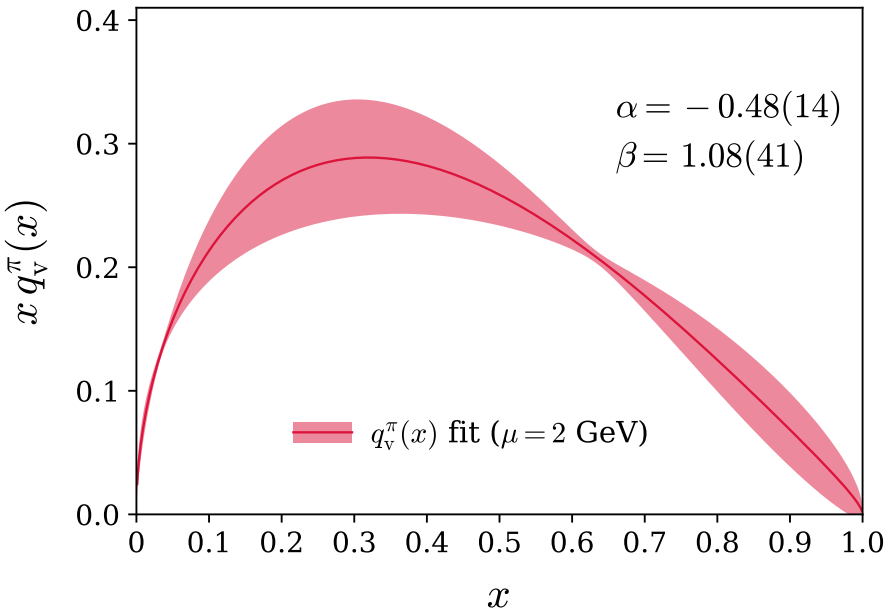

B.Joó, J.Karpie, K.Orginos, A.Radyushkin, D.Richards, R.Sufian, S.Zafeiropoulos, arXiv:1909.08517

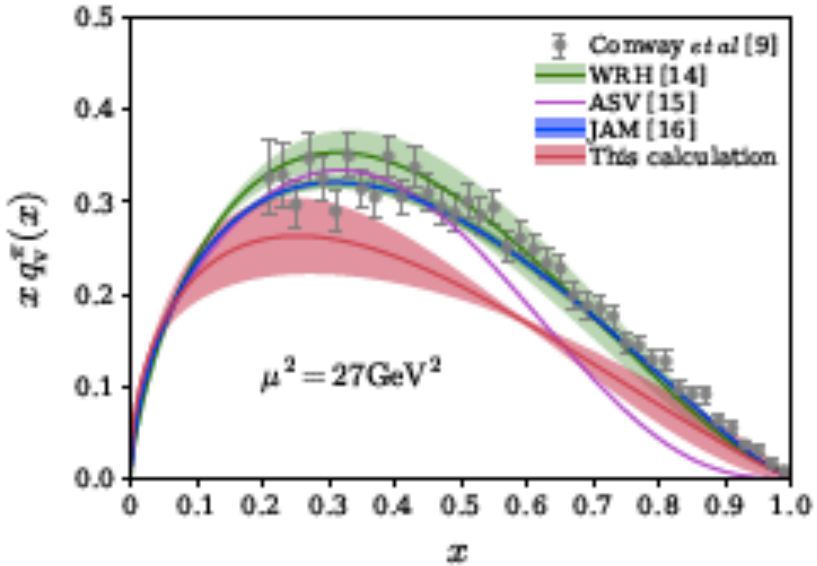
# Pseudo-PDF Moments of Pion PDF



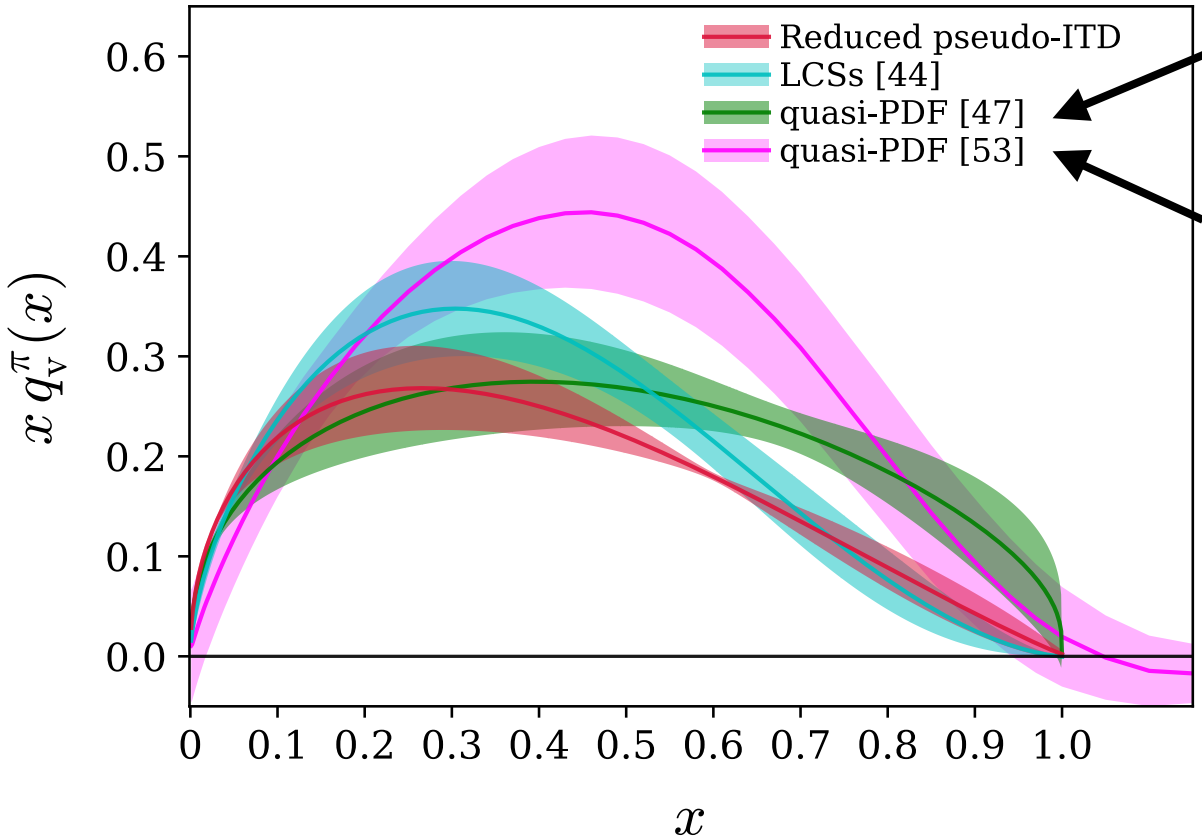



# Pion pPDF - II


$\alpha = -0.48(14)$
$\beta = 1.08(41)$
$q_v^\pi(x)$ fit ($\mu = 2$ GeV)
$x\, q_v^\pi(x)$
$x$


Conway et al [9]
WRH [14]
ASV [15]
JAM [16]
This calculation
$\mu^2 = 27\text{GeV}^2$
$x\, q_v^\pi(x)$
$x$

# Pion pPDF - III




T.Izubuchi *et al.*, Phys. Rev. 100, 034516

J-H Zhang *et al.*, Phys. Rev. D **100**, 034505

# Nucleon pseudo-PDF

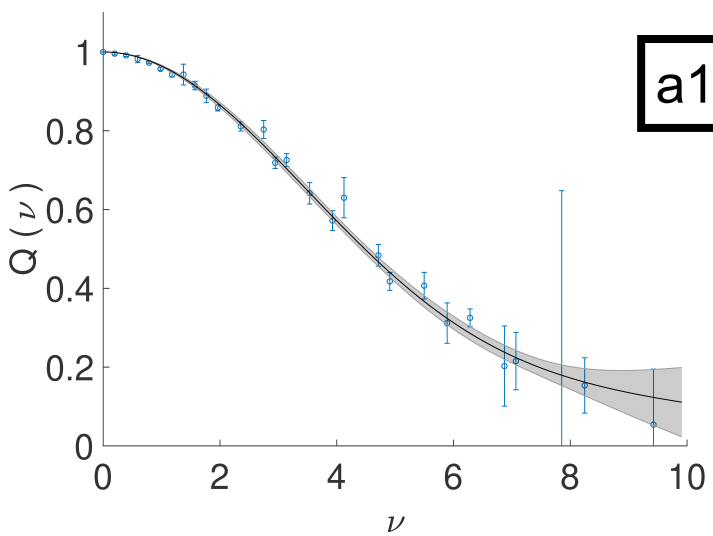
See Savvas Zeiferopoulos, Thursday PM

Ground-breaking quenched calculation: K. Orginos et al., PRD96 (2017), 094503

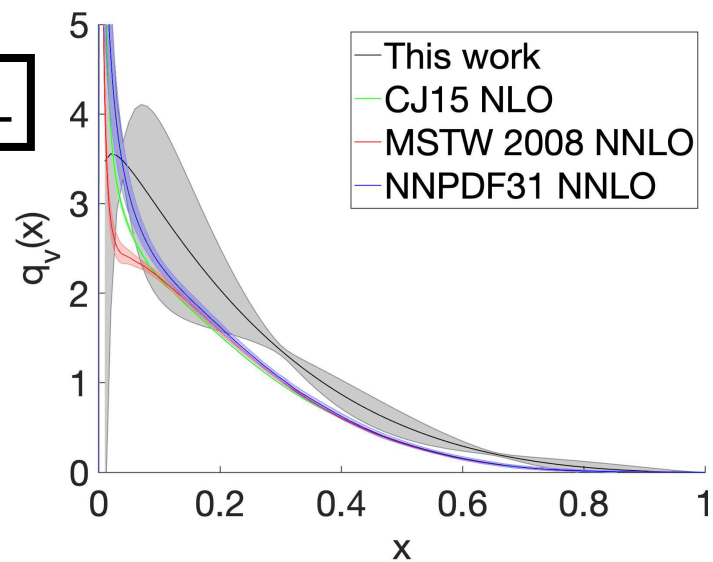
B.Joo et al., arXiv:1908.09771

| ID | $a$(fm) | $M_\pi$(MeV) | $\beta$ | $c_{SW}$ | $am_l$ | $am_s$ | $L^3 \times T$ | $N_{cfg}$ |
|---|---|---|---|---|---|---|---|---|
| $a127m415$ | 0.127(2) | 415(23) | 6.1 | 1.24930971 | -0.2800 | -0.2450 | $24^3 \times 64$ | 2147 |
| $a127m415L$ | 0.127(2) | 415(23) | 6.1 | 1.24930971 | -0.2800 | -0.2450 | $32^3 \times 96$ | 2560 |
| $a094m390$ | 0.094(1) | 390(71) | 6.3 | 1.20536588 | -0.2350 | -0.2050 | $32^3 \times 64$ | 417 |

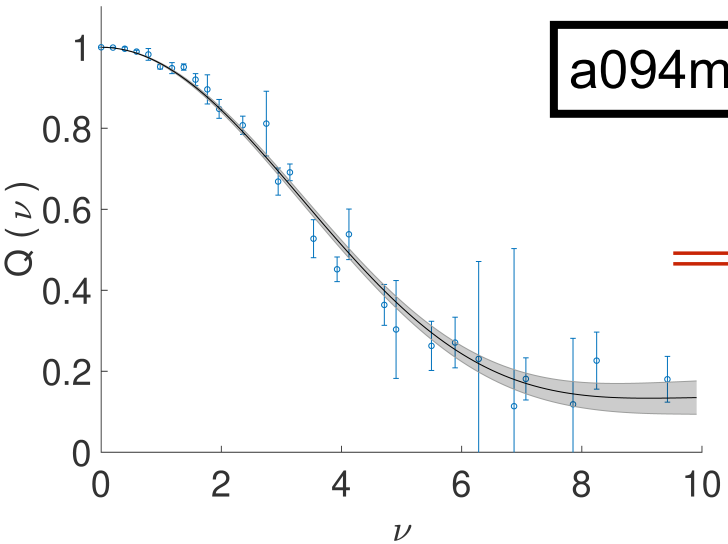
Finer lattice spacing

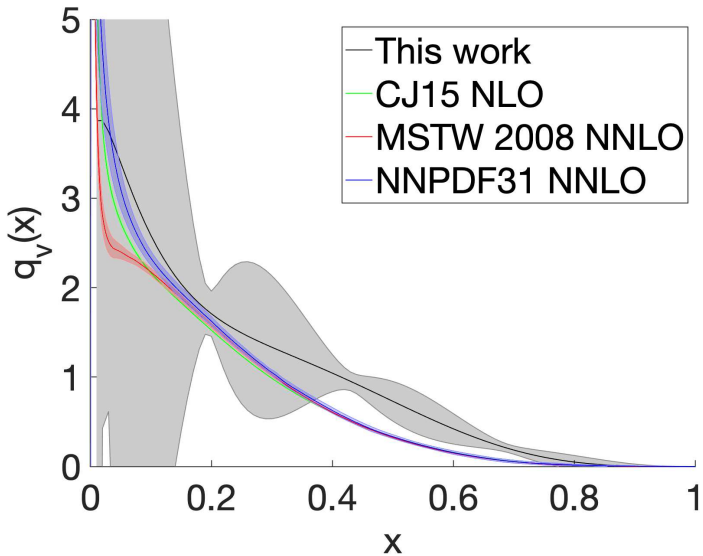


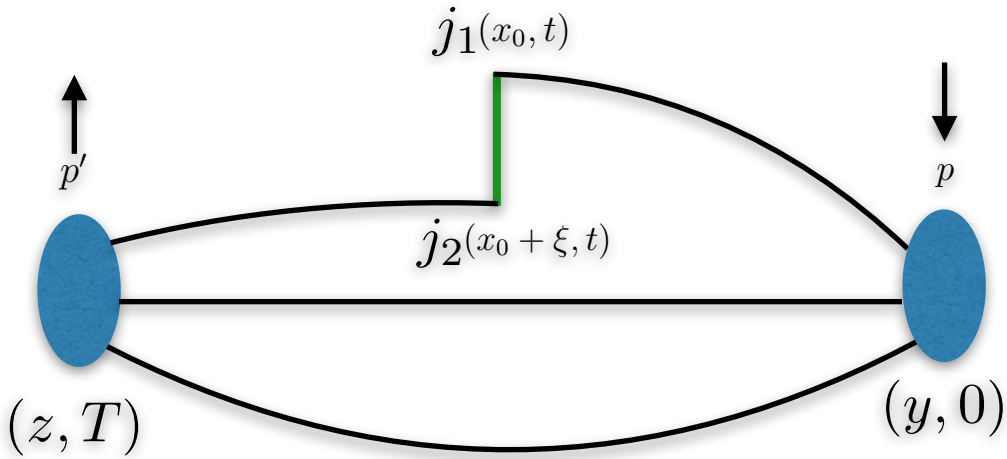

a127m415L



# Nucleon GLCS?




Does not admit sequential source method!

# Summary

- Revolution in the study of x-dependent measures of hadron structure
  - Impact global fitting community? Unclear for valence PDFs BUT
  - First-Principles calculation
- Pseudo-PDF/GLCS approach has a well-defined short-distance scale: factorize short-distance physics from perturbative scale.
- Solution of inverse problem: common to all attempts to extract PDFs. Appeal to global fitting community
- Systematic study of pion PDF using GLCS approach is in preparation; NLO perturbative kernel. Q. Ma
- To control systematics
  - fine lattices - to ensure in perturbative regime
  - large momenta - to provide range in Ioffe time

Extension to 3D imaging through GPDs and TMDs, see Michael Engelhardt - *opportunity to predict and constrain experiment*