Nuclear Femtography - what Lattice QCD can do for you.

David Richards
Jefferson Lab
For Hadstruc Collaboration

Femtography with Hard Exclusive Reactions, August 2023
A New Opportunity in Hadron Structure

Future Electron-Ion Collider

Lattice QCD

JLab@12GeV

Center for Nuclear Femtography

Future Electron-Ion Collider

3D Image of nucleon and nuclei at the femtoscale

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Graduate students, and now post-docs + faculty.
Outline

• Lattice QCD
• Hadron Structure on Euclidean Lattice
• Short-distance factorization and pseudo-PDFs
• Understanding systematic effects
  – Distillation + momentum smearing to reach high momenta
• Precision calculations of isovector PDFs
• Isoscalar structure of the nucleon - *gluon distribution*
• Lattice QCD + Expt → *global analysis of pion*
• Onto 3D Imaging…..
• Summary
Lattice QCD

- Continuum **Euclidean** space time replaced by four-dimensional lattice, or grid, of “spacing” a
- Gauge fields are represented at SU(3) matrices on the links of the lattice - work with the elements rather than algebra

\[ U_\mu(n) = e^{iaT^a A_\mu^a(n)} \]

Quarks \( \psi, \bar{\psi} \) are **Grassmann Variables**, associated with the sites of the lattice

Work in a finite 4D space-time volume
- Volume \( V \) sufficiently big to contain, e.g. proton
- Spacing a sufficiently fine to resolve its structure

\[ V \approx (6 \text{ fm})^4 \]
\[ a \leq 0.1 \text{fm} \]
Rich Menu of calculations….

Isovector Sach’s Form Factor
D. Djukanovic, Lattice 2022

Momentum and spin fractions of nucleon

Axial-vector form factors - neutrino program

Each characterized by matrix element of local operator → calculable on Euclidean lattice.

PDFs, GPDs, TMDs?
Parton Distribution Functions (PDFs)

Describe the *longitudinal momentum distribution* of the partons (quarks and gluons) within the pion
Hadron Structure: No-go Theorem?

• First Challenge:
  – Euclidean lattice precludes calculation of light-cone/time-separated correlation functions

\[
q(x, \mu) = \int \frac{d\xi^-}{4\pi} e^{-ix\xi^- P^+} \left\langle P \mid \bar{\psi}(\xi^-) \gamma^+ e^{-ig \int_{\xi^-}^{0} d\eta^- A^+ (\eta^-)} \psi(0) \mid P \right\rangle
\]

So…. …Use Operator-Product-Expansion to formulate in terms of Mellin Moments with respect to Bjorken x.

\[
\left\langle P \mid \bar{\psi} \gamma_{\mu_1} (\gamma_5) D_{\mu_2} \ldots D_{\mu_n} \psi \mid P \right\rangle \rightarrow P_{\mu_1} \ldots P_{\mu_n} a^{(n)}
\]

• Second Challenge:
  – Discretised lattice: power-divergent mixing for higher moments

Moment Methods


PDFs from Euclidean Lattice

Large-Momentum Effective Theory (LaMET)

"Equal time" correlator


\[ q(x, \mu^2, P^z) = \int \frac{dz}{4\pi} e^{izk^z} \langle P \mid \bar{\psi}(z)\gamma^z e^{-ig \int_0^z dz' A^z(z')} \psi(0) \mid P \rangle \]

\[ + \mathcal{O}((\Lambda^2/(P^z)^2), M^2/(P^z)^2)) \]

\[ q(x, \mu^2, P^z) = \int_x^1 \frac{dy}{y} Z \left( \frac{x}{y}, \frac{\mu}{P^z} \right) q(y, \mu^2) + \mathcal{O}(\Lambda^2/(P^z)^2, M^2/(P^z)^2) \]

"quasi-PDF Approach"
PDFs, GPDs and TMDs

1. GLCS
2. pPDF
3. qPDF

All approaches should give same after:
- Finite volume
- Discretization
- Uncertainties
- Infinite momentum

Light cone reduces to a point

Characterized by short-distance factorization

Same lattice building blocks

Ma and Qiu, Phys. Rev. Lett. 120 022003


Pseudo-PDFs

Lattice “building blocks” that of quasi-PDF approach.

- Pseudo-PDF (pPDF) recognizing generalization of PDFs in terms of Ioffe Time. \( \nu = p \cdot z \)

\[ M^\alpha(p, z) = \langle p \mid \bar{\psi} \gamma^\alpha U(z; 0) \psi(0) \mid p \rangle \]

\[ p = (p^+, m^2/2p^+, 0_T) \]

\[ z = (0, z_-, 0_T) \]

\[ M^\alpha(z, p) = 2p^\alpha \mathcal{M}(\nu, z^2) + 2z^\alpha \mathcal{N}(\nu, z^2) \]

Ioffe-time pseudo-Distribution (pseudo-ITD) generalization to space-like \( z \)

Pseudo-PDFs

To deal with UV divergences, introduce reduced distribution

\[ \mathcal{M}(\nu, z^2) = \int_0^1 du \ K(u, z^2 \mu^2, \alpha_s) Q(u \nu, \mu^2) \]

Computed on lattice

Perturbatively calculable

Ioffe-time Distribution

K. Orginos et al., PRD96 (2017), 094503

Inverse problem

ITD ↔ PDF

Need data for all \( \nu \), or additional physics input

Match data at different \( z \)
Ioffe-Time Distribution to PDF


To extract PDF requires additional information - use a phenomenologically motivated parametrization

\[ f(x) = x^a (1 - x)^b P(x) \quad \text{MSTW, CJ} \]
\[ P(x) = \frac{1 + c\sqrt{x + dx}}{B(a + a, b + 1) + cB(a + 1.5, b + 1) + dB(a + 2, b + 1)} \]

\[ Q(v,\mu^2) \]

B.Joo et al., PRL 125 (2020) 23, 232003
Distillation and Hadron Structure

To control systematic uncertainties, need precise computations over a wide range of momentum.

- Use a low-mode projector to capture states of interest “distillation” M. Peardon et al (Hadspec), Phys Rev D 80 (2009) 054506
- Enables momentum projection at each temporal point.

Momentum projection

Variational basis

G. Bali et al, Phys Rev D 93 (2016) 9, 094515

C. Egerer et al (Hadstruc), Phys Rev D 103, 034502 (2021)
Isovector PDF using Distillation

C. Egerer et al. (hadstruc), JHEP 11 (2021) 148
Expand the x-dependence in terms of (shifted) Jacobi Polynomials

\[ \sigma_n^{(\alpha, \beta)}(\nu, z^2 \mu^2) = \text{Re} \int_0^1 dx \ K_v(x\nu, z^2 \mu^2) x^\alpha (1 - x)^\beta \Omega_n^{(\alpha, \beta)}(x) \]

**Matching kernel**

**Discretization**

**Higher twist**


**m_π \approx 358 \text{ MeV}**
DGLAP Evolution

• Data demonstrate “precious scaling”…
Transversity + Helicity

\[ 2 P^+ S^{\rho \perp} \mathcal{I}(P^+ z^-, \mu) = \langle P, S^{\rho \perp} | \bar{\psi}(z^-) \gamma^+ \gamma^\rho \gamma_5 W_+(z^-, 0) \psi(0) | P, S^{\rho \perp} \rangle \]

\[ h(x, \mu) = \int_{-\infty}^{\infty} \frac{d\nu}{2\pi} e^{-i x \nu} \mathcal{I}(\nu, \mu) \]

In contrast to unpolarized PDF, there is no conserved current - so express in terms of the (renormalized) tensor charge.
Transversity Distribution

$\mu^2 = 2 \text{ GeV}^2$

- JAM18 (SIDIS + lattice $g_T$)
- JAM20 (global fit)
- This work

Isospin symmetric
Helicity Distribution

Valence quark helicity distribution, together with contamination terms

CP-odd helicity distribution, together with contamination terms

Small NS anti-quark helicity
“Understanding the Glue That Binds Us All: The Next QCD Frontier in Nuclear Physics”
Gluon Contribution to unpolarized PDF

T. Khan et al. (Hadstruc), Phys. Rev. D 104 (2021) 9, 094516

\( M_{\mu\alpha;\lambda\beta}(z, p) \equiv \langle p | G_{\mu\alpha}(z) W[z, 0] G_{\lambda\beta}(0) | p \rangle \)

\[ O_g(z) = G_{ji}(z) U(z, 0) G_{ij}(0) U(0, z) - G_{ti}(z) U(z, 0) G_{it}(0) U(0, z). \]

Two-point functions as in isovector case

Reduced matrix element:

\[ \mathcal{M}(\nu, z^2) = \left( \frac{\mathcal{M}(\nu, z^2)}{\mathcal{M}(\nu, 0)|_{z=0}} \right) \left/ \left( \frac{\mathcal{M}(0, z^2)|_{p=0}}{\mathcal{M}(0, 0)|_{p=0, z=0}} \right) \right. \]

Flavor-singlet quantities are subject to severe signal-to-noise problems compared with isovector measures:

- Use distillation and many more measurements per configuration - sampling of lattice
- Use of summed Generalized Eigenvalue Problem (sGEVP) - better control over excited state contributions
- Use of Gradient Flow - smoothing of short-distance fluctuations
ITD to PDF


\[ M(\nu, z^2) = \frac{I_g(\nu, \mu^2)}{I_g(0, \mu^2)} - \frac{\alpha_s N_c}{2\pi} \int_0^1 du \frac{I_g(\nu u, \mu^2)}{I_g(0, \mu^2)} \left\{ \ln \left( \frac{z^2 \mu^2 e^{2\gamma_E}}{4} \right) B_{gg}(u) + 4 \left[ \frac{u + \ln(\bar{u})}{\bar{u}} \right] + \frac{2}{3} \left[ 1 - u^3 \right] \right\} \]

N.B neglecting quark-gluon mixing

Implementation for obtaining the PDFs follows that of the isovector distribution

- Expand in Jacobi Polynomials

\[ x^\alpha (1 - x)^\beta + J_1^{\alpha, \beta} + a/|z| \]
Require normalization of $xg(x)$

$\langle x \rangle_{g}^{\overline{\text{MS}}} (\mu = 2 \text{GeV}) = 0.427(92)$

Gluon Helicity Distribution

- Crucial questions in global analysis - do we need to apply positivity constraint:
  \[ | \Delta g(x) | \leq g(x) \forall x \]

Relaxing constraint leads to new “replicas” in global analysis:

Zhou, Sato and Melnitchouk, Phys. Rev. D 105, 074022 (2022)
FIG. 9. The lattice data points represent the reduced \( \text{Io} \)e-time pseudo-distribution, \( f_M(\chi, z^2) \) in the zero flow-time limit obtained through the subtraction method using \( p = 0 \) matrix elements. The lattice data points and the fit bands are normalized using the gluon momentum fraction, \( \chi \) from [17]. Left panel: the red and cyan bands represent the target mass corrected reduced \( \text{Io} \)e-time pseudo-distribution using the fit of moments in Sec. IV A.

Right panel: the blue band is a fit to the subtracted pseudo-ITD using the functional form in Eq. (20) with \( a_0, a_1, b_1 \) as fit parameters and \( b_0 = 0 \) fixed by construction.

FIG. 10. A comparison between the lattice reduced \( \text{Io} \)e-time pseudo-distribution \( f_M(\chi, z^2) \) in the zero flow-time limit obtained through the subtraction method using the \( p = 0 \) matrix elements, and the gluon helicity ITD constructed from global fits of PDFs. The lattice data points are the same as in Fig. 9, plotted on a smaller vertical scale for better comparison with the phenomenological ITD bands. In the left plot, the red band denotes the ITD constructed from the gluon helicity distribution by the NNPDF collaboration. The green band labeled by \( e^I_p(+) \) and the cyan band labeled by \( e^I_p(+/-) \) represent the gluon helicity ITD determined by the JAM collaboration with and without the positivity constraint on the gluon helicity PDF, respectively.

On the right plot, the gluon helicity ITDs for positive and negative PDFs are compared with the lattice data. The green band labeled by \( e^I_p(+) \) and the maroon band labeled by \( e^I_p(-) \) represent the gluon helicity ITD determined by the JAM collaboration associated with the positive and negative gluon helicity PDF solutions, respectively.

Polarization in the nucleon cannot be properly constrained. In other words, the ITD extracted from the JAM global fit (labeled by JAM \( e^I_p(+/+) \) in Fig. 10) may have a similar or even larger magnitude of uncertainty than our lattice QCD calculation. We show a comparison of the polarized gluon ITDs obtained from global fits and our lattice calculation in Fig. 10. Most importantly, Fig. 10 shows that the ITD data in the \( \chi \) region is primarily controlled by whether the gluon polarization in the nucleon is positive or negative, according to the JAM analysis.

The positivity constraint on the gluon distributions, namely helicity-aligned and helicity-antialigned both being non-negative, in the analysis of experimental data in [11] leads to a substantial reduction of the variance of \( x_g \) in the large-\( x \) region, as seen in Fig. 6 of [11]. Specifically, the PDFs without the positivity assumption were organized into a band of solutions with a negative PDF and a band of solutions with a positive PDF. We compare the ITDs resulting from the two bands with positive and negative \( x_g \) to our results in the right panel of Fig. 10. The current matrix elements, albeit with an unphysical pion mass and finite lattice spacing, are inconsistent within statistical LQCD Calculation of gluon helicity distribution compared with global analyses.

LQCD can inform in advance of EIC!
Lattice QCD + Experiment: Greater than their parts
Pion PDF

Pion PDF has high level of uncertainty - no free-pion targets

"Good Lattice Cross Sections"
Ma and Qiu, Phys. Rev. Lett. 120 022003
\[ O_S(\xi) = \xi^4 Z_S^2 [\bar{\psi}_q \psi_q](\xi)[\bar{\psi}_q \psi](0) \]
\[ O_{V'}(\xi) = \xi^2 Z_{V'}^2 [\bar{\psi}_q \xi \cdot \gamma \psi_q'](\xi)[\bar{\psi}_q' \xi \cdot \gamma \psi](0) \]

\[ q_\pi^\mp(x) = \frac{x^\alpha(1 - x)^\beta(1 + \gamma x)}{B(\alpha + 1, \beta + 1) + \gamma B(\alpha + 2, \beta + 1)} \]

T.Izubuchi et al., Phys. Rev. D 100, 034516
J-H Zhang et al., Phys. Rev. D 100, 034505

Sufian et al., Phys. Rev. D102, 05408 (2020)
Can we use LQCD + expt in global analysis: what is the impact?

\[
\frac{d\sigma}{dx_F d\sqrt{\tau}} = \frac{4\pi\alpha^2}{9 Q^2 S} \sum_{ij} \int_{x_0^\pi}^1 dx_{\pi} \int_{x_0^A}^1 dx_{A} f_i^{\pi}(x_\pi, \mu) f_j^{A}(x_A, \mu) C_{ij}^{DY}(x_\pi, x_0^\pi, x_A, x_0^A, Q, \mu),
\]

Measured Cross Section

\[
f(x, \mu_0^2) = \frac{N_f x^{\alpha_f}(1 - x)^{\beta_f}(1 + \gamma_f x^2)}{B(\alpha_f + 2, \beta_f + 1) + \gamma_f B(\alpha_f + 4, \beta_f + 1)}
\]
Combined analysis for gluon helicity distribution in progress
3D Imaging + GPDs
The Structure of Hadrons

Wigner distributions

B. Musch, M. Englehardt et al

Next Frontier

\( \chi QCD \)
GPDs in pseudo-PDF approach

Thanks to Joe Karpie, Lattice 2023

- GPDs correspond to off-forward matrix elements. In pseudo-PDF framework, our starting point is the Generalized Ioffe Time Distributions

$$I_\mu(p', p, s = s - \mu^2) = \langle p' \mid \bar{q}(-z^-/2)\gamma_\mu W(-z^-/s, z^-/2)q(z^-/2) \mid p \rangle_{\mu^2}$$

Where Ioffe time \( \nu = (p + p')/2 \), \( t = (p - p')^2 \) and skewness \( \xi = q \cdot z/P \cdot z \)

Extends to generalized pseudo-ITD in manner of pseudo ITD.

\[ \downarrow \]

GPDs

Requires solution of inverse problem

Allows us to obtain 3D GITDs/GPDs at discrete values of momentum transfer and skewness, in contrast to \( x = \xi \) in DVCS.
GPDs - II

Accessible values on our “paradigm” lattice

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<th>ID</th>
<th>$a_s$ (fm)</th>
<th>$m_\pi$ (MeV)</th>
<th>$L_s^3 \times N_t$</th>
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<th>$N_{srcs}$</th>
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<td>358(3)</td>
<td>$32^3 \times 64$</td>
<td>349</td>
<td>4</td>
<td>64</td>
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</tbody>
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Introduce double distributions

$$f(x, \xi) = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \delta(x - \beta - \xi \alpha) \tilde{f}(\alpha, \beta)$$


Thanks to Joe Karpie, Lattice 2023
Summary

• Realistic calculation of light-cone distributions from LQCD now available
• Focus on understanding systematic contributions in pseudo-PDF framework
• Distillation + boosting enables both far increased reach in momentum, and improved sampling of lattice
  – *Essential in calculations of gluon contributions*
• Are able to isolate leading twist from higher-twist and discretization contamination
• *Calculation of GPDs Underway*
• Lattice QCD + Expt - global analysis
Quark-Gluon Tomography (QGT)

**Focus areas**

**Theory**
- Theoretical studies of high-momentum transfer processes using perturbative QCD
- Study the properties of GPDs using non-perturbative methods

**Lattice QCD**
- Non-perturbative calculations of Euclidean correlation functions relevant to GPDs

**Phenomenology**
- Global analysis of GPDs based on experimental data, theoretical constraints, and lattice QCD input, using modern data analysis techniques for inference and uncertainty quantification

**Support and training**
- Support 12 postdocs and 6 graduate students
- Provide summer schools and workshops
- Create three bridge positions in nuclear theory

**Thanks to Chris Monahan**
Focus areas: Lattice QCD

Expertise in:
- numerical methods in lattice gauge theories
- simulations of QCD
- non-perturbative renormalization
- numerical calculations for hadron structure

Studying:
- quark and gluon PDFs and GPDs
- gravitational form factors
- nucleon spin, momentum & angular momentum
- quark charge & renormalization
- electric dipole moments
- x-dependence of PDFs & GPDs
- neutrino-nucleus scattering cross-sections
- structure of light nuclei
What do we need?

\[ m(p, z, a) = m_{\text{cont}}(\nu, z^2) + \sum \left( \frac{a}{|z|} \right)^n P_n(\nu) + (a\Lambda_{\text{QCD}})^n R_n(\nu). \]

\[ m_{\text{cont}}(\nu, z^2) = m_{\text{lt}}(\nu, z^2) + \sum_{n=1}^\infty (z^2\Lambda_{\text{QCD}}^2)^n B_n(\nu). \]
Distillation

Low-rank approximation to (typically) Jacobi-smearing kernel

$$-\nabla^2(t)\xi^{(k)}(t) = \lambda^{(k)}(t)\xi^{(k)}(t)$$

M. Peardon et al., PRD80,054506 (2009)

$$\Box \left( \bar{x}, \bar{y}; t \right)_{ab} = \sum_{k=1}^{\infty} \xi^{(k)}_a(\bar{x}, t)\xi^{(k)\dagger}_b(\bar{y}, t)$$

Components of distillation:

$$\tau^{(l,k)}_{\alpha\beta}(t', t) = \xi^{(l)\dagger}(t') M^{-1}_{\alpha\beta}(t', t) \xi^{(k)}(t)$$  

Perambulators \(\rightarrow\) quark propagation

$$\Phi^{(i,j,k)}_{\alpha\beta\gamma}(t) = \epsilon^{abc} \left( D_1 \xi^{(i)} \right)^a \left( D_2 \xi^{(j)} \right)^b \left( D_3 \xi^{(k)} \right)^c (t) S_{\alpha\beta\gamma}$$  

Elementals \(\rightarrow\) (baryon) operators

$$C_{rs}(t) = \sum_{\bar{x}, \bar{y}} \langle 0 | \mathcal{O}_r(t, \bar{x}) \mathcal{O}_s^{\dagger}(0, \bar{y}) | 0 \rangle \equiv \text{Tr} \left[ \Phi_r(t) \otimes \tau(t, 0) \tau(t, 0) \tau(t, 0) \otimes \Phi_s(0) \right]$$  

Projection to irrep

Extension to 3pt functions straightforward