Gluon Helicity from Lattice QCD

David Richards

Jefferson Lab and Hadstruc Collaboration

HadStruc Collaboration

Robert Edwards, Colin Egerer, Joe Karpie, Jianwei Qiu, David Richards, Eloy Romero, Frank Winter

Jefferson Lab

Carl Carlson, Tanjib Khan*, Christopher Monahan,

Kostas Orginos, Raza Sufian

William and Mary

Wayne Morris*, Anatoly Radyushkin

Old Dominion University

Savvas Zafeiropoulos

Aix Marseille Univ, Marseille, France

Yan-Qing Ma

Peking University, Beijing, China

Balint Joo

ORNL

Graduate students, and now post-docs.

* PhD thesis

PHYSICAL REVIEW D **106,** 094511 (2022)

Toward the determination of the gluon helicity distribution in the nucleon from lattice quantum chromodynamics

Colin Egerer,[1] Bálint Joó,[2] Joseph Karpie,[3] Nikhil Karthik,[1,4] Tanjib Khan,[5] Christopher J. Monahan,[1,4] Wayne Morris,[1,6] Kostas Orginos,[1,4] Anatoly Radyushkin,[1,6] David G. Richards,[1] Eloy Romero,[1] Raza Sabbir Sufian,[1,4] and Savvas Zafeiropoulos[7]

Polarized Gluon Pseudodistributions at Short Distances

Ian Balitsky, Wayne Morris and Anatoly Radyushkin

Old Dominion University, Norfolk,
4600 Elkhorn Ave., Norfolk, VA 23529, USA
Thomas Jefferson National Accelerator Facility,
12000 Jeffers...

PHYSICAL REVIEW D **104,** 094516 (2021)

Unpolarized gluon distribution in the nucleon from lattice quantum chromodynamics

Tanjib Khan,[1] Raza Sabbir Sufian,[1,2] Joseph Karpie,[3] Christopher J. Monahan,[1,2] Colin Egerer,[1,2] Bálint Joó,[4] Wayne Morris,[5,2] Kostas Orginos,[1,2] Anatoly Radyushkin,[5,2] David G. Richards,[2] Eloy Romero,[2] and Savvas Zafeiropoulos[6]

Contents lists available at ScienceDirect

Physics Letters B

ELSEVIER

www.elsevier.com/locate/physletb

Gluon pseudo-distributions at short distances: Forward case

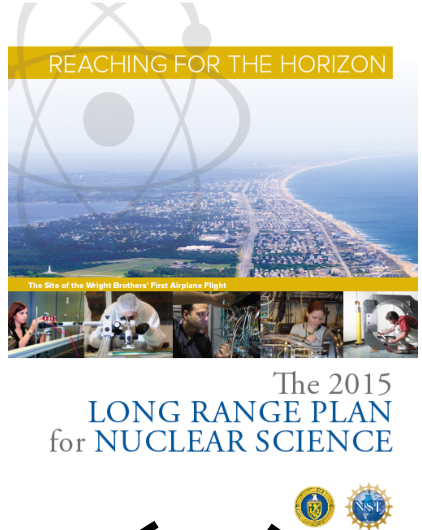
Ian Balitsky[a,b], Wayne Morris[a,b], Anatoly Radyushkin[a,b,*]

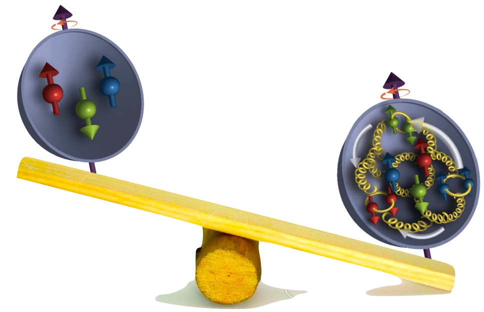
[a] Physics Department, Old Dominion University, Norfolk, VA 23529, USA
[b] Thomas Jefferson National Accelerator Facility, Newport News, VA 23606, USA

- Introduction
- PDFs on the Lattice - “need to know! - *see Monahan and Orginos*
- Unpolarized and helicity gluon PDFs
- Lattice QCD and Global Analysis
- Conclusions

Introduction

"Understanding the Glue That Binds Us All: The Next QCD Frontier in Nuclear Physics"





Ji's sum rule

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma(\mu) + L_q(\mu) + J_g(\mu)$$

Lattice: *Moments of Generalized Form Factors*

Jaffe-Manohar sum rule

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma(\mu) + \Delta G(\mu) + L_{q+g}(\mu)$$

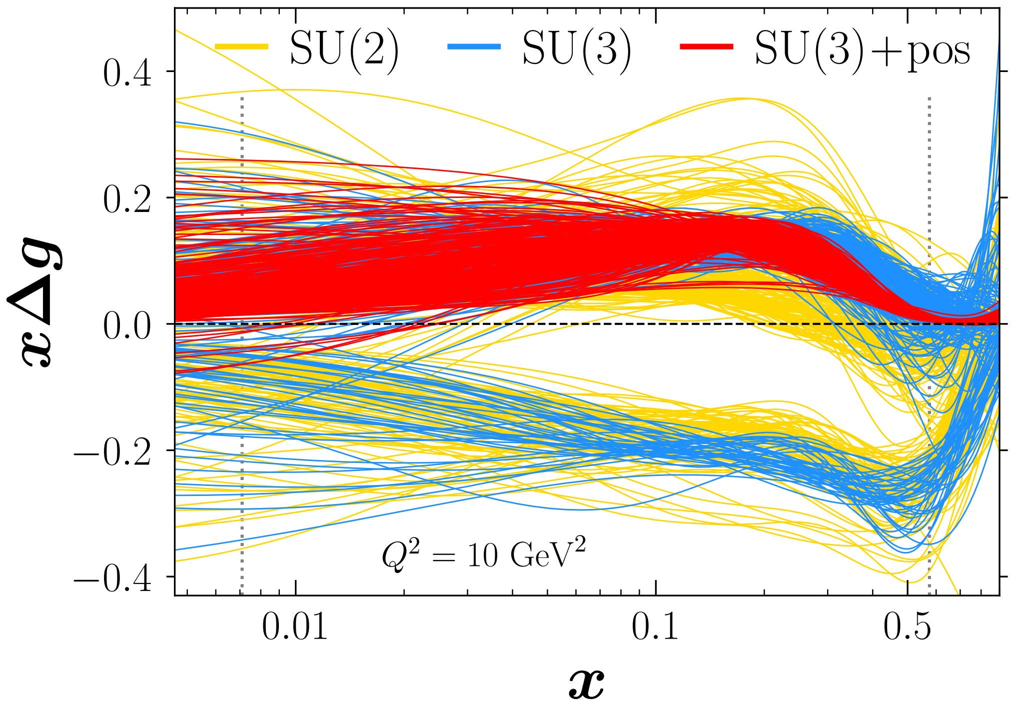
$$\Delta G(\mu) = \int_0^1 dx\, \Delta g(x,\mu)$$

Gluon Helicity Distribution

- Crucial questions in global analysis - do we need to apply positivity constraint:

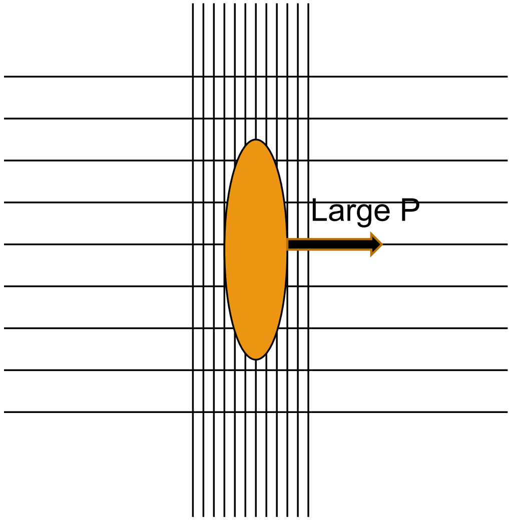
$$|\Delta g(x)| \leq g(x) \forall x$$

Relaxing constraint leads to new “replicas” in global analysis:

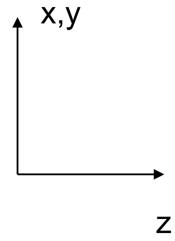


Zhou, Sato and Melnitchouk, Phys. Rev. D 105, 074022 (2022)

PDFs from Euclidean Lattice



Large-Momentum Effective Theory (LaMET)



"Equal time" correlator

X. Ji, Phys. Rev. Lett. 110, 262002 (2013).
X. Ji, J. Zhang, and Y. Zhao, Phys. Rev. Lett. 111, 112002 (2013).
J. W. Qiu and Y. Q. Ma, arXiv:1404.686.

$$q(x,\mu^2,P^z) = \int \frac{dz}{4\pi} e^{izk^z} \langle P \mid \bar{\psi}(z)\gamma^z e^{-ig\int_0^z dz'\, A^z(z')}\psi(0) \mid P > + \mathcal{O}((\Lambda^2/(P^z)^2), M^2/(P^z)^2))$$

$$q(x,\mu^2,P^z) = \int_x^1 \frac{dy}{y} Z\left(\frac{x}{y}, \frac{\mu}{P^z}\right) q(y,\mu^2) + \mathcal{O}(\Lambda^2/(P^z)^2, M^2/(P^z)^2)$$

"quasi-PDF Approach"

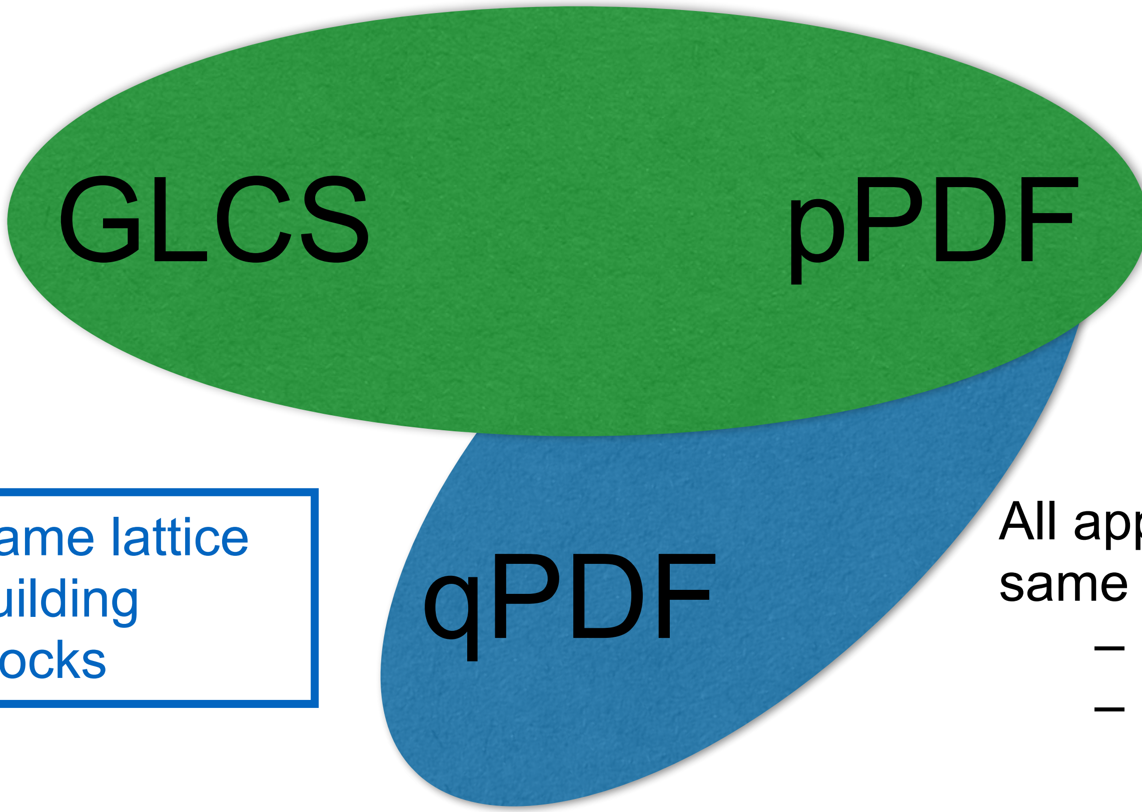
PDFs, GPDs and TMDs

Ma and Qiu, Phys. Rev. Lett. 120 022003

A.Radyushkin, Phys. Rev. D 96, 034025 (2017)

Light cone reduces to a point

Characterized by *short-distance factorization*



Same lattice building blocks

All approaches should give same after:

- Finite volume
- Discretization Uncertainties
- *Infinite momentum*

X. Ji, Phys. Rev. Lett. 110, 262002 (2013).
X. Ji, J. Zhang, and Y. Zhao, Phys. Rev. Lett. 111, 112002 (2013).
J. W. Qiu and Y. Q. Ma, arXiv:1404.686.

Pseudo-PDFs

To deal with UV divergences, introduce reduced distribution

$$\mathfrak{M} = \frac{\mathcal{M}(\nu, z^2)}{\mathcal{M}(0, z^2)} \equiv \left(\frac{\mathcal{M}(\nu, z^2)}{\mathcal{M}(\nu, 0)}\right) / \left(\frac{\mathcal{M}(0, z^2)}{\mathcal{M}(0, 0)}\right)$$

$\nu = p \cdot z$ Ioffe time

z^2 - short-distance scale

$$\mathfrak{M}(\nu, z^2) = \int_0^1 du\, K(u, z^2\mu^2, \alpha_s) Q(u\nu, \mu^2)$$

Computed on lattice

Perturbatively calculable

Ioffe-time Distribution

$$Q(\nu, \mu) = \mathfrak{M}(\nu, z^2) - \frac{\alpha_s C_F}{2\pi} \int_0^1 du \left[\ln\left(z^2\mu^2 \frac{e^{2\gamma_E+1}}{4}\right) B(u) + L(u)\right] \mathfrak{M}(u\nu, z^2).$$

K. Orginos et al., PRD96 (2017), 094503

Match data at different *z*

$$\Downarrow$$

$$Q(\nu) = \int_{-1}^{1} dx\, q(x) e^{i\nu x}$$

$$q(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\nu\, e^{-i\nu x} Q(\nu)$$

Need data for all ν, *or additional physics input*

Inverse problem

ITD ↔ PDF

Ioffe-Time Distribution to PDF

J.Karpie, K.Orginos, A.Radyushkin, S.Zafeiropoulos, Phys.Rev.D 96 (2017)

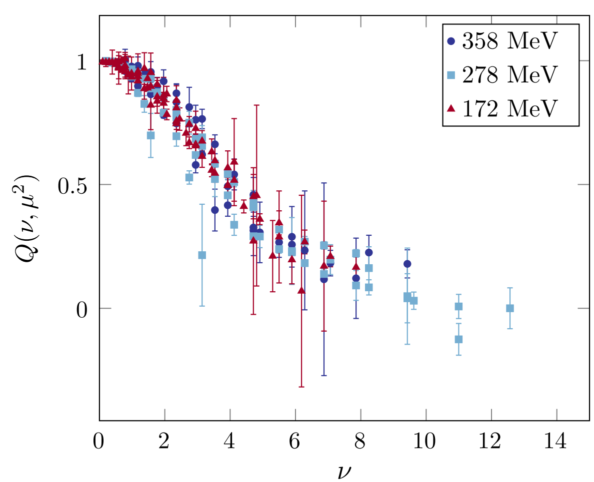
B.Joo *et al.*, HEP 12 (2019) 081, J.Karpie et al., *Phys.Rev.Lett.* 125 (2020) 23, 232003

To extract PDF requires additional information - *use a phenomenologically motivated parametrization*

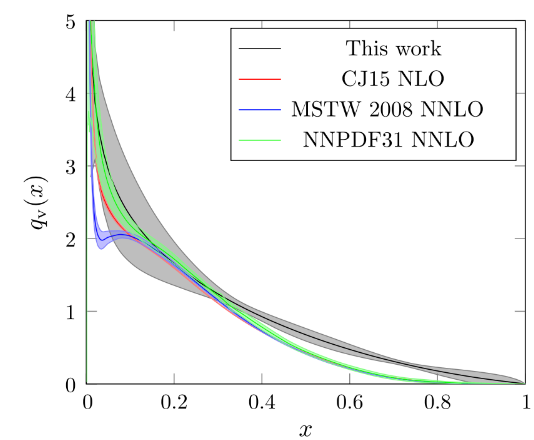
$$f(x) = x^a(1-x)^b P(x)$$ MSTW, CJ

ID	a(fm)	M_π(MeV)	β	c_{SW}	am_l	am_s	$L^3 \times T$	N_{cfg}
$a094m360$	0.094(1)	358(3)	6.3	1.20536588	-0.2350	-0.2050	$32^3 \times 64$	417
$a094m280$	0.094(1)	278(3)	6.3	1.20536588	-0.2390	-0.2050	$32^3 \times 64$	500
$a091m170$	0.091(1)	172(6)	6.3	1.20536588	-0.2416	-0.2050	$64^3 \times 128$	175

$$P(x) = \frac{1 + c\sqrt{x} + dx}{B(a+a, b+1) + cB(a+1.5, b+1) + dB(a+2, b+1)}$$



B.Joo *et al.*,PRL 125 (2020) 23, 232003



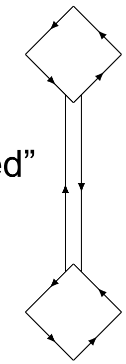
Unpolarized Gluon PDF

Gluon Contribution to unpolarized PDF

c.f. Z.Fan, H-W-Lin, arXiv:2104.06372, arXiv:2007.16113

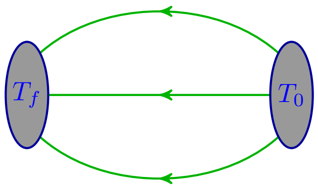
T.Khan *et al.* (Hadstruc), *Phys.Rev.D* 104 (2021) 9, 094516

“disconnected”



$$M_{\mu\alpha;\lambda\beta}(z,p) \equiv \langle p|\, G_{\mu\alpha}(z)\, W[z,0]\, G_{\lambda\beta}(0)\, |p\rangle$$

$$O_g(z) = G_{ji}(z)\, U(z,0)\, G_{ij}(0)\, U(0,z) - \; G_{ti}(z)\, U(z,0)\, G_{it}(0)\, U(0,z).$$



Two-point functions as in isovector case

Reduced matrix element: $$\mathfrak{M}(\nu,z^2) = \left(\frac{\mathcal{M}(\nu,z^2)}{\mathcal{M}(\nu,0)|_{z=0}}\right) \Big/ \left(\frac{\mathcal{M}(0,z^2)|_{p=0}}{\mathcal{M}(0,0)|_{p=0,z=0}}\right)$$

Flavor-singlet quantities are subject to severe signal-to-noise problems compared with isovector measures:

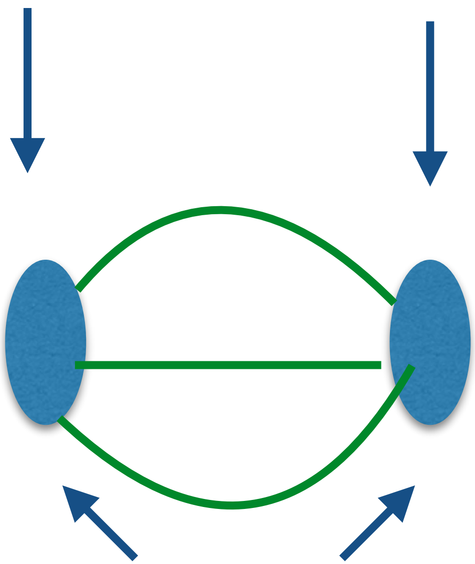
- Use distillation and many more measurements per configuration - *sampling of lattice*
- Use of summed Generalized Eigenvalue Problem (sGEVP) - *better control over excited state contributions*
- Use of *Gradient Flow - smoothing of short-distance fluctuations*

Distillation and Hadron Structure

To control systematic uncertainties, need precise computations over a wide range of momentum.

- Use a low-mode projector to capture states of interest "distillation" M.Peardon *et al* (Hadspec), Phys.Rev.D 80 (2009) 054506
- Enables momentum projection at each temporal point.

Momentum projection

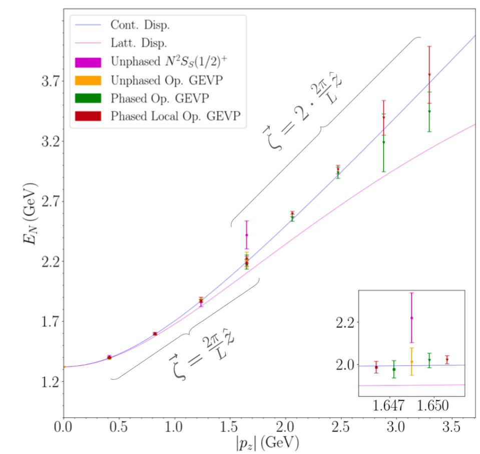


Variational basis

+ momentum smearing

G.Bali *et al*, Phys.Rev.D 93 (2016) 9, 094515

C.Egerer *et al* (Hadstruc), Phys. Rev. D 103, 034502 (2021)

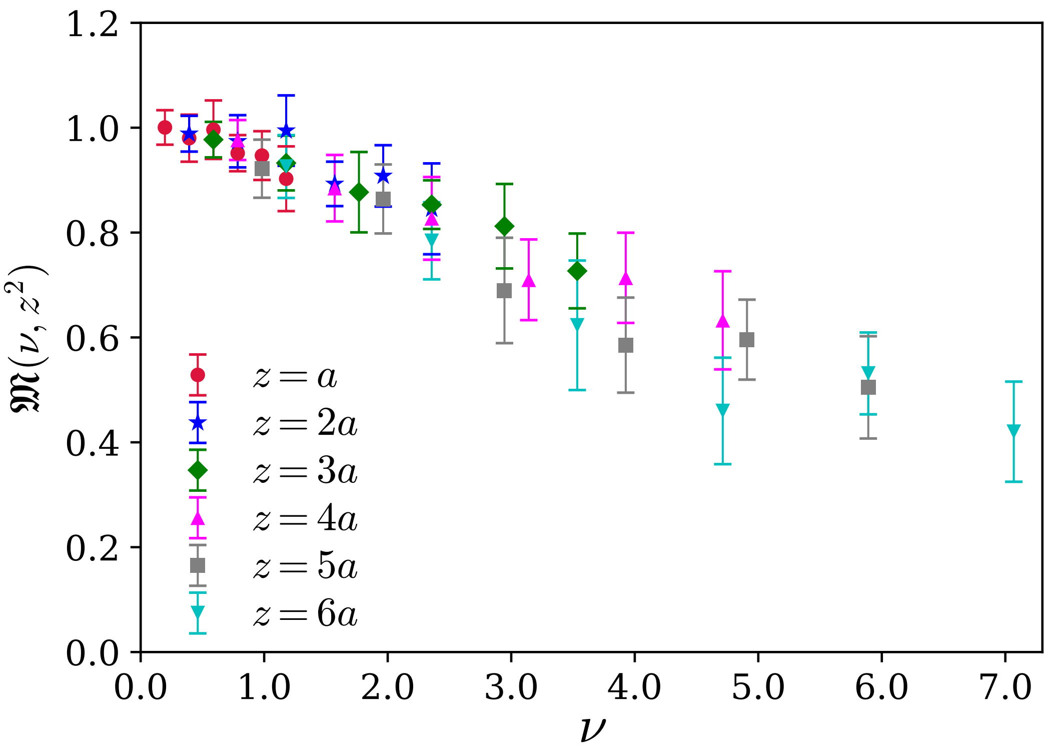


Spatial momentum	Interpolators
$\vec{p} = \vec{0}$	$N\,^2S_S\,\frac{1}{2}^+,\ N\,^2S_M\,\frac{1}{2}^+,\ N\,^4D_M\,\frac{1}{2}^+,$ $N\,^2P_A\,\frac{1}{2}^+,\ N\,^4P^*_M\,\frac{1}{2}^+,\ N\,^2P^*_M\,\frac{1}{2}^+$
$\vec{p} \neq \vec{0}$	$N\,^2P_M\,\frac{1}{2}^-,\ N\,^2P_M\,\frac{3}{2}^-,\ N\,^4P_M\,\frac{1}{2}^-,$ $N\,^4P_M\,\frac{3}{2}^-,\ N\,^4P_M\,\frac{5}{2}^-,\ N\,^2S_S\,\frac{1}{2}^+,$ $N\,^2S_M\,\frac{1}{2}^+,\ N\,^2P^*_M\,\frac{1}{2}^+,\ N\,^4P^*_M\,\frac{1}{2}^+$

Ioffe-time distributions

Use Gradient flow - to further reduce UV fluctuations

Insert flowed link variable $\dot{V}_\mu(\tau, x) - -g_0^2\{\partial_{x,\mu} S(V_\mu(\tau, x))V_\mu(\tau, x)\}V_\mu(\tau, x)$



$\tau \longrightarrow 0$

ITD to PDF

Matching: I.Balitsky,W.Morris,A.Radyushkin,Phys.Lett.B 808 (2020) 135621

$$\mathfrak{M}(\nu, z^2) = \frac{\mathcal{I}_g(\nu,\mu^2)}{\mathcal{I}_g(0,\mu^2)} - \frac{\alpha_s N_c}{2\pi} \int_0^1 du \, \frac{\mathcal{I}_g(u\nu,\mu^2)}{\mathcal{I}_g(0,\mu^2)} \left\{ \ln\left(\frac{z^2\mu^2 e^{2\gamma_E}}{4}\right) B_{gg}(u) + 4\left[\frac{u+\ln(\bar{u})}{\bar{u}}\right]_+ + \frac{2}{3}\left[1-u^3\right]_+ \right\}$$

N.B neglecting quark-gluon mixing

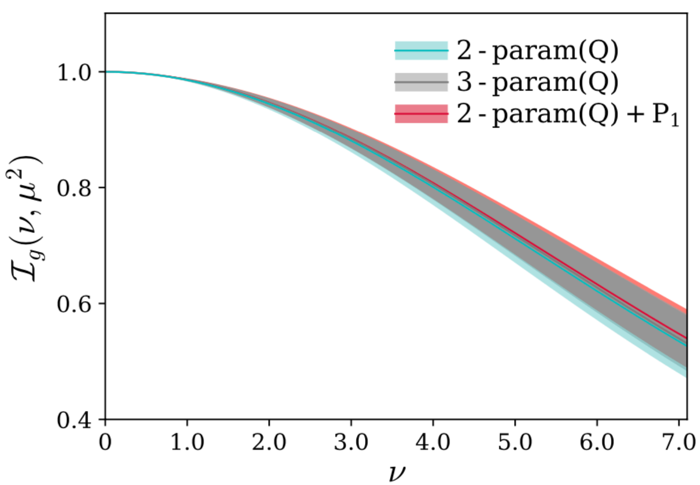
Implementation for obtaining the PDFs follows that of the isovector distribution

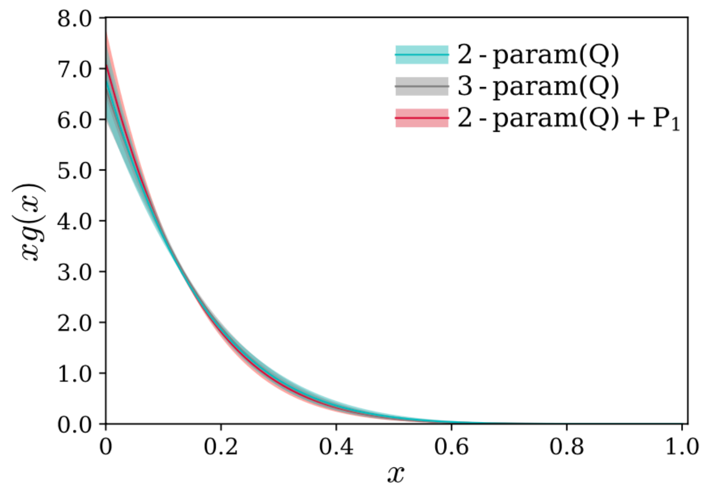
– *Expand in Jacobi Polynomials*

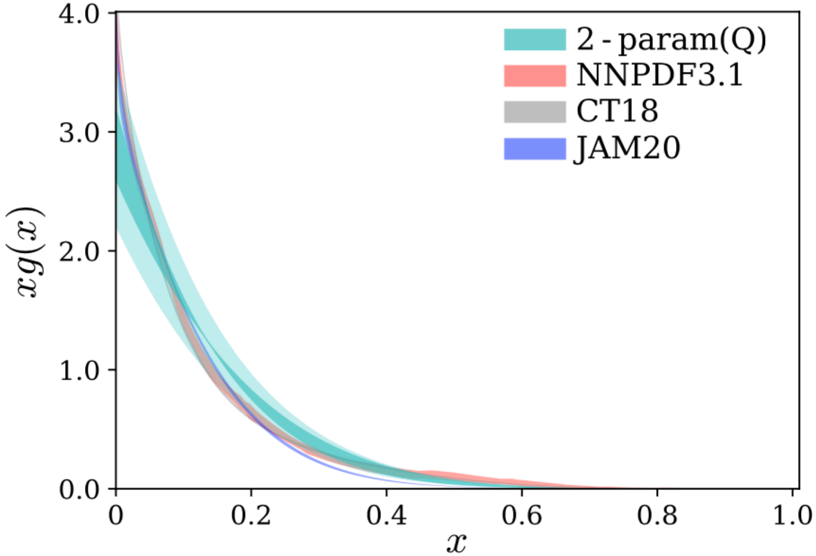
$$x^\alpha(1-x)^\beta$$

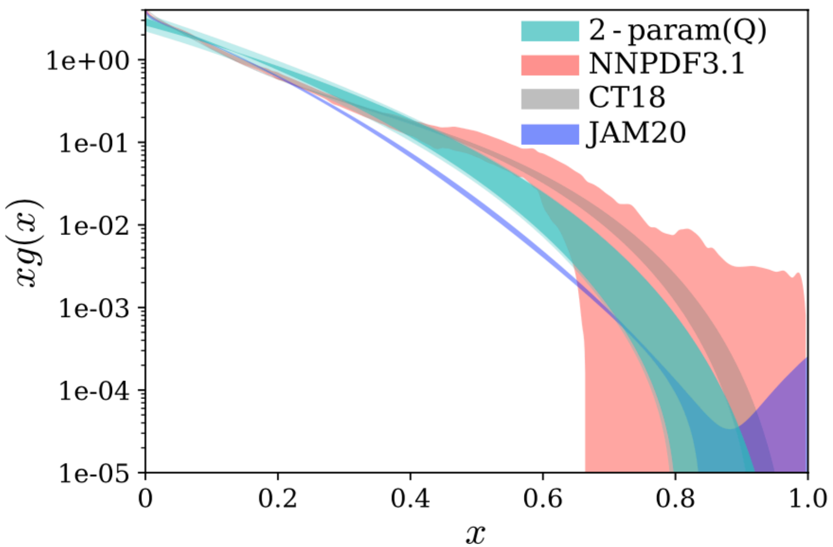
$$+J_1^{\alpha,\beta}$$

$$+a/|z|$$









Require normalization of $xg(x)$ $\langle x \rangle_g^{\overline{\mathrm{MS}}}(\mu = 2\,\mathrm{GeV}) = 0.427(92)$

C.Alexandrou et al., Phys. Rev. Lett. 119, 142002 (2017)

Helicity Gluon PDF

Matrix elements of spatially separated gluon fields

$$\tilde{m}_{\mu\alpha;\lambda\beta} = \langle p, s \,|\, G_{\mu\alpha}(x) W[z,0] \tilde{G}_{\alpha\beta}(0) \,|\, p, s \rangle$$

Combination corresponding to polarized gluon distribution

$$\tilde{M}_{\mu\alpha;\lambda\beta}(z,p,s) = \tilde{m}_{\mu\alpha;\lambda\beta}(z,p,s) - \tilde{m}_{\mu\alpha;\lambda\beta}(-z,p,s)$$

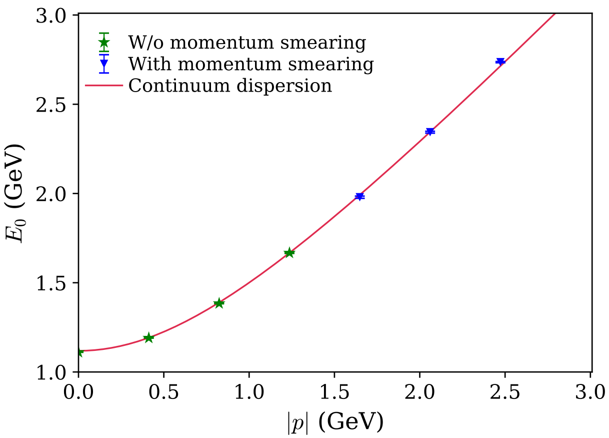
Ioffe-time distribution is related to gluon distribution through inverse problem

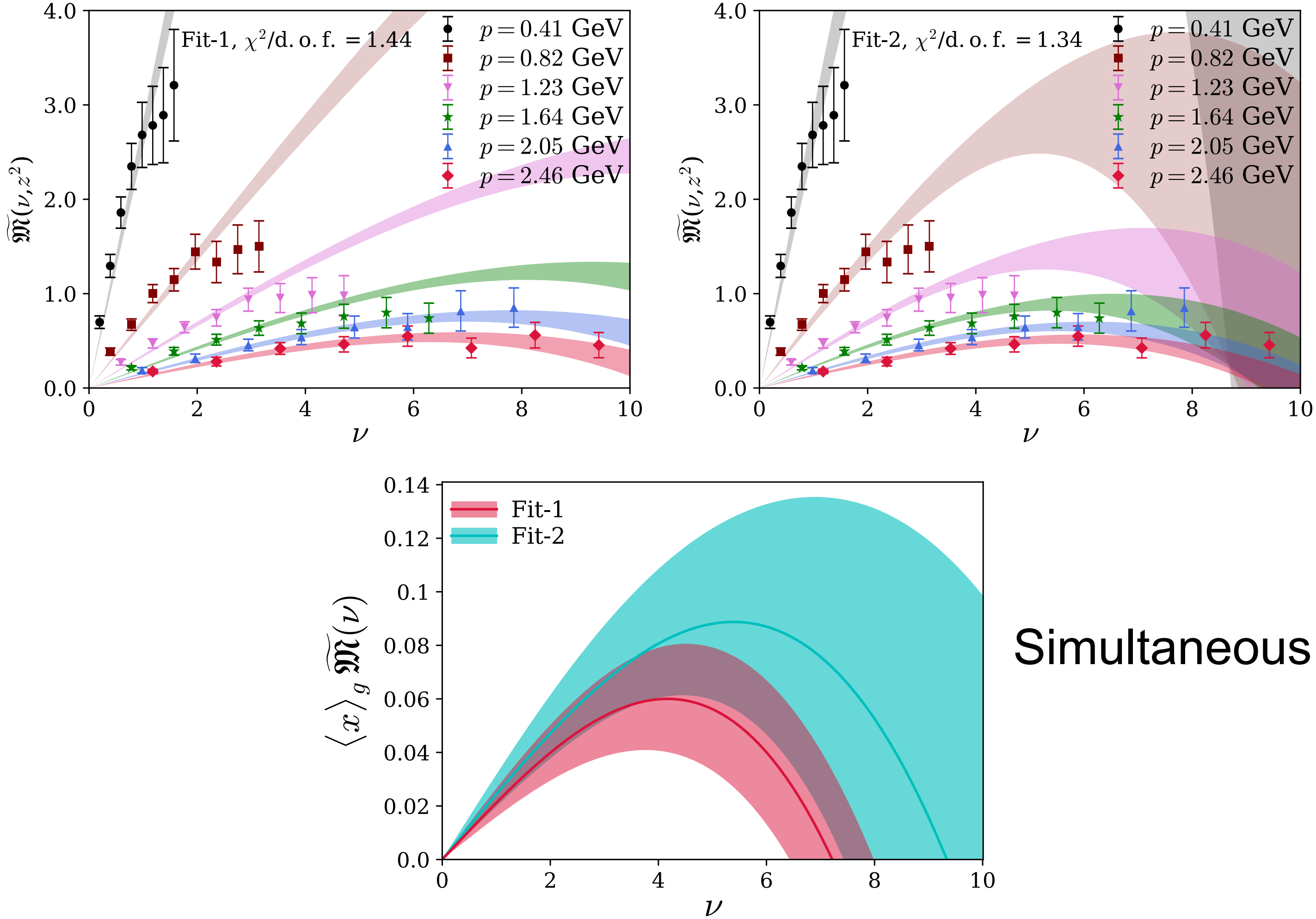
$$\tilde{\mathcal{I}}(\nu) = \frac{i}{2} \int_{-1}^{1} e^{-ix\nu} x \Delta g(x)$$

ID	a (fm)	m_π (MeV)	$L^3 \times N_t$	$N_{\rm cfg}$	$N_{\rm srcs}$
$a094m358$	0.094(1)	358(3)	$32^3 \times 64$	1901	64

$$\widetilde{\mathfrak{M}}(\nu, z^2) = \left[\widetilde{\mathcal{M}}^{(+)}_{sp}(\nu, z^2) - \nu \widetilde{\mathcal{M}}_{pp}(\nu, z^2) \right] + \frac{m_p^2}{p_z^2} \nu \widetilde{\mathcal{M}}_{pp}(\nu, z^2) \,,$$

"Nuisance term"



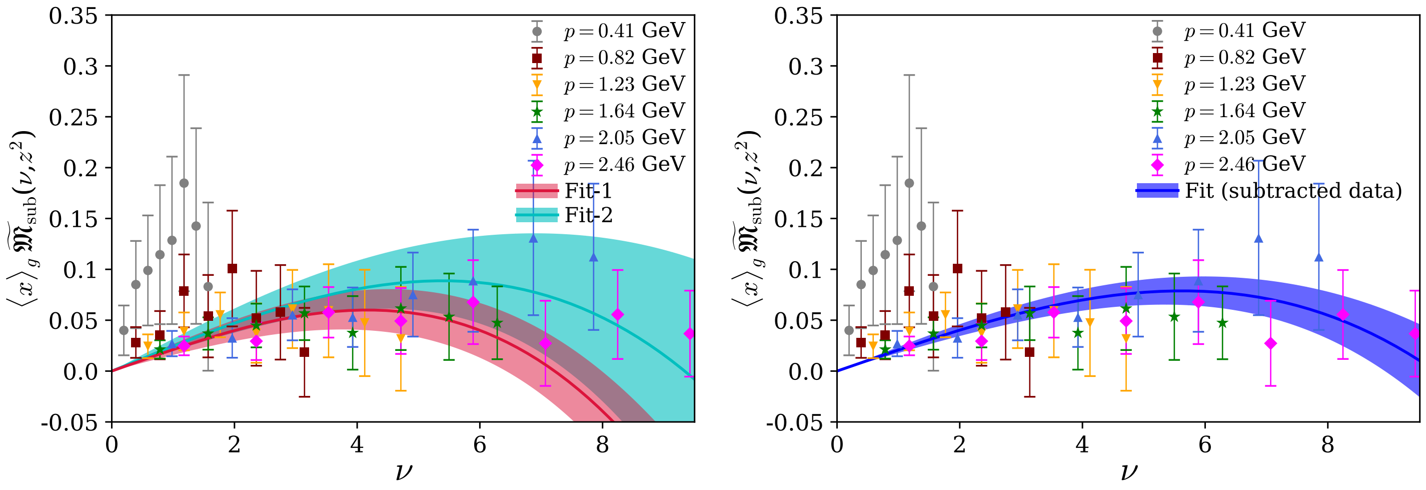


Simultaneous fit to all p

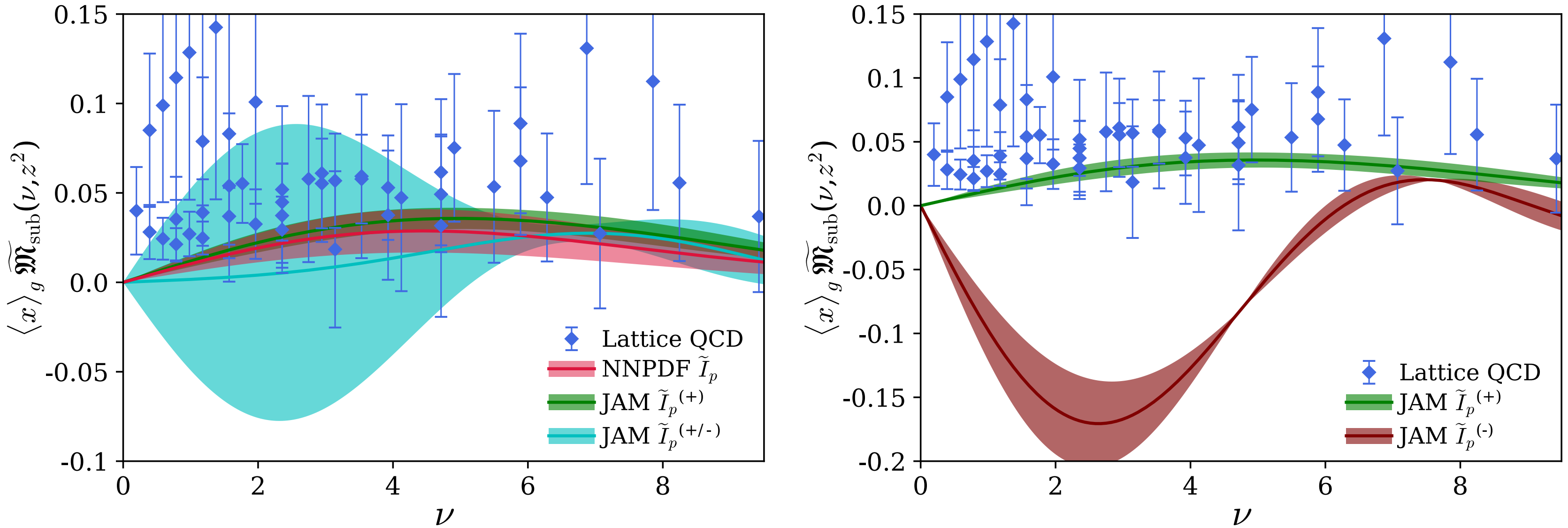
Rather than fitting to $\tilde{\mathscr{M}}$ directly define subtracted matrix element

$$\widetilde{\mathcal{M}}_{\rm sub}(z,p_z) = \widetilde{\mathcal{M}}^{(+)}_{sp}(\nu,z^2) - \nu\widetilde{\mathcal{M}}_{pp}(\nu,z^2) - \nu\frac{m_p^2}{p_z^2}\left[\widetilde{\mathcal{M}}_{pp}(\nu,z^2) - \widetilde{\mathcal{M}}_{pp}(\nu=0,z^2)\right]$$

Still contains nuisance term - but smaller



Recall ITD ↔ PDF



C.Egerer *et al. (HadStruc)*, Phys.Rev.D 106 (2022) 9, 094511

LQCD Calculation of gluon helicity distribution compared with global analyses

Caveat! Mixing with sea quarks not yet included

Lattice QCD + Experiment: Greater than their parts

Pion PDF

Pion PDF has high level of uncertainty - *no free-pion targets*

$$q_v^\pi(x) = \frac{x^\alpha (1-x)^\beta (1+\gamma x)}{B(\alpha+1, \beta+1) + \gamma B(\alpha+2, \beta+1)}$$

"Good Lattice Cross Sections"

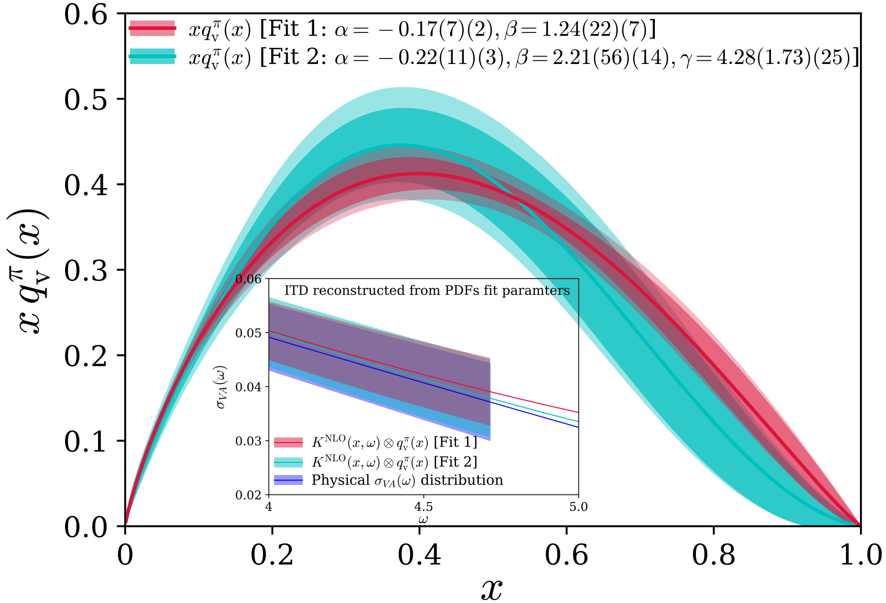
Ma and Qiu, Phys. Rev. Lett. 120 022003

$$\mathcal{O}_S(\xi) = \xi^4 Z_S^2 [\bar{\psi}_q \psi_q](\xi)[\bar{\psi}_q \psi](0)$$

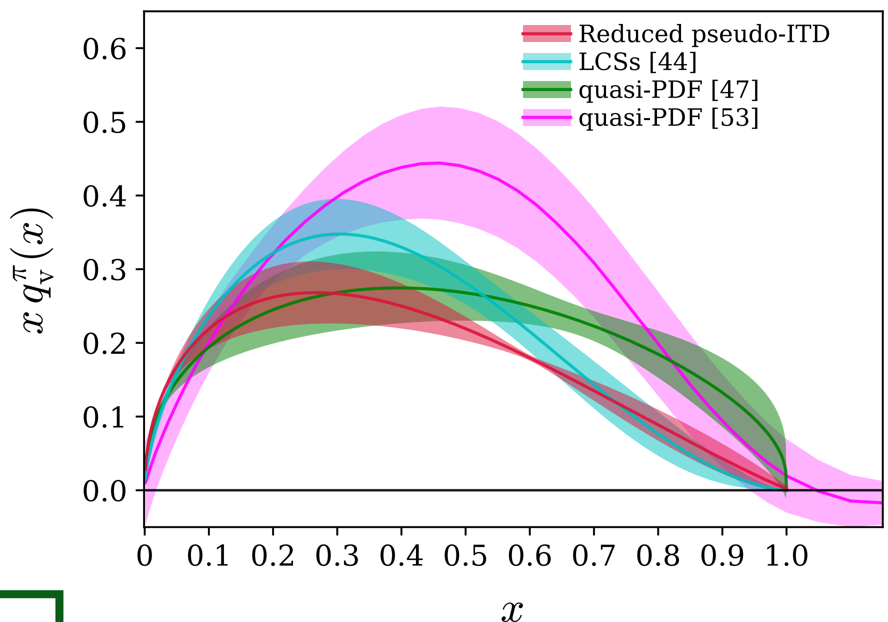
$$\mathcal{O}_{V'}(\xi) = \xi^2 Z_{V'}^2 [\bar{\psi}_q \xi \cdot \gamma \psi_{q'}](\xi)[\bar{\psi}_{q'} \xi \cdot \gamma \psi](0)$$

T.Izubuchi et al., Phys. Rev. D 100, 034516

J-H Zhang et al., Phys. Rev. D 100, 034505



Sufian *et al.*, Phys. Rev. D102, 05408 (2020)



Back to expt........

PHYSICAL REVIEW D **105,** 114051 (2022)

Complementarity of experimental and lattice QCD data on pion parton distributions

P. C. Barry,[1] C. Egerer,[1] J. Karpie,[2] W. Melnitchouk,[1] C. Monahan,[1,3] K. Orginos,[1,3] Jian-Wei Qiu,[1,3] D. Richards,[1] N. Sato,[1] R. S. Sufian,[1,3] and S. Zafeiropoulos[4]

(Jefferson Lab Angular Momentum (JAM) and HadStruc Collaborations)

Can we use LQCD + expt in global analysis: what is the impact?

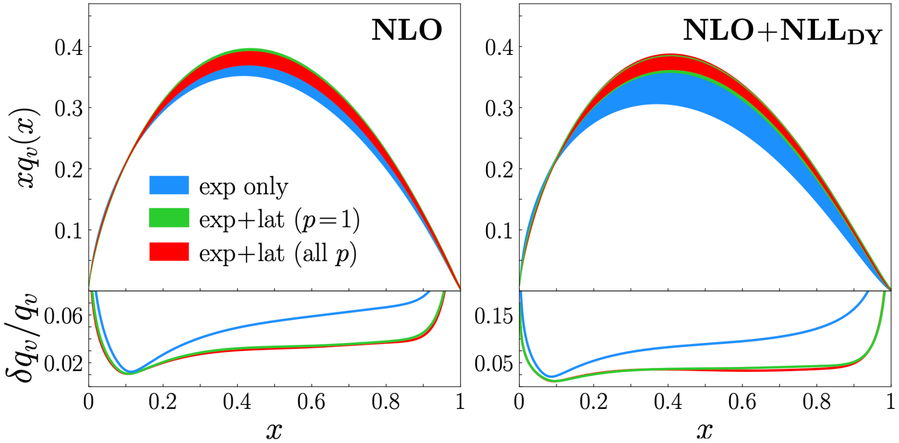
$$\frac{d\sigma}{dx_F d\sqrt{\tau}} = \frac{4\pi\alpha^2}{9\,Q^2 S}\sum_{ij}\int_{x_\pi^0}^{1} dx_\pi \int_{x_A^0}^{1} dx_A\, f_i^\pi(x_\pi,\mu)\, f_j^A(x_A,\mu)\, C_{ij}^{\rm DY}(x_\pi, x_\pi^0, x_A, x_A^0, Q, \mu),$$

Measured Cross Section

PDF

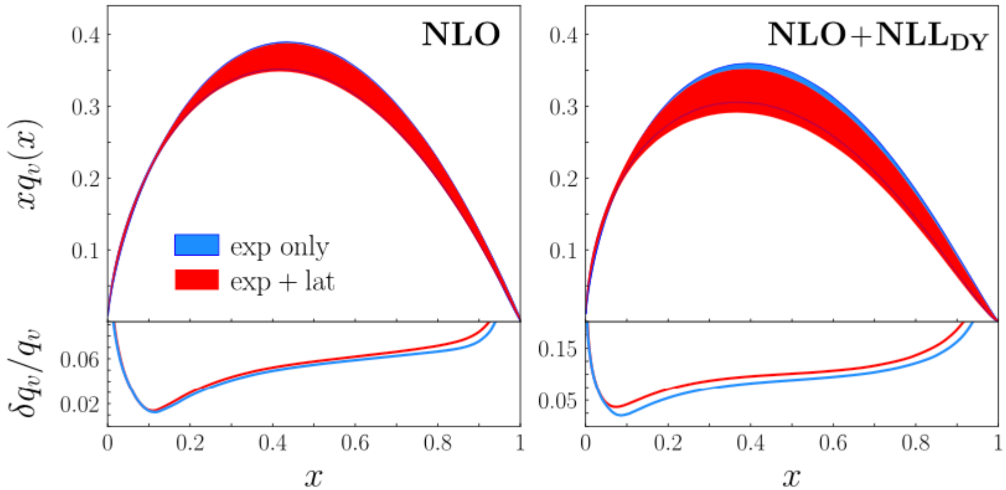
Hard Process

$$f(x,\mu_0^2) = \frac{N_f\, x^{\alpha_f}(1-x)^{\beta_f}(1+\gamma_f x^2)}{B(\alpha_f+2,\beta_f+1)+\gamma_f B(\alpha_f+4,\beta_f+1)}$$



From *pseudo-PDF* *data*

From *Good Lattice Cross Section* *data*



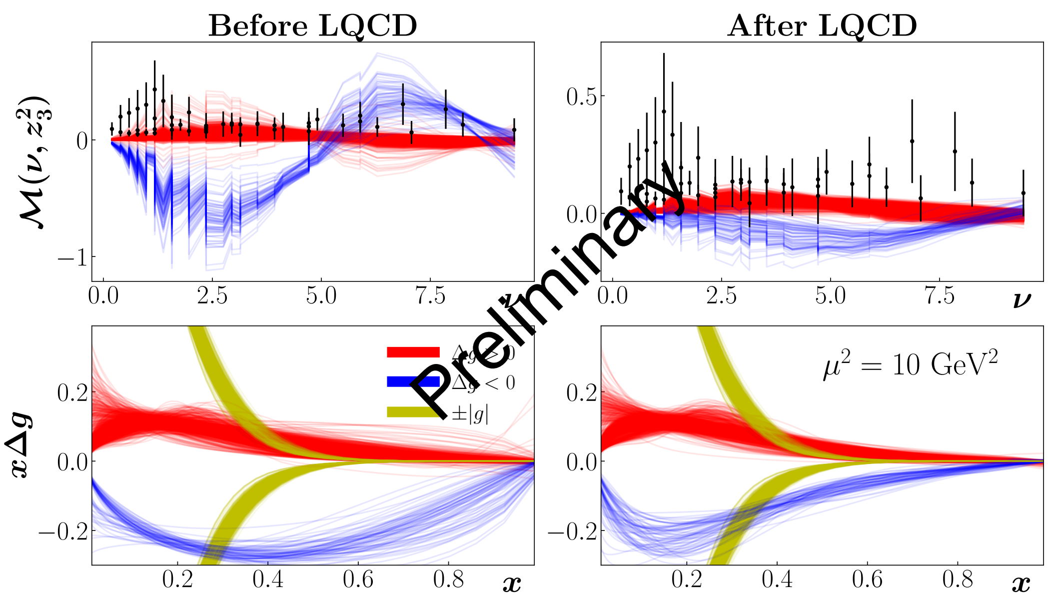
Significance of Δg-Sensitive Ioffe-Time Distributions in QCD Global Analysis

R. M. Whitehill,[1] J. Karpie,[2] W. Melnitchouk,[2] C. Monahan,[2,3]
K. Orginos,[2,3] J.-W. Qiu,[2,3] D. R. Richards,[2] N. Sato,[2] and S. Zafeiropoulos[4]

[1] *Department of Physics, Old Dominion University, Norfolk, Virginia 23529, USA*
[2] *Jefferson Lab, Newport News, Virginia 23606, USA*
[3] *Department of Physics, William & Mary, Williamsburg, Virginia 23185, USA*
[4] *Aix Marseille Univ, Université de Toulon, CNRS, CPT, Marseille, France*

JAM and HadStruc Collaborations

Work in progress



Summary

- The gluon PDF is both a theoretical and computational challenge.
- Distillation + boosting enables both far increased reach in momentum, and improved sampling of lattice
  - *Essential in calculations of gluon contributions*
- *Inclusion of sea-quark/disconnected contributions - work in progress.*
- Lattice QCD + Expt - global analysis; *what calculations would have greatest impact?*