Structure of Hadrons from Lattice QCD using Pseudo-distributions, and implications for Mesons

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Graduate students, and now post-docs/Faculty/Industry

* GPDs and GFFs
Outline

• Hadron Structure on Euclidean Lattice
• Short-distance factorization and pseudo-PDFs/current-current correlators
• PDF of Pion
  – Lattice QCD + Expt - greater than sum of parts
• Pion DA
• Precision: Distillation + momentum smearing
• 3D Structure - GPDs and GFFs
• Summary
Hierarchy of Computations

**Capability Computing - Gauge Generation**

- e.g. Frontier at ORNL

\[ P[U] \propto \det M[U] e^{-S_G[U]} \]

Several \( V, a, T, m_\pi \)

**Capacity Computing - Observable Calculation**

- e.g. Cluster at JLab + Frontier

\[ \langle \mathcal{O} \rangle = \frac{1}{N} \sum_{n=1}^{N} \mathcal{O}(U^n, G[U^n]) \]

\[ C(t) = \sum_{\bar{x}} \langle N(\bar{x}, t) \bar{N}(0) \rangle \]

**“Desktop” Computing - Physical Parameters**

- e.g. Mac at your desk

\[ C(t) = \sum_{n} A_n e^{-E_n t} \]

\[ M_N(a, m_\pi, V) \]
Goldstone bosons of the theory. Pion and Kaon assume a fundamental role in QCD as the (pseudo) Goldstone bosons of the theory.

**Pion and Kaon structure at the electron-ion collider**

Craig D. Roberts, David G. Richards, Tanja Horn, Lei Chang

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Review

Insights into the emergence of mass from studies of pion and kaon structure

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Why the Pion?

Pion and Kaon assume a fundamental role in QCD as the (pseudo) Goldstone bosons of the theory.
Why Lattice QCD?

• We’ve seen it is *physically important*.
• It *may* be computationally less demanding than the nucleon
  – Few Wick contractions
  – Simpler Lorentz decomposition of matrix elements - spin 0
  – Better signal-to-noise ratio - *see later!*

Sullivan process
Distribution along Light Cone (PDFs)

Describe the *longitudinal momentum distribution* of the partons (quarks and gluons) within the hadron, e.g. nucleon, pion, …
Hadron Structure: No-go Theorem?

• First Challenge:
  – Euclidean lattice precludes calculation of light-cone/time-separated
    correlation functions

\[
q(x, \mu) = \int \frac{d\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P | \bar{\psi}(\xi^-) \gamma^+ e^{-ig \int_{\xi^-}^{\xi^+} d\eta^- A^+(\eta^-)} \psi(0) | P \rangle
\]

So…. …Use **Operator-Product-Expansion to formulate in terms of Mellin Moments** with respect to Bjorken x.

\[
\langle P | \bar{\psi} \gamma_{\mu_1} (\gamma_5) D_{\mu_2} \ldots D_{\mu_n} \psi | P \rangle \rightarrow P_{\mu_1} \ldots P_{\mu_n} a^{(n)}
\]

• Second Challenge:
  – Discretised lattice: power-divergent mixing for higher moments

**Moment Methods**
PDFs from Euclidean Lattice

Large-Momentum Effective Theory (LaMET)

"Equal time" correlator

\[ q(x, \mu^2, P^z) = \int \frac{dz}{4\pi} e^{izkz} \langle P | \bar{\psi}(z) \gamma^z e^{-ig \int_0^z dz' A^z(z')} \psi(0) | P > \]
\[ + O((\Lambda^2/(P^z)^2), M^2/(P^z)^2)) \]

\[ q(x, \mu^2, P^z) = \int_x^1 \frac{dy}{y} Z \left( \frac{x}{y}, \frac{\mu}{P^z} \right) q(y, \mu^2) + O(\Lambda^2/(P^z)^2, M^2/(P^z)^2) \]

"quasi-PDF Approach"

Pseudo-PDFs

Wilson line


Pseudo-PDF (pPDF) recognizing generalization of PDFs in terms of loffe Time.

\[ \nu = p \cdot z \]


\[ M^\alpha(p, z) = \langle p \mid \psi \gamma^\alpha U(z; 0)\psi(0) \mid p \rangle \]

\[ p = (p^+, m^2/2p^+, 0_T) \]

\[ z = (0, z_-, 0_T) \]

\[ M^\alpha(z, p) = 2p^\alpha \mathcal{M}(\nu, z^2) + 2z^\alpha \mathcal{N}(\nu, z^2) \]

lofffe-time pseudo-Distribution (pseudo-ITD) generalization to space-like \( z \)

Lattice “building blocks” that of quasi-PDF approach.
Pseudo-PDFs

To deal with UV divergences, introduce reduced distribution

\[ \mathcal{M}(\nu, z^2) = \frac{\mathcal{M}(\nu, z^2)}{\mathcal{M}(0, z^2)} \equiv \frac{\mathcal{M}(\nu, 0)}{\mathcal{M}(0, 0)} \]

\[ \mathcal{M}(\nu, z^2) = \int_0^1 du \, K(u, z^2 \mu^2, \alpha_s) Q(u \nu, \mu^2) \]

- Computed on lattice
- Perturbatively calculable
- Ioffe-time Distribution

\[ Q(\nu, \mu) = \mathcal{M}(\nu, z^2) - \frac{\alpha_s C_F}{2\pi} \int_0^1 du \left[ \ln \left( z^2 \mu^2 \frac{e^{2\gamma_E+1}}{4} \right) B(u) + L(u) \right] \mathcal{M}(u \nu, z^2). \]

- Match data at different z

K. Orginos et al., PRD96 (2017), 094503

Inverse problem

\[ Q(\nu) = \int_{-1}^1 dx \, q(x) e^{i \nu x} \]

\[ q(x) = \frac{1}{2\pi} \int_{-\infty}^\infty d\nu \, e^{-i \nu x} Q(\nu) \]

Need data for all \( \nu \), or additional physics input

Thomas Jefferson National Accelerator Facility
"Good Lattice Cross Sections"

\[ \sigma_n(\nu, \xi^2, P^2) = \langle P \mid T\{O_n(\xi)\} \mid P \rangle \]

where

\[ \sigma_n(\nu, \xi^2, P^2) = \sum_a \int_{-1}^{1} \frac{dx}{x} f_a(x, \mu^2) K_n^a(x\nu, \xi^2, x^2 P^2, \mu^2) + O(\xi^2 \Lambda_{QCD}^2) \]

Ma and Qiu, Phys. Rev. Lett. 120 022003

**Expressed in coordinate space**

**Calculated in perturbation theory ("process dependent")**

**Encompasses qPDF/pPDF**

**Gauge-Invariant Currents**

\[ O(\xi) = \bar{\psi}(0) \Gamma W(0, 0 + \xi) \psi(\xi) \]

\[ O_S(\xi) = \xi^4 Z^2_S [\bar{\psi}_q \gamma_q](\xi) [\bar{\psi}_q \psi](0) \]

\[ O_{V'}(\xi) = \xi^2 Z^2_{V'} [\bar{\psi}_q \xi \cdot \gamma \psi_{q'}](\xi) [\bar{\psi}_{q'} \xi \cdot \gamma \psi](0) \]

**Flavor-changing**

\[ + \text{ analogous gluon operators} \]
PDFs, GPDs and TMDs

Same lattice building blocks

GLCS

pPDF

qPDF

All approaches should give same after:
- Finite volume
- Discretization
- Uncertainties
- Infinite momentum

Light cone reduces to a point

Characterized by short-distance factorization


Ma and Qiu, Phys. Rev. Lett. 120 022003

Good Lattice Cross Section

Sequential-Source Approach

Momentum projection

Process, i.e. current, dependent

\[ \frac{1}{2} [\sigma_{V,A}^{\mu\nu}(\xi, p) + \sigma_{A,V}^{\mu\nu}(\xi, p)] \]

Perturbative kernel:

\[ \equiv e^{\mu\nu\alpha\beta} \xi_\alpha p_\beta T_1(\nu, \xi^2) + (p_\mu \xi_\nu - \xi_\mu p^\nu) T_2(\nu, \xi^2) \]

N.B. We’re inconsistent \( \omega \leftrightarrow \nu! \)

Expansion in \( \alpha_s \)

Y-Q Ma


Thomas Jefferson National Accelerator Facility
Lattice Cross Sections

\[
\sigma_{VA}(\omega, \xi^2) = \sum_{k=0}^{k_{\text{max}}=4} \lambda_k \tau^k + b_1 m_\pi + b_2 a + b_3 \xi^2 + b_4 a^2 p^2 + b_5 e^{-m_\pi(L-\xi)}
\]

\[
\tau = \frac{\sqrt{\omega_{\text{cut}} + \omega} - \sqrt{\omega_{\text{cut}}}}{\sqrt{\omega_{\text{cut}} + \omega} + \sqrt{\omega_{\text{cut}}}}
\]

“Ioffe Time Distribution”

“Z-expansion fit”
Pion PDF

“Inverse Problem” - ill-posed inverse Fourier transform.

$$\sigma_n(\nu, \xi^2, P^2) = \sum_a \int_{-1}^{1} \frac{dx}{x} f_a(x, \mu^2) K_n^a(x\nu, \xi^2, x^2 P^2, \mu^2) + \mathcal{O}(\xi^2 \Lambda_{QCD}^2)$$

Calculate on Lattice  Extract PDF?  Calculate in PQCD

Similar challenge to global fitting community!

$$q_\pi^\nu(x) = \frac{x^\alpha(1-x)^\beta(1+\gamma x)}{B(\alpha+1, \beta+1) + \gamma B(\alpha+2, \beta+1)}$$
Pion Valence Quark Distribution at Large $x$ from Lattice QCD

Raza Sabbir Sufian, Colin Egerer, Joseph Karpie, Robert G. Edwards, Bálint Joó, Yan-Qing Ma, Costas Orginos, Jian-Wei Qiu, and David G. Richards

Sufian et al., Phys. Rev. D102, 05408 (2020)
Determine large-x behavior $\rightarrow$ need for finer resolution and reach in Ioffe time.
Pseudo-PDF Approach

As for GLCS, this is a short-distance expansion - have to map to scale $\mu$


$\mu = 2\text{ GeV}$
Crucial take-away - *should aim for consistency after systematic uncertainties under control*
Can we use LQCD + expt in global analysis: what is the impact?

\[
\frac{d\sigma}{dx_F d\sqrt{\tau}} = \frac{4\pi\alpha^2}{9 Q^2 S} \sum_{ij} \int_{x_0^\pi}^1 dx_\pi \int_{x_0^A}^1 dx_A f_i^\pi (x_\pi, \mu) f_j^A (x_A, \mu) C_{ij}^{DY} (x_\pi, x_0^\pi, x_A, x_0^A, Q, \mu),
\]

PDF Hard Process

\[
f(x, \mu_0^2) = \frac{N_f x^{\alpha_f} (1 - x)^{\beta_f} (1 + \gamma_f x^2)}{B(\alpha_f + 2, \beta_f + 1) + \gamma_f B(\alpha_f + 4, \beta_f + 1)}
\]
From pseudo-PDF data

From CC data
Pion Distribution Amplitude

Pion Distribution Amplitude describes internal structure in Exclusive Processes, e.g. Electromagnetic Form Factors, at high momentum Transfers; DVMP

$$\langle 0 | \bar{d}(z_2 n) \gamma \cdot n \gamma_5 u(z_1 n) | \pi(p) \rangle = i f_\pi (p \cdot n) \int_0^1 dx e^{-i(z_1 x + z_2 (1-x)) p \cdot n} \phi_\pi(x, \mu^2)$$

**Spatially separated**

LaMET or Quasi-PDF

$$\langle 0 | \bar{d}(-z/2) \gamma_\rho \gamma_5 u(z/2) | \pi(p) \rangle$$

SDF or Pseudo-PDF

LaMET calculation with chiral fermions and physical pion mass on a fine lattice

E.Baker et al. arXiv:2405.20120

JLab@12 GeV and EIC


E.Baker et al. arXiv:2405.20120

H.T.Ding et al., arXiv:2404.04412
Daniel Kovner et al., arXiv:2401.06858

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$$M^{\alpha, \text{mix}}(p, z) = z^\beta (p^\alpha z^\beta - p^\beta z^\alpha) M^{\text{mix}}(\nu, -z^2) = p^\alpha [z^2 M^{\text{mix}}(\nu, -z^2)] + z^\alpha [\nu M^{\text{mix}}(\nu, -z^2)].$$

x-dependent DAs in preparation
Distillation and Improved Sampling
Challenges of Higher Momenta

Achieving high momenta in a lattice calculation presents several challenges:
- Discretization errors
- "Compression" of energy spectrum as spatial momentum increased
- Reduced symmetries for states in motion - parities are mixed, helicity defines the basis
- Poor overlaps of e.g. Jacobi smearing on states in motion - poor signal-to-noise ratio.

**Neat solution**  
Boosted interpolating operators  

Now essentially ubiquitous

Can we combine momentum smearing with distillation to address some of the other issues?

N.B Bali et al does indeed suggest application to distillation.

Look at:
- Nucleon energies and dispersion relation
- Nucleon charges
Distillation

Low-rank approximation to (typically) Jacobi-smearing kernel

\[-\nabla^2(t)\xi^{(k)}(t) = \lambda^{(k)}(t)\xi^{(k)}(t)\]

\[\Box(\vec{x}, \vec{y}; t)_{ab} = \sum_{k=1}^{\text{Rank}} \xi^{(k)}_a(\vec{x}, t) \xi^{(k)\dagger}_b(\vec{y}, t),\]

Components of distillation:

\[\tau^{(l,k)}_{\alpha\beta}(t', t) = \xi^{(l)\dagger}(t') M^{-1}_{\alpha\beta}(t', t) \xi^{(k)}(t)\]

\[\Phi^{(i,j,k)}_{\alpha\beta\gamma}(t) = \epsilon^{abc} \left( D_1 \xi^{(i)} \right)^a \left( D_2 \xi^{(j)} \right)^b \left( D_3 \xi^{(k)} \right)^c (t) S_{\alpha\beta\gamma}\]

\[C_{rs}(t) = \sum_{\vec{x}, \vec{y}} \langle 0 | \mathcal{O}_r(t, \vec{x}) \mathcal{O}_s^\dagger(0, \vec{y}) | 0 \rangle \equiv \text{Tr} [\Phi_r(t) \otimes \tau(t, 0) \tau(t, 0) \tau(t, 0) \otimes \Phi_s(0)]\]

Extension to 3pt functions straightforward

Perambulators → quark propagation
Elementals → (baryon or meson) operators
Projection to irrep


Spatial Volume!
High-precision calculations

To control systematic uncertainties, need precise computations over a wide range of momentum.

- Use a low-mode projector to capture states of interest "distillation"
- + momentum smearing

G. Bali et al, Phys. Rev. D 93 (2016) 9, 094515

Challenges related to renormalization and matching of operators - concentrate on nucleon

SULi Student
Preliminary: Sam Bevins, Eloy Romero, Daniel Kovner, Colin Egerer

2.5 GeV
Expand the x-dependence in terms of (shifted) Jacobi Polynomials

\[ \sigma_n^{(\alpha,\beta)}(\nu, z^2 \mu^2) = \Re \int_0^1 dx \ K_V(x\nu, z^2 \mu^2) x^\alpha (1 - x)^\beta \Omega_n^{(\alpha,\beta)}(x) \]

Matching kernel

\[ \Re M_{\text{fit}}(\nu, z^2) = \sum_{n=0}^\infty \sigma_n^{(\alpha,\beta)}(\nu, z^2 \mu^2) C_{v,n}^{\text{lt}}(\alpha,\beta) + \left( \frac{a}{z} \right) \sum_{n=1}^\infty \sigma_n^{(\alpha,\beta)}(\nu) C_{v,n}^{\text{az}}(\alpha,\beta) + z^2 \Lambda_{\text{QCD}}^2 \sum_{n=1}^\infty \sigma_n^{(\alpha,\beta)}(\nu) C_{v,n}^{\text{t4}}(\alpha,\beta) \]

Discretization

Higher twist

Systematics of isovector PDF

\[ m_\pi \approx 358 \ \text{MeV} \]
Gluon Contribution to unpolarized Proton PDF


T.Khan et al. (Hadstruc), Phys.Rev.D 104 (2021) 9, 094516

\[ M_{\mu\alpha;\lambda\beta}(z, p) \equiv \langle p | G_{\mu\alpha}(z) W[z, 0] G_{\lambda\beta}(0) | p \rangle \]

\[ O_g(z) = G_{ji}(z) U(z, 0) G_{ij}(0) U(0, z) - G_{ti}(z) U(z, 0) G_{it}(0) U(0, z). \]

Two-point functions as in isovector case

Reduced matrix element:

\[
\mathcal{M}(\nu, z^2) = \left( \frac{\mathcal{M}(\nu, z^2)}{\mathcal{M}(\nu, 0)|_{z=0}} \right) / \left( \frac{\mathcal{M}(0, z^2)|_{p=0}}{\mathcal{M}(0, 0)|_{p=0, z=0}} \right)
\]

Flavor-singlet quantities are subject to severe signal-to-noise problems compared with isovector measures:

- Use distillation and many more measurements per configuration - sampling of lattice
- Use of summed Generalized Eigenvalue Problem (sGEVP) - better control over excited state contributions
- Use of Gradient Flow - smoothing of short-distance fluctuations

\[ M(\nu, z^2) = \frac{I_g(\nu, \mu^2)}{I_g(0, \mu^2)} - \frac{\alpha_s N_c}{2\pi} \int_0^1 du \frac{I_g(u\nu, \mu^2)}{I_g(0, \mu^2)} \left\{ \ln \left( \frac{z^2 \mu^2 e^{2\gamma_E}}{4} \right) B_{gg}(u) + 4 \left[ \frac{u + \ln(\bar{u})}{\bar{u}} \right] + \frac{2}{3} \left[ 1 - u^3 \right] \right\} \]

*N.B* neglecting quark-gluon mixing

Implementation for obtaining the PDFs follows that of the isovector distribution

- **Expand in Jacobi Polynomials**

\[ x^\alpha (1 - x)^\beta + \gamma^{\alpha, \beta} + \delta / \beta \]

Require normalization of \( xg(x) \)

FIG. 9. The lattice data points represent the reduced $I_{\text{oe}}$-time pseudo-distribution, $f_{\text{M}}(\epsilon, z^2)$ in the zero flow-time limit obtained through the subtraction method using $p=0$ matrix elements. The lattice data points and the fit bands are normalized using the gluon momentum fraction, $x^g$ from [17]. Left panel: the red and cyan bands represent the target mass corrected reduced $I_{\text{oe}}$-time pseudo-distribution using the fit of moments in Sec. IV A. Right panel: the blue band is a fit to the subtracted pseudo-ITD using the functional form in Eq. (20) with $a_0, a_1, b_1$ as fit parameters and $b_0 = 0$ fixed by construction.

FIG. 10. A comparison between the lattice reduced $I_{\text{oe}}$-time pseudo-distribution $f_{\text{M}}(\epsilon, z^2)$ in the zero flow-time limit obtained through the subtraction method using the $p=0$ matrix elements, and the gluon helicity ITD constructed from global fits of PDFs. The lattice data points are the same as in Fig. 9, plotted on a smaller vertical scale for better comparison with the phenomenological ITD bands. In the left plot, the red band denotes the ITD constructed from the gluon helicity distribution by the NNPDF collaboration. The green band labeled by $e^I_{+p}$ and the cyan band labeled by $e^I_{(+/-)}p$ represent the gluon helicity ITD determined by the JAM collaboration with and without the positivity constraint on the gluon helicity PDF, respectively.

On the right plot, the gluon helicity ITDs for positive and negative helicity PDFs are compared with the lattice data. The green band labeled by $e^I_{+p}$ and the maroon band labeled by $e^I_{(-)}p$ represent the gluon helicity ITD determined by the JAM collaboration associated with the positive and negative gluon helicity PDF solutions, respectively.

Polarization in the nucleon cannot be properly constrained. In other words, the ITD extracted from the JAM global fit (labeled by JAM $e^I_{(+/-)}p$ in Fig. 10) may have a similar or even larger magnitude of uncertainty than our lattice QCD calculation. We show a comparison of the polarized gluon ITDs obtained from global fits and our lattice calculation in Fig. 10. Most importantly, Fig. 10 shows that the ITD data in the $\epsilon, \nu$ region is primarily controlled by whether the gluon polarization in the nucleon is positive or negative, according to the JAM analysis.

The positivity constraint on the gluon distributions, namely helicity-aligned and helicity-antialigned both being non-negative, in the analysis of experimental data in [11] leads to a substantial reduction of the variance of $x^g$ in the large-$x$ region, as seen in Fig. 6 of [11]. Specifically, the PDFs without the positivity assumption were organized into a band of solutions with a negative PDF and a band of solutions with a positive PDF. We compare the ITDs resulting from the two bands with positive and negative $x^g$ to our results in the right panel of Fig. 10. The current matrix elements, albeit with an unphysical pion mass and finite lattice spacing, are inconsistent within statistical LQCD Calculation of gluon helicity distribution compared with global analyses.

Caveat! Mixing with sea quarks not yet included.

Recall ITD ↔ PDF

C.Egerer et al. (HadStruc), Phys.Rev.D 106 (2022) 9, 094511

LQCD Calculation of gluon helicity distribution compared with global analyses

Caveat! Mixing with sea quarks not yet included
The culmination of QGT is a framework where LQCD + Expt can provide a more faithful description of hadron structure than either alone.

Does QCD admit negative solutions $\Delta g(x) < 0$?

Before LQCD

After LQCD

Neural network analysis of lattice calculation

Significant change in parametrization but insufficient to exclude negative solute in global analysis

Impact of LQCD on global analysis

RHIC polarized jet, large-x JLab + LQCD eliminate negative solutions: N.Hunt-Smith et al., arXiv:2403.08117
3D Hadron Structure

Generalized Parton Distributions (GPDs) provide 3D description in terms of longitudinal momentum fraction and (2D) transverse displacement
- Orbital Angular Moment
- Integrated Generalized Form Factors: distribution of mass, charge, pressure
Trace-Anomaly Form Factors (L3.1)

- Understanding the distribution of charge, spin and mass within a hadron is a fundamental goal of NP.
- Knowledge about internal mechanical properties encapsulated in the energy-momentum tensor

\[
\hat{T}_{g}^{\mu\nu} = 2 \text{Tr} \left[ -F_{\mu\alpha}^{\nu} F_{\alpha}^{\nu} + \frac{1}{4} g_{\mu\nu} F_{\alpha\beta}^{\alpha\beta} \right] ; \hat{T}_{q}^{\mu\nu} = 2 \sum_{f} i \bar{\psi}_{f} D_{\nu\gamma}^{\mu\gamma} \psi_{f}
\]

Key role is played by the trace-anomaly and its form factors

\[
\langle p' | T_{\mu}^{\mu} | p \rangle = m_{\pi} F(Q^{2}) \quad \text{for case of pion, and the mass at } Q^{2} = 0.
\]

\(\chi\text{QCD Collaboration, B.Wang et al., arXiv:2401.05496, PRD (to appear)}\)

Gluon trace anomaly in

Pion and Nucleon

Requested DOE Highlight
Gravitational Form Factors (L4.3)

Gravitational form factors are given by the matrix elements of the (symmetrised) energy-momentum tensor within a hadron.

\[ \hat{T}_g^{\mu\nu} = 2 \text{Tr} \left[ -F_{\mu\alpha} F_{\nu}^{\alpha} + \frac{1}{4} g^{\mu\nu} F_{\alpha\beta} F_{\alpha\beta} \right] ; \hat{T}_q^{\mu\nu} = 2 \sum_f i \bar{\psi}_f D^{\{\nu \gamma \nu\}} \psi_f \]

We can look at the Lorentz structure to learn more about the mechanical properties.

For pion, they decompose as

\[ \langle \pi(p') \mid \hat{T}^{\mu\nu} \mid \pi(p) \rangle = 2 p^\mu p^\nu A^{\pi}(t) + \frac{1}{2} [\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2] D^{\pi}(t) \]

Clover-fermion computation at \( m_\pi = 170 \text{ MeV} \)

**Complete flavor and gluon decomposition; renormalized to RI-mom**

Observation: smaller mass radius than charge radius


D.C.Hackett et al., arXiv:2310.08484

Joint Theory-Expt Review

V.Burkert et al., Rev. Mod. Phys. 95 (2023) 4, 041002

**Requested DOE Highlight**
(Pseudo)-GPDs from Lattice QCD

H.Dutrieux et al., (HadStruc), arXiv:2405.10304  See also S.Bhattacharya et al., Phys.Rev.D 108 (2023) 1 014507

GPDs described by \textit{off-forward} matrix elements of operators \textit{along light cone}

\[
F^q (x, p_f, p_i) = \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \times \langle N (p_f, \lambda_f) | \bar{\psi}^q \left( -\frac{z}{2} \right) \gamma^+ \hat{W} \left( -\frac{z}{2}, \frac{z}{2}; A \right) \psi^q \left( \frac{z}{2} \right) | N (p_i, \lambda_i) \rangle |_{z^+=0, z_\perp=0_\perp},
\]

\[
= \frac{1}{2P^+} \bar{u} (p_f, \lambda_f) \left[ \gamma^+ H^q (x, \xi, t) + \frac{i\sigma^{+\nu}q_\nu}{2m} E^q (x, \xi, t) \right] u (p_i, \lambda_i),
\]

Kinematic variables

\[
P \equiv \frac{1}{2}(p + p'), \quad q \equiv p' - p, \quad t \equiv q^2, \quad \xi \equiv -\frac{q^+}{2P^+}.
\]

\textbf{Skewness}

As for the case of the PDFs, we calculate matrix elements at \textit{space-like separations} \( z \).

We can then express skewness as

\[
\xi = -\frac{q \cdot z}{2P \cdot z} = -\frac{q \cdot z}{2\nu}. 
\]
Moments of GPDs

For this first study, we will focus on calculations of the moments of GPDs

\[ \int_{-1}^{1} dx \, x^{n-1} \left( \frac{H_{u-d}}{E_{u-d}} \right) (x, \xi, t) = \sum_{k=0}^{n-1} \left( \frac{A_{n,k}(t)}{B_{n,k}(t)} \right) \xi^k. \]

Moments can be obtained by \( \nu \) expansion of the Ioffe-time distribution

\[ F(\nu, \xi, t, z^2) = \sum_{n=0}^{\infty} \frac{(-i\nu)^n}{n!} F_{n+1}(\xi, t, z^2) \quad \text{where} \quad F_n(\xi, t, z^2) \equiv \int_{-1}^{1} dx \, x^{n-1} F(x, \xi, t, z^2) \]

We fit the resulting GFFs to a dipole \( A_{n,k}(t) = A_{n,k}(t = 0) \left( 1 - \frac{t}{\Lambda_{n,k}^2} \right)^{-2} \)

More rigorously, use the so-called \( z \)-expansion.
Pion mass = 0.36 GeV - Proton mass = 1.12 GeV
No continuum limit - signs of discretization errors / light-cone uncertainty
Matching at 2 GeV with leading logarithmic accuracy

<table>
<thead>
<tr>
<th>Value at $t = 0$</th>
<th>Dipole mass (GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>GPD $H^{u-d}$</strong></td>
<td><strong>GPD $E^{u-d}$</strong></td>
</tr>
<tr>
<td>(A_{1,0}) 0.97(2)</td>
<td>(B_{1,0}) 3.44(4)</td>
</tr>
<tr>
<td>(A_{2,0}) 0.204(4)</td>
<td>(B_{2,0}) 0.36(2)</td>
</tr>
<tr>
<td>(A_{3,0}) 0.062(4)</td>
<td>(B_{3,0}) 0.07(2)</td>
</tr>
<tr>
<td>(A_{3,2}) 0.42(7)</td>
<td>(B_{3,2}) 0.9(6)</td>
</tr>
<tr>
<td>(A_{4,0}) 0.06(1)</td>
<td>(B_{4,0}) 0.06(4)</td>
</tr>
<tr>
<td>(A_{4,2}) 0.5(2)</td>
<td>(B_{4,2}) 1.2(9)</td>
</tr>
<tr>
<td><strong>GPD $H^{u-d}$</strong></td>
<td><strong>GPD $E^{u-d}$</strong></td>
</tr>
<tr>
<td>(A_{1,0}) 1.25(2)</td>
<td>(B_{1,0}) 0.982(6)</td>
</tr>
<tr>
<td>(A_{2,0}) 1.86(6)</td>
<td>(B_{2,0}) 1.41(8)</td>
</tr>
<tr>
<td>(A_{3,0}) 2.2(4)</td>
<td>(A_{3,2}) 1.07(9)</td>
</tr>
<tr>
<td>(A_{3,2}) 2.4(9)</td>
<td>(B_{3,0}) 1.0(3)</td>
</tr>
<tr>
<td>(A_{4,0})</td>
<td>(A_{4,2}) 1.2(2)</td>
</tr>
<tr>
<td>(B_{4,0}) Unreliable</td>
<td>(B_{4,2}) 1.1(2)</td>
</tr>
</tbody>
</table>
Figure 18. The skewness-dependent generalized form factors. Same caption as Fig. 17.
Summary

- Realistic calculation of light-cone distributions from LQCD now available
- Numerous calculations of pion structure within the short-distance factorization framework.
- Distillation + boosting enables both far increased reach in momentum, and improved sampling of lattice
  - Allows factorization of quark propagation from states
  - Applied to nucleon properties - next step to pion and kaon.
- Are able to isolate leading twist from higher-twist and discretization contamination
- 3D Hadron Structure through GPDs
  - Moment calculation allows higher moments than from local operators
  - Direct calculation of x dependence in progress
  - Next frontier - flavor singlet. Provides access to so-called D-term
FIRST INTERNATIONAL SCHOOL OF HADRON FEMTOGRAPHY

Jefferson Lab | September 16 - 25, 2024

The Center for Nuclear Femtography (CNF) and the Quark and Gluon Tomography (QGT) collaboration have joined forces to launch the First International School of Hadron Femtography. The school will take place at Jefferson Lab September 16-25, 2024. The program is designed to offer comprehensive lectures aimed at early-career experimental and theoretical scientists, including graduate students and post-doctoral researchers.

Acceptance to the program is through competitive application. Support will be provided for accepted participants, funded by CNF, supported by the Commonwealth of Virginia, and QGT, supported by the US Department of Energy. Participants will be housed on site at Jefferson Lab with ample opportunity for interactions with lecturers, and with other participants. Applications are now open, and for full consideration applications must be received by June 24, 2024.

Topics:
- QCD Analysis - Theory & Experiment
- Processes, DVCS, DVMP and multiparticle final states
- Lattice QCD
- Imaging Structure & Dynamics
- GPD analysis as an Inverse problem
- Experimental methodologies
- AI for nuclear femtography

Organizing Committee
- Martha Constantinou | Temple University, Co-Chair
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- Wally Melnitchouk | Jefferson Lab
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Further details can be found at:
https://www.jlab.org/conference/HadronFemtographySchool
Email: femtoschool@jlab.org