Hadronic Form Factors

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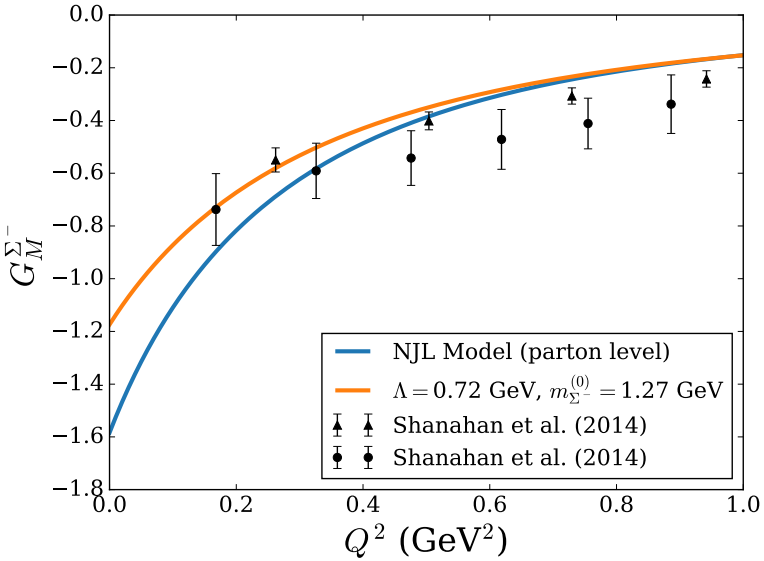
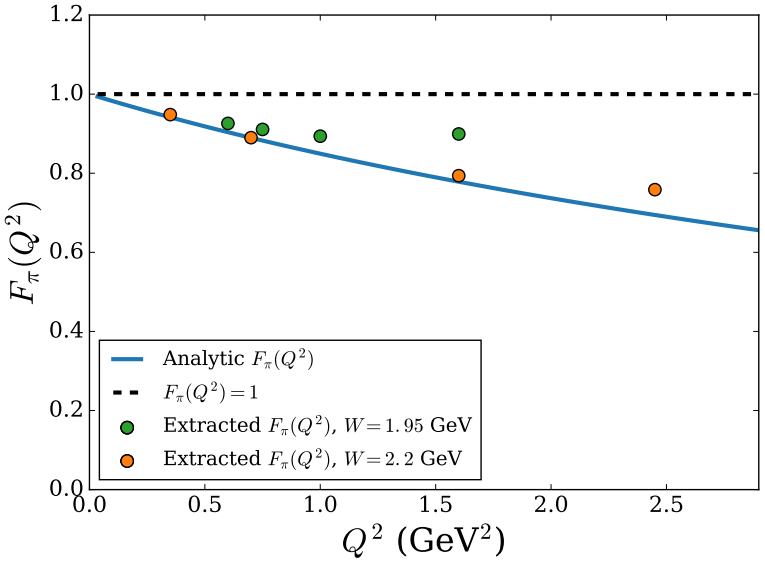
Talk Outline

- Motivation: Bridging QCD and low energy hadronic properties.
- Introduction to Form Factors
- Chiral Corrections to Baryon Electromagnetic Form Factors, arXiv:1703.01032
- Model Dependence of the Pion Form Factor Extracted from Pion Electro-production, arXiv:1811.09356
- Conclusion.





The QCD Lagrangian

- Lagrangian useful for understanding symmetries etc

$$\mathcal{L}_{\mathrm{QCD}} = \overline{q}(i\not{D} - m)q - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a$$

- Non-abelian, $SU(3)$ gauge field theory.
- Many questions remain.
- Masses of light quarks $\mathcal{O}(5\text{ MeV})$
- Lightest baryon ~ 1 GeV (approx. 2 orders of magnitude larger!)
- Where does this nucleon mass come from?
  - Emergent property of QCD.
  - Dynamical generation of mass contributes more than 95 percent of hadronic mass.

Strongly Coupled Physics

- In low energy region, QCD coupling runs, and theory becomes non-perturbative.
- Strongly coupled theory leads to emergent behavior.
  - Dynamical chiral symmetry breaking and mass generation.
  - Confinement.

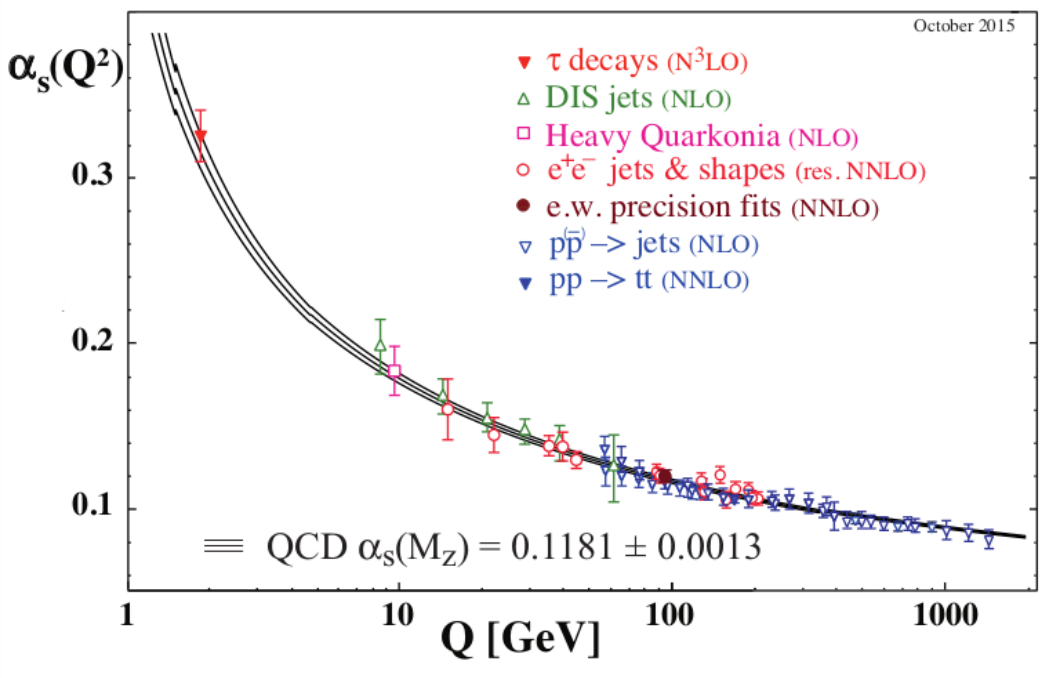


Figure 1: PDG, 2015

Open Questions Remain

The Central Goal of Hadronic Physics

- Central Goal of theoretical hadronic physics: Bridging the gap between $\mathcal{L}_{\text{QCD}}$ and observed hadronic properties.

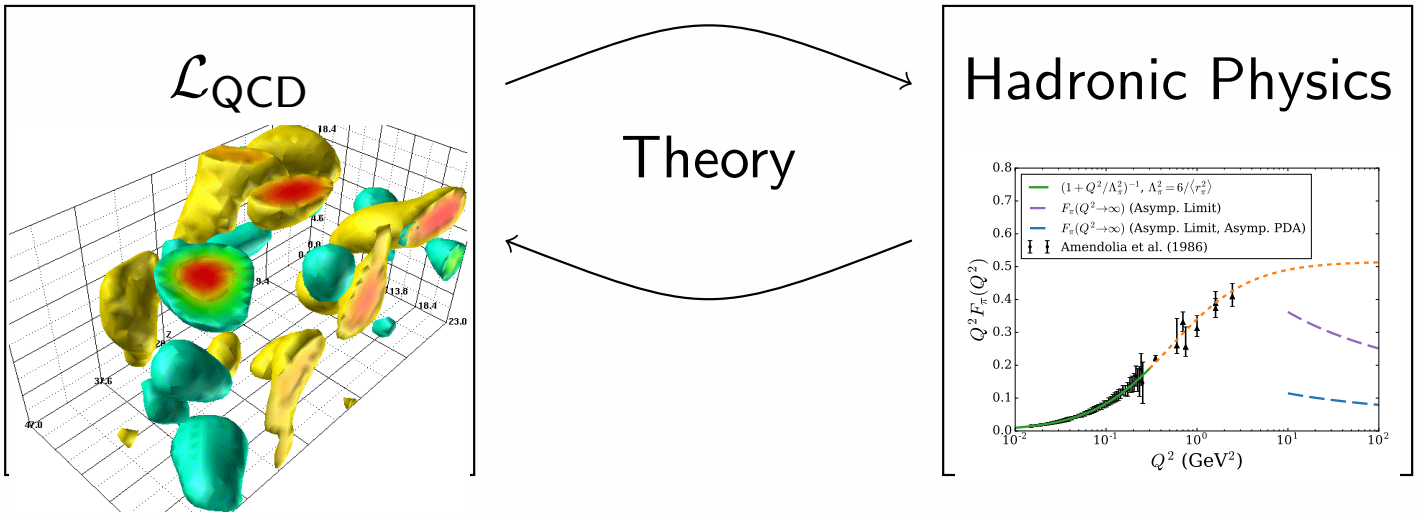


Figure 2: Image of gauge field configuration taken from J. Charvetto.

- Quark models, Chiral EFT, Lattice QCD, Schwinger-Dyson Equations.

Electromagnetic Form Factors

Historical Perspective

- Form factor introduced in 50's to explain proton scattering data.
- Introduce charge density $\rho(\vec{r})$.
- Form factor proportional to Fourier Transform of charge density (in NR limit): Extended structure.

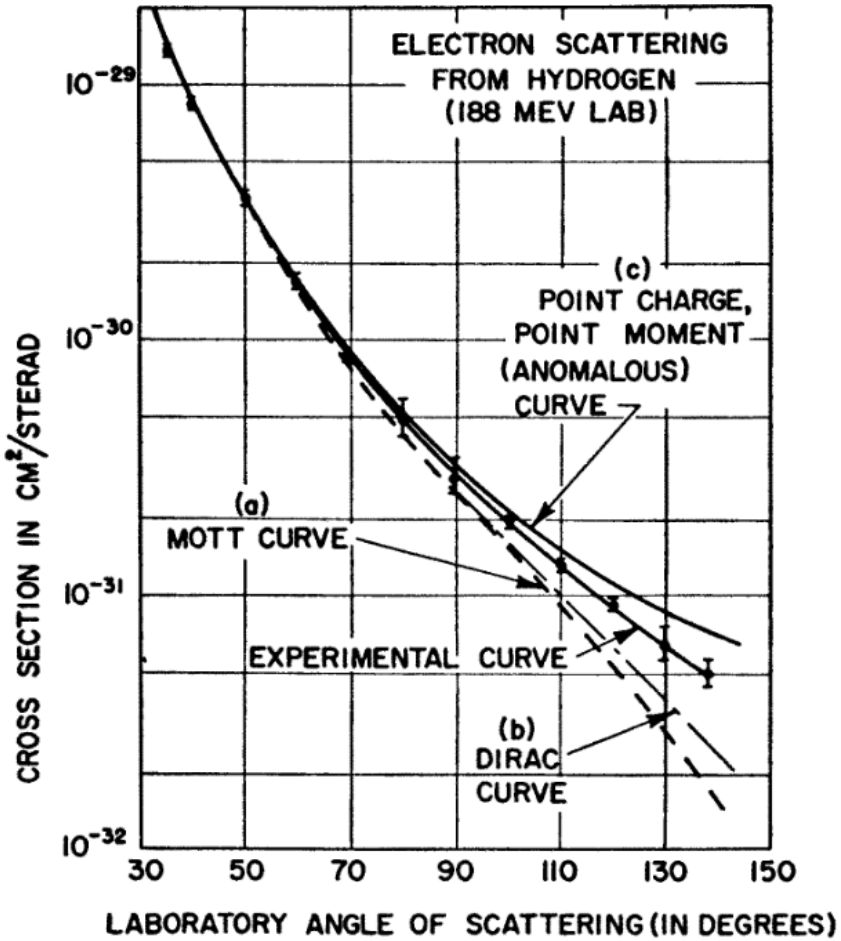
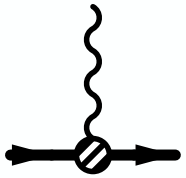


Figure 3: Figure taken from R. Hofstadter et al. (1958))

Baryon Electromagnetic Form Factors

- Contain information about the structure of the baryon.



$$= \overline{u}(p')\Gamma^{\mu}(p',p)u(p) = \overline{u}(p')\left[\gamma^{\mu}F_1(q^2) + \frac{i\sigma^{\mu\nu}q_{\nu}}{2m}F_2(q^2)\right]u(p)$$

- $Q^2 = -q^2$
- Common to use the Sachs Parametrisation.

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4m_N^2}F_2(Q^2)$$

$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

- 3D charge Radius for $i = E, M$

$$\langle r^2 \rangle = -\frac{6}{G_i(0)}\frac{d}{dQ^2}G_i(Q^2)\bigg|_{Q^2=0}$$

- Magnetic moment for $i = p, n$ (units of μ_N)

$$\mu_i = G_M^i(0)$$

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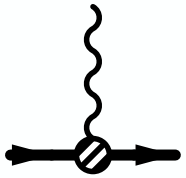

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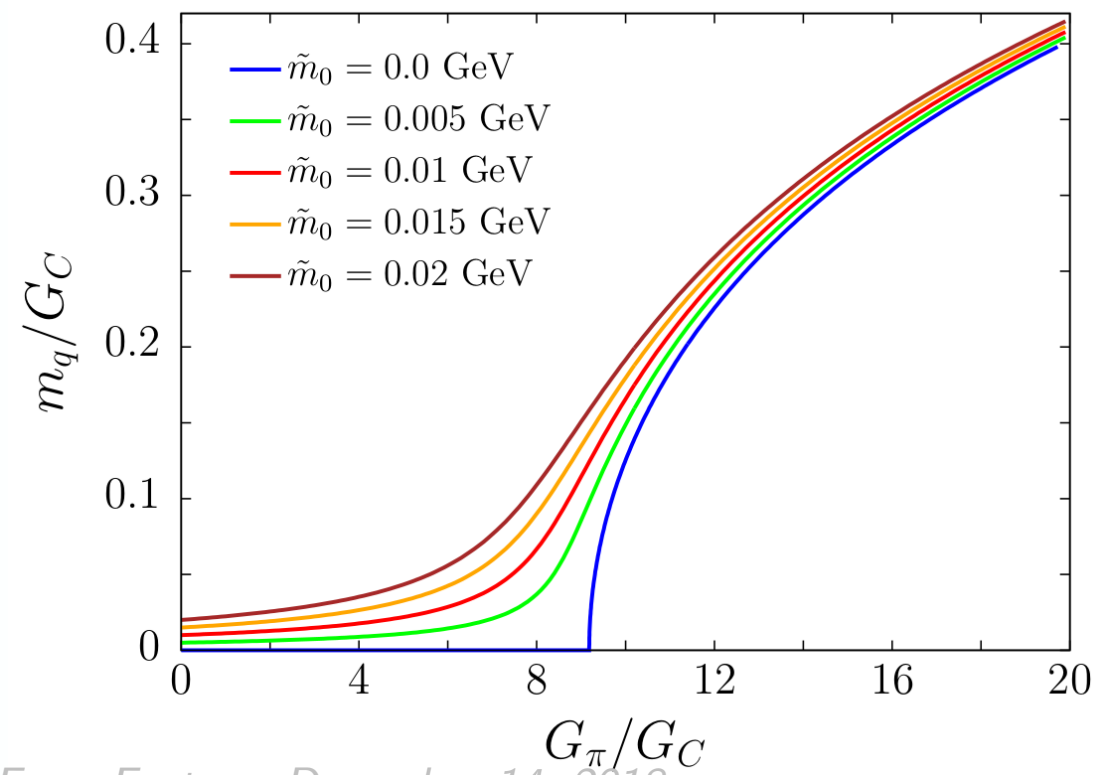
Calculating Electromagnetic Form Factors in the Nambu–Jona-Lasinio (NJL) Model

The Nambu–Jona-Lasinio (NJL) Model

- Low energy approximation of QCD: 4 fermion contact interaction

$$\mathcal{L} = \overline{\psi}(i\partial\!\!\!/ - \hat{m})\psi + \frac{1}{2}G_\pi\left[(\overline{\psi}\psi)^2 - (\overline{\psi}\gamma_5\vec{\tau}\psi)^2\right] - \frac{1}{2}G_\omega(\overline{\psi}\gamma^\mu\psi)^2 - \frac{1}{2}G_\rho\left[(\overline{\psi}\gamma^\mu\lambda_i\psi)^2 + (\overline{\psi}\gamma^\mu\gamma_5\lambda_i\psi)^2\right]$$

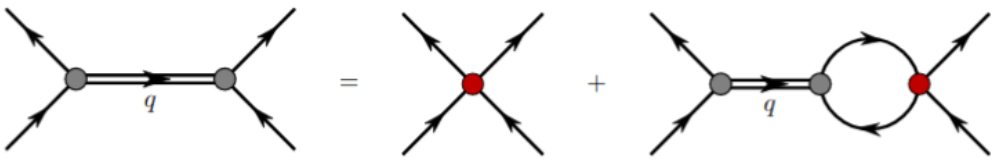


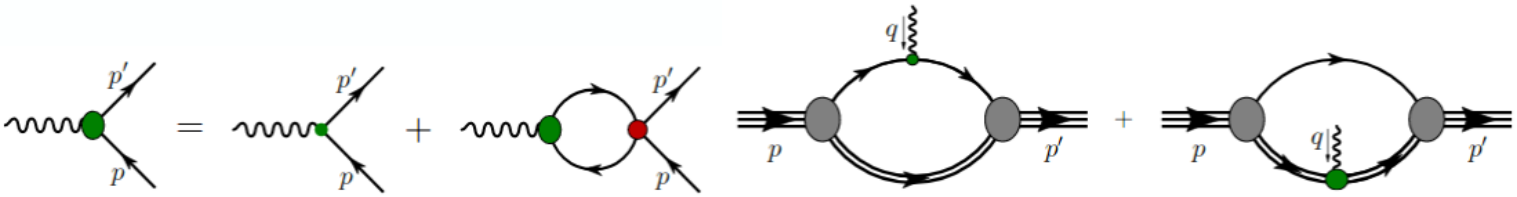
Confinement

- Confinement failure of basic model, but imposed via Proper Time Regularisation & infra-red cutoff.

$$\frac{1}{X^n} = \frac{1}{(n-1)!}\int_{1/\Lambda_{UV}^2}^{1/\Lambda_{IR}^2} d\tau \tau^{n-1} e^{-\tau X}$$

- Prevents singularities in the spectrum from on-shell quarks $\Longrightarrow$ confinement
- Calculate BSE equation





Chiral Corrections to Baryon Electromagnetic Form Factors

arXiv:1703.01032

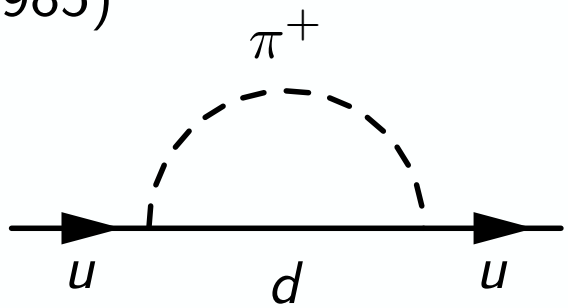
Pionic Corrections

- Long known that pion required dof in quark model calculations.
- Modern understanding of the pion as a pseudo-Goldstone Boson.
  - Result of dynamical chiral symmetry breaking.
- Formalized framework: χPT.
- Long distance (IR) properties are same as UV theory.
  - Must be respected in any model of QCD.
- A variety of ways to incorporate their effects.
  - Previously calculated in the NJL Model as a dressing on quark propagator.

Incorporating Pion Effects

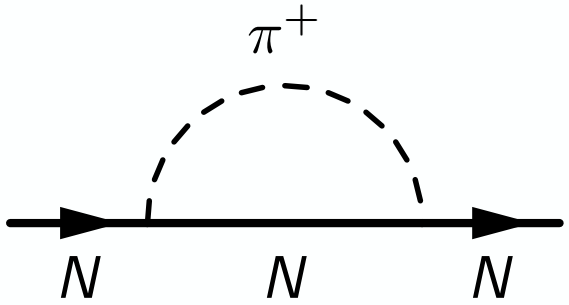
Quark Level

- Calculate pion effects from quark-pion coupling
- Idea goes back to Manohar and Georgi: *Chiral Quarks and the Non-Relativistic Quark Model* (1985)



Hadron Level

- Calculate pion loop corrections in (chiral) nucleon-pion EFT.



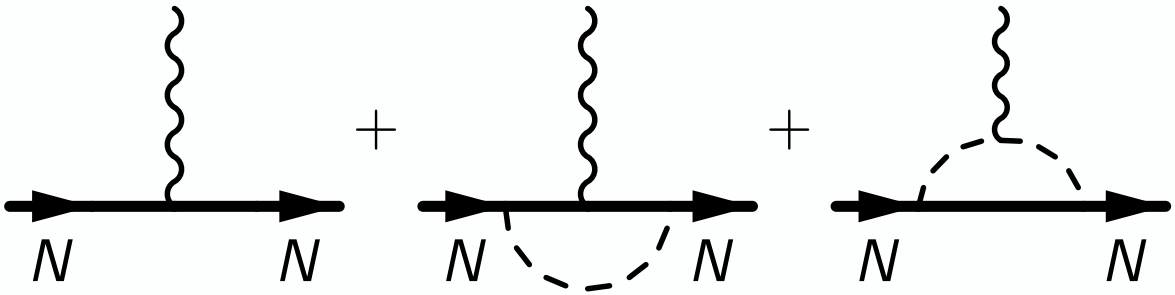
- Take guidance from χPT
- Correct LNA behvaior of nucleon mass only obtained in hadron level approach. (Model independent)
- We can examine the differences between the two approaches.

Pion-Nucleon Effective Field Theory

- Use chiral EFT.
- Work with a pseudoscalar pion-nucleon interaction:

$$\mathcal{L}_{N\pi} = -ig_{\pi N}\overline{\psi}_N \gamma_5 \vec{\tau} \cdot \vec{\pi} \psi_N$$

- After minimal substitution, one has three diagrams at first loop order.



$$F_i^H(Q^2) = Z\left[F_{i,a}^H(Q^2) + F_{i,b}^H(Q^2) + F_{i,c}^H(Q^2)\right]$$

Baryon Self Energy

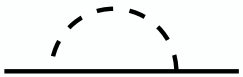
Bare Calculation

Calculate bare (pionless) form factors in NJL Model

$$= \frac{i}{\not{p} - m_N^{(0)} + i\epsilon}$$

Dressed State

Calculate chiral loops



$$= \frac{iZ}{\not{p} - m_N + i\epsilon}$$

- Must fit NJL model parameters to Bare Mass
- Related to physical mass via

$$m_N = m_N^{(0)} + \Sigma(\not{p})\big|_{\not{p}=m_N}$$

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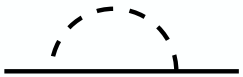

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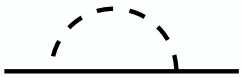

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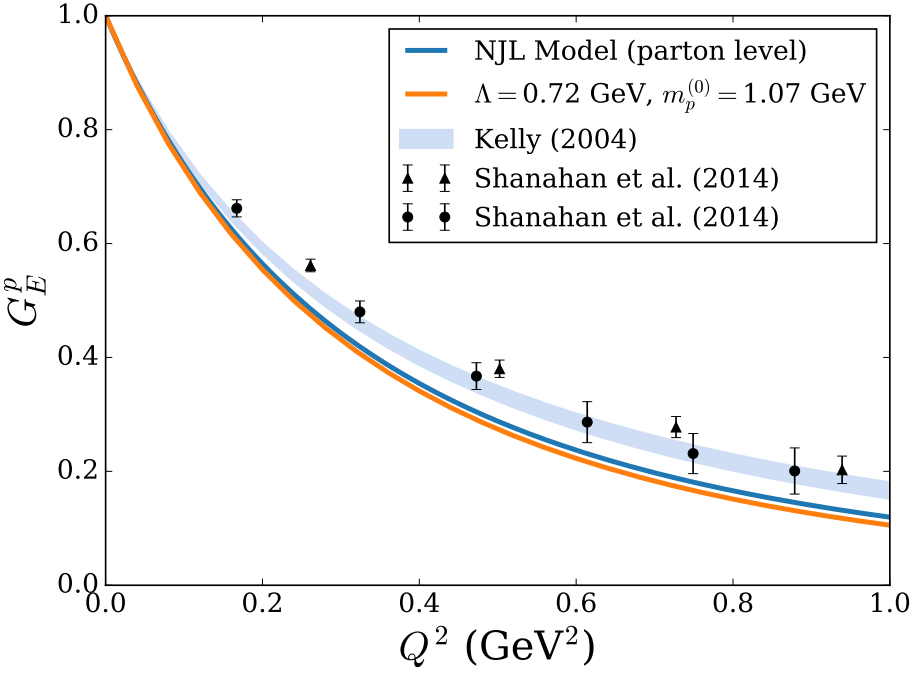
Nucleon Results



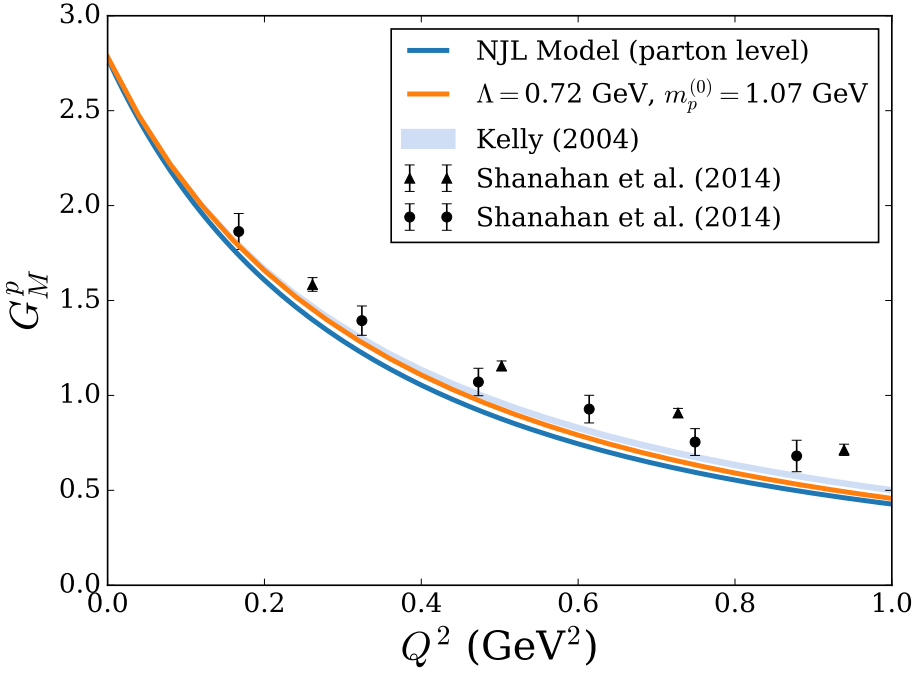


Figure 5: G_E^p and G_M^p. Data from lattice studies

- Similar!

Nucleon Results

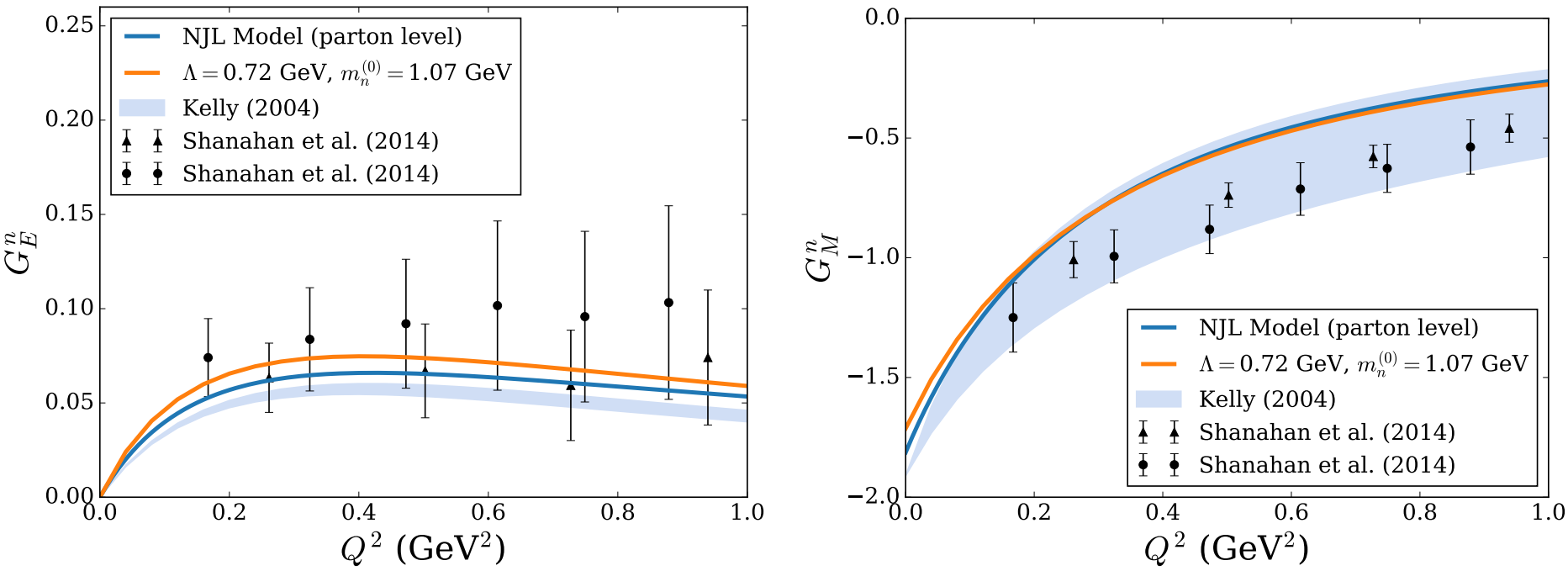


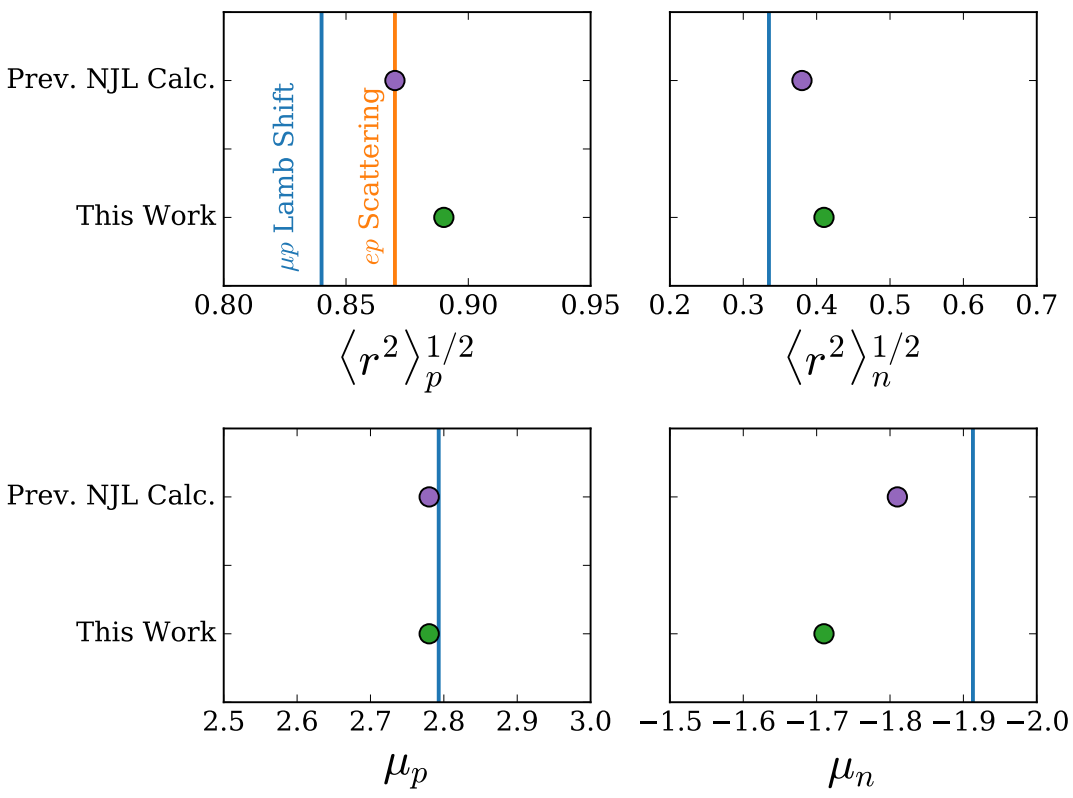
Figure 6: G_E^n and G_M^n. Data from lattice studies

- Similar!

Comparison

$\langle r^2 \rangle^{\frac{1}{2}}$	p	n
Prev. NJL Calc.	0.87	0.38
This Work	0.89	0.41
Exp.	0.84 [3]	0.335

μ	p	n
Prev. NJL Calc.	2.78	-1.81
This Work	2.78	-1.71
Exp.	2.793	-1.913



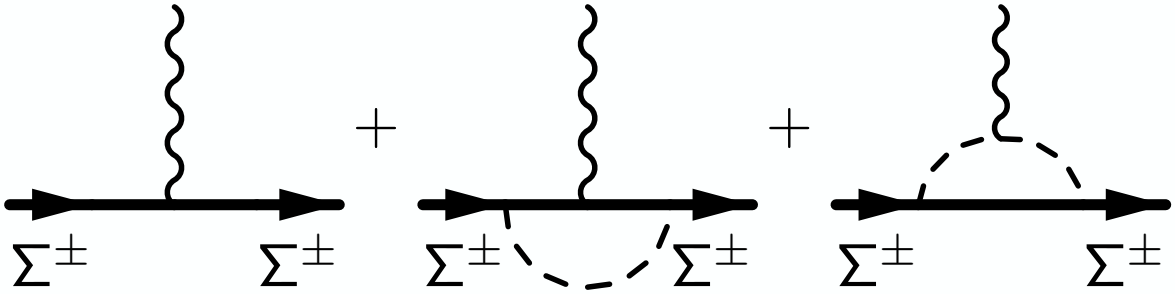
Generalizing Results to Hyperons

Generalization to Hyperons

- Due to approximate $SU(3)_F$ symmetry, one has relations between nucleon-pion and hyperon-pion couplings.

$$g_{\Lambda\Sigma\pi} = \frac{2}{\sqrt{3}}(1-\alpha)g_{NN\pi}; \qquad g_{\Sigma\Sigma\pi} = 2\alpha g_{NN\pi}$$

- Although the particles themselves are different, topology of contributing diagrams are the same.



- Simple replacements in equations allows generalization of the equations to consider the hyperons.

Hyperon Results

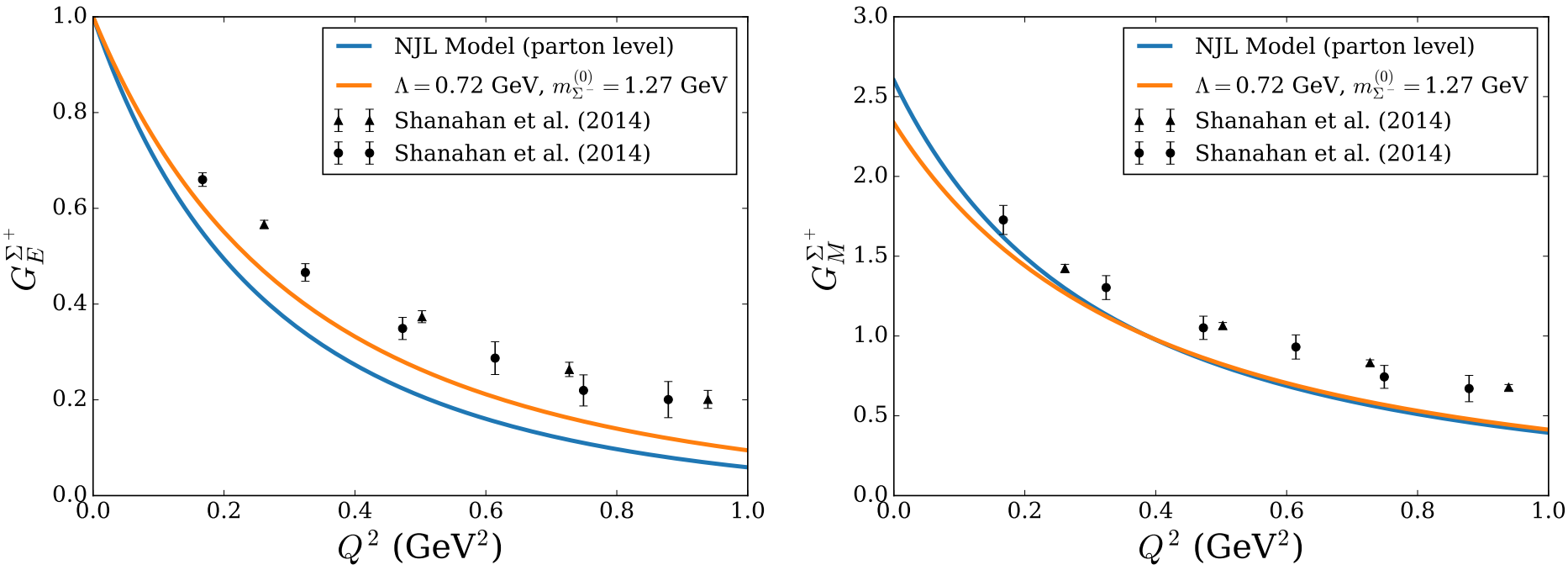


Figure 7: $G_E^{\Sigma^+}$ and $G_M^{\Sigma^+}$, data from lattice studies.

Hyperon Results

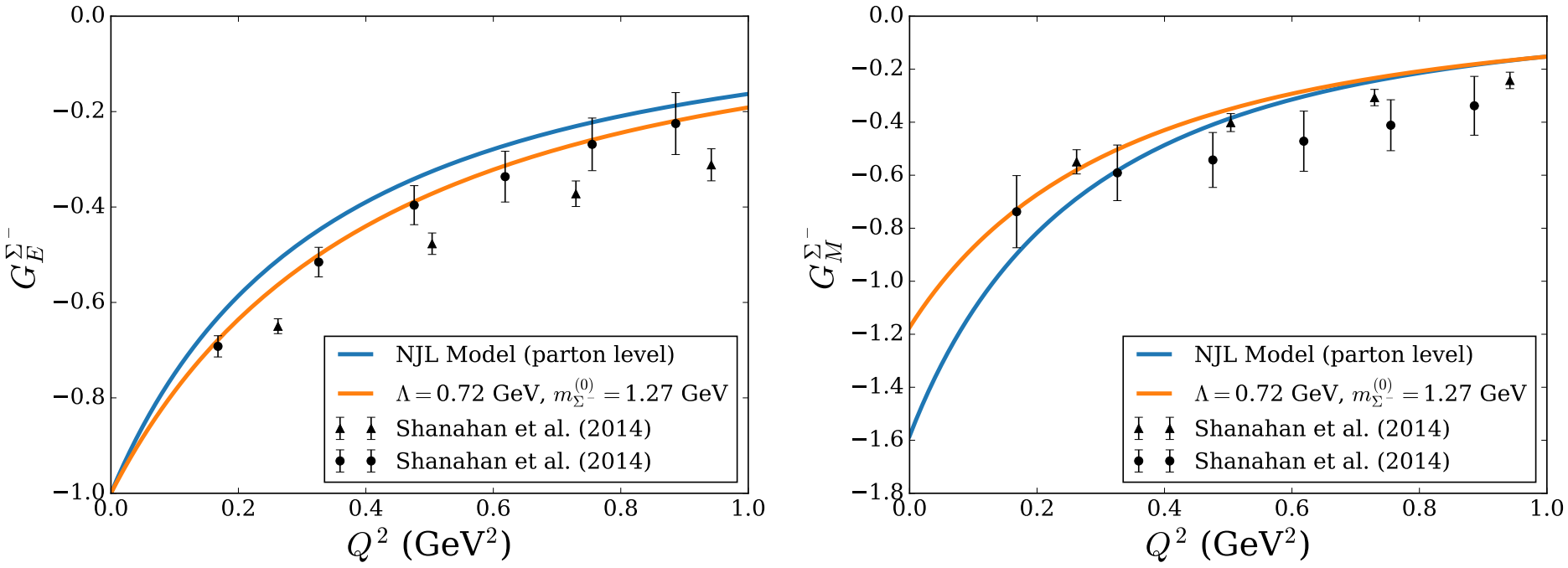
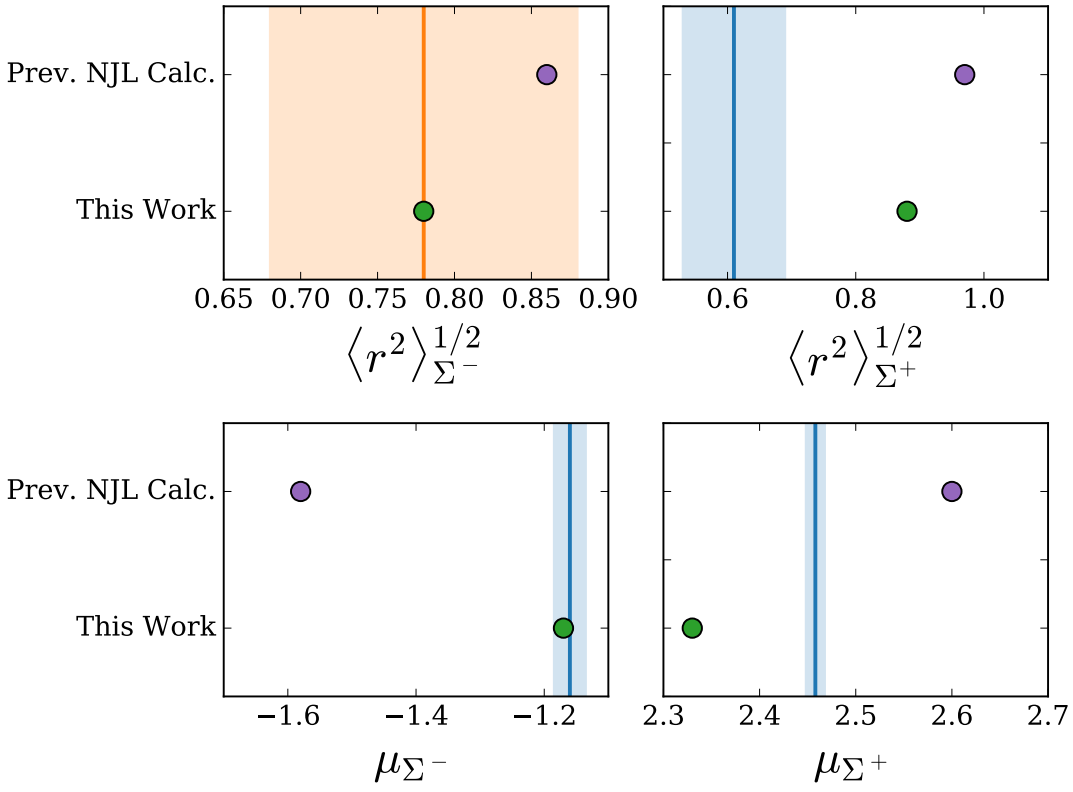


Figure 8: $G_E^{\Sigma^-}$ and $G_M^{\Sigma^-}$, data from lattice studies.

Comparison

	$\langle r^2 \rangle^{\frac{1}{2}}$ Σ^-	Σ^+
Prev. Calc	0.86	0.97
This Work	0.78	0.88
Exp.	0.780	0.61(8) [4]

	μ Σ^-	Σ^+
Prev. Calc.	-1.58	2.60
This Work	-1.17	2.33
Exp.	-1.160(25)	2.458(10)



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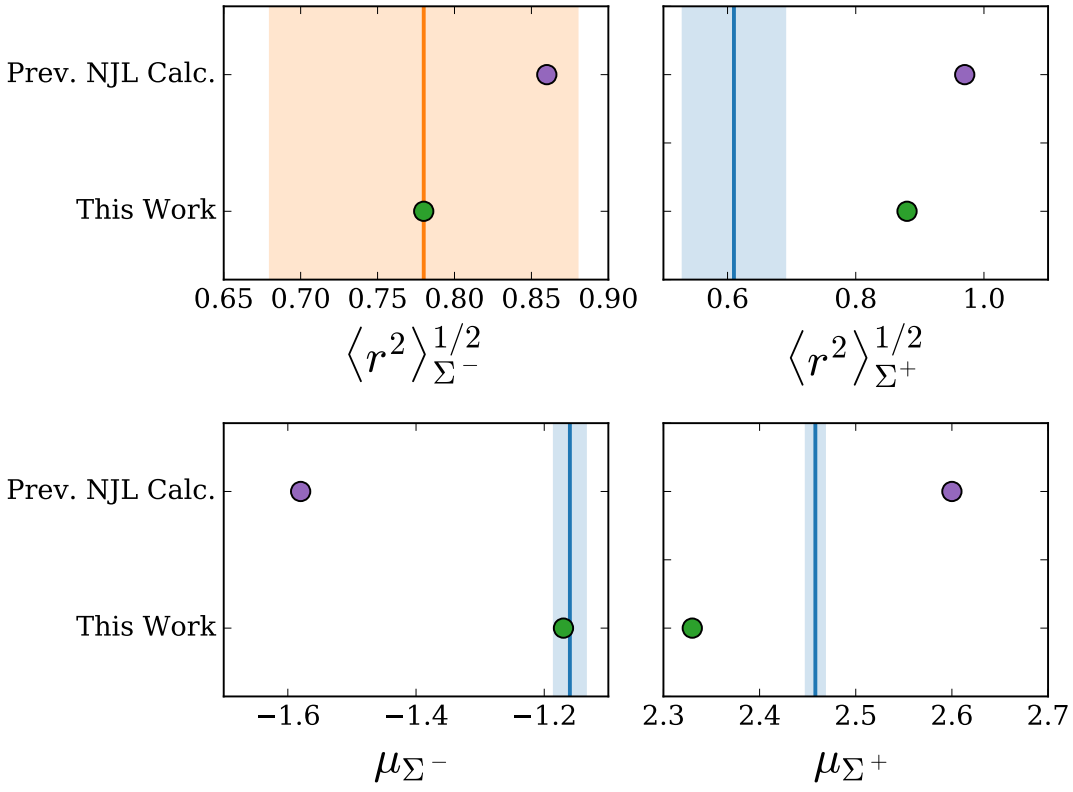

Quark Model

- Large contribution comes from π^- cloud on d quark ($e_{\pi^-} = -1$, $e_d = -1/3$).
- Sigma minus:

$$\left|\Sigma^-\right\rangle = \frac{1}{\sqrt{18}}[2\left|d_\uparrow d_\uparrow s_\downarrow\right\rangle + \text{perm.} - \left|d_\uparrow d_\downarrow s_\uparrow\right\rangle + \text{perm.}]$$

- Leads to coherent enhancement.

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Summary

To summarize...

- χPT gives model independent information on IR physics.
- Calculated chiral loop corrections to the NJL model at the Hadron Level.
- Nucleon system insensitive to approach, but
- Hyperon system sensitive to implementation of pion loops: improvement of Σ^- magnetic moment.

The Pion Electromagnetic Form Factor

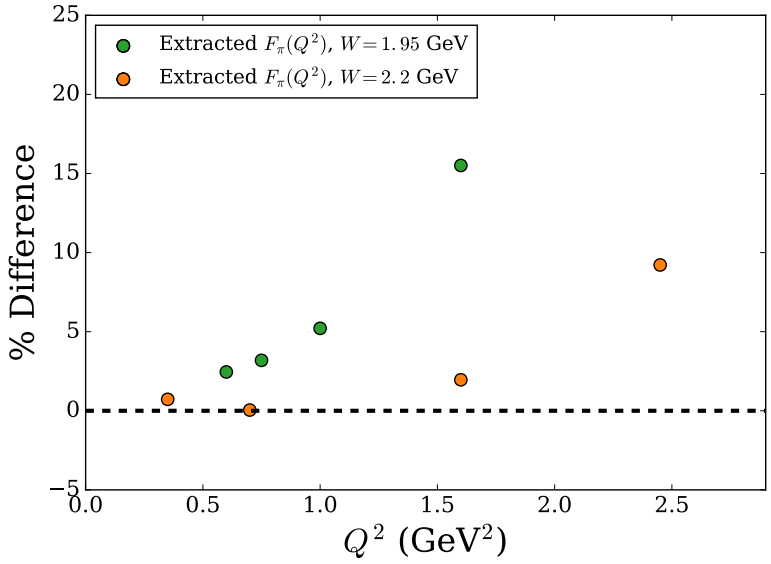


Figure 9: F_π extracted from simple model of pion electro-production.

The pion is special

- χPT $\implies$ important for low energy hadronic physics.
- Simplest QCD system: 'Hydrogen Atom of QCD': Excellent testing ground.
- Form factor spans large energy range: forces us to use a number of approaches.
- Must understand the model used to extract the form factor well.
  - Based on some theoretical arguments, we wanted to check the model dependence of the extracted pion form factor.

Theoretical Approaches

Low Energy

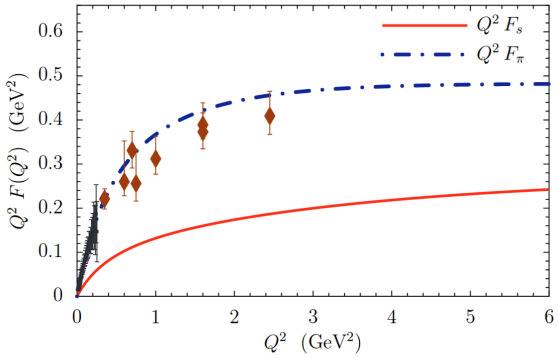


Figure 10: Cloët et al. (2014)

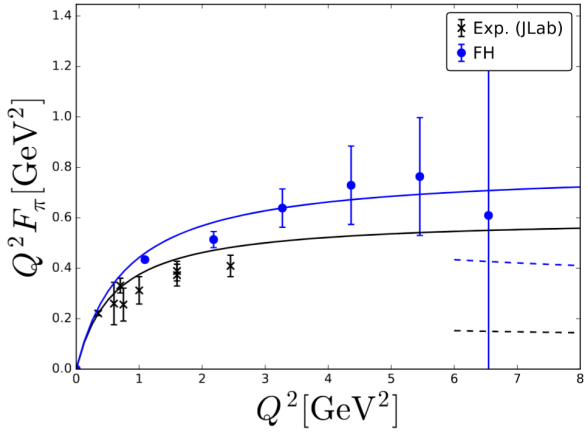


Figure 11: FH approach, $m_\pi = 470$ MeV, Chambers et al. (2017)

High Energy

Lepage and Brodsky:

$$Q^2 F_\pi(Q^2) \to 16\pi f_\pi^2 \alpha_s(Q^2)\omega_\phi^2, \text{ for } Q^2 > Q_0^2$$

Historically, $\omega_\phi = 1$:

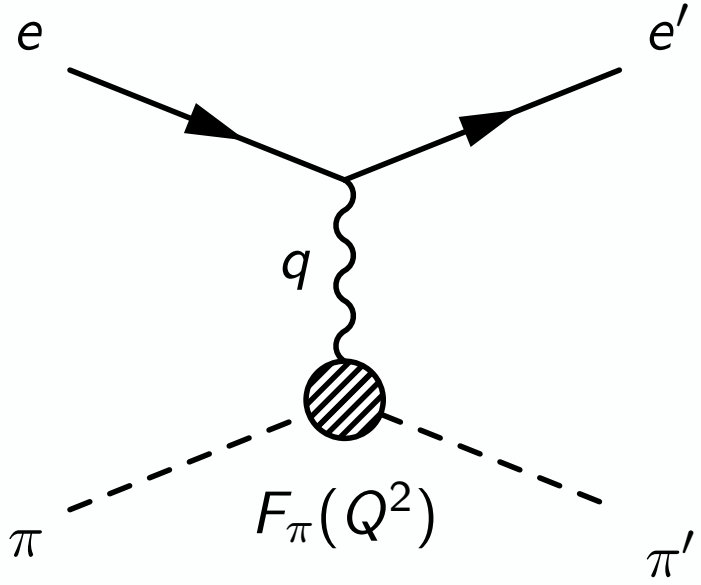
$$\omega_\phi = \frac{1}{3}\int_0^1 dx \frac{1}{x}\phi_\pi(x)$$

$$\lim_{Q^2\to\infty} \phi_\pi(x) = 6x(1-x)$$

Measuring the pion form factor

Experimental Measurements

- At low energy (~ 0.3 GeV^2), scatter pion beam from electrons in liquid hydrogen target.
- Measure recoiling pion and electron.

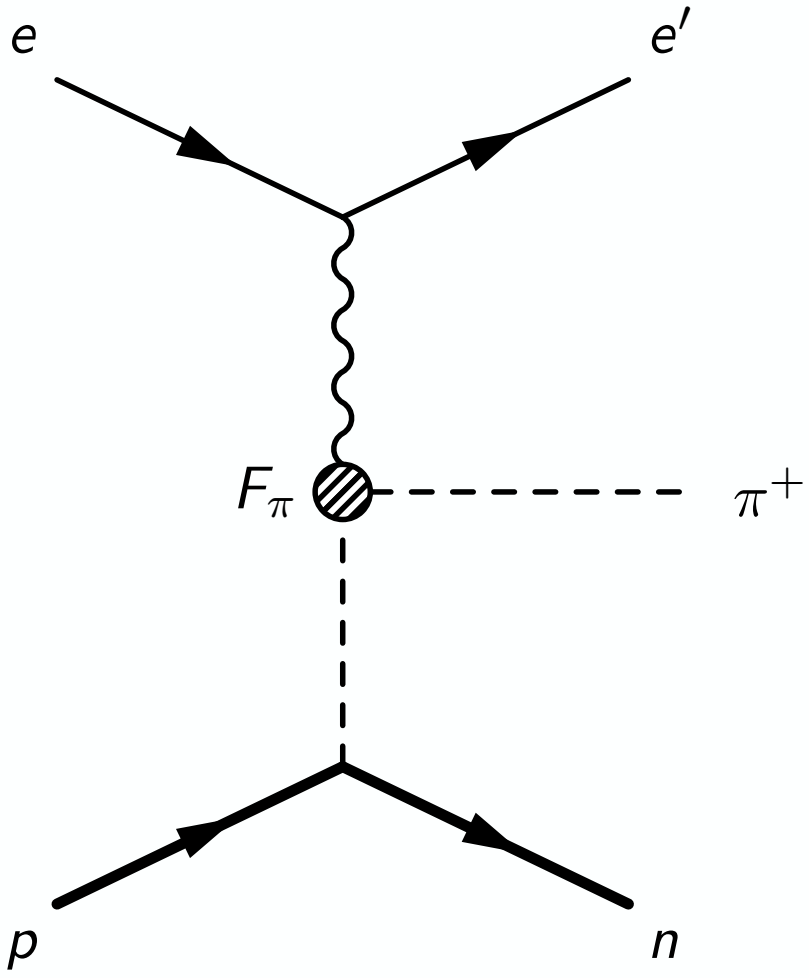


- Differential cross section is

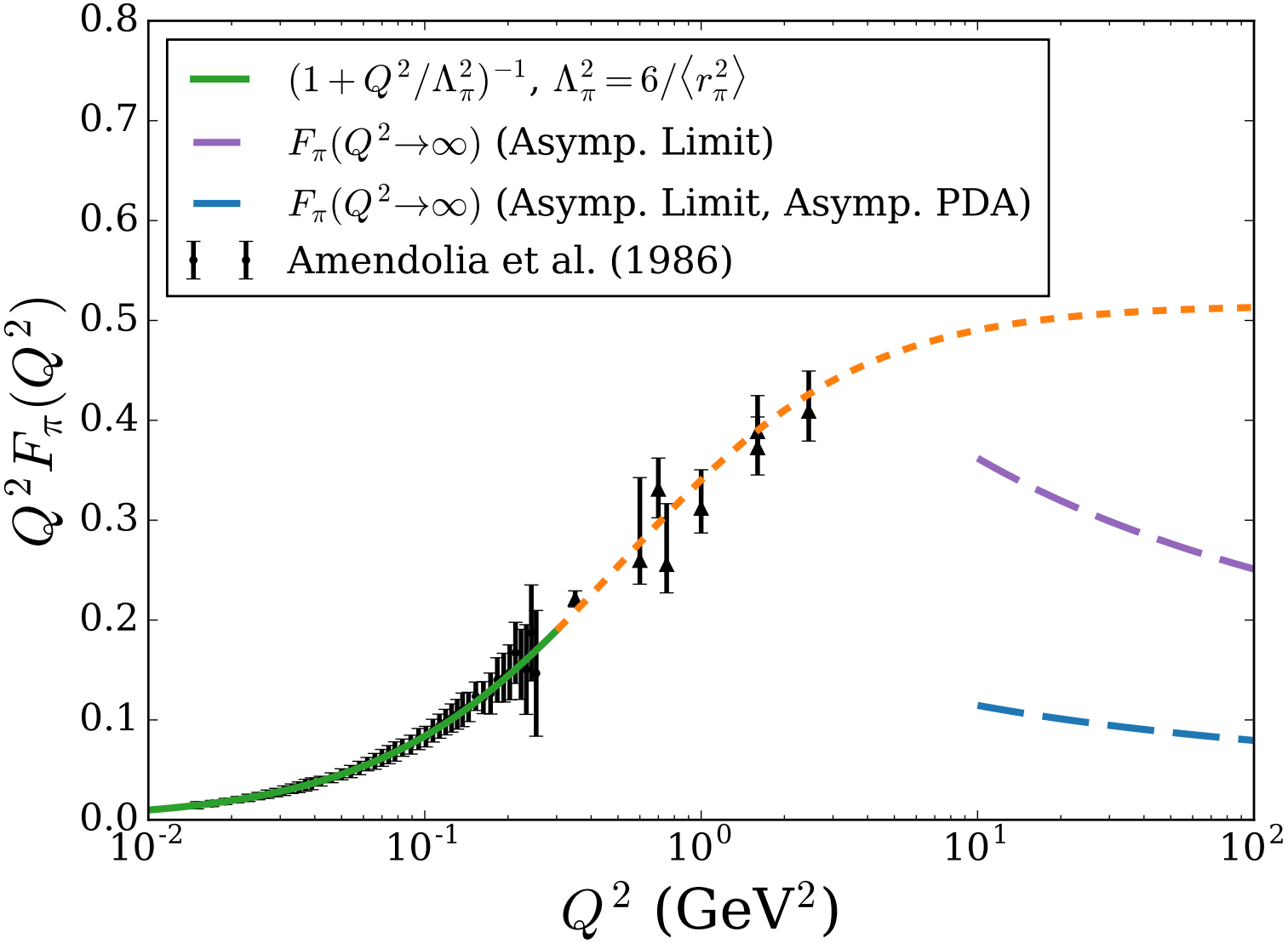
$$\frac{d\sigma}{dq^2} \propto |F_\pi|^2 \frac{1}{q^4}\left(1 - \frac{q^2}{q^2_{\text{max}}}\right)$$

Pion Electro-production

- Scatter electron off liquid hydrogen target.
- Knock pion out of nucleon's virtual meson cloud.
- Measure recoiling electron and produced pion.
- Two theoretical questions:
  1. How does F_π enter cross section?
  2. How does the 'off-shellness' effect the measurement of F_π?
- Must understand how extraction is currently performed.



The State of the Art



Understanding the F_π Measurement

PHYSICAL REVIEW C **78**, 045202 (2008)

Charged pion form factor between $Q^2 = 0.60$ and 2.45 GeV^2. I. Measurements of the cross section for the ^{1}H$(e, e'\pi^+)n$ reaction

H. P. Blok,[1,2] T. Horn,[3,4] G. M. Huber,[5] E. J. Beise,[3] D. Gaskell,[4] D. J. Mack,[4] V. Tadevosyan,[6] J. Volmer,[7,8] D. Abbott,[4] K. Aniol,[9] H. Anklin,[4,10] C. Armstrong,[11] J. Arrington,[12] K. Assamagan,[13] S. Avery,[13] O. K. Baker,[4,13] B. Barrett,[14] C. Bochna,[15] W. Boeglin,[10] E. J. Brash,[5] H. Breuer,[3] C. C. Chang,[3] N. Chant,[3] M. E. Christy,[13] J. Dunne,[4] T. Eden,[4,16] R. Ent,[4] H. Fenker,[4] E. Gibson,[17] R. Gilman,[4,18] K. Gustafsson,[3] W. Hinton,[13] R. J. Holt,[12] H. Jackson,[12] S. Jin,[19] M. K. Jones,[11] C. E. Keppel,[4,13] P. H. Kim,[19] W. Kim,[19] P. M. King,[3] A. Klein,[20] D. Koltenuk,[21] V. Kovaltchouk,[5] M. Liang,[4] J. Liu,[3] G. J. Lolos,[5] A. Lung,[4] D. J. Margaziotis,[9] P. Markowitz,[10] A. Matsumura,[22] D. McKee,[23] D. Meekins,[4] J. Mitchell,[4] T. Miyoshi,[22] H. Mkrtchyan,[6] B. Mueller,[12] G. Niculescu,[24] I. Niculescu,[24] Y. Okayasu,[22] L. Pentchev,[11] C. Perdrisat,[11] D. Pitz,[25] D. Potterveld,[12] V. Punjabi,[16] L. M. Qin,[20] P. Reimer,[12] J. Reinhold,[10] J. Roche,[4] P. G. Roos,[3] A. Sarty,[14] I. K. Shin,[19] G. R. Smith,[4] S. Stepanyan,[6] L. G. Tang,[4,13] V. Tvaskis,[7] R. L. J. van der Meer,[5] K. Vansyoc,[20] D. Van Westrum,[26] S. Vidakovic,[5] W. Vulcan,[4] G. Warren,[4] S. A. Wood,[4] C. Xu,[5] C. Yan,[4] W.-X. Zhao,[27] X. Zheng,[12] and B. Zihlmann[4,28]
(Jefferson Lab F_π Collaboration)

PHYSICAL REVIEW C **78**, 045203 (2008)

Charged pion form factor between $Q^2 = 0.60$ and 2.45 GeV^2. II. Determination of, and results for, the pion form factor

G. M. Huber,[1] H. P. Blok,[2,3] T. Horn,[4,5] E. J. Beise,[4] D. Gaskell,[5] D. J. Mack,[5] V. Tadevosyan,[6] J. Volmer,[2,7] D. Abbott,[5] K. Aniol,[8] H. Anklin,[5,9] C. Armstrong,[10] J. Arrington,[11] K. Assamagan,[12] S. Avery,[12] O. K. Baker,[5,12] B. Barrett,[13] C. Bochna,[14] W. Boeglin,[9] E. J. Brash,[1] H. Breuer,[4] C. C. Chang,[4] N. Chant,[4] M. E. Christy,[12] J. Dunne,[5] T. Eden,[5,15] R. Ent,[5] H. Fenker,[5] E. F. Gibson,[16] R. Gilman,[5,17] K. Gustafsson,[4] W. Hinton,[12] R. J. Holt,[11] H. Jackson,[11] S. Jin,[18] M. K. Jones,[10] C. E. Keppel,[5,12] P. H. Kim,[18] W. Kim,[18] P. M. King,[4] A. Klein,[19] D. Koltenuk,[20] V. Kovaltchouk,[1] M. Liang,[5] J. Liu,[4] G. J. Lolos,[1] A. Lung,[5] D. J. Margaziotis,[8] P. Markowitz,[9] A. Matsumura,[21] D. McKee,[22] D. Meekins,[5] J. Mitchell,[5] T. Miyoshi,[21] H. Mkrtchyan,[6] B. Mueller,[11] G. Niculescu,[23] I. Niculescu,[23] Y. Okayasu,[21] L. Pentchev,[10] C. Perdrisat,[10] D. Pitz,[24] D. Potterveld,[11] V. Punjabi,[15] L. M. Qin,[19] P. E. Reimer,[11] J. Reinhold,[9] J. Roche,[5] P. G. Roos,[4] A. Sarty,[13] I. K. Shin,[18] G. R. Smith,[5] S. Stepanyan,[6] L. G. Tang,[5,12] V. Tvaskis,[2,3] R. L. J. van der Meer,[1] K. Vansyoc,[19] D. Van Westrum,[25] S. Vidakovic,[1] W. Vulcan,[5] G. Warren,[5] S. A. Wood,[5] C. Xu,[1] C. Yan,[5] W.-X. Zhao,[26] X. Zheng,[11] and B. Zihlmann[5,27]
(Jefferson Lab F_π Collaboration)

Decomposing the Cross Section

- Cross section described in terms of
  - Q^2: photon virtuality.
  - W: Invariant mass of virtual photon proton system.
  - $t = (p_\pi - q)^2$: Expresses virtuality of pion.
- Cross section may be decomposed into 4 structure functions.

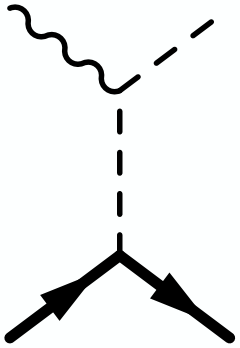
$$(2\pi)\frac{d^2\sigma}{dtd\phi} = \frac{d\sigma_T}{dt} + \epsilon\frac{d\sigma_L}{dt} + \sqrt{2\epsilon(\epsilon+1)}\frac{d\sigma_{LT}}{dt}\cos\phi + \epsilon\frac{d\sigma_{TT}}{dt}\cos 2\phi.$$

ϵ is a measure of the virtual photon polarization

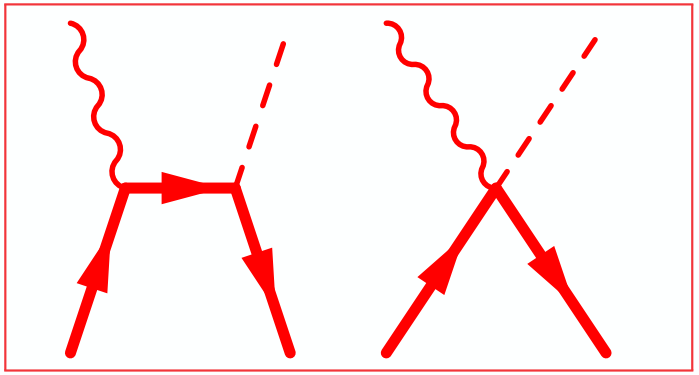
- Important, as is known that Longitudinal cross section dominated by t-channel pion exchange.
  - A good reconstruction of this structure function gives us a good change of extracting the pion form factor.
  - The modern extraction uses the Vanderhaeghen Guidal and Laget (VGL) Model.

Born Term Model of Electro-production

- VGL Model based on Born Term pion exchange diagram:



(a) t-channel pion exchange



(b) Required to restore gauge invariance

- Model is not gauge invariant, so one must include the s-channel diagram and KR term (when using a PV coupling) to restore gauge invariance.
- This is $i\mathcal{M}^{\mu}_{\text{BTM}}$

Gauge Invariance in BTM Model

- WTI requires $q_\mu \mathcal{M}^\mu_{\text{BTM}} = 0$

$$iq_\mu \mathcal{M}^\mu_{\text{BTM}} \propto \bar{u}_N \left[\gamma_5 \not{p}_\pi \frac{(\not{p}_s + m_N)}{s - m_N^2} \not{q} + \gamma_5 \not{p}_t \frac{q \cdot (p_t + p_\pi)}{t - m_\pi^2} - \gamma_5 \not{q} \right] u_N$$

$$\propto \bar{u}_N \gamma_5 \left[\not{p}_\pi - (\not{p}_\pi - \not{q}) - \not{q} \right] u_N$$

- Delicate cancellation required.
- Limits the ways we can modify this amplitude.

Improving Agreement with Data: Reggeizing Amplitude

- Agreement between the model and data may be improved by Reggeizing the amplitude.
  - Replace the Feynman Propagator for the t-channel pion exchange by its Reggeized version

$$S_{\mathrm{R}}^{\pi}(t) = i(\alpha'_{\pi} W^2)^{\alpha_{\pi}(t)} \frac{\pi \alpha'_{\pi} \phi(t)}{\sin(\pi \alpha_{\pi}(t) \Gamma(1 + \alpha_{\pi}(t)))}$$

- Unless the s-channel and contact terms are also modified, gauge invariance will be broken again.
- This is done in the VGL Model by multiplying these terms by a factor $S_F^{\pi -1}(t) S_{\mathrm{R}}^{\pi}(t)$
- One can also understand this Reggeization as multiplication of $i\mathcal{M}^{\mu}_{\mathrm{BTM}}$ by this overall factor:

$$i\mathcal{M}^{\mu}_{\mathrm{R}} = S_F^{\pi -1}(t) S_{\mathrm{R}}^{\pi}(t) \times [i\mathcal{M}^{\mu}_{\mathrm{BTM}}]$$

The VGL Model

- The pion structure is incorporated my multiplying this amplitude by a factor of the pion form factor.
- To summarize:

$$\begin{aligned} i\mathcal{M}^{\mu}_{\text{VGL}} =& F_{\pi}(Q^2) \times [i\mathcal{M}^{\mu}_{\text{R}}] \\ =& F_{\pi}(Q^2) \times S_F^{\pi -1}(t) S^{\pi}_{\text{R}}(t) \times [i\mathcal{M}^{\mu}_{\text{BTM}}] \end{aligned}$$

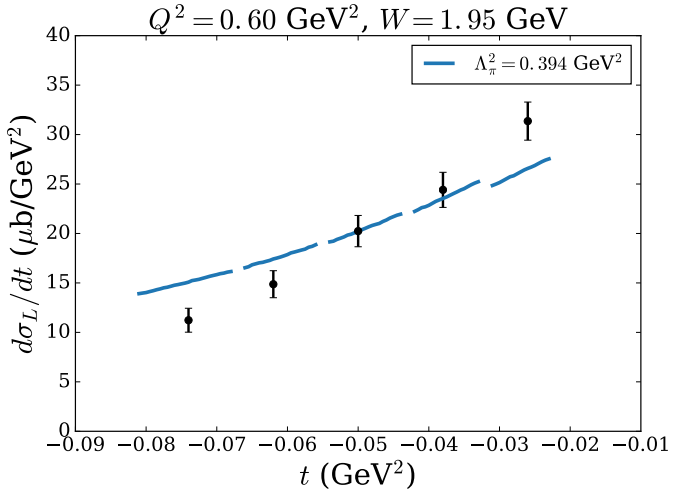
- In theory, one would expect s-channel diagram to be proportional to $F_1^p(Q^2)$, but this breaks gauge invariance.
- Only possible to have single form factor. Amounts to $F_1^p(Q^2) \approx F_{\pi}(Q^2)$.

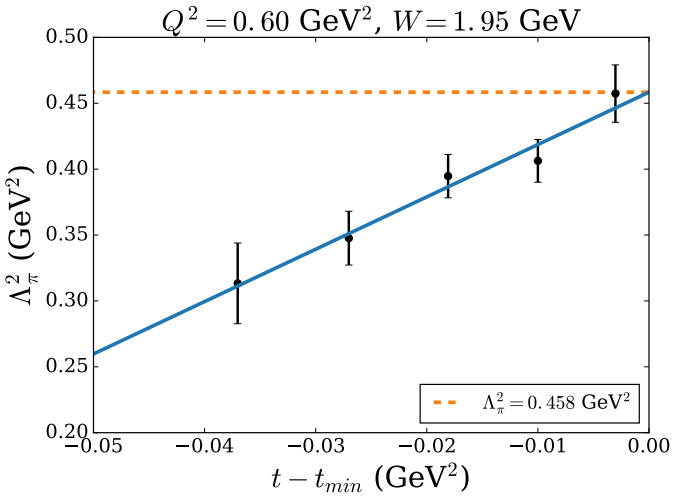
Extracting Pion Form Factor from Data

- Measure cross section at a range of t values for fixed Q^2 and W.
- Longitudinal cross section is

$$\frac{d\sigma_L}{dt} \propto |F_\pi|^2$$

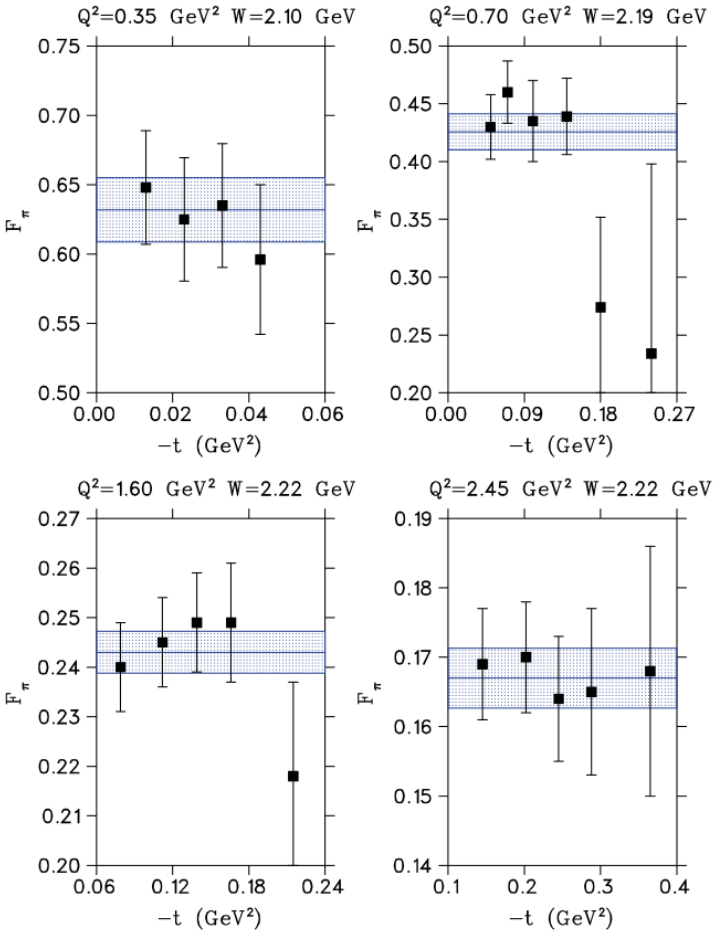
- $F_\pi(Q^2) = (1 + Q^2/\Lambda_\pi^2)^{-1}$
- Fit model to cross section.
- If required...
  - Fit each data point.
  - Extrapolate these points to $t = t_{\text{min}}$, where there is least contamination from interfering backgrounds not included in the VGL model.





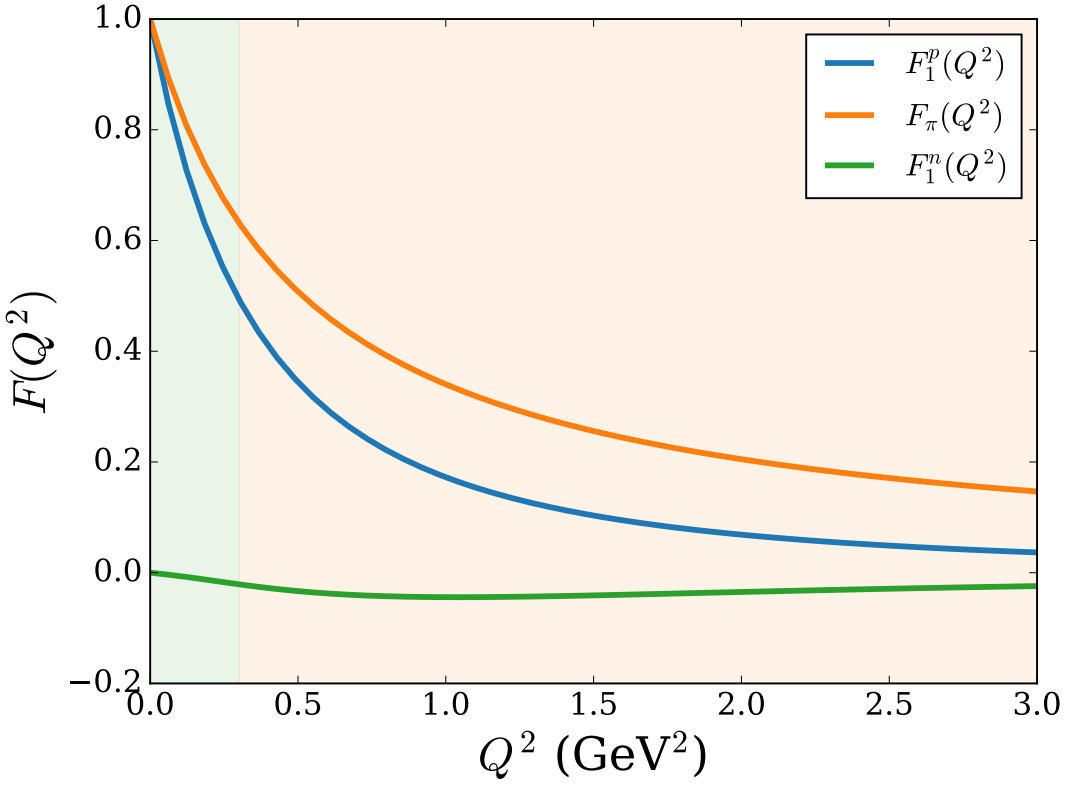
Sanity Checks of Extraction

- Clearly some simplifications in this model.
- How do we know we are extracting the pion form factor?



Key Questions

1. Currently, $F_1^p = F_\pi$: can we do better?
2. Can we incorporate the 'off-shellness' of particles?
3. What are the implications for the current measured data points?



Model Dependence of the Pion Form Factor Extracted from Pion Electro-production Data

arXiv:1811.09356

Our Approach

1. ~~Currently, form factors are all the same: can we do better?~~
2. ~~Can we incorporate the 'off-shellness' of particles?~~
3. What are the implications for the current measured data points?
   - Generate cross section in model (pseudodata), and then attempt to extract form factor using VGL-like Model.

	Current Extraction	This Analysis
Model	$i\mathcal{M}^{\mu}_{\mathrm{VGL}} = F_{\pi}(Q^2) S_F^{\pi-1} S_{\pi}^{\mathrm{R}}(t)[i\mathcal{M}^{\mu}_{\mathrm{BTM}}]$	$i\mathcal{M}^{\mu} = F_{\pi}(Q^2)[i\mathcal{M}^{\mu}_{\mathrm{BTM}}]$
	$\downarrow$ fit to... $\downarrow$	$\downarrow$ fit to... $\downarrow$
Data	$^1\mathrm{H}(e, e'\pi^+)n$	Pseudodata

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	$\downarrow$ fit to... $\downarrow$	$\downarrow$ fit to... $\downarrow$
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Model	$i\mathcal{M}^{\mu}_{\mathrm{VGL}} = F_{\pi}(Q^2) S_F^{\pi-1} S_{\pi}^{\mathrm{R}}(t)[i\mathcal{M}^{\mu}_{\mathrm{BTM}}]$	$i\mathcal{M}^{\mu} = F_{\pi}(Q^2)[i\mathcal{M}^{\mu}_{\mathrm{BTM}}]$
	$\downarrow$ fit to... $\downarrow$	$\downarrow$ fit to... $\downarrow$
Data	${}^{1}\mathrm{H}(e, e'\pi^{+})n$	Pseudodata

A Bosonic Model of Pion Electro-production

- Inspired by a simple model due to Miller.

PHYSICAL REVIEW C **80**, 045210 (2009)

Electromagnetic form factors and charge densities from hadrons to nuclei

Gerald A. Miller
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(Received 18 August 2009; published 22 October 2009)

A simple exact covariant model in which a scalar particle Ψ is modeled as a bound state of two different particles is used to elucidate relativistic aspects of electromagnetic form factors $F(Q^2)$. The model form factor is computed using an exact covariant calculation of the lowest order triangle diagram. The light-front

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \Phi_N)^2 - \frac{1}{2} m_N^2 \Psi_N^2 + \frac{1}{2}(\partial_\mu \boldsymbol{\pi})^2 - \frac{1}{2} m_\pi^2 \boldsymbol{\pi}^2 - g_{\pi N} \Psi_N^\dagger \boldsymbol{\tau} \cdot \boldsymbol{\pi} \Psi_N$$

Our Approach

1. ~~Currently, form factors are all the same: can we do better?~~
2. ~~Can we incorporate the 'off-shellness' of particles?~~
3. What are the implications for the current measured data points?
   - Generate cross section in model (pseudodata), and then attempt to extract form factor using VGL-like Model.

	Current Extraction	This Analysis
Model	$i\mathcal{M}^{\mu}_{\text{VGL}} = F_{\pi}(Q^2) S_F^{\pi-1} S_{\pi}^{\text{R}}(t)[i\mathcal{M}^{\mu}_{\text{BTM}}]$	$i\mathcal{M}^{\mu} = F_{\pi}(Q^2)[i\mathcal{M}^{\mu}_{\text{BTM}}]$
	$\downarrow$ fit to... $\downarrow$	$\downarrow$ fit to... $\downarrow$
Data	$^1\text{H}(e, e'\pi^+)n$	$i\mathcal{M}^{\mu}_{\text{1-Loop}}$

Pros and Cons

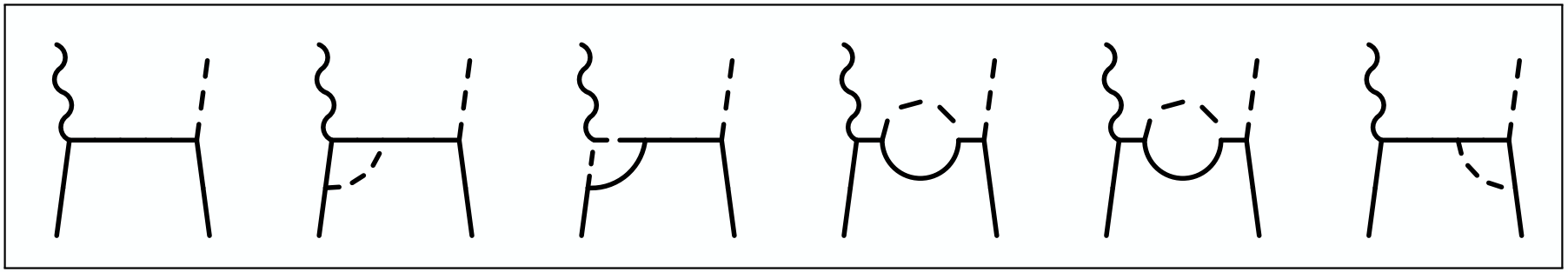
Pros

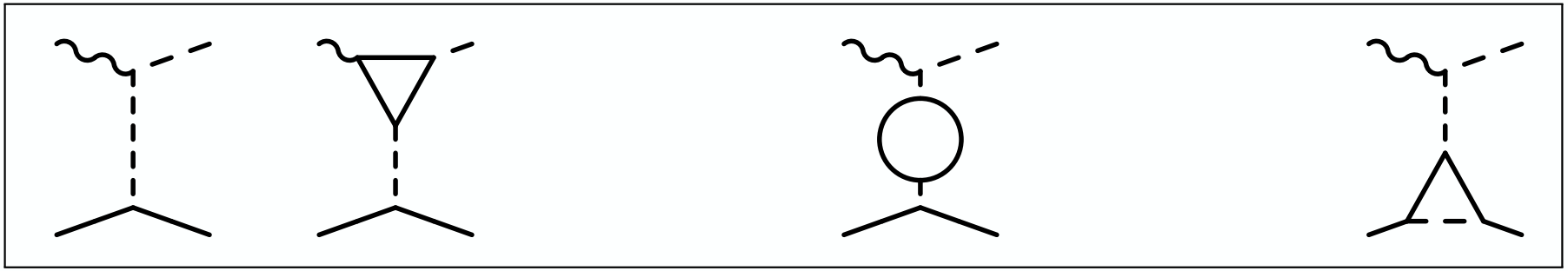
- Perturbative calculation: gauge invariant.
- Calculate to 1-loop order: obtain (different) form factors at vertices.
- Simple.

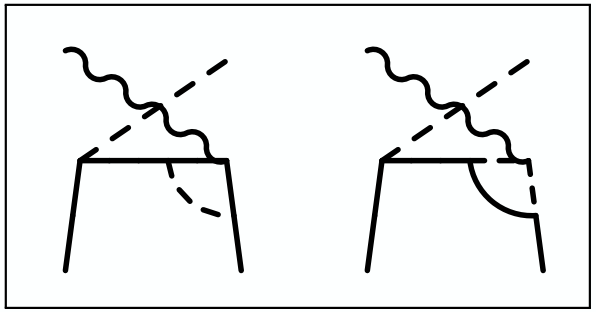
Cons

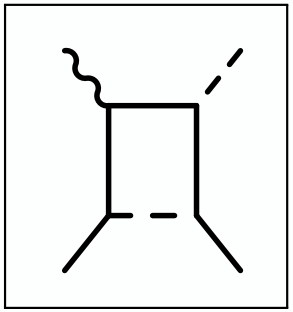
- Perturbative calculation doesn't generally give form factors enough q^2 dependence.
- Connection to QCD is tenuous
  - Prevents quantitative conclusions.

Generating Pseudodata

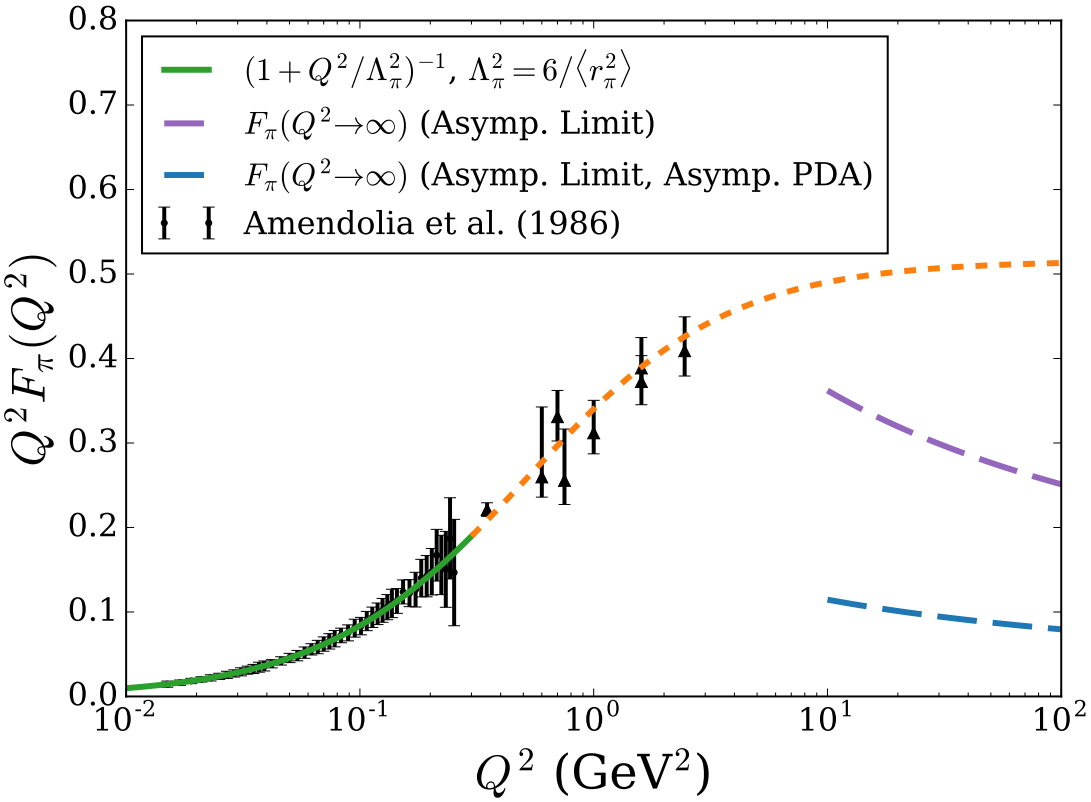








Kinematic Points

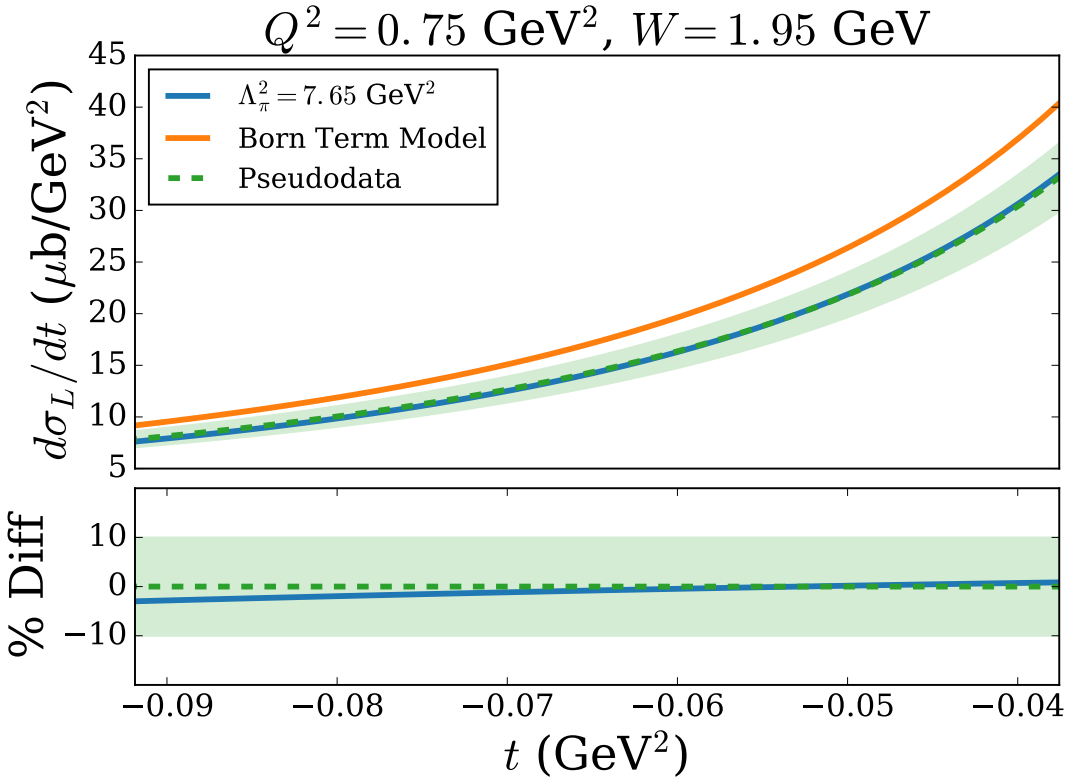


Q^2 (GeV^2)	W (GeV)
0.35	2.10
0.60	1.95
0.70	2.19
0.75	1.95
1.00	1.95
1.60	1.9
1.60	2.22
2.45	2.22

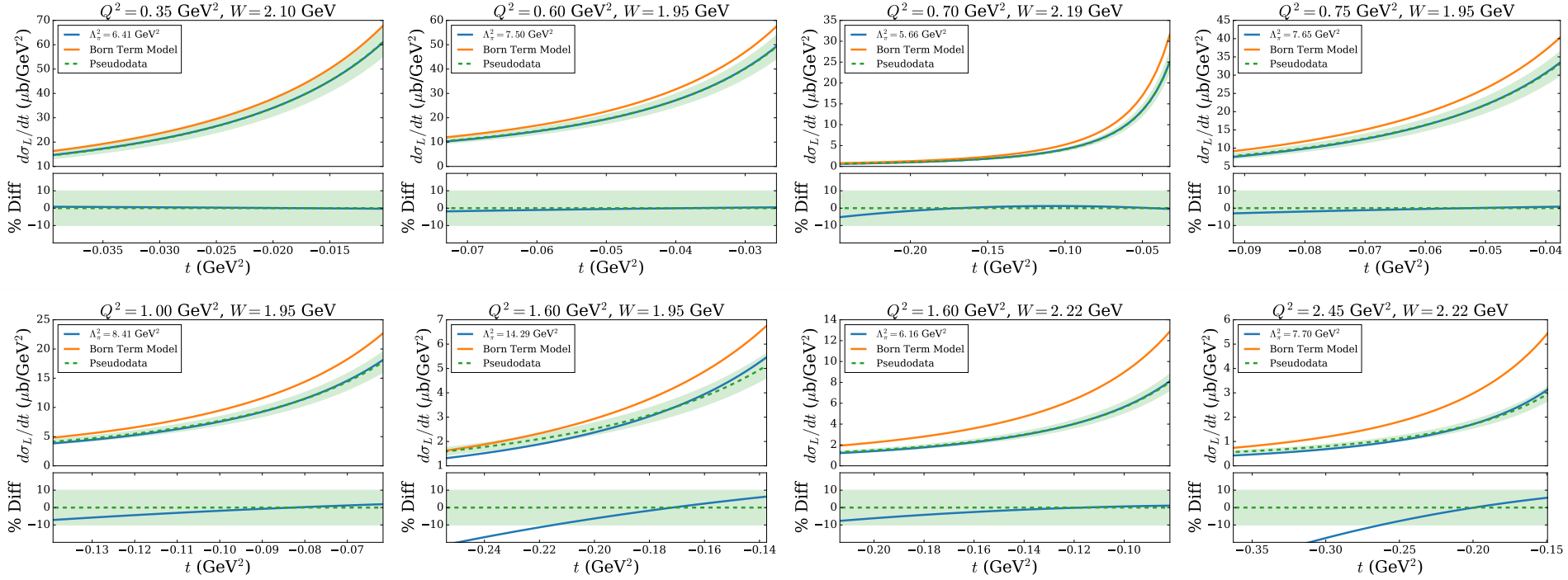
Pseudodata: A Specific Example

- Pseudodata: $i\mathcal{M}^{\mu}_{\text{1-Loop}}$
- Model:
  $i\mathcal{M}^{\mu} = F_{\pi}(Q^2)[i\mathcal{M}^{\mu}_{\text{BTM}}]$

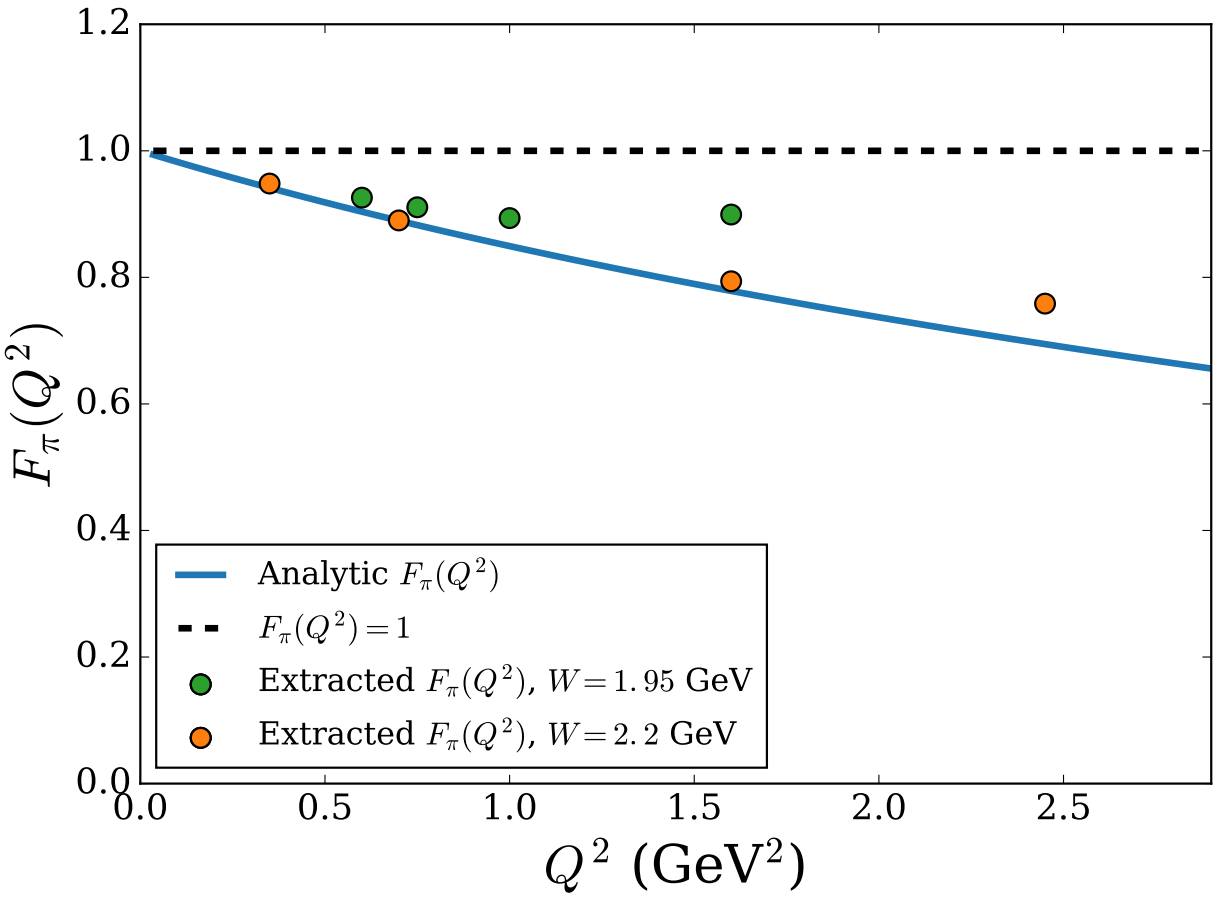
  $F_{\pi}(Q^2) = (1+Q^2/\Lambda_{\pi}^2)^{-1}$
- t range chosen to be same as experiment.



Cross Section

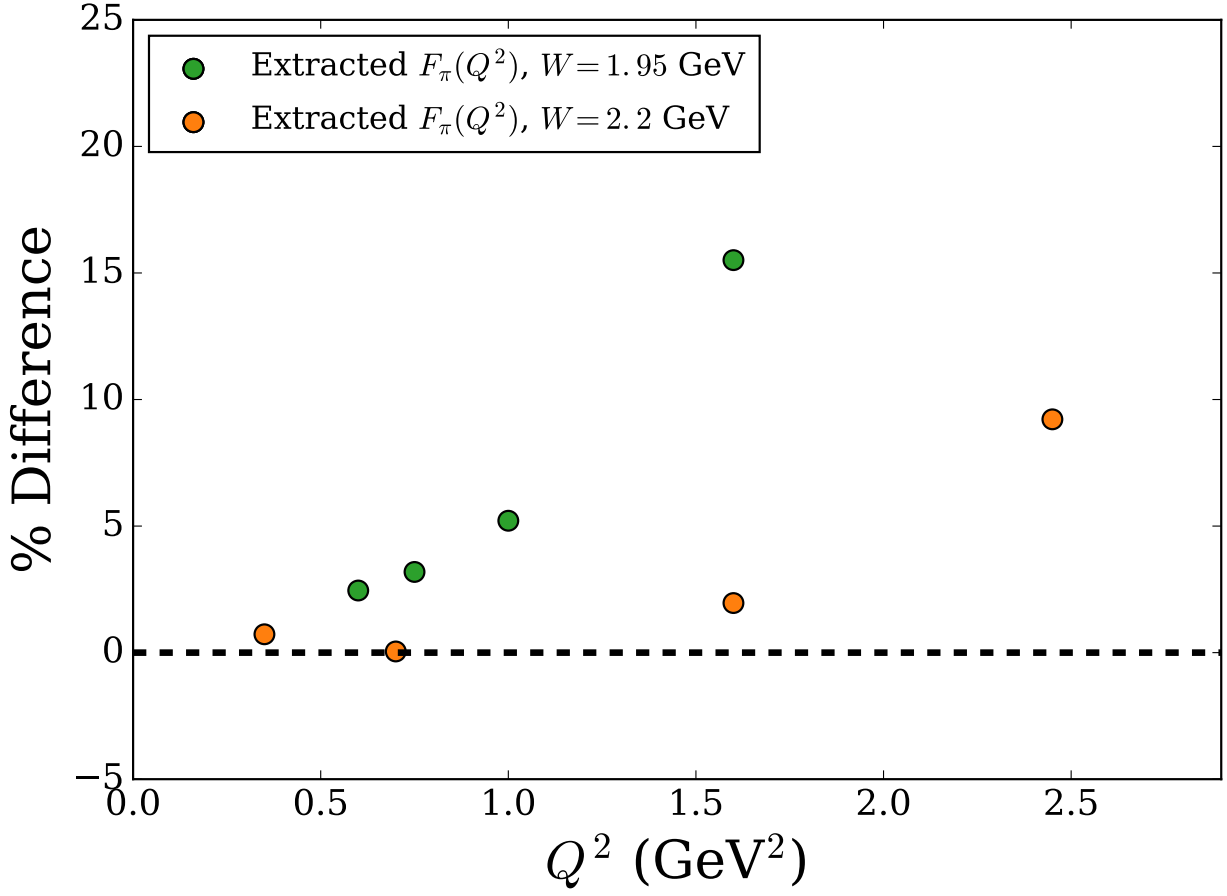


Results



- Apart from possibly point at $(Q^2, W) = (1.6, 1.95)$, results look ok.

Systematic Overestimate?



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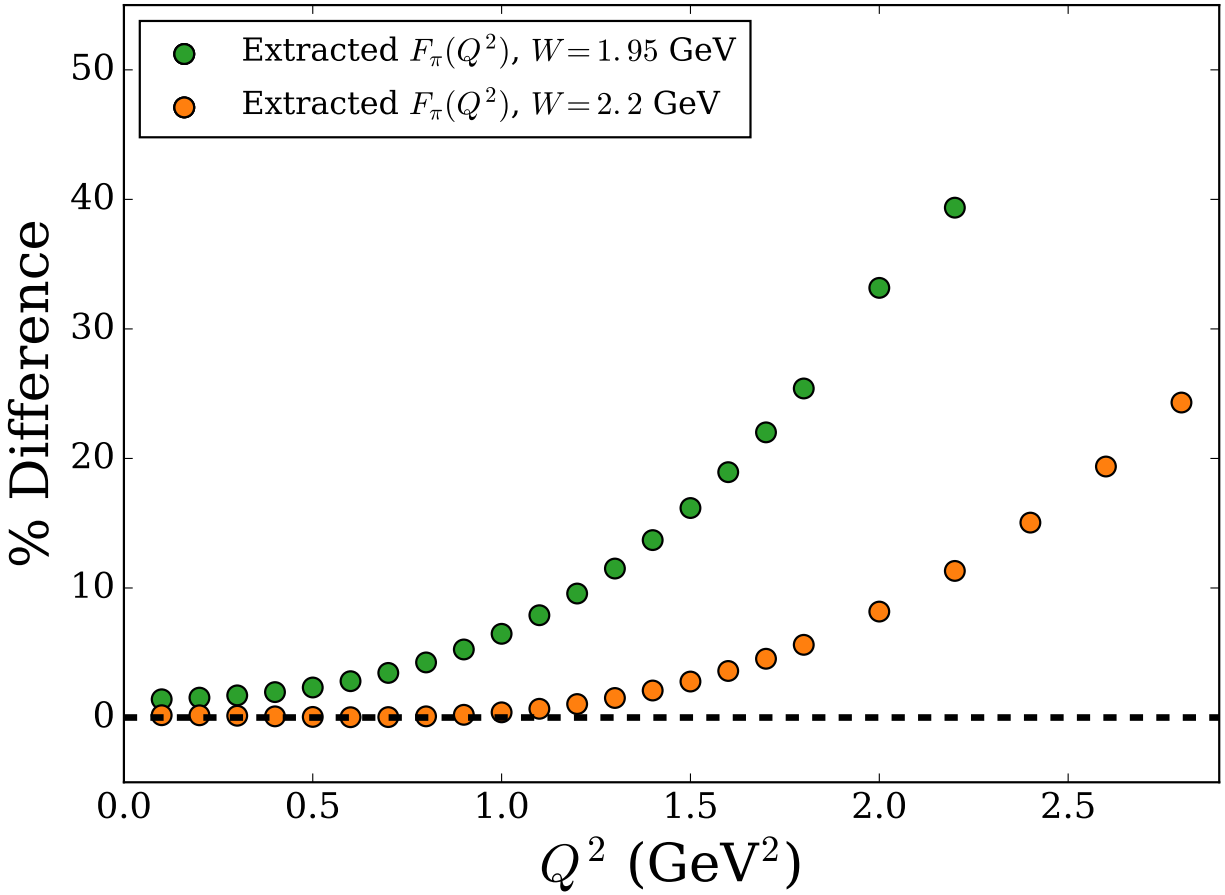
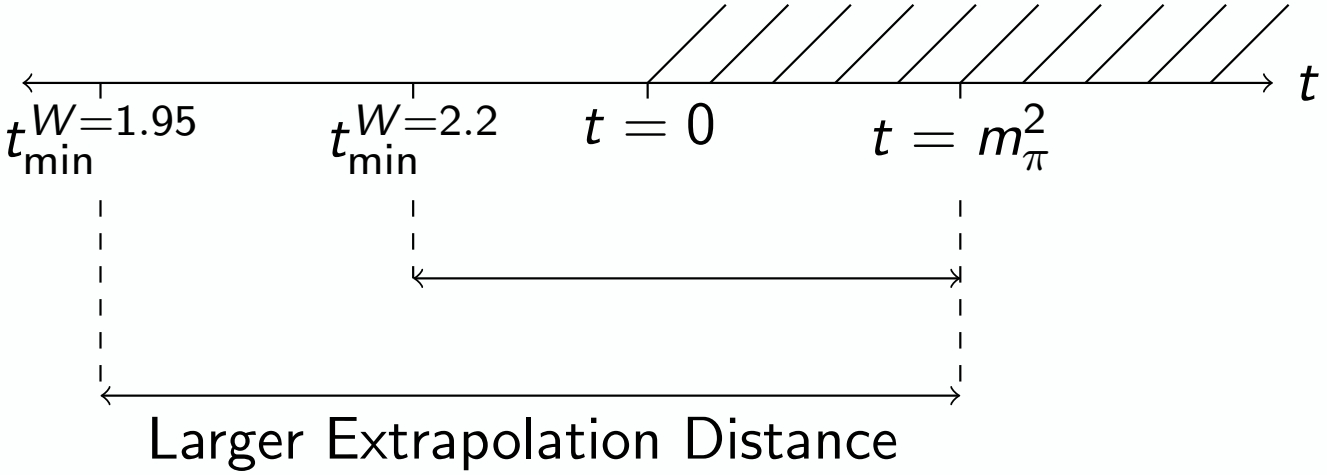


- Fit first 5% of allowed t.

How Do We Understand the W Dependence?

- A kinematic argument. Ideally, we would measure this process at $t = m_\pi^2$.
- $t < 0$ for electro-production.
- t_{min} more negative for increasing Q^2
- t_{min} more negative for decreasing W
- Larger W at the same Q^2 will allow a smaller (negative) $|t_{\text{min}}|$
- Closer to the pion pole. So interpretation of F_π as pion form factor better.



Conclusion

- Pion electro-production allows us to measure the pion form factor at higher Q^2.
- We tested extraction method in simple model.
- Results seem to imply a reasonably accurate extraction is possible, except at certain kinematics.
- Important to choose kinematics wisely to minimize extrapolation to pion pole.

Further Work

- Model extremely simple. A more complicated calculation including fermions is underway.
- Lattice QCD:

$$\left\langle N(p')\pi(p_\pi)\right| J^\mu(q) \left|N(p)\right\rangle$$

$$\left\langle \pi(k')\right| J^\mu(q) \left|\pi(k)\right\rangle = (k+k')F_\pi(Q^2)$$

Thanks

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Phys. Rev., D90:034502, 2014.

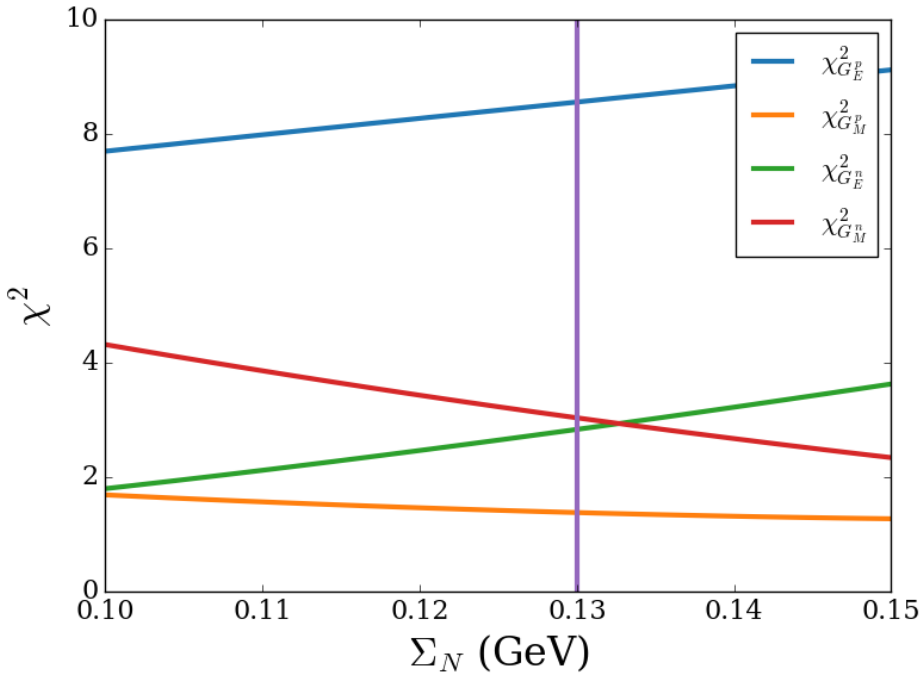
Spare Slides

Describing Mesons and Baryons in the NJL Model

- Mesons in NJL Model are quark–anti-quark bound states
  - Solve BSE
$$\tau(q) = \kappa + \int \frac{d^4k}{(2\pi)^4} \kappa S(k+q) S(k) \tau(q)$$
- Baryons are quark di-quark bound states
  - Solve Faddeev Equation
- Calculate form factors by calculating BSE for $q\overline{q}\gamma$ vertex.

Variation of Nucleon Self Energy

- How do fits change if we vary our self energy?
  - This corresponds to modifying the parameter Λ in our pion-nucleon form factor $G(t)$.
- Results are reasonable stable for self energies between 100 and 150 MeV.
- Our choice of $\Sigma = 130$ MeV is seen to be reasonable.



Cutoff Dependence

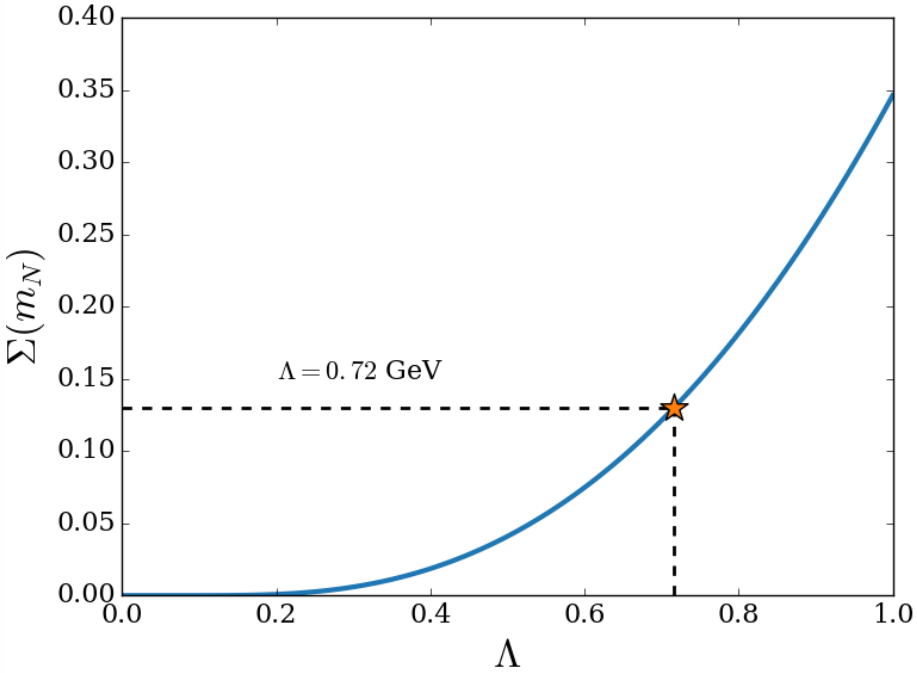
- Incorporate strong form factor at πN vertex
  - Utilize a t-dependent Form Factor

$$G(t) = \exp\left[\frac{(t - m_\pi^2)}{\Lambda^2}\right]$$

  to parameterize extended particle structure.
- All observables become cutoff (Λ) dependent.
  - Self energy cutoff dependent.
  - $m_N^{(0)} = m_N - \Sigma(m_N, \Lambda)$
  - Bare mass (used in NJL Calculation) varies with Λ.

Fitting Model Parameters

- Self Energy not observable, so must take guidance from other models.
- CBM, Dyson-Schwinger, Lattice QCD suggest self-energy from process $N \to N\pi$ is of order 0.1 to 0.15 GeV
- In practice, this, along with the light quark mass (required to be chosen in the NJL Model) were scanned over.



Magnetic Moments and Charge Radii

	$\langle r^2 \rangle^{\frac{1}{2}}$			
	p	n	Σ^-	Σ^+
This Work	0.89	0.41	0.78	0.88
Prev. NJL Calc.	0.87	0.38	0.86	0.97
Exp.	0.84 [3]	0.335	0.780	0.61(8) [4]

Table 1: Experimental results are taken from [1, 2, 3], except for the Σ^+ charge radius, for which there is currently no experimental value. In this case, a recent lattice QCD result [4] is given instead. Charge radii are quoted in femtometres.

	μ			
	p	n	Σ^-	Σ^+
This Work	2.78	-1.71	-1.17	2.33
Prev. NJL Calc.	2.78	-1.81	-1.58	2.60
Exp.	2.793	-1.913	-1.160(25)	2.458(10)

Table 2: Comparison of the predicted magnetic moments to experimental results for the proton, neutron, Σ^- and Σ^+ baryons. Experimental results are taken from [1, 2]. Magnetic moments are in units of nuclear magnetons ($\mu_N = e/2m_N$).

Magnetic Moments and Charge Radii

	$\langle r^2 \rangle^{\frac{1}{2}}$			
	p	n	Σ^-	Σ^+
This Work	0.89	0.41	0.78	0.88
Prev. NJL Calc.	0.87	0.38	0.86	0.97
Exp.	0.84 [3]	0.335	0.780	0.61(8) [4]

Table 1: Experimental results are taken from [1, 2, 3], except for the Σ^+ charge radius, for which there is currently no experimental value. In this case, a recent lattice QCD result [4] is given instead. Charge radii are quoted in femtometres.

	μ			
	p	n	Σ^-	Σ^+
This Work	2.78	-1.71	-1.17	2.33
Prev. NJL Calc.	2.78	-1.81	-1.58	2.60
Exp.	2.793	-1.913	-1.160(25)	2.458(10)

Table 2: Comparison of the predicted magnetic moments to experimental results for the proton, neutron, Σ^- and Σ^+ baryons. Experimental results are taken from [1, 2]. Magnetic moments are in units of nuclear magnetons ($\mu_N = e/2m_N$).

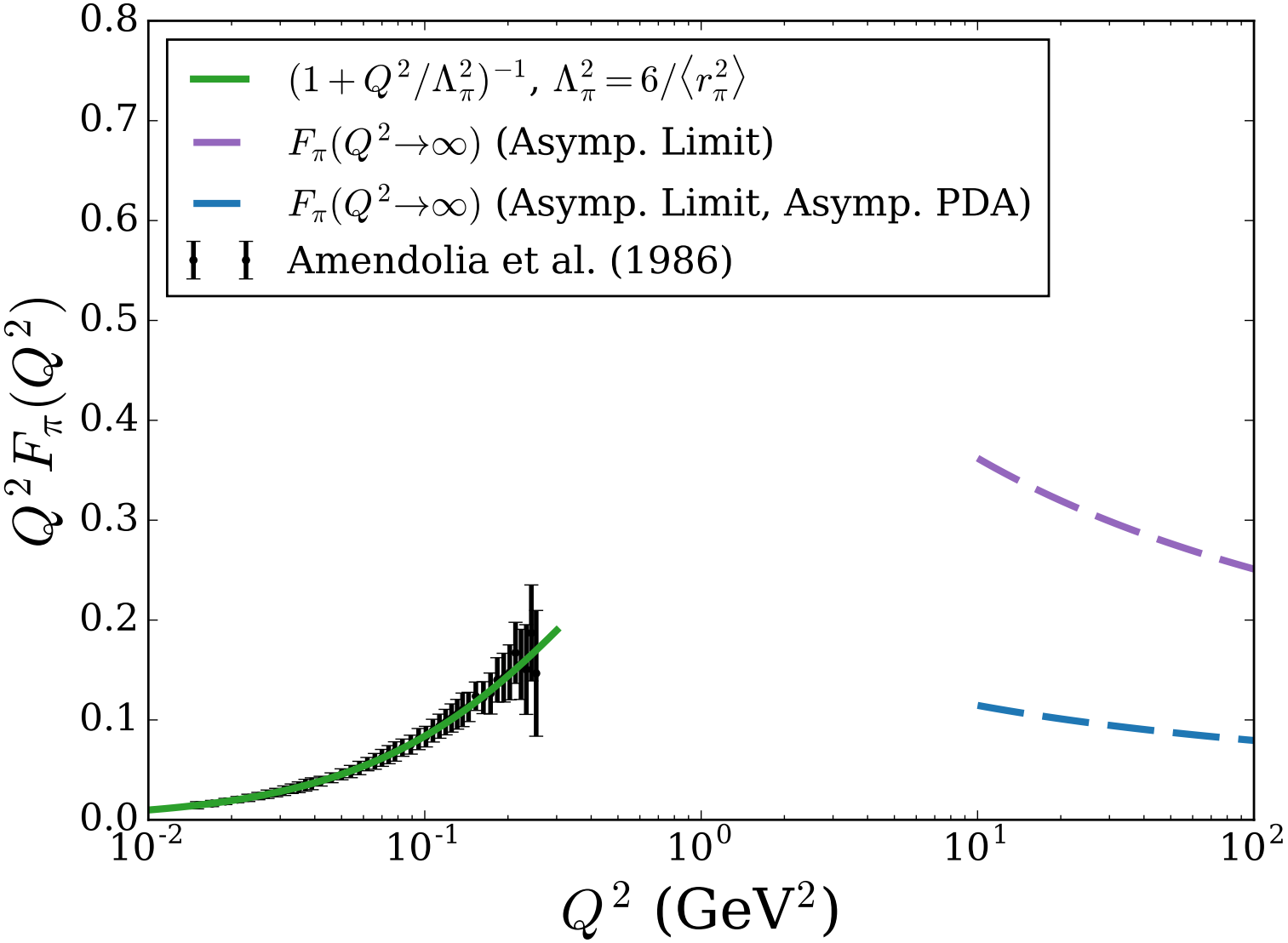
Kinematic Limitations

- Direct measurement has kinematic limitation.

$$\frac{d\sigma}{dq^2} \propto |F_\pi|^2 \frac{1}{q^4}\left(1 - \frac{q^2}{q^2_{\text{max}}}\right)$$

- Where q^2_{max} corresponds to backward scattering in CM frame.
  - roughly proportional to pion beam momentum
- For 300 GeV pion beam, $q^2_{\text{max}} = 0.288$ GeV^2.
- Close to this momentum, the cross section is suppressed, and an extraction becomes difficult.
- Thus could only measure pion form factor up to about 0.3 GeV.

The State of the Art



A Difficult Measurement (2008)

PHYSICAL REVIEW C **78**, 045202 (2008)

Charged pion form factor between $Q^2 = 0.60$ and 2.45 GeV^2. I. Measurements of the cross section for the ^{1}H$(e, e'\pi^+)n$ reaction

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Cross sections for the reaction ^{1}H$(e, e'\pi^+)n$ were measured in Hall C at Thomas Jefferson National Accelerator Facility (JLab) using the high-intensity Continuous Electron Beam Accelerator Facility (CEBAF) to determine the charged pion form factor. Data were taken for central four-momentum transfers ranging from $Q^2 = 0.60$ to 2.45 GeV^2 at an invariant mass of the virtual photon-nucleon system of $W = 1.95$ and 2.22 GeV. The measured cross sections were separated into the four structure functions σ_L, σ_T, σ_{LT}, and σ_{TT}. The various parts of the experimental setup and the analysis steps are described in detail, including the calibrations and systematic studies, which were needed to obtain high-precision results. The different types of systematic uncertainties are also discussed. The results for the separated cross sections as a function of the Mandelstam variable t at the different values of Q^2 are presented. Some global features of the data are discussed, and the data are compared with the results of some model calculations for the reaction ^{1}H$(e, e'\pi^+)n$.

DOI: 10.1103/PhysRevC.78.045202 PACS number(s): 14.40.Aq, 11.55.Jy, 13.40.Gp, 13.60.Le

PHYSICAL REVIEW C **78**, 045203 (2008)

Charged pion form factor between $Q^2 = 0.60$ and 2.45 GeV^2. II. Determination of, and results for, the pion form factor

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(Received 11 July 2008; published 15 October 2008)

The charged pion form factor, $F_\pi(Q^2)$, is an important quantity that can be used to advance our knowledge of hadronic structure. However, the extraction of F_π from data requires a model of the ^{1}H$(e, e'\pi^+)n$ reaction and thus is inherently model dependent. Therefore, a detailed description of the extraction of the charged pion form factor from electroproduction data obtained recently at Jefferson Lab is presented, with particular focus given to the dominant uncertainties in this procedure. Results for F_π are presented for $Q^2 = 0.60$–2.45 GeV^2. Above $Q^2 = 1.5$ GeV^2, the F_π values are systematically below the monopole parametrization that describes the low Q^2 data used to determine the pion charge radius. The pion form factor can be calculated in a wide variety of theoretical approaches, and the experimental results are compared to a number of calculations. This comparison is helpful in understanding the role of soft versus hard contributions to hadronic structure in the intermediate Q^2 regime.

DOI: 10.1103/PhysRevC.78.045203 PACS number(s): 14.40.Aq, 13.40.Gp, 13.60.Le, 25.30.Rw

I. INTRODUCTION

There is much interest in trying to understand the structure of hadrons, both mesons and baryons, in terms of their constituents, the quarks and gluons. However, this structure is too complicated to be calculated rigorously in quantum chromodynamics (QCD) because perturbative QCD (pQCD)

Kinematics and Conventions

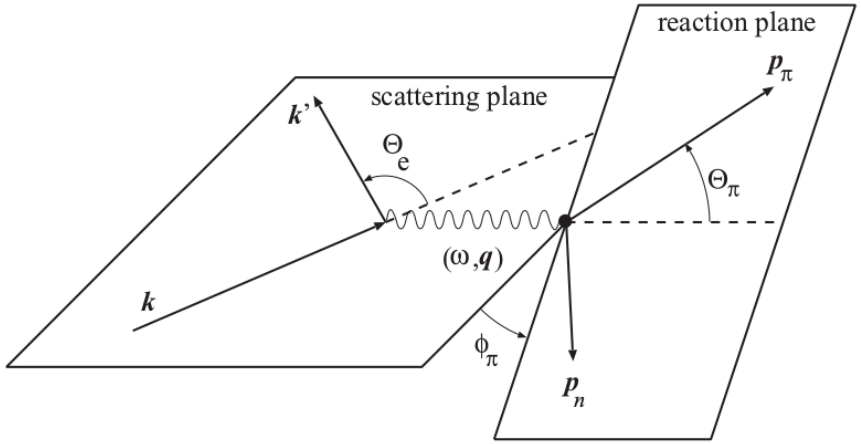


Figure 14: Blok et al., 2008

Mandelstam Variables:

$$s = p_s^2 = (p+q)^2 = (p' + p_\pi)^2 \equiv W^2$$
$$t = p_t^2 = (p_\pi - q)^2 = (p - p')^2 < 0$$
$$u = p_u^2 = (p - p_\pi)^2 = (p' - q)^2$$

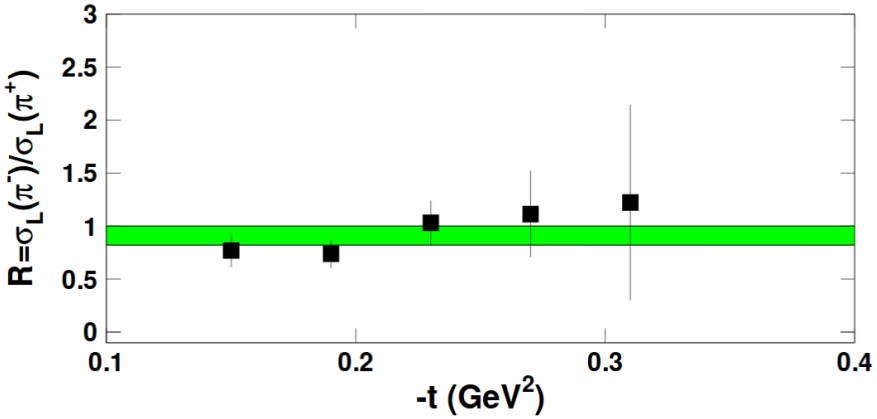
Experimentally, use Q^2, W and t.

Sanity Checks of Extraction

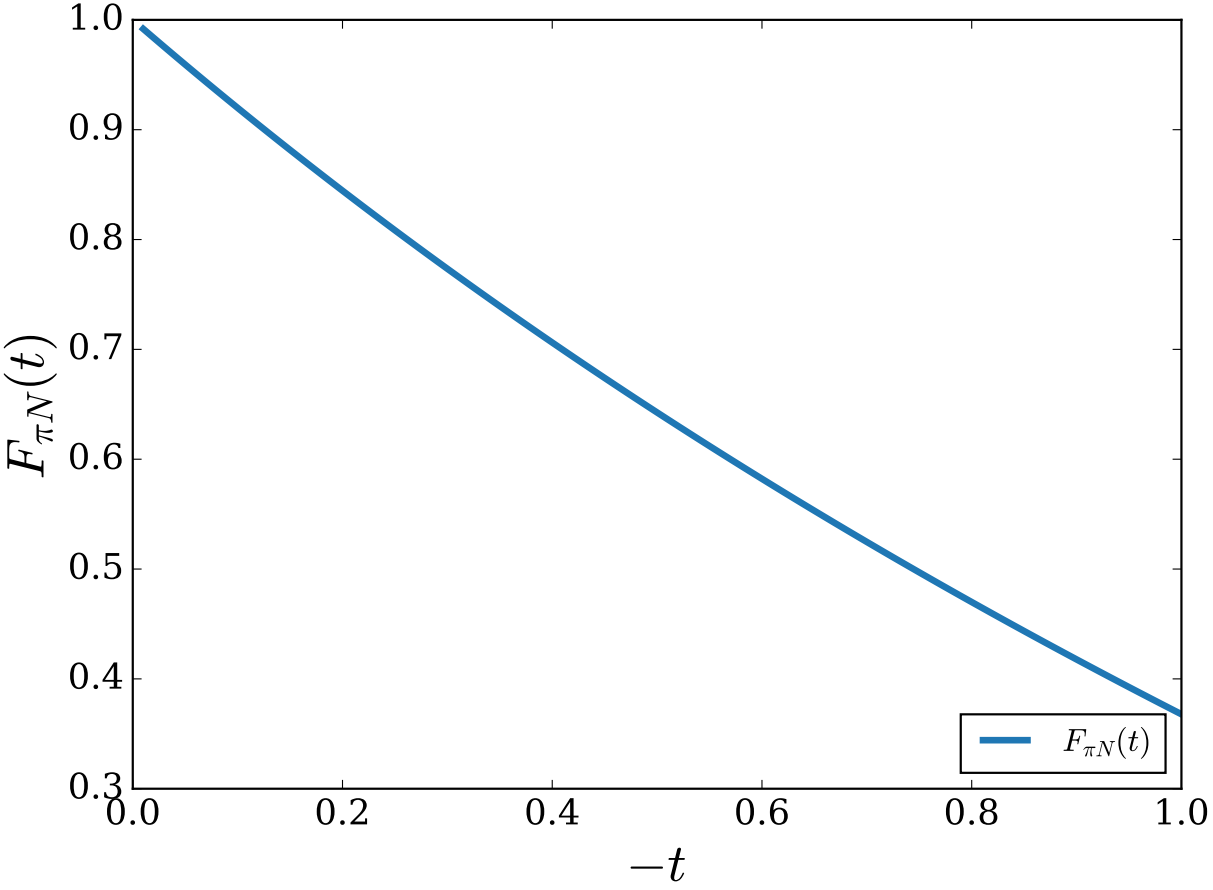
- Construct ratio:

$$R_L = \frac{\gamma_L^* n \to \pi^- p}{\gamma_L^* p \to \pi^+ n} = \frac{|A_v - A_s|^2}{|A_v + A_s|^2}$$

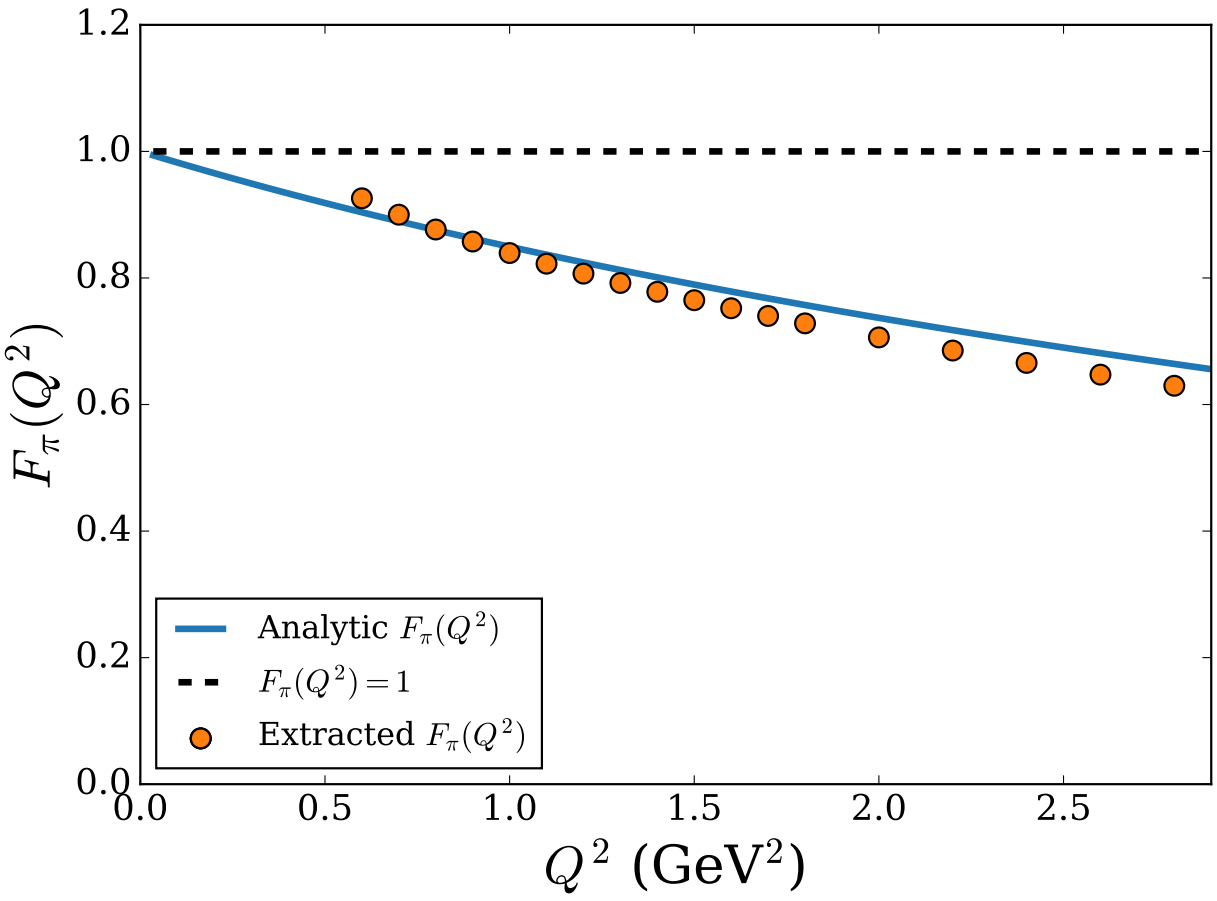
- A_s: Isoscalar amplitude, A_v: Isovector amplitude. t-channel pion amplitude *isovector*.



t Dependent Form Factor



Constant W Extraction



Evidence for Enhanced Form Factor

PHYSICAL REVIEW C **97**, 015203 (2018)

Off-shell persistence of composite pions and kaons

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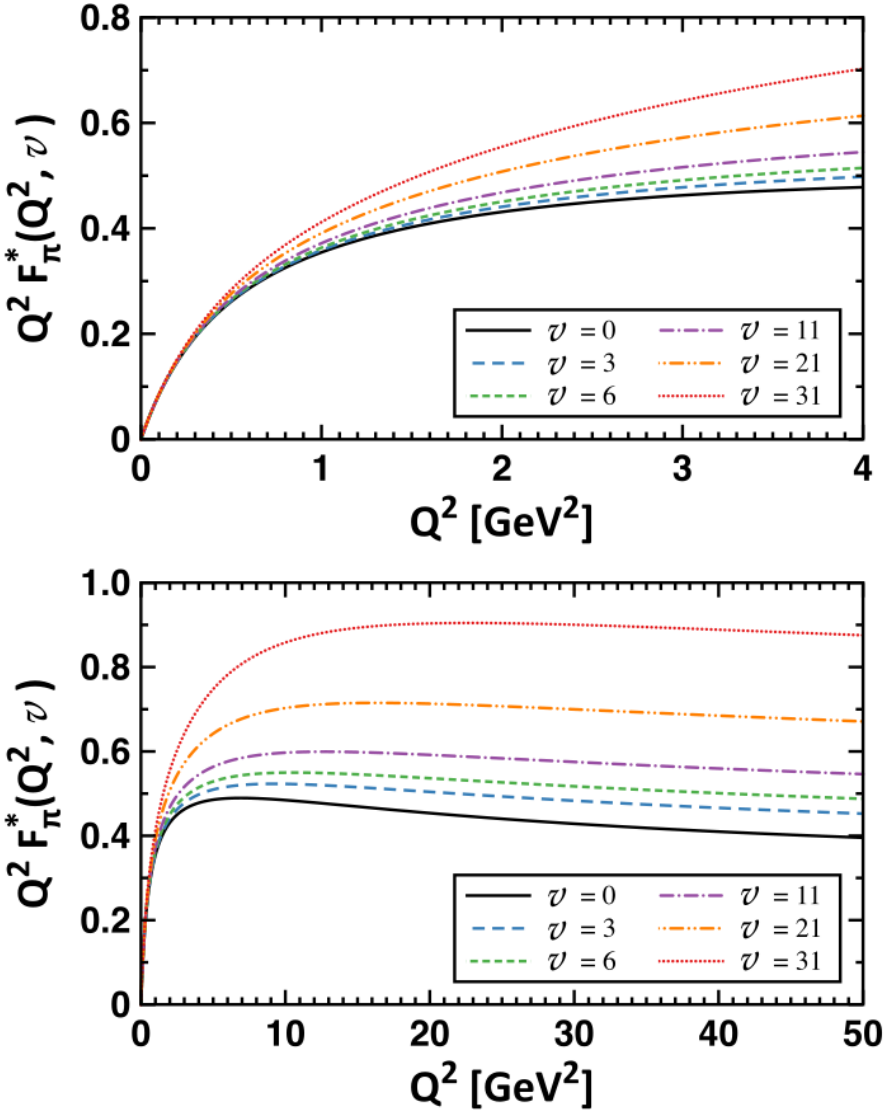
In order for a Sullivan-like process to provide reliable access to a meson target as t becomes spacelike, the pole associated with that meson should remain the dominant feature of the quark-antiquark scattering matrix and the wave function describing the related correlation must evolve slowly and smoothly. Using continuum methods for the strong-interaction bound-state problem, we explore and delineate the circumstances under which these conditions are satisfied: for the pion, this requires $-t \lesssim 0.6\,\mathrm{GeV}^2$, whereas $-t \lesssim 0.9\,\mathrm{GeV}^2$ will suffice for the kaon. These results should prove useful in planning and evaluating the potential of numerous experiments at existing and proposed facilities.

DOI: 10.1103/PhysRevC.97.015203

Evidence for Enhanced Form Factor

- How does pion form factor vary off-shell?
- Although off-shell pion is not well defined, can attempt to address question using BSE.
- $v \geq 0$ parameterizes "off-shellness" in units of m_π^2
- $t = 0.015 \approx m_\pi^2$ GeV^2, $v = 1$
- $t = 0.35 \approx 18 m_\pi^2$ GeV^2, $v = 18$



Model Independent Extraction?

- Can attempt a model independent extraction of pion form factor.
- Since $t < 0$, we want to extrapolate to $t = m_\pi^2$
- Form of extrapolation is unknown.
- Linear, quadratic, higher order all fit data well.
- The modern extraction is model dependent and uses the VGL Model.

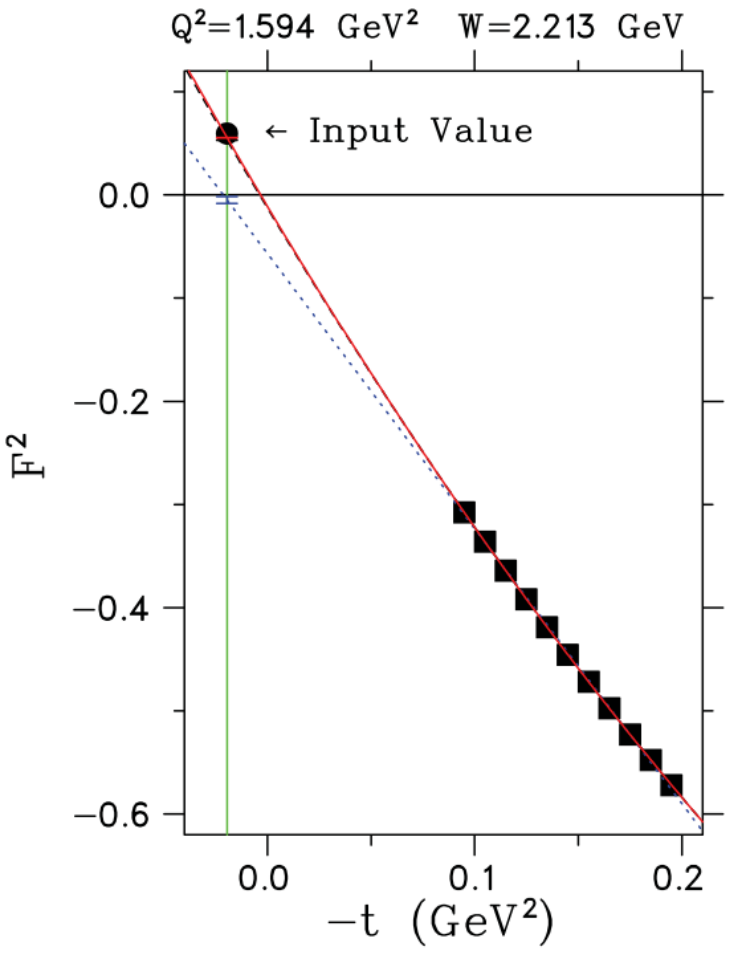
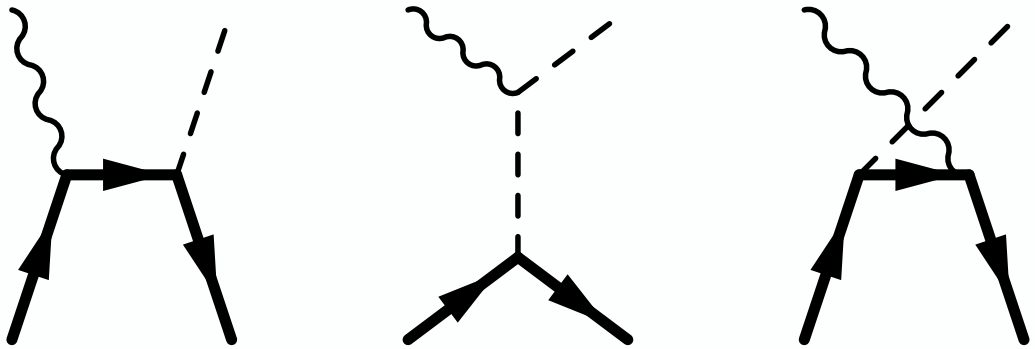


Figure 15: Linear (dotted blue), quadratic (dashed black) and cubic (solid red) fits to data.

Not quite so simple...

1. Also have interference from s- and u- channel terms.
   - Proportional to Nucleon Form Factors



(a) s-channel (b) t-channel (c) u-channel

2. Pion is initially off-shell. What does this mean for the extraction of F_π?
   - Theoretically?
   - Empirically?