Positivity of renormalized parton distributions

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Based on recent work with John Collins and Nobuo Sato: arXiv:2111.01170

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Factorization and hadron structure

- Factorization:
  - Separate perturbative and nonperturbative parts
  - What is the nonperturbative part exactly?

- Factorization in HEP phenomenology
  - What reliable constraints can be reliably imposed on pdfs extractions?

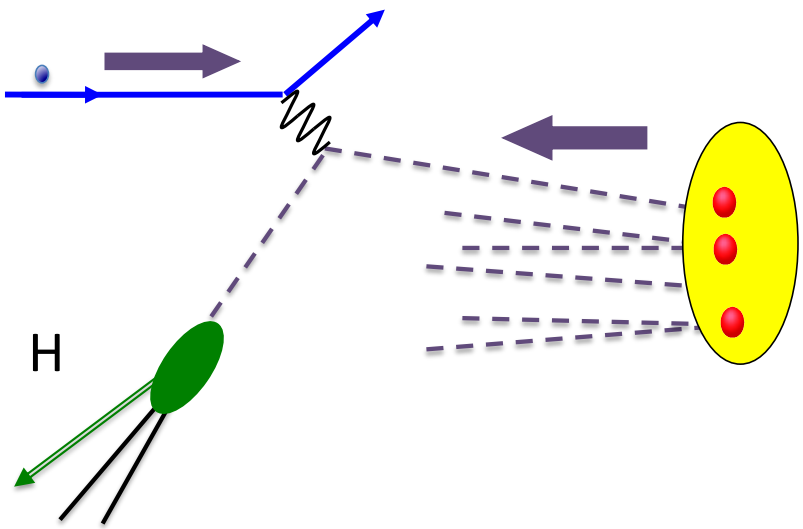
- Factorization in hadron structure
  - New types of correlation functions: GPDs, TMDs, etc…
  - Evolution from very low input scales

SIDIS

- Collinear factorization

$$\mathrm{d}\sigma \sim \mathrm{d}\hat{\sigma} \otimes f(x_{\mathrm{bj}}/\xi) \otimes d(z/\zeta)$$

$$O\left(\Lambda_{\mathrm{QCD}}^2/Q^2\right)$$

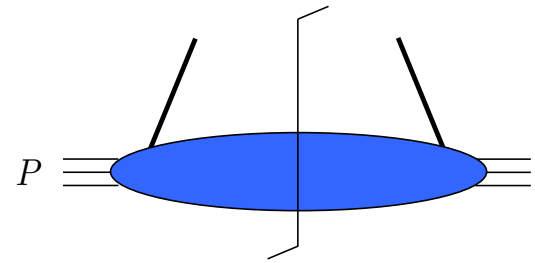


- TMD factorization

$$\mathrm{d}\sigma \sim \mathrm{d}\hat{\sigma} \otimes f(x_{\mathrm{bj}}, \mathbf{q}_T - \mathbf{k}_T) \otimes d(z, \mathbf{k}_T)$$

$$O\left(\Lambda_{\mathrm{QCD}}^2/Q^2, \mathbf{q}_T^2/Q^2\right)$$

Number densities



- Collinear

$$f(\xi) \equiv \int \frac{\mathrm{d}w^-}{2\pi}\, e^{-i\xi p^+ w^-}\ \langle p|\, \bar{\psi}(0, w^-, \mathbf{0}_T)\frac{\gamma^+}{2}\psi(0,0,\mathbf{0}_T)\,|p\rangle$$

  - *Wilson lines/gauge invariance?*
  - *Divergences?*

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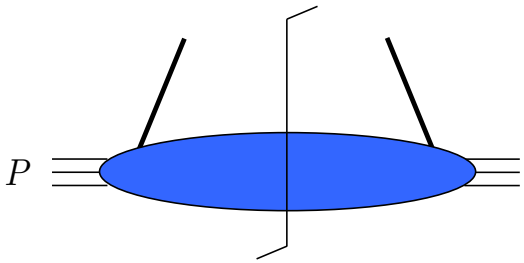

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- TMD

$$f(\xi, \mathbf{k}_T) \equiv \int \frac{\mathrm{d}w^- \mathrm{d}^2\mathbf{w}_T}{(2\pi)^3}\, e^{-i\xi p^+ w^- + i\mathbf{k}_T\cdot\mathbf{w}_T}\ \langle p|\, \bar{\psi}(0, w^-, \mathbf{w}_T) \frac{\gamma^+}{2} \psi(0, 0, \mathbf{0}_T)\, |p\rangle$$

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Two approaches to pdfs and (collinear) factorization

- Track A:
  - Define pdf in terms of ultraviolet renormalization of bare number density operator.

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- Track A:
  - Define pdf in terms of ultraviolet renormalization of bare number density operator.
- Track B:
  - Calculate higher order hard scattering amplitudes. "Absorb" collinear divergences into pdf.

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- Operator definition of the pdf from the beginning.
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- Factorization (e.g., inclusive DIS):
  - Obtained from general region analysis.
  - Beyond parton model: Higher order hard scattering constructed from nested subtractions.

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$$f^{\mathrm{bare,a}}(\xi) \equiv \int \frac{\mathrm{d}w^-}{2\pi}\, e^{-i\xi p^+ w^-}\, \langle p|\, \bar{\psi}_0(0, w^-, \mathbf{0}_\mathrm{T}) \frac{\gamma^+}{2} W[0, w^-] \psi_0(0, 0, \mathbf{0}_\mathrm{T})\, |p\rangle$$

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$$f^{\mathrm{renorm,a}}(\xi) \equiv Z^a \otimes f^{\mathrm{bare,a}} \qquad\qquad Z^a = \delta(1-\xi) + \sum_{j=1}^{\infty} C_j \left(\frac{S_\epsilon}{\epsilon}\right)^j$$

Track B:

- Assert(?): $\mathrm{d}\sigma = f_{\text{“bare,b”}} \otimes \mathrm{d}\hat{\sigma}$

Massless partonic

E.g., Curci, Furmanski, Petronzio (1980)

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- Then:

$$\mathrm{d}\sigma = f \otimes \mathrm{d}\hat{\sigma}_{\text{finite}}$$

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Track B:

- Questions:
  - Derivation of factorization for step 1 ($\mathrm{d}\sigma = f_{\text{“bare,b”}} \otimes \mathrm{d}\hat{\sigma}$) ?
  - Bare pdf ($f_{\text{“bare,b”}}$) of step 1 is undefined
  - Interpretation of collinear divergences?
  - Can we reverse engineer $f_{\text{“bare,b”}}$?

Track A vs. Track B Logic

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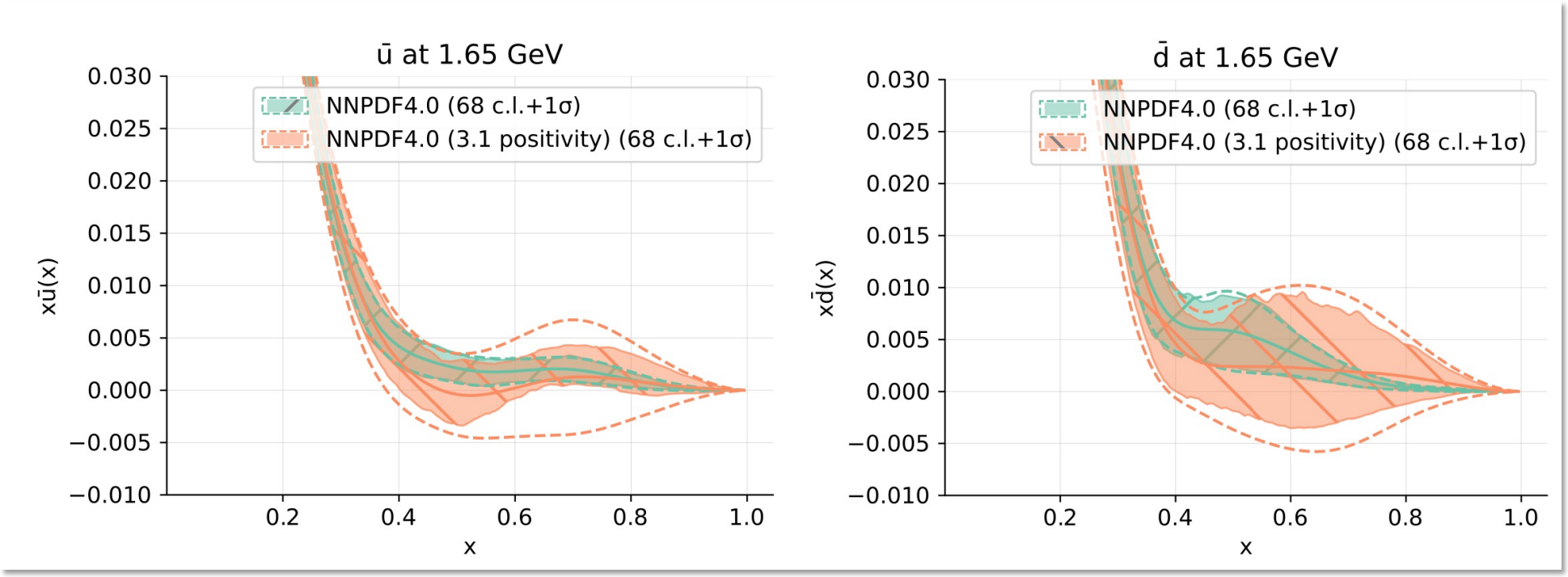
- <u>Example 1:</u> Track-B leads to arguments that pdf positivity is an absolute property of pdfs in certain schemes (MS-bar).

$f(x;\mu) \geq 0$ *A. Candido, S. Forte, and F. Hekhorn (2020), 2006.07377*

Track A vs. Track B Logic

R. Ball et al (NNPDF), arXiv:2109.02653



Good lattice cross sections

- Example 2:
  - How to relate lattice calculations to light-cone pdfs
  - Factorization:
    - Derivations must follow track A

      (Note, e.g., *Phys.Rev.D 98 (2018) 7, 074021* uses track B)

Example

- Stress-test assertions about DIS factorization in other finite-range renormalizable theories.

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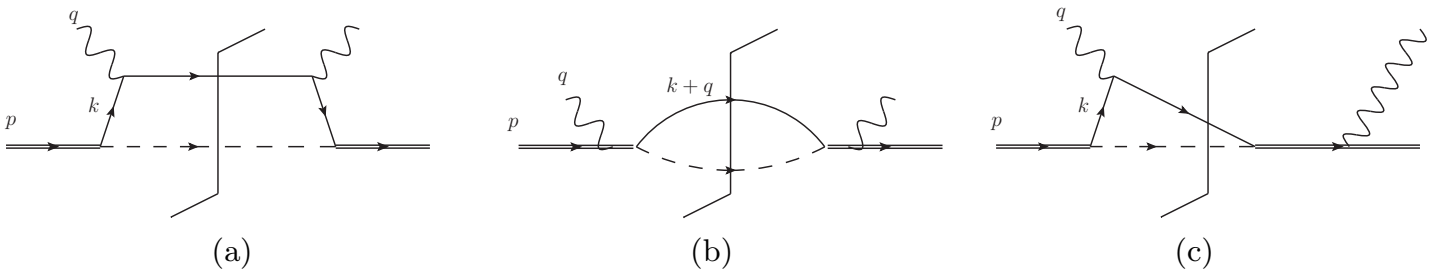

$$\mathcal{L}_{\rm int} = -\lambda\, \overline{\Psi}_N\, \psi_q\, \phi \;+\; \text{H.C.}$$

- Exact $O(\lambda^2)$ DIS cross section is easy to calculate exactly.

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Example

Collinear Factorization

$$\Delta(\xi)^2 \equiv \xi m_s^2 + (1-\xi)m_q^2 - \xi(1-\xi)m_p^2$$

$$F_1(x,Q) = \sum_f \int_x^1 \frac{\mathrm{d}\xi}{\xi}$$

$$\times \underbrace{\frac{1}{2}\left\{\delta\left(1-\frac{x}{\xi}\right)\delta_{qf} + a_\lambda(\mu)\left(1-\frac{x}{\xi}\right)\left[\ln(4) - \frac{\left(\frac{x}{\xi}\right)^2 - 3\frac{x}{\xi} + \frac{3}{2}}{\left(1-\frac{x}{\xi}\right)^2} - \ln\frac{4x\mu^2}{Q^2(\xi-x)}\right]\delta_{pf}\right\}}_{\hat{F}_{1,q/f}(x/\xi,\mu/Q;a_\lambda(\mu))} \times$$

$$\times \underbrace{\left\{\delta(1-\xi)\,\delta_{fp} + a_\lambda(\mu)(1-\xi)\left[\frac{(m_q+\xi m_p)^2}{\Delta(\xi)^2} + \ln\left(\frac{\mu^2}{\Delta(\xi)^2}\right) - 1\right]\delta_{fq}\right\}}_{f_{f/p}(\xi;\mu)} .$$

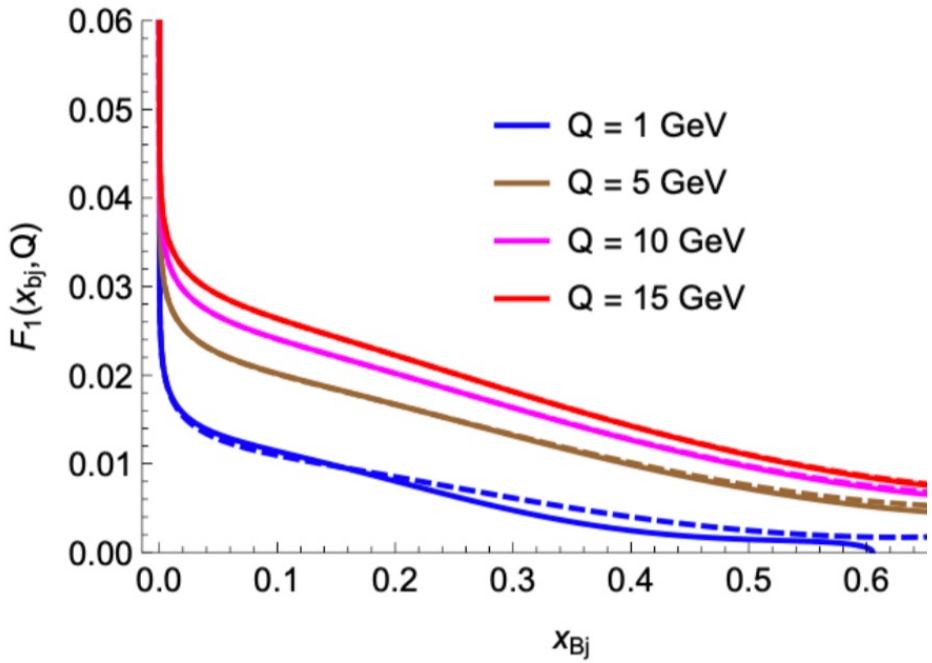
Partonic structure function

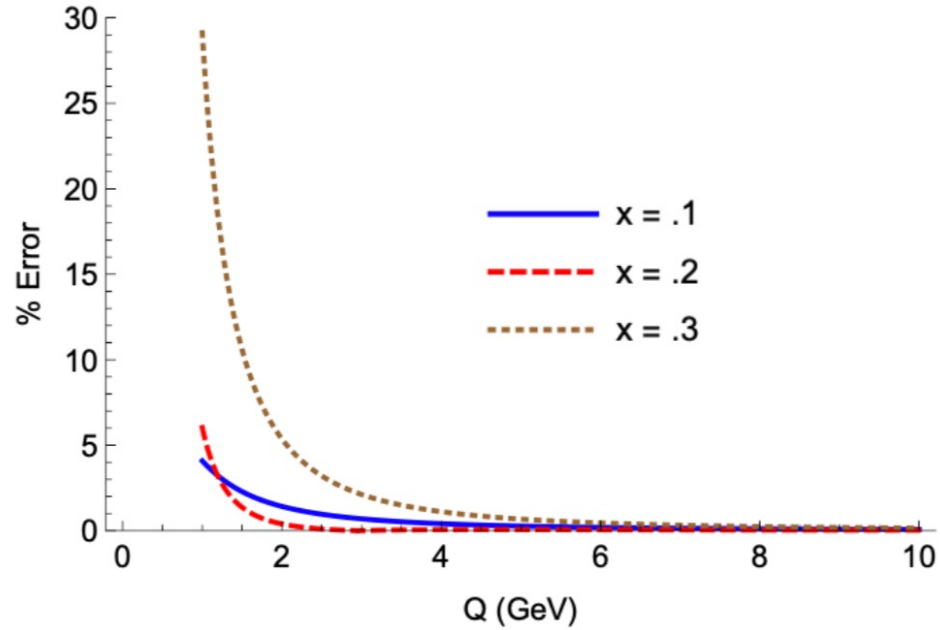
Parton Distribution

$$\overline{\mathrm{MS}}\ \ \mathrm{C.T.} = -a_\lambda(\mu)(1-\xi)\frac{S_\epsilon}{\epsilon}$$

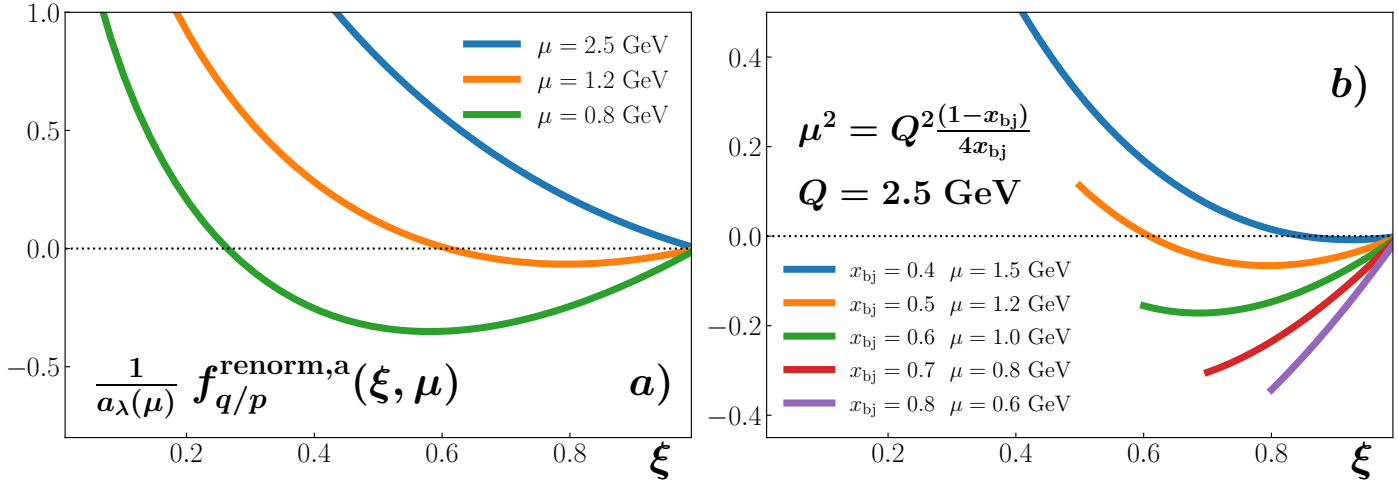
Example

Collinear Factorization

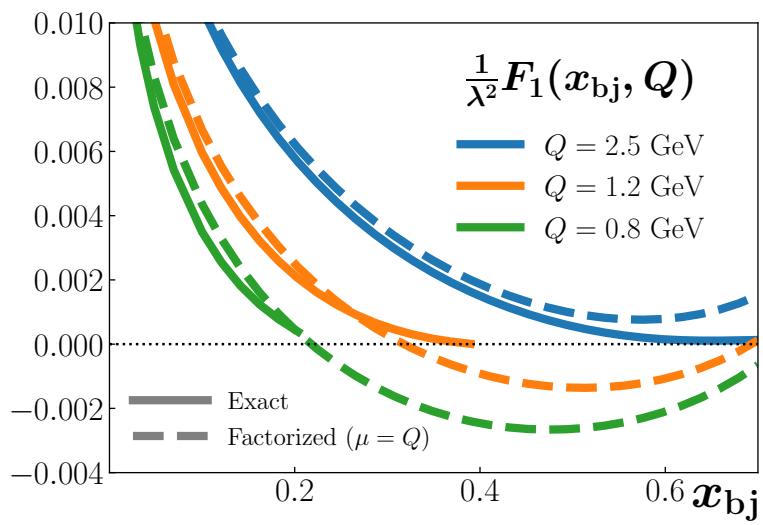




Positivity?



$m_q = 0.3$ GeV
$m_p = 1.0$ GeV
$m_s = 1.5$ GeV



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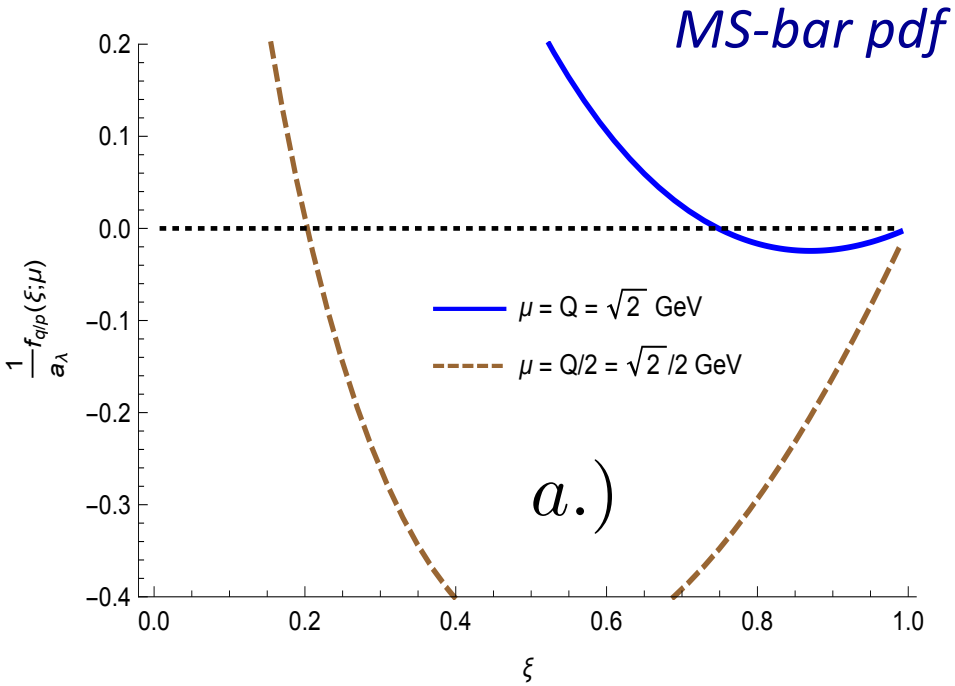

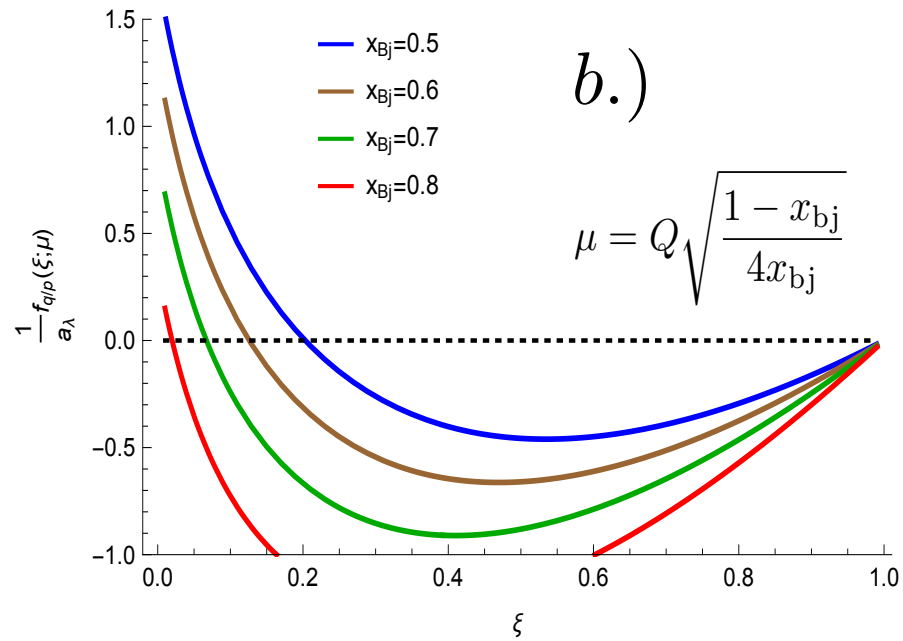

Track B bare pdf: Can we reverse engineer it?

- $f_{\text{“bare,b”}}$ must be a renormalized pdf, not a bare one

$$\mathrm{d}\sigma = f_{\text{“bare,b”}} \otimes \mathrm{d}\hat{\sigma}$$

?

- Factorization is target-independent
- Take the target to be a massless, on shell parton
- Infer the renormalization scheme for $f_{\text{“bare,b”}}$

Track B bare pdf: Can we reverse engineer it?

- $\mathrm{d}\sigma = f_{\text{“bare,b”}} \otimes \mathrm{d}\hat{\sigma} \implies \mathrm{d}\hat{\sigma} = f_{\text{“bare,b”}} \otimes \mathrm{d}\hat{\sigma}$
- $f_{\text{“bare,b”},ij} = \delta_{ij}\delta(1-x)$

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- $f_{\text{“bare,b”},ij} = \delta_{ij}\delta(1-x)$
- Analogy with Bogoliubov-Parasiuk-Hepp-Zimmermann (BPH(Z)) renormalization *(used by Wilson-Zimmermann for UV divergences)*
  - BPH(Z): Subtract at zero external momentum
  - BPH(Z)’: Subtract at x = 1, masses = 0

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Note: $f_{\text{“bare,b”}}$ *is not actually bare*

Track B bare pdf: Can we reverse engineer it?

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- $f_{\text{“bare,b”},ij} = \delta_{ij}\delta(1-x)$
- BPHZ’
- Now generalize to a non-partonic target: $f_{\text{“bare,b”}}$ has a collinear divergence

Return to positivity

- How to think about $f^{\text{“bare,b”}}$ generally (in any scheme)?
- It is not (necessarily) positive
- Track B situation for general regulators:

$$F(Q, x_{\rm bj}) = \left[F^{\rm partonic}\right]_{\substack{\rm R_1,\\ \rm IRR}} \otimes \left[f^{\rm bare,B}\right]_{\substack{\rm R_2,\\ \rm IRR}}$$

Return to positivity

- Dimensional regularization violates positivity

$$\int \mathrm{d}^{2-2\epsilon}\mathbf{k}_T \, \frac{(k_T^2 - Q^2)^2}{k_T^2 (k_T^2 + Q^2)^2} \overset{\epsilon \to 0}{=} -4\pi$$

- Vanishing of dimensionless integrals

Recall: Track B:

- Assert(?): $\mathrm{d}\sigma = f_{\text{“bare,b”}} \otimes \mathrm{d}\hat{\sigma}$

  Massless partonic

- Collinear divergences! $\mathrm{d}\hat{\sigma} = \mathcal{C} \otimes \mathrm{d}\hat{\sigma}_{\text{finite}}$

- So…

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- Absorb:

$$f = f_{\text{“bare,b”}} \otimes \mathcal{C}$$

- Then:

$$\mathrm{d}\sigma = f \otimes \mathrm{d}\hat{\sigma}_{\text{finite}}$$

Breakdown in positivity in track B: Summary

$$F(Q, x_{\rm bj}) = \left[F^{\rm partonic}\right]_{\substack{\rm R_1,\\ \rm IRR}} \otimes \left[f^{\rm bare,B}\right]_{\substack{\rm R_2,\\ \rm IRR}}$$

1) Equation must be valid

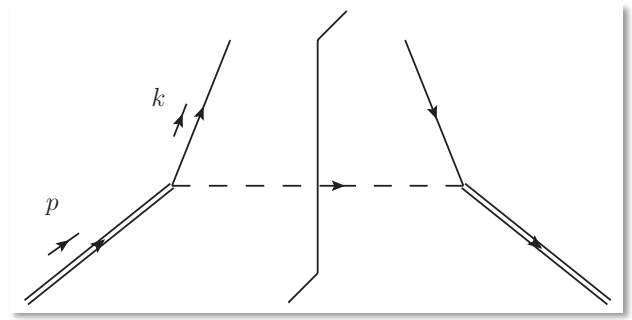
2) All bare pdfs must have guaranteed positivity

3) Partonic structure functions (Fs) must have guaranteed positivity

One of these must be false in Msbar/dim reg

Rescuing positivity?

Yukawa theory



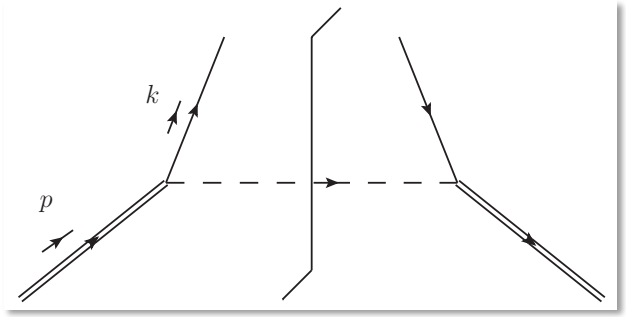
- $\overline{\text{MS}}$ pdf obtained from

$$\frac{a_\lambda(\mu)}{\pi}\int \mathrm{d}^{2-2\epsilon}\boldsymbol{k}_\mathrm{T} \times \frac{(1-\xi)\left[k_\mathrm{T}^2+(m_q+\xi m_p)^2\right]}{\left[k_\mathrm{T}^2+\xi m_s^2+(1-\xi)\,m_q^2+\xi(\xi-1)m_p^2\right]^2}$$

Rescuing positivity?

- Instead try cutoff scheme (but be careful!)

Yukawa theory



$$a_\lambda(\mu)\int_0^{k^2_{\mathrm{cut,T}}} \mathrm{d}k^2_{\mathrm{T}} \times \frac{(1-\xi)\left[k^2_{\mathrm{T}}+(m_q+\xi m_p)^2\right]}{\left[k^2_{\mathrm{T}}+\xi m_s^2+(1-\xi)\,m_q^2+\xi(\xi-1)m_p^2\right]^2}$$

To convert to $\overline{MS}$, subtract

$$a_\lambda(\mu)(1-\xi)\int_{\mu^2}^{k^2_{\mathrm{cut,T}}} \frac{\mathrm{d}k^2_{\mathrm{T}}}{k^2_{\mathrm{T}}}$$

Summary

- Historically, two alternative ways to view divergences and their role in pdf definitions.
  - Track A: UV renormalization – no collinear divergences
  - Track B: Collinear absorption – absorb collinear divergences
- Track A is more complete. Differences between tracks have practical consequences.
- Minimal requirement for track B argument: The BPHZ' renormalization scheme
  - Shows that positivity is not guaranteed
- Positivity is not a <u>general</u> property of MS-bar renormalized parton densities
- Important for details of hadron structure at $Q \approx$ few GeVs (or lower)
- Other ways to get positivity? Connection to TMD functions?