Ultraviolet Structure of Integrated TMD Observables

Ted Rogers
Jefferson Lab and Old Dominion University

LBL Seminar, March 2021

What happens when you integrate transverse momentum?

- Positivity/Sum Rules for ordinary pdfs? $f(x;\mu) \geq 0$

A. Candido, S. Forte, and F. Hekhorn (2020), 2006.07377

What happens when you integrate transverse momentum?

- Positivity/Sum Rules for ordinary pdfs? $f(x;\mu) \geq 0$

A. Candido, S. Forte, and F. Hekhorn (2020), 2006.07377

- TMD/Twist-3 correspondence

$$\int d^2\mathbf{k}_T \frac{k_T^2}{2M^2} f_{1\perp}(x, k_T) = f_{1\perp}^{(1)}(x) \approx \frac{1}{M} T(x)$$

Boer, Mulders, Pijlman, Nucl. Phys.B667, 201 (2003)

What happens when you integrate transverse momentum?

- Positivity/Sum Rules for ordinary pdfs? $f(x;\mu) \geq 0$

A. Candido, S. Forte, and F. Hekhorn (2020), 2006.07377

- TMD/Twist-3 correspondence

$$\int d^2\mathbf{k}_T \frac{k_T^2}{2M^2} f_{1\perp}(x, k_T) = f_{1\perp}^{(1)}(x) \approx \frac{1}{M} T(x)$$

Boer, Mulders, Pijlman, Nucl. Phys.B667, 201 (2003)

- Matching small and large transverse momentum

What happens when you integrate transverse momentum?

- Positivity/Sum Rules for ordinary pdfs? $f(x;\mu) \geq 0$

A. Candido, S. Forte, and F. Hekhorn (2020), 2006.07377

- TMD/Twist-3 correspondence

$$\int d^2\mathbf{k}_T \frac{k_T^2}{2M^2} f_{1\perp}(x, k_T) = f_{1\perp}^{(1)}(x) \approx \frac{1}{M} T(x)$$

Boer, Mulders, Pijlman, Nucl. Phys.B667, 201 (2003)

- Matching small and large transverse momentum

- Lorentz-invariance relations

$$g_T(x) = g_1(x) + \frac{d}{dx} g_{1T}^{(1)}(x),$$

Mulders, Tangerman, Nucl. Phys. B461, 197 (1996)

$$g_{1T}^{(1)}(x) \equiv \int d^2\boldsymbol{k}_T \, \frac{k_T^2}{2\,M^2} g_{1T}(x, k_T)$$

What happens when you integrate transverse momentum?

- Positivity/Sum Rules for ordinary pdfs? $f(x;\mu) \geq 0$

A. Candido, S. Forte, and F. Hekhorn (2020), 2006.07377

- TMD/Twist-3 correspondence

$$\int d^2\mathbf{k}_T \frac{k_T^2}{2M^2} f_{1\perp}(x, k_T) = f_{1\perp}^{(1)}(x) \approx \frac{1}{M} T(x)$$

Boer, Mulders, Pijlman, Nucl. Phys.B667, 201 (2003)

- Matching small and large transverse momentum

- Lorentz-invariance relations

$$g_T(x) = g_1(x) + \frac{d}{dx} g_{1T}^{(1)}(x),$$

Mulders, Tangerman, Nucl. Phys. B461, 197 (1996)

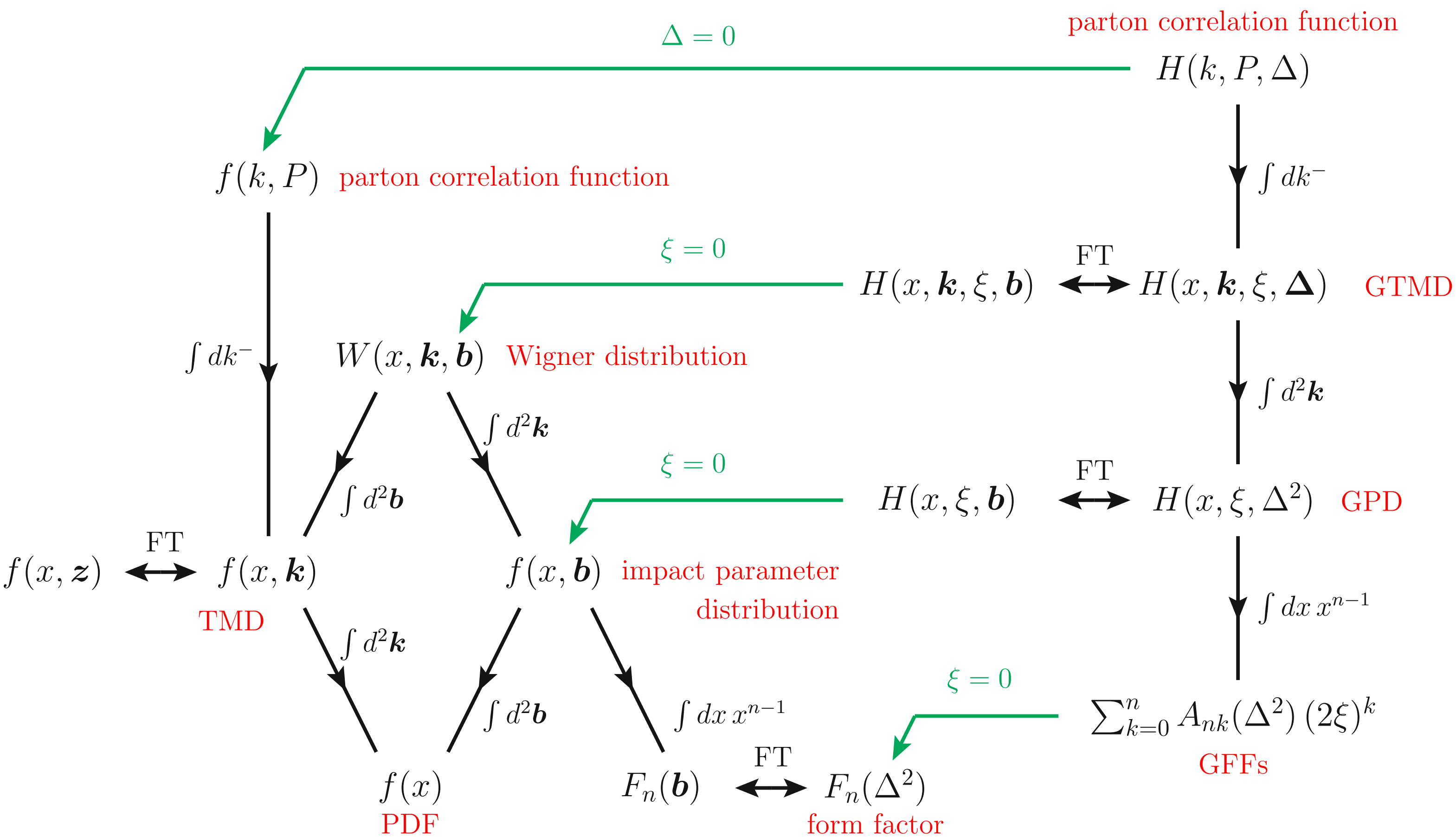
$$g_{1T}^{(1)}(x) \equiv \int d^2\boldsymbol{k}_T \; \frac{k_T^2}{2\, M^2} g_{1T}(x, k_T)$$

- Equations of motion relations

What happens when you integrate transverse momentum?

Proton / Quark	Unpolarized	Longitudinally polarized	Transversely polarized
Unpolarized	$f_1(x, k_T)$		$f_{1T}^{\perp}(x, k_T)$
Longitudinally polarized		$g_{1L}(x, k_T)$	$g_{1T}(x, k_T)$
Transversely polarized	$h_1^{\perp}(x, k_T)$	$h_{1L}(x, k_T)$	$h_{1T}(x, k_T)$ $h_{1T}^{\perp}(x, k_T)$

Tiers of transverse momentum dependence



M. Diehl, Eur.Phys.J.A 52 (2016) 6, 149

Phenomenology

- TMD / higher-twist frequently used interchangeably:

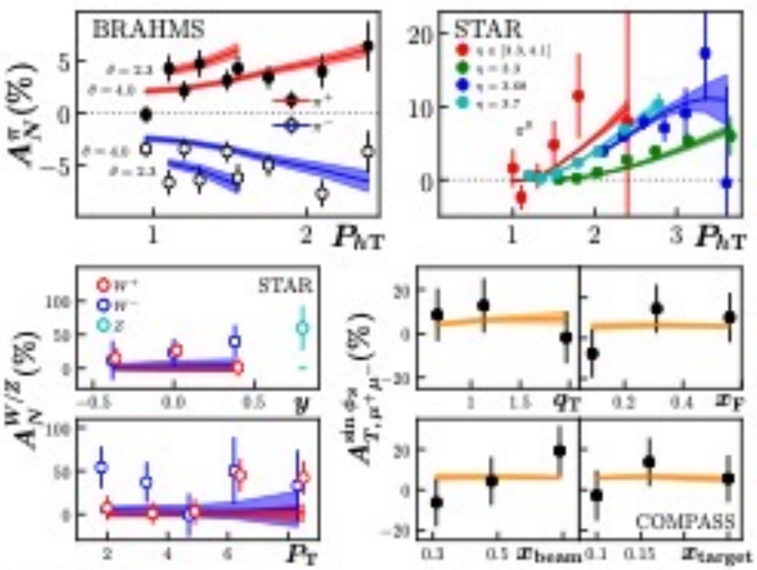


FIG. 5. Theory compared to experiment for A_N^{π} and A_{DY}^{Siv}.

J. Cammarota et al, Phys.Rev.D 102 (2020) 5, 054002

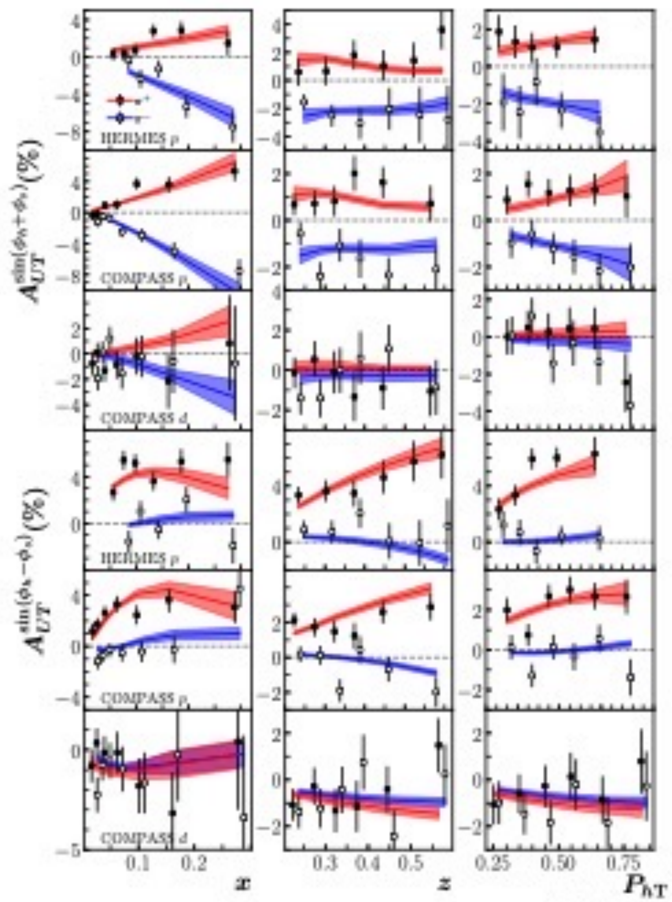


FIG. 4. Theory compared to experiment for $A_{SIDIS}^{Col/Siv}$.

Two views of factorization

Collins, Rogers, Sato, In Preparation
J. Collins, "Foundations of Perturbative QCD", Sec. 9.11

- Track A:

- Track B:

Two views of factorization

- Track B:
  - Assert: $\underline{\mathrm{d}\sigma} = f_{\text{``bare''}} \otimes \mathrm{d}\hat{\sigma}$

Two views of factorization

- Track B:
  - Assert: $\mathrm{d}\hat{\sigma} = f_{\text{``bare''}} \otimes \mathrm{d}\hat{\sigma}$
  - Collinear divergences! $\mathrm{d}\hat{\sigma} = C \otimes \mathrm{d}\hat{\sigma}_{\mathrm{finite}}$

Two views of factorization

- Track B:
  - Assert: $\mathrm{d}\cancel{\hat{\sigma}} = f_{\text{“bare”}} \otimes \mathrm{d}\hat{\sigma}$
  - Collinear divergences! $\mathrm{d}\hat{\sigma} = C \otimes \mathrm{d}\hat{\sigma}_{\text{finite}}$
  - So…

$$\mathrm{d}\cancel{\hat{\sigma}} = f_{\text{“bare”}} \otimes C \otimes \mathrm{d}\hat{\sigma}_{\text{finite}}$$

Two views of factorization

- Track B:
  - Assert: $\mathrm{d}\hat{\sigma} = f_{\text{“bare”}} \otimes \mathrm{d}\hat{\sigma}$
  - Collinear divergences! $\mathrm{d}\hat{\sigma} = C \otimes \mathrm{d}\hat{\sigma}_{\text{finite}}$
  - So…
    $$\mathrm{d}\hat{\sigma} = f_{\text{“bare”}} \otimes C \otimes \mathrm{d}\hat{\sigma}_{\text{finite}}$$
  - Absorb
    $$f = f_{\text{“bare”}} \otimes C$$

Two views of factorization

- Track B:
  - Assert: $\mathrm{d}\hat{\sigma} = f_{\text{“bare”}} \otimes \mathrm{d}\hat{\sigma}$
  - Collinear divergences! $\mathrm{d}\hat{\sigma} = C \otimes \mathrm{d}\hat{\sigma}_{\text{finite}}$
  - So…

$$\mathrm{d}\hat{\sigma} = f_{\text{“bare”}} \otimes C \otimes \mathrm{d}\hat{\sigma}_{\text{finite}}$$

  - Absorb

$$f = f_{\text{“bare”}} \otimes C$$

  - Then

$$\mathrm{d}\sigma = f_{\cancel{\text{“bare”}}} \otimes \mathrm{d}\hat{\sigma}_{\text{finite}}$$

Two views of factorization

- Track B: *Problems*
  - ``Bare'' pdf is ill-defined
  - Collinear divergences viewed as physical?

Two views of factorization

- Track A:
  - Start with the operator definition of the pdf
    - Deal with UV divergences with renormalization

$$f^{\rm bare,a}(\xi) \equiv \int \frac{{\rm d}w^-}{2\pi}\, e^{-i\xi p^+ w^-}\, \langle p|\, \bar{\psi}_0(0,w^-,\mathbf{0}_{\rm T})\, \frac{\gamma^+}{2} W[0,w^-]\psi_0(0,0,\mathbf{0}_{\rm T})\, |p\rangle$$

$$f^{\rm renorm,a}(\xi) \equiv Z^a \otimes f^{\rm bare,a}$$

Two views of factorization

- Track A:
  - Start with the operator definition of the pdf
    - Deal with UV divergences with renormalization
  - Derive factorization by analyzing dominant regions

Two views of factorization

- Track A:
  - Start with the operator definition of the pdf
    - Deal with UV divergences with renormalization
  - Derive factorization by analyzing dominant regions
  - Higher orders are constructed from nested subtractions

Positivity

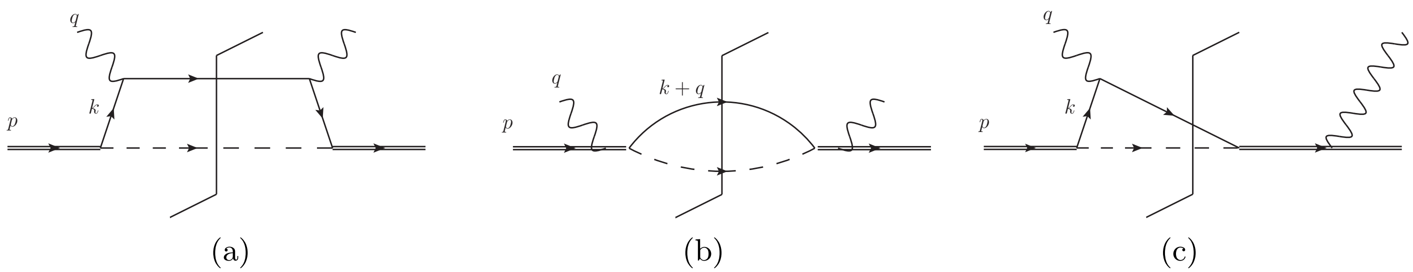
- Practical consequences?

- Example: Track-B leads to arguments that pdf positivity as an absolute property of pdfs in certain schemes

A. Candido, S. Forte, and F. Hekhorn (2020), 2006.07377

Example

- Stress-test assertions in any finite-range renormalizable theory



$$\mathcal{L}_{\text{int}} = -\lambda\, \overline{\Psi}_N\, \psi_q\, \phi \;+\; \text{H.C.}$$

- Exact $O(\lambda^2)$ DIS cross section is easy to calculate exactly

Example

- Collinear Factorization

$$F_1(x,Q) = \sum_f \int_x^1 \frac{d\xi}{\xi}$$

$$\times \underbrace{\frac{1}{2}\left\{\delta\left(1-\frac{x}{\xi}\right)\delta_{qf} + a_\lambda(\mu)\left(1-\frac{x}{\xi}\right)\left[\ln(4) - \frac{\left(\frac{x}{\xi}\right)^2 - 3\frac{x}{\xi} + \frac{3}{2}}{\left(1-\frac{x}{\xi}\right)^2} - \ln\frac{4x\mu^2}{Q^2(\xi - x)}\right]\delta_{pf}\right\}}_{\hat{F}_{1,q/f}(x/\xi,\mu/Q;a_\lambda(\mu))} \times$$

$$\times \underbrace{\left\{\delta(1-\xi)\,\delta_{fp} + a_\lambda(\mu)(1-\xi)\left[\frac{(m_q+\xi m_p)^2}{\Delta(\xi)^2} + \ln\left(\frac{\mu^2}{\Delta(\xi)^2}\right) - 1\right]\delta_{fq}\right\}}_{f_{f/p}(\xi;\mu)} .$$

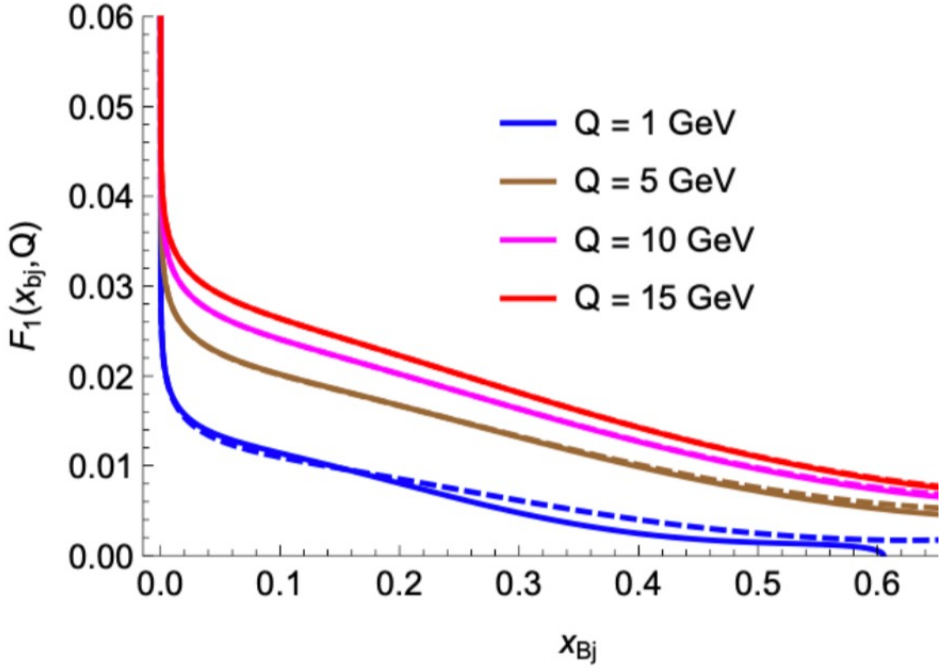
Partonic structure function

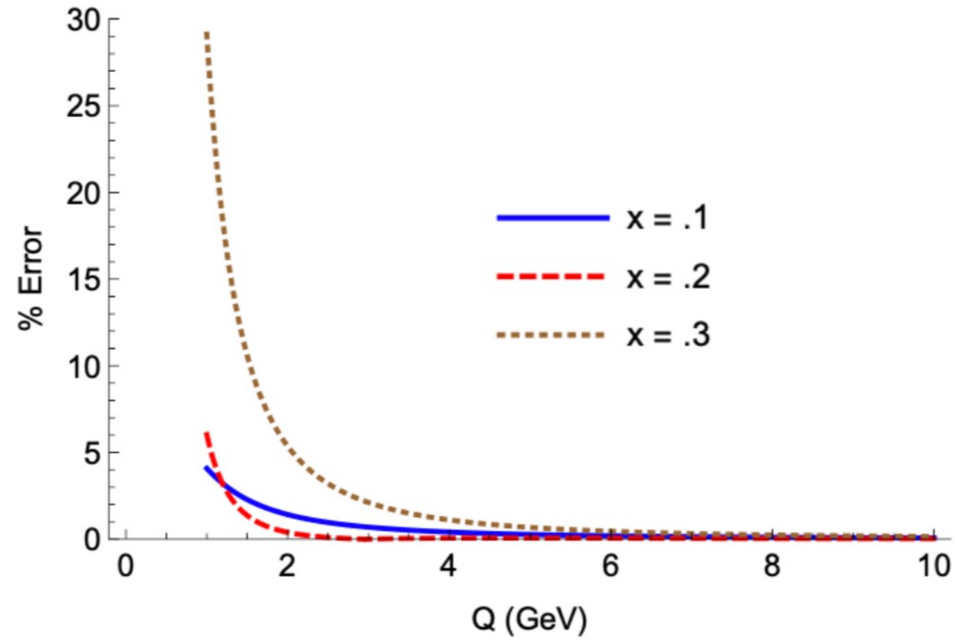
Parton Distribution

$$+ \mathcal{O}\left(\frac{\Lambda_{QCD}}{Q}\right)$$

Example

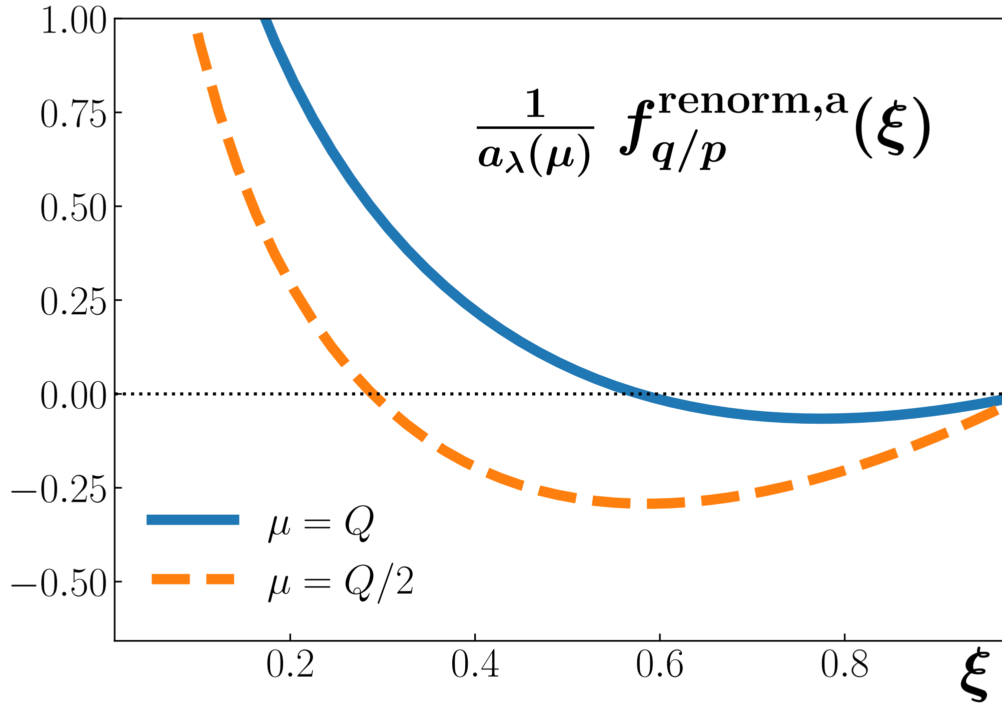
- Collinear Factorization





Example

- Negative pdfs



$\overline{MS}$ ren

Transverse momentum in correlation functions and in cross sections

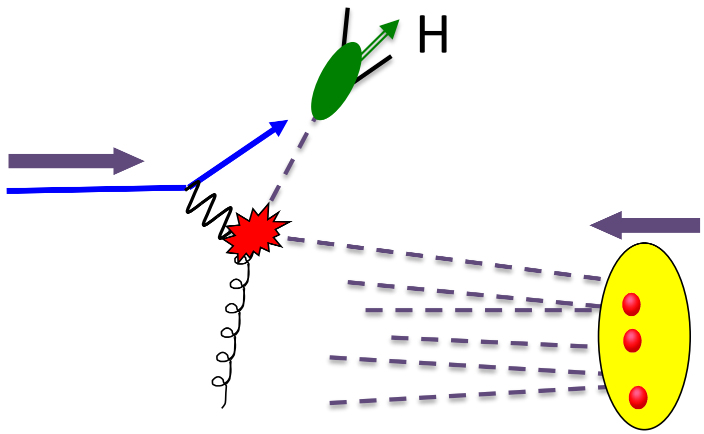
- Correlation functions
  - Parton densities (pdfs), fragmentation functions, others…

Factorization

- Cross sections
  - Semi-inclusive deep inelastic scattering, Drell-Yan, etc…

Semi-inclusive deep inelastic scattering

$$\frac{d\sigma^{\mathrm{SIDIS}}}{dx\,dy\,dz\,d^2\mathbf{P}_{hT}}$$

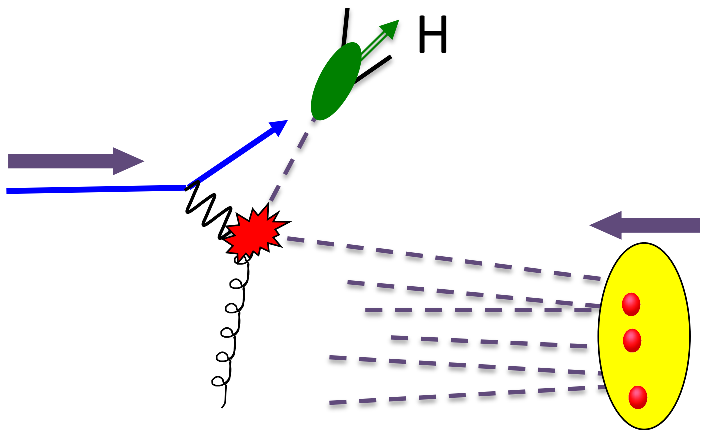


- *Large P_T, insensitive to intrinsic parton transverse momentum.*

➡ *Collinear factorization and evolution (DGLAP, etc)*

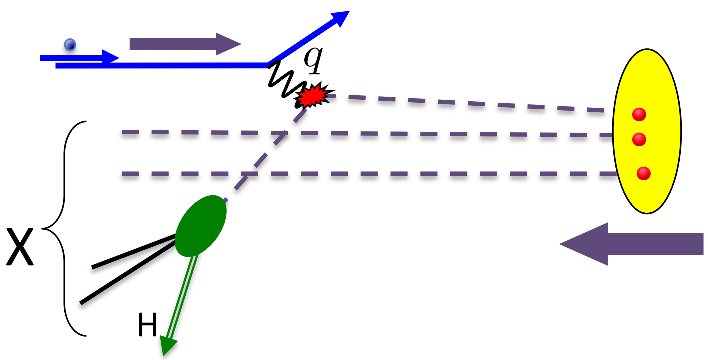
Semi-inclusive deep inelastic scattering

$$\frac{\mathrm{d}\sigma^{\mathrm{SIDIS}}}{\mathrm{d}x\mathrm{d}y\mathrm{d}z\mathrm{d}^2\mathbf{P}_{h\mathrm{T}}}$$



- *Large P_T, insensitive to intrinsic parton transverse momentum.*

→ *Collinear factorization and evolution (DGLAP, etc)*



- *Small P_T, access to intrinsic parton transverse momentum*

→ *TMD factorization, TMD evolution, Sudakov, etc*

Transverse momentum dependence and factorization

$$\frac{d\sigma}{dx\,dz\,dQ\,d\boldsymbol{q}_T} = \underbrace{H(Q) f(x, \boldsymbol{k}_{1T}) \otimes d(z, z\boldsymbol{k}_{2T})}_{\substack{\text{Small } q_T/\text{Q} \\ \text{TMD factorization}}} + \underbrace{Y(x, z, \boldsymbol{q}_T, Q)}_{\substack{q_T \sim Q \\ \text{collinear factorization} \\ (\text{Small } m/q_T)}} + O(m/Q)$$

Transverse momentum dependence and factorization

Extra scales for TMD evolution

Collinear / DGLAP evolution scale, μ

$$\frac{d\sigma}{dx\,dz\,dQ\,d\boldsymbol{q}_T} = \underbrace{H(Q)\,f(x,\boldsymbol{k}_{1T}) \otimes d(z, z\boldsymbol{k}_{2T})}_{\substack{\text{Small } q_T/\text{Q} \\ \text{TMD factorization}}} + \underbrace{Y(x,z,\boldsymbol{q}_T,Q)}_{\substack{q_T \sim Q \\ \text{collinear factorization} \\ (\text{Small } m/q_T)}} + O(m/Q)$$

Transverse momentum dependence and factorization

Extra scales for TMD evolution

Collinear / DGLAP evolution scale, μ

$$\frac{d\sigma}{dx\,dz\,dQ\,d\boldsymbol{q}_T} = \underbrace{H(Q) f(x, \boldsymbol{k}_{1T}) \otimes d(z, z\boldsymbol{k}_{2T})}_{\substack{\text{Small } q_T/\text{Q} \\ \text{TMD factorization}}} + \underbrace{Y(x, z, \boldsymbol{q}_T, Q)}_{\substack{q_T \sim Q \\ \text{collinear factorization} \\ (\text{Small } m/q_T)}} + O(m/Q)$$

- *There is an overlapping collinear/TMD description for $m \ll q_T \ll Q$*

For single-spin asymmetries:

X. Ji, J.-W. Qiu, W. Vogelsang, and F. Yuan, Phys. Rev. Lett. 97, 082002 (2006)
X. Ji, J.-W. Qiu, W. Vogelsang, and F. Yuan, Phys. Rev. D73, 094017 (2006)
I. Scimemi, A. Tarasov, and A. Vladimirov, JHEP 05, 125 (2019)

Integrated observables

$$\frac{d\sigma}{dx\,dz\,dQ\,d\boldsymbol{q}_T} = H(Q) f(x, \boldsymbol{k}_{1T}) \otimes d(z, z\boldsymbol{k}_{2T}) + Y(x, z, \boldsymbol{q}_T, Q) + O(m/Q)$$

- *Unpolarized*

$$\int d^2\boldsymbol{q}_T \frac{d\sigma}{dx\,dz\,dQ\,d\boldsymbol{q}_T} = \int_x^1 \frac{d\xi}{\xi} \int_z^1 \frac{d\zeta}{\zeta^2} H(Q, x/\xi\,, z/\zeta) f(\xi) d(\zeta) + O(m/Q)$$

Integrated observables

$$\frac{d\sigma}{dx\,dz\,dQ\,d\boldsymbol{q}_T} = H(Q) f(x, \boldsymbol{k}_{1T}) \otimes d(z, z\boldsymbol{k}_{2T}) + Y(x, z, \boldsymbol{q}_T, Q) + O(m/Q)$$

- *Unpolarized*

$$\int d^2\boldsymbol{q}_T \frac{d\sigma}{dx\,dz\,dQ\,d\boldsymbol{q}_T} = \int_x^1 \frac{d\xi}{\xi} \int_z^1 \frac{d\zeta}{\zeta^2} H(Q, x/\xi, z/\zeta) f(\xi) d(\zeta) + O(m/Q)$$

Collinear / DGLAP evolution scale, μ

$O(\alpha_s)$

Integrated observables

$$\frac{d\sigma}{dx\, dz\, dQ\, d\boldsymbol{q}_T} = H(Q) f(x, \boldsymbol{k}_{1T}) \otimes d(z, z\boldsymbol{k}_{2T}) + Y(x, z, \boldsymbol{q}_T, Q) + O(m/Q)$$

- *Unpolarized, approximated*

$$\int d^2\boldsymbol{q}_T \frac{d\sigma}{dx\, dz\, dQ\, d\boldsymbol{q}_T} \approx H(Q) f(x) \otimes d(z)$$

$$\int d^2 k_{1T}\, f(x, k_{1T}) \approx f(x)$$

$$\int d^2 k_{2T}\, d(z, z k_{2T}) \approx d(x)$$

Integrated observables

$$\frac{d\sigma}{dx\,dz\,dQ\,d\boldsymbol{q}_T} = H(Q) f(x, \boldsymbol{k}_{1T}) \otimes d(z, z\boldsymbol{k}_{2T}) + Y(x, z, \boldsymbol{q}_T, Q) + O(m/Q)$$

- *Unpolarized, approximated*

$$\int d^2\boldsymbol{q}_T \frac{d\sigma}{dx\,dz\,dQ\,d\boldsymbol{q}_T} \approx H(Q) f(x) \otimes d(z)$$

$$\int d^2 k_{1T}\, f(x, k_{1T}) \approx f(x)$$

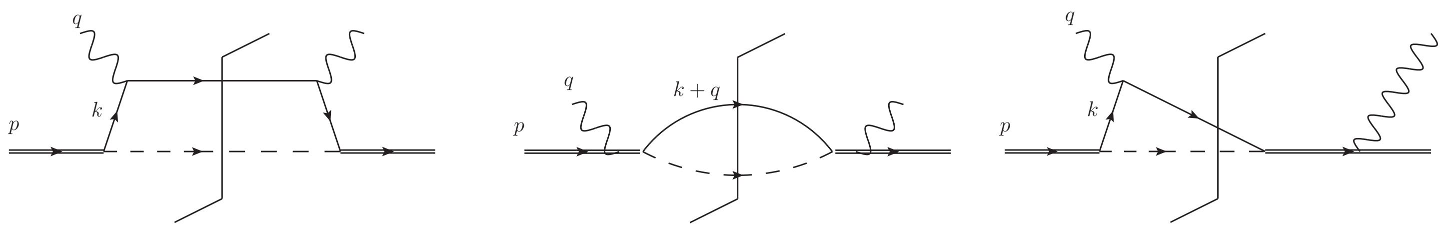
$$\int d^2 k_{2T}\, d(z, z k_{2T}) \approx d(x)$$

$$\left[+O(\alpha_s(k_c)) \right]$$

k_c = transverse momentum cutoff

Example

- $q_T \approx Q$ outside the region where TMD factorization is applicable
- Still needed for TMD pdf identification



(a) (b) (c)

$$\mathcal{L}_{\text{int}} = -\lambda\, \overline{\Psi}_N\, \psi_q\, \phi \;+\; \text{H.C.}$$

- Exact $\mathrm{O}(\lambda^2)$ cross section is easy to calculate

Example

- Collinear Factorization

$$F_1(x,Q) \doteq \sum_f \int_x^1 \frac{\mathrm{d}\xi}{\xi}$$

$$\times \underbrace{\frac{1}{2}\left\{\delta\left(1-\frac{x}{\xi}\right)\delta_{qf} + a_\lambda(\mu)\left(1-\frac{x}{\xi}\right)\left[\ln(4) - \frac{\left(\frac{x}{\xi}\right)^2 - 3\frac{x}{\xi} + \frac{3}{2}}{\left(1-\frac{x}{\xi}\right)^2} - \ln\frac{4x\mu^2}{Q^2(\xi - x)}\right]\delta_{pf}\right\}\times}_{\hat{F}_{1,q/f}(x/\xi,\mu/Q;a_\lambda(\mu))}$$

$$\times \underbrace{\left\{\delta(1-\xi)\,\delta_{fp} + a_\lambda(\mu)(1-\xi)\left[\frac{(m_q+\xi m_p)^2}{\Delta(\xi)^2} + \ln\left(\frac{\mu^2}{\Delta(\xi)^2}\right) - 1\right]\delta_{fq}\right\}}_{f_{f/p}(\xi;\mu)} .$$

Partonic structure function

Parton Distribution

Effect from integrating $k_T \to \infty$ cancels

Example

- TMD Factorization

$$F_1^W(x,z,\boldsymbol{k}_{\mathrm{T}},Q) = \hat{F}_1^W \int \mathrm{d}^2\boldsymbol{k}_{1\mathrm{T}}\mathrm{d}^2\boldsymbol{k}_{2\mathrm{T}}\delta^{(2)}(\boldsymbol{k}_{1\mathrm{T}}+\boldsymbol{k}_{\mathrm{T}}-\boldsymbol{k}_{2\mathrm{T}})f(x,\boldsymbol{k}_{1\mathrm{T}};\mu)d(z,z\boldsymbol{k}_{2\mathrm{T}};\mu)$$

$$\hat{F}_1^W = \frac{1}{2}$$

$$f(x,\boldsymbol{k}_{1\mathrm{T}};\mu) = \frac{a_\lambda(\mu)}{\pi}\frac{(1-x)\left[k_{\mathrm{T}}^2+(m_q+xm_p)^2\right]}{\left[k_{\mathrm{T}}^2+xm_s^2+(1-x)\,m_q^2+x(x-1)m_p^2\right]^2}$$

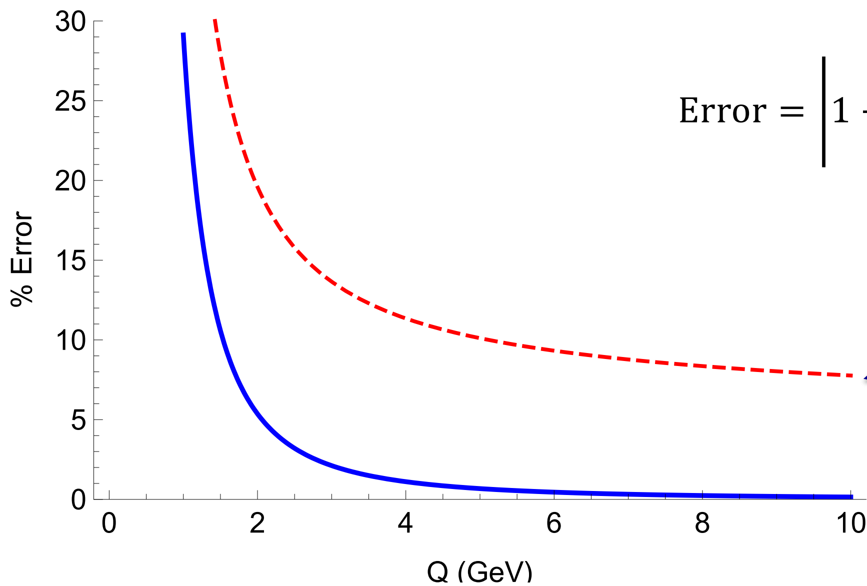
$$d(z,z\boldsymbol{k}_{2\mathrm{T}};\mu) = \delta(1-z)\delta^{(2)}(z\boldsymbol{k}_{2\mathrm{T}})$$

Example

- Accuracy

Collinear Factorization

Integrated TMD factorization



$$\text{Error} = \left| 1 - \frac{F_1^{\text{Factorized}}}{F_1^{\text{Exact}}} \right|$$

$$\int_0^Q d^2 k_T \, [F_1(x, k_T)]_{\text{TMD fact.}}$$

Effect of Large Transverse Momentum

- Integrated cross section in generalized parton model

$$\int d^2\boldsymbol{q}_T \left(\frac{d\sigma}{dx\ dz\ dQ\ dq_T}\right) = H(Q)\left(\int d^2\mathbf{k}_{1T}\ f(x, k_{1T})\right)\left(\int d^2\mathbf{k}_{2T}\ d(z, z\ k_{2T})\right)$$

$$= H(Q)f(x)d(z)$$

- Full integral

$$\int d^2\boldsymbol{q}_T \left(\frac{d\sigma}{dx\ dz\ dQ\ dq_T}\right) = \int d^2\boldsymbol{q}_T \left(H(Q)f(x, k_{1T}) \otimes d(z, z\ k_{2T}) + Y(x, z, q_T)\right)$$

Cutoff dependence cancels between terms

Weighted integrals

$$\int \mathrm{d}^2\mathbf{P}_{h\mathrm{T}}\,(P^{\alpha}_{h\mathrm{T}})^n\,\frac{\mathrm{d}\sigma^{\mathrm{SIDIS}}}{\mathrm{d}x\mathrm{d}y\mathrm{d}z\mathrm{d}^2\mathbf{P}_{h\mathrm{T}}}$$

Quark \ Proton	Unpolarized	Longitudinally polarized	Transversely polarized
Unpolarized	$f_1(x,k_T)$		$f_{1T}^{\perp}(x,k_T)$
Longitudinally polarized		$g_{1L}(x,k_T)$	$g_{1T}(x,k_T)$
Transversely polarized	$h_1^{\perp}(x,k_T)$	$h_{1L}(x,k_T)$	$h_{1T}(x,k_T)$ $h_{1T}^{\perp}(x,k_T)$

Weighted integrals

$$\int \mathrm{d}^2\mathbf{P}_{h\mathrm{T}}\,(P_{h\mathrm{T}}^{\alpha})^n\,\frac{\mathrm{d}\sigma^{\mathrm{SIDIS}}}{\mathrm{d}x\mathrm{d}y\mathrm{d}z\mathrm{d}^2\mathbf{P}_{h\mathrm{T}}}$$

- T-odd effects

$$\int d^2\mathbf{k}_T\,\frac{k_T^2}{2M^2}f_{1\perp}(x,k_T) = f_{1\perp}^{(1)}(x) \approx \frac{1}{M}T(x)$$

Boer, Mulders, Pijlman, Nucl. Phys.B667, 201 (2003)

Proton / Quark	Unpolarized	Longitudinally polarized	Transversely polarized
Unpolarized	$f_1(x,k_T)$	✗	$f_{1T}^{\perp}(x,k_T)$
Longitudinally polarized	✗	$g_{1L}(x,k_T)$	$g_{1T}(x,k_T)$
Transversely polarized	$h_1^{\perp}(x,k_T)$	$h_{1L}(x,k_T)$	$h_{1T}(x,k_T)$ $h_{1T}^{\perp}(x,k_T)$

Weighted integrals

$$\int \mathrm{d}^2\mathbf{P}_{h\mathrm{T}}\,(P_{h\mathrm{T}}^{\alpha})^n\,\frac{\mathrm{d}\sigma^{\mathrm{SIDIS}}}{\mathrm{d}x\mathrm{d}y\mathrm{d}z\mathrm{d}^2\mathbf{P}_{h\mathrm{T}}}$$

- T-odd effects

$$\int d^2\mathbf{k}_T\,\frac{k_T^2}{2M^2}f_{1\perp}(x,k_T) = f_{1\perp}^{(1)}(x) \approx \frac{1}{M}T(x)$$

Boer, Mulders, Pijlman, Nucl. Phys.B667, 201 (2003)

$$T_{i(g)/H}(x) = g_s\,\epsilon^{S_T\alpha}g_{\alpha\beta}$$
$$\times\int\frac{\mathrm{d}\xi^-\mathrm{d}\eta^-}{4\pi}e^{ixP^+\xi^-}\langle P,S|\bar{\psi}_i(0)G^{\beta+}(\eta^-)\gamma^+\psi_i(\xi^-)|P,S\rangle$$

Weighted integrals

$$\int \mathrm{d}^2\mathbf{P}_{h\mathrm{T}}\,(P^{\alpha}_{h\mathrm{T}})^n\,\frac{\mathrm{d}\sigma^{\mathrm{SIDIS}}}{\mathrm{d}x\mathrm{d}y\mathrm{d}z\mathrm{d}^2\mathbf{P}_{h\mathrm{T}}}$$

- T-odd effects

$$\int d^2\mathbf{k}_T\,\frac{k_T^2}{2M^2}f_{1\perp}(x,k_T) = f_{1\perp}^{(1)}(x) \approx \frac{1}{M}T(x)$$

Boer, Mulders, Pijlman, Nucl. Phys.B667, 201 (2003)

- Lorentz-invariant relations

$$g_T(x) = g_1(x) + \frac{d}{dx}g_{1T}^{(1)}(x),$$

$$g_{1T}^{(1)}(x) \equiv \int d^2\boldsymbol{k}_T\,\frac{k_T^2}{2\,M^2}g_{1T}(x,k_T)$$

Mulders, Tangerman, Nucl. Phys. B461, 197 (1996)

Proton / Quark	Unpolarized	Longitudinally polarized	Transversely polarized
Unpolarized	$f_1(x,k_T)$	✗	$f_{1T}^{\perp}(x,k_T)$
Longitudinally polarized	✗	$g_{1L}(x,k_T)$	$g_{1T}(x,k_T)$
Transversely polarized	$h_1^{\perp}(x,k_T)$	$h_{1L}(x,k_T)$	$h_{1T}(x,k_T)$ $h_{1T}^{\perp}(x,k_T)$

Weighted integrals

$$\int \mathrm{d}^2\mathbf{P}_{h\mathrm{T}}\,(P^{\alpha}_{h\mathrm{T}})^n\,\frac{\mathrm{d}\sigma^{\mathrm{SIDIS}}}{\mathrm{d}x\mathrm{d}y\mathrm{d}z\mathrm{d}^2\mathbf{P}_{h\mathrm{T}}}$$

- T-odd effects

$$\int d^2\mathbf{k}_T\,\frac{k_T^2}{2M^2} f_{1\perp}(x,k_T) = f_{1\perp}^{(1)}(x) \approx \frac{1}{M}T(x)$$

Boer, Mulders, Pijlman, Nucl. Phys.B667, 201 (2003)

- Lorentz-invariant relations

$$g_T(x) = g_1(x) + \frac{d}{dx} g_{1T}^{(1)}(x),$$

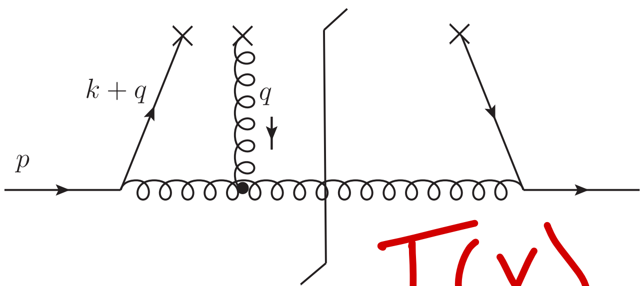
$$g_{1T}^{(1)}(x) \equiv \int d^2\boldsymbol{k}_T\,\frac{k_T^2}{2\,M^2} g_{1T}(x,k_T)$$

Mulders, Tangerman, Nucl. Phys. B461, 197 (1996)

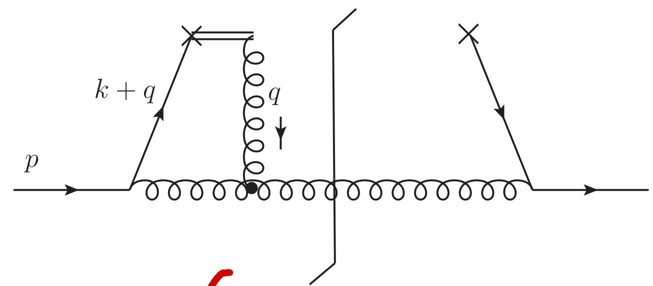
- *Which kind of evolution?*

Proton / Quark	Unpolarized	Longitudinally polarized	Transversely polarized
Unpolarized	$f_1(x,k_T)$	✗	$f_{1T}^{\perp}(x,k_T)$
Longitudinally polarized	✗	$g_{1L}(x,k_T)$	$g_{1T}(x,k_T)$
Transversely polarized	$h_1^{\perp}(x,k_T)$	$h_{1L}(x,k_T)$	$h_{1T}(x,k_T)$ $h_{1T}^{\perp}(x,k_T)$

Renormalization and regularization



T(x)

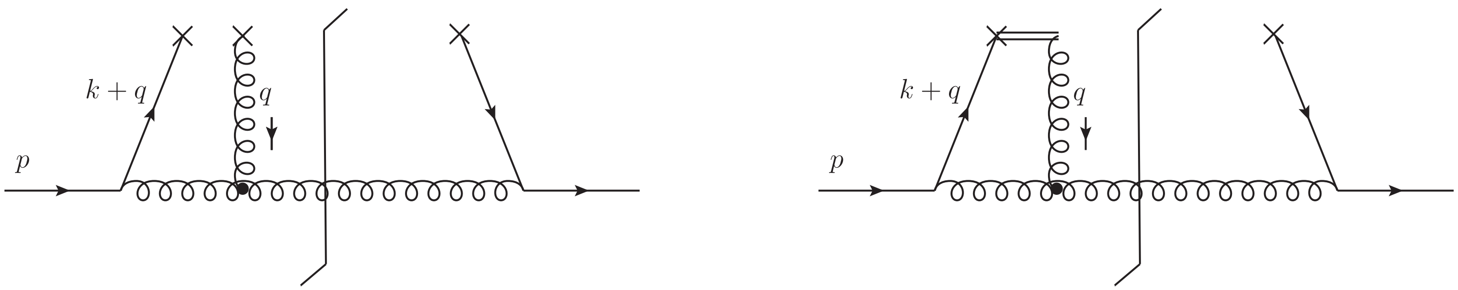


Sivers

$$\left[f_{1\perp}^{(1)}(x)\right]^{\text{cutoff}} - \left[\frac{1}{M}T(x)\right]^{\overline{\text{MS}}\text{ renorm}} \sim \alpha_s^2(k_c)\ln^2\left(\frac{k_c^2}{m^2}\right)$$

With Jianwei Qiu and Bowen Wang:
Phys.Rev.D 101 (2020) 11, 116017, 2004.13193
and 2008.05351

Renormalization and regularization



$$\left[f_{1\perp}^{(1)}(x)\right]^{\text{cutoff}} - \left[\frac{1}{M}T(x)\right]^{\overline{\text{MS}}\text{ renorm}} \sim \alpha_s^2(k_c)\ln^2\left(\frac{k_c^2}{m^2}\right)$$

$$\alpha_s^2(k_c) \sim 1/\ln^2(k_c^2/m^2)$$

Difference unsuppressed by asymptotic freedom

With Jianwei Qiu and Bowen Wang:
Phys.Rev.D 101 (2020) 11, 116017, [2004.13193](2004.13193)
and [2008.05351](2008.05351)

Discussion

- Ultraviolet divergences in transversely-integrated quantities relate to:
  - Identifying intrinsic versus process-specific effects
  - Evolution

Discussion

- Ultraviolet divergences in transversely-integrated quantities relate to:
  - Identifying intrinsic versus process-specific effects
  - Evolution
- Collinear pdfs: Positivity?
- Does asymptotic freedom does always suppress errors to the naïve number density interpretation?

Discussion

- Ultraviolet divergences in transversely-integrated quantities relate to:
  - Identifying intrinsic versus process-specific effects
  - Evolution
- Collinear pdfs: Positivity?
- Does asymptotic freedom does always suppress errors to the naïve number density interpretation?
- When access to the intrinsic transverse momentum is the objective:

Discussion

- Ultraviolet divergences in transversely-integrated quantities relate to:
  - Identifying intrinsic versus process-specific effects
  - Evolution
- Collinear pdfs: Positivity?
- Does asymptotic freedom does always suppress errors to the naïve number density interpretation?
- When access to the intrinsic transverse momentum is the objective:
  - Limit transverse momentum in weighted integrals and use TMD evolution
- How to merge collinear higher twist and TMD in integrated quantities?
- How to merge collinear HT and TMD in integrated quantities?

Discussion

- Ultraviolet divergences in transversely-integrated quantities relate to:
  - Identifying intrinsic versus process-specific effects
  - Evolution
- Collinear pdfs: Positivity?
- Does asymptotic freedom does always suppress errors to the naïve number density interpretation?
- When access to the intrinsic transverse momentum is the objective:
  - Limit transverse momentum in weighted integrals and use TMD evolution
- How to merge collinear higher twist and TMD in integrated quantities?

Discussion

- Ultraviolet divergences in transversely-integrated quantities relate to:
  - Identifying intrinsic versus process-specific effects
  - Evolution
- Collinear pdfs: Positivity?
- Does asymptotic freedom does always suppress errors to the naïve number density interpretation?
- When access to the intrinsic transverse momentum is the objective:
  - Limit transverse momentum in weighted integrals and use TMD evolution
- How to merge collinear higher twist and TMD in integrated quantities?

Thanks!

Lorentz Invariance Relations

With F. Aslan and L. Gamberg

- Divergences

$$g_T(x) = g_1(x) + \frac{d}{dx} g_{1T}^{(1)}(x),$$

Mulders, Tangerman, Nucl. Phys. B461, 197 (1996)

$$g_{1T}^{(1)}(x) \equiv \int^{\Lambda} d^2\boldsymbol{k}_T \; \frac{k_T^2}{2\,M^2} g_{1T}(x, k_T)$$

Divergent integral

$$\Rightarrow \quad g_T(x) - g_1(x) - \frac{d}{dx} g_{1T}^{(1)}(x) \neq 0$$