Transverse momentum integrals and the positivity violations of collinear pdfs

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Based on recent work with John Collins and Nobuo Sato: arXiv:2111.01170,
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Track A Factorization:

- Operator definition of the pdf from the beginning.
  - The only divergences are ultraviolet.
  - Deal with them using standard UV renormalization techniques.
- Factorization (e.g., inclusive DIS):
  - Obtained from general region analysis.
  - Beyond parton model: Higher order hard scattering constructed from nested subtractions.

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$$f^{\mathrm{bare,a}}(\xi) \equiv \int \frac{\mathrm{d}w^-}{2\pi}\, e^{-i\xi p^+ w^-} \,\langle p|\, \bar{\psi}_0(0, w^-, \mathbf{0}_\mathrm{T}) \frac{\gamma^+}{2} W[0, w^-] \psi_0(0, 0, \mathbf{0}_\mathrm{T})\, |p\rangle$$

$$f^{\mathrm{renorm,a}}(\xi) \equiv Z^a \otimes f^{\mathrm{bare,a}} \qquad\qquad Z^a = \delta(1-\xi) + \sum_{j=1}^{\infty} C_j \left(\frac{S_\epsilon}{\epsilon}\right)^j$$

Track B Factorization:

- Assert(?): $\mathrm{d}\sigma = f_{\text{“bare,b”}} \otimes \mathrm{d}\hat{\sigma}$

  Massless partonic

- Collinear divergences! $\mathrm{d}\hat{\sigma} = \mathcal{C} \otimes \mathrm{d}\hat{\sigma}_{\text{finite}}$

- So…

$$\mathrm{d}\sigma = f_{\text{“bare,b”}} \otimes \mathcal{C} \otimes \mathrm{d}\hat{\sigma}_{\text{finite}}$$

- Absorb:

$$f = f_{\text{“bare,b”}} \otimes \mathcal{C}$$

- Then:

$$\mathrm{d}\sigma = f \otimes \mathrm{d}\hat{\sigma}_{\text{finite}}$$

E.g., Curci, Furmanski, Petronzio (1980)

Track B:

- Questions:
  - Derivation of factorization for step 1 ($\mathrm{d}\sigma = f_{\text{“bare,b”}} \otimes \mathrm{d}\hat{\sigma}$) ?
  - Bare pdf ($f_{\text{“bare,b”}}$) of step 1 is undefined
  - Interpretation of collinear divergences?
  - Can we reverse engineer $f_{\text{“bare,b”}}$?

Track A vs. Track B Logic

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- <u>Example 1:</u> Track-B leads to arguments that pdf positivity is an absolute property of pdfs in certain schemes (MS-bar).

$f(x;\mu) \geq 0$ *A. Candido, S. Forte, and F. Hekhorn (2020), 2006.07377*

Example

- Stress-test assertions about DIS factorization in other finite-range renormalizable theories.

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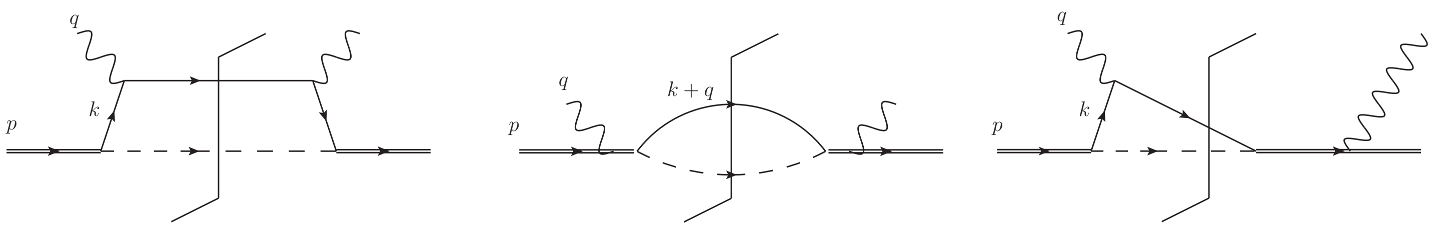

(a) (b) (c)

$$\mathcal{L}_{\rm int} = -\lambda\, \overline{\Psi}_N\, \psi_q\, \phi \; + \; \text{H.C.}$$

- Exact $O(\lambda^2)$ DIS cross section is easy to calculate exactly.

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$$f^{\mathrm{renorm,a}}(\xi) \equiv Z^a \otimes f^{\mathrm{bare,a}} \qquad\qquad Z^a = \delta(1-\xi) + \sum_{j=1}^{\infty} C_j \left(\frac{S_\epsilon}{\epsilon} \right)^j$$

Example

Collinear Factorization

$$\Delta(\xi)^2 \equiv \xi m_s^2 + (1-\xi)m_q^2 - \xi(1-\xi)m_p^2$$

$$F_1(x,Q) \doteq \sum_f \int_x^1 \frac{\mathrm{d}\xi}{\xi}$$

$$\times \underbrace{\frac{1}{2}\left\{\delta\left(1-\frac{x}{\xi}\right)\delta_{qf} + a_\lambda(\mu)\left(1-\frac{x}{\xi}\right)\left[\ln(4) - \frac{\left(\frac{x}{\xi}\right)^2 - 3\frac{x}{\xi} + \frac{3}{2}}{\left(1-\frac{x}{\xi}\right)^2} - \ln\frac{4x\mu^2}{Q^2(\xi-x)}\right]\delta_{pf}\right\}}_{\hat{F}_{1,q/f}(x/\xi,\mu/Q;a_\lambda(\mu))} \times$$

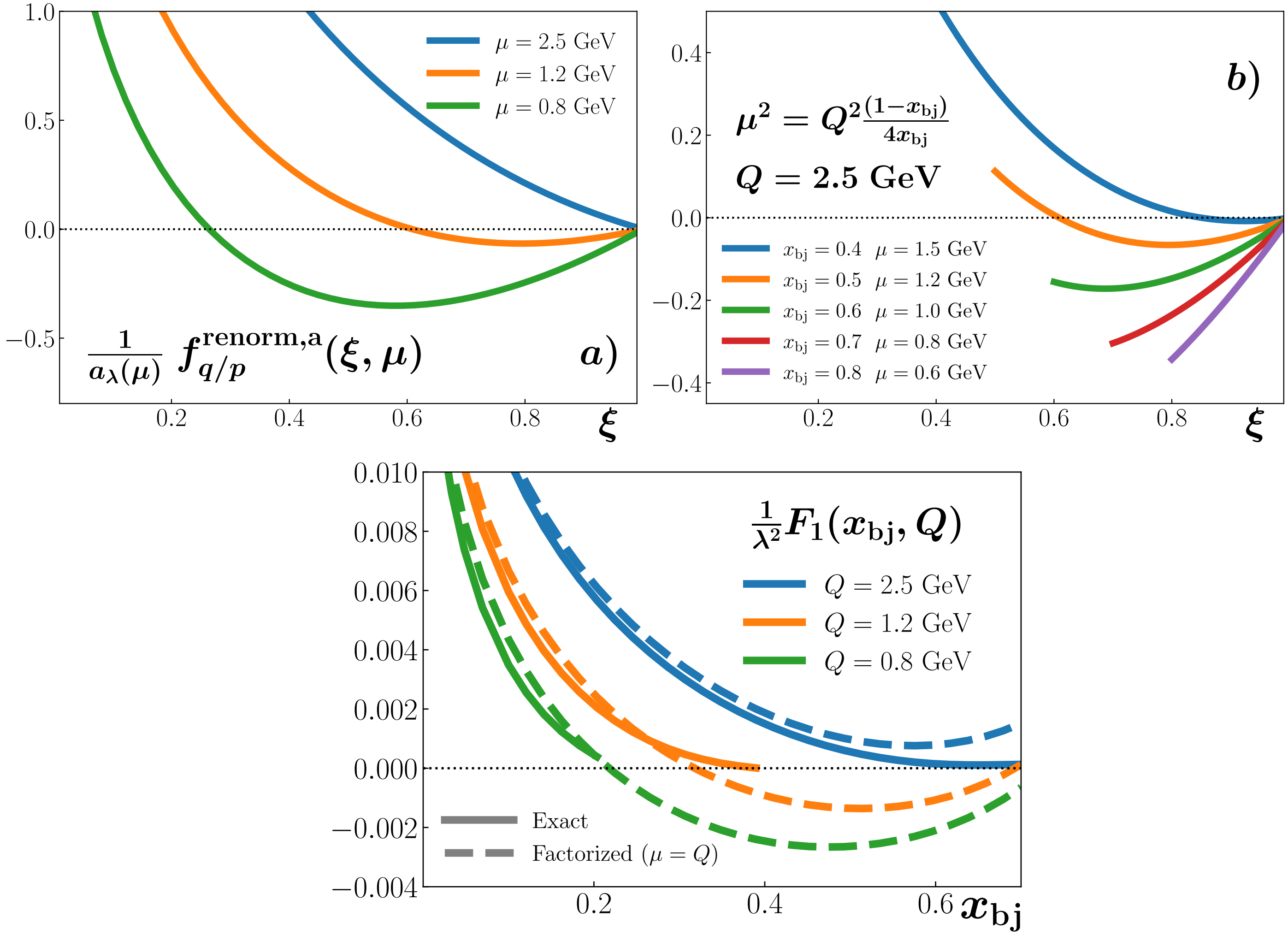
$$\times \underbrace{\left\{\delta(1-\xi)\delta_{fp} + a_\lambda(\mu)(1-\xi)\left[\frac{(m_q+\xi m_p)^2}{\Delta(\xi)^2} + \ln\left(\frac{\mu^2}{\Delta(\xi)^2}\right) - 1\right]\delta_{fq}\right\}}_{f_{f/p}(\xi;\mu)}.$$

Partonic structure function

Parton Distribution

$$\overline{\mathrm{MS}}\ \ \mathrm{C.T.} = -a_\lambda(\mu)(1-\xi)\frac{S_\epsilon}{\epsilon}$$

Positivity?



$m_q = 0.3$ GeV

$m_p = 1.0$ GeV

$m_s = 1.5$ GeV

Return to positivity

- Why does track B seem to imply properties like positivity?
- ”Bare” pdf is not “parton model pdf.” It inherits the properties of the UV regulator.
- Dimensional regularization violates positivity

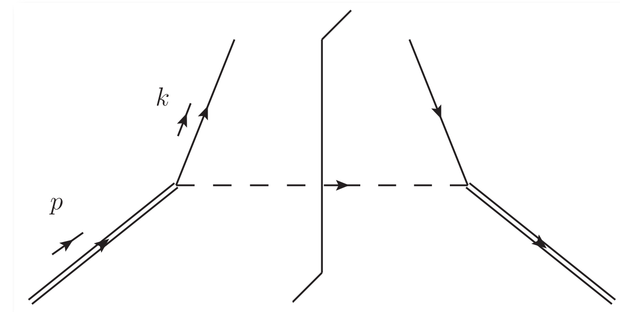
$$\int \mathrm{d}^{2-2\epsilon}\mathbf{k}_T \, \frac{(k_T^2 - Q^2)^2}{k_T^2(k_T^2+Q^2)^2} \overset{\epsilon \to 0}{=} -4\pi$$

- Vanishing of dimensionless integrals

Rescuing positivity?

- Instead try cutoff scheme (but be careful!)

Yukawa theory



$$a_\lambda(\mu) \int_0^{k^2_{\mathrm{cut,T}}} \mathrm{d}k_\mathrm{T}^2$$
$$\times \frac{(1-\xi)\left[k_\mathrm{T}^2 + (m_q + \xi m_p)^2\right]}{\left[k_\mathrm{T}^2 + \xi m_s^2 + (1-\xi)\, m_q^2 + \xi(\xi-1) m_p^2\right]^2}$$

To convert to $\overline{MS}$, subtract

$$a_\lambda(\mu)(1-\xi) \int_{\mu^2}^{k^2_{\mathrm{cut,T}}} \frac{\mathrm{d}k_\mathrm{T}^2}{k_\mathrm{T}^2}$$

Summary

- Historically, two alternative ways to view divergences and their role in pdf definitions.
  - Track A: UV renormalization – no collinear divergences
  - Track B: Collinear absorption – absorb collinear divergences
- Track A is more complete. Differences between tracks have practical consequences.
- Positivity is not a <u>general</u> property of MS-bar renormalized parton densities
- Other ways to get positivity via TMD functions?