A hadron structure oriented (HSO) approach to TMD phenomenology

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Based on:

- J.O. Gonzalez, TCR, N. Sato, Phys.Rev.D 106 (2022) 3, 034002
- J.O. Gonzalez, T. Rainaldi, TCR (2023), arXiv:2303.04921, To appear in PRD

&

- Work in progress with F. Aslan, M. Boglione, J.O. Gonzalez, T. Rainaldi, A. Simonelli

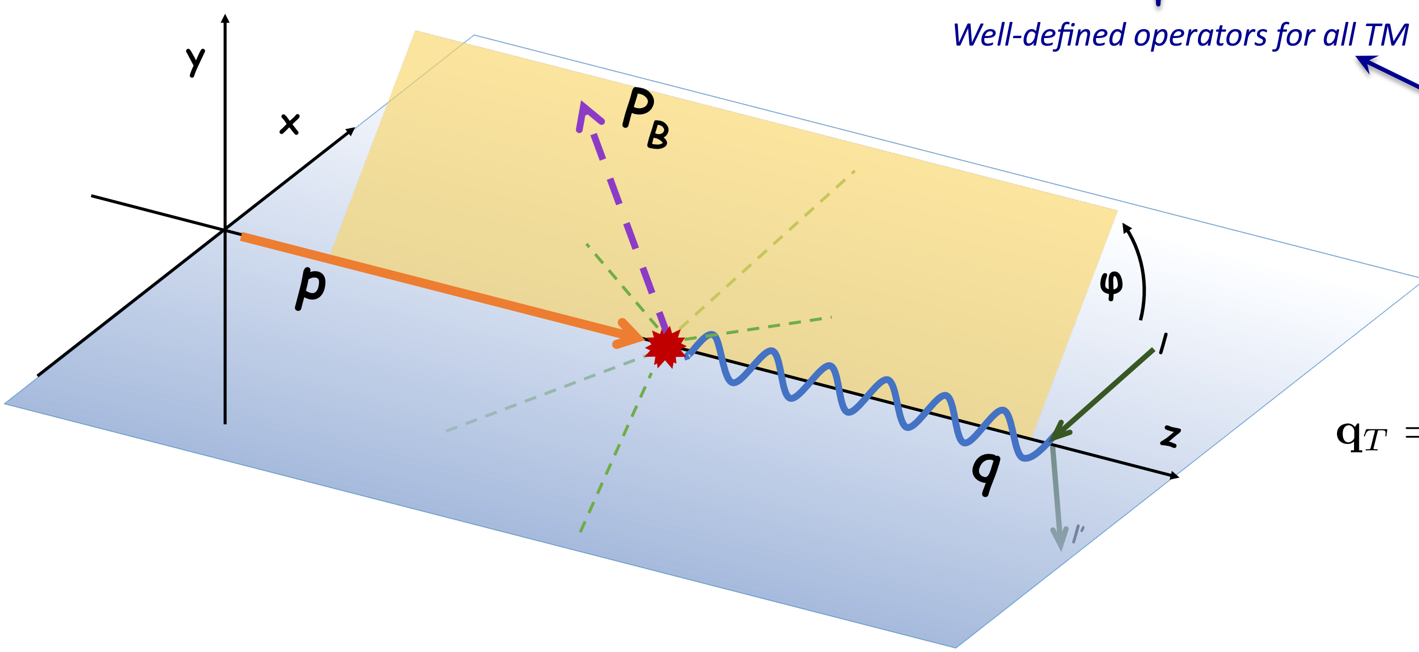
Cake seminar, May 10, 2023

TMD factorization & SIDIS $\left(for\ \frac{q_T}{Q} \ll 1\right)$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q\,\mathrm{d}^2\mathbf{q}_T\,\mathrm{d}x\,\mathrm{d}z} = H(Q/\mu_Q)\int \mathrm{d}^2\mathbf{k}_{1T}\mathrm{d}^2\mathbf{k}_{2T}\underbrace{f_{j/p}(x,\mathbf{k}_{1T};\mu_Q,Q^2)D_{h/j}(z,z\mathbf{k}_{2T};\mu_Q,Q^2)}_{\text{Well-defined operators for all TM}}\delta^{(2)}(\mathbf{q}_T+\mathbf{k}_{1T}-\mathbf{k}_{2T})$$

+ power suppressed

The "W-term"



$$\mathbf{q}_T = -\frac{\mathbf{P}_{BT}}{z}$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q\,\mathrm{d}^2\mathbf{q}_T\,\mathrm{d}x\,\mathrm{d}z} = H(Q/\mu_Q)\int \frac{\mathrm{d}^2\mathbf{b}_T}{(2\pi)^2}e^{-i\mathbf{b}_T\cdot\mathbf{q}_T}\tilde{f}_{j/p}(x,\mathbf{b}_T;\mu_Q,Q^2)\tilde{D}_{h/j}(z,\mathbf{b}_T;\mu_Q,Q^2)$$

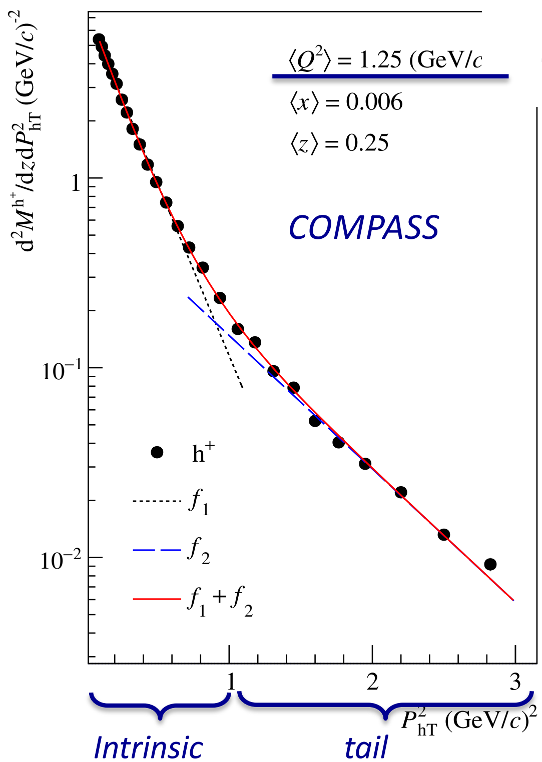
+ power suppressed

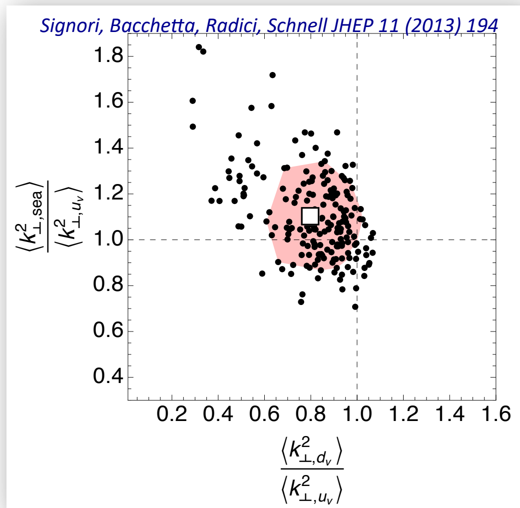
Nonperturbative structures in pheno

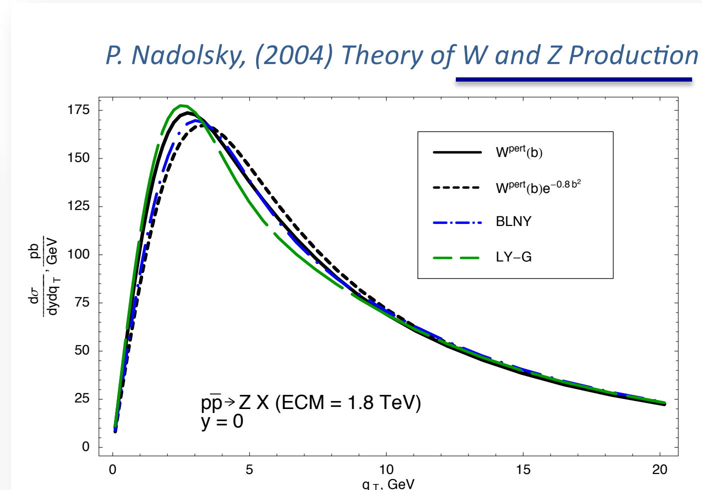
"While significant effort has been put into the study of W(b) ***at large b*** *[36, 42, 43, 44], none … adequately describe the observed Z boson distribution without introducing free parameters."*

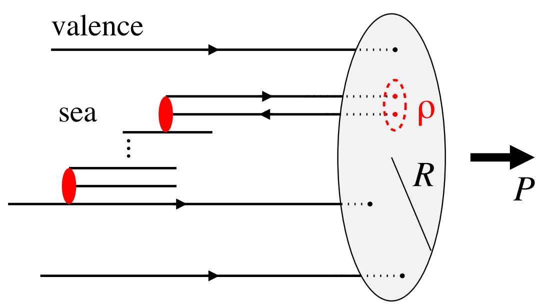
PHYSICAL REVIEW D **97,** 032006 (2018)

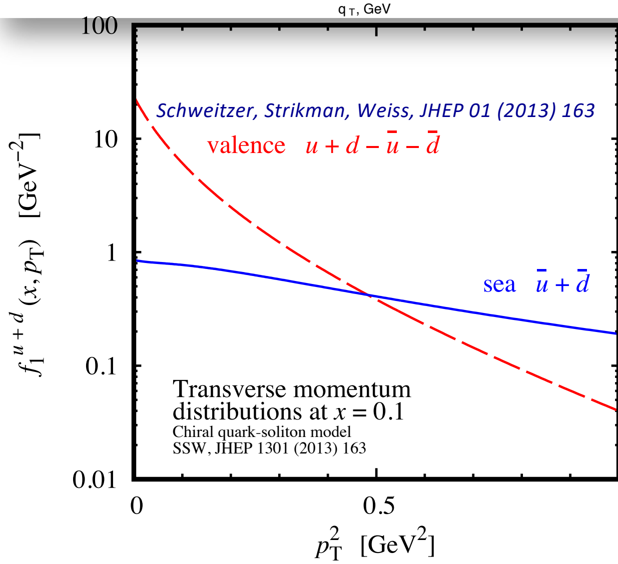
Transverse-momentum-dependent multiplicities of charged hadrons in muon-deuteron deep inelastic scattering







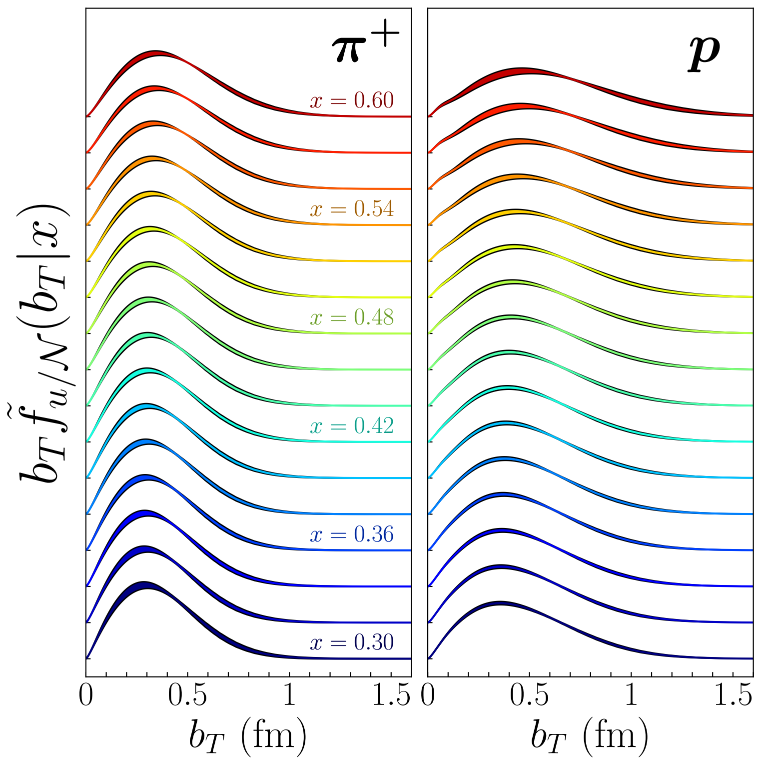




"… the two exponential functions in our parametrizations F_1 can be attributed to two completely different underlying physics mechanisms that overlap in the region $P_{hT} \simeq 1\ \mathrm{GeV}^2$."

Nonperturbative structures in pheno

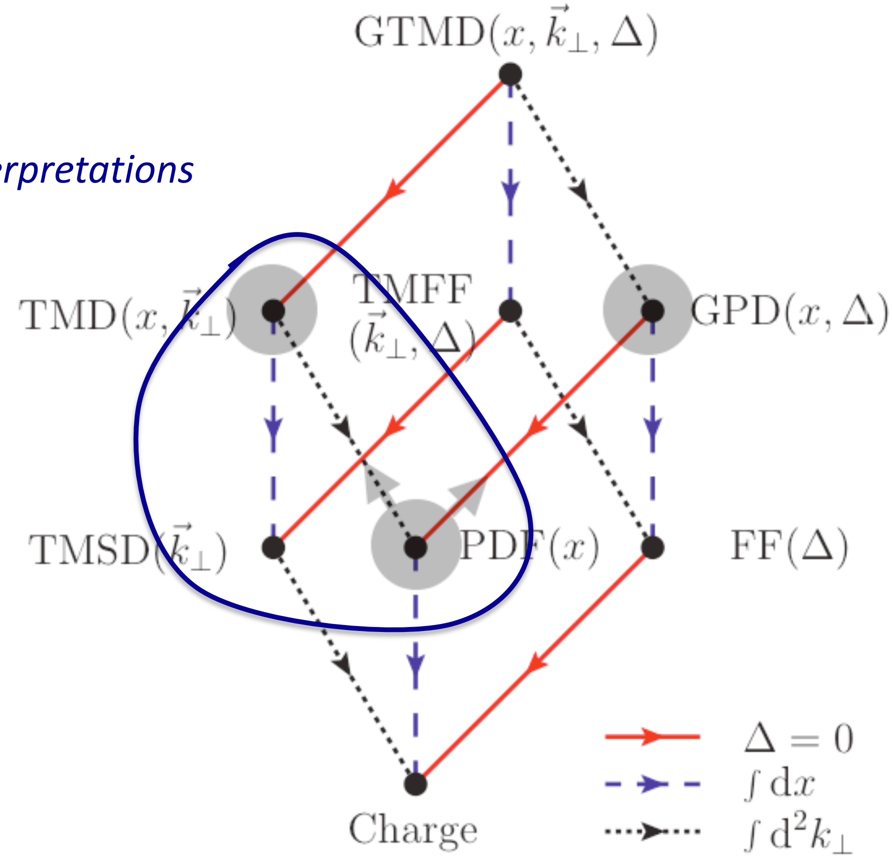
JAM: Barry, et al (2302.01192)



??

"Importantly, we have checked that the differences between the proton and pion ⟨bT |x⟩ are completely due to the nonperturbative TMD structure, independent of the collinear PDFs." *??*

Interpretations



These integrals are often divergent

The conventional organization

1) Solve evolution equations to relate overall SIDIS hard scale ($\mu_Q = Q$) to input scale ($\mu_{Q_0} = Q_0$)

$$\frac{\partial \ln \tilde{f}_{j/p}(x, b_T; \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T; \mu)$$

$$\frac{\mathrm{d} \ln \tilde{f}_{j/p}(x, b_T; \mu, \zeta)}{\mathrm{d} \ln \mu} = \gamma(\alpha_s(\mu); \zeta/\mu^2)$$

$$\frac{\mathrm{d}\tilde{K}(b_T; \mu)}{\mathrm{d} \ln \mu} = -\gamma_K(\alpha_s(\mu))$$

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1) Solve evolution equations to relate overall SIDIS hard scale ($\mu_Q = Q$) to input scale ($\mu_{Q_0} = Q_0$)
2) How to use small $b_T \ll 1/\Lambda_{QCD}$ collinear factorization?
   - Partition small ($b_T < b_{\max}$) & large ($b_T > b_{\max}$) regions with a b_*
   - Define hard scale $\mu_{b_*} \sim 1/b_*$
3) Evolve again to relate Q_0 to μ_{b_*}

$$\frac{\partial \ln \tilde{f}_{j/p}(x, b_T; \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T; \mu)$$

$$\frac{\mathrm{d} \ln \tilde{f}_{j/p}(x, b_T; \mu, \zeta)}{\mathrm{d} \ln \mu} = \gamma(\alpha_s(\mu); \zeta/\mu^2)$$

$$\frac{\mathrm{d}\tilde{K}(b_T; \mu)}{\mathrm{d} \ln \mu} = -\gamma_K(\alpha_s(\mu))$$

$$\tilde{f}_{j/p}(x, b_T; \mu, \zeta) = \int_x^1 \frac{\mathrm{d}\xi}{\xi} \tilde{C}_{j/k}(x/\xi, b_T; \zeta, \mu, \alpha_s(\mu)) f_{k/p}(\xi; \mu) + O(b_T \Lambda_{\mathrm{QCD}})$$

Or

$$f_{j/p}(x, k_T; \mu, \zeta) = \frac{1}{k_T^2} \left[\int_x^1 \frac{\mathrm{d}\xi}{\xi} C_{j/k}(x/\xi, k_T; \zeta, \mu, \alpha_s(\mu)) f_{k/p}(\xi; \mu) + O\left(\frac{\Lambda_{\mathrm{QCD}}}{k_T}\right)\right]$$

$$\tilde{K}(b_T; \mu) = \tilde{K}_{\mathrm{pert}}(b_T \mu, \alpha_s(\mu)) + O(b_T \Lambda_{\mathrm{QCD}})$$

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3) Evolve again to relate Q_0 to μ_{b_*}
4) **Place remaining NP parts in an exponent:**

$$\tilde{f}_{j/p}(x, \boldsymbol{b}_{\mathrm{T}}; \mu_Q, \zeta) = \tilde{f}_{j/p}(x, \boldsymbol{b}_*; \mu_Q, \zeta) \underbrace{\frac{\tilde{f}_{j/p}(x, \boldsymbol{b}_{\mathrm{T}}; \mu_Q, \zeta)}{\tilde{f}_{j/p}(x, \boldsymbol{b}_*; \mu_Q, \zeta)}}_{??}$$

$$\frac{\partial \ln \tilde{f}_{j/p}(x, b_T; \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T; \mu)$$

$$\frac{\mathrm{d} \ln \tilde{f}_{j/p}(x, b_T; \mu, \zeta)}{\mathrm{d} \ln \mu} = \gamma(\alpha_s(\mu); \zeta/\mu^2)$$

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<u>Or</u>

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4) **Place remaining NP parts in an exponent:**

$$-g_{j/p}(x, b_{\mathrm{T}}) \equiv \ln\left(\frac{\tilde{f}_{j/p}(x, \boldsymbol{b}_{\mathrm{T}}; \mu_{Q_0}, Q_0^2)}{\tilde{f}_{j/p}(x, \boldsymbol{b}_*; \mu_{Q_0}, Q_0^2)}\right)$$

$$g_K(b_{\mathrm{T}}) \equiv \tilde{K}(b_*; \mu) - \tilde{K}(b_{\mathrm{T}}; \mu)$$

5) **Perform small-b_T expansions & drop $O(\Lambda_{QCD}\, b_{max})$ errors**
6) **Ansatz for g-functions**

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⬇

$$\int \frac{d^2\boldsymbol{b}_{\rm T}}{(2\pi)^2}\, e^{-i\boldsymbol{q}_{\rm T}\cdot\boldsymbol{b}_{\rm T}}\, \tilde{f}_{j/p}^{\rm OPE}(x_{\rm bj}, \boldsymbol{b}_*; \mu_{b_*}, \mu_{b_*}^2)\, \tilde{D}_{h/j}^{\rm OPE}(z_{\rm h}, \boldsymbol{b}_*; \mu_{b_*}, \mu_{b_*}^2)$$
$$\times \exp\left\{2\int_{\mu_{b_*}}^{\mu_Q} \frac{d\mu'}{\mu'}\left[\gamma(\alpha_s(\mu'); 1) - \ln\frac{Q}{\mu'}\gamma_K(\alpha_s(\mu'))\right] + \ln\frac{Q^2}{\mu_{b_*}^2}\tilde{K}(b_*; \mu_{b_*})\right\}$$
$$\times \exp\left\{-g_{j/p}(x_{\rm bj}, b_{\rm T}) - g_{h/j}(z_{\rm h}, b_{\rm T}) - g_K(b_{\rm T}) \ln\left(\frac{Q^2}{Q_0^2}\right)\right\} + O\left(b_{\rm max}\Lambda_{\rm QCD}\right)$$

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$$\Downarrow$$

$$\int \frac{d^2\boldsymbol{b}_T}{(2\pi)^2}\, e^{-i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}\, \tilde{f}^{\rm OPE}_{j/p}(x_{\rm bj}, \boldsymbol{b}_*; \mu_{b_*}, \mu_{b_*}^2)\, \tilde{D}^{\rm OPE}_{h/j}(z_h, \boldsymbol{b}_*; \mu_{b_*}, \mu_{b_*}^2)$$
$$\times \exp\left\{2\int_{\mu_{b_*}}^{\mu_Q} \frac{d\mu'}{\mu'}\left[\gamma(\alpha_s(\mu'); 1) - \ln\frac{Q}{\mu'}\gamma_K(\alpha_s(\mu'))\right] + \ln\frac{Q^2}{\mu_{b_*}^2}\tilde{K}(b_*; \mu_{b_*})\right\}$$
$$\times \exp\left\{-g_{j/p}(x_{\rm bj}, b_T) - g_{h/j}(z_h, b_T) - g_K(b_T)\ln\left(\frac{Q^2}{Q_0^2}\right)\right\} + O(b_{\max}\Lambda_{\rm QCD})$$

$$\tilde{f}_{j/p}(x, b_T; \mu, \zeta) = \underbrace{\int_x^1 \frac{d\xi}{\xi}\tilde{C}_{j/k}(x/\xi, b_T; \zeta, \mu, \alpha_s(\mu)) f_{k/p}(\xi; \mu)}_{\tilde{f}^{\rm OPE}_{j/p}(x, b_T; \mu, \zeta)} + O(b_T\Lambda_{\rm QCD})$$

How to test consistency

- $\int d^2\mathbf{k}_{1T} d^2\mathbf{k}_{2T} f_{j/p}(x,\mathbf{k}_{1T};\mu_Q,Q^2) D_{h/j}(z,z\mathbf{k}_{2T};\mu_Q,Q^2)\delta^{(2)}(\mathbf{q}_T+\mathbf{k}_{1T}-\mathbf{k}_{2T})$

  is uniquely determined by its operator definition

- At $q_T \approx Q$

$$= \underbrace{\frac{1}{q_T^2}\left[\mathcal{C}\left(q_T/\mu_Q,\alpha_s\left(\mu_Q\right)\right)\otimes f_{j/p}(x;\mu_Q)\otimes d_{h/j}(z;\mu_Q)\right.}_{q_T\sim Q,\, Q\to\infty \text{ asymptote}}\left.+O\left(\frac{\Lambda_{\rm QCD}}{q_T}\right)\right]$$

- For TMD pdfs & ffs

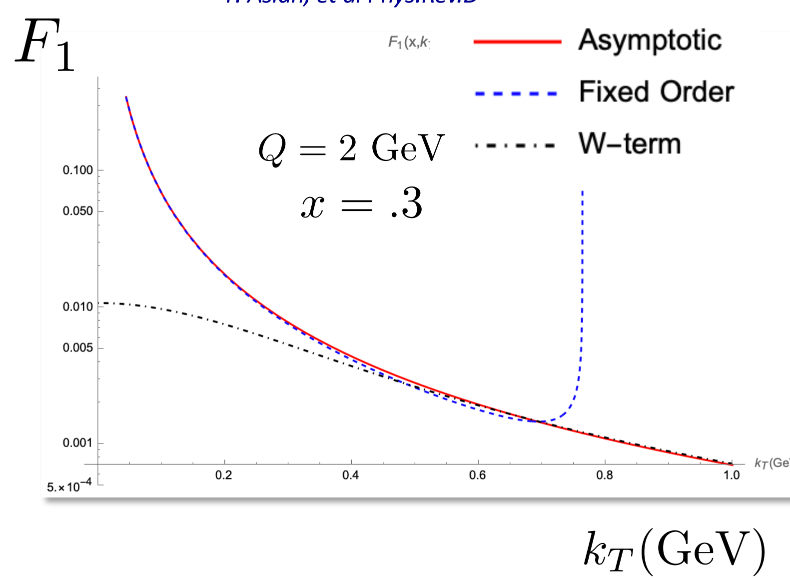
$$f_{j/p}(x,k_T\approx Q;\mu_Q,Q^2)=\frac{1}{k_T^2}\left[\int_x^1\frac{d\xi}{\xi}C_{j/k}(x/\xi,k_T/Q,\alpha_s(Q))f_{k/p}(\xi;Q)+O\left(\frac{\Lambda_{\rm QCD}}{k_T}\right)\right]$$

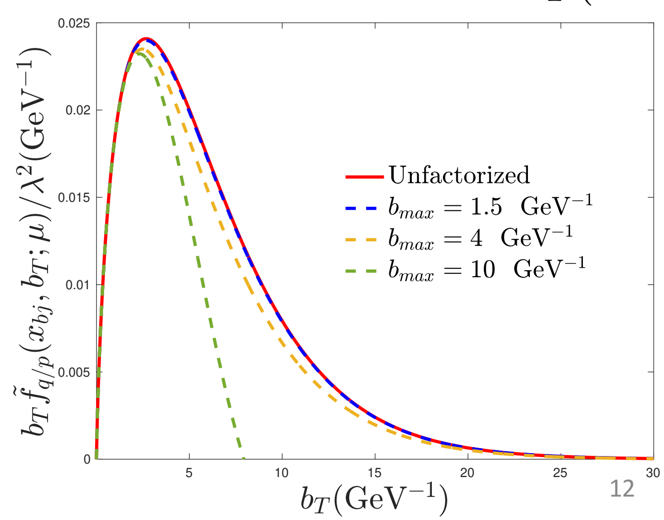
&

$$\int d^2\mathbf{k}_T f_{j/p}(x,k_T;\mu_Q,Q^2)\approx f_{j/p}(x;\mu_Q)$$

- $\frac{d}{db_{\max}}\left(\frac{d\sigma}{d^2\boldsymbol{q}_T\ldots}\right)=0$ *for* $b_{\max}\ll 1/\Lambda_{QCD}$

F. Aslan, et al Phys.Rev.D

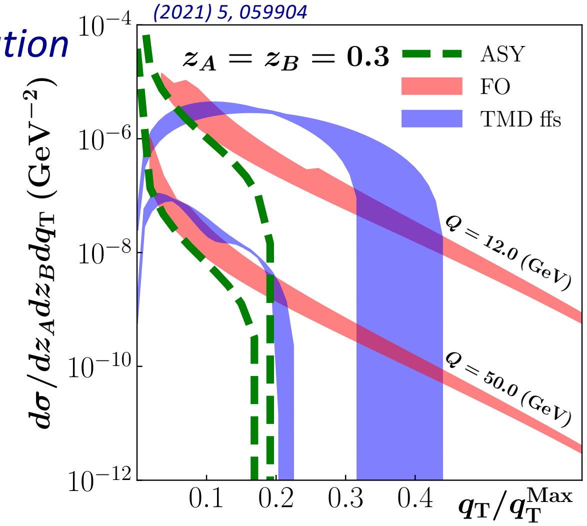




How to test consistency

e+e- annihilation

E. Moffat, T. Rogers, N. Sato, A. Signori Phys.Rev.D 104 (2021) 5, 059904

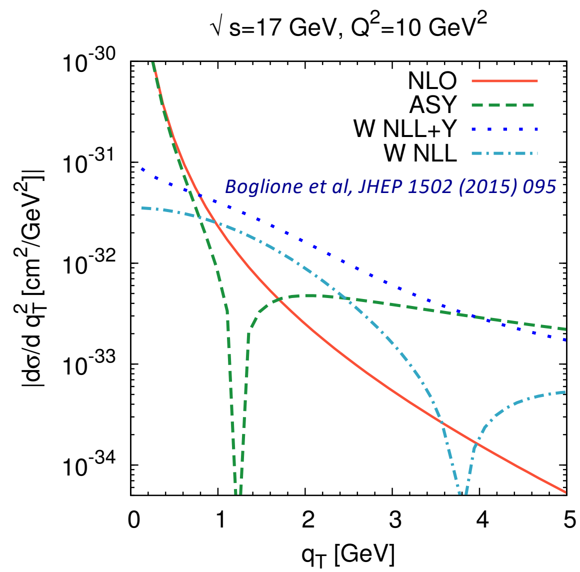


- What does $q_T \approx Q$ mean?
  - No sensitivity to parameters related nonperturbative transverse momentum
  - $\Lambda_{QCD} \ll q_T \ll Q$? Look for matching between fixed order x-section and asymptotic term
    - Is there a region where both

$$\frac{\Lambda_{QCD}}{q_T} \;\&\; \frac{q_T}{Q}$$

powers are simultaneously negligible?

  - No large logarithms: Look for node in asymptotic term

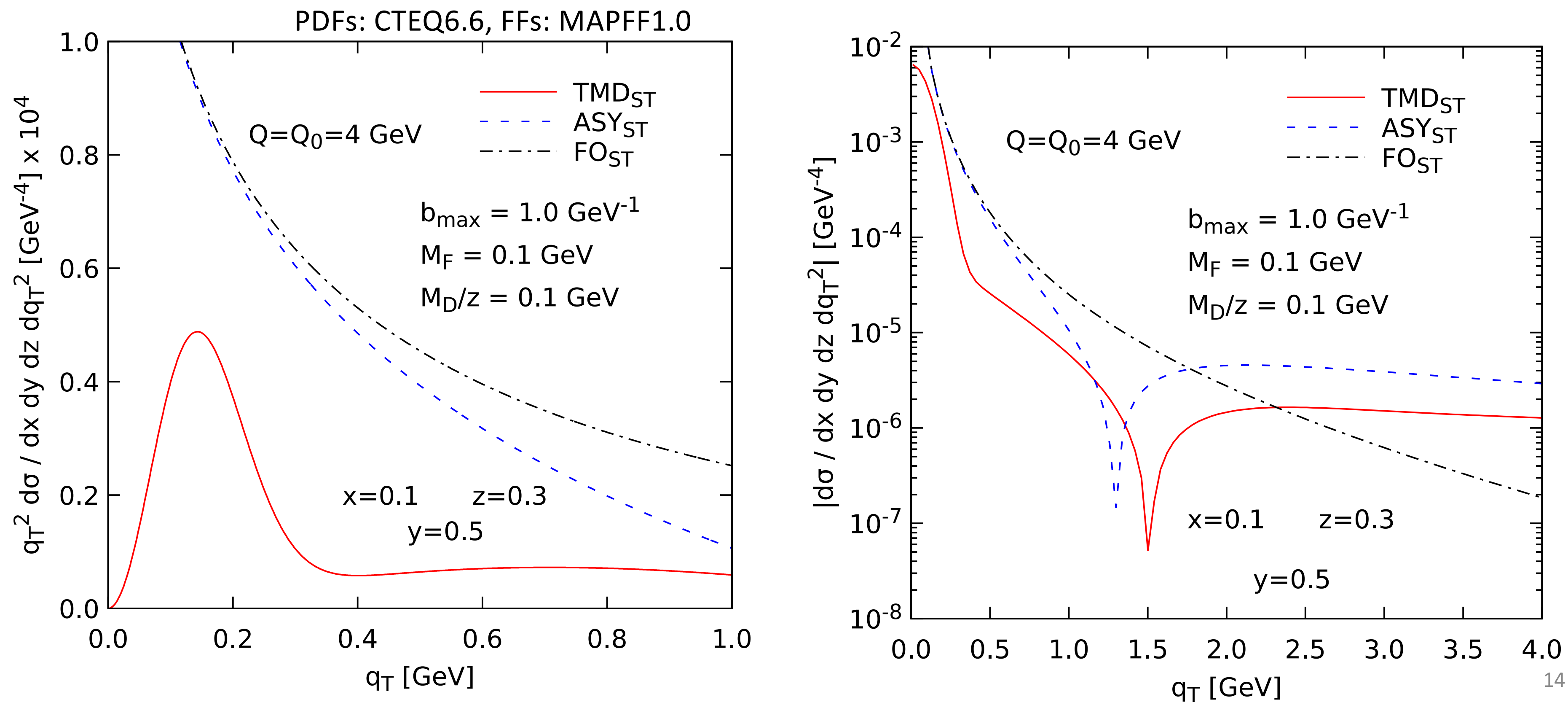


Conventional organization & complications

(An example typical of conventional approach)

$$g_{j/p}(x, b_{\mathrm{T}}) = \frac{1}{4} M_F^2 \, b_{\mathrm{T}}^2$$

$$g_{h/j}(z, b_{\mathrm{T}}) = \frac{1}{4 \, z^2} M_D^2 \, b_{\mathrm{T}}^2$$



Conventional organization & complications

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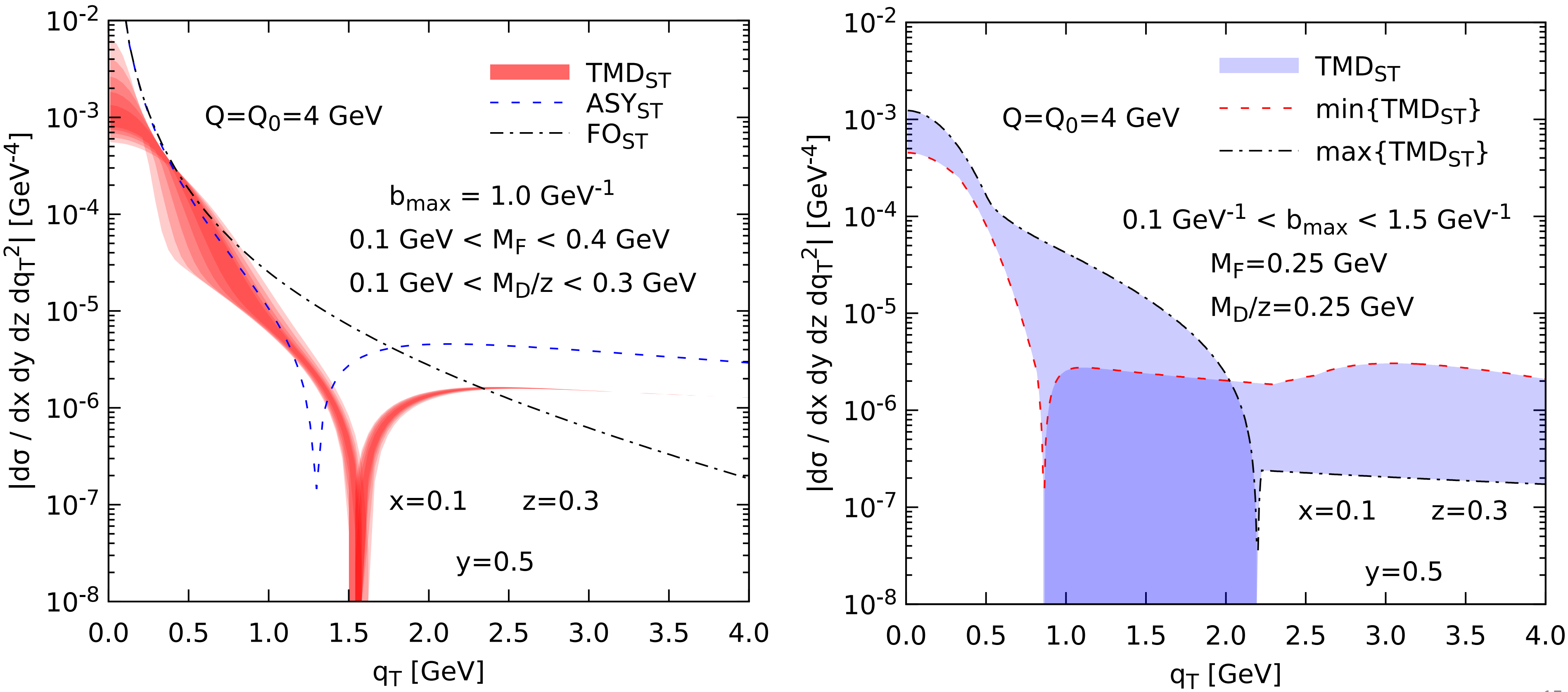

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q\,\mathrm{d}^2\mathbf{q}_T\,\mathrm{d}x\,\mathrm{d}z} = H(Q/\mu_Q)\underbrace{\int \mathrm{d}^2\mathbf{k}_{1T}\mathrm{d}^2\mathbf{k}_{2T} f_{j/p}(x,\mathbf{k}_{1T};\mu_Q,Q^2) D_{h/j}(z,z\mathbf{k}_{2T};\mu_Q,Q^2)}_{\text{Well-defined operators for all TM}}\delta^{(2)}(\mathbf{q}_T+\mathbf{k}_{1T}-\mathbf{k}_{2T}) + \text{power suppressed}$$

Diagnosis

1) Consistency tests will generally fail for a g-function ansatz unless constraints are imposed

2) Fixed order perturbation theory should work fine for $q_T \approx Q_0$, but evol. factors have a large effect. What is going on?

3) $\exists$ no region at input scale $Q = Q_0$ where $\Lambda_{QCD} \ll q_T \ll Q_0$

4) Backwards evolution…
No large, perturbative $\ln\frac{Q_0}{q_T}$.

5) $\int \mathrm{d}^2\mathbf{k}_T f_{j/p}(x,k_T;\mu_Q,Q^2) \approx f_{j/p}(x;\mu_Q)$
Very badly violated at moderate scales

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q\,\mathrm{d}^2\mathbf{q}_T\,\mathrm{d}x\,\mathrm{d}z} = H(Q/\mu_Q)\underbrace{\int \mathrm{d}^2\mathbf{k}_{1T}\mathrm{d}^2\mathbf{k}_{2T} f_{j/p}(x,\mathbf{k}_{1T};\mu_Q,Q^2) D_{h/j}(z,z\mathbf{k}_{2T};\mu_Q,Q^2)}_{\textit{Well-defined operators for all TM}}\delta^{(2)}(\mathbf{q}_T+\mathbf{k}_{1T}-\mathbf{k}_{2T}) + \text{power suppressed}$$

Diagnosis

1) Consistency tests will generally fail for a g-function ansatz unless constraints are imposed

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q\,\mathrm{d}^2\mathbf{q}_T\,\mathrm{d}x\,\mathrm{d}z} =$$

$$H(Q/\mu_Q)\int \frac{\mathrm{d}^2\boldsymbol{b}_\mathrm{T}}{(2\pi)^2}\, e^{-i\boldsymbol{q}_\mathrm{T}\cdot\boldsymbol{b}_\mathrm{T}}\ \tilde{f}_{j/p}(x,\boldsymbol{b}_\mathrm{T};\mu_{Q_0},Q_0^2)\ \tilde{D}_{h/j}(z,\boldsymbol{b}_\mathrm{T};\mu_{Q_0},Q_0^2)$$

$$\times\exp\left\{\tilde{K}(b_\mathrm{T};\mu_{Q_0})\ln\left(\frac{Q^2}{Q_0^2}\right)+\int_{\mu_{Q_0}}^{\mu_Q}\frac{\mathrm{d}\mu'}{\mu'}\left[2\gamma(\alpha_s(\mu');1)-\ln\frac{Q^2}{\mu'^2}\gamma_K(\alpha_s(\mu'))\right]\right\}$$

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No large, perturbative $\ln\frac{Q_0}{q_T}$.

5) $\int \mathrm{d}^2\mathbf{k}_T f_{j/p}(x,k_T;\mu_Q,Q^2) \approx f_{j/p}(x;\mu_Q)$
Very badly violated at moderate scales

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q\,\mathrm{d}^2\mathbf{q}_T\,\mathrm{d}x\,\mathrm{d}z} = H(Q/\mu_Q)\underbrace{\int \mathrm{d}^2\mathbf{k}_{1T}\mathrm{d}^2\mathbf{k}_{2T} f_{j/p}(x,\mathbf{k}_{1T};\mu_Q,Q^2) D_{h/j}(z,z\mathbf{k}_{2T};\mu_Q,Q^2)\delta^{(2)}(\mathbf{q}_T+\mathbf{k}_{1T}-\mathbf{k}_{2T})}_{\text{Well-defined operators for all TM}} + \text{power suppressed}$$

Diagnosis

1) Consistency tests will generally fail for a g-function ansatz unless constraints are imposed

2) Fixed order perturbation theory should work fine for $q_T \approx Q_0$, but evol. factors have a large effect. What is going on?

3) $\exists$ no region at input scale $Q = Q_0$ where $\Lambda_{QCD} \ll q_T \ll Q_0$

4) Backwards evolution… No large, perturbative $\ln \frac{Q_0}{q_T}$.

5) $\int \mathrm{d}^2\mathbf{k}_T f_{j/p}(x,k_T;\mu_Q,Q^2) \approx f_{j/p}(x;\mu_Q)$ Very badly violated at moderate scales

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q\,\mathrm{d}^2\mathbf{q}_T\,\mathrm{d}x\,\mathrm{d}z} =$$

$$H(Q/\mu_Q)\int \frac{\mathrm{d}^2\boldsymbol{b}_\mathrm{T}}{(2\pi)^2} e^{-i\boldsymbol{q}_\mathrm{T}\cdot\boldsymbol{b}_\mathrm{T}} \tilde{f}_{j/p}(x,\boldsymbol{b}_\mathrm{T};\mu_{Q_0},Q_0^2)\ \tilde{D}_{h/j}(z,\boldsymbol{b}_\mathrm{T};\mu_{Q_0},Q_0^2)$$

$$\times \exp\left\{\tilde{K}(b_\mathrm{T};\mu_{Q_0})\ln\left(\frac{Q^2}{Q_0^2}\right) + \int_{\mu_{Q_0}}^{\mu_Q}\frac{\mathrm{d}\mu'}{\mu'}\left[2\gamma(\alpha_s(\mu');1) - \ln\frac{Q^2}{\mu'^2}\gamma_K(\alpha_s(\mu'))\right]\right\}$$

$$\tilde{D}_{h/j}(z,\boldsymbol{b}_\mathrm{T};\mu_{Q_0},Q_0^2) = \tilde{D}_{\mathrm{inpt},h/j}(z,\boldsymbol{b}_\mathrm{T};\mu_{\overline{Q}_0},\overline{Q}_0^2)E(\overline{Q}_0/Q_0,b_\mathrm{T})$$

$$\tilde{f}_{i/p}(x,\boldsymbol{b}_\mathrm{T};\mu_{Q_0},Q_0^2) = \tilde{f}_{\mathrm{inpt},i/p}(x,\boldsymbol{b}_\mathrm{T};\mu_{\overline{Q}_0},\overline{Q}_0^2)E(\overline{Q}_0/Q_0,b_\mathrm{T})$$

$$\lim_{b_\mathrm{T}\to 0}\overline{Q}_0 \sim 1/b_\mathrm{T}$$

Instead, characterize the full range of k_T behavior of TMD functions at the input scale

A hadron structure oriented (HSO) reorganization

1) Use the uniquely determined TMDs for all k_T

2) Smoothly interpolate between nonperturbative TM dependence at small TM ($k_T \approx \Lambda_{QCD}$) & perturbative (collinear) TM at large TM ($k_T \approx Q$)

3) (Approximate) probability interpretation

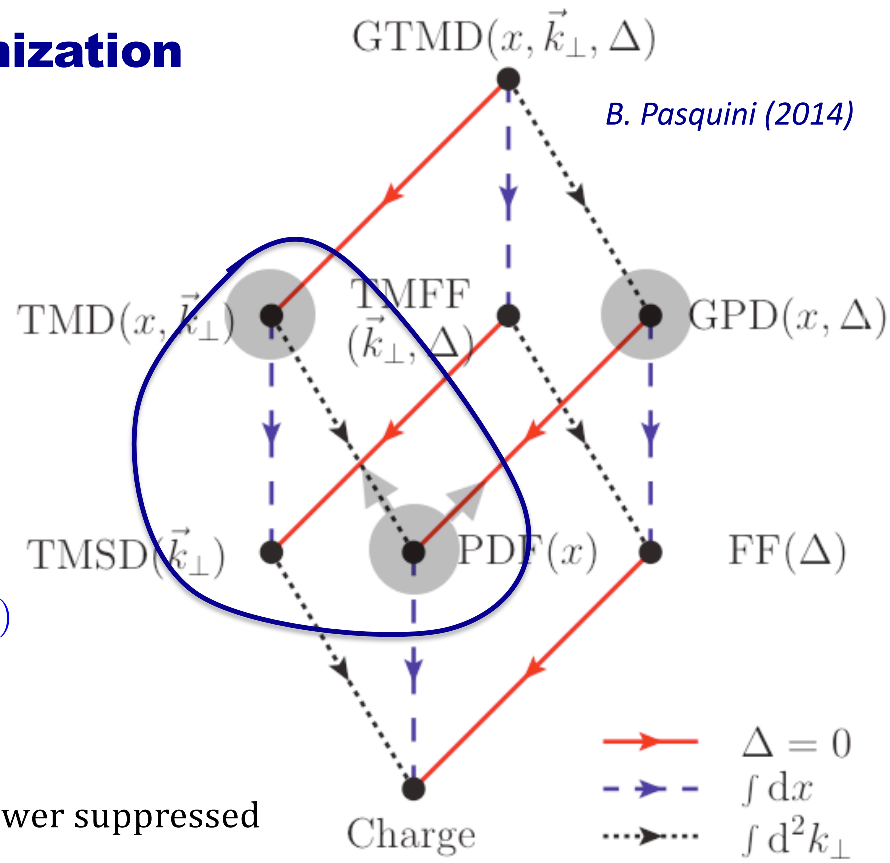
- Parton model: $\int \mathrm{d}^2\mathbf{k}_T f_{j/p}(x, k_T; \mu_Q, Q^2) = f_{j/p}(x; \mu_Q)$

- QCD:

$$\pi \int^{\mu_Q^2} \mathrm{d}k_T^2 f_{j/p}(x, k_T; \mu_Q, Q^2) = f_{j/p}(x; \mu_Q) + \Delta_{j/p} \ + \text{power suppressed}$$

The "W-term"

$\overline{\mathrm{MS}}$ $\qquad \underbrace{\sum_{j'} \mathcal{C}^{\Delta}_{j/j'} \otimes f_{j'/p}}_{O(\alpha_s)}$



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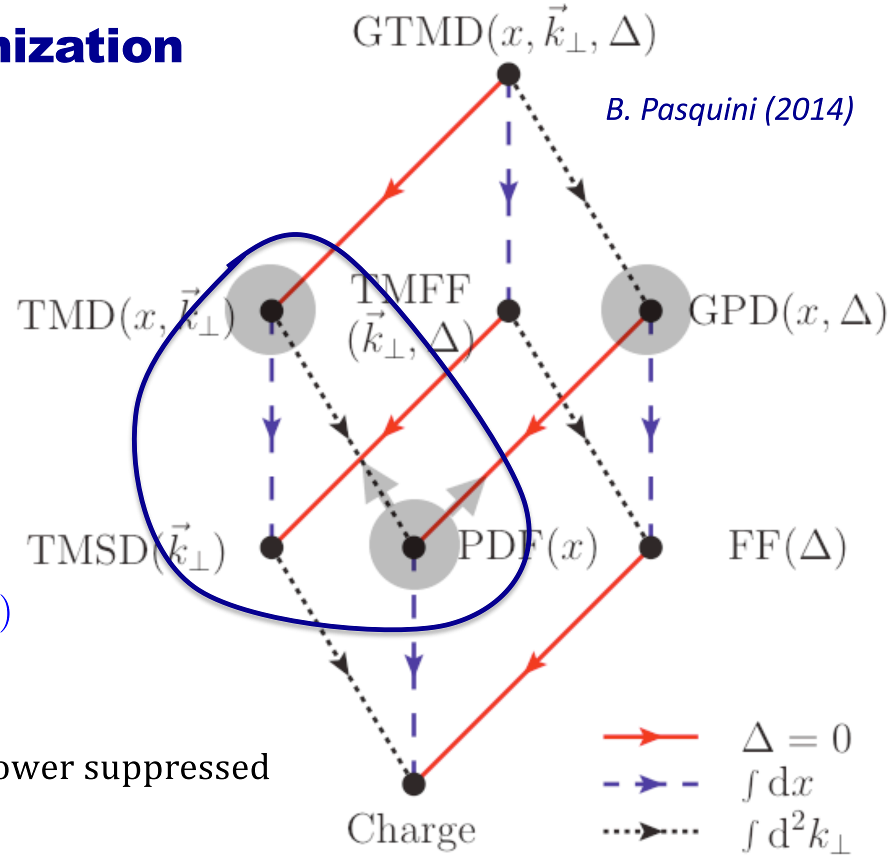
4) All should apply at an input scale, Q_0

5) Pheno requirement: Must be simple to swap one model/parametrization for another while still satisfying 1-4



A hadron structure oriented (HSO) reorganization

- We provide a recipe to transform a NP TMD parametrization into an evolved parametrization at other scales:
  - *Sec. VI of Phys.Rev.D 106 (2022) 3, 034002*

Called "bottom up" approach here

- No b_{max} or b_* necessary

- HSO approach is equivalent to standard TMD factorization, CSS, etc, just with additional consistency constraints on the g-functions

- It is straightforward to translate between standard treatment and HSO

  - *Sec. IX of Phys.Rev.D 106 (2022) 3, 034002*

An $O(\alpha_s)$ example with $\overline{\text{MS}}$ pdfs and ffs

$$f_{\text{inpt},i/p}(x,\boldsymbol{k}_\text{T};\mu_{Q_0},Q_0^2) = \frac{1}{2\pi}\frac{1}{k_\text{T}^2+m_{f_{i,p}}^2}\left[A_{i/p}^f(x;\mu_{Q_0}) + B_{i/p}^f(x;\mu_{Q_0})\ln\frac{Q_0^2}{k_\text{T}^2+m_{f_{i,p}}^2}\right] + \frac{1}{2\pi}\frac{1}{k_\text{T}^2+m_{f_{g,p}}^2}A_{i/p}^{f,g}(x;\mu_{Q_0})$$
$$+ C_{i/p}^f\, f_{\text{core},i/p}(x,\boldsymbol{k}_\text{T};Q_0^2)$$

$$D_{\text{inpt},h/j}(z,z\boldsymbol{k}_\text{T};\mu_{Q_0},Q_0^2) = \frac{1}{2\pi z^2}\frac{1}{k_\text{T}^2+m_{D_{h,j}}^2}\left[A_{h/j}^D(z;\mu_{Q_0}) + B_{h/j}^D(z;\mu_{Q_0})\ln\frac{Q_0^2}{k_\text{T}^2+m_{D_{h,j}}^2}\right] + \frac{1}{2\pi z^2}\frac{1}{k_\text{T}^2+m_{D_{h,g}}^2}A_{h/j}^{D,g}(z;\mu_{Q_0})$$
$$+ C_{h/j}^D\, D_{\text{core},h/j}(z,z\boldsymbol{k}_\text{T};Q_0^2)$$

- C^f & C^D constrained by:

$$f_{i/p}^c(x;\mu_{Q_0}) \equiv 2\pi\int_0^{\mu_{Q_0}} \text{d}k_\text{T}\, k_\text{T} f_{i/p}(x,\boldsymbol{k}_\text{T};\mu_{Q_0},Q_0^2) = f_{i/p}(x;\mu_{Q_0}) + \mathcal{C}_{i/i'}^{\Delta^f}\otimes f_{i'/p} + \text{p.s.}$$
$$d_{h/j}^c(z;\mu_{Q_0}) \equiv 2\pi z^2\int_0^{\mu_{Q_0}} \text{d}k_\text{T}\, k_\text{T} D_{h/j}(z,z\boldsymbol{k}_\text{T};\mu_{Q_0},Q_0^2) = d_{h/j}(z;\mu_{Q_0}) + \mathcal{C}_{j'/j}^{\Delta^d}\otimes d_{h/j'} + \text{p.s.}$$

An $O(\alpha_s)$ example with $\overline{\text{MS}}$ pdfs and ffs

- Parametrizing the very small transverse momentum

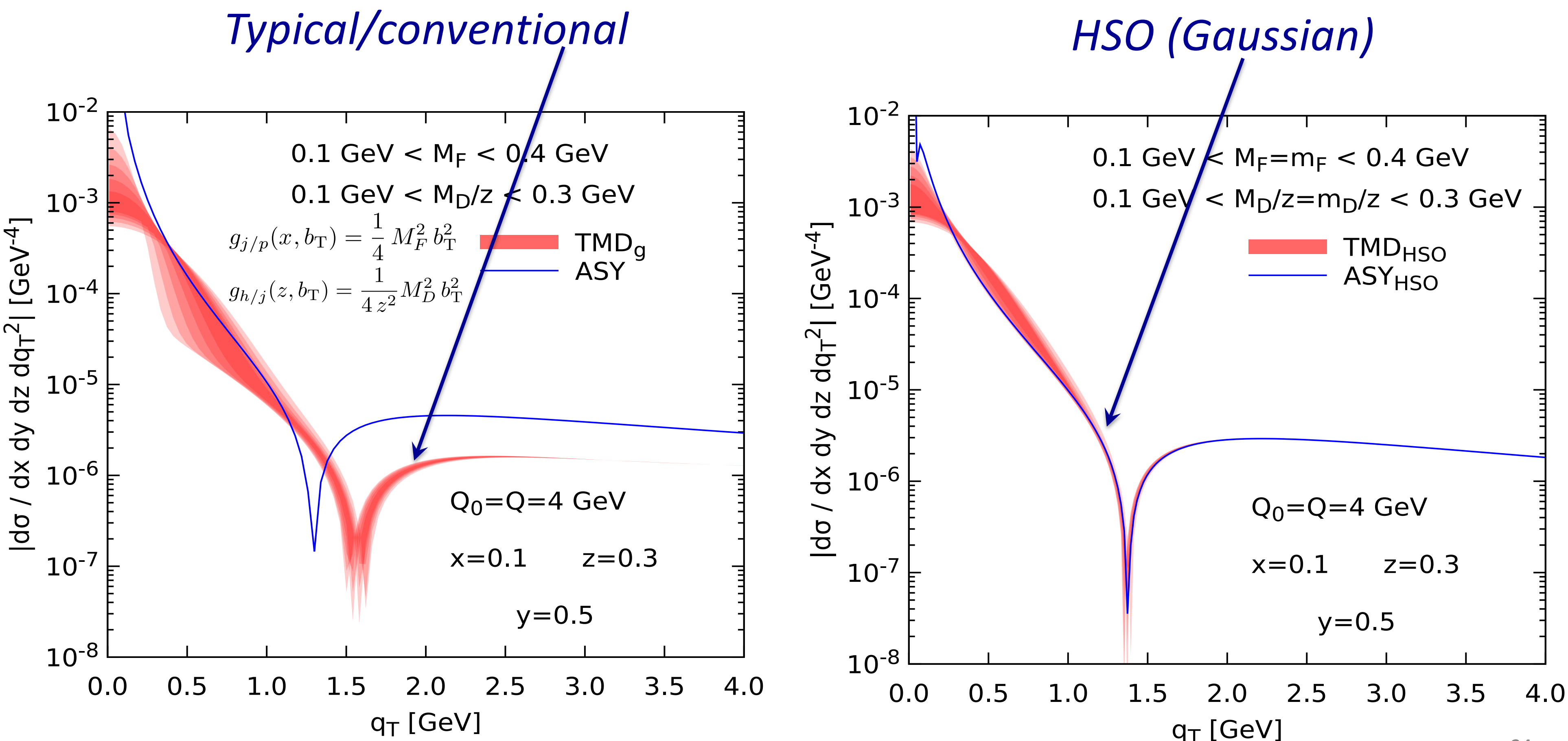
A. Gaussian model (very commonly used)

$$f^{\text{Gauss}}_{\text{core},i/p}(x,\boldsymbol{k}_\text{T};Q_0^2) = \frac{e^{-k_\text{T}^2/M_\text{F}^2}}{\pi M_\text{F}^2}\,, \qquad D^{\text{Gauss}}_{\text{core},h/j}(z,z\boldsymbol{k}_\text{T};Q_0^2) = \frac{e^{-z^2\,k_\text{T}^2/M_\text{D}^2}}{\pi M_\text{D}^2}$$

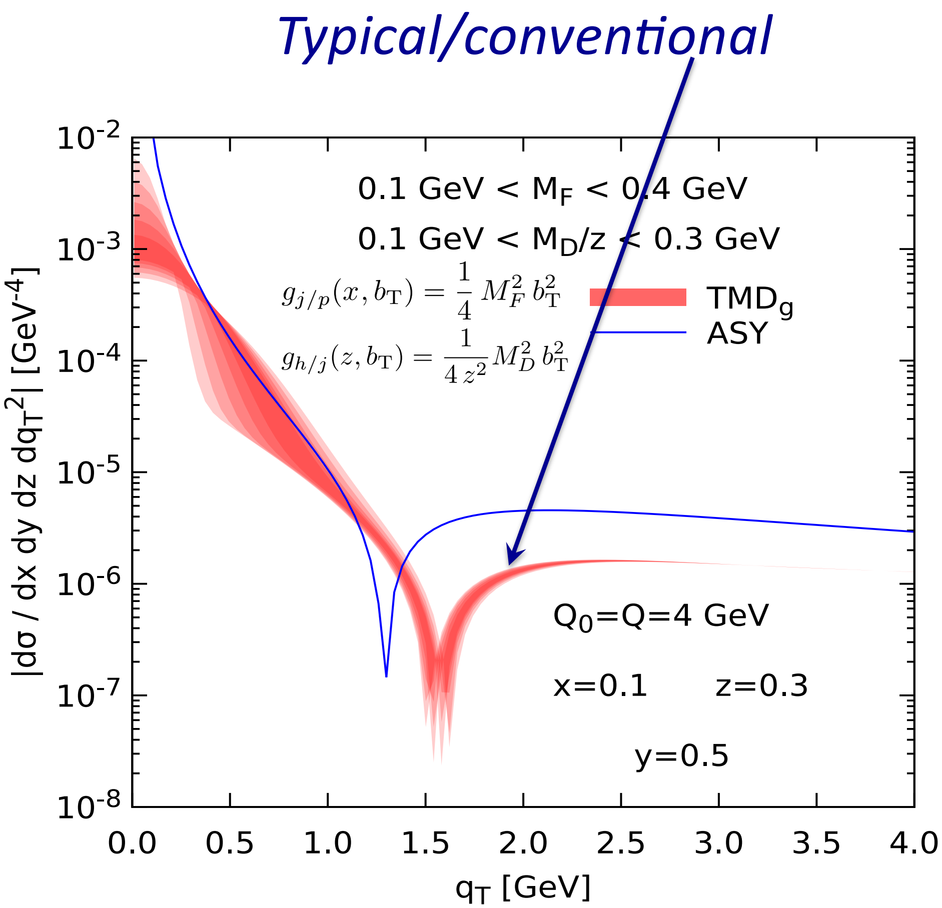
B. Spectator model

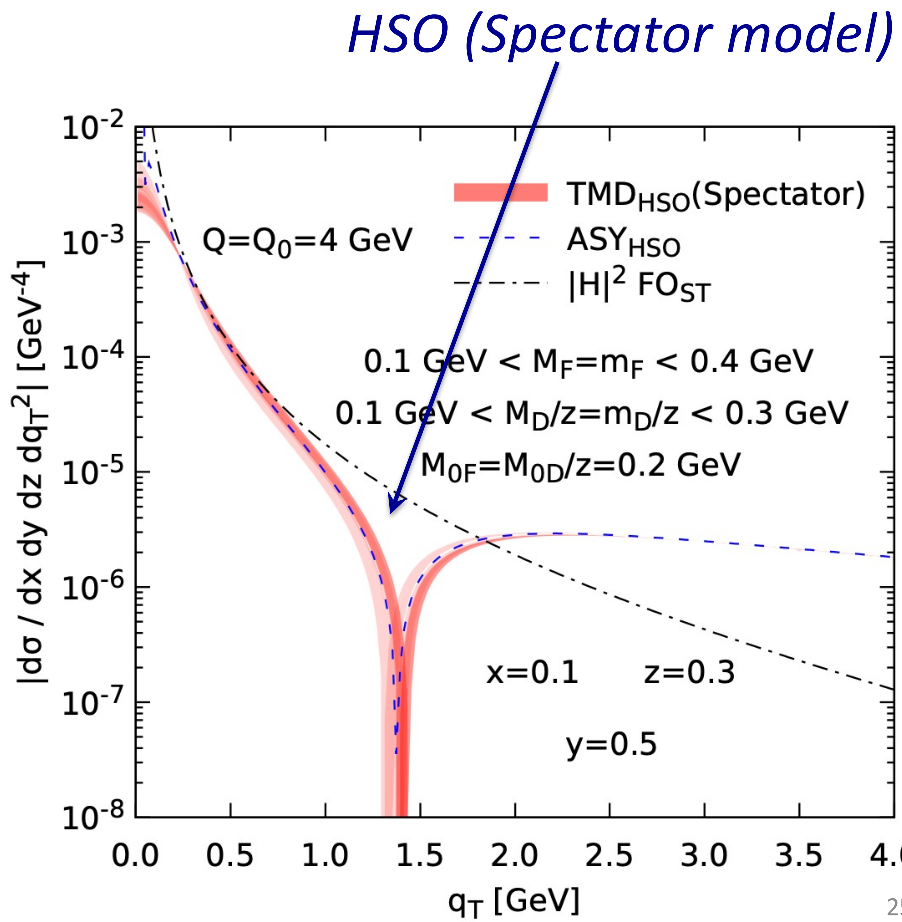
$$f^{\text{Spect}}_{\text{core},i/p}(x,\boldsymbol{k}_\text{T};Q_0^2) = \frac{6M_{0\text{F}}^6}{\pi\,(2M_\text{F}^2+M_{0\text{F}}^2)}\,\frac{M_\text{F}^2+k_\text{T}^2}{(M_{0\text{F}}^2+k_\text{T}^2)^{\,4}}\,, \quad D^{\text{Spect}}_{\text{core},h/j}(z,z\boldsymbol{k}_\text{T};Q_0^2) = \frac{2M_{0\text{D}}^4}{\pi\,(M_\text{D}^2+M_{0\text{D}}^2)}\,\frac{M_\text{D}^2+k_\text{T}^2z^2}{(M_{0\text{D}}^2+k_\text{T}^2z^2)^{\,3}}$$

Compare standard/unconstrained with HSO ($O(\alpha_s)$)

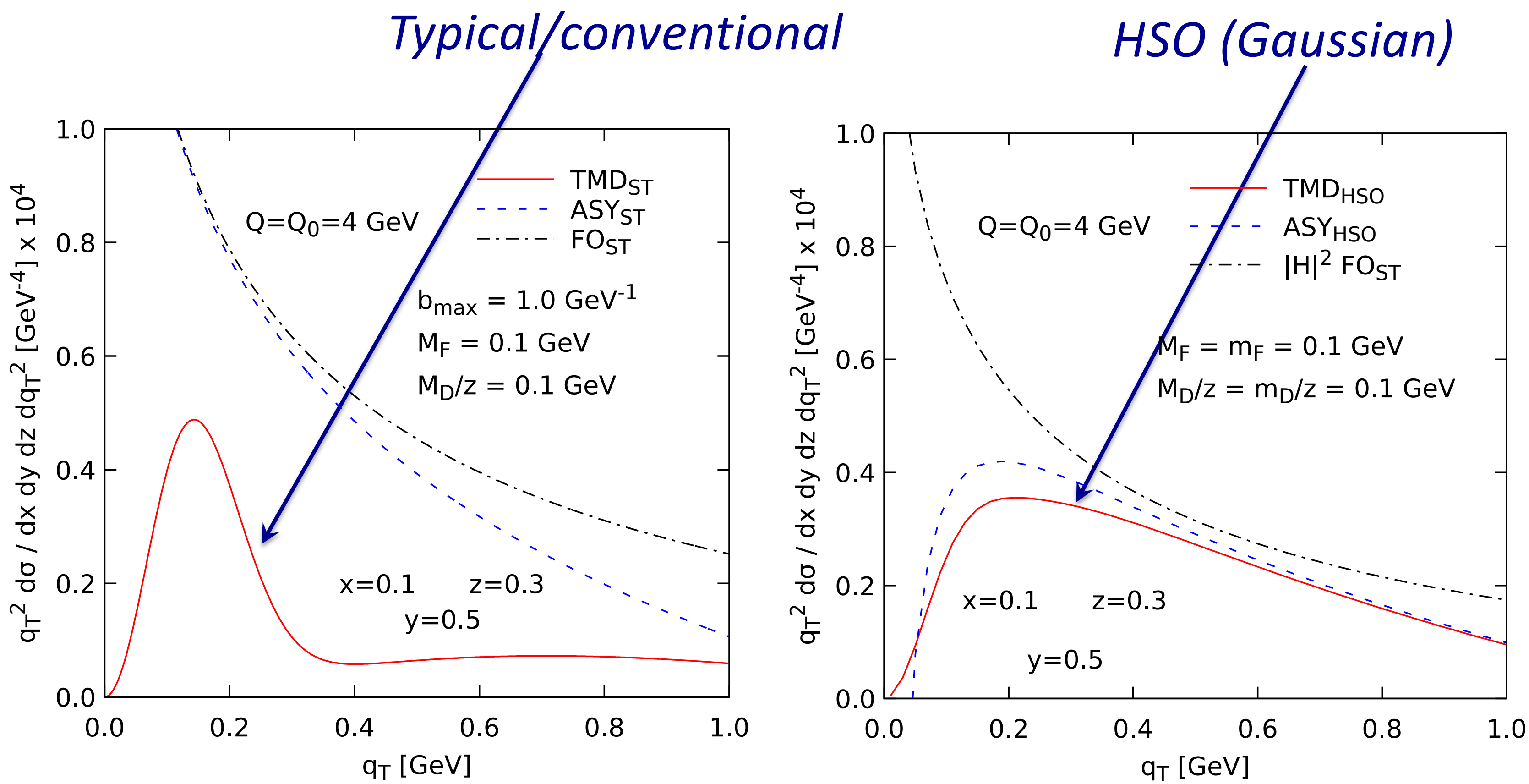


Compare standard/unconstrained with HSO ($O(\alpha_s)$)





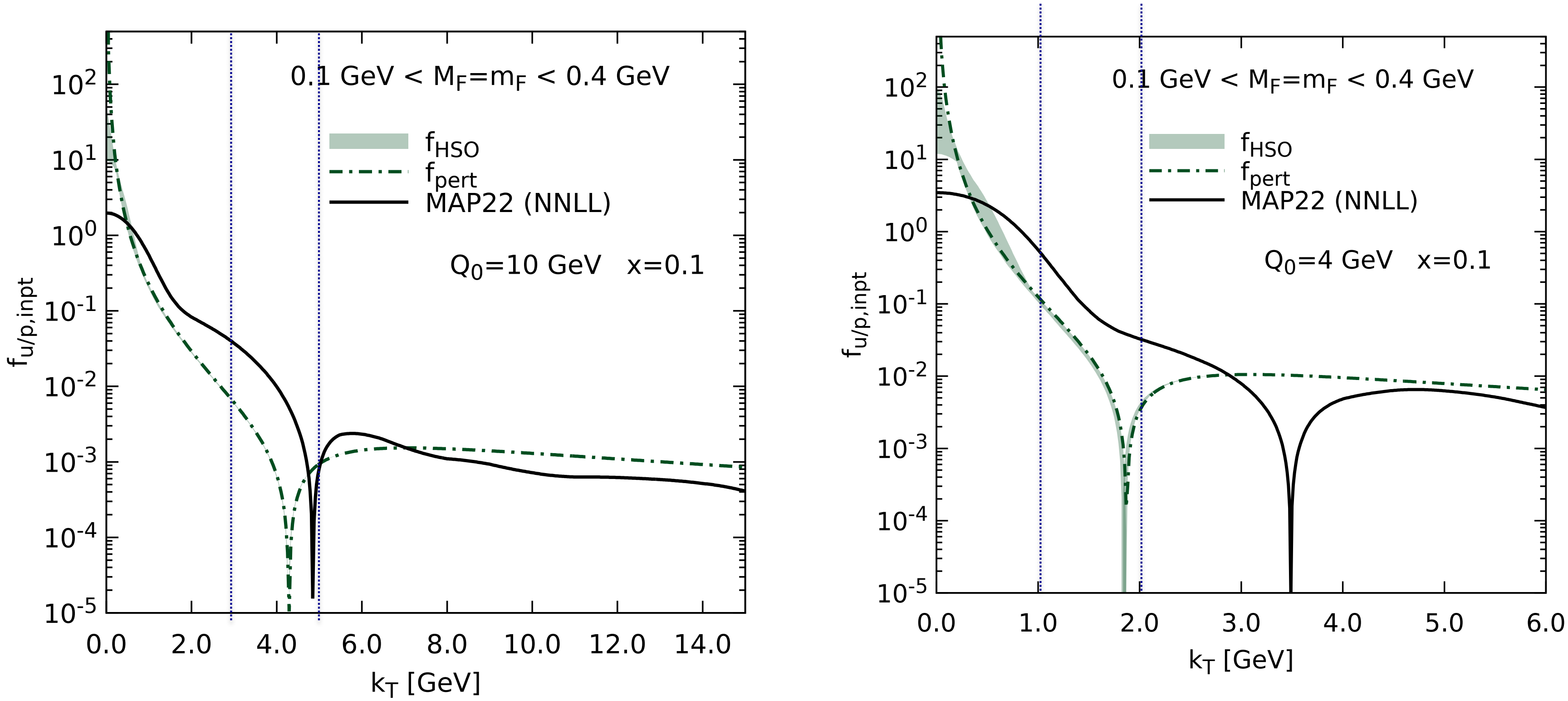
Compare standard/unconstrained with HSO ($O(\alpha_s)$)



What about the individual TMD pdfs?

Proton pdfs MAP (2023)

Bacchetta et al, JHEP 10 (2022) 127, 2206.07598 [hep-ph]



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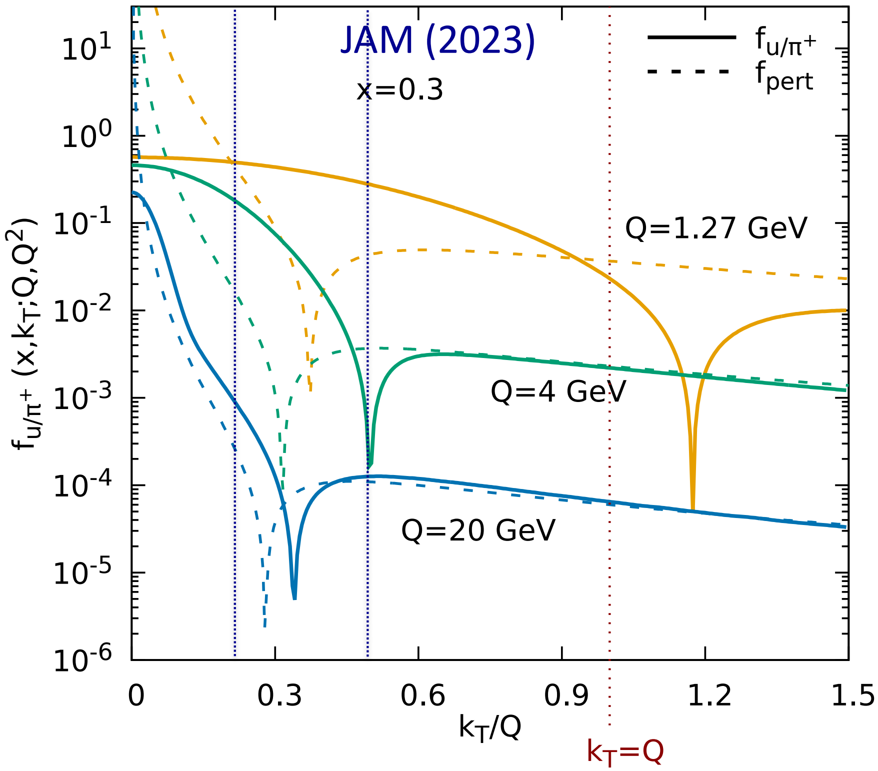
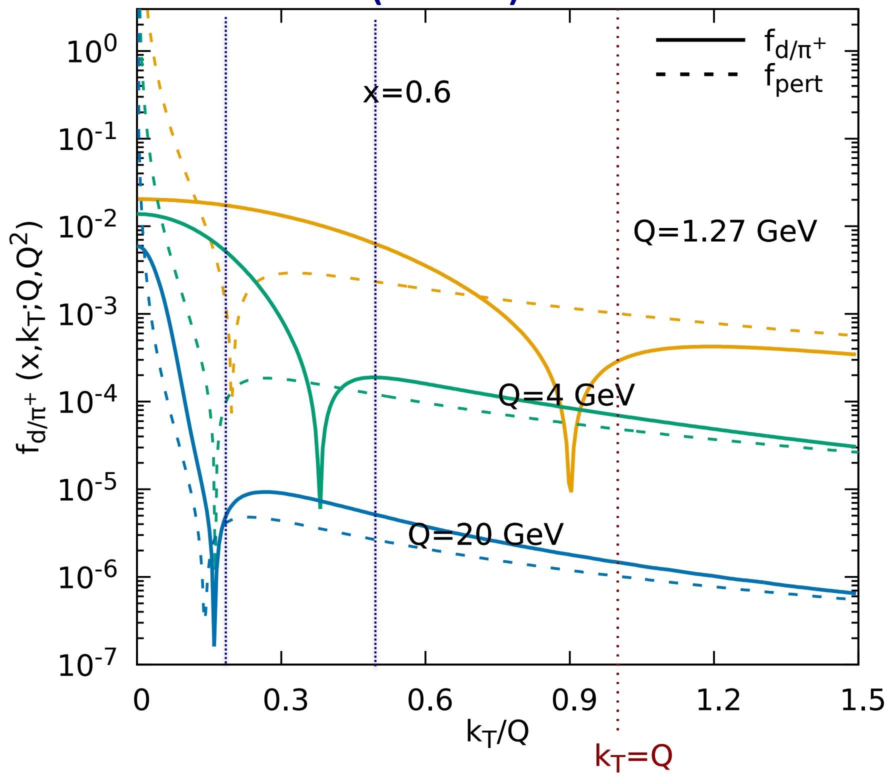
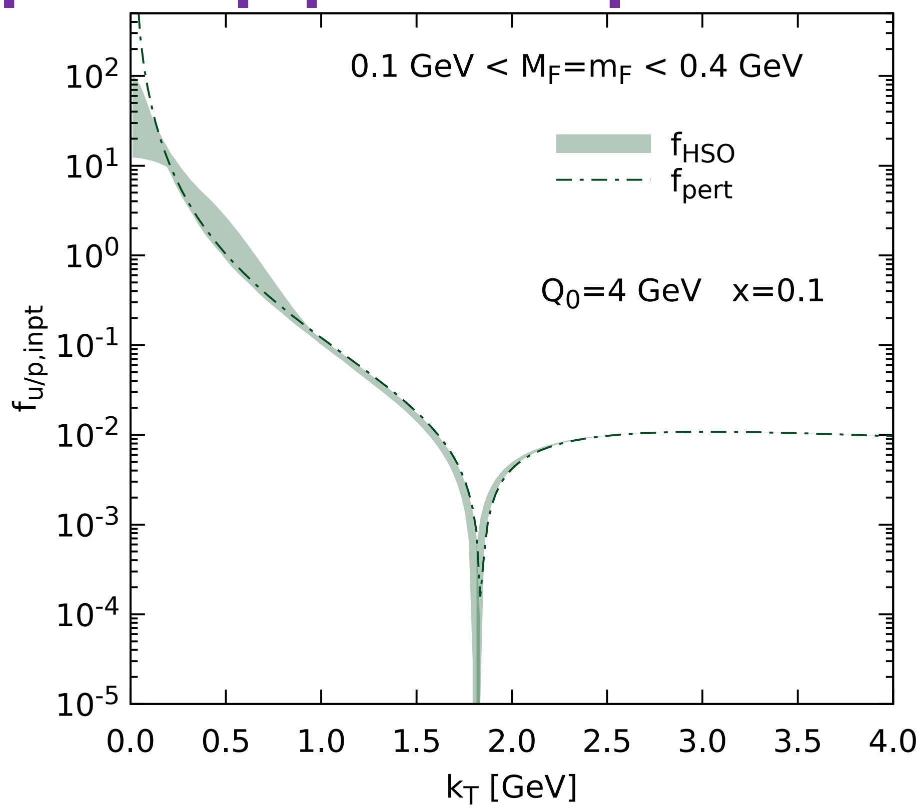


JAM (2023) 2302.01192 [hep-ph]

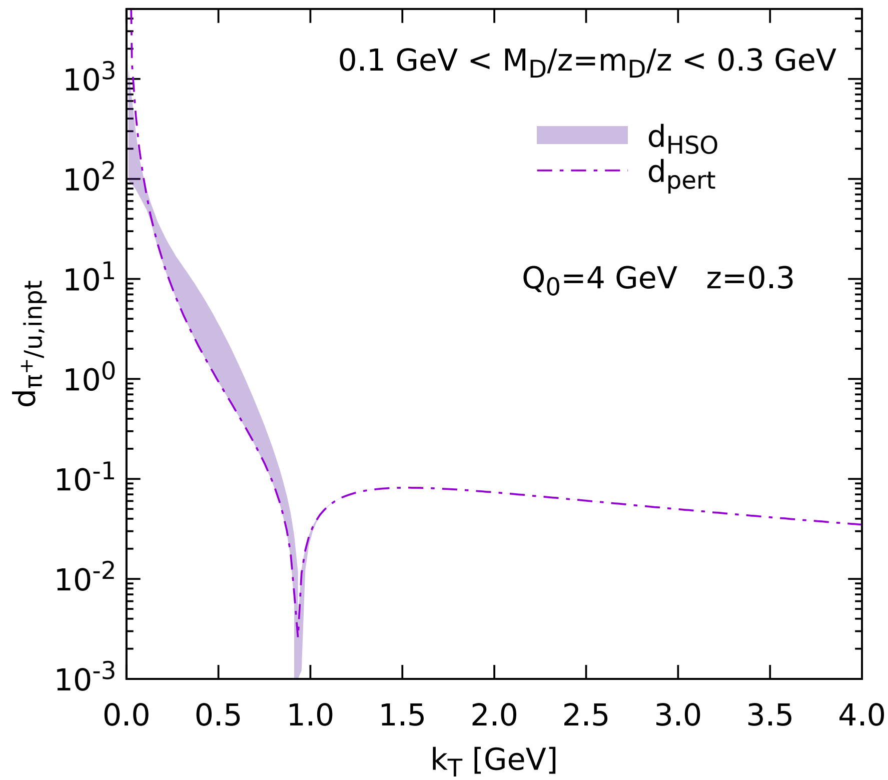


What about the individual TMD pdfs?

proton up-quark TMD pdfs



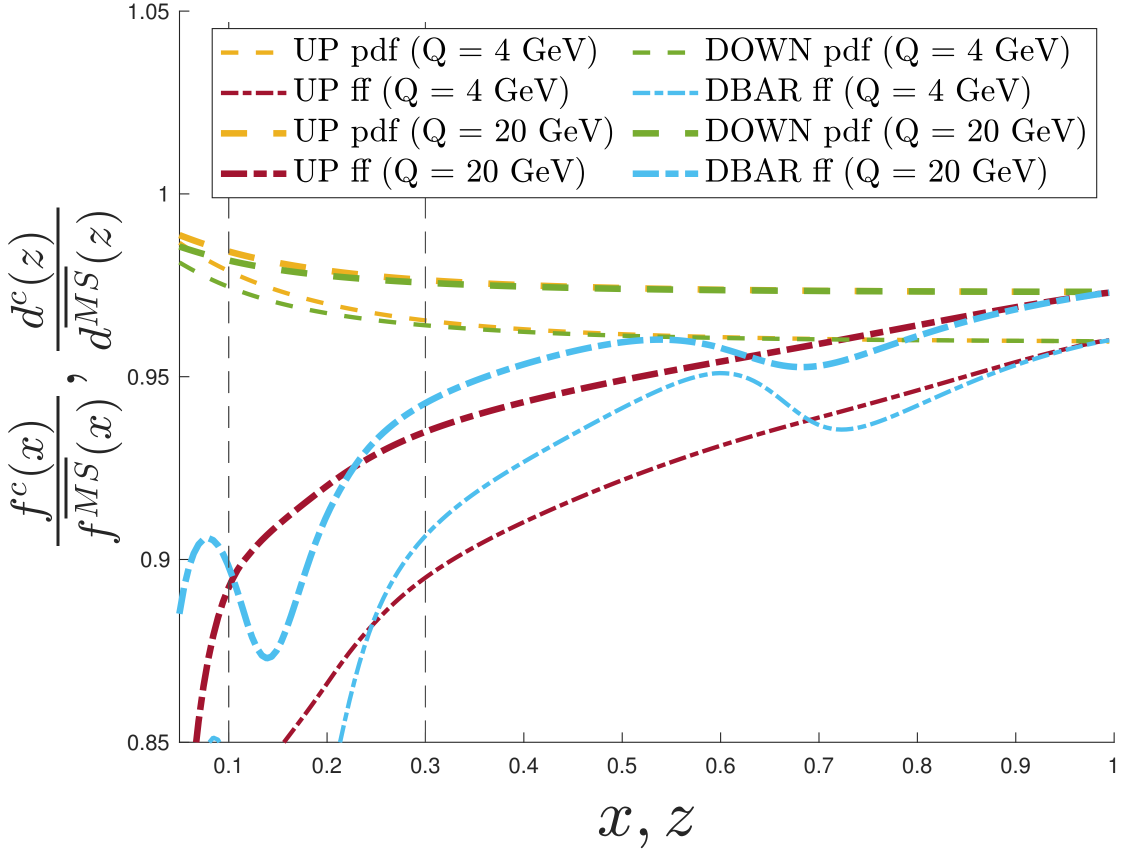
π^+ from up-quark TMD ff



HSO (Gaussian)

Improvements beyond the TMD pdfs

- Cutoff TMD versus renormalized collinear pdfs?



Summary

- Switching to a hadron-structure-oriented approach to pheno with TMD factorization improves consistency in the large transverse behavior of TMD correlation functions

- Necessary for understanding the shapes of nonperturbative distributions, separating perturbative and nonpeturbative parts, etc

- Necessary for transforming claims about nonperturbative TM physics into testable/falsifiable hypotheses

- HSO is not a new formalism; **HSO = "standard CSS"!**

- Next:
  - Applications
  - Higher orders
  - Incorporating NP calculations (lattice, EFTs, models etc)
  - Spin dependent TMDs

Backup

$$A^f_{i/p}(x;\mu_{Q_0}) \equiv \sum_{ii'} \delta_{i'i} \frac{\alpha_s(\mu_{Q_0})}{\pi} \left\{ \left[(P_{i'i} \otimes f_{i'/p})(x;\mu_{Q_0})\right] - \frac{3C_F}{2} f_{i'/p}(x;\mu_{Q_0}) \right\} ,$$

$$B^f_{i/p}(x;\mu_{Q_0}) \equiv \sum_{i'i} \delta_{i'i} \frac{\alpha_s(\mu_{Q_0}) C_F}{\pi} f_{i'/p}(x;\mu_{Q_0}) ,$$

$$A^{f,g}_{i/p}(x;\mu_{Q_0}) \equiv \frac{\alpha_s(\mu_{Q_0})}{\pi} \left[(P_{ig} \otimes f_{g/p})(x;\mu_{Q_0})\right] ,$$

$$C^f_{i/p} \equiv \frac{1}{N^f_{i/p}} \left[f_{i/p}(x;\mu_{Q_0}) - A^f_{i/p}(x;\mu_{Q_0}) \ln\left(\frac{\mu_{Q_0}}{m_{f_{i,p}}}\right) - B^f_{i/p}(x;\mu_{Q_0}) \ln\left(\frac{\mu_{Q_0}}{m_{f_{i,p}}}\right) \ln\left(\frac{Q_0^2}{\mu_{Q_0} m_{f_{i,p}}}\right) ,\right.$$

$$\left. - A^{f,g}_{i/p}(x;\mu_{Q_0}) \ln\left(\frac{\mu_{Q_0}}{m_{f_{g,p}}}\right) + \frac{\alpha_s(\mu_{Q_0})}{2\pi} \left\{ \sum_{ii'} \delta_{i'i} [\mathcal{C}_\Delta^{i/i'} \otimes f_{i'/p}](x;\mu_{Q_0}) + [\mathcal{C}_\Delta^{i/g} \otimes f_{g/p}](x;\mu_{Q_0}) \right\} \right] .$$

$$P_{ig}(x) = T_F \left[x^2 + (1-x)^2\right] ,$$

$$\mathcal{C}_\Delta^{i/i}(x) = C_F(1-x) - C_F \frac{\pi^2}{12} \delta(1-x) ,$$

$$\mathcal{C}_\Delta^{g/p}(x) = 2T_F x(1-x) ,$$

$$N^f_{i/p} \equiv 2\pi \int_0^\infty dk_T k_T \, f_{\text{core},i/p}(x, \boldsymbol{k}_T; Q_0^2)$$

Backup

$$A^D_{h/j}(z;\mu_{Q_0}) \equiv \sum_{jj'} \delta_{j'j} \frac{\alpha_s(\mu_{Q_0})}{\pi} \left\{ [(P_{jj'} \otimes d_{h/j'})(z;\mu_{Q_0})] - \frac{3C_F}{2} d_{h/j'}(z;\mu_{Q_0}) \right\},$$

$$B^D_{h/j}(z;\mu_{Q_0}) \equiv \sum_{jj'} \delta_{j'j} \frac{\alpha_s(\mu_{Q_0}) C_F}{\pi} d_{h/j'}(z;\mu_{Q_0}),$$

$$A^{D,g}_{h/j}(z;\mu_{Q_0}) \equiv \frac{\alpha_s(\mu_{Q_0})}{\pi} [(P_{gj} \otimes d_{h/g})(z;\mu_{Q_0})] ,$$

$$C^D_{h/j} \equiv \frac{1}{N^D_{h/j}} \Bigg[d_{h/j}(z;\mu_{Q_0}) - A^D_{h/j}(z;\mu_{Q_0}) \ln\left(\frac{\mu_{Q_0}}{m_{D_{h,j}}}\right) - B^D_{h/j}(z;\mu_{Q_0}) \ln\left(\frac{\mu_{Q_0}}{m_{D_{h,j}}}\right) \ln\left(\frac{Q_0^2}{\mu_{Q_0} m_{D_{h,j}}}\right),$$

$$- A^{D,g}_{h/j}(z;\mu_{Q_0}) \ln\left(\frac{\mu_{Q_0}}{m_{D_{h,g}}}\right) + \frac{\alpha_s(\mu_{Q_0})}{2\pi} \left\{ \sum_{jj'} \delta_{j'j} [\mathcal{C}^{j'/j}_{\Delta} \otimes d_{h/j'}](z;\mu_{Q_0}) + [\mathcal{C}^{g/j}_{\Delta} \otimes d_{h/g}](z;\mu_{Q_0}) \right\} \Bigg].$$

$$P_{qq}(z) = P_{\bar{q}\bar{q}}(z) = C_F \left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z) \right],$$

$$P_{gq}(z) = C_F \frac{1+(1-z)^2}{z},$$

$$\mathcal{C}^{q/q}_{\Delta}(z) = 2P_{qq}(z)\ln z + C_F(1-z) - C_F \frac{\pi^2}{12}\delta(1-z),$$

$$\mathcal{C}^{g/q}_{\Delta}(z) = 2P_{gq}(z)\ln z + C_F z,$$

$$N^D_{h/j} \equiv 2\pi z^2 \int_0^\infty dk_T k_T \, D_{\text{core},h/j}(z, z\boldsymbol{k}_T; Q_0^2).$$

Backup

$$
\begin{aligned}
f^{c}_{\mathrm{inpt},i/p}(x;\mu_{Q_0}) =& 2\pi \int_0^{\mu_{Q_0}} \mathrm{d}k_{\mathrm{T}}\, k_{\mathrm{T}} f_{\mathrm{inpt},i/p}(x,\boldsymbol{k}_{\mathrm{T}};\mu_{Q_0},Q_0^2) = \\
& C^{f}_{i/p}\, f^{c}_{\mathrm{core},i/p}(x;\mu_{Q_0}) + \frac{1}{2} A^{f,g}_{i/p}(x;\mu_{Q_0}) \ln\left(1+\frac{\mu^2_{Q_0}}{m^2_{f_{g,p}}}\right) \\
& + \frac{1}{2} A^{f}_{i/p}(x;\mu_{Q_0}) \ln\left(1+\frac{\mu^2_{Q_0}}{m^2_{f_{i,p}}}\right) + \frac{1}{4} B^{f}_{i/p}(x;\mu_{Q_0}) \left[\ln^2\left(\frac{m^2_{f_{i,p}}}{Q_0^2}\right) - \ln^2\left(\frac{\mu^2_{Q_0}+m^2_{f_{i,p}}}{Q_0^2}\right)\right] \\
& = f_{i/p}(x;\mu_{Q_0}) + O\left(\alpha_s(\mu_0), \frac{m^2}{Q_0^2}\right) ,
\end{aligned}
$$

$$
\begin{aligned}
d^{c}_{\mathrm{inpt},h/j}(z;\mu_{Q_0}) =& 2\pi z^2 \int_0^{\mu_{Q_0}} \mathrm{d}k_{\mathrm{T}}\, k_{\mathrm{T}} D_{\mathrm{inpt},h/j}(z,z\boldsymbol{k}_{\mathrm{T}};\mu_{Q_0},Q_0^2) = \\
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& + \frac{1}{2} A^{D}_{h/j}(z;\mu_{Q_0}) \ln\left(1+\frac{\mu^2_{Q_0}}{m^2_{D_{h,j}}}\right) + \frac{1}{4} B^{D}_{h/j}(z;\mu_{Q_0}) \left[\ln^2\left(\frac{m^2_{D_{h,j}}}{Q_0^2}\right) - \ln^2\left(\frac{\mu^2_{Q_0}+m^2_{D_{h,j}}}{Q_0^2}\right)\right] \\
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\end{aligned}
$$