# The hadron structure oriented approach to TMD phenomenology

Ted Rogers
Jefferson Lab & Old Dominion University

Based on:

- J.O. Gonzalez, TCR, N. Sato, Phys.Rev.D 106 (2022) 3, 034002
- J.O. Gonzalez, T. Rainaldi, TCR (2023), Phys.Rev.D 107 (2023) 9, 094029

&

- Work in progress with F. Aslan, M. Boglione, J.O. Gonzalez, T. Rainaldi, A. Simonelli

JLab EIC Meeting, October 13, 2023

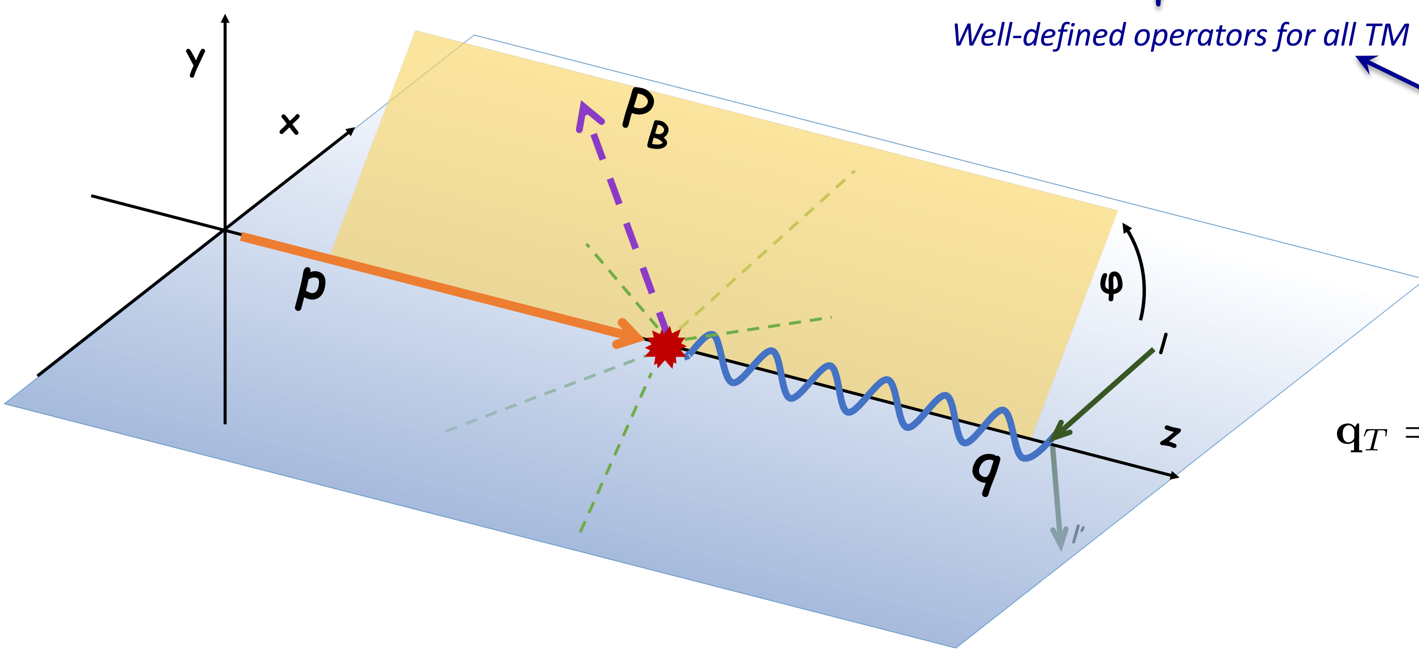
# Transverse momentum dependent (TMD) factorization & SIDIS $\left(for\ \frac{q_T}{Q} \ll 1\right)$

$$\frac{d\sigma}{dQ\,d^2\mathbf{q}_T\,dx\,dz} = H(Q/\mu_Q)\int d^2\mathbf{k}_{1T}d^2\mathbf{k}_{2T}\,f_{j/p}(x,\mathbf{k}_{1T};\mu_Q,Q^2)D_{h/j}(z,z\mathbf{k}_{2T};\mu_Q,Q^2)\delta^{(2)}(\mathbf{q}_T+\mathbf{k}_{1T}-\mathbf{k}_{2T})$$

+ power suppressed

*Well-defined operators for all TM*

*The "W-term"*




$$\mathbf{q}_T = -\frac{\mathbf{P}_{BT}}{z}$$

## Simplistic parton model

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q\ \mathrm{d}x} \sim \sum_i f_{i/p}(x)$$

**QCD**

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q\ \mathrm{d}x} = \sum_i \int_x^1 \frac{\mathrm{d}\xi}{\xi} H_i(x/\xi, \alpha_s(Q)) f_{i/p}(\xi; Q)$$

# Simplistic parton model

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q\,\mathrm{d}^2\mathbf{q}_T\,\mathrm{d}x\,\mathrm{d}z} = \sum_j \int \mathrm{d}^2\mathbf{k}_{1T}\mathrm{d}^2\mathbf{k}_{2T} f_{j/p}(x,\mathbf{k}_{1T}) D_{h/j}(z, z\mathbf{k}_{2T})\delta^{(2)}(\mathbf{q}_T + \mathbf{k}_{1T} - \mathbf{k}_{2T})$$

# QCD

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q\,\mathrm{d}^2\mathbf{q}_T\,\mathrm{d}x\,\mathrm{d}z} = H(\alpha_s(Q)) \int \mathrm{d}^2\mathbf{k}_{1T} \sum_j \mathrm{d}^2\mathbf{k}_{2T} f_{j/p}(x,\mathbf{k}_{1T};Q^2) D_{h/j}(z, z\mathbf{k}_{2T};Q^2)\delta^{(2)}(\mathbf{q}_T + \mathbf{k}_{1T} - \mathbf{k}_{2T})$$

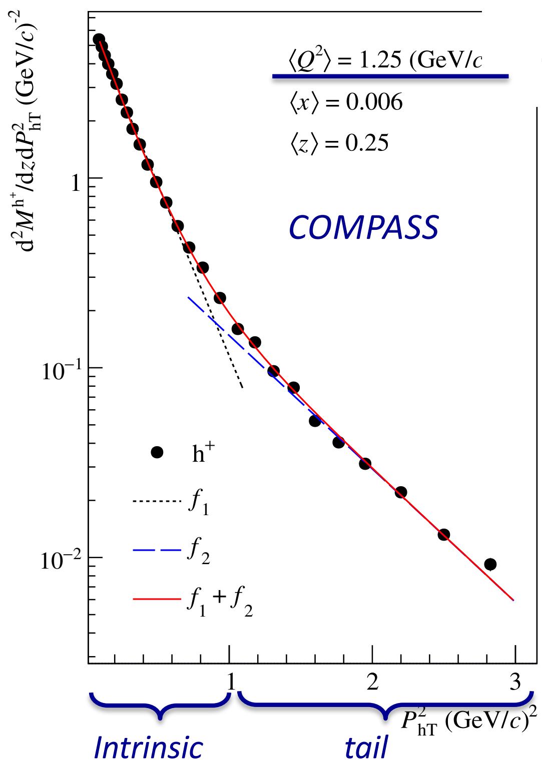
$+ O\left(\frac{q_T^2}{Q^2}\right)$ corrections (the “Y-term”)

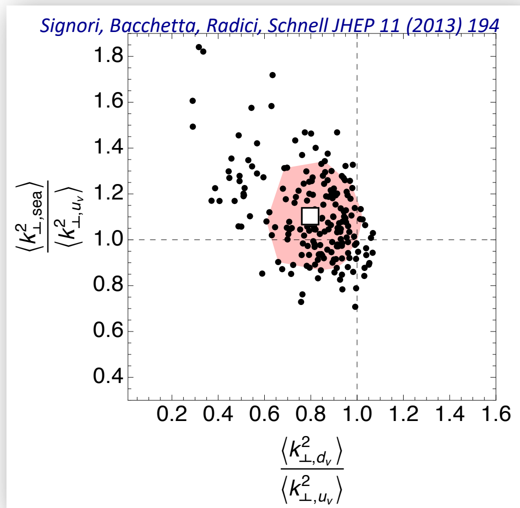
# Nonperturbative structures in pheno

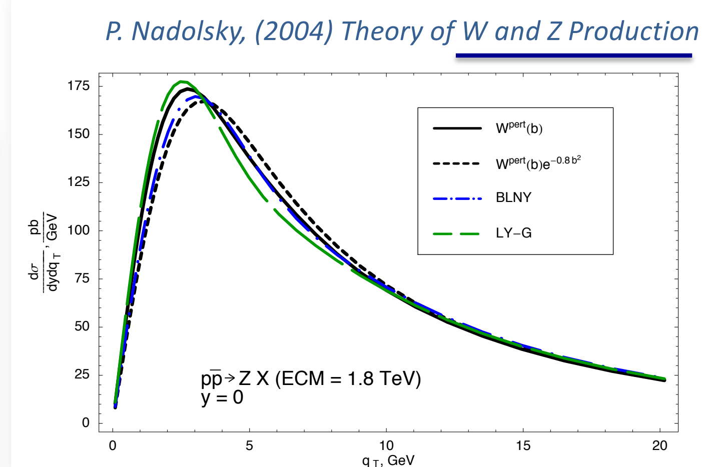
*"While significant effort has been put into the study of W(b)* ***at large b*** *[36, 42, 43, 44], none … adequately describe the observed Z boson distribution without introducing free parameters."*

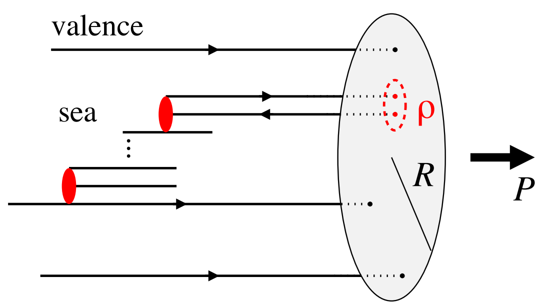
PHYSICAL REVIEW D **97,** 032006 (2018)

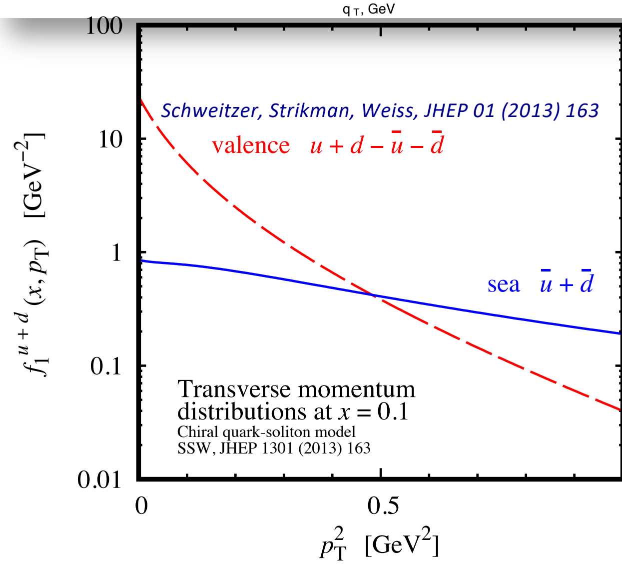
**Transverse-momentum-dependent multiplicities of charged hadrons in muon-deuteron deep inelastic scattering**



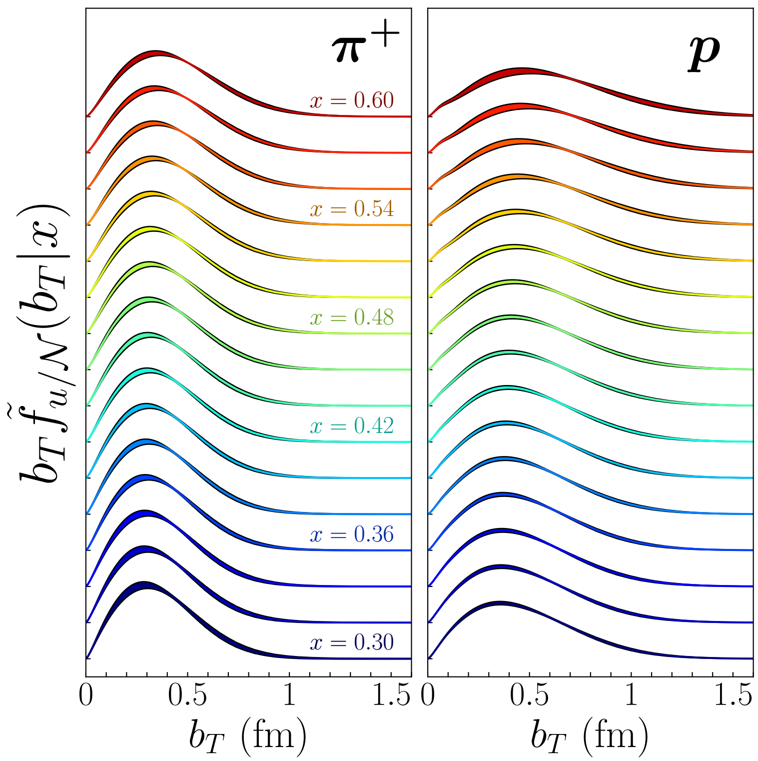




*"… the two exponential functions in our parametrizations $F_1$ can be attributed to two completely different underlying physics mechanisms that overlap in the region $P_{hT} \simeq 1\ \mathrm{GeV}^2$."*

# Nonperturbative structures in pheno

*JAM: Barry, et al (2302.01192)*

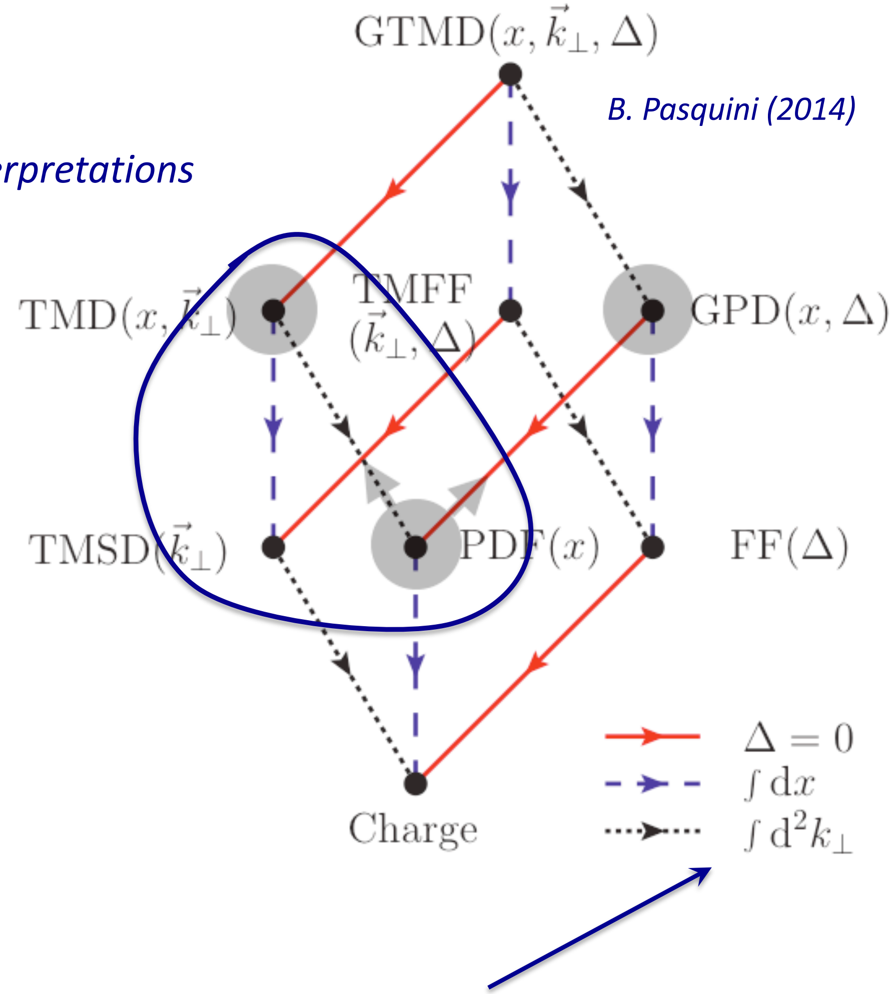



*??*

*“Importantly, we have checked that the differences between the proton and pion ⟨bT |x⟩ are completely due to the nonperturbative TMD structure, independent of the collinear PDFs.”* *??*

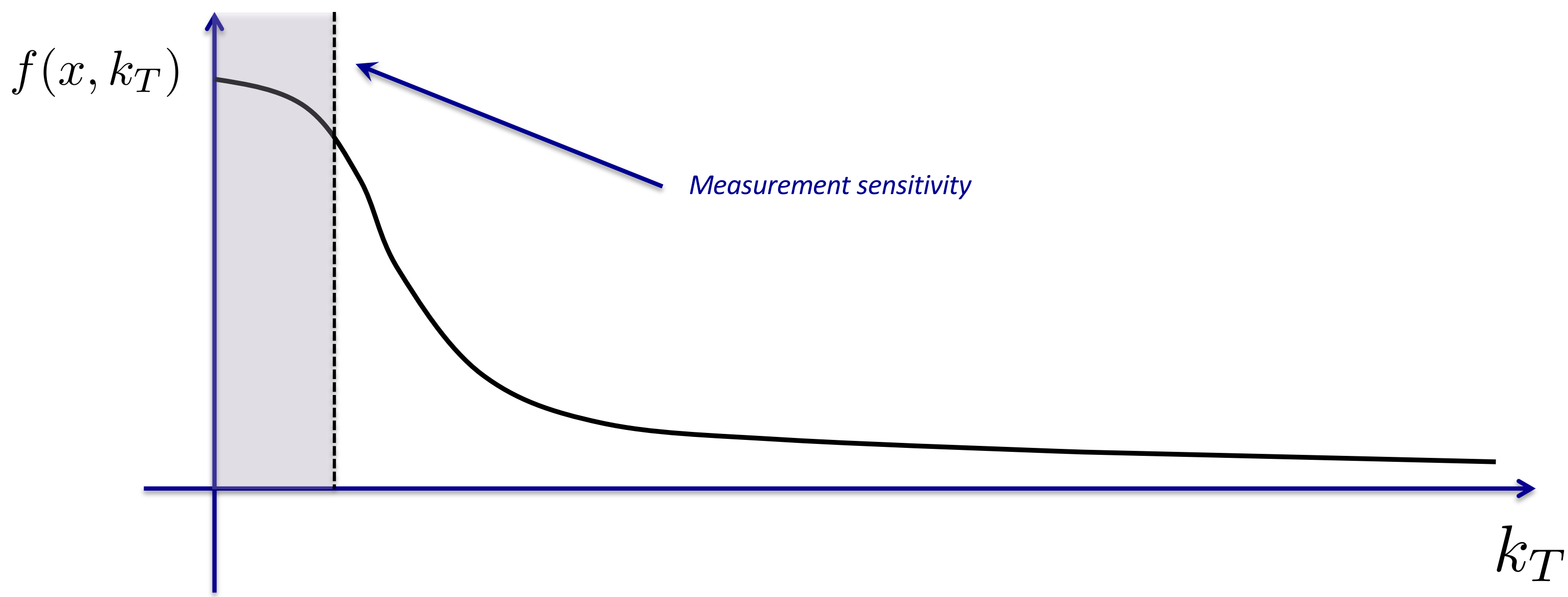
*Interpretations*

*B. Pasquini (2014)*




*These integrals are often divergent in real QCD!*

## Why worry about "large" $k_T$?




- TMD pdf exists for all $k_T$. Large/small $k_T$, consistency constrains small $k_T$!
- How to compare low/moderate Q TMD pdf to large Q pdf?

## Where is the hadron structure!?

$$
\begin{aligned}
\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\,\mathrm{d}y\,\mathrm{d}q_{\mathrm{T}}^2} = \frac{4\pi^2\alpha^2}{9Q^2 s} \sum_{j,j_A,j_B} e_j^2 \int \frac{\mathrm{d}^2\boldsymbol{b}_{\mathrm{T}}}{(2\pi)^2} e^{i\boldsymbol{q}_{\mathrm{T}}\cdot\boldsymbol{b}_{\mathrm{T}}} \\
&\times \int_{x_A}^{1} \frac{\mathrm{d}\xi_A}{\xi_A} f_{j_A/A}(\xi_A;\mu_{b_*})\ \tilde{C}_{j/j_A}^{\mathrm{CSS1,\ DY}}\left(\frac{x_A}{\xi_A}, b_*; \mu_{b_*}^2, \mu_{b_*}, C_2, a_s(\mu_{b_*})\right) \\
&\times \int_{x_B}^{1} \frac{\mathrm{d}\xi_B}{\xi_B} f_{j_B/B}(\xi_B;\mu_{b_*})\ \tilde{C}_{\bar{\jmath}/j_B}^{\mathrm{CSS1,\ DY}}\left(\frac{x_B}{\xi_B}, b_*; \mu_{b_*}^2, \mu_{b_*}, C_2, a_s(\mu_{b_*})\right) \\
&\times \exp\left\{ - \int_{\mu_{b_*}^2}^{\mu_Q^2} \frac{\mathrm{d}\mu'^2}{\mu'^2} \left[ A_{\mathrm{CSS1}}(a_s(\mu'); C_1) \ln\left(\frac{\mu_Q^2}{\mu'^2}\right) + B_{\mathrm{CSS1,\ DY}}(a_s(\mu'); C_1, C_2) \right] \right\} \\
&\times \exp\Big[ -g_{j/A}^{\mathrm{CSS1}}(x_A, b_{\mathrm{T}}; b_{\max}) - g_{\bar{\jmath}/B}^{\mathrm{CSS1}}(x_B, b_{\mathrm{T}}; b_{\max}) - g_K^{\mathrm{CSS1}}(b_{\mathrm{T}}; b_{\max}) \ln(Q^2/Q_0^2) \Big] \\
&+ \text{suppressed corrections.}
\end{aligned}
$$

*CSS1 = Collins-Soper-Sterman (≈1985)*

## What is meant by “hadron structure oriented?”

- Adhere strictly to logic of TMD factorization
- Preserve parton model interpretation
- Preserve relationships between TMD and collinear factorization
- Allow to compare/contrast different models and descriptions of hadron structure

## The conventional organization

1) Solve evolution equations to relate overall SIDIS hard scale ($\mu_Q = Q$) to input scale ($\mu_{Q_0} = Q_0$)
2) How to use small $b_T \ll 1/\Lambda_{QCD}$ collinear factorization?
   - Partition small ($b_T < b_{\max}$) & large ($b_T > b_{\max}$) regions with a $b_*$
   - Define hard scale $\mu_{b_*} \sim 1/b_*$
3) Evolve again to relate $Q_0$ to $\mu_{b_*}$
4) **Place remaining NP parts in an exponent:**

$$-g_{j/p}(x, b_\mathrm{T}) \equiv \ln \left( \frac{\tilde{f}_{j/p}(x, \boldsymbol{b}_\mathrm{T}; \mu_{Q_0}, Q_0^2)}{\tilde{f}_{j/p}(x, \boldsymbol{b}_*; \mu_{Q_0}, Q_0^2)} \right)$$

$$g_K(b_\mathrm{T}) \equiv \tilde{K}(b_*; \mu) - \tilde{K}(b_\mathrm{T}; \mu)$$

5) **Perform small-$\boldsymbol{b_T}$ expansions & drop $\boldsymbol{O(\Lambda_{QCD}\, b_{max})}$ errors**
6) **Ansatz for g-functions**

$$\frac{\partial \ln \tilde{f}_{j/p}(x, b_T; \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T; \mu)$$

$$\frac{\mathrm{d} \ln \tilde{f}_{j/p}(x, b_T; \mu, \zeta)}{\mathrm{d} \ln \mu} = \gamma(\alpha_s(\mu); \zeta/\mu^2)$$

$$\frac{\mathrm{d}\tilde{K}(b_T; \mu)}{\mathrm{d} \ln \mu} = -\gamma_K(\alpha_s(\mu))$$

$$\tilde{f}_{j/p}(x, b_T; \mu, \zeta) = \int_x^1 \frac{\mathrm{d}\xi}{\xi} \tilde{C}_{j/k}(x/\xi, b_T; \zeta, \mu, \alpha_s(\mu)) f_{k/p}(\xi; \mu) + O\left(b_T \Lambda_\mathrm{QCD}\right)$$

***Or***

$$f_{j/p}(x, k_T; \mu, \zeta) = \frac{1}{k_T^2} \left[ \int_x^1 \frac{\mathrm{d}\xi}{\xi} C_{j/k}(x/\xi, k_T; \zeta, \mu, \alpha_s(\mu)) f_{k/p}(\xi; \mu) + O\left(\frac{\Lambda_\mathrm{QCD}}{k_T}\right) \right]$$

$$\tilde{K}(b_T; \mu) = \tilde{K}_\mathrm{pert}\left(b_T \mu, \alpha_s(\mu)\right) + O\left(b_T \Lambda_\mathrm{QCD}\right)$$

# The conventional organization

1) Solve evolution equations to relate overall SIDIS hard scale ($\mu_Q = Q$) to input scale ($\mu_{Q_0} = Q_0$)
2) How to use small $b_T \ll 1/\Lambda_{QCD}$ collinear factorization?
   - Partition small ($b_T < b_{\max}$) & large ($b_T > b_{\max}$) regions with a $b_*$
   - Define hard scale $\mu_{b_*} \sim 1/b_*$
3) Evolve again to relate $Q_0$ to $\mu_{b_*}$
4) **Place remaining NP parts in an exponent:**

$$-g_{j/p}(x, b_{\mathrm{T}}) \equiv \ln \left( \frac{\tilde{f}_{j/p}(x, \boldsymbol{b}_{\mathrm{T}}; \mu_{Q_0}, Q_0^2)}{\tilde{f}_{j/p}(x, \boldsymbol{b}_*; \mu_{Q_0}, Q_0^2)} \right)$$

$$g_K(b_{\mathrm{T}}) \equiv \tilde{K}(b_*; \mu) - \tilde{K}(b_{\mathrm{T}}; \mu)$$

5) **Perform small-$b_T$ expansions & drop $O(\Lambda_{QCD}\, b_{max})$ errors**
6) **Ansatz for g-functions**

$$\int \mathrm{d}^2\mathbf{k}_{1T}\mathrm{d}^2\mathbf{k}_{2T} f_{j/p}(x, \mathbf{k}_{1T}; \mu_Q, Q^2) D_{h/j}(z, z\mathbf{k}_{2T}; \mu_Q, Q^2)\delta^{(2)}(\mathbf{q}_T + \mathbf{k}_{1T} - \mathbf{k}_{2T})$$

$$\Downarrow$$

$$\int \frac{\mathrm{d}^2\boldsymbol{b}_{\mathrm{T}}}{(2\pi)^2}\, e^{-i\boldsymbol{q}_{\mathrm{T}}\cdot\boldsymbol{b}_{\mathrm{T}}}\, \tilde{f}^{\mathrm{OPE}}_{j/p}(x_{\mathrm{bj}}, \boldsymbol{b}_*; \mu_{b_*}, \mu^2_{b_*})\, \tilde{D}^{\mathrm{OPE}}_{h/j}(z_{\mathrm{h}}, \boldsymbol{b}_*; \mu_{b_*}, \mu^2_{b_*})$$
$$\times \exp\left\{ 2\int_{\mu_{b_*}}^{\mu_Q} \frac{d\mu'}{\mu'}\left[\gamma(\alpha_s(\mu'); 1) - \ln\frac{Q}{\mu'}\gamma_K(\alpha_s(\mu'))\right] + \ln\frac{Q^2}{\mu^2_{b_*}}\tilde{K}(b_*; \mu_{b_*}) \right\}$$
$$\times \exp\left\{ -g_{j/p}(x_{\mathrm{bj}}, b_{\mathrm{T}}) - g_{h/j}(z_{\mathrm{h}}, b_{\mathrm{T}}) - g_K(b_{\mathrm{T}}) \ln\left(\frac{Q^2}{Q_0^2}\right) \right\} + O(b_{\max}\Lambda_{\mathrm{QCD}})$$

$$\tilde{f}_{j/p}(x, b_T; \mu, \zeta) = \underbrace{\int_x^1 \frac{\mathrm{d}\xi}{\xi}\tilde{C}_{j/k}(x/\xi, b_T; \zeta, \mu, \alpha_s(\mu)) f_{k/p}(\xi; \mu)}_{\tilde{f}^{\mathrm{OPE}}_{j/p}(x, b_T; \mu, \zeta)} + O(b_T \Lambda_{\mathrm{QCD}})$$

## How to test consistency

- $\int \mathrm{d}^2\mathbf{k}_{1T}\mathrm{d}^2\mathbf{k}_{2T} f_{j/p}(x,\mathbf{k}_{1T};\mu_Q,Q^2) D_{h/j}(z,z\mathbf{k}_{2T};\mu_Q,Q^2)\delta^{(2)}(\mathbf{q}_T+\mathbf{k}_{1T}-\mathbf{k}_{2T})$

  is uniquely determined by its operator definition

- At $q_T \approx Q$

$$= \underbrace{\frac{1}{q_{\mathrm{T}}^2}\left[\mathcal{C}\left(q_{\mathrm{T}}/\mu_Q, \alpha_s\left(\mu_Q\right)\right) \otimes f_{j/p}(x;\mu_Q) \otimes d_{h/j}(z;\mu_Q)\right.}_{q_T \sim Q,\, Q \to \infty \text{ asymptote}} \left. + O\left(\frac{\Lambda_{\mathrm{QCD}}}{q_{\mathrm{T}}}\right)\right]$$

- For TMD pdfs & ffs

$$f_{j/p}(x,k_T \approx Q;\mu_Q,Q^2) = \frac{1}{k_T^2}\left[\int_x^1 \frac{\mathrm{d}\xi}{\xi} C_{j/k}(x/\xi, k_T/Q, \alpha_s(Q)) f_{k/p}(\xi;Q) + O\left(\frac{\Lambda_{\mathrm{QCD}}}{k_T}\right)\right]$$

&

$$\int \mathrm{d}^2\mathbf{k}_T f_{j/p}(x,k_T;\mu_Q,Q^2) \approx f_{j/p}(x;\mu_Q)$$

- $\frac{\mathrm{d}}{\mathrm{d}b_{\max}}\left(\frac{\mathrm{d}\sigma}{\mathrm{d}^2\boldsymbol{q}_{\mathrm{T}}\ldots}\right) = 0$ *for* $b_{\max} \ll 1/\Lambda_{QCD}$

# How to test consistency

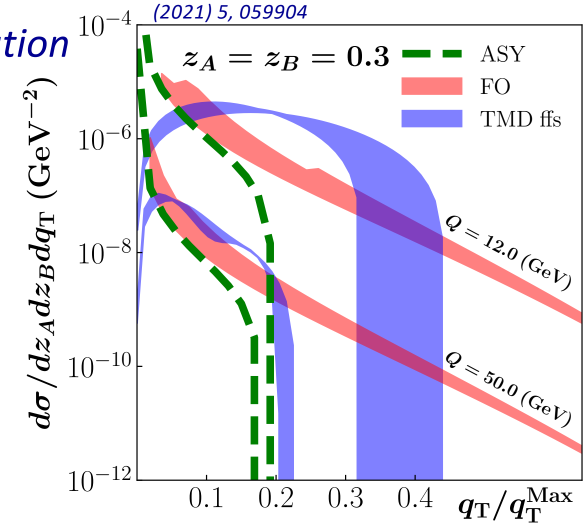
- What does $q_T \approx Q$ mean?
  - No sensitivity to parameters related nonperturbative transverse momentum
  - $\Lambda_{QCD} \ll q_T \ll Q$? Look for matching between fixed order x-section and asymptotic term
    - Is there a region where both

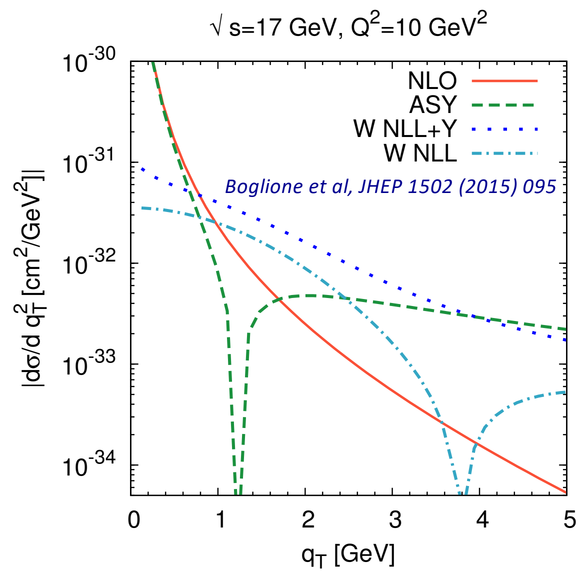
$$\frac{\Lambda_{QCD}}{q_T} \;\&\; \frac{q_T}{Q}$$

      powers are simultaneously negligible?
  - No large logarithms: Look for node in asymptotic term

*e+e- annihilation*

*E. Moffat, T. Rogers, N. Sato, A. Signori Phys.Rev.D 104 (2021) 5, 059904*



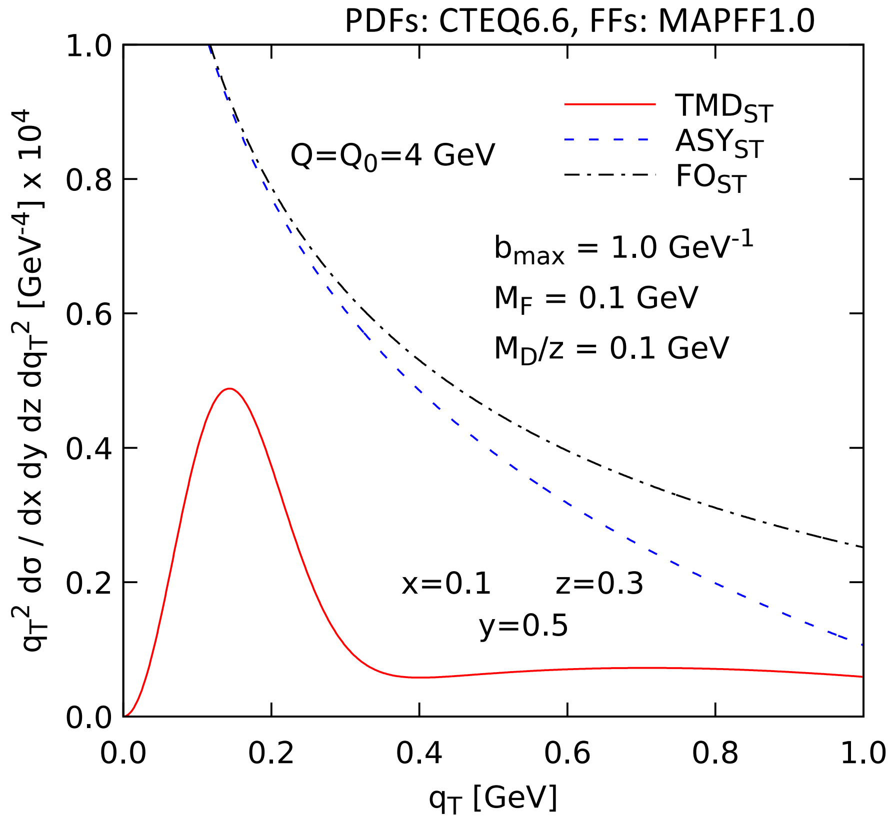



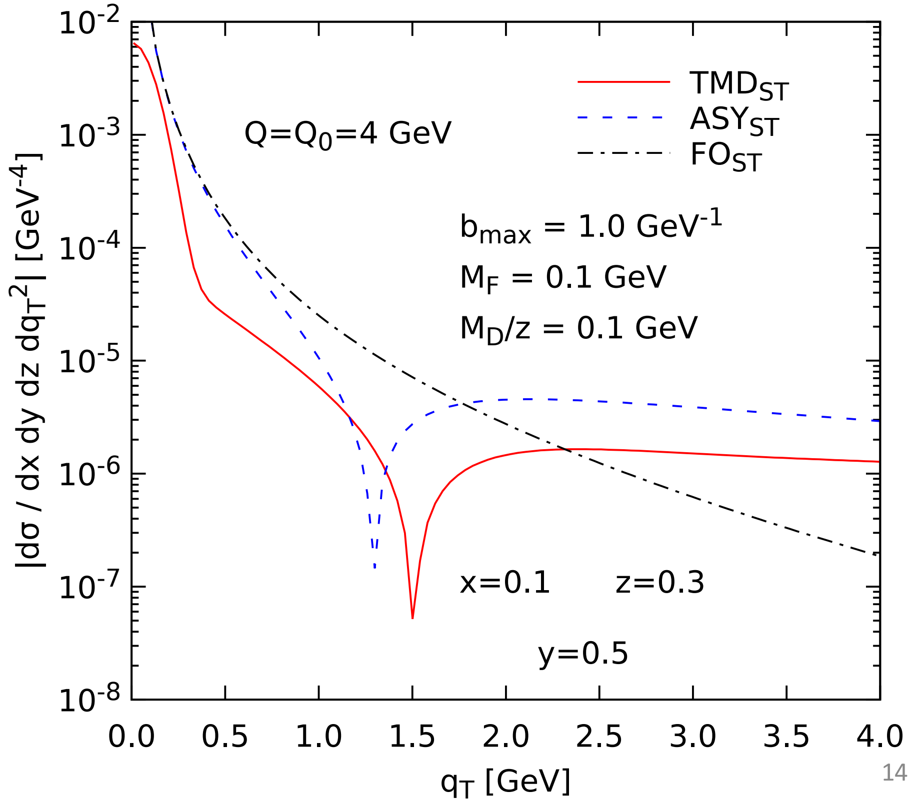
# Conventional organization & complications

*(An example typical of conventional approach)*

$$g_{j/p}(x, b_T) = \frac{1}{4} M_F^2 \, b_T^2$$

$$g_{h/j}(z, b_T) = \frac{1}{4 z^2} M_D^2 \, b_T^2$$

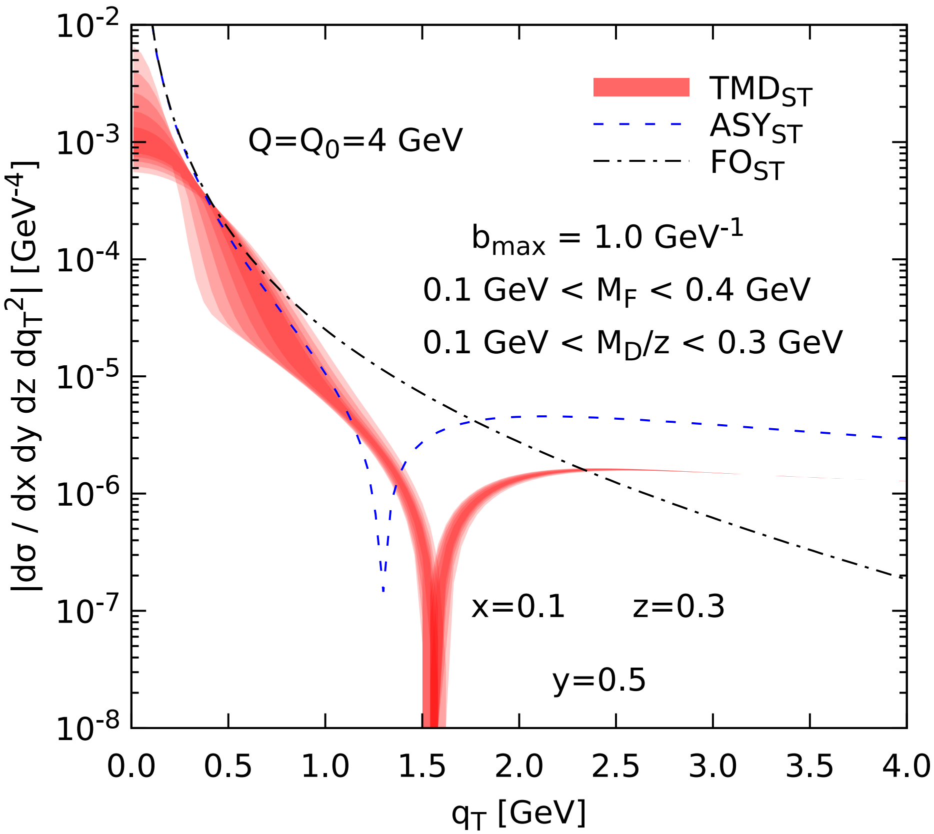


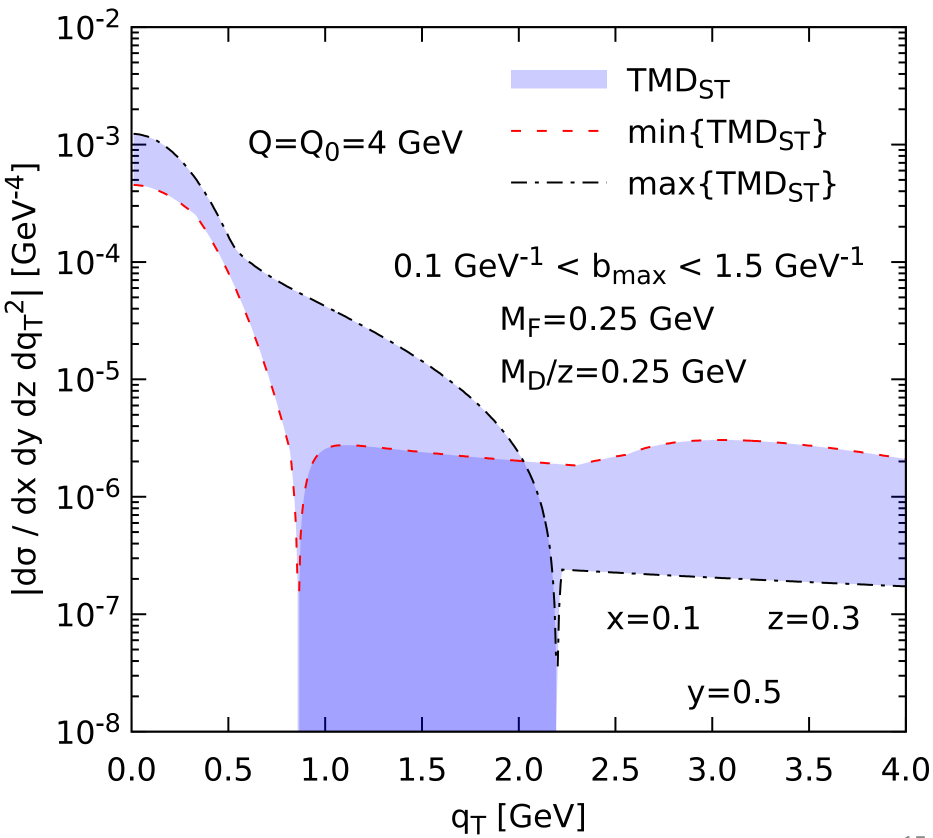



# Conventional organization & complications

$$g_{j/p}(x, b_{\mathrm{T}}) = \frac{1}{4} M_F^2\, b_{\mathrm{T}}^2$$

$$g_{h/j}(z, b_{\mathrm{T}}) = \frac{1}{4\, z^2} M_D^2\, b_{\mathrm{T}}^2$$



$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q\,\mathrm{d}^2\mathbf{q}_T\,\mathrm{d}x\,\mathrm{d}z} = H(Q/\mu_Q) \underbrace{\int \mathrm{d}^2\mathbf{k}_{1T}\mathrm{d}^2\mathbf{k}_{2T} f_{j/p}(x, \mathbf{k}_{1T}; \mu_Q, Q^2) D_{h/j}(z, z\mathbf{k}_{2T}; \mu_Q, Q^2)}_{\text{Well-defined operators for all TM}} \delta^{(2)}(\mathbf{q}_T + \mathbf{k}_{1T} - \mathbf{k}_{2T}) + \text{power suppressed}$$

# Diagnosis

1) Consistency tests will generally fail for a g-function ansatz unless constraints are imposed

2) Fixed order perturbation theory should work fine for $q_T \approx Q_0$, but evol. factors have a large effect. What is going on?

3) $\exists$ no region at input scale $Q = Q_0$ where $\Lambda_{QCD} \ll q_T \ll Q_0$

4) Backwards evolution…
No large, perturbative $\ln \frac{Q_0}{q_T}$.

5) $\int \mathrm{d}^2\mathbf{k}_T f_{j/p}(x, k_T; \mu_Q, Q^2) \approx f_{j/p}(x; \mu_Q)$
Very badly violated at moderate scales

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q\,\mathrm{d}^2\mathbf{q}_T\,\mathrm{d}x\,\mathrm{d}z} = H(Q/\mu_Q)\underbrace{\int \mathrm{d}^2\mathbf{k}_{1T}\mathrm{d}^2\mathbf{k}_{2T} f_{j/p}(x,\mathbf{k}_{1T};\mu_Q,Q^2) D_{h/j}(z,z\mathbf{k}_{2T};\mu_Q,Q^2)}_{\text{Well-defined operators for all TM}}\delta^{(2)}(\mathbf{q}_T+\mathbf{k}_{1T}-\mathbf{k}_{2T}) + \text{power suppressed}$$

# Diagnosis

1) Consistency tests will generally fail for a g-function ansatz unless constraints are imposed

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q\,\mathrm{d}^2\mathbf{q}_T\,\mathrm{d}x\,\mathrm{d}z} =$$

$$H(Q/\mu_Q)\int \frac{\mathrm{d}^2\boldsymbol{b}_{\mathrm{T}}}{(2\pi)^2}\, e^{-i\boldsymbol{q}_{\mathrm{T}}\cdot\boldsymbol{b}_{\mathrm{T}}}\ \tilde{f}_{j/p}(x,\boldsymbol{b}_{\mathrm{T}};\mu_{Q_0},Q_0^2)\ \tilde{D}_{h/j}(z,\boldsymbol{b}_{\mathrm{T}};\mu_{Q_0},Q_0^2)$$

$$\times \exp\left\{\tilde{K}(b_{\mathrm{T}};\mu_{Q_0})\ln\left(\frac{Q^2}{Q_0^2}\right)+\int_{\mu_{Q_0}}^{\mu_Q}\frac{\mathrm{d}\mu'}{\mu'}\left[2\gamma(\alpha_s(\mu');1)-\ln\frac{Q^2}{\mu'^2}\gamma_K(\alpha_s(\mu'))\right]\right\}$$

2) Fixed order perturbation theory should work fine for $q_T \approx Q_0$, but evol. factors have a large effect. What is going on?

3) $\exists$ no region at input scale $Q = Q_0$ where $\Lambda_{QCD} \ll q_T \ll Q_0$

4) Backwards evolution…
   No large, perturbative $\ln\frac{Q_0}{q_T}$.

5) $\int \mathrm{d}^2\mathbf{k}_T f_{j/p}(x,k_T;\mu_Q,Q^2) \approx f_{j/p}(x;\mu_Q)$
   Very badly violated at moderate scales

$$\frac{d\sigma}{dQ\, d^2\mathbf{q}_T\, dx\, dz} = H(Q/\mu_Q) \underbrace{\int d^2\mathbf{k}_{1T} d^2\mathbf{k}_{2T} f_{j/p}(x, \mathbf{k}_{1T}; \mu_Q, Q^2) D_{h/j}(z, z\mathbf{k}_{2T}; \mu_Q, Q^2) \delta^{(2)}(\mathbf{q}_T + \mathbf{k}_{1T} - \mathbf{k}_{2T})}_{\text{Well-defined operators for all TM}} + \text{power suppressed}$$

## Diagnosis

1) Consistency tests will generally fail for a g-function ansatz unless constraints are imposed

2) Fixed order perturbation theory should work fine for $q_T \approx Q_0$, but evol. factors have a large effect. What is going on?

3) $\exists$ no region at input scale $Q = Q_0$ where $\Lambda_{QCD} \ll q_T \ll Q_0$

4) Backwards evolution…
No large, perturbative $\ln \frac{Q_0}{q_T}$.

5) $\int d^2\mathbf{k}_T f_{j/p}(x, k_T; \mu_Q, Q^2) \approx f_{j/p}(x; \mu_Q)$
Very badly violated at moderate scales

$$\frac{d\sigma}{dQ\, d^2\mathbf{q}_T\, dx\, dz} =$$

$$H(Q/\mu_Q) \int \frac{d^2\boldsymbol{b}_T}{(2\pi)^2} e^{-i\boldsymbol{q}_T\cdot\boldsymbol{b}_T} \tilde{f}_{j/p}(x, \boldsymbol{b}_T; \mu_{Q_0}, Q_0^2)\, \tilde{D}_{h/j}(z, \boldsymbol{b}_T; \mu_{Q_0}, Q_0^2)$$

$$\times \exp\left\{ \tilde{K}(b_T; \mu_{Q_0}) \ln\left(\frac{Q^2}{Q_0^2}\right) + \int_{\mu_{Q_0}}^{\mu_Q} \frac{d\mu'}{\mu'}\left[2\gamma(\alpha_s(\mu'); 1) - \ln\frac{Q^2}{\mu'^2}\gamma_K(\alpha_s(\mu'))\right]\right\}$$

$$\tilde{D}_{h/j}(z, \boldsymbol{b}_T; \mu_{Q_0}, Q_0^2) = \tilde{D}_{\text{inpt},h/j}(z, \boldsymbol{b}_T; \mu_{\overline{Q}_0}, \overline{Q}_0^2) E(\overline{Q}_0/Q_0, b_T)$$

$$\tilde{f}_{i/p}(x, \boldsymbol{b}_T; \mu_{Q_0}, Q_0^2) = \tilde{f}_{\text{inpt},i/p}(x, \boldsymbol{b}_T; \mu_{\overline{Q}_0}, \overline{Q}_0^2) E(\overline{Q}_0/Q_0, b_T)$$

$$\lim_{b_T\to 0} \overline{Q}_0 \sim 1/b_T$$

*Instead, characterize the full range of $k_T$ behavior of TMD functions at the input scale*

# A hadron structure oriented (HSO) reorganization

1) Use the uniquely determined TMDs for all $k_T$

2) Smoothly interpolate between nonperturbative TM dependence at small TM ($k_T \approx \Lambda_{QCD}$) & perturbative (collinear) TM at large TM ($k_T \approx Q$)

3) (Approximate) probability interpretation

- Parton model: $\int d^2\mathbf{k}_T f_{j/p}(x, k_T; \mu_Q, Q^2) = f_{j/p}(x; \mu_Q)$

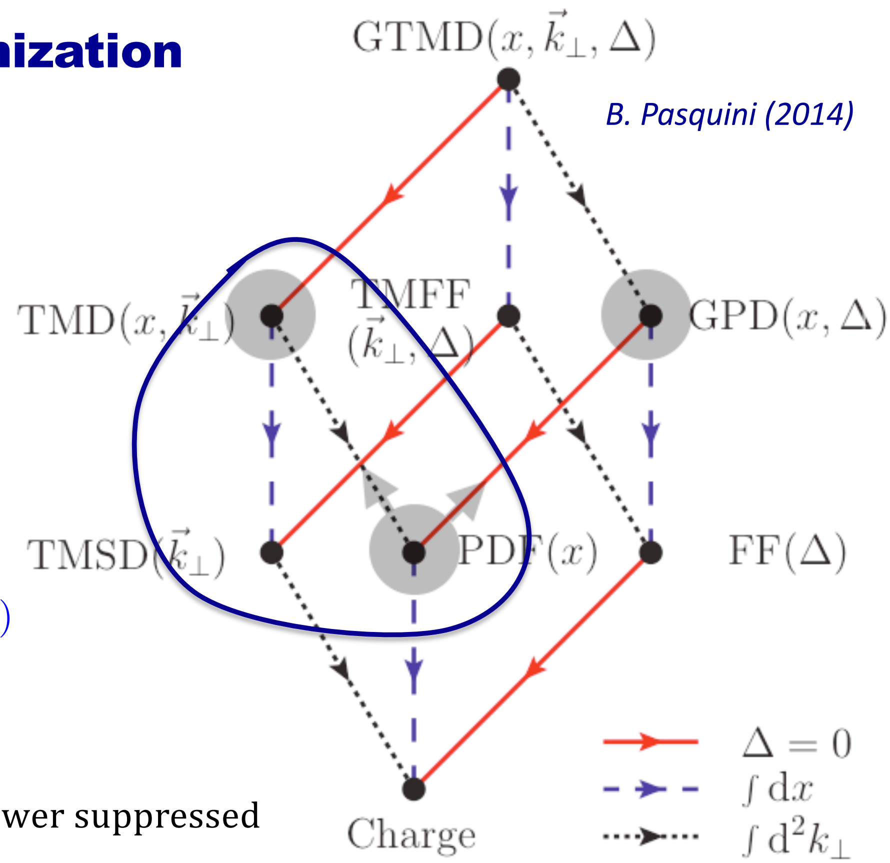
- QCD:

$$\pi \int^{\mu_Q^2} dk_T^2 f_{j/p}(x, k_T; \mu_Q, Q^2) = f_{j/p}(x; \mu_Q) + \Delta_{j/p} + \text{power suppressed}$$

*The "W-term"*

$\overline{\text{MS}}$

$\underbrace{\sum_{j'} \mathcal{C}^{\Delta}_{j/j'} \otimes f_{j'/p}}_{O(\alpha_s)}$



# A hadron structure oriented (HSO) reorganization

1) Use the uniquely determined TMDs for all $k_T$

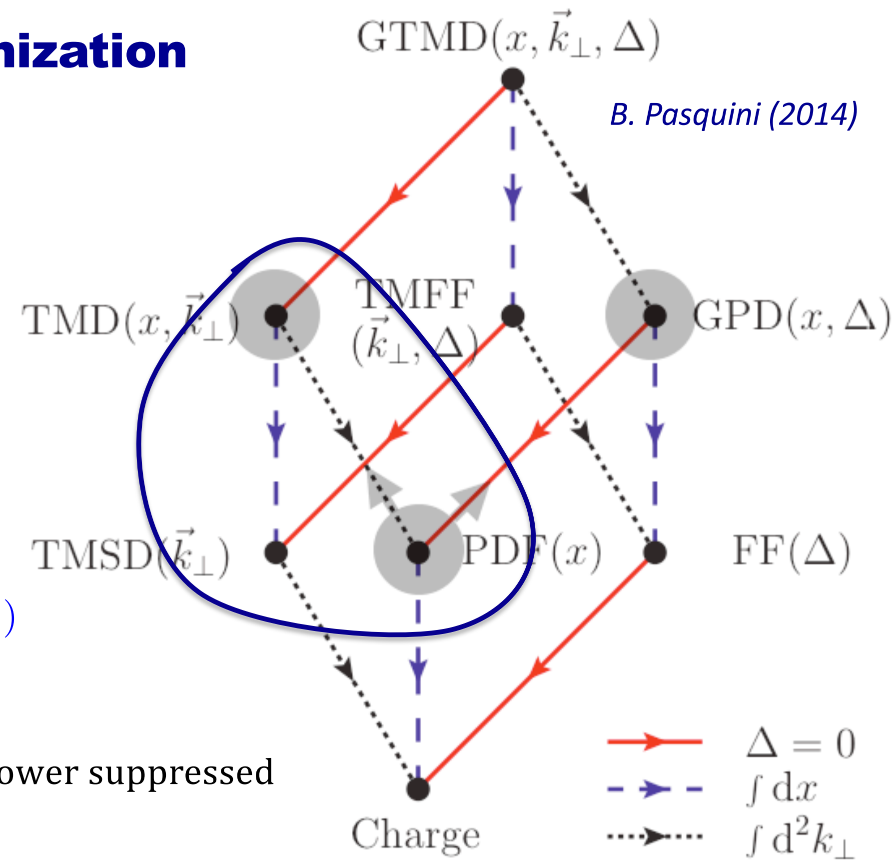
2) Smoothly interpolate between nonperturbative TM dependence at small TM ($k_T \approx \Lambda_{QCD}$) & perturbative (collinear) TM at large TM ($k_T \approx Q$)

3) (Approximate) probability interpretation

- Parton model: $\int d^2\mathbf{k}_T f_{j/p}(x, k_T; \mu_Q, Q^2) = f_{j/p}(x; \mu_Q)$
- QCD: $\pi \int^{\mu_Q^2} dk_T^2 f_{j/p}(x, k_T; \mu_Q, Q^2) = f_{j/p}(x; \mu_Q) + \Delta_{j/p}$ + power suppressed

4) All should apply at an input scale, $Q_0$

5) Pheno requirement: Must be simple to swap one model/parametrization for another while still satisfying 1-4



## A hadron structure oriented (HSO) reorganization

- Provides a recipe to transform a NP TMD parametrization into an evolved parametrization at other scales:
  - *Sec. VI of Phys.Rev.D 106 (2022) 3, 034002*

*Called "bottom up" approach here*

- No $b_{max}$ or $b_*$ necessary

- HSO approach is equivalent to standard TMD factorization, CSS, etc, just with additional consistency constraints on the g-functions

- It is straightforward to translate between standard treatment and HSO

  - *Sec. IX of Phys.Rev.D 106 (2022) 3, 034002*

## An $O(\alpha_s)$ example with $\overline{\text{MS}}$ pdfs and ffs

- Parametrizing the very small transverse momentum

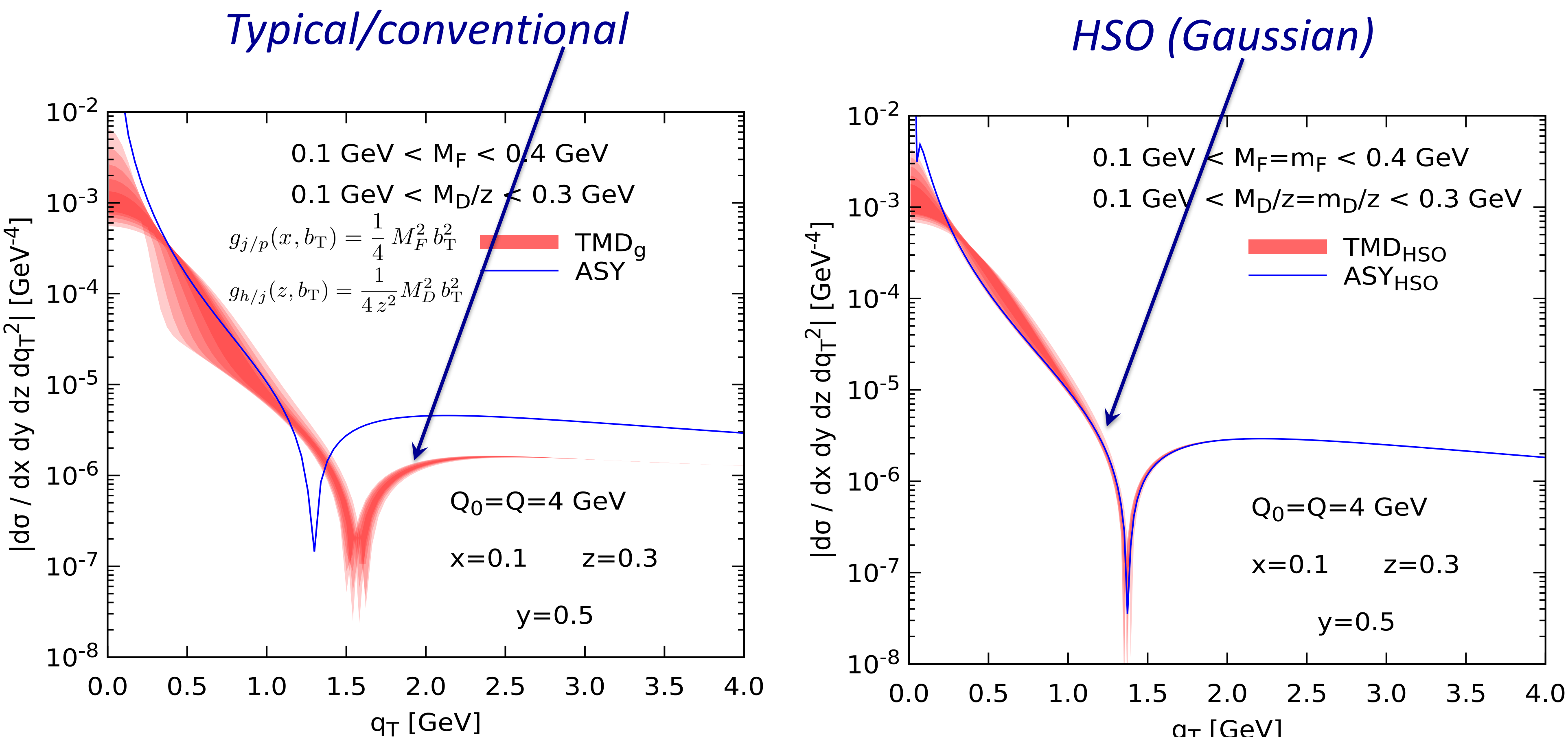
A. Gaussian model (very commonly used)

$$f^{\text{Gauss}}_{\text{core},i/p}(x,\boldsymbol{k}_{\text{T}};Q_0^2) = \frac{e^{-k_{\text{T}}^2/M_{\text{F}}^2}}{\pi M_{\text{F}}^2}\,, \qquad D^{\text{Gauss}}_{\text{core},h/j}(z,z\boldsymbol{k}_{\text{T}};Q_0^2) = \frac{e^{-z^2\,k_{\text{T}}^2/M_{\text{D}}^2}}{\pi M_{\text{D}}^2}$$

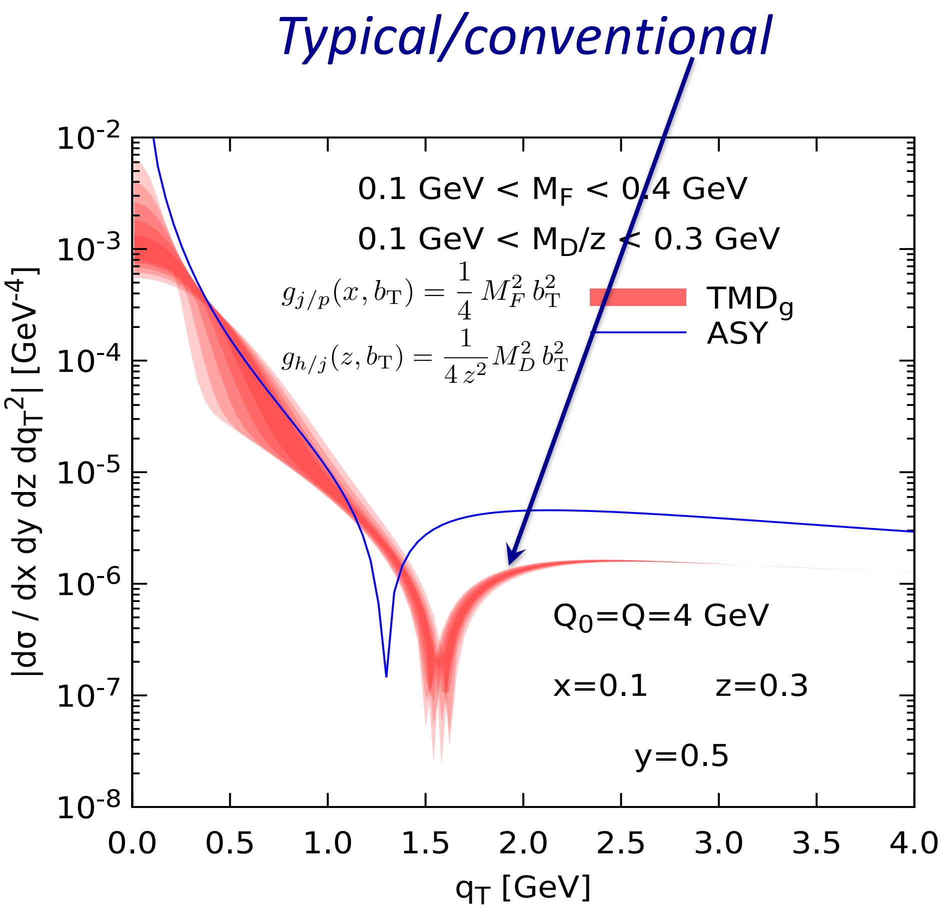
B. Spectator model

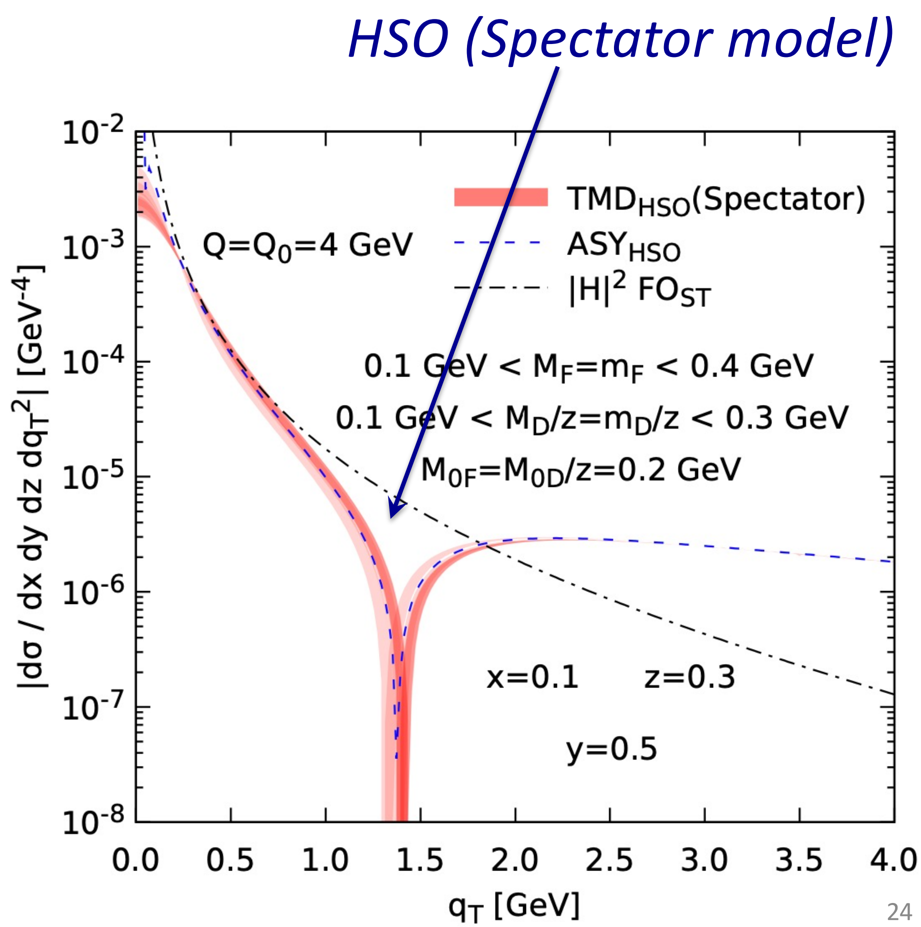
$$f^{\text{Spect}}_{\text{core},i/p}(x,\boldsymbol{k}_{\text{T}};Q_0^2) = \frac{6M_{0\text{F}}^6}{\pi\,(2M_{\text{F}}^2+M_{0\text{F}}^2)}\,\frac{M_{\text{F}}^2+k_{\text{T}}^2}{(M_{0\text{F}}^2+k_{\text{T}}^2)^{\,4}}\,, \quad D^{\text{Spect}}_{\text{core},h/j}(z,z\boldsymbol{k}_{\text{T}};Q_0^2) = \frac{2M_{0\text{D}}^4}{\pi\,(M_{\text{D}}^2+M_{0\text{D}}^2)}\,\frac{M_{\text{D}}^2+k_{\text{T}}^2 z^2}{(M_{0\text{D}}^2+k_{\text{T}}^2 z^2)^{\,3}}$$

# Compare standard/unconstrained with HSO ($O(\alpha_s)$)

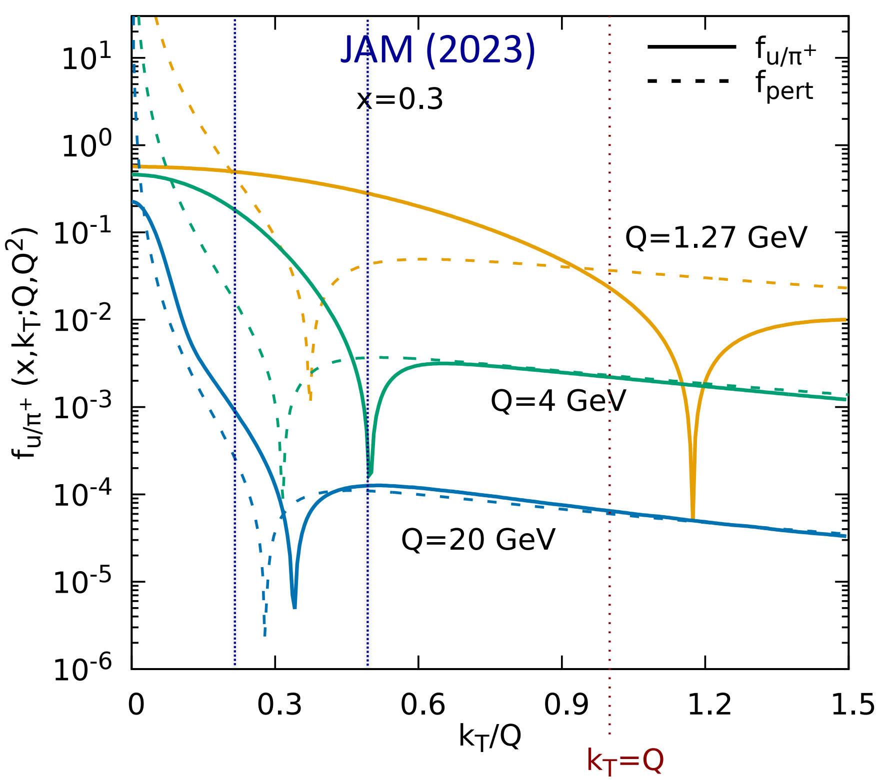


# Compare standard/unconstrained with HSO ($O(\alpha_s)$)

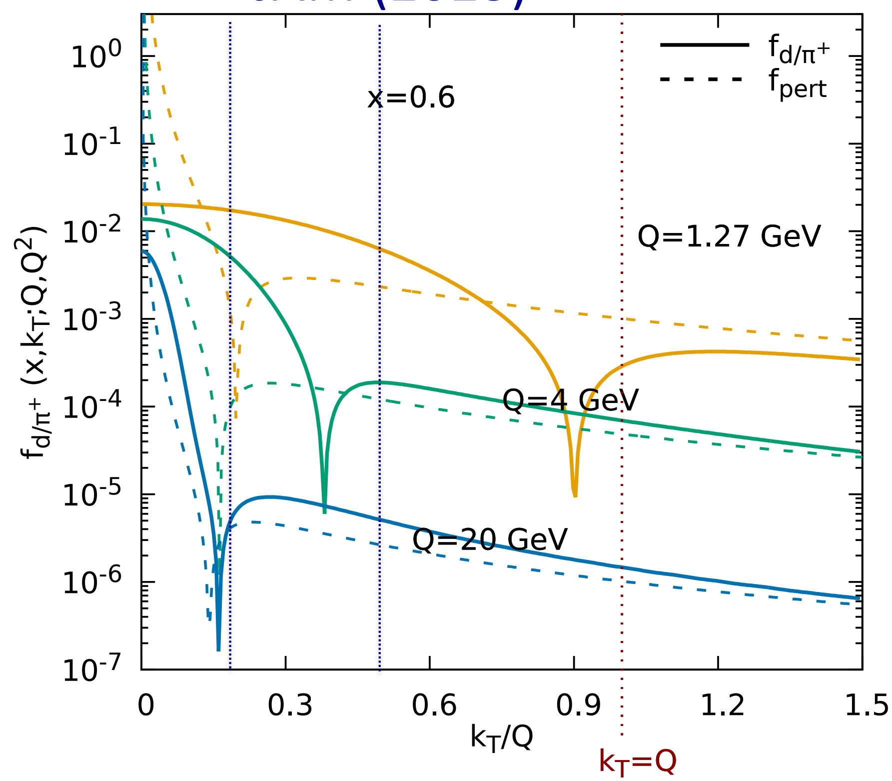


# What about the individual TMD pdfs?




JAM (2023) 2302.01192 [hep-ph]



## Summary

- Switching to a hadron-structure-oriented approach to pheno with TMD factorization improves consistency in the large transverse behavior of TMD correlation functions

- Necessary for understanding the shapes of nonperturbative distributions, separating perturbative and nonpeturbative parts, etc

- Necessary for transforming calculations of nonperturbative TM physics into testable/falsifiable predictions

- HSO is not a new formalism; **HSO = "standard CSS"!**

- Next:
  - Applications
  - Higher orders
  - Incorporating NP calculations (lattice, EFTs, models etc)
  - Spin dependent TMDs