

# The ${}^6\text{He}$ $\beta$ -decay ( ${}^6\text{He} \rightarrow {}^6\text{Li} + e^- + \bar{\nu}$ )

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- $\beta$ -decay rates of large nuclei smaller than free neutron [1]  
 $g_A$  quenching
- Two-body currents contribution can explain this [2]  
(Studies based on NCSM and CC methods)
- QMC calculations show a different sign for the two-body currents [3]
- They both use  $\chi$ EFT (see next slides)

## Tensions between the two calculations

[1] I.S. Towner, Phys. Rep. **155**, 263 (1985)

[2] P. Gysbers, *et al.* Nature Phys. **15**, 428 (2019)

[3] G.B. King, *et al.* Phys. Rev. C **102**, 025501 (2020)

$$T_{fi} = \frac{G_F^2}{\pi^2} F(p_e, p_\nu, x_{e\nu}) M_{fi}$$

- $G_F$  Fermi constant
- $F(p_e, p_\nu, x_{e\nu})$  phase space factor
- $M_{fi} = \frac{1}{2J_i+1} \sum_{M_i, M_f} | \langle f | J_5 | i \rangle |^2 \Rightarrow$  Nuclear matrix element

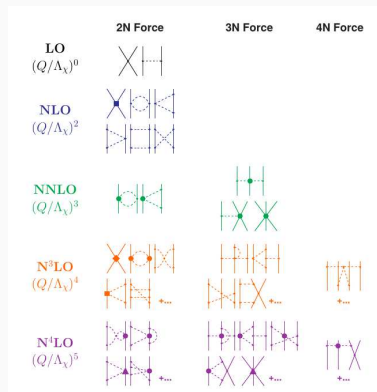
$$H = \sum_i \frac{p_i^2}{2M} + \sum_{i<j} V(i,j) + \sum_{i<j<k} W(i,j,k) + \dots$$

- Nuclear model
  - chiral effective field theory  $\chi$ EFT
- Search for accurate solution of  $H\Psi = E\Psi$ 
  - Variational approach
  - Expansion of  $\Psi$  on the basis of **Hyperspherical Harmonic (HH)** functions  
[\[L.E. Marcucci, \*et al.\*, \*Front. Phys.\* \*\*8\*\*, 69 \(2020\)\]](#)

# Chiral interaction ( $\chi$ EFT)

$$\text{QCD} \xrightarrow{\text{chiral symmetry}} \chi\text{EFT}$$

- Low Energy Theory ( $\Lambda_\chi \sim 1 \text{ GeV}$ )
  - $N, \pi$  as d.o.f.
  - high energy d.o.f. integrated out  $\rightarrow$   
**Low Energy Constants**
- Perturbative expansion ( $\propto (Q/\Lambda_\chi)^\nu$ )
- Non-relativistic expansion  $\Rightarrow$  Nuclear potential
- **Regularization with a cutoff**  
( $\Lambda = 400 - 600 \text{ MeV}$ )
- LECs fitted to the experimental data



D.R. Entem, *et al.* Phys. Rev. C **96**, 024004 (2017)

E. Epelbaum, *et al.* Phys. Rev. Lett. **115**, 122301

(2015)

# The HH wave function

- Jacobi vectors  $\vec{\xi}_1, \dots, \vec{\xi}_N \Rightarrow$  CoM completely decoupled
- Hyperangular variables  $\rho = \sum_{k=1}^5 (\xi_k)^2$ ,  $\Omega = \{\hat{\xi}_i, \phi_i\}$ ,  $\cos \phi_k = \frac{\xi_k}{\sqrt{\xi_1^2 + \dots + \xi_k^2}}$

$$T = -\frac{\hbar^2}{m} \left( \frac{\partial^2}{\partial \rho^2} + \frac{D-1}{\rho} \frac{\partial}{\partial \rho} - \frac{L^2(\Omega)}{\rho^2} \right)$$

- Expansion on a base  $\Rightarrow$  **Hyperspherical Harmonics (HH)**

$$L^2(\Omega) \mathcal{Y}_{[K]}(\Omega) = K(K+13) \mathcal{Y}_{[K]}(\Omega)$$

- The variational wave function

$$\psi_A = \sum_{l, [K]} A_{l, [K]} f_l(\rho) \mathcal{Y}_{[K]}(\Omega_{A-1}) [\chi_S \otimes \chi_T] = \sum_{[\alpha]} A_{[\alpha]} \Phi_{[\alpha]},$$

- **Check convergence on  $K$**

# The HH wave function

- Sum over the permutations  $\Rightarrow$  antisymmetrization
- Transformation Coefficients

$$\sum_{perm} \mathcal{Y}_{[\alpha]}^{KLSTJ}(\Omega_{perm}) = \sum_{[\alpha']} a_{[\alpha],[\alpha']}^{KLSTJ} \mathcal{Y}_{[\alpha']}^{KLSTJ}(\Omega)$$

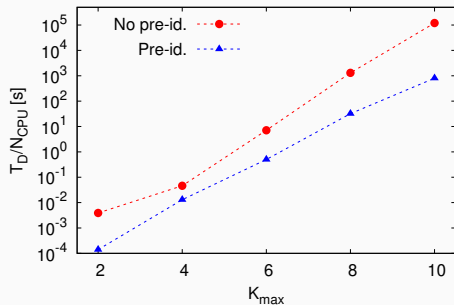
- Evaluation of the matrix elements

$$H_{\alpha,\beta} = \langle \Phi_{[\alpha]} | H | \Phi_{[\beta]} \rangle$$

- Kinetic energy immediate
- Nuclear potential

$$\langle \Phi_{[\alpha]} | \sum_{i < j} v(i,j) | \Phi_{[\beta]} \rangle = \frac{A(A-1)}{2} \sum_{[\alpha'], [\beta']} a_{[\alpha],[\alpha']} a_{[\beta],[\beta']} V_{[\alpha'], [\beta']}(1,2)$$

$\Rightarrow$  By using the sum over the permutations and the TC

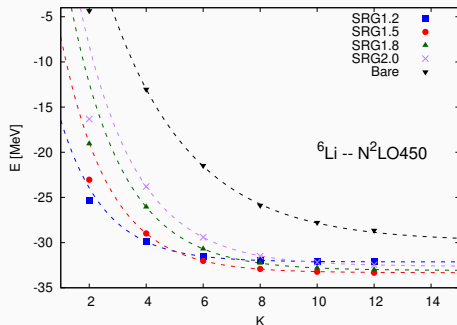


- $T_D$  = time for computing all the  $\langle \Phi_\alpha | \sum_{i < j} V(i, j) | \Phi_\beta \rangle$  with  $K(\alpha) = K(\beta)$
- Exponential behavior
- Blue use a new algorithm

[A.G., L.E. Marcucci and M. Viviani, Phys. Rev. C **102**, 014001 (2020)]



# Convergence studies



	${}^6\text{Li}$	${}^6\text{He}$
SRG1.2	-32.13(2)	-28.87(3)
SRG1.5	-33.33(3)	-30.17(4)
SRG1.8	-33.07(1)	-30.08(2)
SRG2.0	-32.62(7)	-29.73(2)
Bare	-29.8(4)	-27.6(3)
Exp.	-31.99	-29.27

Note that it is a rough estimate!

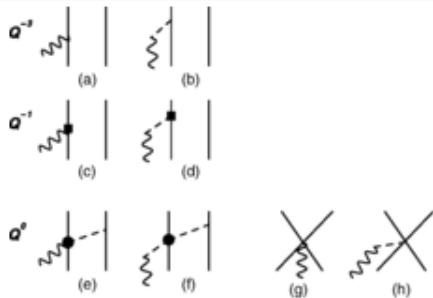
	$B_{full}[2]$	$B(\infty)[2]$	Ref. [1]
N3LO500-SRG1.2 ( ${}^6\text{Li}$ )	31.78	31.81(1)	31.85(5)
N3LO500-SRG1.5 ( ${}^6\text{Li}$ )	32.79	32.91(2)	33.00(5)
N3LO500-SRG1.8 ( ${}^6\text{Li}$ )	32.25	32.68(9)	32.8(1)
N2LO450-bare ( ${}^6\text{Li}$ )	28.51	30.2(3)	
N2LO450-bare ( ${}^6\text{He}$ )	26.07	27.5(4)	

- All the energies are in MeV
- Results of Ref. [1] from No Core Shell Model
- Experimental value  $B_{6\text{Li}} = 31.99$  MeV and  $B_{6\text{He}} = -29.27$

[1] E.D Jungerson, P. Navrátil and R.J. Furnstahl, PRC **83**, 034301 (2011)

[2] A.G., L.E. Marcucci and M. Viviani, Phys. Rev. C **102**, 014001 (2020)

# Nuclear axial currents ( $J_5$ )



A. Baroni, *et al.*, Phys. Rev. C **93**, 015501 (2016)

- $Q^{-3}$  Gamow-Teller (GT)  $\Rightarrow$   
 $g_A \approx 1.27$
- $Q^{-1}$  Relativistic Corrections (RC)
- $Q^0$  One pion exchange (OPE)  $\Rightarrow$   
 $c_3, c_4$  from Nuclear Potential
- $Q^0$  Contact Term (CT)  $\Rightarrow$   
 $Z_0(c_D, c_3, c_4)$   
Fitted on the  ${}^3\text{H} \rightarrow {}^3\text{He} + e^- + \bar{\nu}$

## Preliminary results

$$\text{RME} = \frac{\sqrt{2J_f + 1}}{g_A} \frac{\langle J_f M | J_5 | J_i M \rangle}{\langle J_i M, 10 | J_f M \rangle}$$

	LO(GT)	NLO(RC)	N2LO(OPE)	N2LO(CT)	Tot.
SRG1.2	2.342	-0.020	-0.040	-0.039	2.244
SRG1.5	2.335	-0.022	-0.028	-0.044	2.242
SRG1.8	2.320	-0.022	-0.018	-0.047	2.234
SRG2.0	2.321	-0.023	-0.012	-0.047	2.239
bare	2.299	-0.022	0.003	-0.049	2.231
Exp.					2.1609(40)

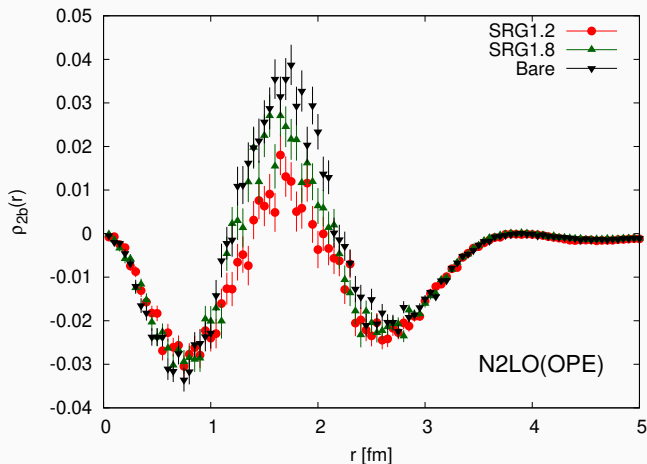
- $\text{RME}(K = 12) \simeq \text{RME}(\infty)$
- Preliminary results consistent with [1]
- Sign of CT is the same as QMC calculations [2]
- Large variability on OPE contribution

[1] P. Gysbers, *et al.* Nature Phys. **15**, 428 (2019)

[2] G.B. King, *et al.* Phys. Rev. C **102**, 025501 (2020)

## Two-body density

$$\rho_{2b}(r) \propto \langle J_f M | \sum_{i < j} \delta(r - r_{ij}) J_5(ij) | J_i M \rangle \quad \text{RME}(2b) = \int_0^\infty \rho_{2b}(r) dr$$



- Overview of the **HH method**
  - Inclusion of three-body forces
  - Scattering states
  
- **Study of electro-weak interactions in light nuclei**
  - Beta-decay
  - Muon capture reaction
  - Electromagnetic transition in  $A = 6$  nuclei
  
- **Study of electro-weak interactions in medium-mass nuclei**  
AFDMC (in coll. with A. Lovato)

The logo for Jefferson Lab features the text "Jefferson Lab" in a bold, black, sans-serif font. A red, stylized orbital path with a small red sphere at its end curves around the word "Jefferson".

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# Cluster form factor $\alpha + d$

$$\frac{f_L(r)}{r} = \langle \Psi_{\alpha+d}^{(L)} | \Psi_{6\text{Li}} \rangle, \quad |\Psi_{\alpha+d}^{(L)}\rangle = [(\Psi_\alpha \otimes \Psi_d)_S Y_L(\hat{r})]_J$$

with  $S = 1$ ,  $J = 1$  and  $L = 0$  ( ${}^3S_1$ )

