Mapping Hadronic Structure at Small x

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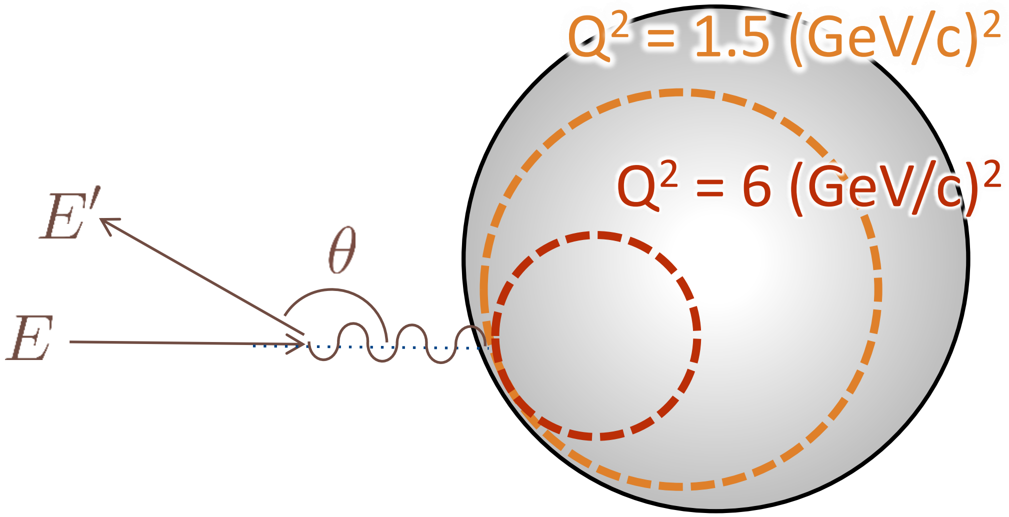
What Do We Learn From Small x?

- In DIS, Bjorken x measures the **Ioffe "exposure time"** of the virtual photon

  Ioffe, Phys. Lett. ***B****30 (1969) 123*

- The **small-x limit** is equivalent to the **high-energy limit** :

- **Time dilation** at higher energies can reveal **more ephemeral** quantum fluctuations



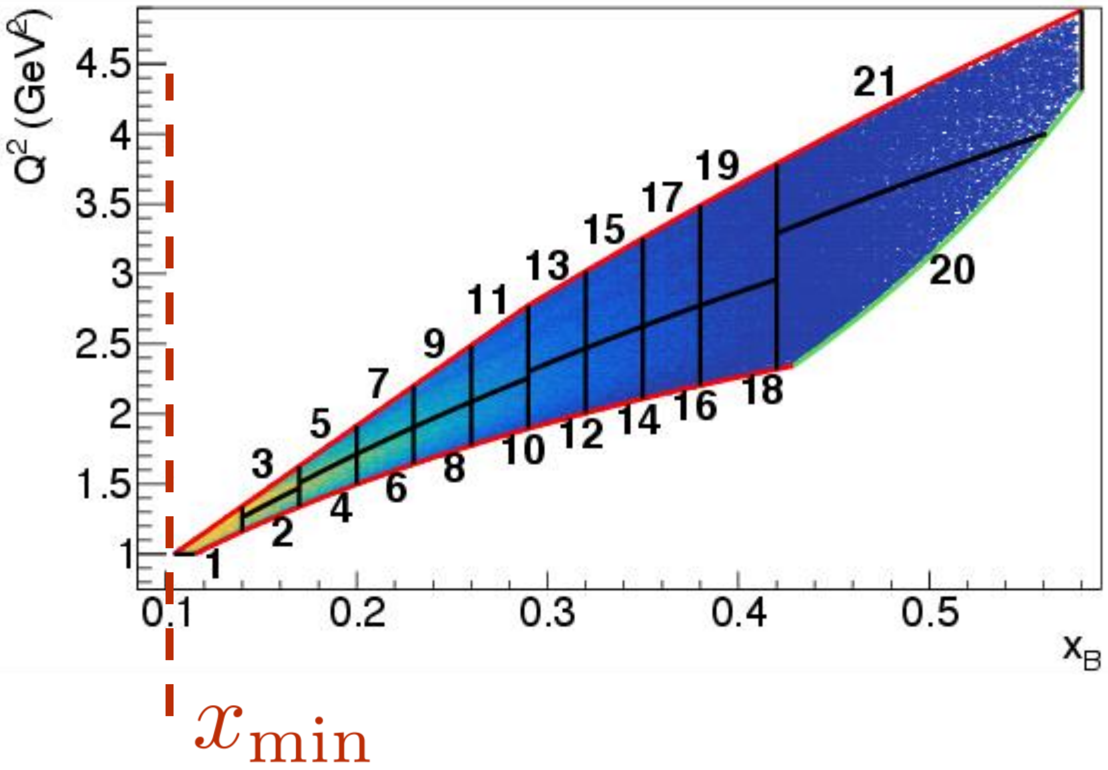
"Exposure time"

$$x = \frac{Q^2}{s + Q^2} \qquad \Delta t \sim \frac{1}{mx}$$

The Limits of Finite Energy

- Any **finite-energy experiment** is limited to a **minimum value of x**
- There are always **small-x tails** of structure functions which are **inaccessible to experiment**

CLAS Collaboration, Phys. Rev. C98 (2018) 045203



What Could be Hiding at Small x?

- **Hidden contributions** to the **proton spin budget**

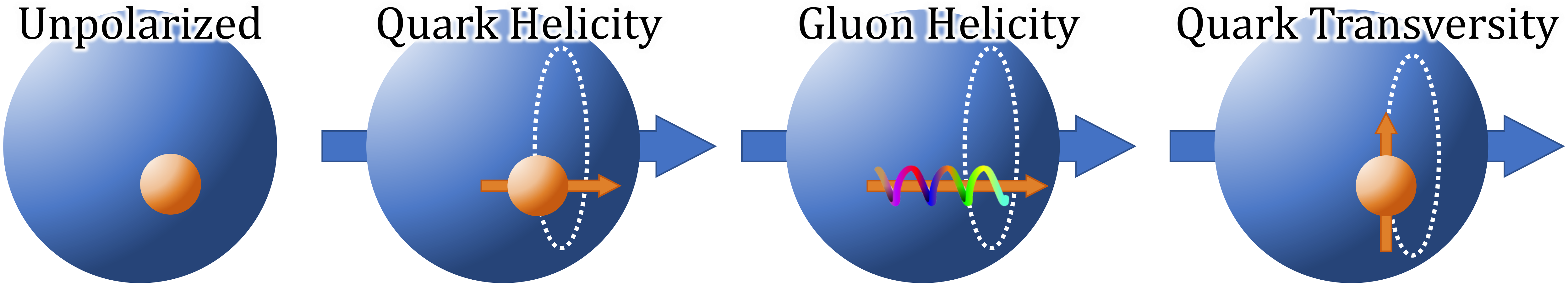
- **Unitarization** of QCD into a **UV complete** theory

- Exotic **gluon-dominated phase** of nuclear matter

- **Bridge** between local operators calculated in **lattice QCD** and nonlocal structure functions measured in **experiment**

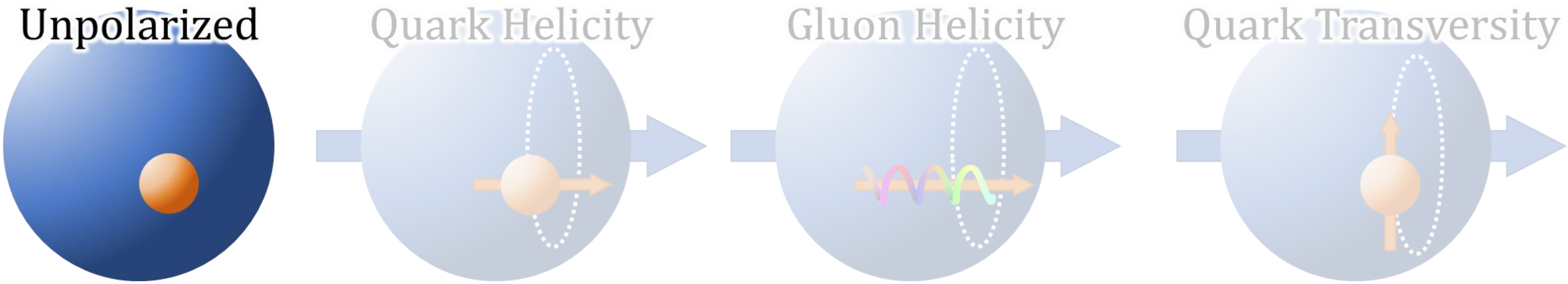
x_{min}

Outline: Small-x Asymptotics

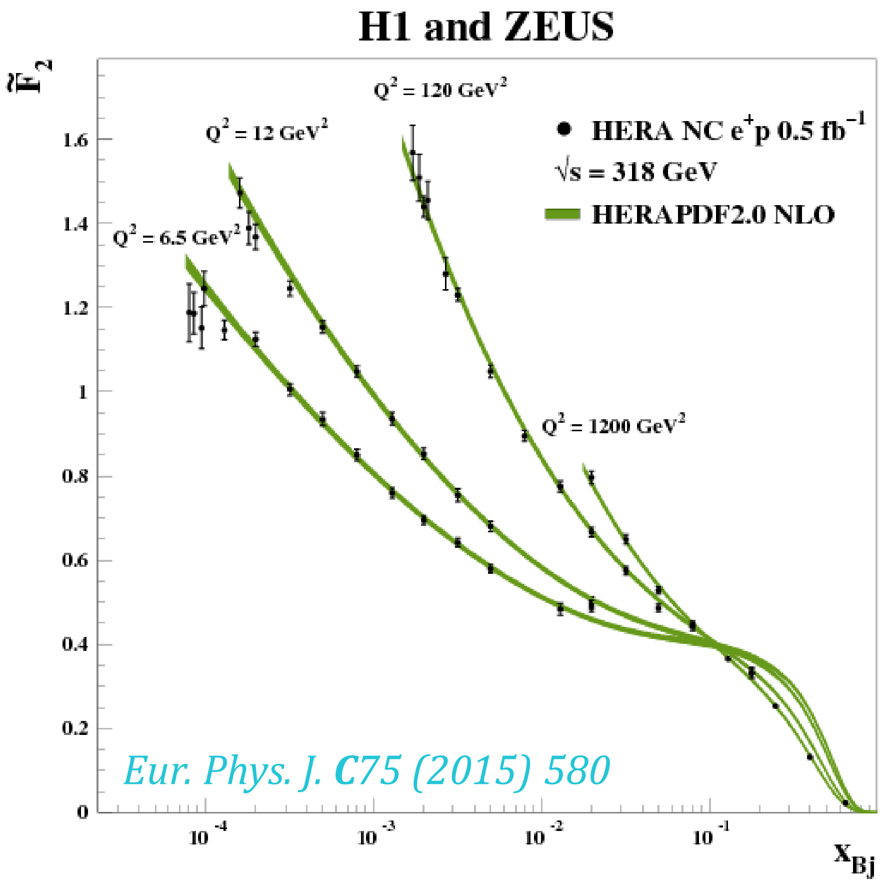


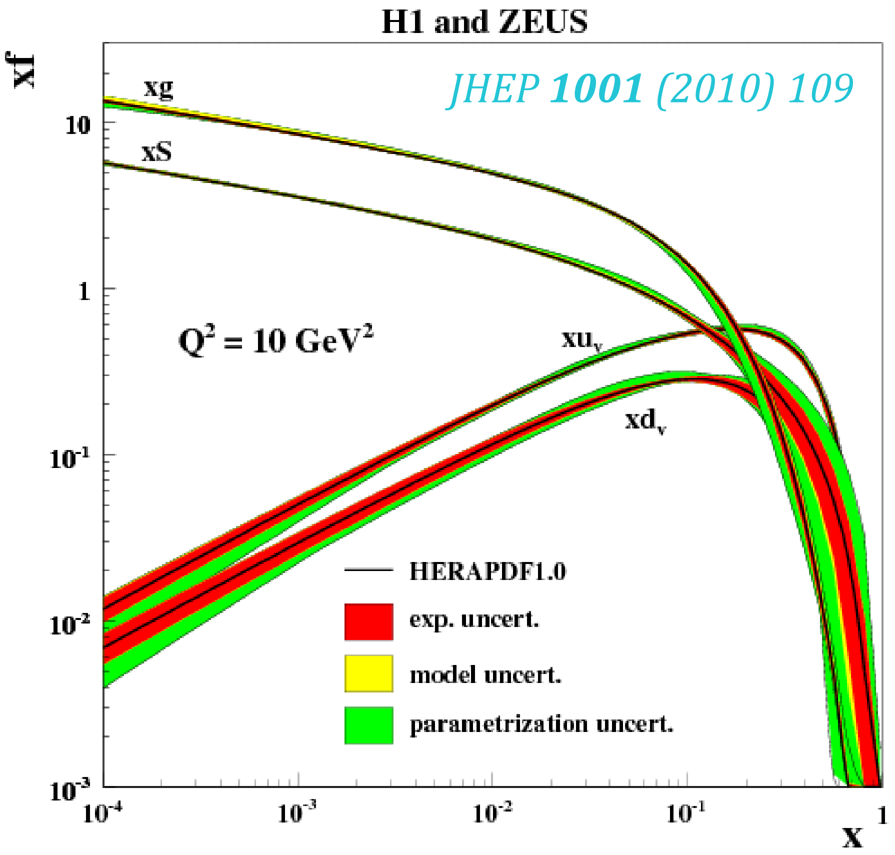
1. Unpolarized Quarks and Gluons
2. Quark Helicity
3. Gluon Helicity
4. Quark Transversity

Unpolarized Quarks and Gluons: The Conceptual Part



Unpolarized Structure Functions from HERA





- At HERA, the proton structure functions **increase strongly at small x**
- Reflects a **power-law growth** of **gluon** and **sea quark densities**

Non-Abelian Bremsstrahlung at High Energies

- At high energies, **QCD radiates soft gluons uniformly** around **mid-rapidity**

➢ Intrinsic feature of **non-Abelian** field theories

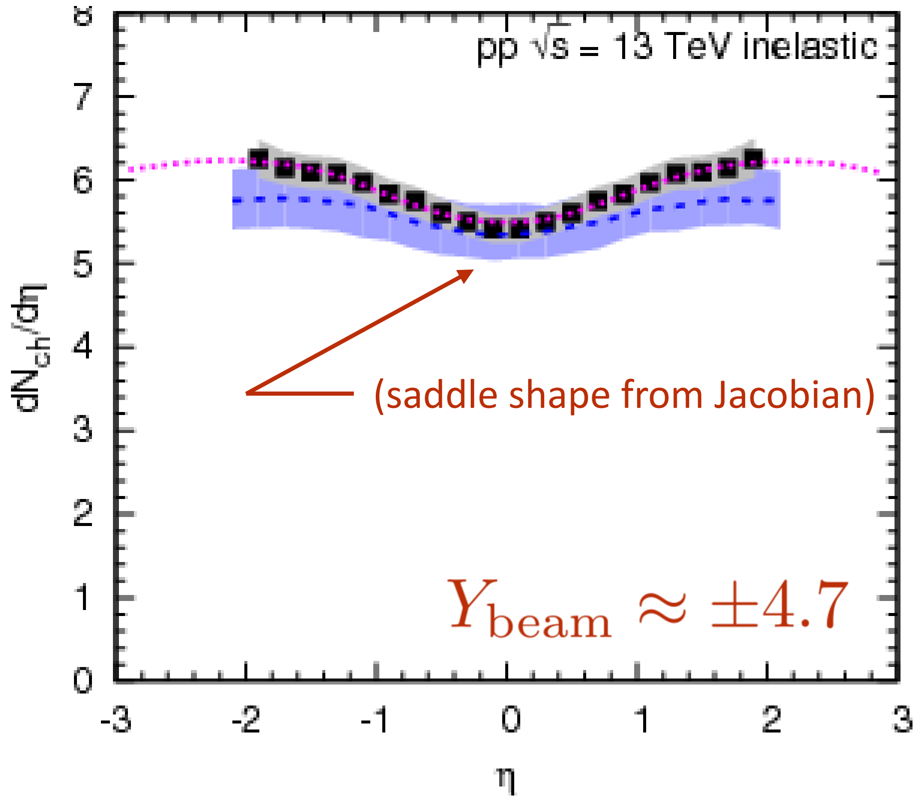
$$\frac{d\sigma^G}{dy} = \alpha_s \times (\text{const})$$

$$\frac{\sigma_{tot}^{(pp+G)}}{\sigma_{tot}^{(pp)}} \propto \alpha_s \, \Delta Y_{tot}$$

(small coupling)

(large phase space)

CMS Collaboration, Phys. Lett. ***B****751 (2015) 143*

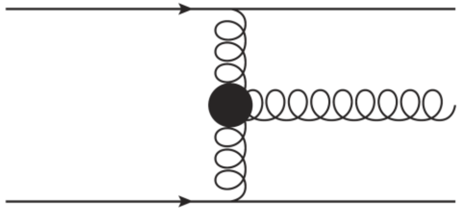


$$\Delta Y \sim \ln \frac{s}{\Lambda^2} \sim \ln \frac{1}{x}$$

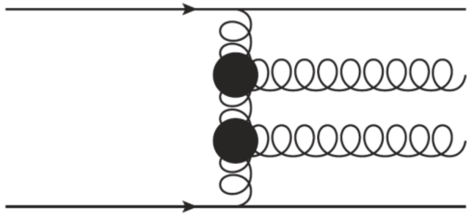
A Large Phase Space for Soft Gluons

- Perturbation theory in pQCD relies on a **hierarchy of contributions**

$$\underset{\text{LO}}{\alpha_s} \gg \underset{\text{NLO}}{\alpha_s^2} \gg \cdots$$



$\gg$



$\gg \cdots$

$$\langle N_G \rangle_{LO} \sim \alpha_s \, \Delta Y \qquad \langle N_G \rangle_{NLO} \sim (\alpha_s \, \Delta Y)^2$$

$$\langle N_g \rangle \ll 1$$

A Large Phase Space for Soft Gluons

- Perturbation theory in pQCD relies on a **hierarchy of contributions**

LO NLO

$$(\alpha_s \Delta Y) \sim (\alpha_s \Delta Y)^2 \sim \cdots$$

$\langle N_G \rangle_{LO} \sim \alpha_s \, \Delta Y$ $\qquad$ $\langle N_G \rangle_{NLO} \sim (\alpha_s \, \Delta Y)^2$

$$\langle N_g \rangle \gtrsim 1$$

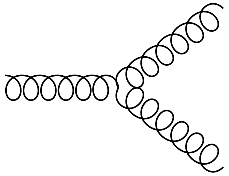
$$\Delta Y \sim \ln \frac{1}{x}$$

- At high energies (small x), the **large logarithmic phase space enhances** the probability of **soft gluon radiation**

$$\alpha_s \ln \frac{1}{x} \sim \mathcal{O}(1)$$

- Recast the systematic enhancement as a **differential equation**

$$\frac{d\langle N_G\rangle}{dY} = \langle N_G\rangle \times \left[\quad \right]$$

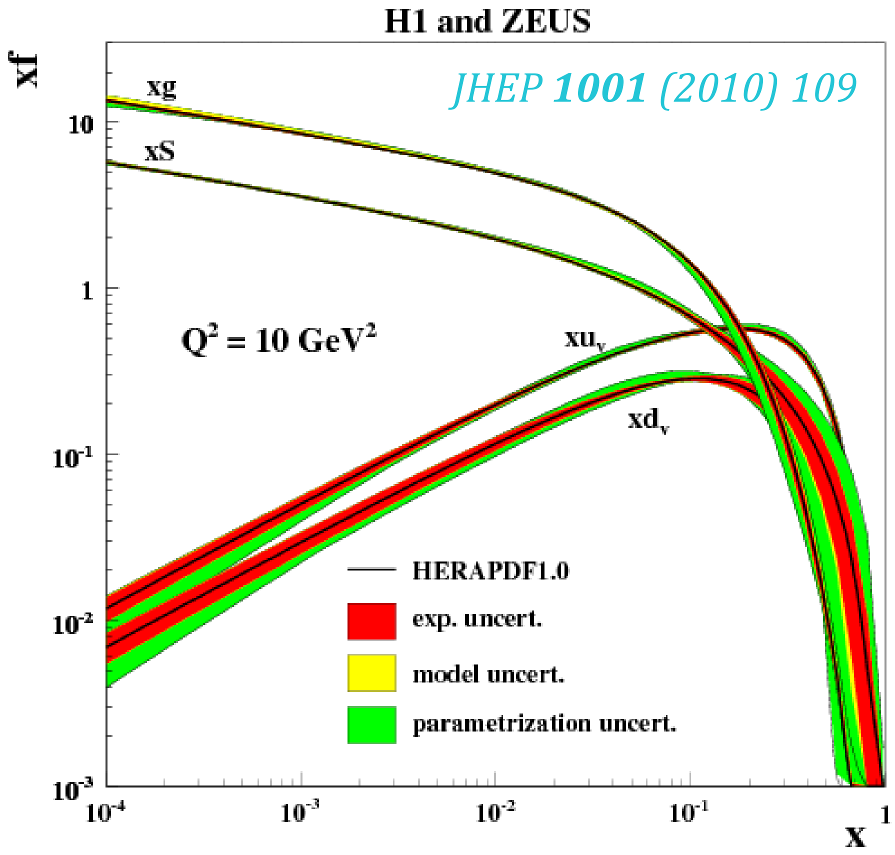


➢ **Power-law growth** of the **gluon density** at small x

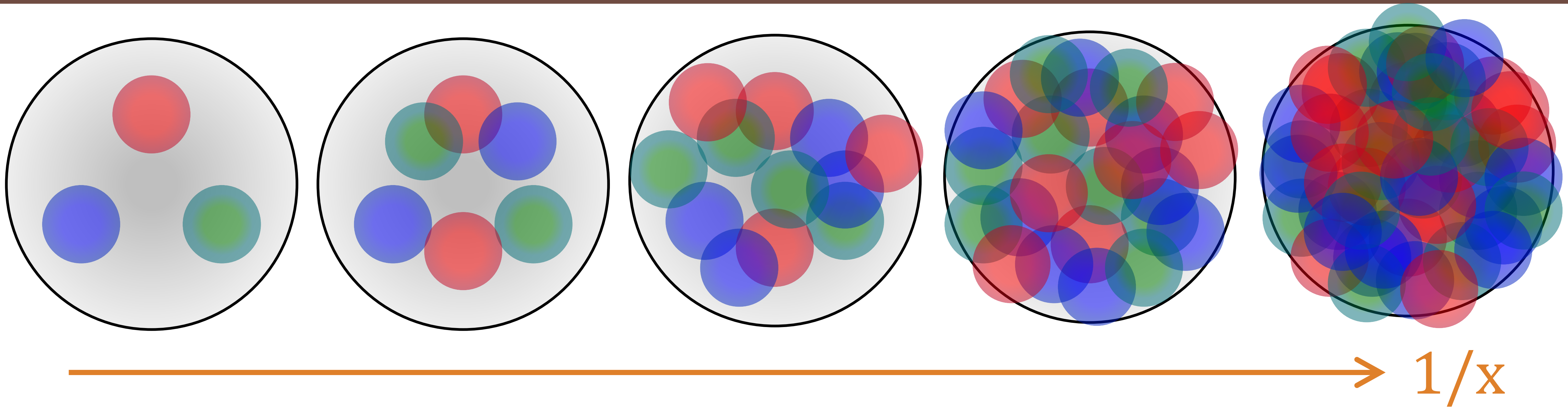
*Kuraev, Lipatov, and Fadin, Sov. Phys. JETP **45** (1977) 199*
*Balitsky and Lipatov, Sov. J. Nucl. Phys. **28** (1978) 822*

$$\langle N_G\rangle \sim \left(\frac{1}{x}\right)^{(2.65\,\alpha_s)}$$

"Pomeron Intercept"



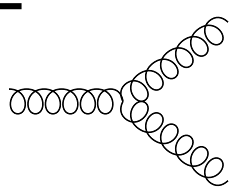
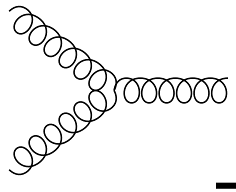
Color Screening at High Densities



- Emission of **many gluons** with **random colors**: **dynamical color screening**
- The size of **coherent color domains shrinks** as we approach **small x**
- The **growing color charge density** defines an **emergent length scale**

- At **high enough densities**, gluon **recombination** competes with bremsstrahlung
  - **Saturation** of the gluon density

L. V. Gribov, E. M. Levin, and M. G. Ryskin, Phys. Rept. ***100*** *(1983) 1*
A. H. Mueller and J. W. Qiu, Nucl. Phys. ***B268*** *(1986) 427*

$$\frac{d\langle N_G\rangle}{dY} = \langle N_G\rangle \times \Big[$$

$$\Big] - \langle N_G\rangle^2 \Big[$$

$$\Big]$$

$$\frac{d\rho_G}{dY\,dQ^2} = \left(\frac{\alpha_s N_c}{\pi}\right)\frac{\rho_G}{Q^2} - \left(\frac{\alpha_s^2 N_c \pi}{2C_F}\right)\left(\frac{\rho_G}{Q^2}\right)^2$$

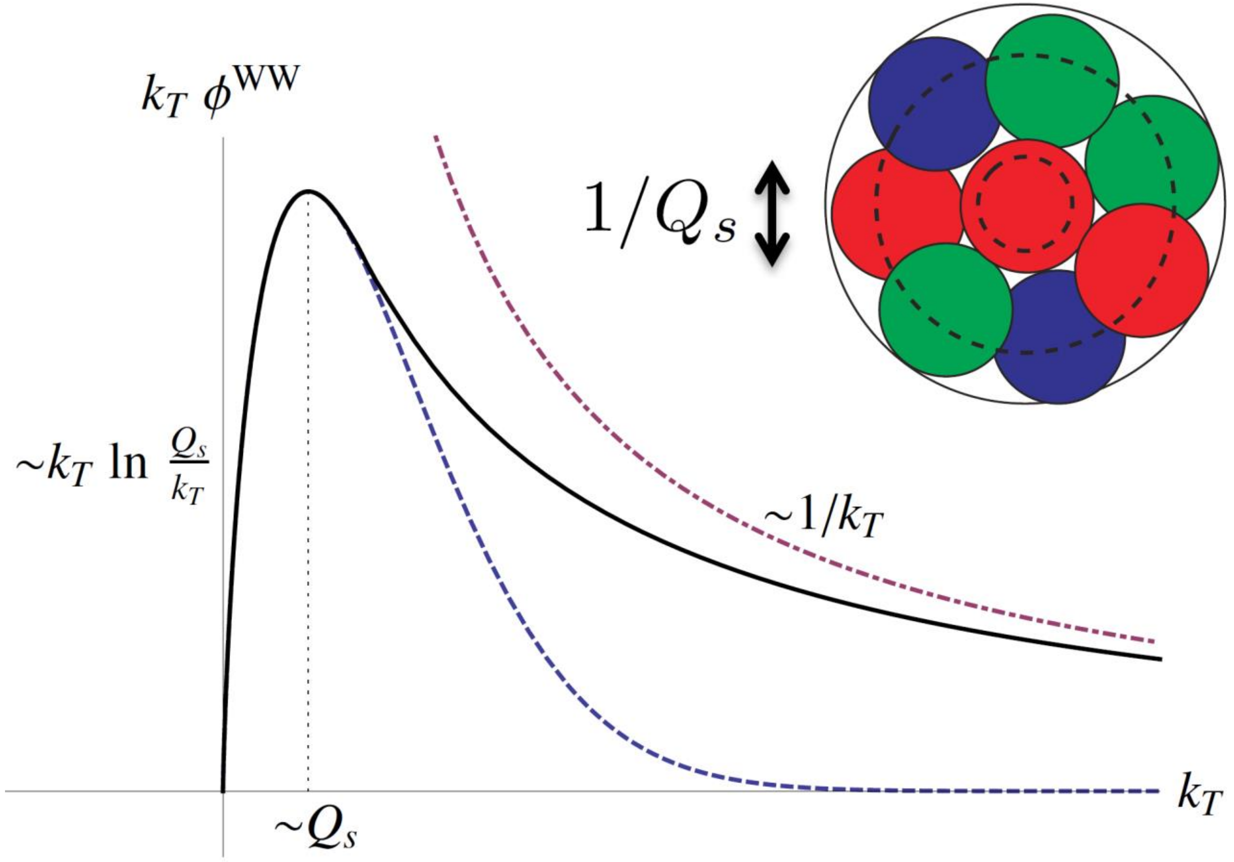
Equilibrium point: Q_s^2

- The **saturation momentum scale** **grows with the density**

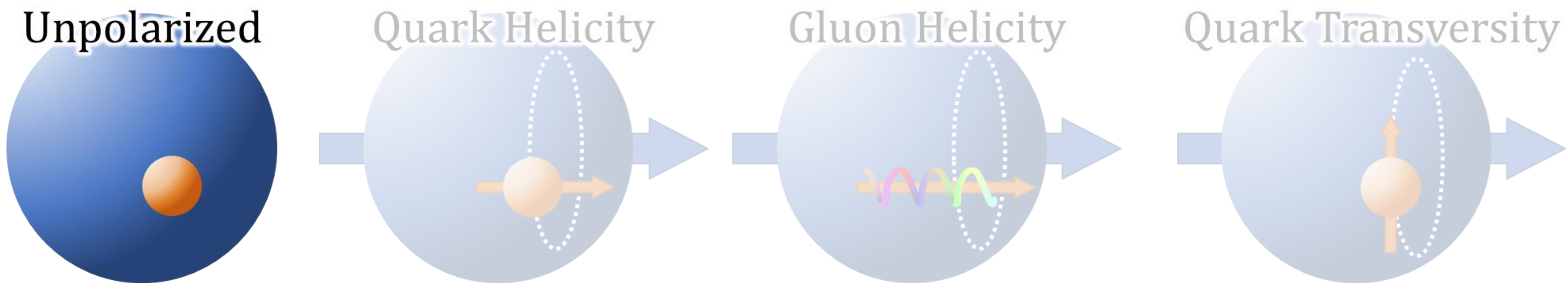
$$Q_s^2(Y) \sim \alpha_s\,\rho(Y)$$

The Perturbative High-Density Limit

- Parton transverse momentum distributions are **dynamically screened below Q_s**
- If the **density is large enough** that Q_s becomes a (semi)hard scale, the dynamics become **perturbative**
- With high energies and heavy nuclei, a future **Electron-Ion Collider** may peek into this regime.

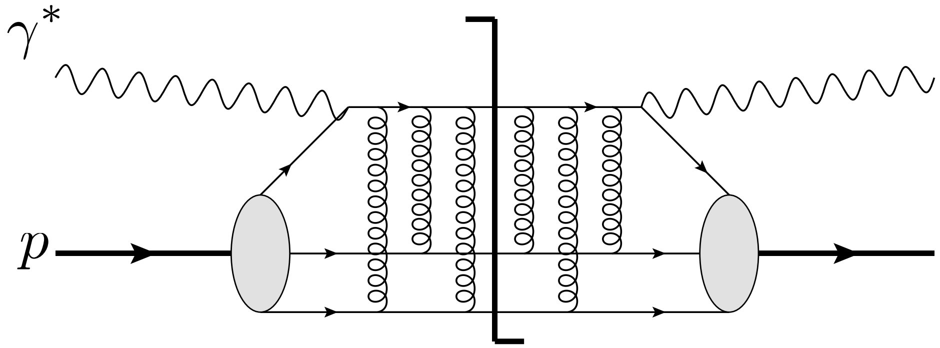


Unpolarized Quarks and Gluons: The Technical Part

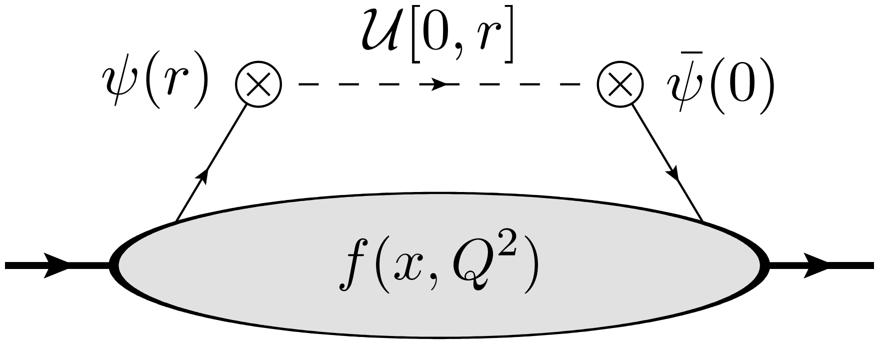


- Collinear (or TMD) **factorization** provides a one-to-one correspondence between the (SI)DIS **cross section** and hadronic structure: **PDFs / TMDs**

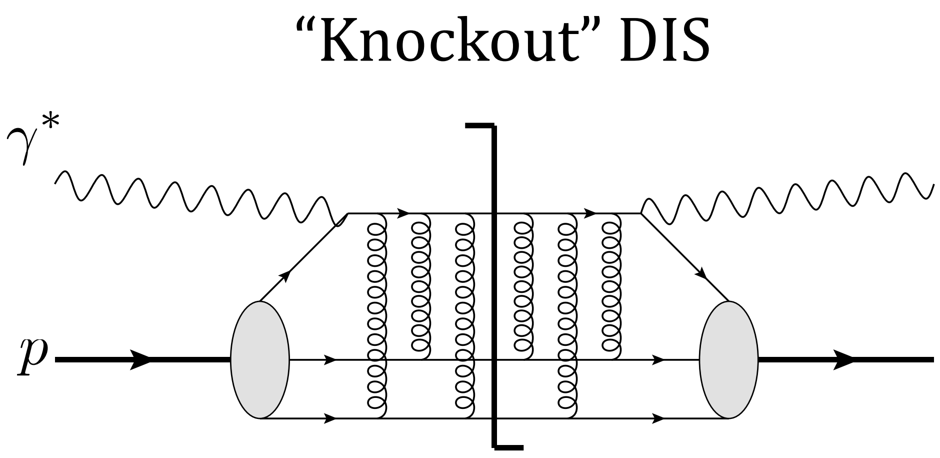
$$\frac{Q^2}{4\pi^2 \alpha_{EM}} \frac{d\sigma^{(\gamma^* p)}}{dx\, dQ^2} = F_2(x, Q^2) \overset{L.O.}{=} \sum_f e_f^2 \, x q_f(x, Q^2)$$

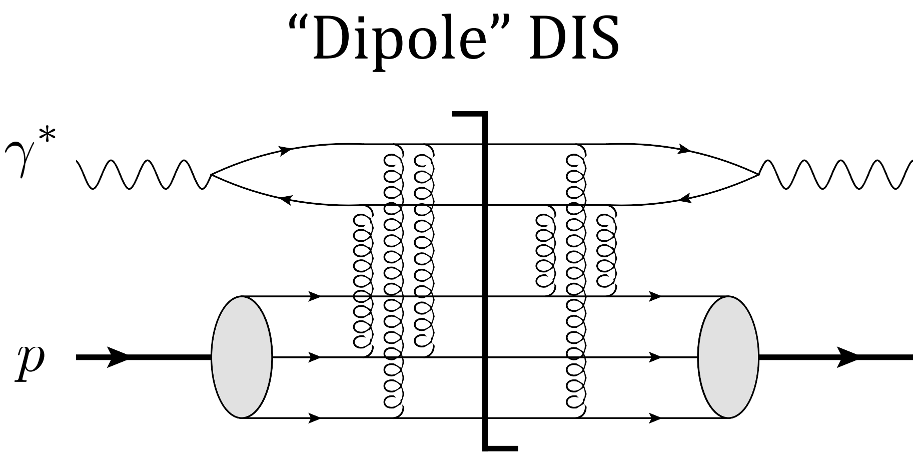


$$q(x, Q^2) = \int \frac{dr^-}{2\pi} e^{ixp^+ r^-} \langle p | \bar{\psi}(0) \, \mathcal{U}[0, r^-] \frac{\gamma^+}{2} \psi(r^-) | p \rangle$$



Dipole DIS at Small x





- At small x, the lifetime of the DIS photon becomes much larger than the size of the proton

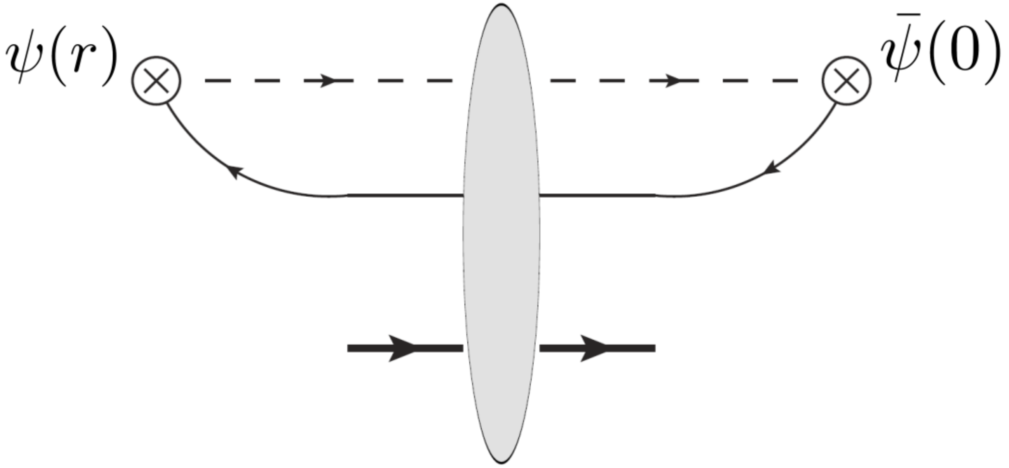
$$\Delta t \sim \frac{1}{mx} \gg \frac{1}{m}$$

- Photon fluctuates into a long-lived $q\bar{q}$ dipole when then scatters on the proton

- The **PDF operator** also looks different at small x

(Note: Light-cone gauge $A^- = 0$)

“Dipole” PDF



- ➢ The proton is **Lorentz-contracted** to $\delta(x^-)$

- ➢ The operators **create / annihilate antiquarks** instead, which propagate through the proton fields as **Wilson lines**

$$V_{\underline{x}} = \mathcal{P} \exp \left[ig \int dz^- \, \hat{A}^+(0^+, z^-, \underline{x}) \right]$$

From PDFs to Dipoles

- The PDF operator is reformulated in terms of **dipole scattering amplitudes**

$$xq_f(x,Q^2) = \frac{Q^2 N_c}{4\pi^2 \alpha_{EM}} \int \frac{d^2x_{10}\, dz}{4\pi z(1-z)} \sum_{L,T} \left|\Psi(x_{10}^2, z)\right|^2 \int d^2b_{10} \left[2 - \frac{1}{N_c}\left\langle \mathrm{tr}\left[V_0 V_1^\dagger\right]\right\rangle_{(zs)} - \frac{1}{N_c}\left\langle \mathrm{tr}\left[V_1 V_0^\dagger\right]\right\rangle_{(zs)}\right]$$

Photon splitting wave functions

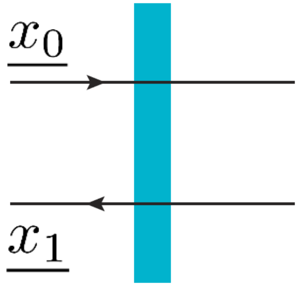
Non-interacting terms

Dipole amplitudes

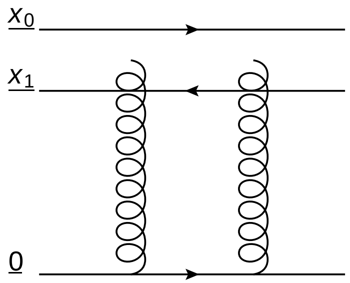
The Small-x Operator: The Dipole Amplitude

- The **x-dependence of the PDF** (TMD) is governed by the **energy dependence of the dipole amplitude**

$$\frac{1}{N_c}\left\langle \mathrm{tr}\left[V_0 V_1^\dagger\right]\right\rangle_{(zs)}$$



- Arises from the **phase-space enhanced quantum corrections** in the **background field** of the proton



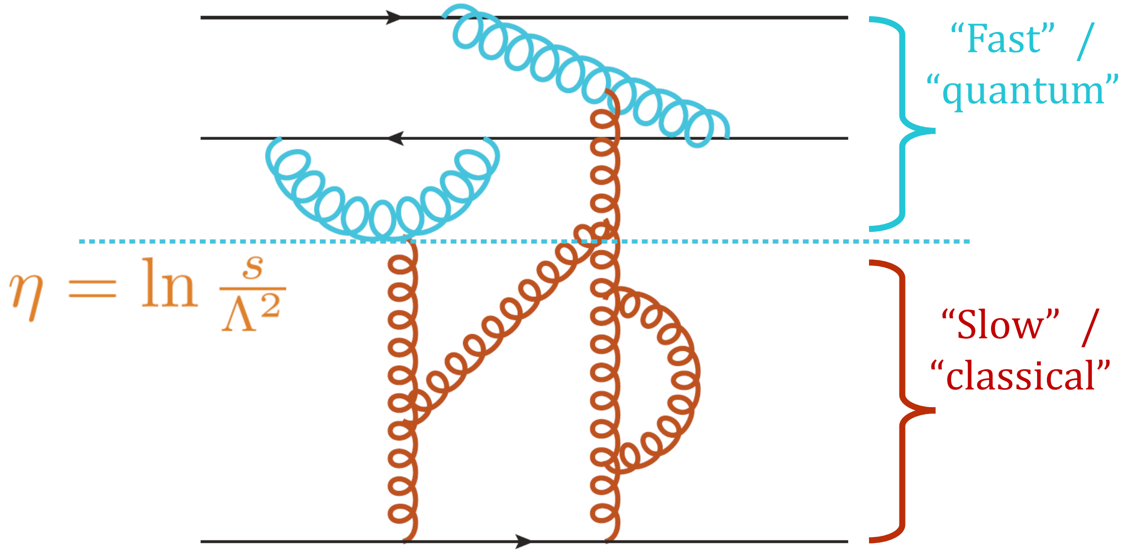
- The **initial conditions** can be taken from PDF fits at large x or, e.g.) the quark target model

$$\frac{1}{N_c}\left\langle \mathrm{tr}\left[V_0 V_1^\dagger\right]\right\rangle_{(zs)}^{(0)} = \frac{2\alpha_s^2 C_F}{N_c} \ln^2 \frac{x_{0T}}{x_{1T}}$$

Quantum Evolution in a Classical Background

- Quantum evolution is obtained through the **background field method**

$$A^\mu(x) = A^\mu_{\text{classical}}(x) + a^\mu_{\text{quantum}}(x)$$



- **Abitrary rapidity cut η** between “fast, quantum modes” and “slow, classical modes.”

- Compute corrections from the **“quantum” fields** in the **“classical” background** if they cross the proton.

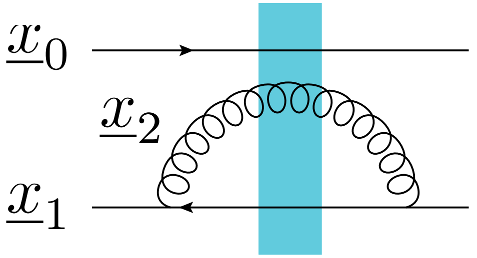
- **RG evolution** with respect to the **arbitrary cutoff** generates quantum evolution

I. Balitsky, Nucl. Phys. ***B463*** *(1996) 99*
I. Balitsky, Phys. Rev. ***D60*** *(1999) 014020*
I. Balitsky and A. Tarasov, JHEP ***1510*** *(2015) 017*

Types of Corrections: Real and Virtual

- **"Real" gluon emissions** propagate through the **classical background** of the proton

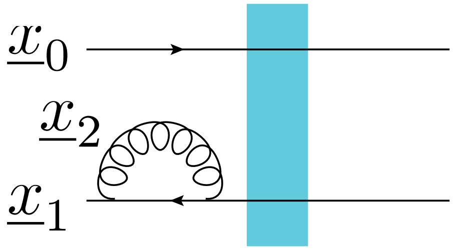
$$\frac{1}{N_c} \left\langle \mathrm{tr} \left[V_0 t^a V_1^\dagger t^b \right] U_2^{ba} \right\rangle_{(z's)}$$



$$(x_2^-)_i < 0^- < (x_2^-)_f$$

- **"Virtual" gluon emissions** propagate through the **vacuum**

$$-\frac{C_F}{N_c} \left\langle \mathrm{tr} \left[V_0 V_1^\dagger \right] \right\rangle_{(z's)}$$



+ "c.c."

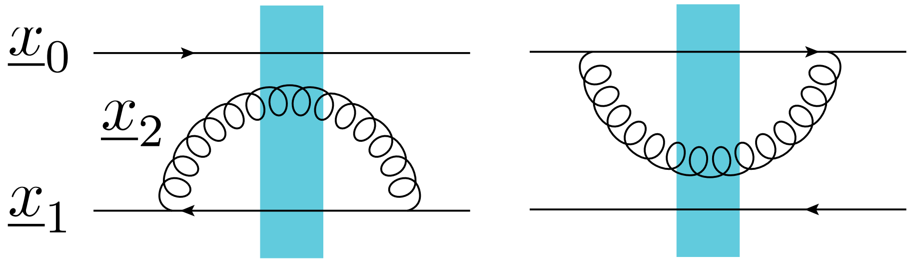
$$(x_2^-)_i \, , \, (x_2^-)_f < 0^-$$

$$0^- < (x_2^-)_i \, , \, (x_2^-)_f$$

Types of Corrections: Ladder and Non-Ladder

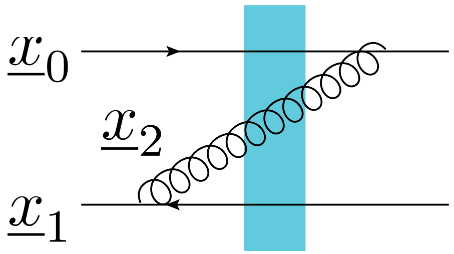
- **"Ladder" emissions** are emitted and absorbed by the same parton

$$\frac{\alpha_s}{\pi^2}\int\limits_{\frac{\Lambda^2}{s}}^{z}\frac{dz'}{z'}\int d^2x_2\left(\frac{1}{x_{21}^2}+\frac{1}{x_{20}^2}\right)\times\Big[\text{operator}\Big]$$

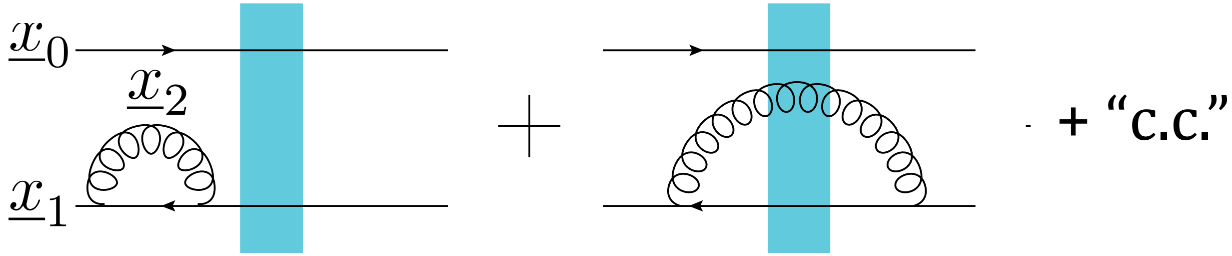


- **"Non-ladder" emissions** are emitted and absorbed by different partons

$$\frac{\alpha_s}{\pi^2}\int\limits_{\frac{\Lambda^2}{s}}^{z}\frac{dz'}{z'}\int d^2x_2\left(-2\,\frac{\underline{x}_{21}\cdot\underline{x}_{20}}{x_{21}^2\,x_{20}^2}\right)\times\Big[\text{operator}\Big]$$



Color Transparency of Small Fluctuations



$$\frac{\alpha_s N_c}{2\pi^2} \int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int \frac{d^2 x_2}{x_{21}^2} \times \left[\frac{1}{N_c^2} \left\langle \mathrm{tr}\left[V_2 V_1^\dagger\right] \mathrm{tr}\left[V_0 V_2^\dagger\right] \right\rangle_{(z's)} - \frac{1}{N_c} \left\langle \mathrm{tr}\left[V_0 V_1^\dagger\right] \right\rangle_{(z's)} \right]$$

- "Ladder" emissions of **small-sized fluctuations are enhanced**
- Potentially divergent… a **second logarithm?**
- **No**: **Cancellation** of real + virtual diagrams due to **color transparency**

The Balitsky Operator Hierarchy

$$\frac{1}{N_c}\left\langle \mathrm{tr}\left[V_0 V_1^\dagger\right]\right\rangle_{(zs)} = \frac{1}{N_c}\left\langle \mathrm{tr}\left[V_0 V_1^\dagger\right]\right\rangle_{(zs)}^{(0)}$$

$$+\frac{\alpha_s N_c}{2\pi^2}\int\limits_{\frac{\Lambda^2}{s}}^{z}\frac{dz'}{z'}\int d^2x_2\,\frac{x_{10}^2}{x_{20}^2\,x_{21}^2}\left[\frac{1}{N_c^2}\left\langle \mathrm{tr}\left[V_2 V_1^\dagger\right]\,\mathrm{tr}\left[V_0 V_2^\dagger\right]\right\rangle_{(z's)} - \frac{1}{N_c}\left\langle \mathrm{tr}\left[V_0 V_1^\dagger\right]\right\rangle_{(z's)}\right]$$

Rapidity Logarithm

BFKL Kernel

New, more complex operator

- The dipole evolves into **increasingly complex operators**….
- Equivalent to a **functional differential equation**….

*I. Balitsky, Nucl. Phys. **B463** (1996) 99*
*I. Balitsky, Phys. Rev. **D60** (1999) 014020*

*Jalilian-Marian et al., Phys. Rev. **D59** (1998) 014015*
*Jalilian-Marian et al., Phys. Rev. **D59** (1998) 014014*
*Iancu et al., Phys. Lett. **B510** (2001) 133*
*Iancu et al., Nucl. Phys. **A692** (2001) 583*

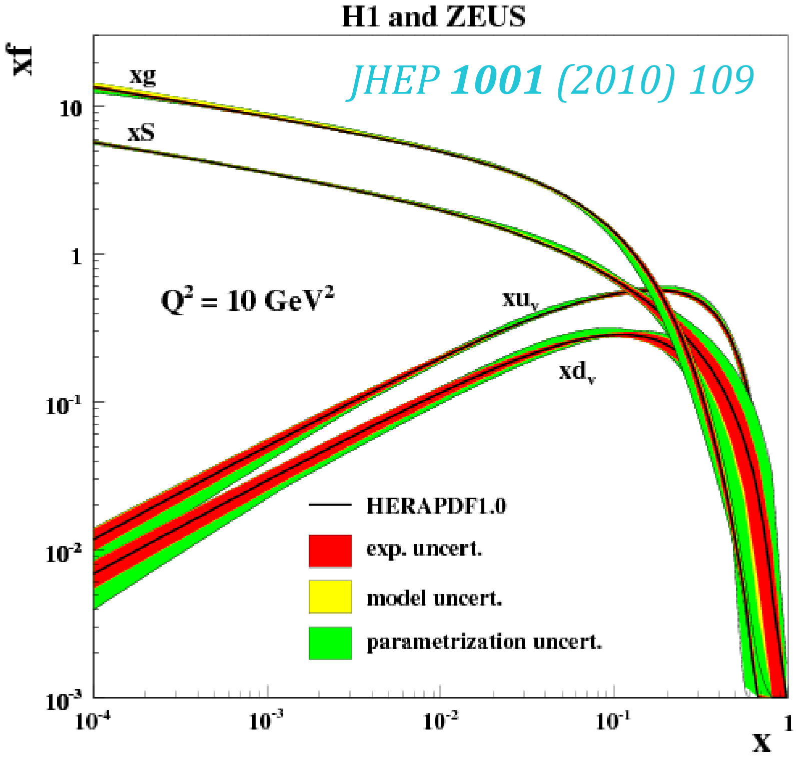
- The equations **linearize** in the **dilute limit** (BFKL)

Kuraev, et al., Sov. Phys. JETP ***45*** *(1977) 199*
Balitsky and Lipatov, Sov. J. Nucl. Phys. ***28*** *(1978) 822*

$$\frac{\alpha_s N_c}{2\pi^2} \int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int d^2 x_2 \, \frac{x_{10}^2}{x_{20}^2 \, x_{21}^2} \left\langle \frac{1}{N_c} \mathrm{tr}\left[V_2 V_1^\dagger\right] + \frac{1}{N_c} \mathrm{tr}\left[V_0 V_2^\dagger\right] - \frac{1}{N_c} \mathrm{tr}\left[V_0 V_1^\dagger\right] - 1 \right\rangle_{(z's)}$$

➢ Leads to **power-law growth** in the PDFs at small x

$$xq(x, Q^2) \sim xG(x, Q^2) \sim \left(\frac{1}{x}\right)^{\frac{4\alpha_s N_c}{\pi} \ln 2}$$



The BFKL and BK Equations

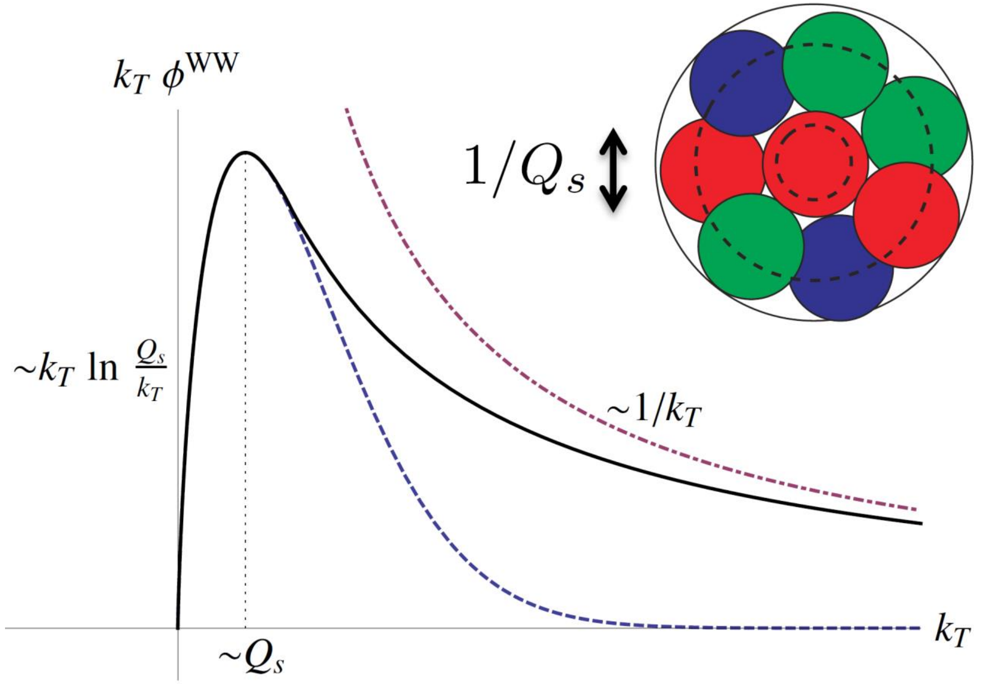
- The operator **hierarchy closes** in **large-Nc limit** (BK)

Balitsky, Nucl. Phys. ***B463*** *(1996) 99*
Balitsky, Phys. Rev. ***D60*** *(1999) 014020*
Kovchegov, Phys. Rev. ***D60*** *(1999) 034008*
Kovchegov, Phys. Rev. ***D61*** *(2000) 074018*

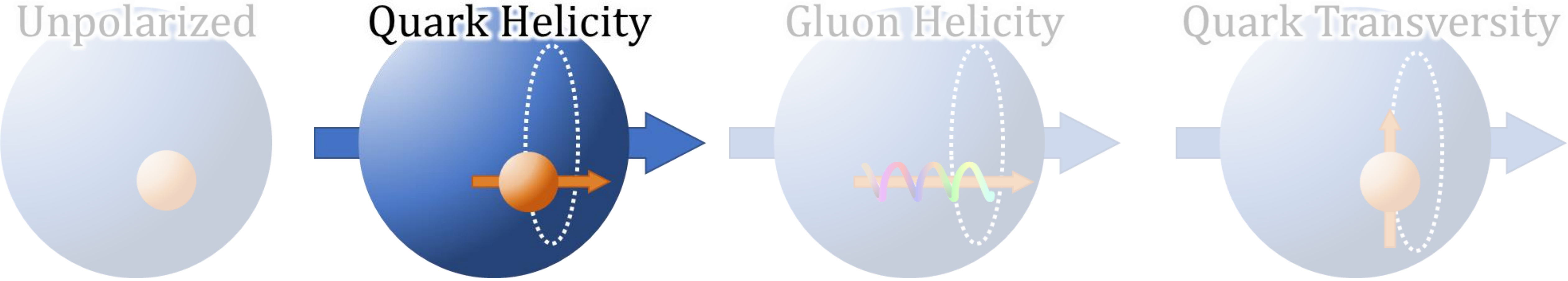
$$\frac{\alpha_s N_c}{2\pi^2} \int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int d^2 x_2 \, \frac{x_{10}^2}{x_{20}^2 \, x_{21}^2} \left[\frac{1}{N_c^2} \left\langle \mathrm{tr} \left[V_2 V_1^\dagger \right] \right\rangle_{(z's)} \times \left\langle \mathrm{tr} \left[V_0 V_2^\dagger \right] \right\rangle_{(z's)} - \frac{1}{N_c} \left\langle \mathrm{tr} \left[V_0 V_1^\dagger \right] \right\rangle_{(z's)} \right]$$

- Nonlinear gluon recombination leads to **saturation** of the small-x PDFs.

$$Q_s^2(x) \sim \left(\frac{1}{x}\right)^{0.3}$$

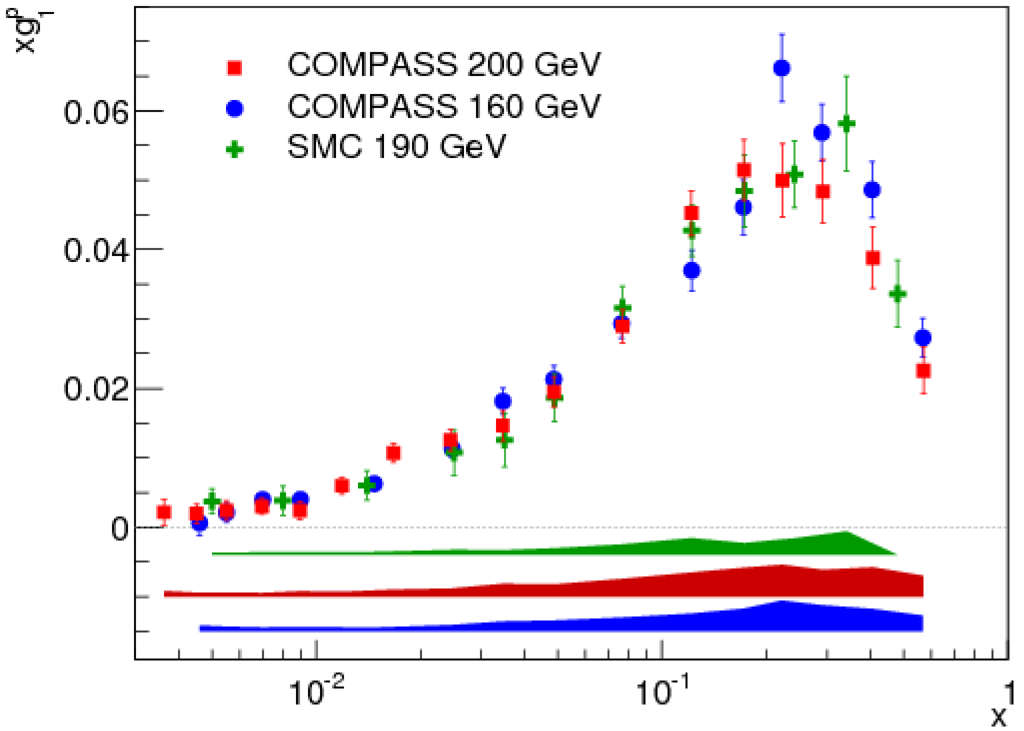


Quark Helicity and the Proton Spin

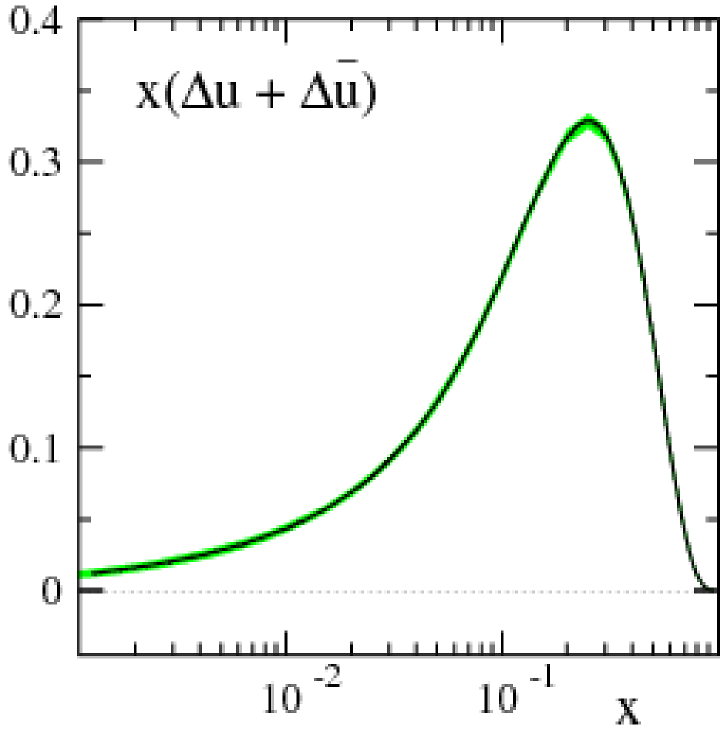


Quark Helicity: What Do We Know?

*COMPASS Collaboration, Phys. Lett. **B753** (2016) 18*



*De Florian et al., Phys. Rev. **D80** (2009) 034030*



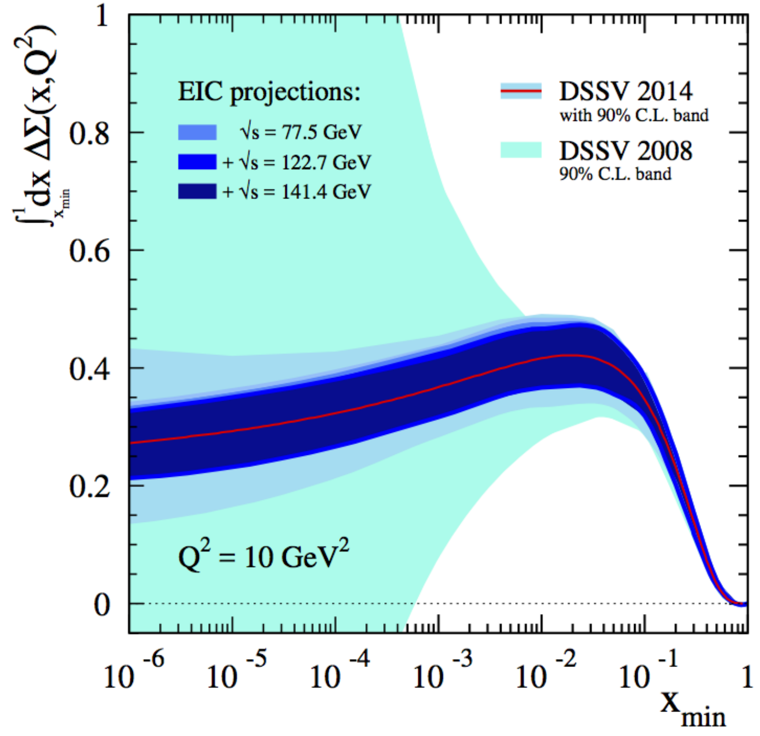
- **Polarized structure functions** are measured e.g.) by COMPASS and JAM

*J. J. Ethier et al., Phys. Rev. Lett. **119** (2017) 132001*

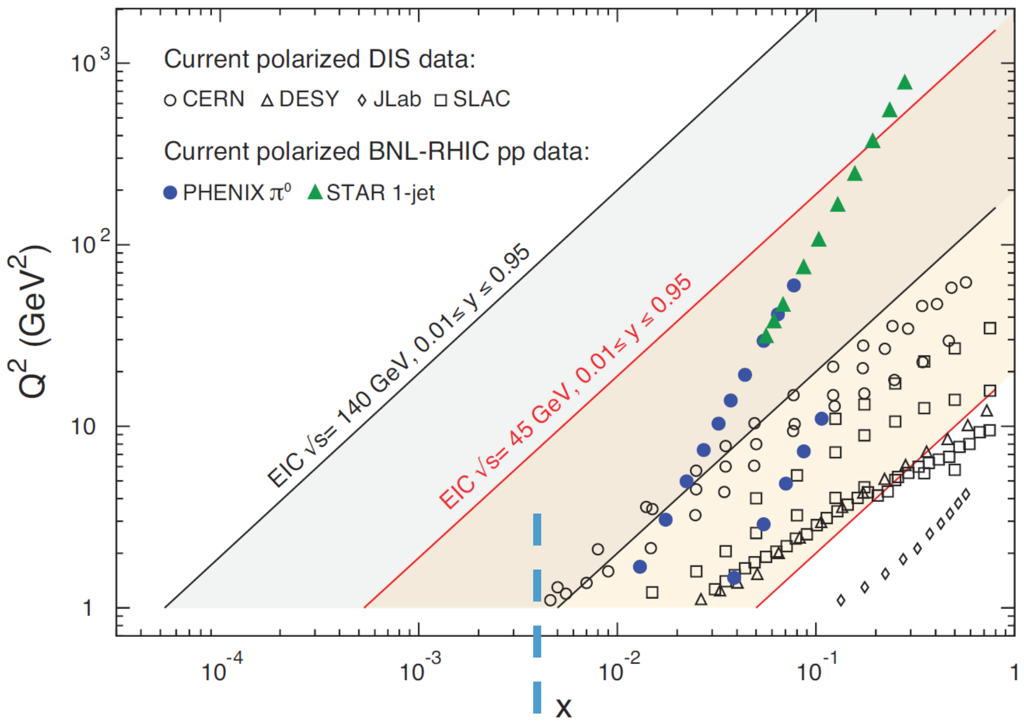
- The polarized structure functions **decay at small x**

How Much Polarization is there at Small x?

*E.-C. Aschenauer et al., Phys. Rev. **D92** (2015) 094030*



*A. Accardi et al., Eur. Phys. J. **A52** (2016) 268*

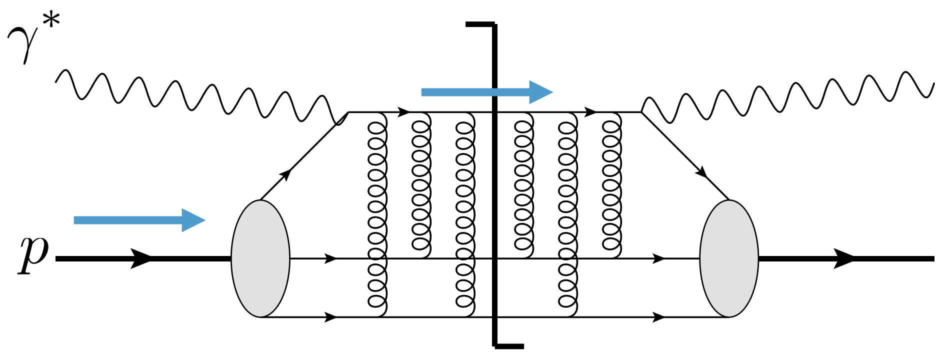


- **Polarized DIS data** runs out below x ~ $few \times 10^{-3}$

- The running integral of ΔΣ is **not converging quickly enough** to strongly constrain the quark contribution to the proton spin budget.

- The **DIS polarized structure functions** are related via factorization to the **quark helicity PDFs**

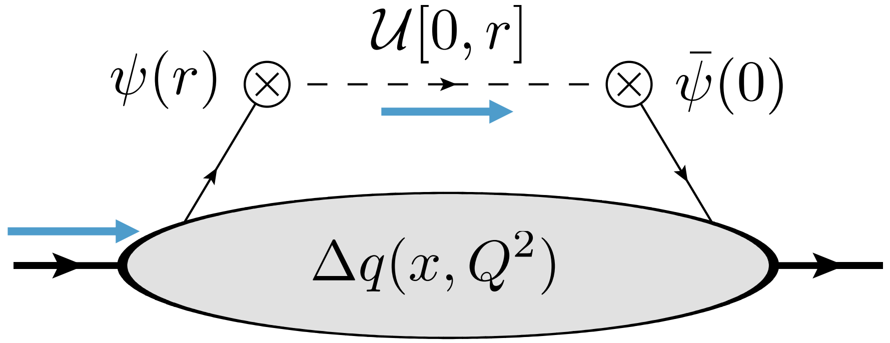
$$\frac{Q^2}{4\pi^2 \alpha_{EM}} \frac{d\,\Delta\sigma^{(\gamma^* p)}}{dx\, dQ^2} = 2x\, g_1(x, Q^2) \overset{L.O.}{=} \sum_f e_f^2\, x \Delta q_f(x, Q^2)$$



- The quark hPDFs are non-local matrix elements of the **axial vector current**

$$\Delta q\ (x, Q^2) = \int \frac{dr^-}{2\pi}\, e^{ixp^+ r^-}\, \langle p|\, \bar\psi(0)\, \mathcal{U}[0, r^-]\, \frac{\gamma^+\gamma^5}{2}\, \psi(r^-)\, |p\rangle$$

Light-Cone Spin Projection: S^+



The Naïve Translation to Dipoles at Small x

- Naively, we get **dipole amplitudes** at small x which are sensitive to **polarization**

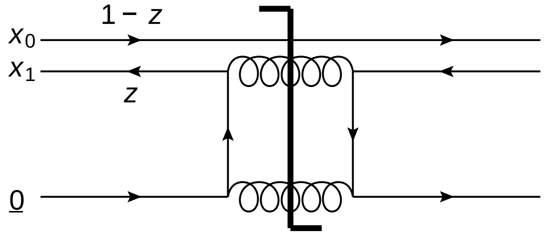
*Y. Kovchegov, D. Pitonyak, **M.S.**, JHEP **1601** (2016) 072*

- The "squared" contributions are **insensitive to the proton spin.**

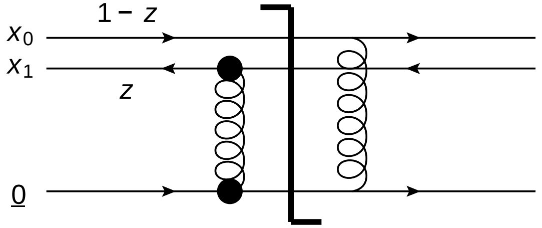
$$x\Delta q_f(x,Q^2) = \frac{Q^2 N_c}{4\pi^2 \alpha_{EM}} \int \frac{d^2 x_{10}\, dz}{4\pi z(1-z)} \sum_{L.T} \left| \Delta\Psi(x_{10}^2, z) \right|^2 \int d^2 b_{10} \left[\frac{1}{N_c} \left\langle \mathrm{tr}\left[V_0 V_1^{pol\,\dagger} \right] \right\rangle_{(zs)} + \frac{1}{N_c} \left\langle \mathrm{tr}\left[V_1^{pol} V_0^{\dagger} \right] \right\rangle_{(zs)} \right]$$

Spin Transfer is Power-Suppressed at Small x

- The leading-power interaction at small x (i.e., a Wilson line) is **spin-independent**.



- The initial conditions for spin-dependent scattering are **suppressed by one power of s** (or x).

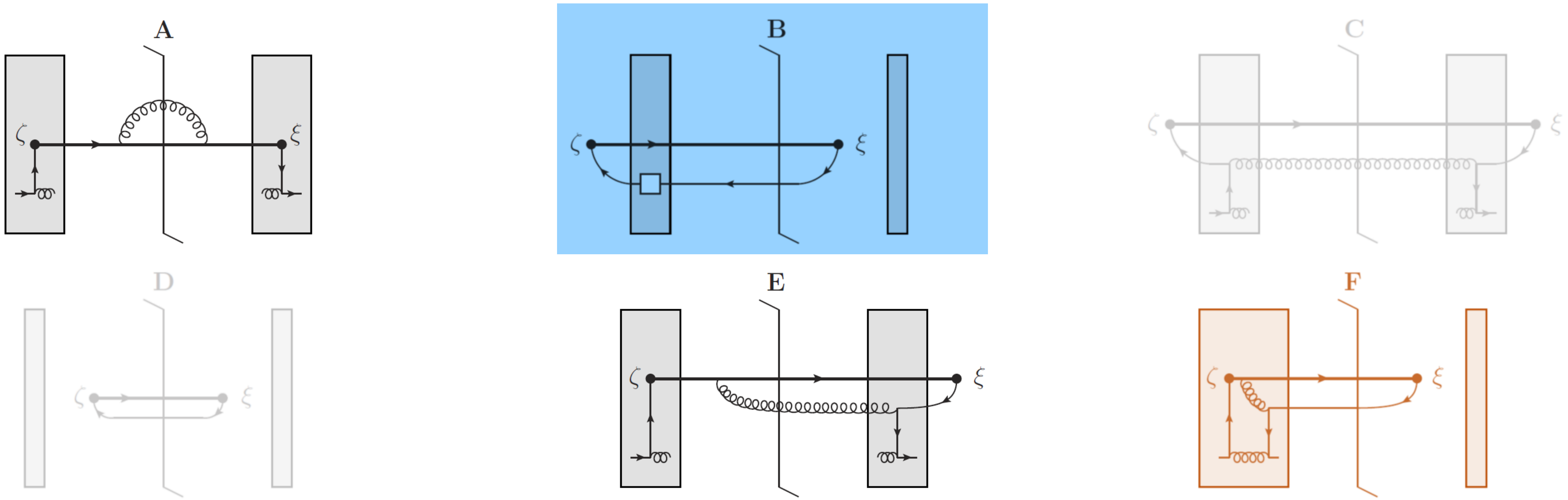


+ perm's

- **Spin transfer is sensitive** to several effects which would be negligible **power-suppressed corrections** to the unpolarized PDFs

$$\frac{d\,\Delta\sigma^{Born}}{d^2b} \sim x \sim \frac{1}{s}$$

  - T-channel **quark exchange** is now a leading contribution

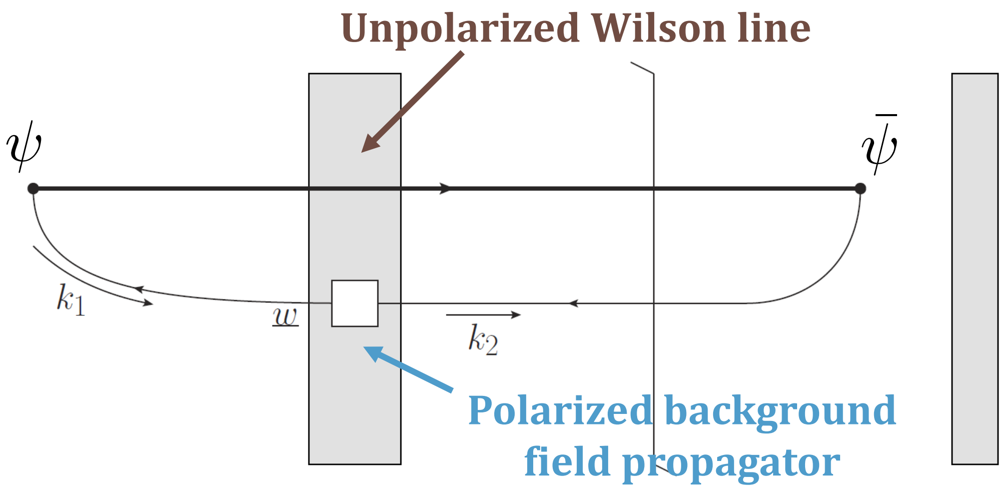


- Since the **hPDF itself is power-suppressed**, the handbag diagram and other **“knockout” processes can contribute**.

*Y. Kovchegov, **M.S**, arXiv: 1808.09010*

- Using the **Ward identity**, the potential evolution corrections coming from “knockout” channels (A + E + cc) **cancel after all**.

- The "polarized dipole" contains an antiquark **exchanging spin** while propagating through the **background field of the proton**



$$\overbrace{\bar\psi^i_\alpha(\xi)\,\psi^j_\beta(\zeta)} = \int d^2w\, \frac{d^4k_1}{(2\pi)^4}\,\frac{d^4k_2}{(2\pi)^4}\, e^{ik_1^+\zeta^-}\, e^{i\underline{k}_1\cdot(\underline{w}-\underline{\zeta})}\, e^{-ik_2^+\xi^-}\, e^{i\underline{k}_2\cdot(\underline{\xi}-\underline{w})}$$

$$\times \left\{ \left[\frac{-i\not{k}_1}{k_1^2+i\epsilon}\right] \left[\left(\hat{V}^\dagger_{\underline{w}}\right)^{ji}\,(2\pi)\,\delta(k_1^- - k_2^-)\right] \left[-\not{k}_2\,(2\pi)\,\delta(k_2^2)\right] \right\}_{\beta\alpha}$$

Free propagator

General Background
Field Propagator
(Dirac matrix)

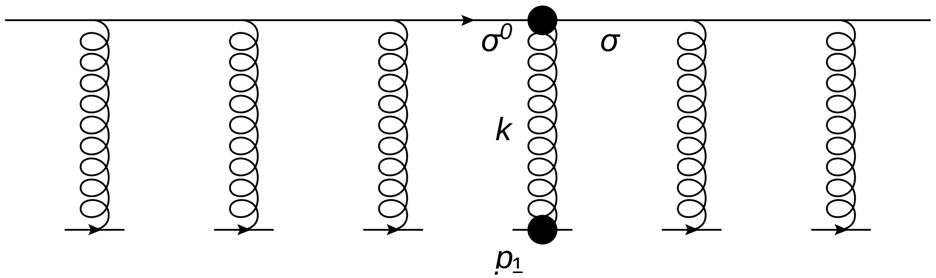
Cut free propagator

*Y. Kovchegov, **M.S**, arXiv: 1808.09010*
G. Chirilli, arXiv: 1807.11435

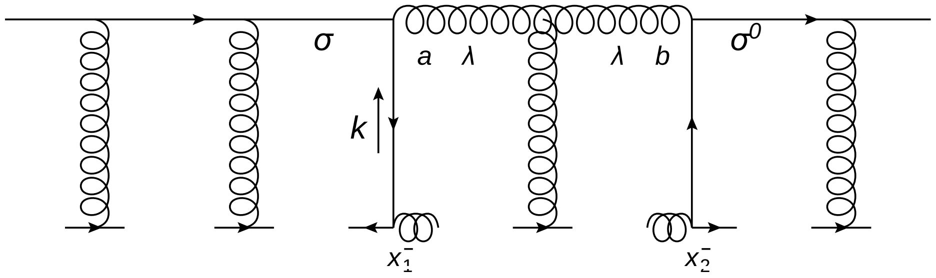
- After performing the k_1 , k_2 integrals and replacing the Dirac structure with spinor sums, we can **pick out any desired spin projections**.

Polarized Wilson Lines

*Y. Kovchegov, D. Pitonyak, **M.S.**, JHEP **1710** (2017) 198*



*Y. Kovchegov, **M.S**, arXiv: 1808.09010*



- **Each spin-dependent coupling** to the background field is **power suppressed**
- A "polarized Wilson line" contains **one spin-dependent operator**, dressed with **$O(1)$ Wilson lines**

$\vec{\mu} \cdot \vec{B}$ coupling to the background gluon field

Flavor-changing Wilson line

$$V_{\underline{x}}^{pol} = \frac{igp_1^+}{s} \int\limits_{-\infty}^{\infty} dx^- \, V_{\underline{x}}[+\infty, x^-] \, F^{12}(x^-, \underline{x}) \, V_{\underline{x}}[x^-, -\infty]$$

$$- \frac{g^2 p_1^+}{s} \int\limits_{-\infty}^{\infty} dx_1^- \int\limits_{x_1^-}^{\infty} dx_2^- \, V_{\underline{x}}[+\infty, x_2^-] \, t^b \, \psi_\beta(x_2^-, \underline{x}) \, U_{\underline{x}}^{ba}[x_2^-, x_1^-] \left[\frac{1}{2}\gamma^+ \gamma^5\right]_{\alpha\beta} \bar{\psi}_\alpha(x_1^-, \underline{x}) \, t^a \, V_{\underline{x}}[x_1^-, -\infty].$$

Helicity-Dependent Propagation in a Background Field

- The quark helicity PDF is built from **polarized dipole amplitudes**, as expected:

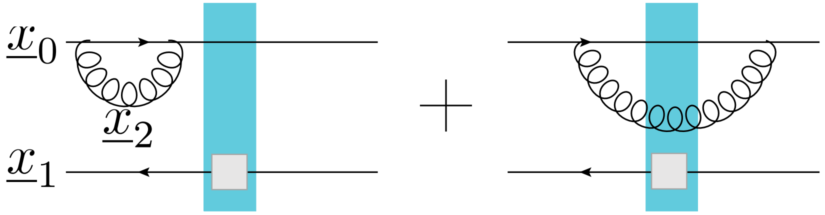
Scale out suppression factor

$$\Delta q(x, Q^2) = \frac{N_c}{8\pi^3} \int\limits_{\Lambda^2/s}^{1} \frac{dz}{z} \int\limits_{1/zs}^{1/zQ^2} \frac{dx_{10}^2}{x_{10}^2} \int d^2 b_{10} \, \underbrace{\left\langle \frac{zs}{N_c} \mathrm{tr}\left[V_0 V_1^{pol\,\dagger} \right] + \frac{zs}{N_c} \mathrm{tr}\left[V_1^{pol} V_0^\dagger \right] \right\rangle_{(zs)}}_{\text{Polarized Dipole Amplitude: } 2\, G_{10}(z\, s)}$$

Extra logarithm…!

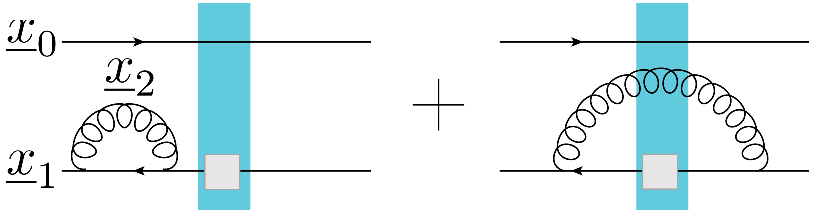
Quantum Evolution: Beyond Color Transparency

- Ladder emissions from the **unpolarized Wilson line** possess **color transparency** at short distances



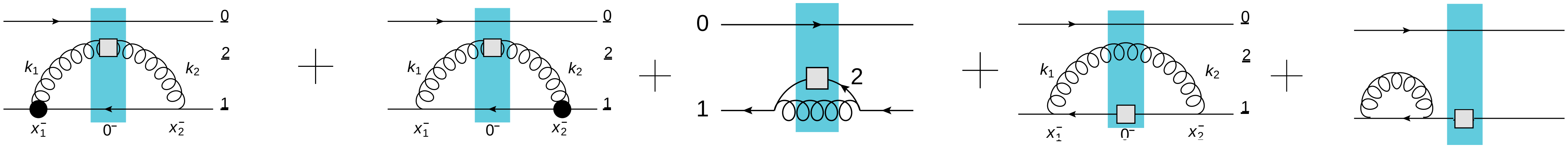
$$\frac{\alpha_s N_c}{2\pi^2} \int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int \frac{d^2 x_2}{x_{20}^2} \times \underbrace{\left[\frac{1}{N_c^2} \left\langle \mathrm{tr}\left[V_2 V_1^{pol\,\dagger} \right] \mathrm{tr}\left[V_0 V_2^{\dagger} \right] \right\rangle_{(z' s)} - \frac{1}{N_c} \left\langle \mathrm{tr}\left[V_0 V_1^{pol\,\dagger} \right] \right\rangle_{(z' s)} \right]}_{\text{Cancels when } \underline{x}_2 \to \underline{x}_0}$$

- But for ladder emissions from the **polarized Wilson line**, color transparency is **violated by spin**



$$\frac{\alpha_s N_c}{2\pi^2} \int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int \frac{d^2 x_2}{x_{21}^2} \times \underbrace{\left[\frac{1}{N_c^2} \left\langle \mathrm{tr}\left[V_2 V_1^{pol\,\dagger} \right] \mathrm{tr}\left[V_0 V_2^{\dagger} \right] \right\rangle_{(z' s)} - \frac{1}{N_c} \left\langle \mathrm{tr}\left[V_0 V_1^{pol\,\dagger} \right] \right\rangle_{(z' s)} \right]}_{\text{Does NOT cancel when } \underline{x}_2 \to \underline{x}_1}$$

Double Logarithmic Helicity Evolution



- Short-distance fluctuations about the polarized Wilson line generate **double logarithms** of the energy.

$$\alpha_s \ln^2 \frac{1}{x} \sim 1$$

- Helicity evolution is **stronger** than unpolarized one evolution, but starts off power-suppressed.

Unpolarized: $x < e^{1/\alpha_s}$

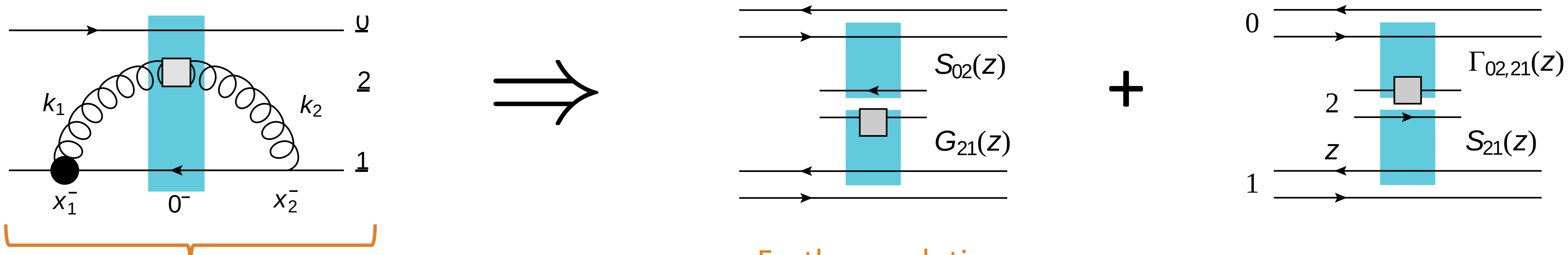
- The transverse logarithm: **more sensitive** to the structure and ordering in the transverse plane

Helicity: $x < e^{1/\sqrt{\alpha_s}}$

- Strict **lifetime ordering** (NLO for unpolarized evolution) is a **leading order effect for helicity**

*Kirschner and Lipatov, Nucl.Phys. **B213** (1983) 122*
*Bartels, Ermolaev, and Ryskin, Z.Phys. **C70** (1996) 273*
*Griffiths and Ross, Eur.Phys.J. **C12** (2000) 277*

The Neighbor Dipole Function



Constrained by lifetime of x_{21}

Further evolution of x_{21} dipole

Evolution of x_{02} dipole is constrained by x_{21}

- These evolution equations describe another **operator hierarchy**

- Just like the BK equation, they do **close in the large-Nc limit** (or large Nc & Nf)

- But because **lifetime ordering** constrains the history of the polarized gluon cascade, **not all dipoles are independent**.

*Y. Kovchegov, D. Pitonyak, **M.S.**, JHEP **1601** (2016) 072*

*Y. Kovchegov, D. Pitonyak, **M.S.**, JHEP **1601** (2016) 072*

$$G(x_{10}^2, z) = G^{(0)}(x_{10}^2, z) + \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{1}{x_{10}^2 s}}^{z} \frac{dz'}{z'} \int\limits_{\frac{1}{z' s}}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \left[\Gamma(x_{10}^2, x_{21}^2, z') + 3G(x_{21}^2, z')\right]$$

$$\Gamma(x_{10}^2, x_{21}^2, z') = G^{(0)}(x_{10}^2, z') + \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{1}{x_{10}^2 s}}^{z'} \frac{dz''}{z''} \int\limits_{\frac{1}{z'' s}}^{\min\left[x_{10}^2, x_{21}^2 \frac{z'}{z''}\right]} \frac{dx_{32}^2}{x_{32}^2} \left[\Gamma(x_{10}^2, x_{32}^2, z'') + 3G(x_{32}^2, z'')\right]$$

- Even at large-Nc, lifetime ordering leads to a **system of coupled equations** through the with **auxiliary "neighbor dipole" function**

Solution: The Quark Helicity Intercept

- After evolving for a few units in rapidity, a **scaling behavior** sets in

$$G(x_{10}^2, zs) \sim G(zs x_{10}^2)$$

- Makes it possible to solve the large-Nc equations **analytically**

$$\left.\begin{aligned} G(x_\perp^2, zs) &\sim (zs)^{\alpha_h^q} \\ g_1(x, k_T^2) &\sim \left(\tfrac{1}{x}\right)^{\alpha_h^q} \\ \Delta q(x, Q^2) &\sim \left(\tfrac{1}{x}\right)^{\alpha_h^q} \end{aligned}\right\}$$

- The x (energy) dependence approaches a **universal power-law behavior:**

$$\boxed{\alpha_h^{q,S} = \frac{4}{\sqrt{3}} \sqrt{\frac{\alpha_s N_c}{2\pi}}}$$

$$\boxed{\alpha_h^{q,NS} = \sqrt{2} \sqrt{\frac{\alpha_s N_c}{2\pi}}}$$

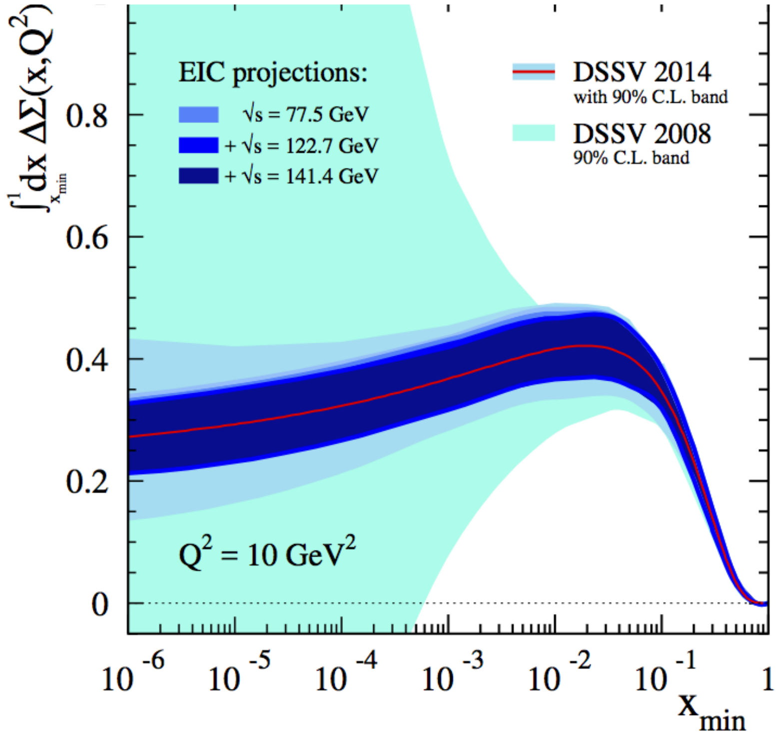
*Y. Kovchegov, D. Pitonyak, **M.S.**, Phys. Rev. Lett. **118** (2017) 052001*

*Y. Kovchegov, D. Pitonyak, **M.S.**, Phys. Lett. **B772** (2017) 136*

*Y. Kovchegov, D. Pitonyak, **M.S.**, Phys. Rev. **D95** (2017) 014033*

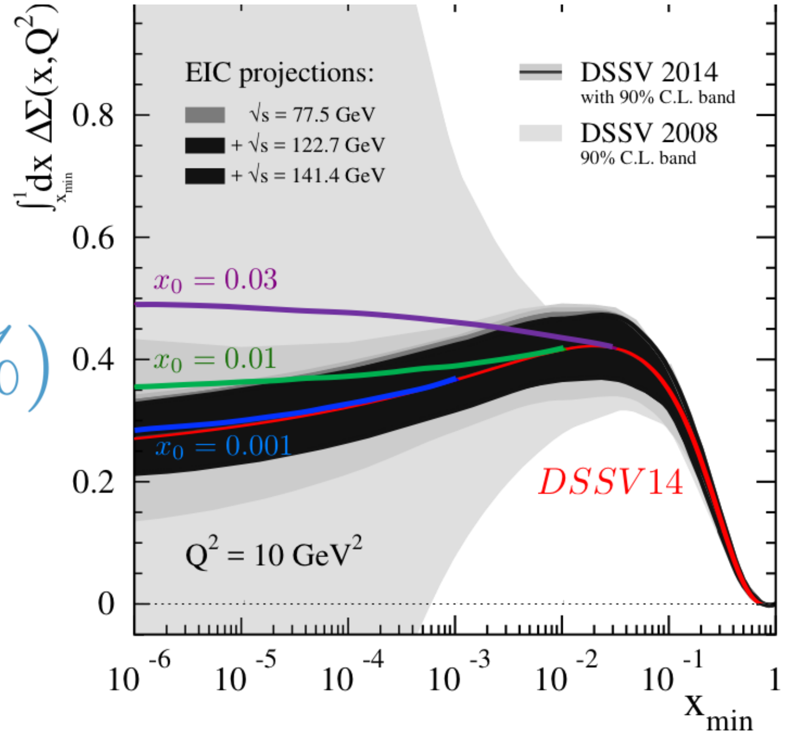
A Crude Phenomenological Estimate

E.-C. Aschenauer et al., Phys. Rev. ***D92*** *(2015) 094030*



Y. Kovchegov, D. Pitonyak, ***M.S.****, Phys. Rev. Lett.* ***118*** *(2017) 052001*

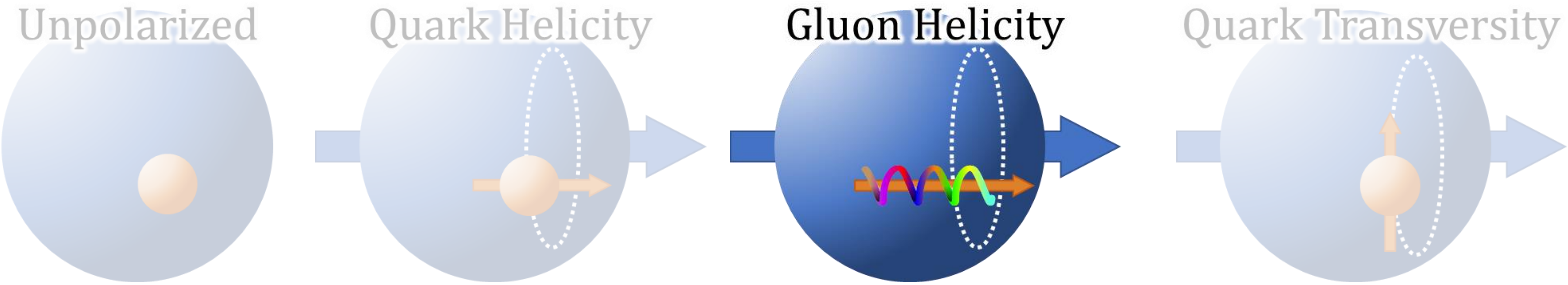
$\mathcal{O}\,(20\%)$



- Depending on when our asymptotic behavior is turned on, the **added contribution to the proton spin** at small x could be **significant**.

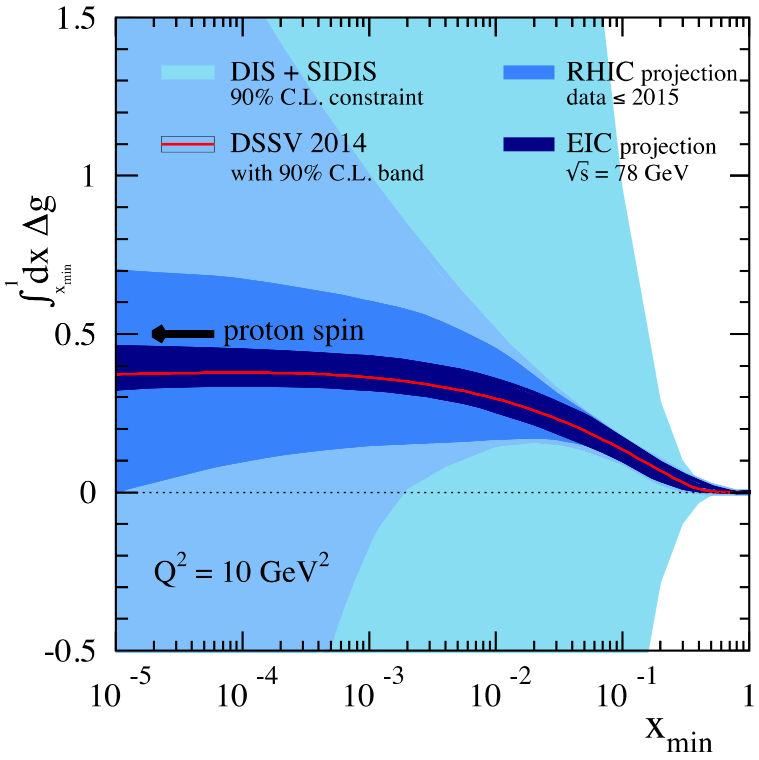
- The theory **doesn't tell you** when the **small-x effects set in**.

Gluon Helicity and the Proton Spin

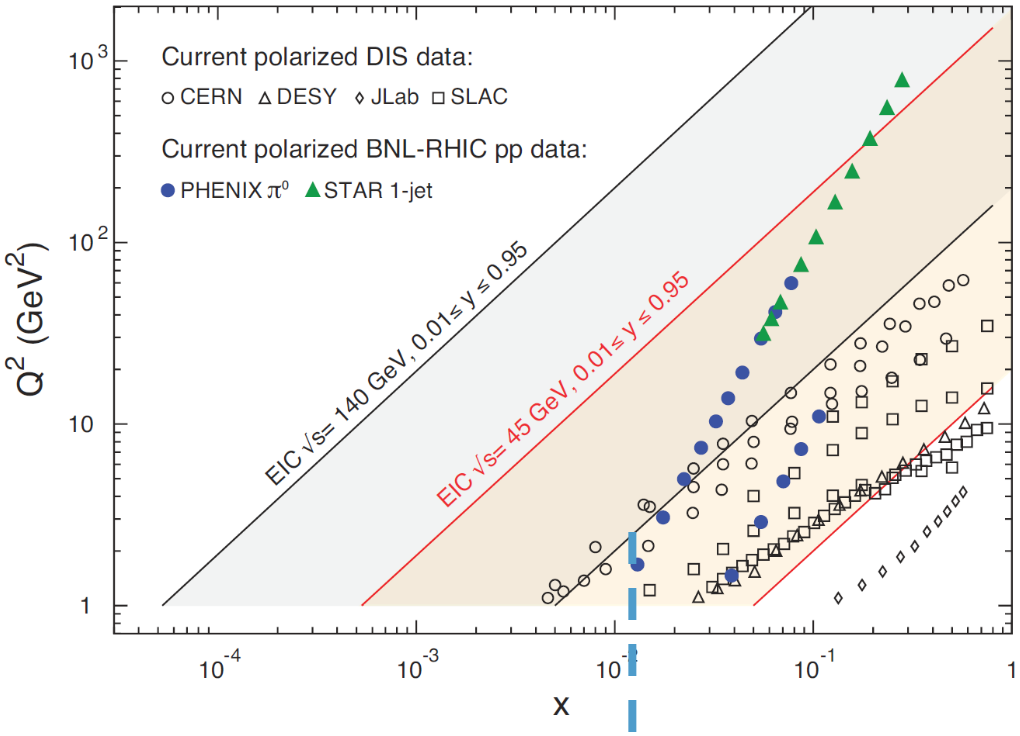


Gluon Helicity: What Do We Know?

E.-C. Aschenauer et al., Phys. Rev. ***D92*** *(2015) 094030*

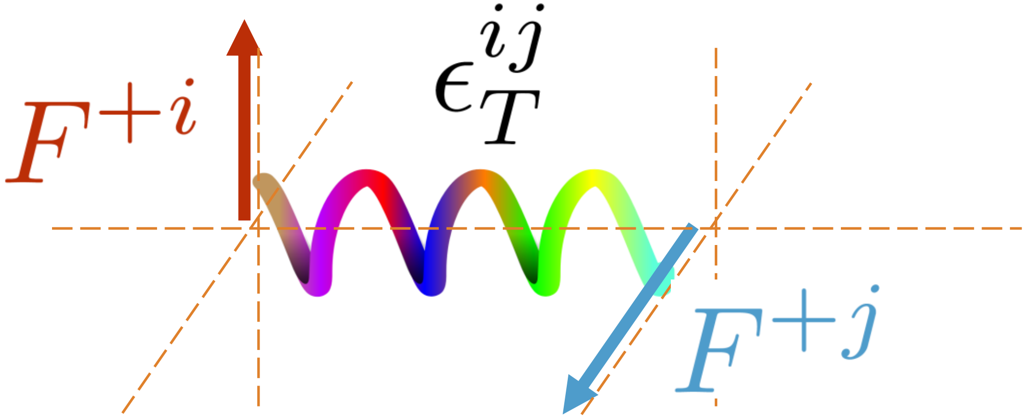


A. Accardi et al., Eur. Phys. J. ***A52*** *(2016) 268*



- **Data on polarized gluons runs out** by $x{\sim}10^{-2}$
- The gluon contribution to the proton spin is **far less constrained** at small x

Definition of Gluon Helicity

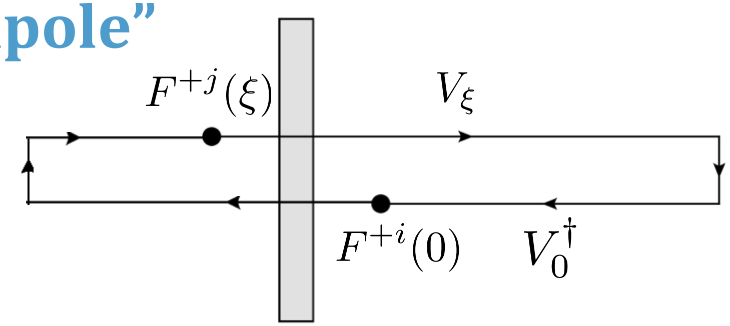


$$g_{1L}^{G}(x,k_T^2) = \frac{-2i}{x\,P^+} \int \frac{d\xi^- \, d^2\xi}{(2\pi)^3} \; e^{ik\cdot\xi} \; \langle P, S_L | \, \underline{\epsilon_T^{ij}} \, \mathrm{tr}\left[\underline{F^{+i}(0)} \, \mathcal{U}[0,\xi] \; \underline{F^{+j}(\xi)} \, \mathcal{U}'[\xi,0] \right] | P, S_L \rangle_{\xi^+=0}$$

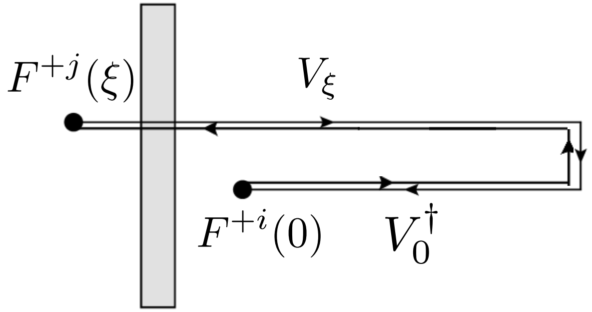
- Gluon helicity is a **very different object** than quark helicity
- A **circular flow** of the gluon field-strength:
  - ➢ Requires **preserving azimuthal correlations**

Two Different Gluon Distributions

"Dipole"



"Weizsäcker-Williams"



	Inclusive	Single inclusive	DIS dijet	γ +jet	dijet in pA
xG_{WW}	×	×	√	×	√
xG_{DP}	√	√	×	√	√

*B.-W. Xiao, Nucl. Phys. **A967** (2017) 257*

- There are **two distinct kinds of gluon TMDs** with different **gauge link structures**.
- The two are **measured experimentally** in **different processes**

The Gluon Helicity Operators at Small x

*Y. Kovchegov, D. Pitonyak, **M.S.**, JHEP **1710** (2017) 198*

$$g_{1L}^{G\,dip}(x, k_T^2) = \frac{-4i}{g^2 (2\pi)^3} \int d^2 x_{10}\, d^2 b_{10}\;\; e^{+i\underline{k}\cdot \underline{x}_{10}}\; \underline{k_\perp^i\, \epsilon_T^{ij}} \left\{ \left\langle \mathrm{tr}\left[V_{\underline{0}}\, \underline{(V_{\underline{1}}^{pol\,\dagger})_\perp^j} \right] \right\rangle + \mathrm{c.c.} \right\}$$

$$g_{1L}^{G\,WW}(x, k_T^2) = \frac{4}{g^2 (2\pi)^3} \int d^2 x_{10}\, d^2 b_{10}\; e^{i\underline{k}\cdot \underline{x}_{10}}\; \underline{\epsilon_T^{ij}} \left\langle \mathrm{tr}\left[\underline{(V_{\underline{1}}^{pol})_\perp^i}\, V_{\underline{1}}^\dagger\;\; V_{\underline{0}} \left(\underline{\frac{\partial}{\partial (x_0)_\perp^j}} V_{\underline{0}}^\dagger \right) \right] + \mathrm{c.c.} \right\rangle$$

- The two different gluon helicity distributions correspond to **different operators at small x**
- Both invoke a **circular flow (curl) of a preferred direction** in the polarized Wilson line

The Difference Between Quark and Gluon Polarization

*Y. Kovchegov, D. Pitonyak, **M.S.**, JHEP **1710** (2017) 198*

- Polarized quarks couple to a **local curl** of the gluon field

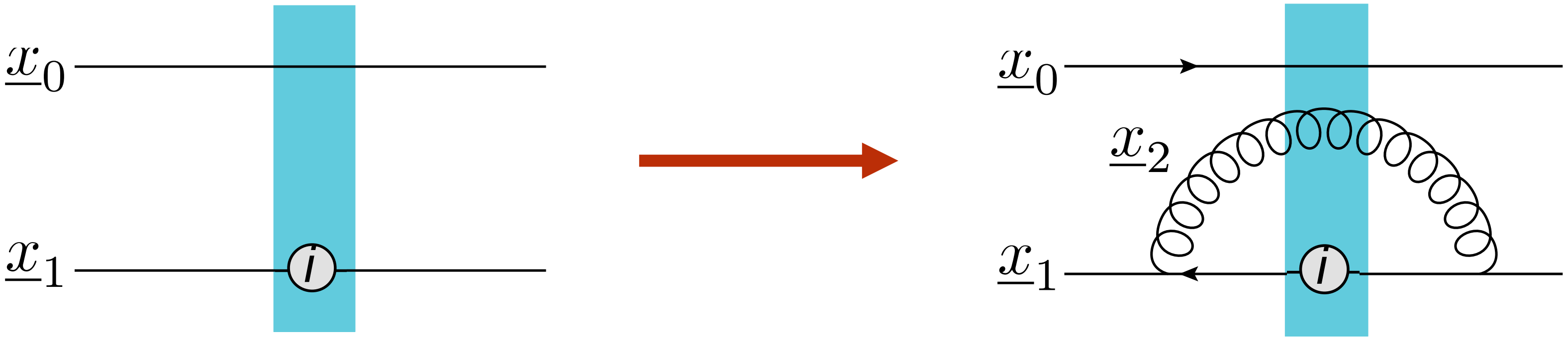
$$V_{\underline{x}}^{pol}\Big|_{\text{quarks}} = \int dx^- \, V_{\underline{x}}[\infty, x^-] \left(igp^+ \, \underline{\nabla} \times \underline{A}(x) \right) V_{\underline{x}}[x^-, -\infty]$$

- Polarized gluons couple to a **global curl**, after multiple rescattering

$$\underline{\nabla} \times \underline{(V_{\underline{x}})}\Big|_{\text{gluons}} = \underline{\nabla} \times \left[\int dx^- \, V_{\underline{x}}^{pol}[\infty, x^-] \left(igp^+ \, \underline{A}(x) \right) V_{\underline{x}}[x^-, -\infty] \right]$$

➢ That **azimuthal correlation** can **get washed out by multiple scattering**.

Quarks are Forever, but Gluons can Forget



- Real unpolarized emissions are **isotropic** and can **wash out the azimuthal correlations** necessary for gluon helicity.
- These **vanish** after **angular averaging**.

$$\int d^2x_2 \to 0$$

- Leads to a **depletion** of the gluon distribution

Gluon Helicity: Evolution Equations

*Y. Kovchegov, D. Pitonyak, **M.S.**, JHEP **1710** (2017) 198*

$$G_2(x_{10}^2, zs) = G_2^{(0)}(x_{10}^2, zs) - \left(\frac{\alpha_s N_c}{3\pi} \frac{1}{\alpha_h^q} G_0 \right) \left(zs\, x_{10}^2 \right)^{\alpha_h^q} \ln \frac{1}{x_{10}\Lambda}$$
$$- \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{1}{x_{10}^2 s}}^{z} \frac{dz'}{z'} \int\limits_{\frac{1}{z' s}}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \Gamma_2(x_{10}^2, x_{21}^2, z' s),$$

Quark helicity evolution

Neighbor dipole

$$\Gamma_2(x_{10}^2, x_{21}^2, z' s) = G_2^{(0)}(x_{10}^2, z' s) - \left(\frac{\alpha_s N_c}{3\pi} \frac{1}{\alpha_h^q} G_0 \right) \left(z' s\, x_{10}^2 \right)^{\alpha_h^q} \ln \frac{1}{x_{10}\Lambda}$$
$$- \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{1}{x_{10}^2 s}}^{z'} \frac{dz''}{z''} \int\limits_{\frac{1}{z'' s}}^{\min\left[x_{10}^2 \, , \, x_{21}^2 \frac{z'}{z''} \right]} \frac{dx_{31}^2}{x_{31}^2} \Gamma_2(x_{10}^2, x_{31}^2, z'' s).$$

- Gluon helicity evolution has **similar structure to quark helicity evolution**
- Receives **feed-in** from fluctuating into a **polarized quark**

Gluon Helicity: Evolution Equations

~20% smaller exponent

$$\left.\begin{aligned} G_2(x_\perp^2, zs) &\sim (zs)^{\alpha_h^G} \\ g_1^{G\,,\,dip}(x, k_T^2) &\sim \left(\tfrac{1}{x}\right)^{\alpha_h^G} \\ \Delta G(x, Q^2) &\sim \left(\tfrac{1}{x}\right)^{\alpha_h^G} \\ g_1^{G\,,\,WW}(x, k_T^2) &\sim \left(\tfrac{1}{x}\right)^{\alpha_h^G} \end{aligned}\right\}$$

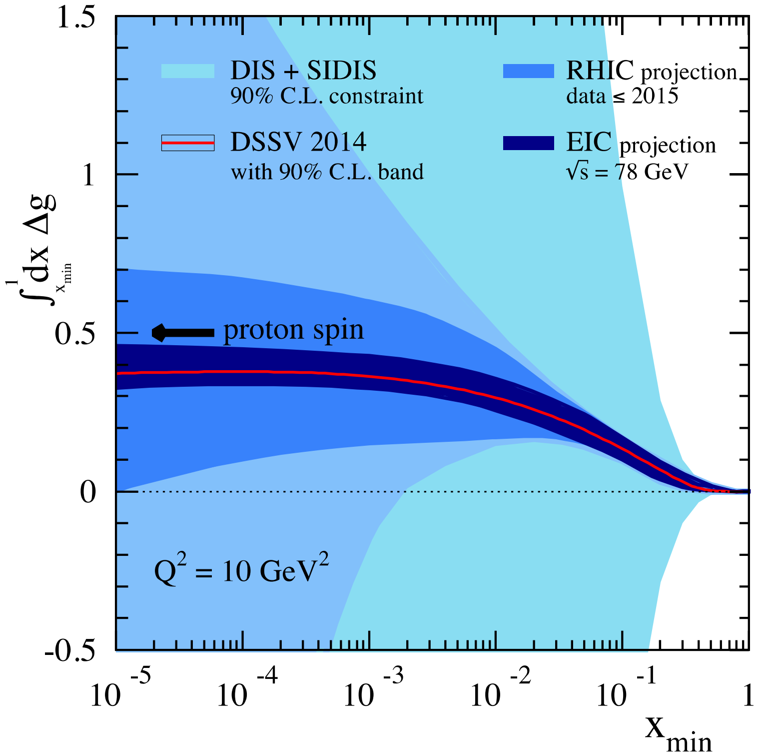
$$\boxed{\alpha_h^G = \frac{13}{4\sqrt{3}} \sqrt{\frac{\alpha_s N_c}{2\pi}}}$$

c.f.

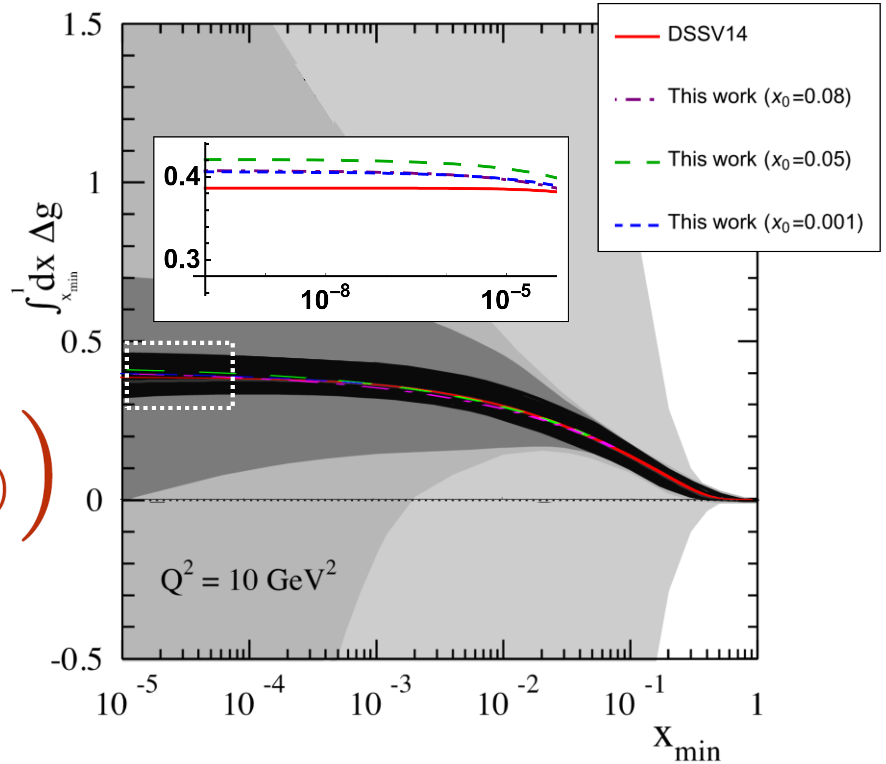
$$\boxed{\alpha_h^{q,S} = \frac{4}{\sqrt{3}} \sqrt{\frac{\alpha_s N_c}{2\pi}}}$$

- Because **multiple scattering dilutes the gluon polarization**, it **decays faster at small x** than for quarks.

E.-C. Aschenauer et al., Phys. Rev. ***D92*** *(2015) 094030*



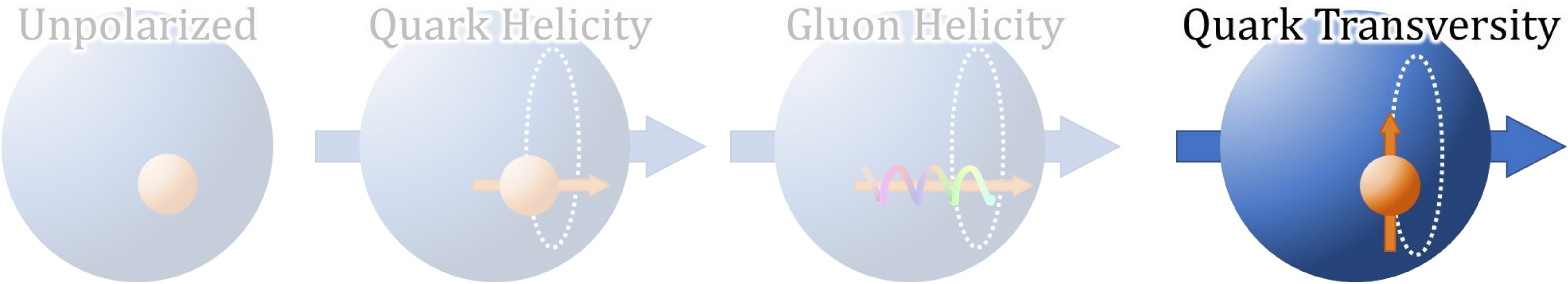
Y. Kovchegov, D. Pitonyak, ***M.S.****, JHEP* ***1601*** *(2016) 072*



$\mathcal{O}\,(5\%)$

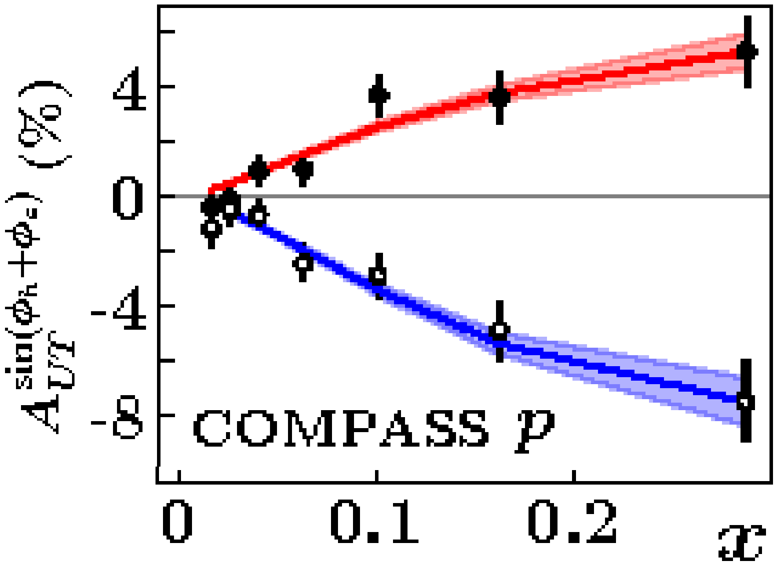
- The **enhancement** of gluon polarization at small x is **much milder** than for quarks
- **Not very important** for constraining the **gluon contribution** to the proton spin

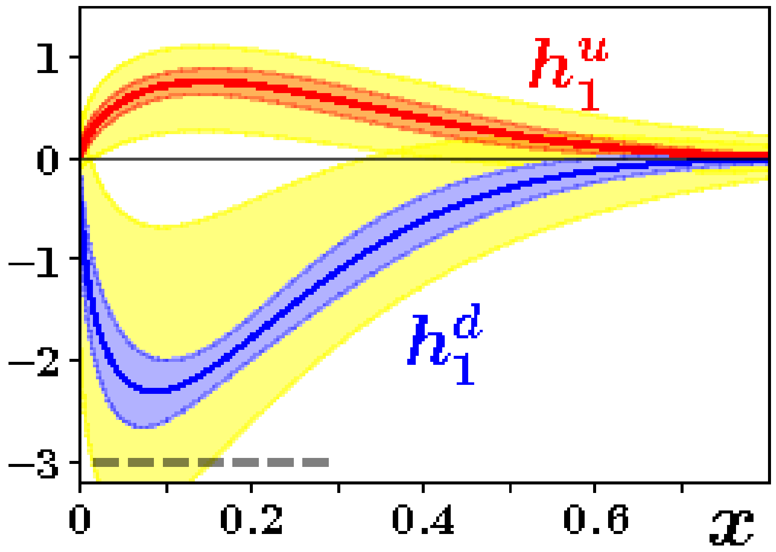
A Taste of Quark Transversity



Quark Transversity: What Do We Know?

H.-W. Lin et al., Phys. Rev. Lett. ***120*** *(2018) 152502*





- Transversity is **notoriously difficult to extract**
- **Chiral odd PDF** convoluted with **another chiral odd distribution** in observables

A. Courtoy et al., Phys. Rev. Lett. ***115*** *(2015) 162001*
T. Bhattacharya et al., Phys. Rev. Lett. ***115*** *(2015) 212002*

Tensor Charge

$$g_T^q(Q^2) = \int_0^1 dx \, \left[h_1^q(x, Q^2) - h_1^{\bar{q}}(x, Q^2) \right]$$

Potential imprints of BSM Physics

Neutron EDM:

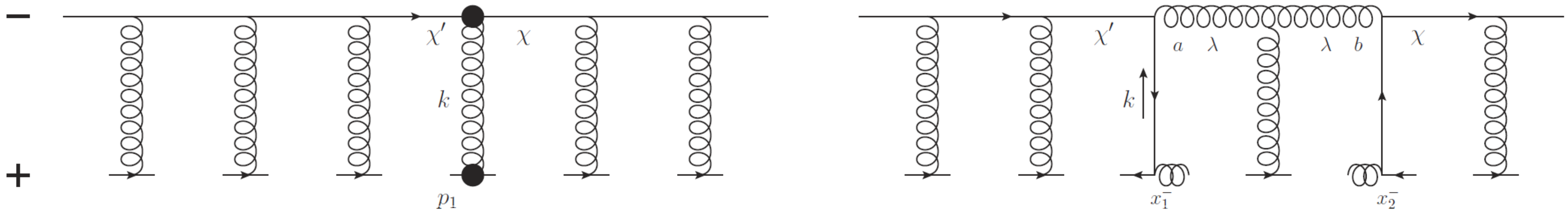
$$\langle n | \, \bar{\psi}(0) \, \sigma^{\mu\nu} \gamma^5 \, \psi(0) \, | n \rangle$$

Neutron Beta Decay:

$$\langle p | \, \bar{u}(0) \, \sigma^{\mu\nu} \gamma^5 \, d(0) \, | n \rangle$$

- The **flavor non-singlet moment** of transversity gives the **tensor charge**
- Like the proton spin sum rules, requires **extrapolation to small x**
- Sensitive to contributions from **Beyond the Standard Model physics**

*Y. Kovchegov and **M. S.**., arXiv: 1808.10354*



Coupling to gluons is proportional to the mass

$$V_{\underline{x}}^{pol,T} = \frac{2g\,m\,(p_1^+)^2}{\underline{s^2}} \int\limits_{-\infty}^{+\infty} dx^- \; V_{\underline{x}}[+\infty, x^-]\, S^i \; \left[i\,\epsilon^{ij}\, F^{-j}(x^-, \underline{x})\right] \; V_{\underline{x}}[x^-, -\infty]$$

$$- \frac{g^2\,(p_1^+)^2}{\underline{2\,s^2}} \int\limits_{-\infty}^{\infty} dx_1^- \int\limits_{x_1^-}^{\infty} dx_2^- \; V_{\underline{x}}[+\infty, x_2^-]\, t^b\, \psi_\beta(x_2^-, \underline{x})\, U_{\underline{x}}^{ba}[x_2^-, x_1^-] \left[\left(i\,\gamma^5\, \underline{S} \cdot \overleftarrow{\underline{D}} - \underline{S} \times \overleftarrow{\underline{D}} \right) \gamma^+ \gamma^- \right.$$

$$\left. + \left(i\,\gamma^5\, \underline{S} \cdot \underline{D} - \underline{S} \times \underline{D} \right) \gamma^- \gamma^+ \right]_{\alpha\beta} \bar{\psi}_\alpha(x_1^-, \underline{x})\, t^a\, V_{\underline{x}}[x_1^-, -\infty].$$

- Coupling to transverse spin is suppressed by **two powers** of energy

The Transversely Polarized Dipole

Flavor singlet / non-singlet

$$h_{1T} \sim h_{1T}^{\perp} \sim \frac{(zs)^2}{2N_c} \mathrm{Re} \left\langle \mathrm{tr} \left[V_0 V_1^{pol\, T\, \dagger} \right] \pm \mathrm{tr} \left[V_1^{pol\, T} V_1^{\dagger} \right] \right\rangle$$

- Transversity (and pretzelosity) are both governed by a similar **transversely-polarized dipole amplitude**
- The flavor singlet and non-singlet transversities evolve **differently**

Nonsinglet Quark Transversity

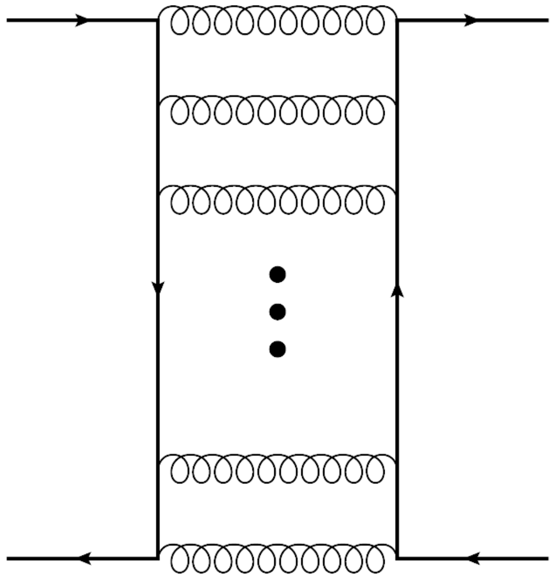
- The **flavor-singlet** evolution is **complicated**…

- But the **flavor non-singlet** evolution for transversity has exactly the same structure as for **non-singlet helicity**

$$h_{1T}^{NS}(x, k_T^2) \sim h_{1T}^{\perp\, NS}(x, k_T^2) \sim \left(\frac{1}{x}\right)^{-1+2\sqrt{\frac{\alpha_s N_c}{2\pi}}}$$

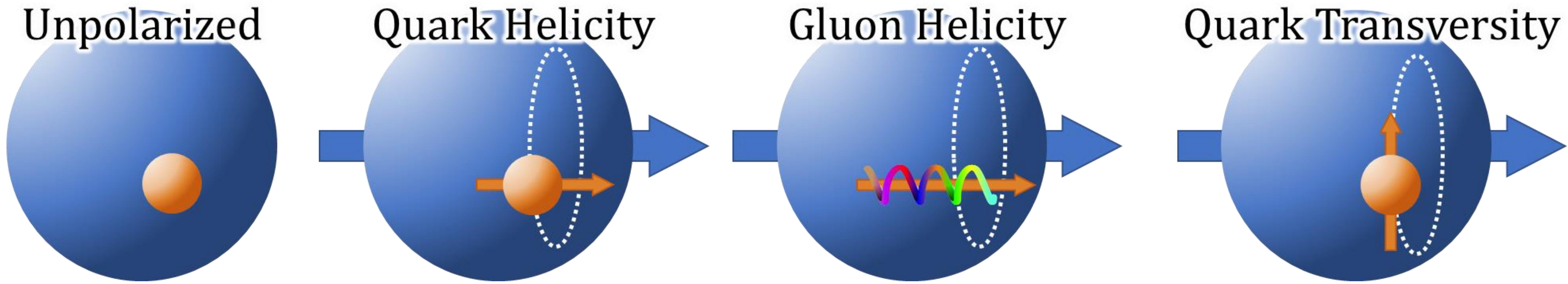
- Phenomenologically: **very small**, not likely to contribute much to the tensor charge.

Itakura et al., Nucl. Phys. ***A730*** *(2004) 160*



$$\sim x^{0.243}$$

Conclusions and Outlook



The General Procedure

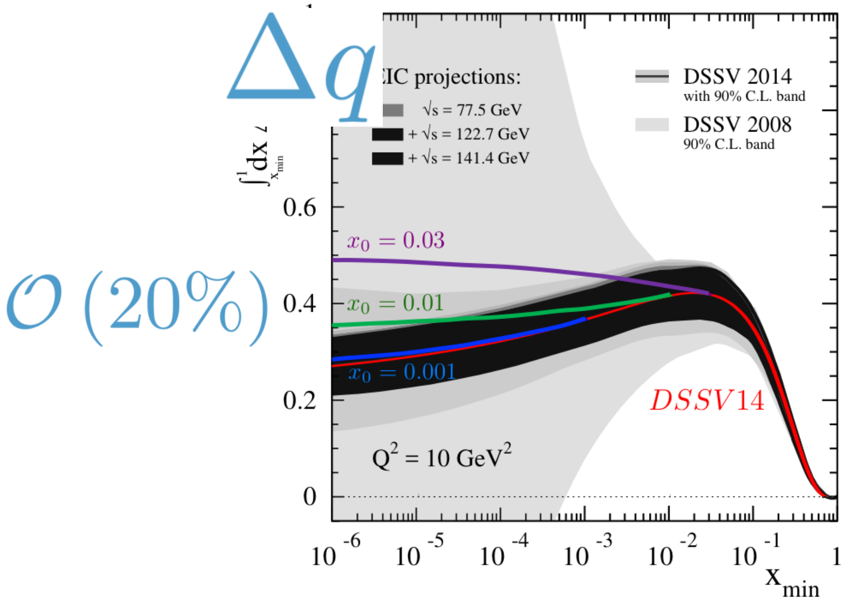
1. Approximate the **general operator** with **small-x kinematics**
2. Construct the appropriate **polarized Wilson lines** and dipole operators
3. Evolve using the **background field method**
4. Try to solve using **Laplace-Mellin** techniques

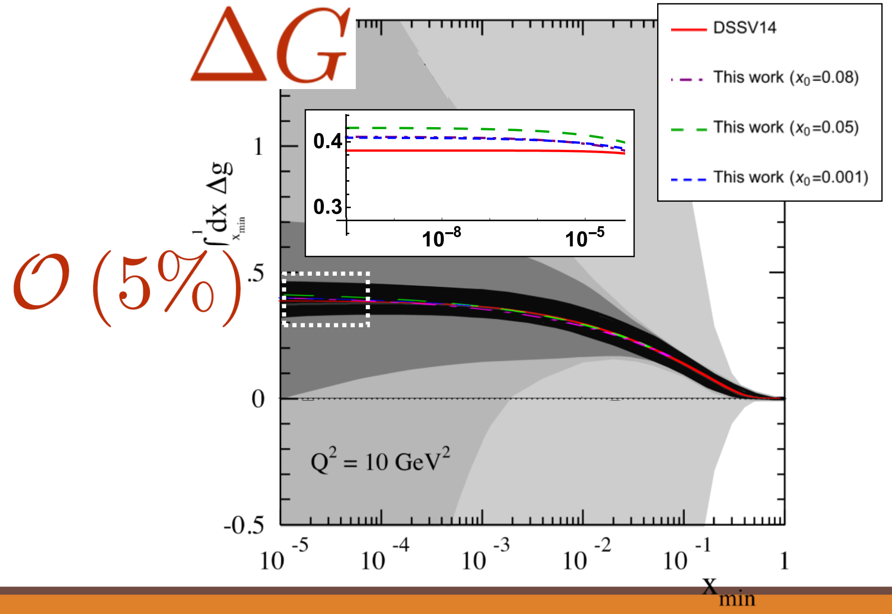
Other Operators of Interest

*Y. Hatta et al., Phys. Rev. **D95** (2017) 114032*
*S. Bhattacharya et al., Phys. Lett. **B771** (2017) 396*

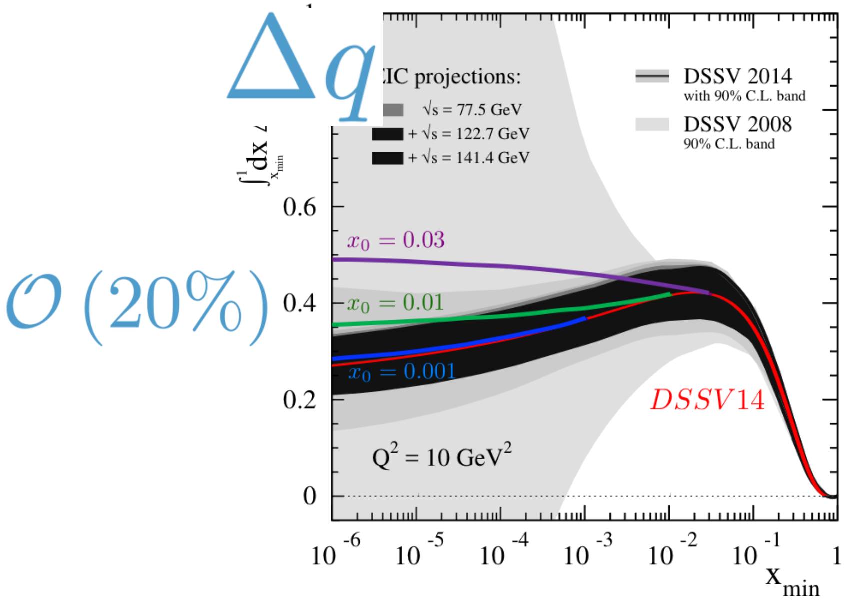
- Quark and gluon **Orbital Angular Momentum**
  - At small x, the operators appear to be governed by the **same polarized dipole as the gluon helicity**
  - Enter with **different weights** in the integrand…
- **Gluonic transversity** (for the deuteron, etc.)
- **Other TMDs**: worm-gear, etc…
  - Do they follow the evolution of the **t-channel spin exchange**?

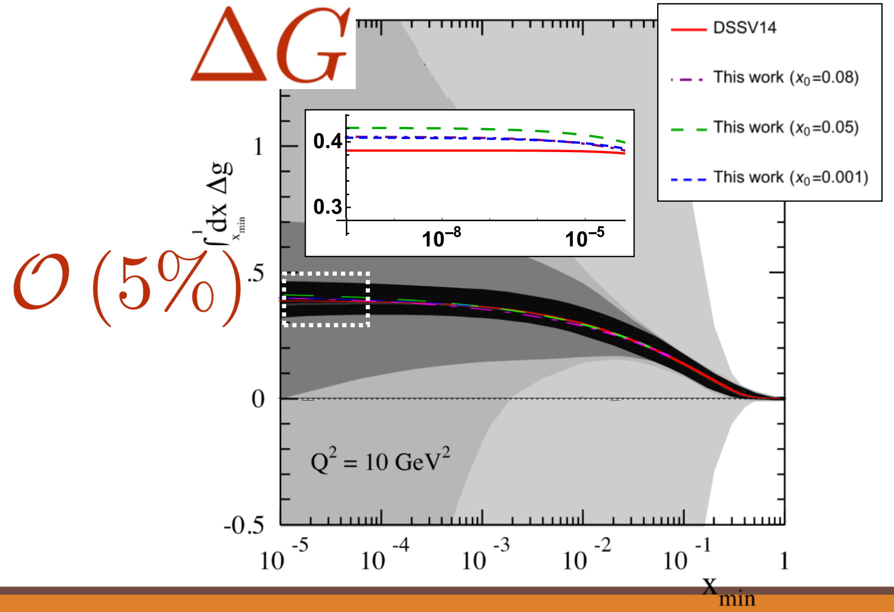
- We are **building a framework** to study the whole **map of hadronic structure at small x**
  - **Quark helicity**: Potentially large contribution
  - **Gluon helicity**: More modest contribution
  - **Non-singlet transversity**: Very small (?)

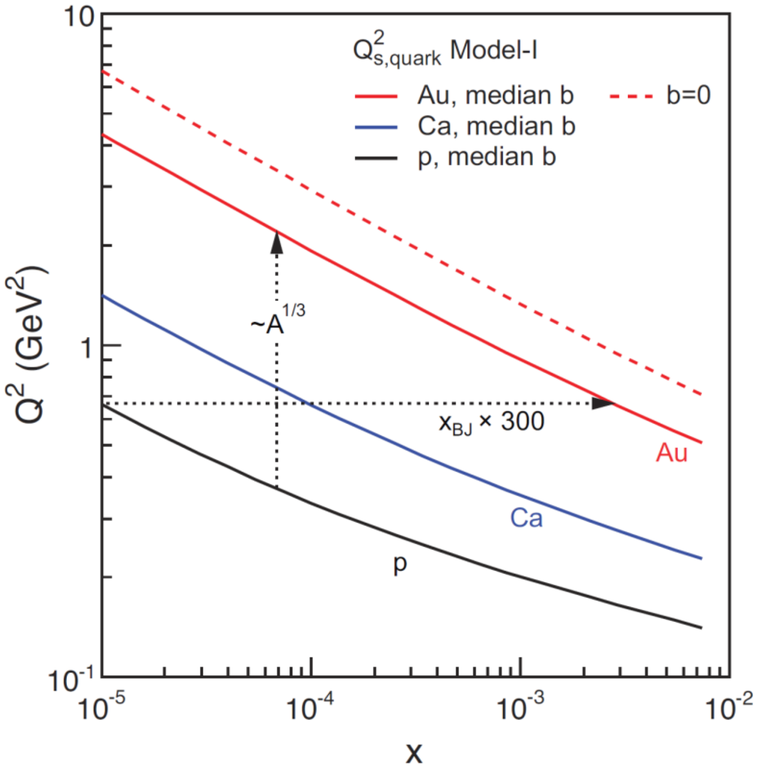


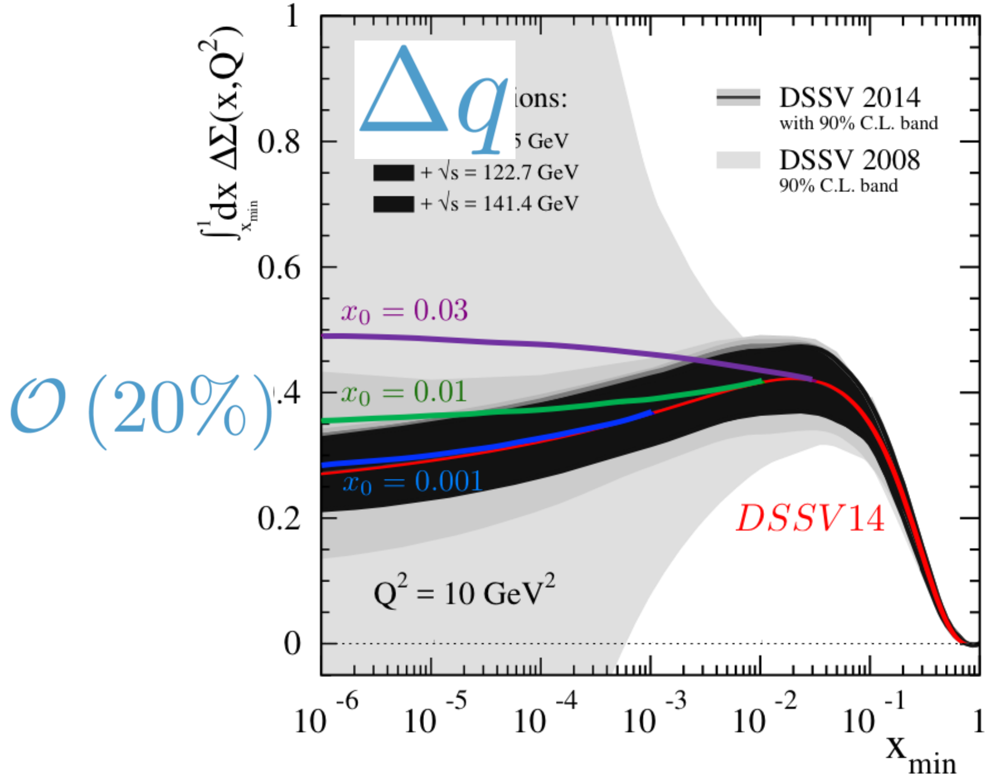


- **Future applications**:
  - **OAM** and other operators
  - Systematically improve the **precision**
  - Connections with **other subfields** (jets, heavy ions, etc.)









- How great a **lever arm in x** do we need to really **solve the Proton Spin Puzzle**?
- Can we **quantify this** to better inform the **design capabilities of an EIC**?
- What **other tools** can help fill the gap? (Quasi-PDFs?)