Calculations of Nucleon EDMs on a Lattice

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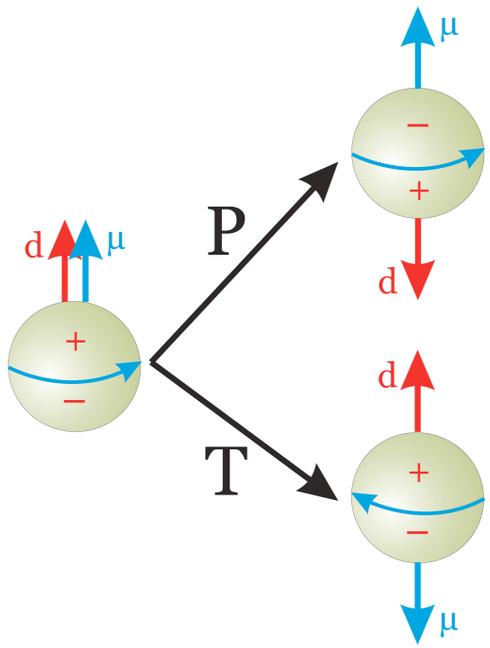
Outline

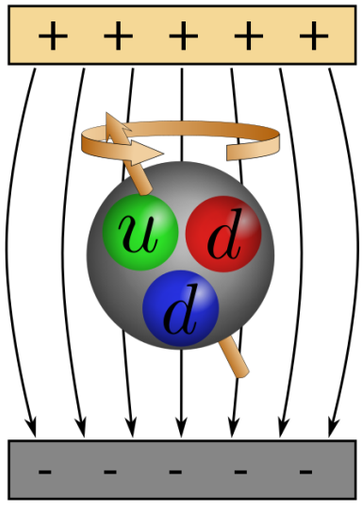
- Nucleon Electric Dipole Moments: Introduction
  - *Motivation*
  - *Experimental status & outlook*
  - *Lattice methodology*
- Studies of θ_{QCD} -induced nucleon EDM
  - *Noise reduction with subvolume top.charge sampling*
  - *Results from lattices with heavy pions $m_\pi \gtrsim 330$ MeV*
  - *Outlook for physical point calculations*
- Novel method for θ_{QCD}-induced physics
  - *Electric dipole moments induced by quark chromo-EDM*
- Summary & Outlook

Nucleon Electric Dipole Moments



$$\vec{d}_N = d_N \frac{\vec{S}}{S}$$

$$\mathcal{H} = -\vec{d}_N \cdot \vec{E}$$



EDMs are the most sensitive probes of CPv:

- Signals for beyond SM physics

  (SM = 10^{-5} of the current exp.bound)
- Prerequisite for Baryogenesis
- Strong ~~CP~~ problem : θ_{QCD}-induced EDM?

A.Sakharov's conditions for baryon asymmetry in the Universe [JETP letters, 1967]

- ~~P~~, ~~CP~~ symmetry violation
- Baryon number violation
- non-equilibrium transition

$$\langle N_{p'}|J^\mu|\bar{N}_p\rangle_{\cancel{CP}} = \bar{u}_{p'}\left[F_1\gamma^\mu + (F_2 + iF_3\gamma_5)\frac{\sigma^{\mu\nu}(p'-p)_\nu}{2m_N}\right]u_p$$

Dirac

Pauli (anom.magnetic)

Electric dipole

Experimental Outlook

Current nEDM limits:

- $|d_n| < 2.9 \times 10^{-26}\, e \cdot \mathrm{cm}$ (stored UC neutrons)
  [Baker et al, PRL97: 131801(2006)]
- $|d_n| < 1.6 \times 10^{-26}\, e \cdot \mathrm{cm}$ (^{199}Hg)
  [Graner et al, PRL116:161601(2016)]
-

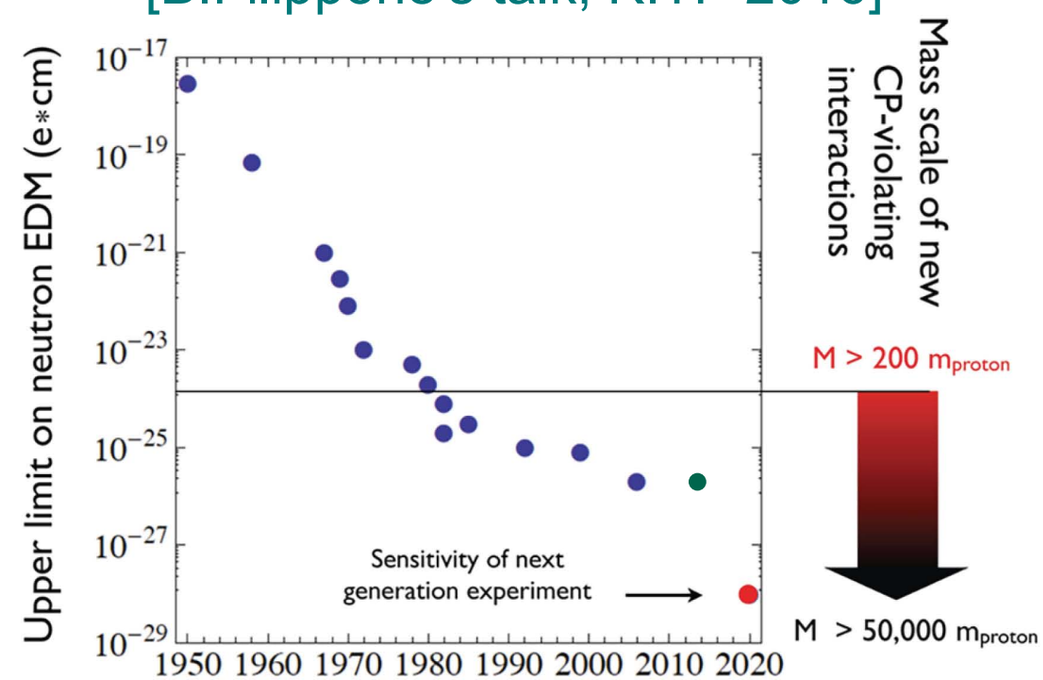
  SM prediction from CKM CP

	10^{-28} *e* cm
CURRENT LIMIT	<300
Spallation Source @ORNL	**< 5**
Ultracold Neutrons @LANL	~30
PSI EDM	<50 (I), **<5 (II)**
ILL PNPI	<10
Munich FRMII	**< 5**
RCMP TRIUMF	<50 (I), **<5 (II)**
JPARC	**< 5**
Standard Model (CKM)	*< 0.001*

Future nEDM sensitivity :

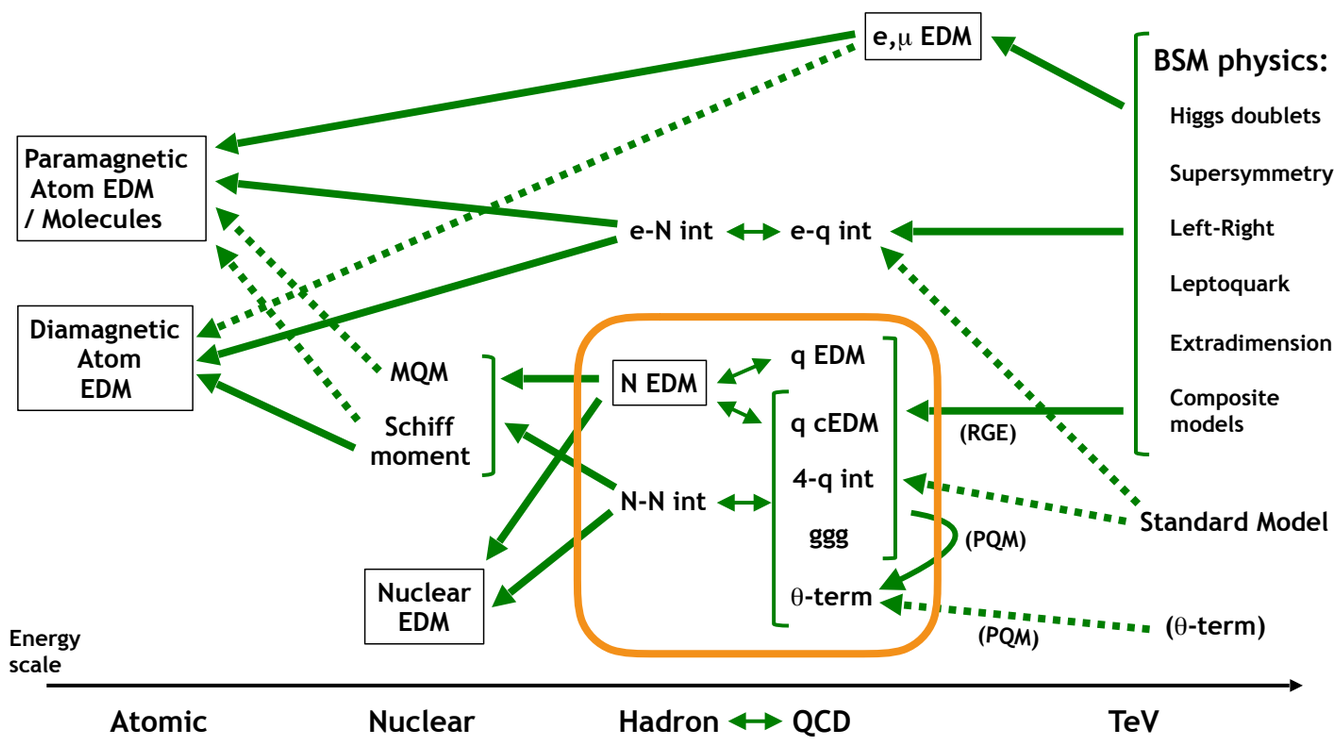
- 1–2 years : next best limit?
- 3–4 years : x10 improvement
- 7-10 years : x100 improvement

[B.Filippone's talk, KITP 2016]



- Other nuclear EDM experiments:
  light nuclei in storage rings, octupole-deformed ^{225}Ra, etc

Nucleon EDMs: a Window into New Physics



[Yamanaka et al, EPJA53:54 (2017)]

- Effective quark-gluon CPv interactions: dimension $\Longleftrightarrow$ scale of BSM physics

  [Engel, Ramsey-Musolf, van Kolck, Prog.Part.Nucl.Phys. 71:21 (2013)]

$$\mathcal{L}_{eff} = \sum_i \frac{c_i}{[\Lambda_{(i)}]^{d_i-4}} \mathcal{O}_i^{[d_i]}$$

$d{=}4$: θ_{QCD}

$d{=}5(6)$: quark EDM, chromo-EDM

$d{=}6$: 4-fermion CPv, 3-gluon (Weinberg)

$$d_{n,p}$$
$$F_3^{n,p}(Q^2)$$

$$c_i \Longleftrightarrow d_{n,p} \quad ?$$

required to constrain θ_{QCD}, c_{cEDM}, ...

- Lattice QCD:

  quark-gluon CPv $\Longrightarrow$ nucleon/pion CPv

  (Nucleon EDM + πNN ~~CP~~ interactions)

$$d_{n,p} = d_{n,p}^{\theta}\theta_{\rm QCD} + d_{n,p}^{cEDM} c_{cEDM} + \dots$$

CPv in QCD on a Lattice

- CP-odd interaction as perturbation
  [S. Aoki et al (2005); F. Berruto et al (2005); A.Shindler et al (2015) ; C. Alexandrou et al (2015) ; E. Shintani et al (2016)]

$$\langle \mathcal{O} \ldots \rangle_{\not{CP}} = \langle \mathcal{O} \ldots \rangle_{CP-even} - i\theta \langle Q \cdot \mathcal{O} \ldots \rangle_{CP-even} + O(\theta^2)$$

$\not{CP}$ coupling

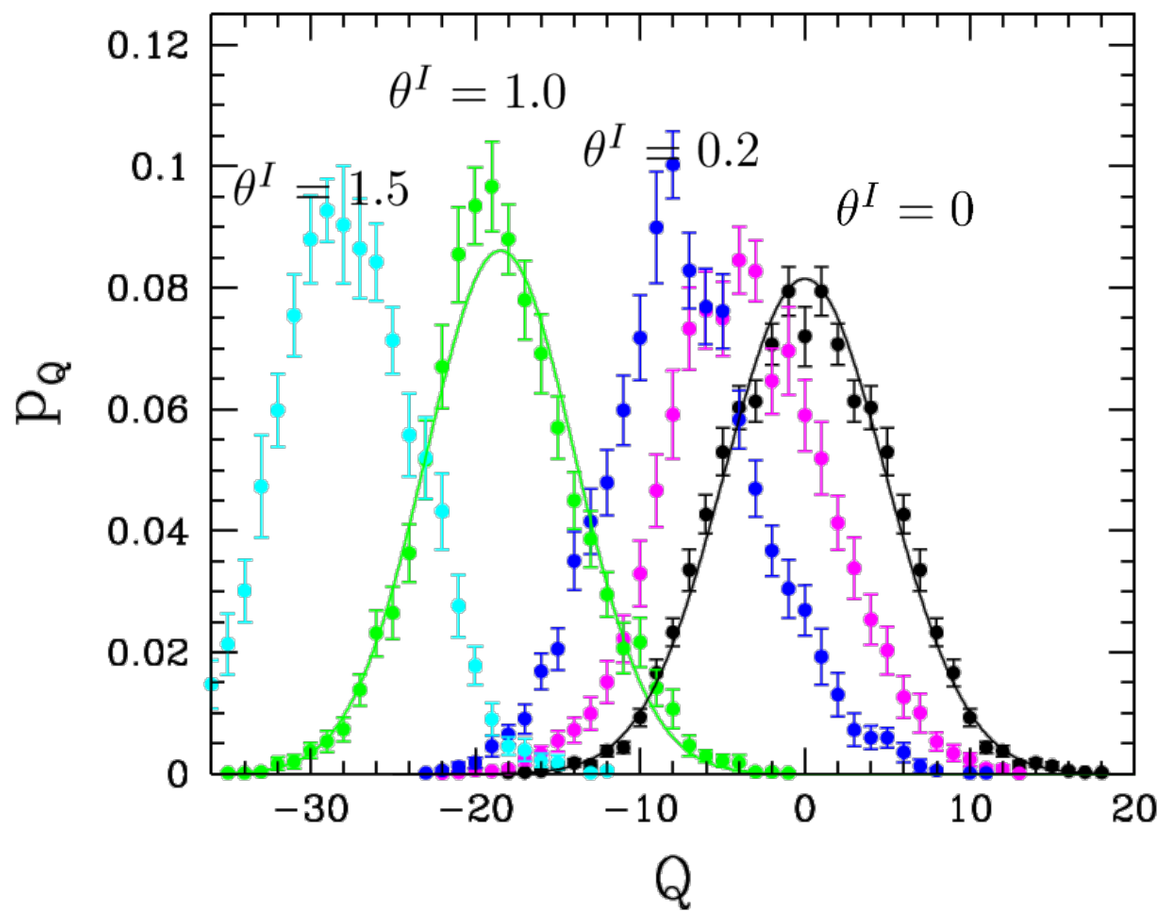
$\not{CP}$ operator: $G\tilde{G}$, cEDM, $GG\tilde{G}$(Weinberg), ...

(n) → (n+1) correlation function

- Finite (imaginary) CPv: θ^I_{QCD}
  [R.Horsley et al (2008) ; F.K.Guo et al (2015)]

$$\langle \mathcal{O} \ldots \rangle_\theta \sim \int \mathcal{D}U \, e^{-S-\theta^I Q} \, (\mathcal{O} \ldots)$$

require dedicated QCD simulation
$\Longrightarrow$ better sampling of Q≠0 sectors

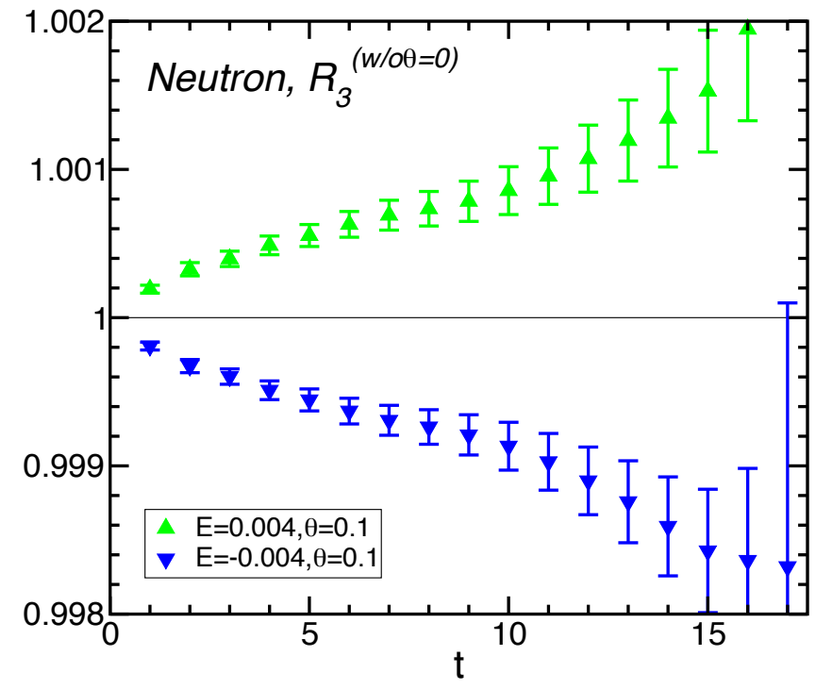


Determination of Nucleon EDM

- Nucleon energy shift in uniform electric field
  [S.Aoki et al '89 ; E.Shintani et al '06;
  E.Shintani et al, PRD75, 034507(2007)]

$$\langle N(t)\bar{N}(0)\rangle_{\theta,\vec{E}} \sim e^{-(E\pm\vec{d}_N\cdot\vec{E})t}$$

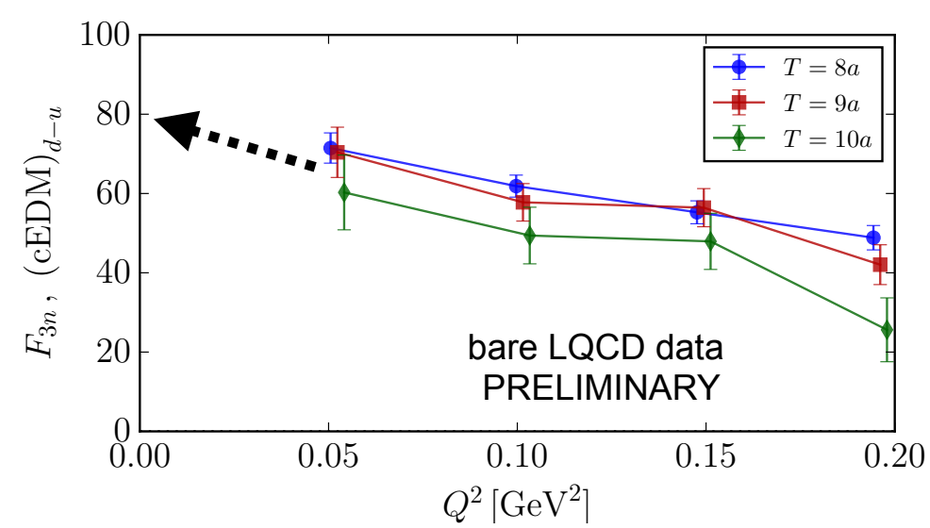
Euclidean lattice:
Real-valued $\bar{E} \Longrightarrow$ time-BC violated
Imag-valued $\bar{E} \Longrightarrow$ imaginary shift in m_N



- P,T-odd Form Factor $d_N=F_3(0)/2m_N$
  [E.Shintani et al '05, '15 ; F.Berruto et al '05 ; A.Shindler et al '15 ; C.Alexandrou et al'15]

$$\langle N_{p'}|\bar{q}\gamma^\mu q|N_p\rangle_{\not{CP}} = \bar{u}_{p'}\left[F_1\gamma^\mu + (F_2 + i\boxed{F_3}\gamma_5)\frac{i\sigma^{\mu\nu}(p'-p)_\nu}{2m_N}\right]u_p$$

Require extrapolation $F_3(Q^2\to 0)$



Nucleon "Parity Mixing"

CPv interaction induces a chiral phase in nucleon wave functions on a lattice

$$\langle \mathrm{vac}|N|p,\sigma\rangle_{\not{CP}} = e^{i\alpha\gamma_5} u_{p,\sigma} = \tilde{u}_{p,\sigma}$$

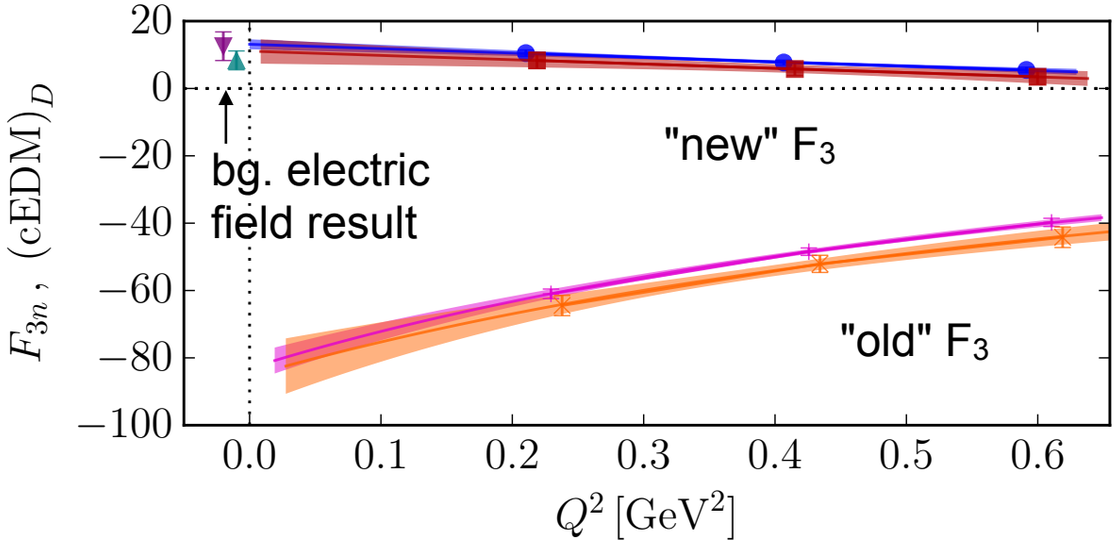
$$N \to u\,[u^T C\gamma_5 d] \qquad \tilde{u}_{p,\sigma} \to \sum_\sigma \tilde{u}_{p,\sigma}\bar{\tilde{u}}_{p,\sigma} \sim \left(-i\not{p}_{\mathcal{E}} + m_N e^{2i\alpha\gamma_5}\right)$$

[M.Abramczyk, S.Aoki, S.N.S, et al (2017) arXiv:1701.07792]

EDM and MDM are defined with positive-parity spinors

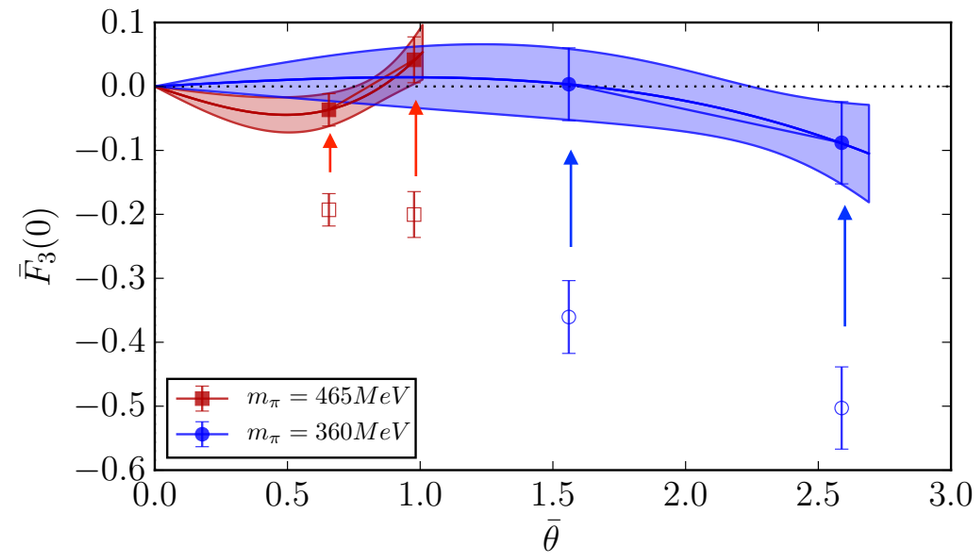
$$\langle N_{p'}|\bar{q}\gamma^\mu q|N_p\rangle_{\not{CP}} = \bar{u}_{p'}\left[F_1\gamma^\mu + (F_2 + iF_3\gamma_5)\frac{i\sigma^{\mu\nu}(p'-p)_\nu}{2m_N}\right]u_p \quad , \quad \text{with} \quad \begin{array}{l}\gamma_4 u = +u \\ \bar{u}\gamma_4 = +\bar{u}\end{array}$$

check: cEDM-induced EDM / EDFF
comparison of form factor F_3
to energy shift in *background* $\bar{E}$=*const*



correction to F_3 results prior to 2017:

$$\text{“}d_{n,p}\text{”} \approx [d_{n,p}]_{\mathrm{true}} - 2\alpha\frac{\kappa_{n,p}}{2m_N}$$



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$$\downarrow \qquad\qquad\qquad \downarrow$$

$$u\,[u^T C\gamma_5 d] \qquad\qquad \sum_\sigma \tilde{u}_{p,\sigma}\bar{\tilde{u}}_{p,\sigma} \sim \left(-i\not{p}_{\mathcal{E}} + m_N e^{2i\alpha\gamma_5}\right)$$

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EDM and MDM are defined with positive-parity spinors

$$\langle N_{p'}|\bar{q}\gamma^\mu q|N_p\rangle_{\not{CP}} = \bar{u}_{p'}\left[F_1\gamma^\mu + (F_2 + iF_3\gamma_5)\frac{i\sigma^{\mu\nu}(p'-p)_\nu}{2m_N}\right]u_p \quad , \quad \text{with} \quad \begin{array}{l}\gamma_4 u = +u \\ \bar{u}\gamma_4 = +\bar{u}\end{array}$$

Θ-nEDM

		m_π [MeV]	m_N [GeV]	F_2	α	$\tilde{F}_3$	F_3
[ETMC 2016]	n	373	1.216(4)	$-1.50(16)^a$	$-0.217(18)$	$-0.555(74)$	0.094(74)
[Shintani et al 2005]	n	530	1.334(8)	$-0.560(40)$	$-0.247(17)^b$	$-0.325(68)$	$-0.048(68)$
	p	530	1.334(8)	0.399(37)	$-0.247(17)^b$	0.284(81)	0.087(81)
[Berruto et al 2006]	n	690	1.575(9)	$-1.715(46)$	$-0.070(20)$	$-1.39(1.52)$	$-1.15(1.52)$
	n	605	1.470(9)	$-1.698(68)$	$-0.160(20)$	0.60(2.98)	1.14(2.98)
[Guo et al 2015]	n	465	1.246(7)	$-1.491(22)^c$	$-0.079(27)^d$	$-0.375(48)$	$-0.130(76)^d$
	n	360	1.138(13)	$-1.473(37)^c$	$-0.092(14)^d$	$-0.248(29)$	$0.020(58)^d$

After removing the spurious contribution,

- *no lattice signal for θ_{QCD}-induced nEDM*
- *RESOLVED conflict with pheno. values, lack of $d_N \sim m_q$ scaling*

Importance of "Parity Mixing" Correction

Exact value of α_5 is critical for correct determination of EDM:

$$F_3^{\rm lat}(Q^2) \approx \frac{m}{q_3} \underbrace{\langle N_\uparrow(0)|\bar{q}\gamma_4 q|N_\uparrow(-q_3)\rangle_{\not{CP}}}_{\text{CPv matrix element}} - \underbrace{\alpha_5 G_E(Q^2)}_{\text{Sachs form factor subtraction}}$$

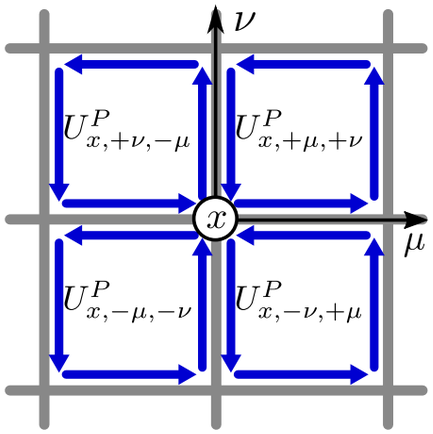
- For proton, nonzero correction ~ α_5
- For neutron, zero correction at $Q^2=0$
  however, may skew $Q^2 \to 0$ extrapolation in practice

Nucleon EDM Induced by Quark Chromo-EDM

P-,T-odd Dim-5 operator (Dim-6 with Higgs vev)

$$\mathcal{L}_{\rm cEDM} = \sum_{q=u,d} \frac{\tilde{\delta}_q}{2}\, \bar{q}\,[G_{\mu\nu}\sigma^{\mu\nu}\gamma_5]\, q \longrightarrow$$

cEDM operator on a lattice



- dim-5 operator : O(a^{-2}) mixing with dim-3 pseudoscalar density
  ⇒ evaluate&subtract p,nEDM induced by PS density $P = \bar{q}\gamma_5 q$
  [T.Bhattacharya et al, 1502.07325]

- *Chiral symmetry is important:*
  O(a) clover term in, e.g., Wilson fermion action ≡ chromo-magnetic DM

$$\mathcal{L}^{\rm clover} = a\frac{c}{4}\bar{q}\,[G_{\mu\nu}\sigma^{\mu\nu}]\, q$$

In presense of CPv, condensate is realigned $q \to e^{i\gamma_5\Omega} q$

so that $\langle {\rm vac}|\mathcal{L}_m + \mathcal{L}_{\not{CP}}|\pi^a\rangle = 0$

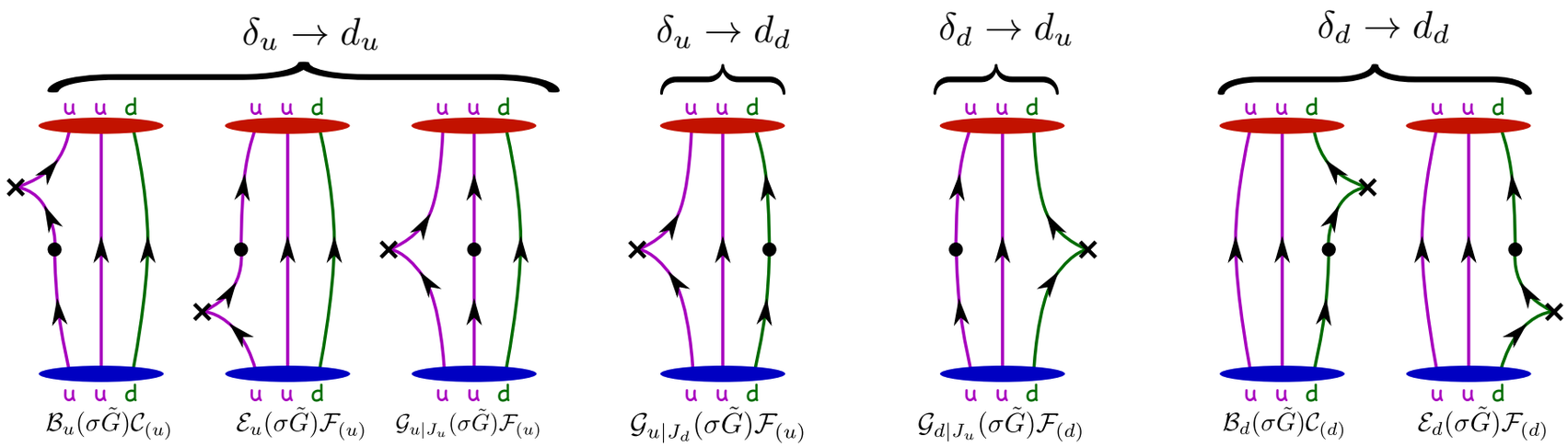
leading to mixing (chromo)EDM ⟺(chromo)MDM:

$$\delta\mathcal{L}_{\rm cEDM} = \delta(\bar{q}\,[\tilde{D}_q G_{\mu\nu}\sigma^{\mu\nu}\gamma_5]\, q) = \bar{q}\,[\{\Omega, \tilde{D}_q\} G_{\mu\nu}\sigma^{\mu\nu}]\, q) \sim \delta\mathcal{L}_{\rm cMDM}$$

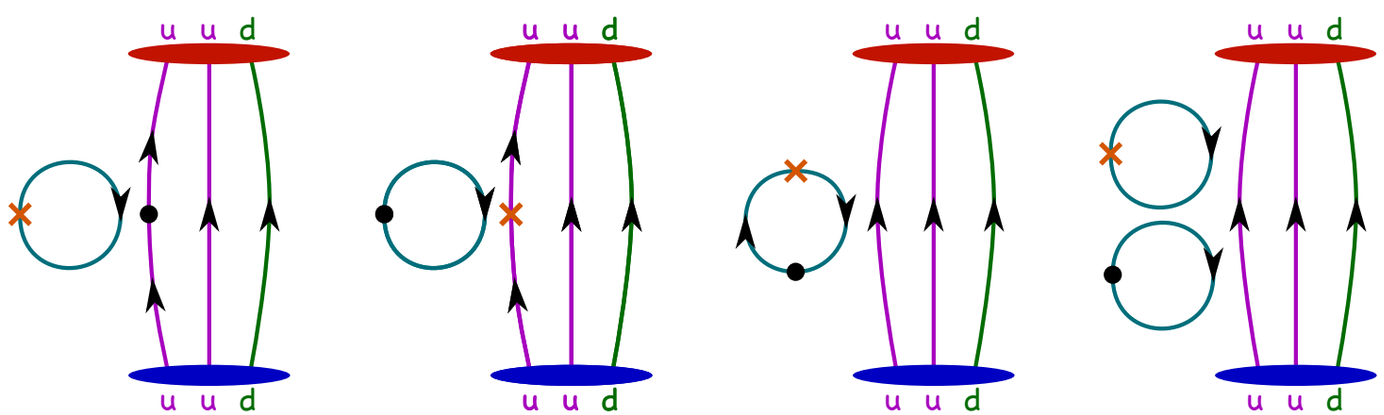
Quark-Gluon EDM: Insertions of dim-5 Operators

$$\mathcal{L}^{(5)} = \sum_q \tilde{d}_q \, \bar{q}(G\cdot\sigma)\gamma_5 q \quad \longrightarrow \quad \begin{cases} \langle N(y)\,\bar{N}(0) \int d^4x\, \bar{q}(G\cdot\sigma)\gamma_5 q\rangle \\ \langle N(y)\,[\bar{\psi}\gamma^\mu\psi]_z\,\bar{N}(0) \int d^4x\, \bar{q}(G\cdot\sigma)\gamma_5 q\rangle \end{cases}$$

- This work: Only quark-connected insertions

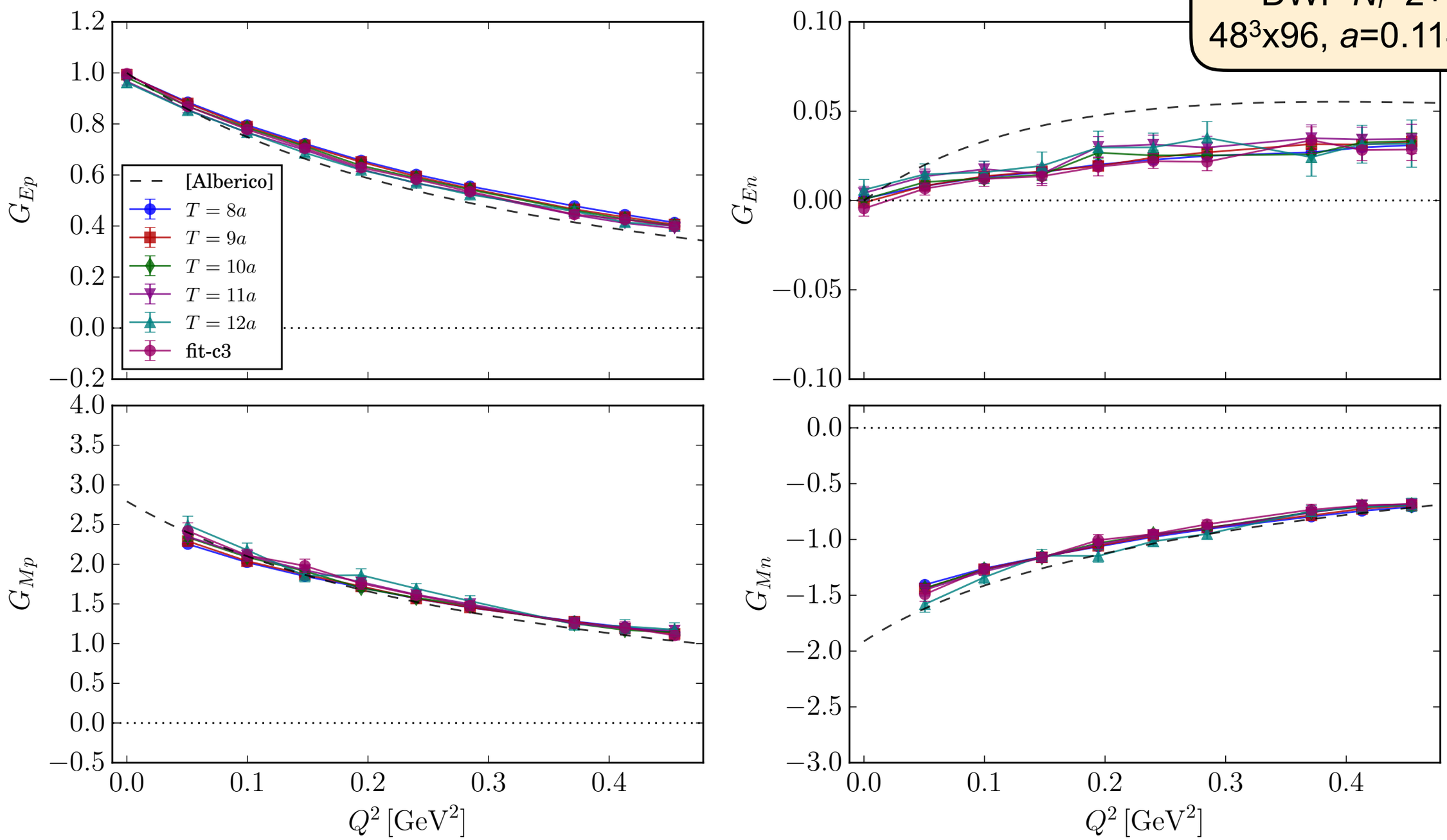


- In future: Single- and double-disconnected diagrams (contribute to isosinglet cEDM, mix with θ-term)



Nucleon Vector (Sachs) Form Factors

Physical point
DWF N_f=2+1
48^3x96, a=0.114 fm



$$G_E = F_1 - \frac{Q^2}{4m_N^2} F_2$$

$$G_M = F_1 + F_2$$

~30 M core*hours @Mira (Bluegene/Q)
[with RBC collaboration, in prep.]

Parity Mixing : cEDM and pseudoscalar(*)

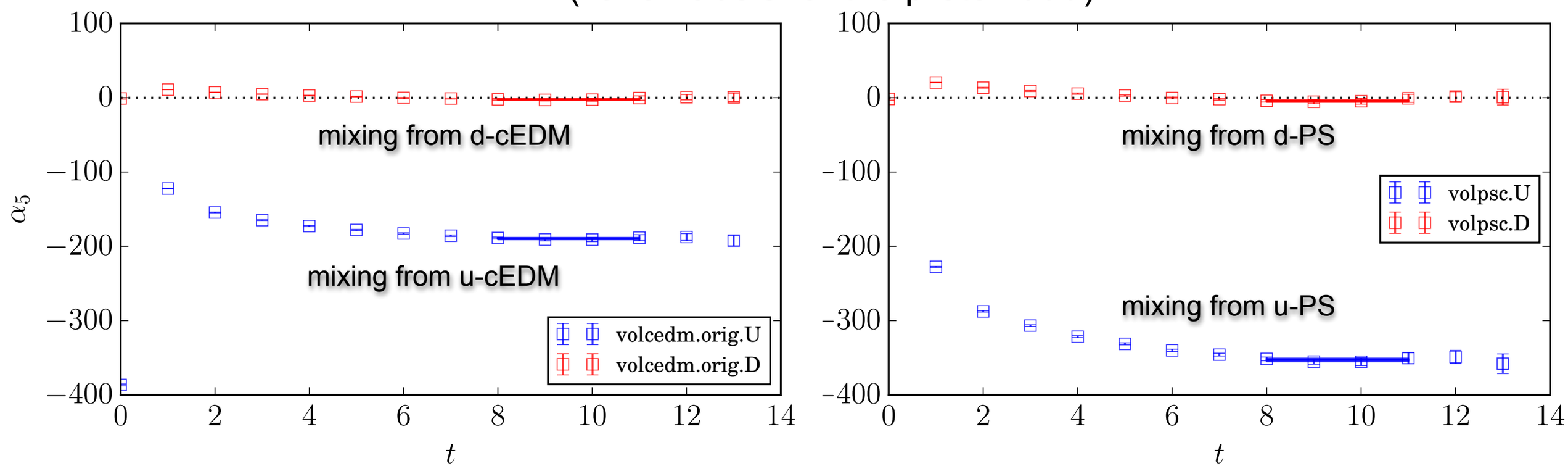
Physical point
DWF N_f=2+1
48^3x96, a=0.114 fm

$$N_\delta = \epsilon^{abc}\, u^a_\delta\, (u^{aT} \mathcal{C}\gamma_5 d^c)$$

$$\langle N(t)\bar{N}(0)\rangle_{\not{CP}} = \frac{-i\not{p} + m_N e^{2i\alpha_5\gamma_5}}{2m_N} e^{-E_N t}$$

$$\hat{\alpha}_5 = \frac{\alpha_5}{\tilde{d}} = -\frac{\mathrm{ReTr}\left[T^+\gamma_5 \cdot \delta^{\overline{CP}} C_{2pt}(t)\right]}{\mathrm{ReTr}\left[T^+ \cdot C_{2pt}(t)\right]}, \quad t \to \infty$$

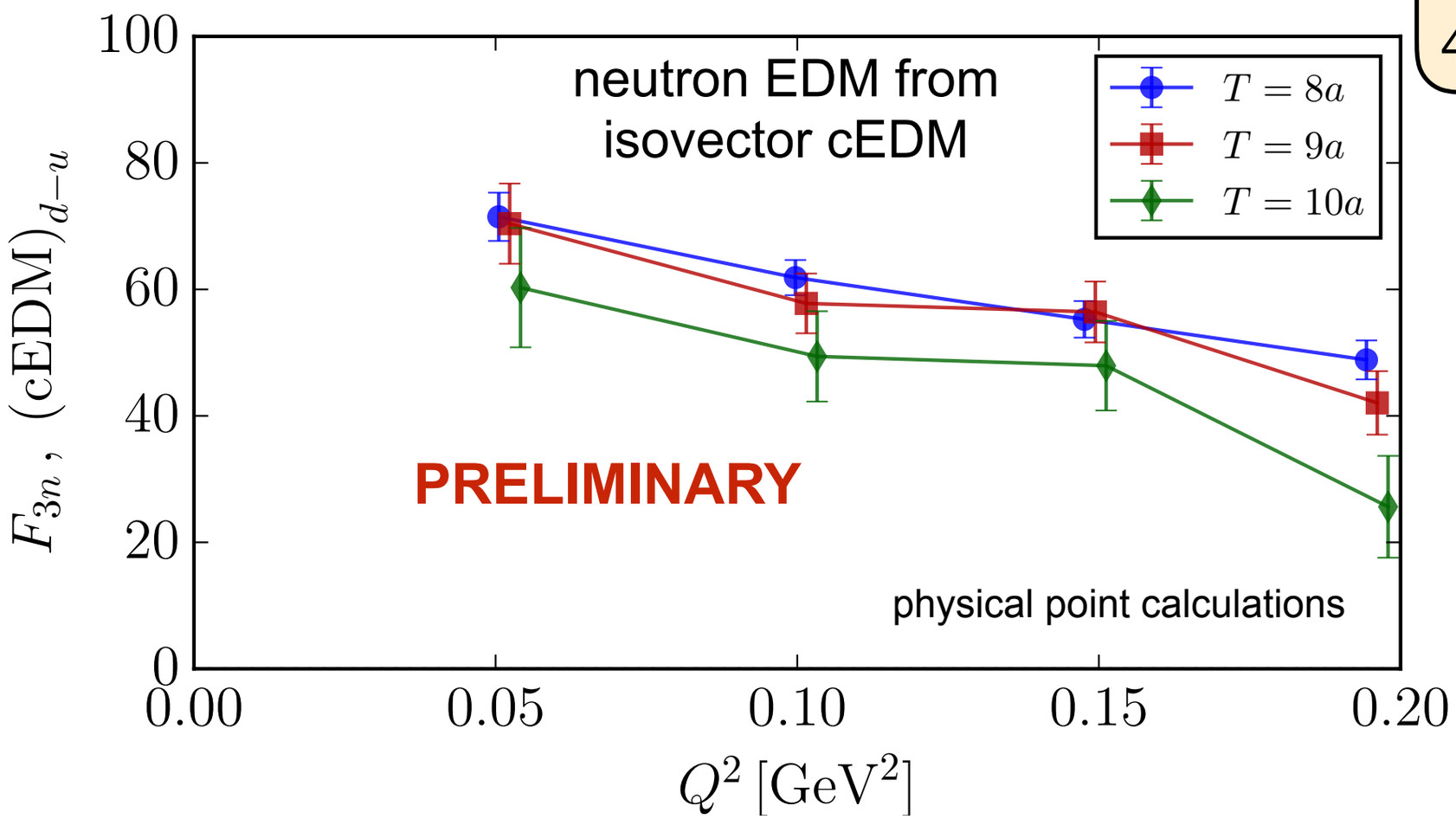
(flavor labels for the proton *uud*)



(*)connected-only, bare cEDM and PS operators

Neutron EDM FF from Isovector Quark cEDM

Physical point
DWF N_f=2+1
48^3x96, a=0.114 fm



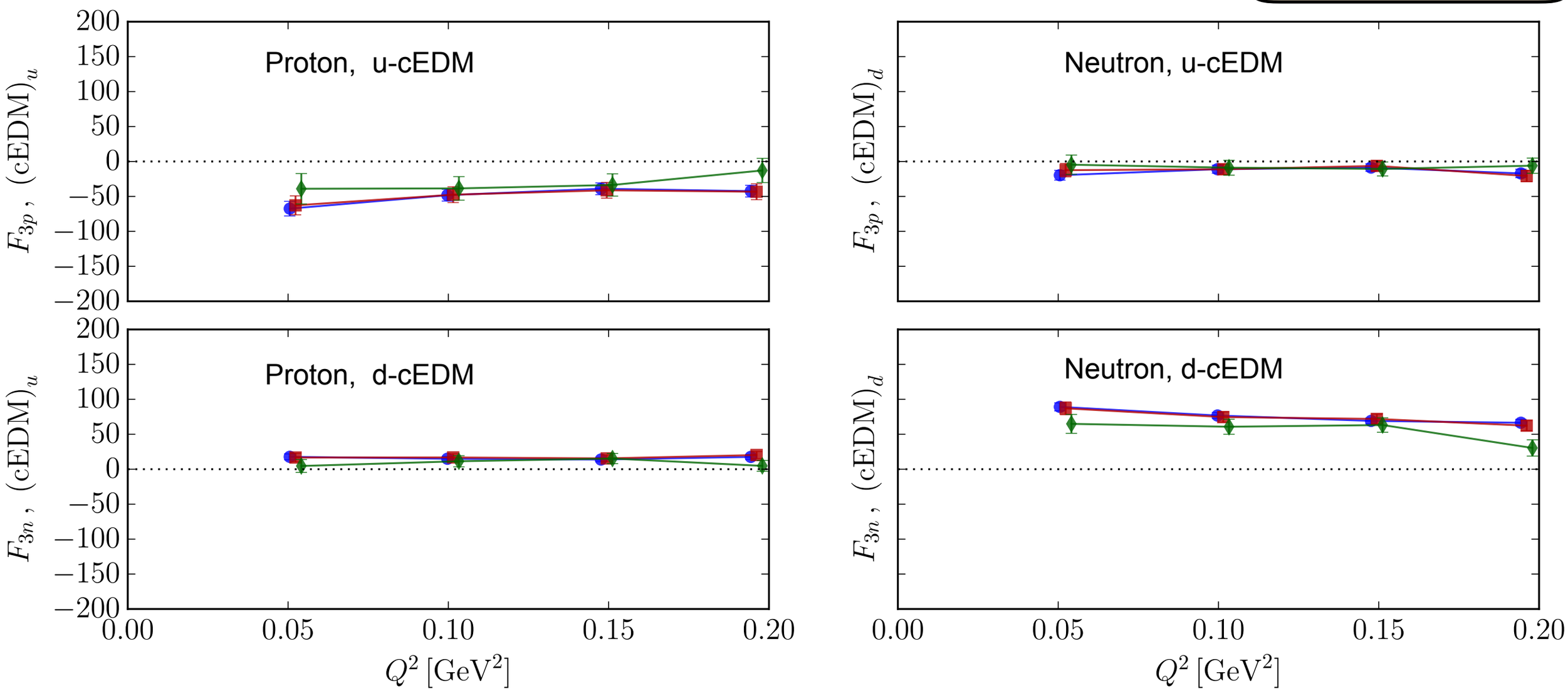
Neutron EDM induced by (u-d) chromoEDM

- isovector CPv interaction: no mixing with θ_{QCD} *term*
- disconnected corrections are required for final answer (although found very small for neutron $F_2(Q^2)$)

(*)connected-only; bare cEDM and PS operators

Proton & Neutron EDM FF from u,d cEDM (*)

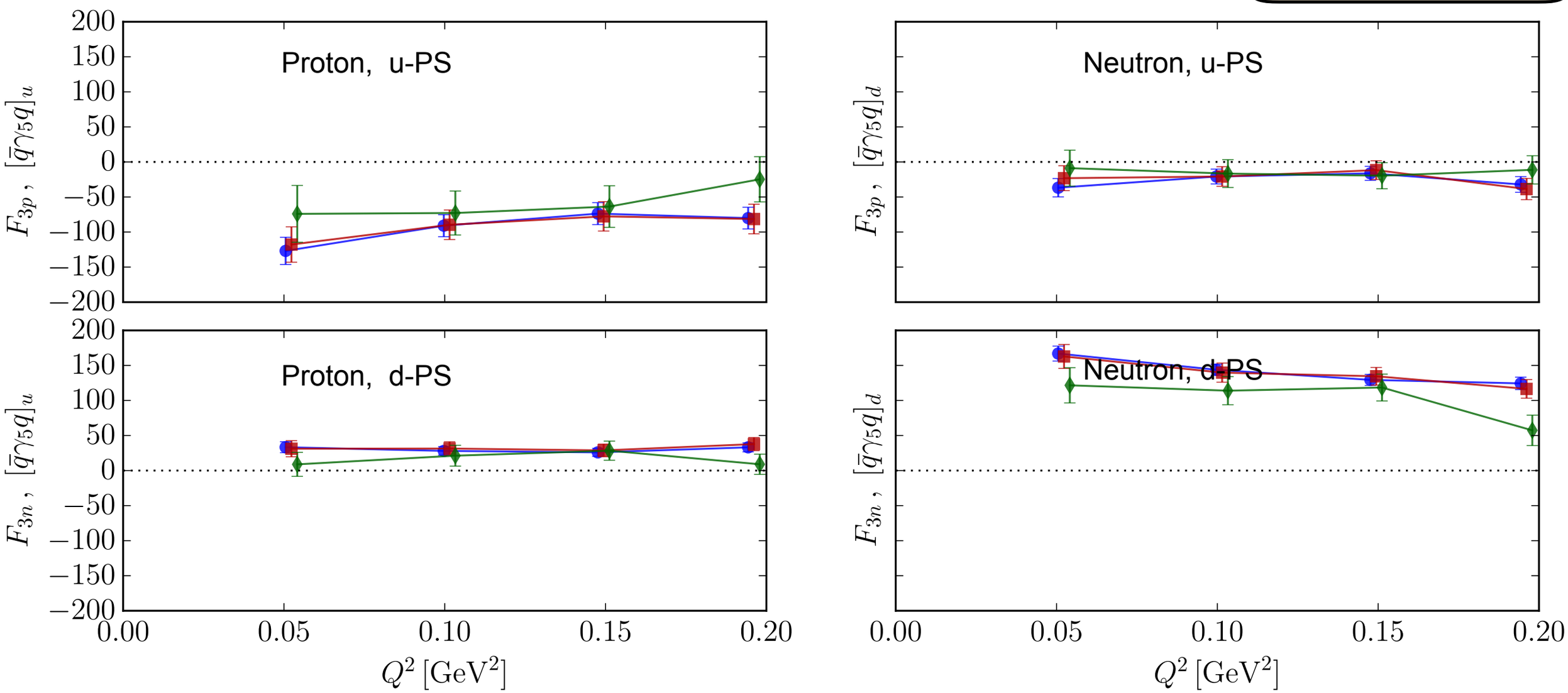
Physical point
DWF N_f=2+1
48^3x96, a=0.114 fm



(*)connected-only; bare cEDM and PS operators

Proton & Neutron EDM FF from u,d Pseudoscalar

Physical point
DWF N_f=2+1
48^3x96, a=0.114 fm



(*)connected-only, bare cEDM and PS operators

Renormalization of cEDM: RI-MOM

- "Momentum-scheme" : matrix elements between virtual quark states in Landau gauge [T.Bhattacharya et al , PRD92(2015) 114026]

Problem: divergent contact terms of gauge-dependent quark fields and operators
$\Longrightarrow$ mixing with gauge-dependent and EoM-vanishing operators [Collins, "Renormalization"]

cEDM →

$G\tilde{G}$ →

mixing in RI-MOM due to fixed gauge

	C	$\partial^2 P$	E	$mF\tilde{F}$	$mG\tilde{G}$	$(m\partial\cdot A)_1$	$(m\partial\cdot A)_2$	$(m^2P)_1$	$(m^2P)_2$	$(m^2P)_3$	P_{EE}	$\partial\cdot A_E$	A_∂	$A_{A(\gamma)}$
C	x	x	x	x	x	x	x	x	x	x	x	x	x	x
$\partial^2 P$		x												
E			x											
$mF\tilde{F}$				x										
$mG\tilde{G}$					x	x								
$(m\partial\cdot A)_1$						x								
$(m\partial\cdot A)_2$							x							
$(m^2P)_1$								x						
$(m^2\hat{P})_2$									x					
$(m^2\hat{P})_3$										x				
P_{EE}											x	x	x	
$\partial\cdot A_E$												x		
A_∂											x	x	x	x
$A_{A(\gamma)}$														x

Total: 14 operators (all flavor combinations)
[T.Bhattacharya et al , PRD92(2015) 114026]

Renormalization of cEDM: Position Space

- Alternative: matching short-distance correlators at $X \approx \mu^{-1} \ll (\Lambda_{QCD})^{-1}$ ("position space scheme"); extending work of [Gimenez et al (2004); Chetyrkin(2010)]

Matching CPv operators
lattice $\Longrightarrow$ MSbar
(isovector)

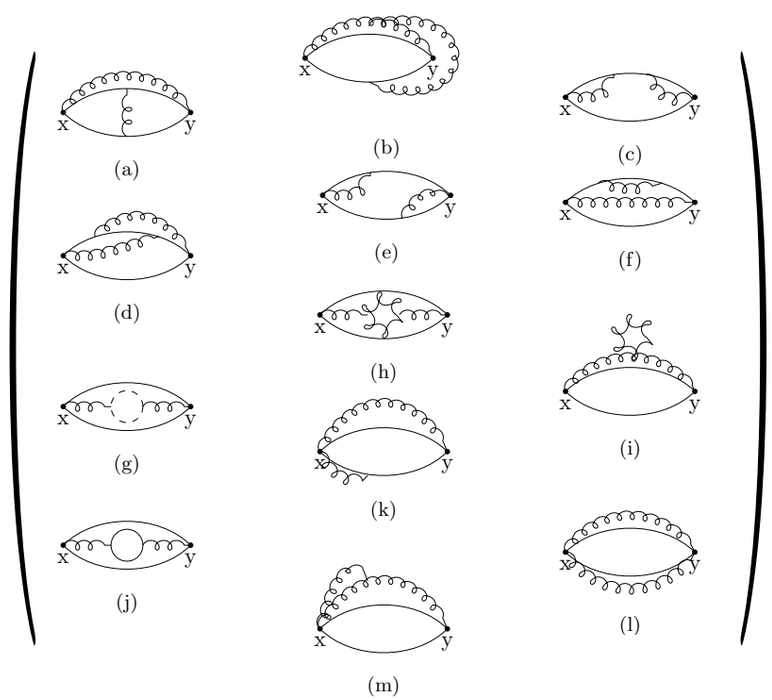
$$\mathcal{C} = \bar{q}\big[\frac{1}{2}(\sigma_{\mu\nu}G^{\mu\nu})\gamma_5\big]q$$
$$\mathcal{P} = \bar{q}\gamma_5 q$$

$$\mathcal{C}^{\overline{MS}} = Z_{\mathcal{C}}[\mathcal{C}]^{\rm lat} + Z_{\mathcal{C}/m^2\mathcal{P}}[m^2\mathcal{P}]^{\rm lat}$$

$$\begin{aligned}\langle\mathcal{C}(r)\mathcal{C}(0)\rangle^{\overline{MS}} &= Z_{\mathcal{C}}^2\,\langle\mathcal{C}(r)\,\mathcal{C}(0)\rangle^{\rm lat} \\ &+ 2Z_{\mathcal{C}}Z_{\mathcal{C}/m^2\mathcal{P}}\,\langle m^2\mathcal{P}(r)\,\mathcal{C}(0)\rangle^{\rm lat} \\ &+ Z_{\mathcal{C}/m^2\mathcal{P}}^2\,\langle m^2\mathcal{P}(r)\,m^2\mathcal{P}(0)\rangle^{\rm lat}\end{aligned}$$

NLO(α_S) in MSbar

$$\langle\mathcal{C}(r)\mathcal{C}(0)\rangle^{(1)} =$$



Also

$$\langle\mathcal{C}(r)\mathcal{P}(0)\rangle$$
$$\langle\mathcal{P}(r)\mathcal{P}(0)\rangle$$

[M.Kellerstein, SNS, *in preparation*]

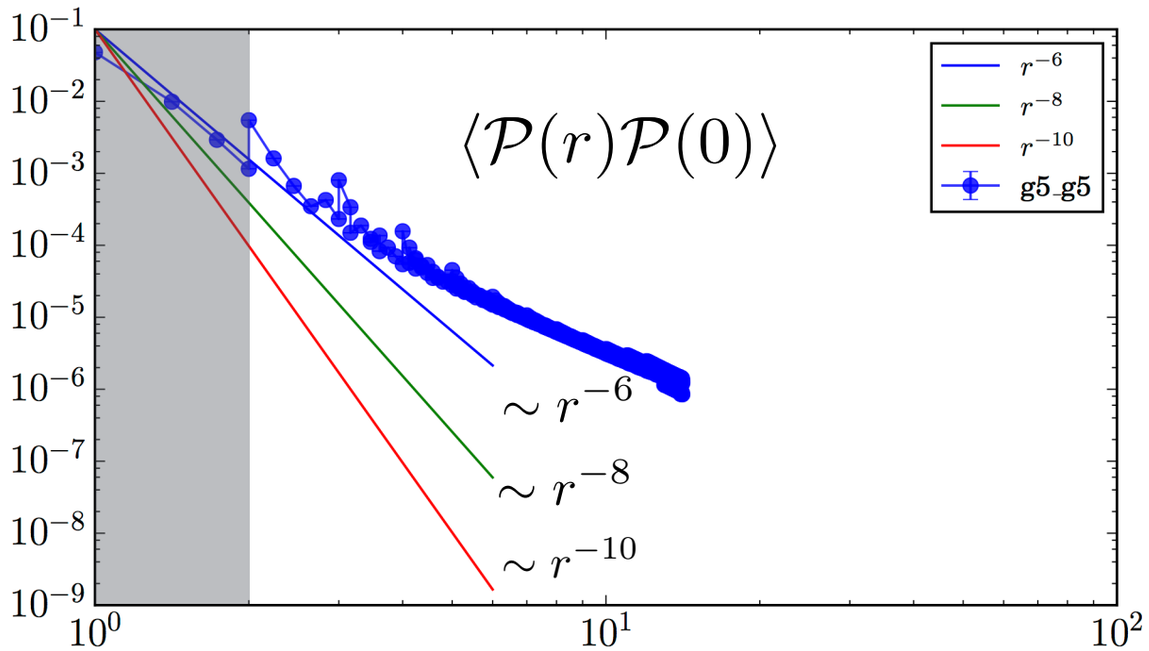
- Isoscalar cEDM (d=5) will *also mix with* $G\tilde{G}$ (d=4) $\Rightarrow$ "noisy" subtraction on a lattice

Renormalization of cEDM: Position Space

Isovector cEDM:
no $G\tilde{G}$ mixing

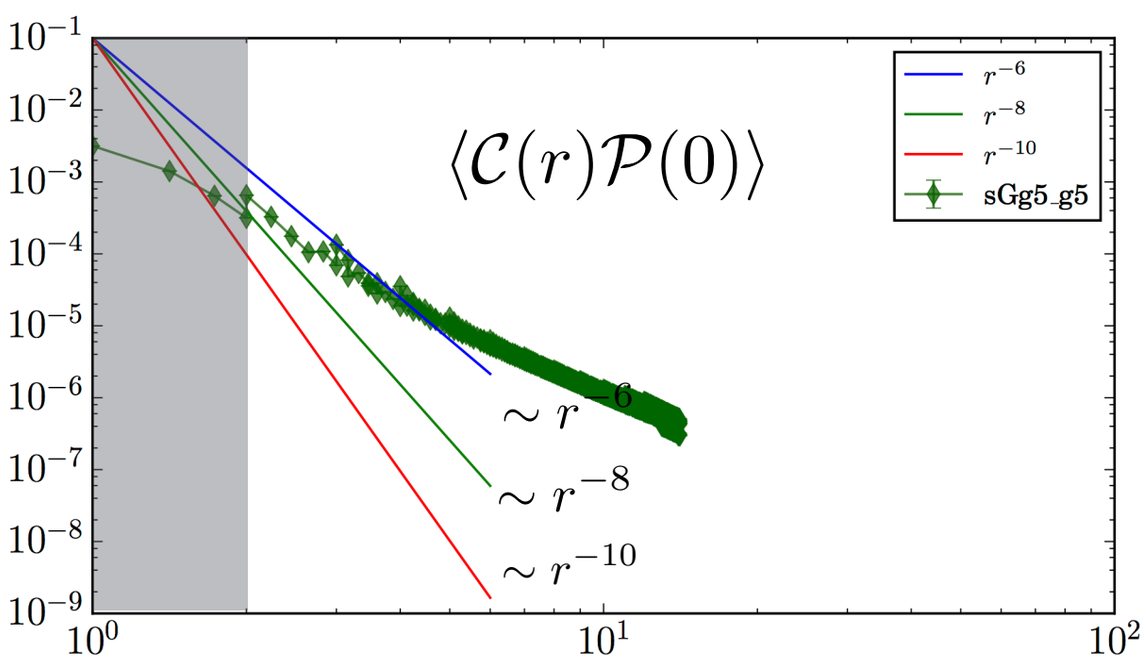
LO scaling of correlators
(dim.analysis)

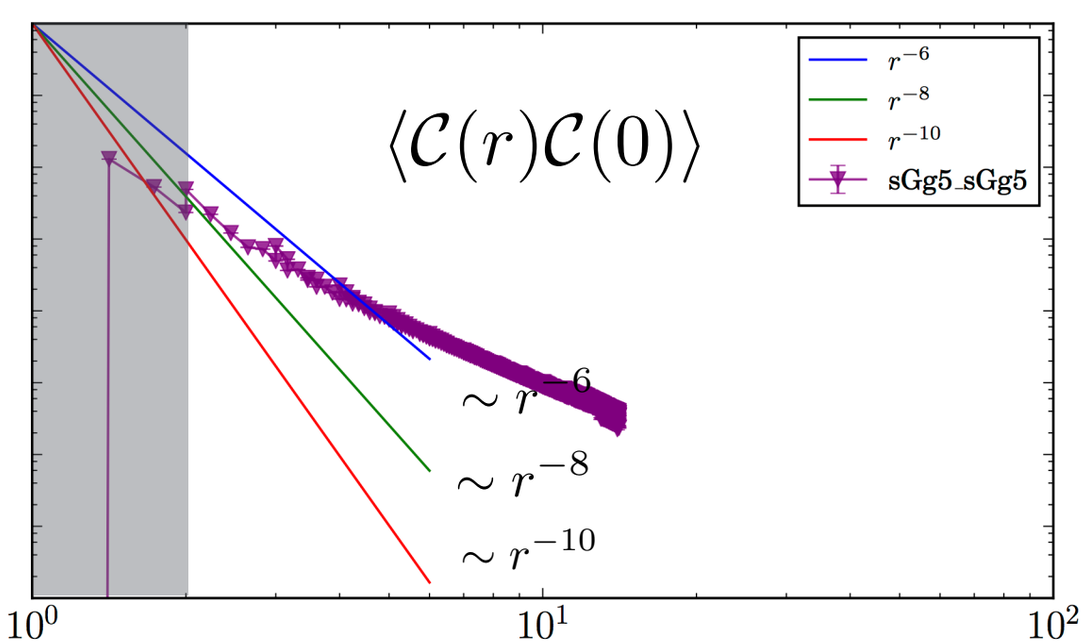
$$\begin{cases} \langle \mathcal{P}(r)\mathcal{P}(0)\rangle^{\rm LO} & \sim r^{-6} \\ \langle \mathcal{C}(r)\mathcal{P}(0)\rangle^{\rm LO} & \sim r^{-8} \\ \langle \mathcal{C}(r)\mathcal{C}(0)\rangle^{\rm LO} & \sim r^{-10} \end{cases}$$



- evident mixing from lattice data
- minimal displacement |r|=2 {1,1,1,1} to avoid contact terms
- ⇒narrow window to avoid NP effects (pions):

  $|r| \lesssim (0.5\ \mathrm{GeV})^{-1} \approx (3..4)\ a$
- isoscalar cEDM will also require disconnected correlators and mix with ($G\tilde{G}$)



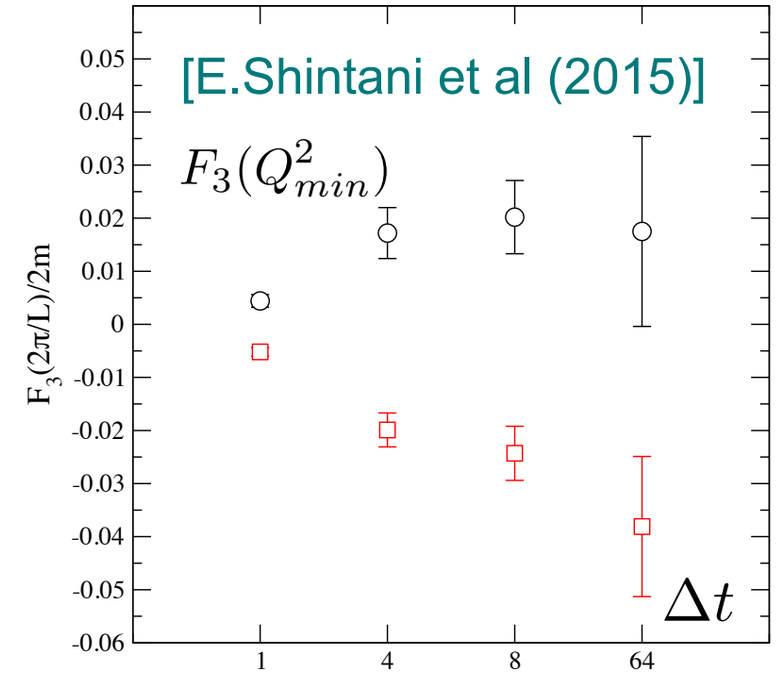


EDM from θ-Term: Noise from Global Q_{top}

Variance of lattice *θ*-induced nEDM signal ~ (*Volume*)$_{4d}$:

$$d_N \sim \langle Q \cdot (N J_\mu \bar{N}) \rangle$$

Top. charge $\quad Q \sim \int_{V_4} (G\tilde{G})\,, \quad$ with $\langle |Q|^2 \rangle \sim V_4$



Constrain Q sum to the fiducial volume

- in time around current, $|t_Q - t_J| < \Delta t$
  [E.Shintani et al (2015); B.Yoon et al (2019)]
- in time around source, $|t_Q - t_{source}| < \Delta t$ [J. Dragos, talk on Tue]
- 4-d sphere around sink, $|x_Q - x_{sink}| < R$ [K.-F.Liu et al, (2017)]:

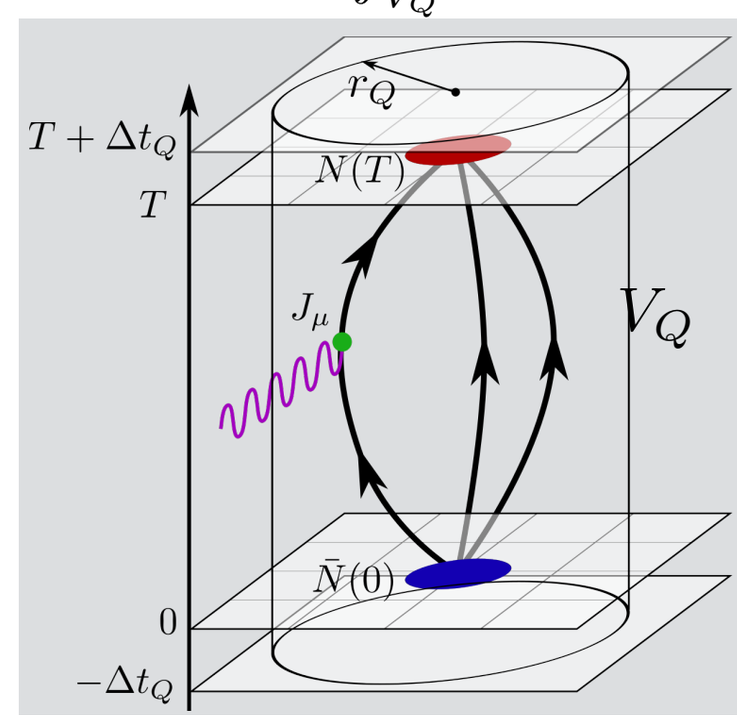
$$Q \approx \int_{V_Q} d^4 z\, q(z)$$

Proper treatment of nucleon parity mixing is critical for correct determination of F_3
$\Longrightarrow$ nucleon must "settle" in the new θ≠0 vacuum

$$N^{(+)} \to \tilde{N}^{(+)} \approx N^{(+)} + i\alpha N^{(-)}$$
$$N^{(-)} \to \tilde{N}^{(-)} \approx N^{(-)} - i\alpha N^{(+)}$$

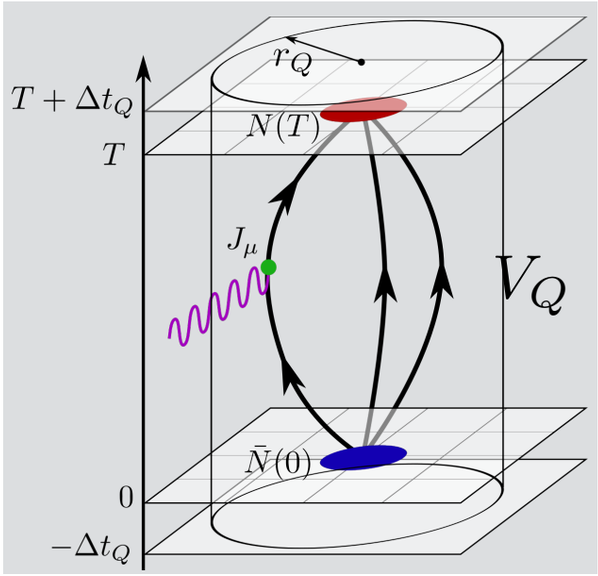
$\Longrightarrow$ constrain time and space differently :

4d "cylinder" $\quad V_Q \;:\; |\vec{z}| < r_Q, \quad -\Delta t_Q < z_0 < T + \Delta t_Q$

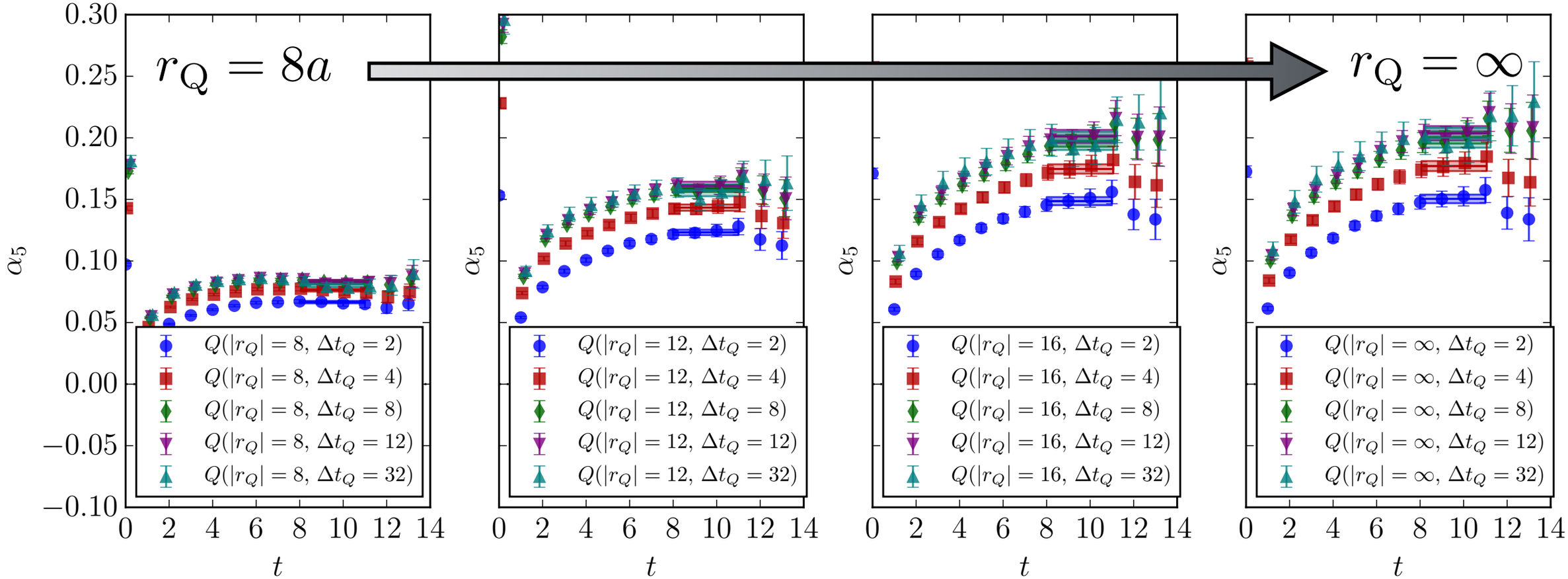


θ-EDM at m_π=330 MeV: Parity Mixing

- N_f=2+1 Domain Wall (RBC/UKQCD) 24^3x64 a = 0.114 fm
- 1400 confiigs * (64sloppy+1exact) samples $\Longrightarrow$ 89.6k stat.
- Top.charge with 5-loop improved $G\tilde{G}$ [P. de Forcrand et al '97] on Wilson-flowed (t=8a^2) gauge links [M.Luscher, 1006.4518]

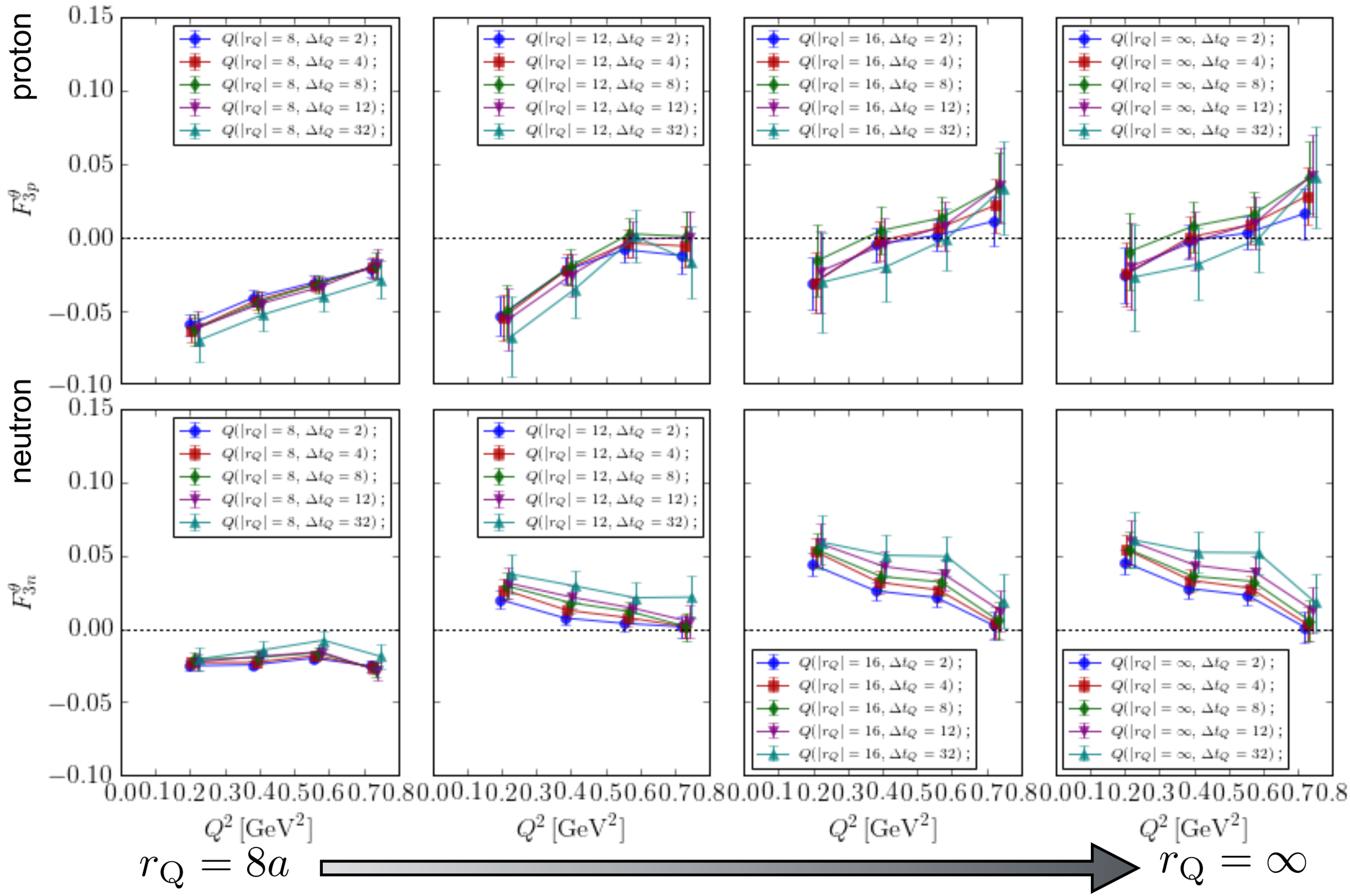


parity mixing angle α_N as a function of r_Q, Δt_Q



- convergence at $r_Q \gtrsim 16a$, $\Delta t_Q \gtrsim 8a$

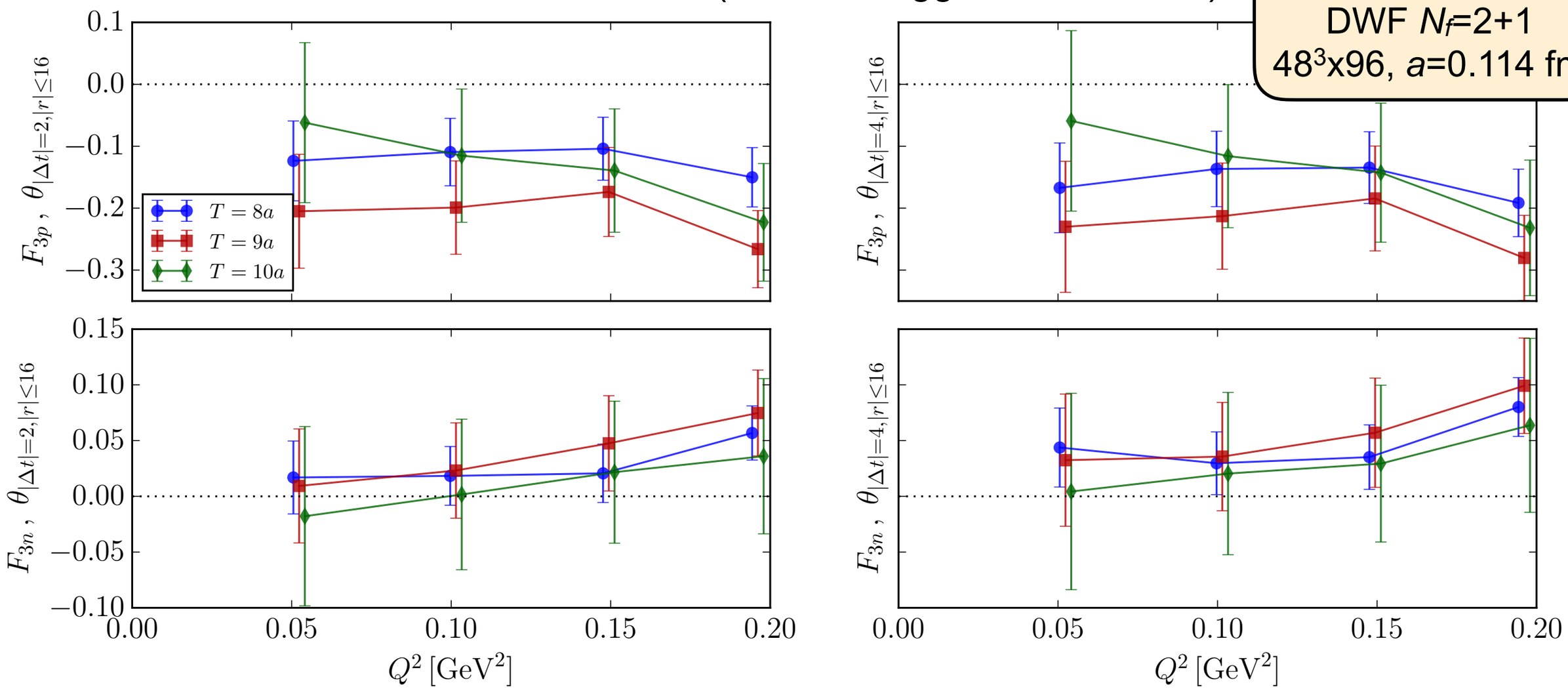
θ_{QCD}-EDM at m_π=330 MeV: EDM Form Factor



θ_{QCD}-EDM at Physical point: EDM Form Factor

EDFF F_3 from constrained Q sum (*the most aggressive* Q cuts)

Physical point
DWF N_f=2+1
48^3x96, a=0.114 fm



- 33k lattice samples, ~ 30 M core-hours on Argonne BlueGene/Q
- connected diagrams only
- result compatible with zero, $|F_{3n}| \leq 0.05$ constraint

From LO ChPT, $d_n \sim m_q \sim (m\pi)^2$: $|F_{3n}|_{phys} \approx 0.01$

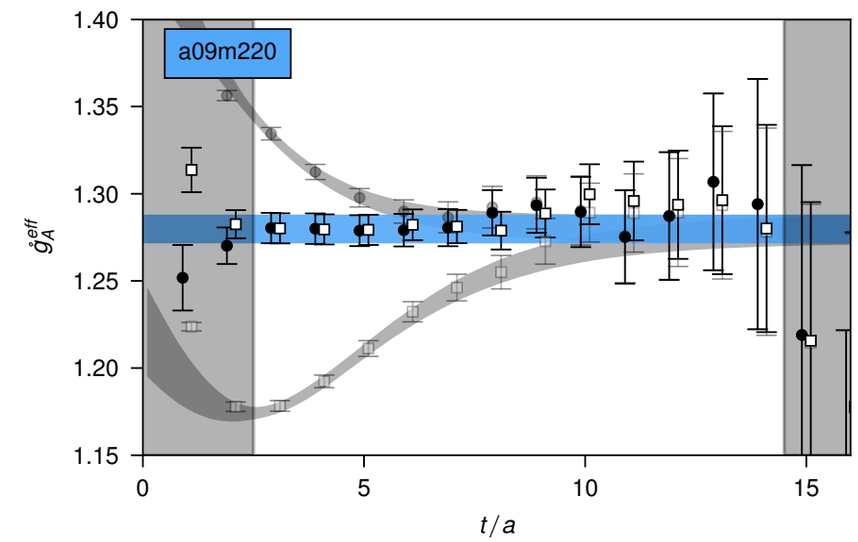
Need x30..100 more statistics

EDM from (rev.) Feynman-Hellman Theorem

- FH relates perturbation's matrix element to energy shift

$$\frac{\partial E_\lambda}{\partial \lambda} = \left\langle \phi_\lambda \middle| \frac{\partial \hat{H}_\lambda}{\partial \lambda} \middle| \phi_\lambda \right\rangle$$

(used successfully to compute g_A to sub-% precision) and other forward hadron matrix elements



[Chang et al (CalLat), Nature 558:91 (2018)]

Nucleon EDM from FH:

- theta-term is a small perturbation:

$$\delta\mathcal{L} = \frac{\theta g^2}{32\pi^2} \int dt \int d^3x G^a_{\mu\nu} \widetilde{G}^a_{\mu\nu}$$

- which leads to a <u>mass shift</u> of a <u>polarized</u> nucleon in background <u>electric field</u>

$$m'_N = m_N - (d^\theta_N \theta)\, \vec{\Sigma} \cdot \vec{\mathcal{E}}$$

- FH $\Longrightarrow$ relation between EDM and matrix element of <u>local top.charge density</u>

$$d^\theta_N \propto \left\langle N_\uparrow \middle| \int d^3x\, G^a_{\mu\nu} \widetilde{G}^a_{\mu\nu} \middle| N_\uparrow \right\rangle_{\mathcal{E}_z}$$

EDM = Top.charge Density in Polarized Nucleon

$$d_N^\theta \propto \left\langle N_\uparrow \middle| \int d^3x\, G^a_{\mu\nu}\widetilde{G}^a_{\mu\nu} \middle| N_\uparrow \right\rangle_{\mathcal{E}_z}$$

in CP-even vacuum, both *spin*- and *charge*-polarization are required for $\langle G\tilde{G}\rangle \neq 0$
(equivalent to background E || B fields)

permanent EDM =
correlation of *spin* and *charge* =

- *charge polarization* of a spin-polarized state in CPv (*θ-*)vacuum, **OR**
- *spin polarization* of a charge-polarized state in CPv (*θ-*)vacuum, **OR**
- "topological" polarization of a charge- and spin-polarized state in CP-even vacuum

NOTE: without background E field, $\langle N|G\tilde{G}|N\rangle$ is zero due to parity

$$\left.\begin{array}{r} \mathcal{P}\big(|N_\uparrow^{(+)}\rangle\big) = +|N_\uparrow^{(+)}\rangle \\ \mathcal{P}\big(G\tilde{G}\big) = -G\tilde{G} \end{array}\right\} \implies \langle N_\uparrow^{(+)}|G\tilde{G}|N_\uparrow^{(+)}\rangle = 0$$

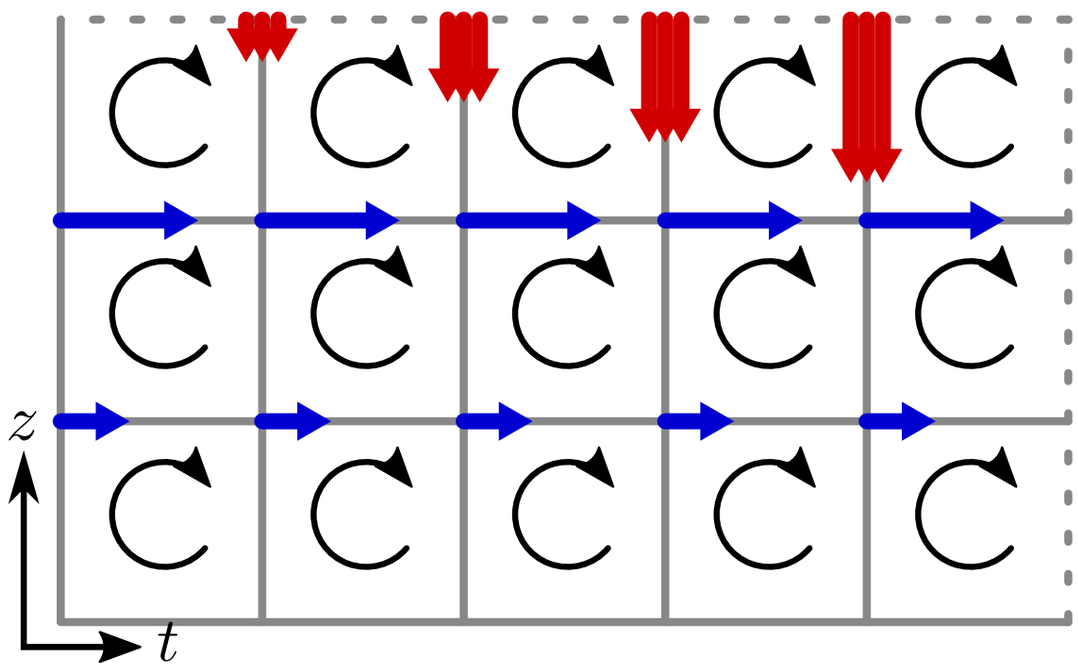
background E polarizes state into a mixed-parity state:

$$|N\rangle_{\mathcal{E}_z} = |N^{(+)}\rangle + O(\mathcal{E}_z)|N^{(-)}\rangle$$

(P-mixed state in CP-even vacuum)

Background Electric Field

Accessing magnetic and electric moments at Q^2=0
Imag.Minkowski/Real Euc. electric field on a lattice
[W.Detmold et al (2009)] : calculation of hadron polarizabilities



$$U_\mu \to e^{iqA_\mu} U_\mu$$

$$A_z(z,t) = n\,\mathcal{E}_{\min} \cdot t$$

$$A_t(z, t = L_t - 1) = -n\,\mathcal{E}_{\min} \cdot L_t z$$

Full flux through the "side" of the periodic box

$$= q\Phi = 2\pi \cdot n$$

Constant Electric field has to be quantized,

$$\mathcal{E}_{\min} = \frac{1}{|q_d|}\frac{2\pi}{L_x L_t}$$

Electric field on a 24^3x64 lattice

$$\mathcal{E} = \frac{6\pi}{L_x L_t} \approx 0.037\ \mathrm{GeV}^2$$

$$\approx 186\ \mathrm{MV/fm}$$

Unambiguous determination of EDM from the energy shift
Straightforward for neutron with Q=0

Topological Charge with Gradient Flow

[M.Luscher, JHEP08:071; 1006.4518]

Gradient flow: covariant *4D-diffusion* of quantum fields with "G.F." time t_{GF}:

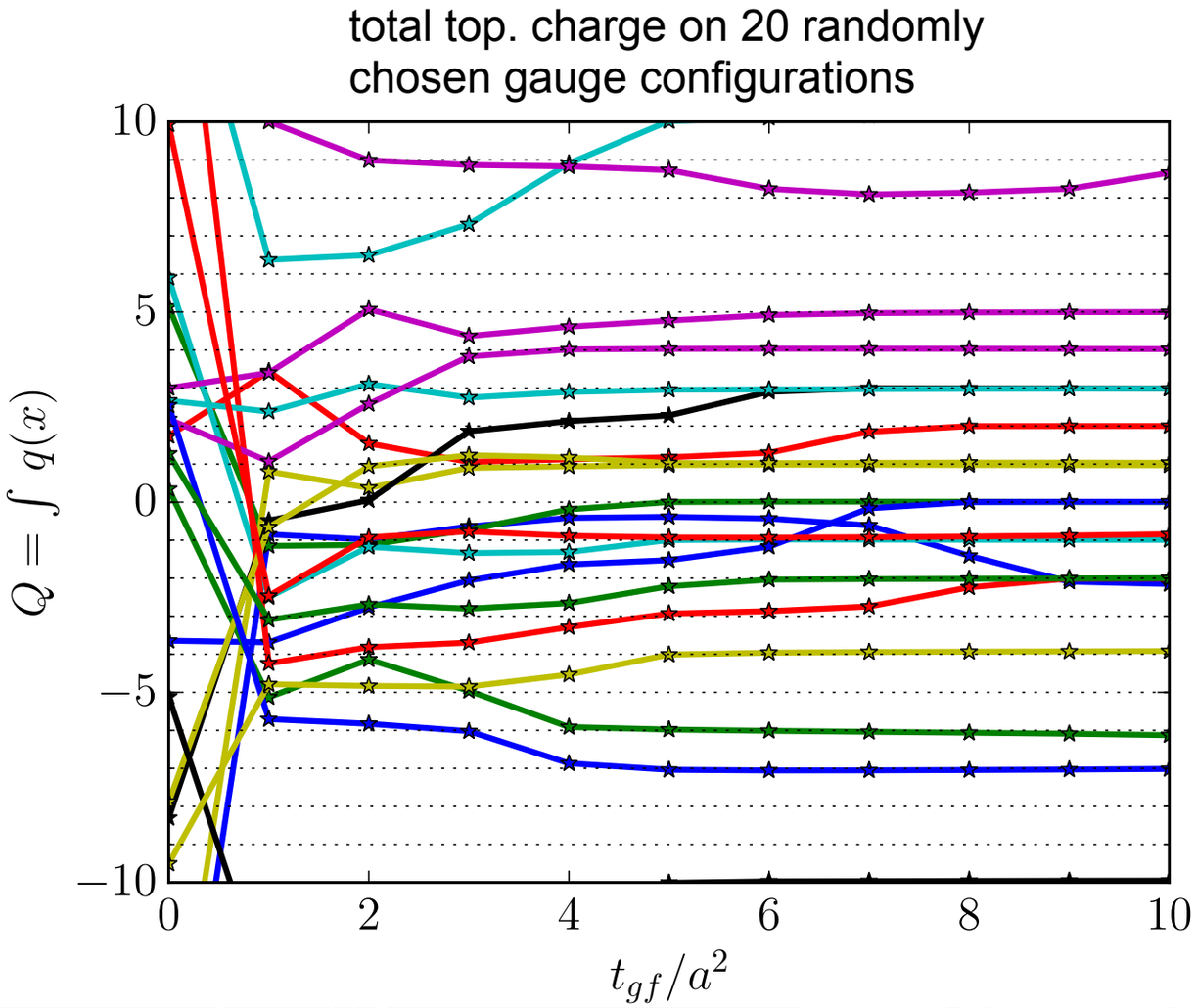
$$\frac{d}{dt_{\rm GF}} B_\mu(t_{\rm GF}) = D_\mu G_{\mu\nu}(t_{\rm GF})\,, \quad B_\mu(0) = A_\mu$$

Tree-level:

$$B_\mu(x, t_{\rm GF}) \propto \int d^4y \, \exp\Big[-\frac{(x-y)^2}{4t_{\rm GF}}\Big] A_\mu(y)$$

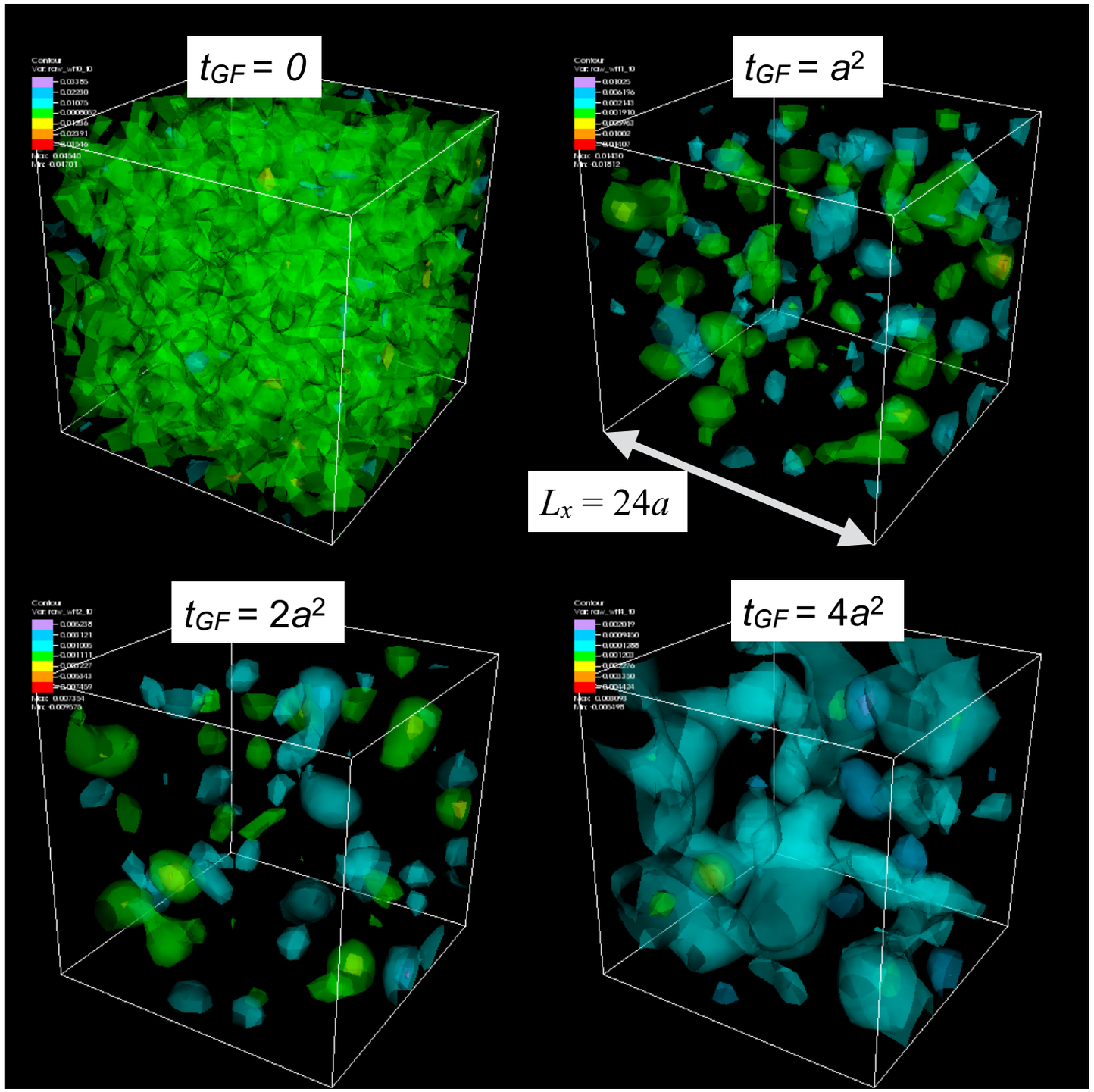
Gradient-flowed topological charge:

$$\tilde{Q}(t_{\rm GF}) = \int d^4x \, \frac{g^2}{32\pi^2} \Big[G_{\mu\nu}\tilde{G}_{\mu\nu}\Big]\Big|_{t_{\rm GF}}$$



- effective scale $\Lambda_{\rm UV} \to (t_{\rm GF})^{-1/2}$
- smooth fields (reduce $|G_{\mu\nu}|$)
  $\Longleftrightarrow$ continuous "cooling"
- remove $G_{\mu\nu}$ dislocations
  $\Rightarrow$dynamical separation of top. sectors
  [M.Luscher, JHEP08:071; 1006.4518]
- diffusion of top.charge density

Gradient-Flowed Topological Charge Density



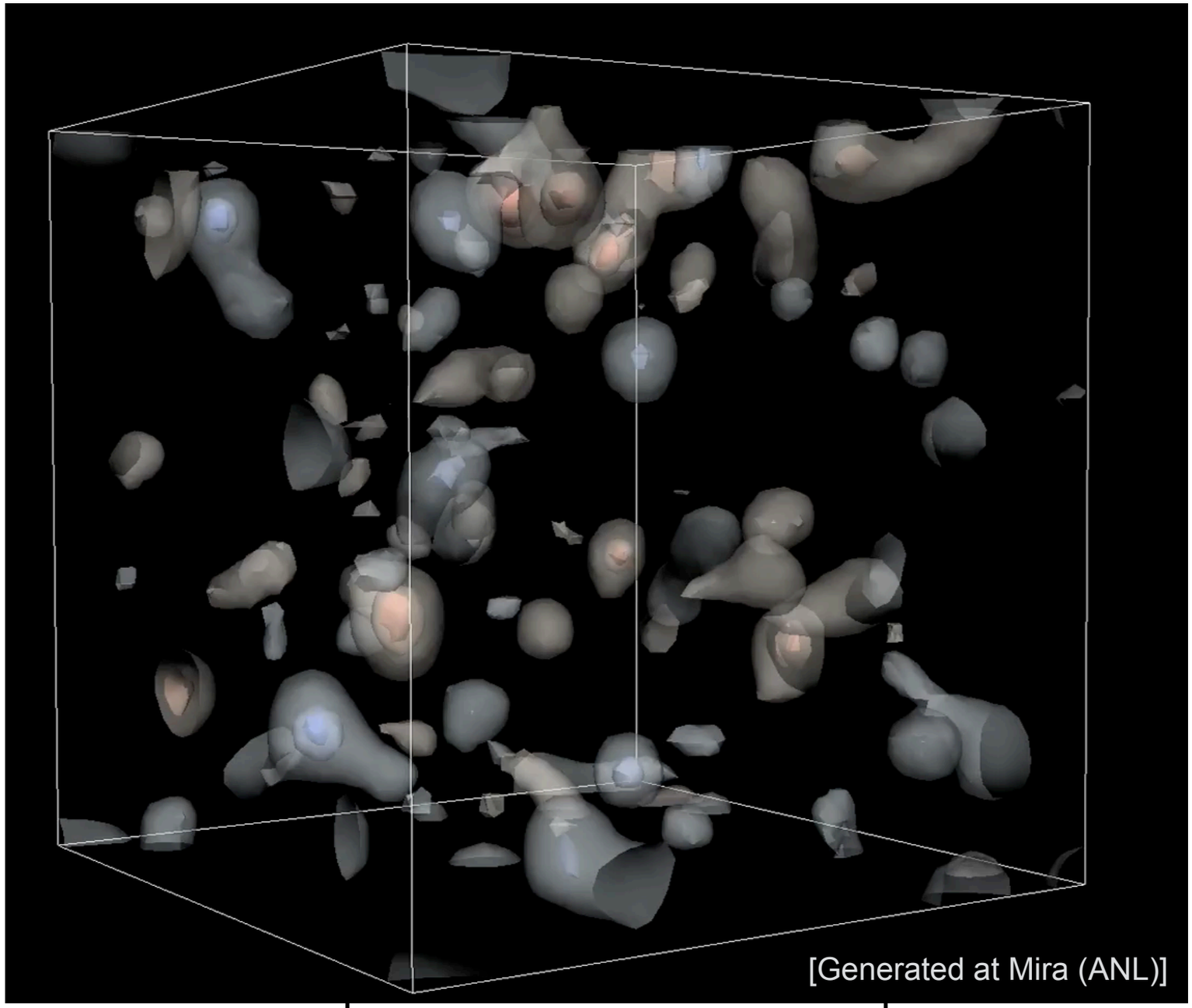
$$q(x) = \frac{g^2}{32\pi^2} G^a_{\mu\nu} \widetilde{G}^a_{\mu\nu}$$
$$\approx \frac{1}{16\pi^2} \frac{1}{a^4} \mathrm{Tr}\left[G^{\mathrm{lat}}_{\mu\nu} \widetilde{G}^{\mathrm{lat}}_{\mu\nu}\right]$$
$$\propto (\mathbf{E} \cdot \mathbf{H})_{\mathrm{color}}$$

Gradient flow:

- effective scale $(\Lambda_{\mathrm{UV}})^{-1} \rightarrow (t_{\mathrm{GF}})^{1/2}$
- make fields smooth (reduce $|G_{\mu\nu}|$)
- remove dislocations⇒dynamical separation of topological sectors [M.Luscher, JHEP08:071; 1006.4518]
- 4D-diffusion (including time) of $q(x)$
  $\langle q(x)q(0) \rangle \sim exp[-(x-y)^2 / 8t_{\mathrm{GF}}]$

$24^3 \times 64$ lattice, $m\pi \approx 340\ MeV$

Tunneling Between Topology Sectors



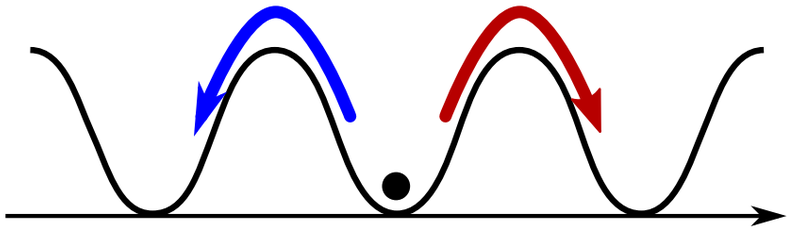
5 fm=5 · 10^{-15}m

6 s video =5 fm / c =1.7 · 10^{-23} s real time

[Lattice QCD at the physical point]

$$q(x) = \frac{g^2}{32\pi^2} G^a_{\mu\nu} \widetilde{G}^a_{\mu\nu}$$
$$\approx \frac{1}{16\pi^2} \frac{1}{a^4} \mathrm{Tr}\left[G^{\mathrm{lat}}_{\mu\nu} \widetilde{G}^{\mathrm{lat}}_{\mu\nu}\right]$$
$$\propto (\mathbf{E} \cdot \mathbf{H})_{\mathrm{color}}$$

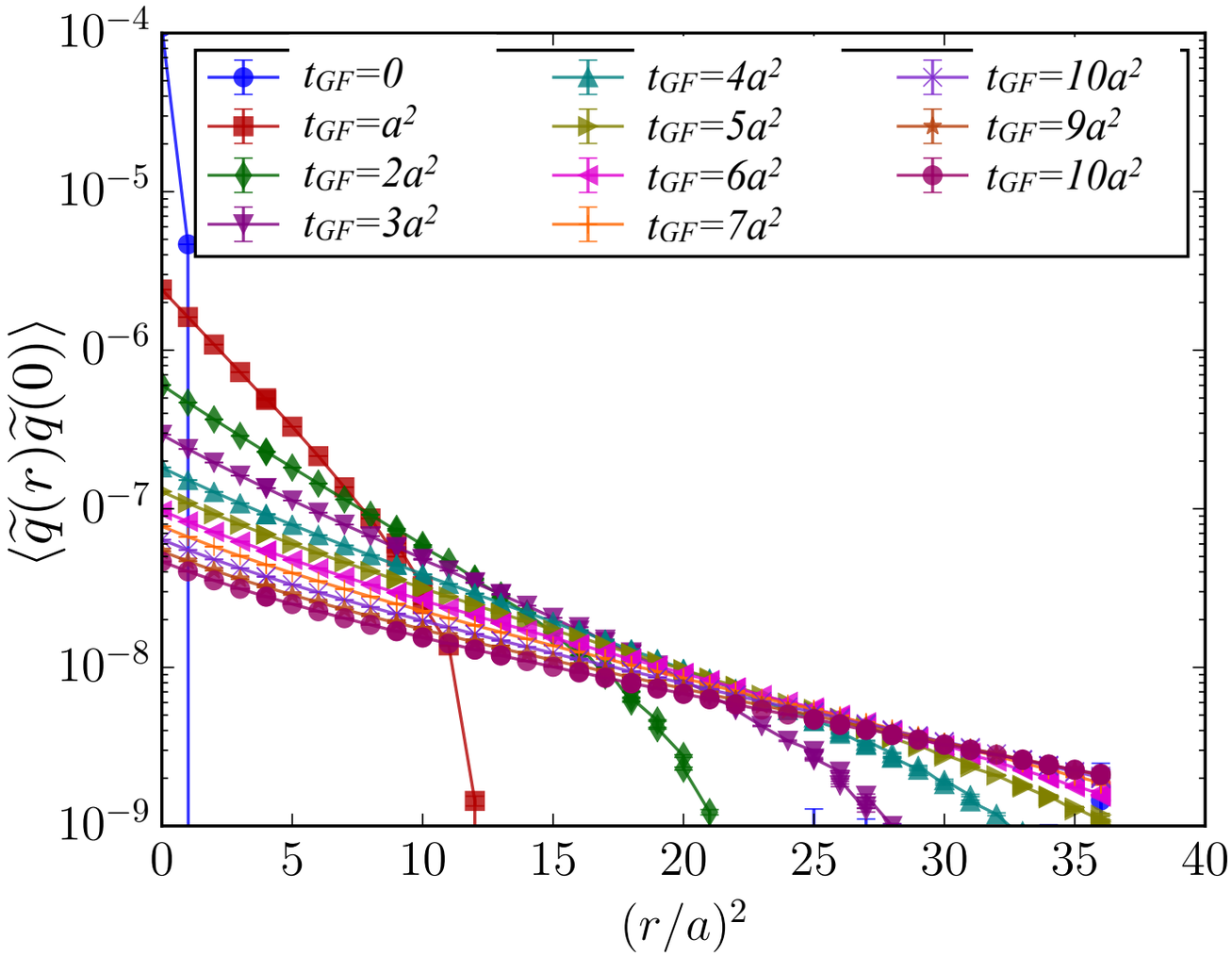
Instantons and Anti-Instantons : Quantum tunneling of gluon field between topological sectors



CPv-QCD Θ-Vacuum :

$$|vac\rangle_\theta = \sum_Q e^{i\theta Q} |Q\rangle$$

How "local" is Top.Charge Density?



- Gradient-flowed topological charge density is nonlocal:

$$\widetilde{q}(x) \propto \int d^D y \, \exp\Big[-\frac{(x-y)^2}{2r_Q^2(t_{\rm GF})}\Big] q(y)$$

Empirically for $r, \sqrt{t_{\rm GF}} \gg m_{\eta'}^{-1}$

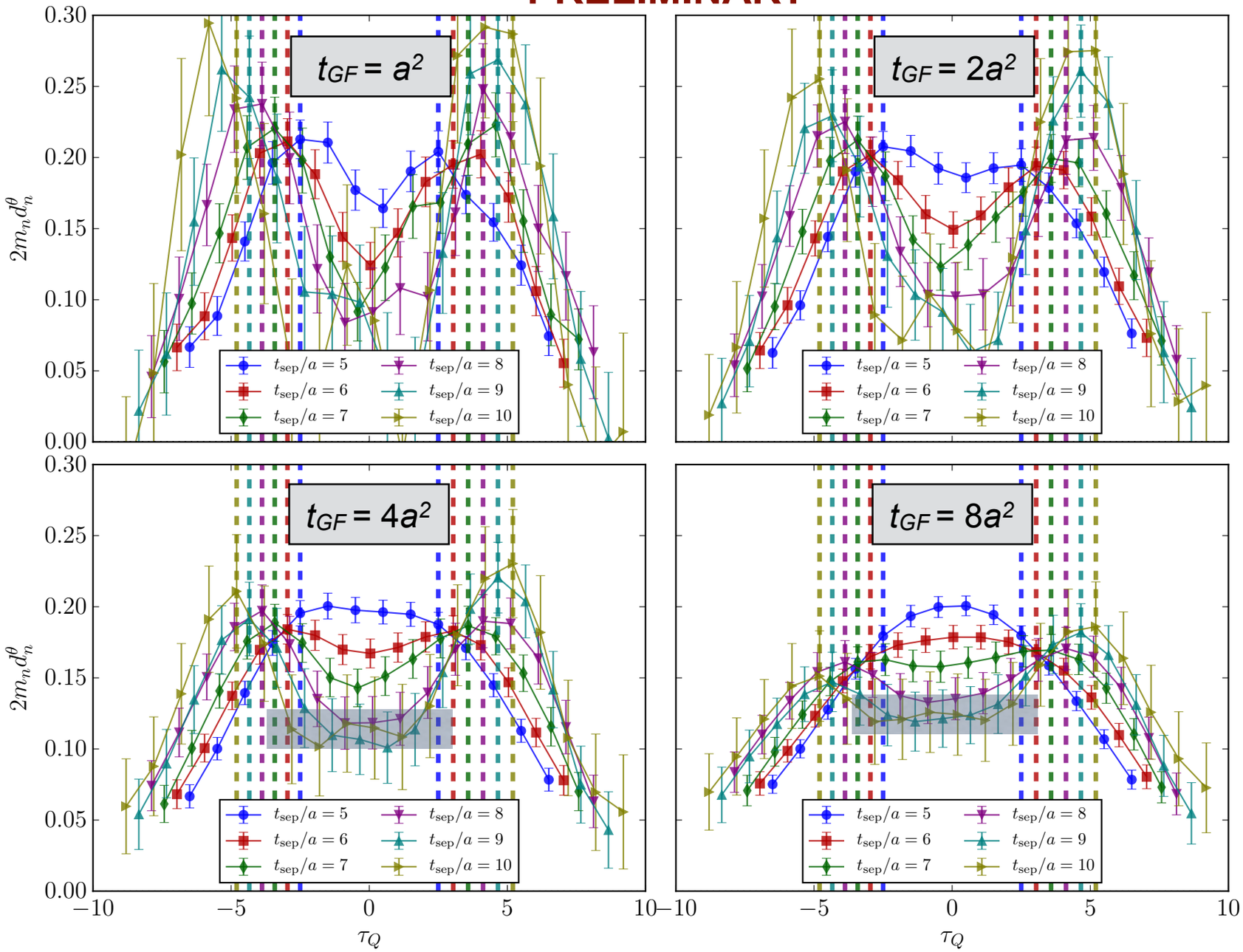
$$\langle \widetilde{q}(r)\widetilde{q}(0)\rangle \propto \exp\Big[-\frac{r^2}{4r_Q^2(t_{\rm GF})}\Big]$$

Gradient flow leads to diffusion of q(x) in Euclidean (lattice) time:

$\Longrightarrow$ *complications for matrix element analysis*

Matrix Elements of Gradient-Flowed Q(t_{GF})

PRELIMINARY



Two effects observed:

1. Convergence to ground state matrix el.
2. Diffusion of top.charge for $t_{sep} \lesssim 7a$

Analysis of (τ_Q, t_{GF}) required to detangle

$$\langle N|G\tilde{G}|N\rangle\,,$$
$$\langle N|G\tilde{G}|N\rangle_{\rm exc}\,,$$
$$\langle {\rm vac}|G\tilde{G}|N\bar{N}\rangle\,,$$
$$\ldots$$

PRELIMINARY
from plateaus
$2md_n = F_3(0) \approx 0.11 \,..\, 0.13$
in agreement with FF. method

$$\langle N_\uparrow(t_{\rm sep})\,\widetilde{Q}(\tau_Q)\,\overline{N}_\uparrow(0)\rangle_{\varepsilon_z} \;\propto\; \exp\Big[-\frac{(\tau_Q-\tau_Q')^2}{4t_{GF}^2}\Big] \;\otimes\; \langle N_\uparrow(t_{\rm sep})\,\widetilde{Q}(\tau_Q')\,\overline{N}_\uparrow(0)\rangle_{\varepsilon_z}$$

Summary & Outlook

- Encouraging results for nucleon EDM induced by quark chromo-EDM physical-point
  - *~20% stochastic uncertainty for quark cEDM-induced EDM*
  - *Renormalization & mixing subtractions are underway*
  - *Full flavor dependence will require disconnected diagrams & θ_{QCD} -term*
- Clear signal for θ_{QCD}-induced nEDM at m_π = 330 MeV
  - *Variance-reduction for Q sampling is essential*
- Novel method to compute nEDM from local topological charge
  - *Potential method for physical-point calculations*

BACKUP

Computing θ-nEDM at the Physical Point?

- chiral fermions, m_π = 330 MeV [this work]

$|2m_n d_n| = |F_{3n}(0)| \approx 0.05 \cdot \theta$

- Wilson fermions, mπ=360 MeV [Guo et al 2015] *after correction*

$|2m_n d_n| = |F_{3n}(0)| \lesssim 0.06 \cdot \theta$

- best guess for the physical point with LO ChPT,

$|d_n| \sim m_q \sim (m_\pi)^2 \Longrightarrow$ phys.point

$|F_{3n}(0)| \approx 0.01 \cdot \theta, \;\; |d_n| \approx 0.001 \cdot \theta \; e$ fm

$$|F_{3n}^{\rm phys}(0)| \sim O(10^{-2})\,\theta, \quad |d_n| \sim O(10^{-3})\, e\,{\rm fm}\,\theta$$

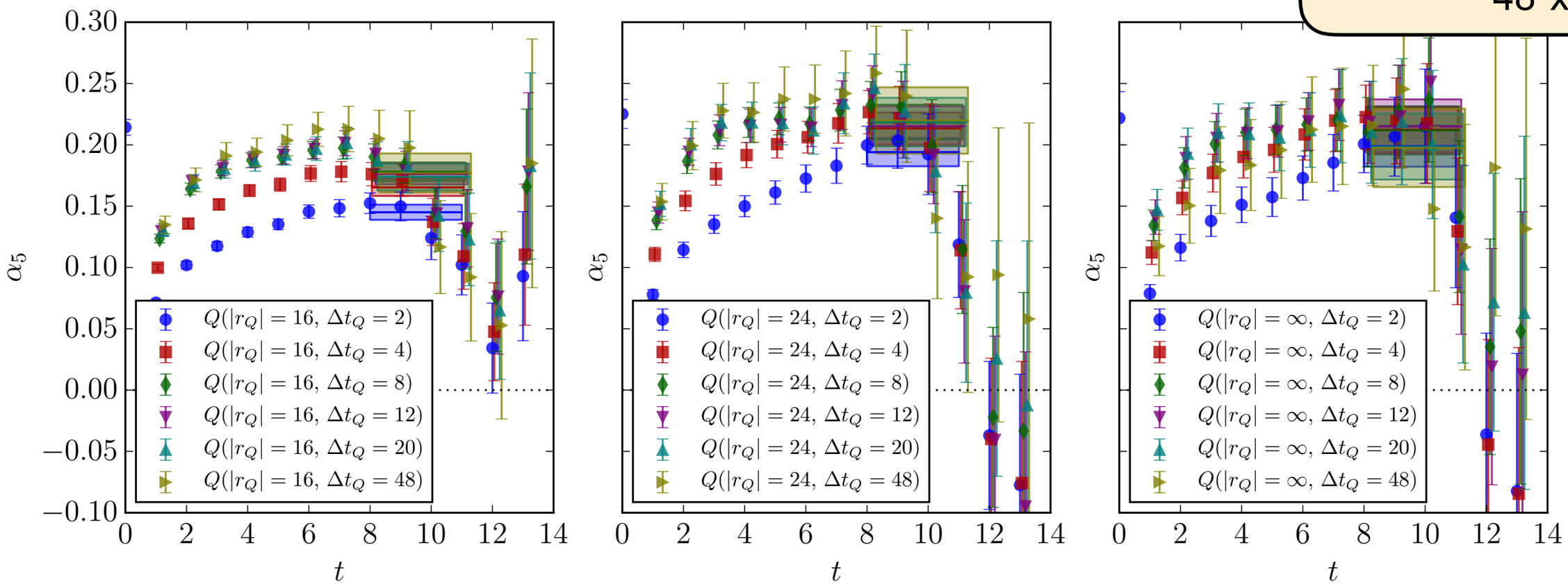
θ_{QCD} from ***estimated*** $|F_{3n}(0)| \approx 10^{-2} \cdot \theta$:

- from neutron: $|d_n| \lesssim 2.9 \cdot 10^{-26}\; e \cdot$cm [Baker et al (2006)] : $|\theta_{QCD}| \lesssim 2.9 \cdot 10^{-10}$
- from ^{199}Hg: $|d_n| \lesssim 1.6 \cdot 10^{-26}\; e \cdot$cm [Graner *et al* (2016)] : $|\theta_{QCD}| \lesssim 1.6 \cdot 10^{-10}$

Physical point : θ_{QCD}-induced Parity Mixing α_N

Parity-mixing angle from constrained Q sum

Physical poi
DWF N_f=2+1
48^3x96, a=0.



48^3x96 m_π=139 MeV (P

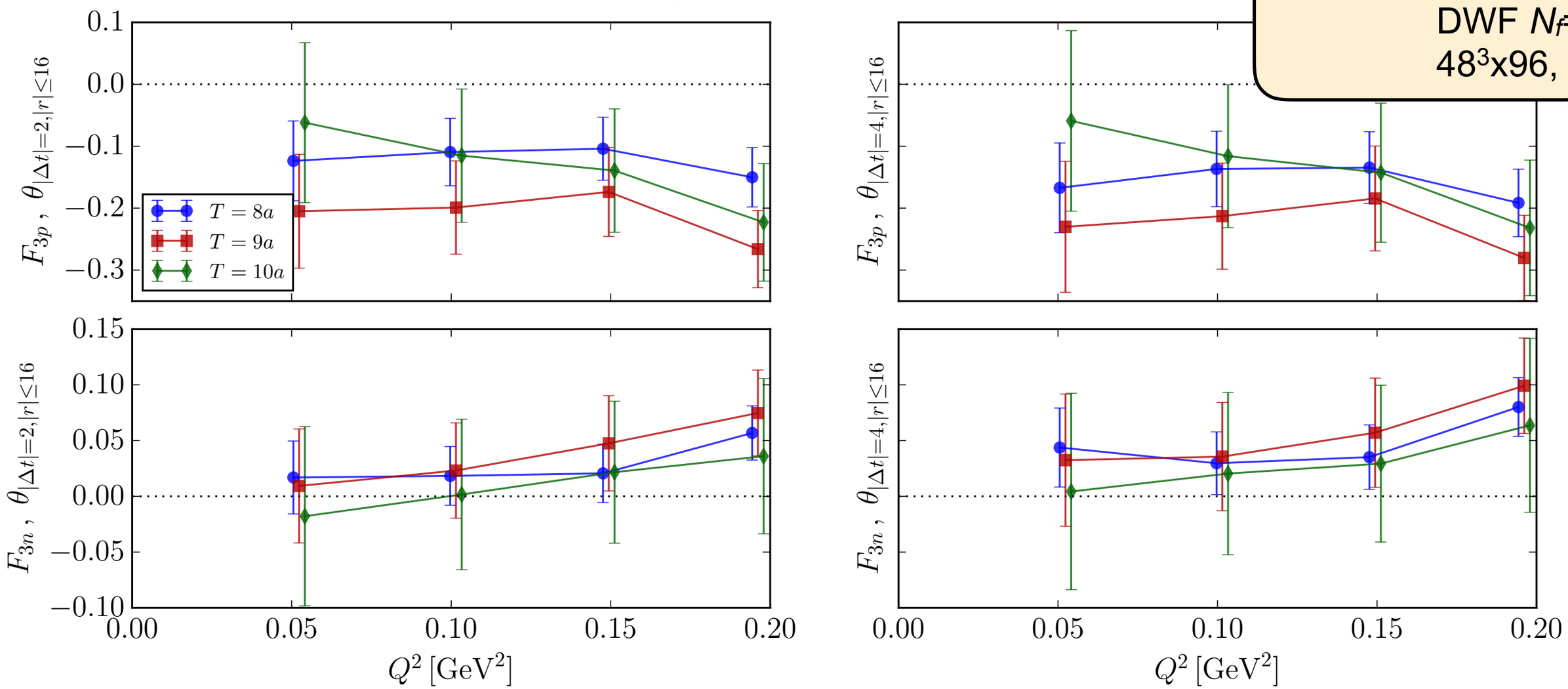
Reassuring results for noise reduction at the physical point

- time region $\Delta t_Q \gtrsim 8a \approx 1.2\,\mathrm{fm}$
- spatial region $r_Q \gtrsim 20a \approx 2.3\,\mathrm{fm}$

Physical point : θ_{QCD}-induced EDFF F_3

EDFF F_3 from constrained Q sum (*the most aggressive* Q cuts)

Physical poi
DWF N_f=2+1
48^3x96, a=0.



48^3x96 m_π=139 MeV (P

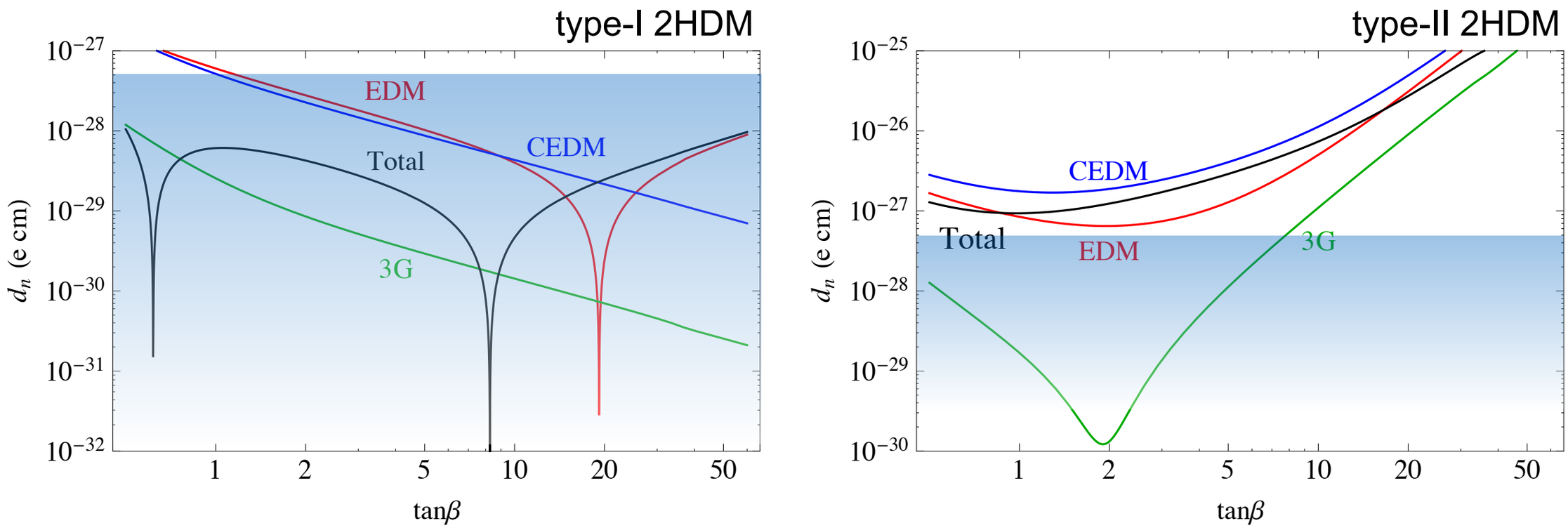
- 33k lattice samples, ~ 30 M core-hours on Argonne BlueGene/Q
- connected diagrams only
- result compatible with zero, $|F_{3n}| \leq 0.05$ constraint

Need x30..100 more statistics to constrain $|F_{3n}| \approx 0.$
θ-nEDM remains difficult at the physical point

Example: Impact on 2 Higgs-Doublet Model

Exp.constraint $d_n \lesssim 1.6 \cdot 10^{-26}\,\mathrm{e}\cdot\mathrm{cm}$

Expected in 10yr $d_n \lesssim 5 \cdot 10^{-28}\,\mathrm{e}\cdot\mathrm{cm}$



Contributions to the neutron EDM in flavor-conserving 2HDM

[S.Inoue, M.Ramsey-Musolf, Y.Zhang, PRD89:115023(2014)]

- *Assuming* opposite signs of EDM and CEDM contributions -> cancellation
- *Assuming* "ballpark" estimate for CEDM matrix elements similar to EDM

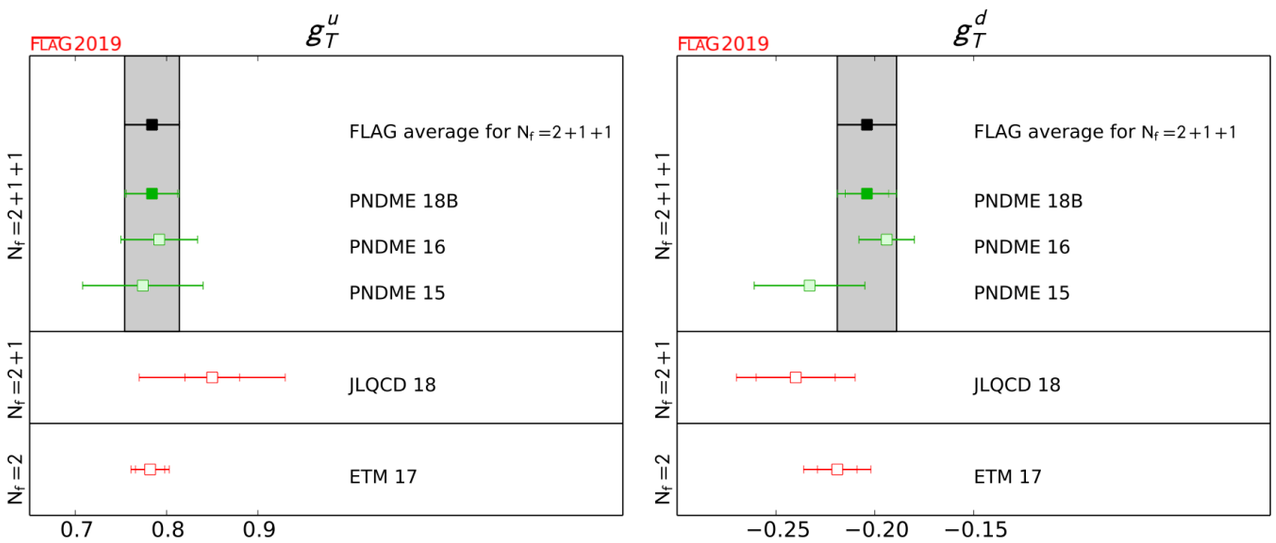
qCEDM-induced Nucleon EDMs induced by qCEDM, EDM, 3G
== sensitivity of nEDM to CPv mechanisms in SM extensions

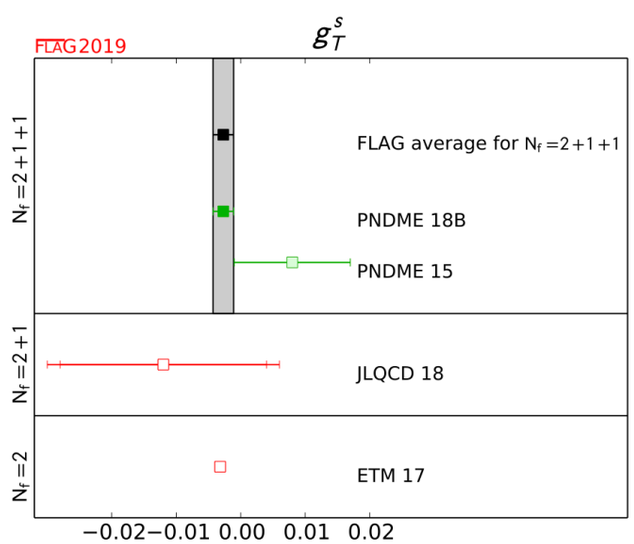
Quark EDMs

Quark → nucleon EDM: "tensor charge" $\delta\mathcal{L}_{\rm CPV} \supset -\frac{ie}{2}\sum_{f=u,d,s,e} d_f\ \bar{f}\sigma_{\mu\nu}\gamma_5 F^{\mu\nu} f$

$$d_N = g_T^u d_u + g_T^d d_d + g_T^s d_s$$

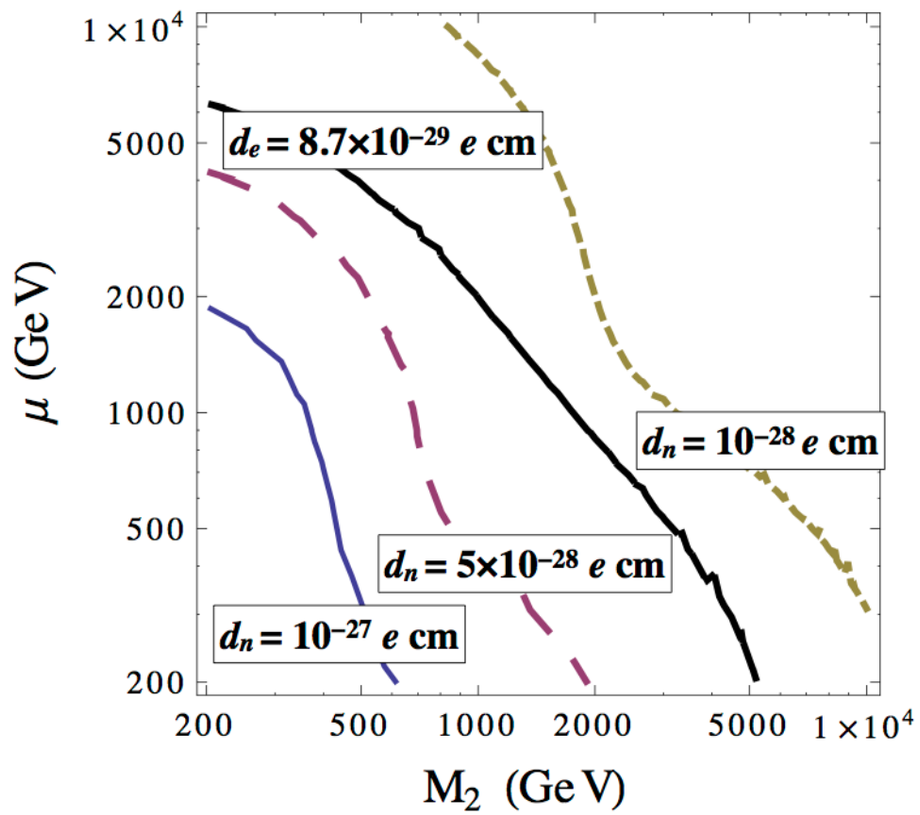
$$\langle N|\bar{q}\sigma^{\mu\nu}q|N\rangle = d_T^q\ \bar{u}\sigma^{\mu\nu}u$$





[Y.Aoki et al, FLAG 2019 review 1902.08191]

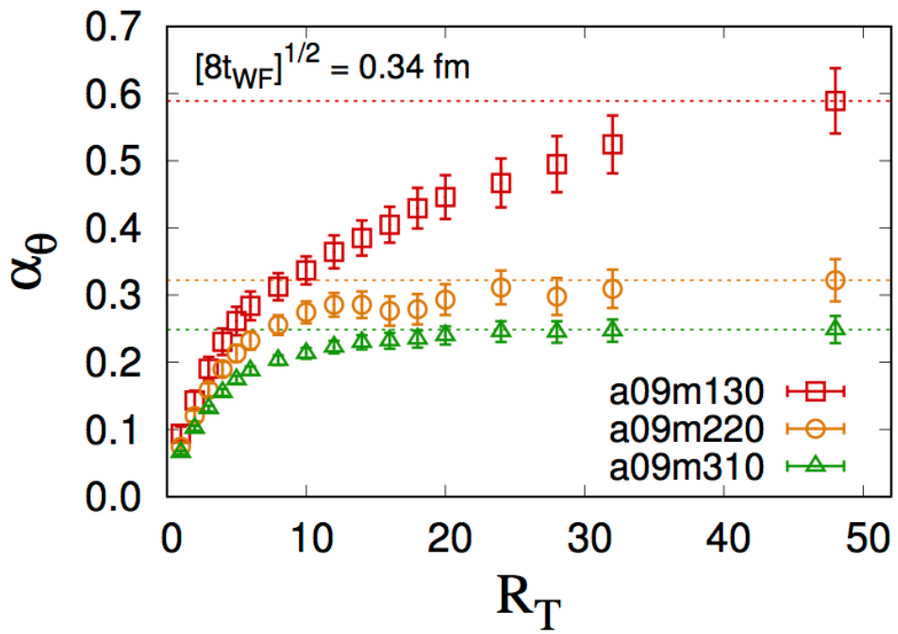
Constraints on split-SUSY model

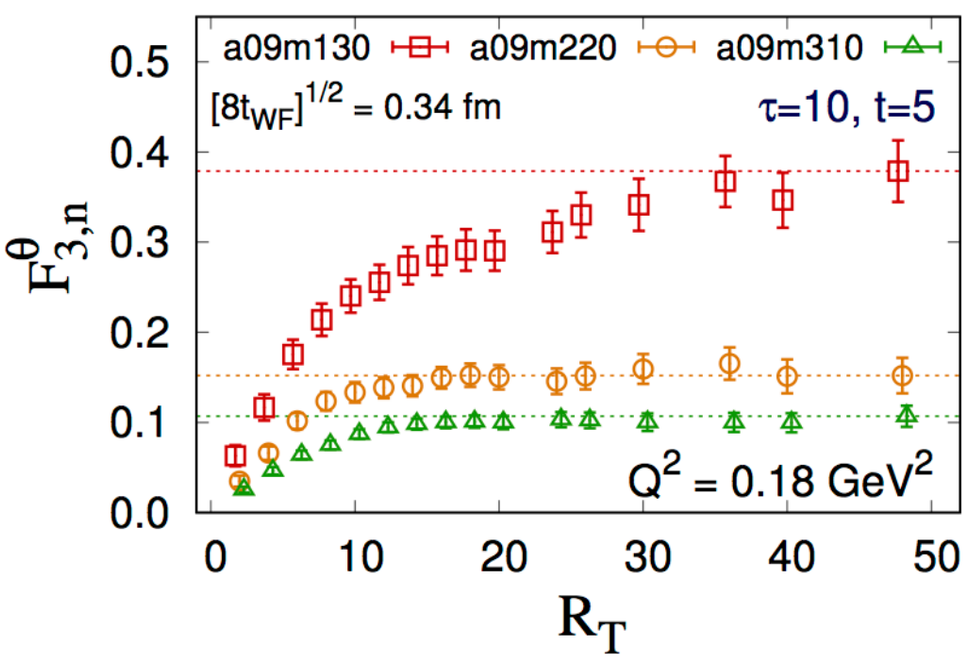


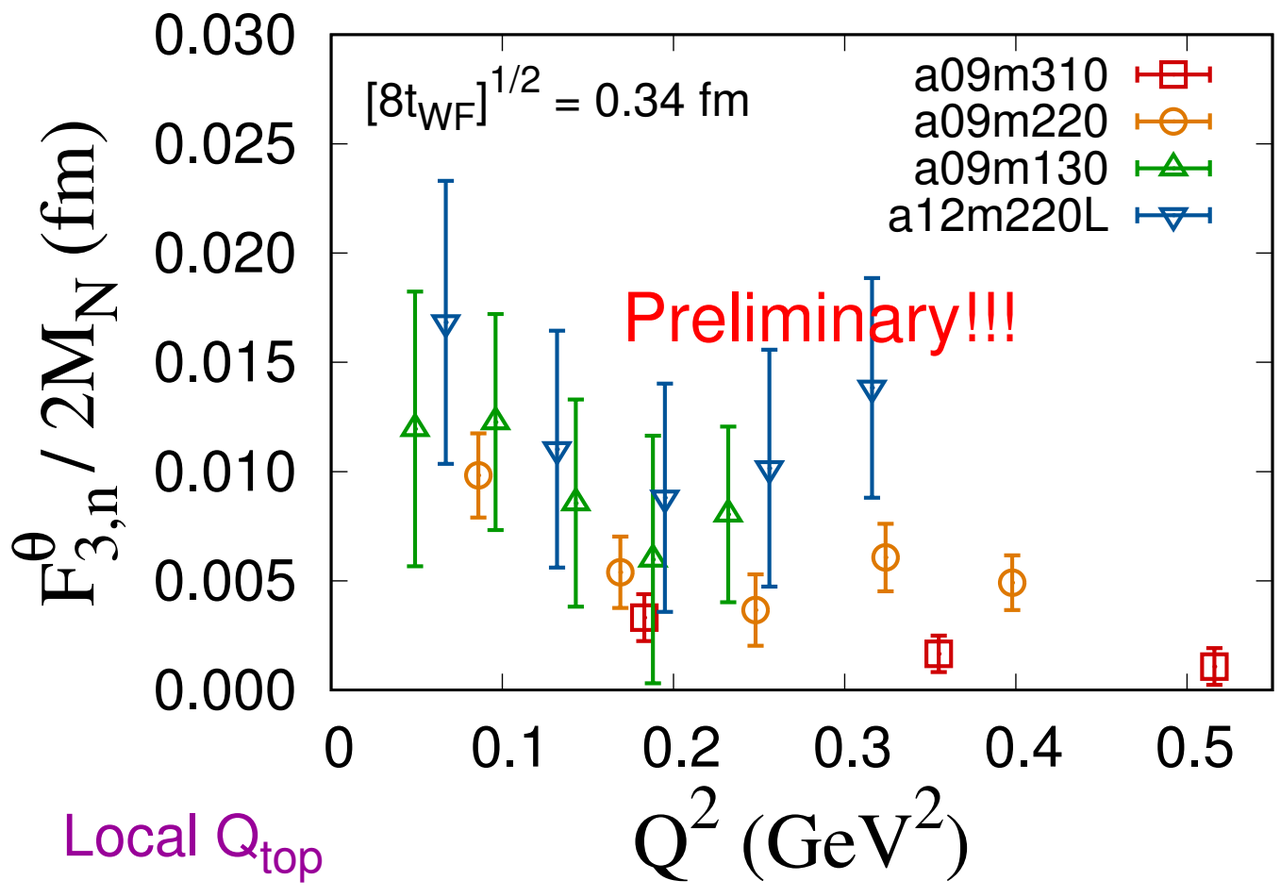
[Bhattacharya et al, PRL115: 212002 (2019)]

Theta-nEDM with Clover fermions

[B.Yoon et al, LATTICE 2019]







Local Q_{top}

R_T-convergence need further exploration at the physical point