Calculations of Nucleon EDMs on a Lattice

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Jefferson Lab Theory Seminar
Newport News, VA, December 9, 2019
Outline

- Nucleon Electric Dipole Moments: Introduction
  - Motivation
  - Experimental status & outlook
  - Lattice methodology

- Studies of $\theta_{\text{QCD}}$-induced nucleon EDM
  - Noise reduction with subvolume top.charge sampling
  - Results from lattices with heavy pions $m_\pi \gtrsim 330$ MeV
  - Outlook for physical point calculations

- Novel method for $\theta_{\text{QCD}}$-induced physics
  - Electric dipole moments induced by quark chromo-EDM

- Summary & Outlook
Nucleon Electric Dipole Moments

EDMs are the most sensitive probes of CPv:

- Signals for beyond SM physics
  
  \( \text{SM} = 10^{-5} \) of the current exp. bound

- Prerequisite for Baryogenesis

- Strong CP problem: \( \theta_{\text{QCD}} \)-induced EDM?

\[
\langle N_{p'} | J^\mu | \bar{N}_p \rangle_{\mathcal{CP}} = \bar{u}_{p'} [F_1 \gamma^\mu + (F_2 + iF_3 \gamma_5) \frac{\sigma^{\mu\nu} (p' - p)_\nu}{2m_N}] u_p
\]

*Dirac*  
*Pauli*  
*Electric dipole*  
*(anom. magnetic)*

A. Sakharov's conditions for baryon asymmetry in the Universe [JETP letters, 1967]

- \( \mathcal{P}, \mathcal{CP} \) symmetry violation
- Baryon number violation
- Non-equilibrium transition
Experimental Outlook

Current nEDM limits:

- $|d_n| < 2.9 \times 10^{-26} \, e \cdot cm$ (stored UC neutrons)  
  [Baker et al, PRL97: 131801(2006)]
- $|d_n| < 1.6 \times 10^{-26} \, e \cdot cm$ (199Hg)  
  [Graner et al, PRL116:161601(2016)]

- SM prediction from CKM CP

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<th>10^{-28} , e \cdot cm</th>
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<td>CURRENT LIMIT</td>
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<td>Spallation Source @ORNL</td>
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<td>Munich FRMII</td>
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<td>RCMP TRIUMF</td>
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<td>Standard Model (CKM)</td>
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Future nEDM sensitivity:

- 1–2 years: next best limit?
- 3–4 years: x10 improvement
- 7-10 years: x100 improvement

Other nuclear EDM experiments:
- light nuclei in storage rings, octupole-deformed $^{225}\text{Ra}$, etc
**Effective quark-gluon CPv interactions:**
dimension $\iff$ scale of BSM physics

\[ L_{\text{eff}} = \sum_i \frac{c_i}{\Lambda(i)^d} \mathcal{O}[d_i] \]

$d=4$ : $\theta_{QCD}$

$d=5(6)$ : quark EDM, chromo-EDM

$d=6$ : 4-fermion CPv, 3-gluon (Weinberg)

**Lattice QCD:**

quark-gluon CPv $\implies$ nucleon/pion CPv

(Nucleon EDM + $\pi NN$ CP interactions)

\[ d_{n,p} = d_{n,p}^\theta \theta_{QCD} + d_{n,p}^{cEDM} c_{cEDM} + \ldots \]

required to constrain $\theta_{QCD}$, $c_{cEDM}$, ...
CPv in QCD on a Lattice

- CP-odd interaction as perturbation
  
  \[ \langle \mathcal{O} \ldots \rangle_{\mathcal{CP}} = \langle \mathcal{O} \ldots \rangle_{\mathcal{CP-even}} - i\theta \langle Q \cdot \mathcal{O} \ldots \rangle_{\mathcal{CP-even}} + O(\theta^2) \]

  - \( \mathcal{CP} \) coupling
  - \( \mathcal{CP} \) operator: \( G\bar{G}, \ cEDM, \ G\bar{G}\bar{G}(\text{Weinberg}), \ldots \)

- Finite (imaginary) CPv: \( \theta^{QCD} \)
  
  \[ \langle \mathcal{O} \ldots \rangle_{\theta} \sim \int \mathcal{D}U \ e^{-S - \theta^I Q} (\mathcal{O} \ldots) \]

  require dedicated QCD simulation
  
  \( \implies \) better sampling of \( Q\neq0 \) sectors
Determination of Nucleon EDM

- Nucleon energy shift in uniform electric field
  [S.Aoki et al '89; E.Shintani et al '06; E.Shintani et al, PRD75, 034507(2007)]
  \[
  \langle N(t)\bar{N}(0)\rangle_{\theta,E} \sim e^{-(E\pm\vec{d}_N\cdot\vec{E})t}
  \]
  Euclidean lattice:
  **Real**-valued \(\vec{E}\) \(\implies\) time-BC violated
  **Imag**-valued \(\vec{E}\) \(\implies\) imaginary shift in \(m_N\)

- P,T-odd Form Factor \(d_N=F_3(0)/2m_N\)
  [E.Shintani et al '05, '15; F.Berruto et al '05; A.Shindler et al '15; C.Alexandrou et al'15]
  \[
  \langle N_{p'}|\bar{q}\gamma^\mu q|N_p\rangle_{QP} = \bar{u}_{p'}\left[F_1\gamma^\mu + (F_2 + iF_3\gamma_5)\frac{i\sigma^{\mu\nu}(p' - p)\nu}{2m_N}\right]u_p
  \]
  Require extrapolation \(F_3(Q^2\to0)\)
Nucleon "Parity Mixing"

CPv interaction induces a chiral phase in nucleon wave functions on a lattice

\[
\langle \text{vac} | N | p, \sigma \rangle_{\text{CP}} = e^{i\alpha_5} u_{p,\sigma} = \bar{u}_{p,\sigma}
\]

\[
\sum_{\sigma} \bar{u}_{p,\sigma} \tilde{u}_{p,\sigma} \sim (-i\Sigma + m_N e^{2i\alpha_5})
\]

![Diagram showing the comparison of form factor \(F_3\) to energy shift in background \(E=\text{const}\)]

**Check:** cEDM-induced EDM / EDFF comparison of form factor \(F_3\)

**Correction to \(F_3\) results prior to 2017:**

\[
\tilde{d}_{n,p} \approx [d_{n,p}]_{\text{true}} - 2\alpha \frac{\kappa_{n,p}}{2m_N}
\]

**Graphs showing the comparison of \(F_3\) values with different masses:**
- \(m = 465\text{MeV}\)
- \(m = 360\text{MeV}\)
Nucleon "Parity Mixing"

CPv interaction induces a chiral phase in nucleon wave functions on a lattice

\[
\langle \text{vac} | N | p, \sigma \rangle_{CP} = e^{i\alpha \gamma_5} u_{p,\sigma} = \tilde{u}_{p,\sigma} \\
\sum_{\sigma} \tilde{u}_{p,\sigma} \tilde{u}_{p,\sigma} \sim (- i \phi_{\epsilon} + m_N e^{2i\alpha \gamma_5})
\]

[Error! Reference source not found.]

EDM and MDM are defined with positive-parity spinors

\[
\langle N_{p^{'}} | q \gamma^{\mu} q | N_p \rangle_{CP} = \bar{u}_{p^{'}} \left[F_1 \gamma^{\mu} + (F_2 + iF_3) \gamma_5 \right] \frac{i\sigma^{\mu\nu} (p' - p)_{\nu}}{2m_N} u_p,
\]

\[
\gamma_4 u = +u, \quad \bar{u}\gamma_4 = +\bar{u}
\]

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<th>(m_\pi ) [MeV]</th>
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<td>-1.473(37) (^c)</td>
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<td>-0.248(29)</td>
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After removing the spurious contribution,

- no lattice signal for \(\theta_{QCD}\)-induced nEDM
- RESOLVED conflict with pheno. values, lack of \(d_N \sim m_q\) scaling
Importance of "Parity Mixing" Correction

Exact value of $\alpha_5$ is critical for correct determination of EDM:

$$F_{3}^{\text{lat}}(Q^2) \approx \frac{m}{q_3} \left\langle N_{\uparrow}(0) \bar{q}_4 q \right| N_{\uparrow}(-q_3) \left| \bar{q}_4 q \right\rangle_{CPV} - \alpha_5 G_E(Q^2)$$

- CPV matrix element
- Sachs form factor subtraction

- For proton, nonzero correction $\sim \alpha_5$
- For neutron, zero correction at $Q^2=0$
  however, may skew $Q^2\rightarrow0$ extrapolation in practice
Nucleon EDM Induced by Quark Chromo-EDM

P-,T-odd Dim-5 operator (Dim-6 with Higgs vev)

\[ \mathcal{L}_{c\text{EDM}} = \sum_{q=u,d} \frac{\delta_q}{2} \bar{q} \left[ G_{\mu\nu} \sigma^{\mu\nu} \gamma_5 \right] q \]

- dim-5 operator: \( O(a^2) \) mixing with dim-3 pseudoscalar density
  \[ P = \bar{q} \gamma_5 q \]
  \[ \text{[T.Bhattacharya et al, 1502.07325]} \]

- Chiral symmetry is important:
  \( O(a) \) clover term in, e.g., Wilson fermion action \( \equiv \) chromo-magnetic DM

\[ \mathcal{L}^{\text{clover}} = a \frac{c}{4} \bar{q} \left[ G_{\mu\nu} \sigma^{\mu\nu} \right] q \]

In presence of CPv, condensate is realigned

\[ q \rightarrow e^{i\gamma_5 \Omega} q \]

so that

\[ \langle \text{vac} | \mathcal{L}_m + \mathcal{L}_{GP} | \pi^a \rangle = 0 \]

leading to mixing (chromo)EDM \( \leftrightarrow \) (chromo)MDM:

\[ \delta \mathcal{L}_{c\text{EDM}} = \delta (\bar{q} \left[ \tilde{D}_q G_{\mu\nu} \sigma^{\mu\nu} \gamma_5 \right] q) = \bar{q} \left[ \left\{ \Omega, \tilde{D}_q \right\} G_{\mu\nu} \sigma^{\mu\nu} \right] q \sim \delta \mathcal{L}_{c\text{MDM}} \]
Quark-Gluon EDM: Insertions of dim-5 Operators

\[ \mathcal{L}^{(5)} = \sum_q \tilde{d}_q \bar{q}(G \cdot \sigma) \gamma_5 q \]

\[ \langle N(y) \bar{N}(0) \int d^4 x \bar{q}(G \cdot \sigma) \gamma_5 q \rangle \]

- This work: Only quark-connected insertions
- In future: Single- and double-disconnected diagrams (contribute to isosinglet cEDM, mix with \( \theta \)-term)
Nucleon Vector (Sachs) Form Factors

\[ G_E = F_1 - \frac{Q^2}{4m_N^2} F_2 \]

\[ G_M = F_1 + F_2 \]

Physical point
DWF \( N_f=2+1 \)
48^3 \times 96, \( a=0.114 \) fm

~30 M core*hours @Mira (Bluegene/Q)
[with RBC collaboration, in prep.]
Parity Mixing: cEDM and pseudoscalar(*)

\[ N_\delta = \varepsilon^{abc} u_\delta^a (u^{aT} C \gamma_5 d^c) \]

\[ \langle N(t)\bar{N}(0) \rangle_{\text{CP}} = \frac{-i\psi + m_N e^{2i\alpha_5 \gamma_5}}{2m_N} e^{-E_N t} \]

\[ \alpha_5 = \frac{1}{\tilde{d}} = -\frac{\text{ReTr}[T^+ \gamma_5 \cdot \delta C^P C_{2pt}(t)]}{\text{ReTr}[T^+ \cdot C_{2pt}(t)]}, \quad t \to \infty \]

(flavor labels for the proton uud)

(*)connected-only, bare cEDM and PS operators

Physical point
DWF \( N_f=2+1 \)
48\(^3\) x 96, \( a=0.114 \) fm
Neutron EDM induced by (u-d) chromoEDM

- isovector CPv interaction: no mixing with $\theta_{QCD}$ term
- disconnected corrections are required for final answer
  (although found very small for neutron $F_2(Q^2)$)

(*) connected-only; bare cEDM and PS operators

Physical point
DWF $N_f=2+1$
$48^3\times96$, $a=0.114$ fm
Proton & Neutron EDM FF from u,d cEDM (*)

Physical point
DWF \( N_f=2+1 \)
48^3 \times 96, a=0.114 \text{ fm}

\( (*) \) connected-only; bare cEDM and PS operators
Proton & Neutron EDM FF from u,d Pseudoscalar

(*)connected-only, bare cEDM and PS operators

Physical point
DWF $N_f=2+1$
$48^3 \times 96$, $a=0.114$ fm
Renormalization of cEDM: RI-MOM

"Momentum-scheme": matrix elements between virtual quark states in Landau gauge

Problem: divergent contact terms of gauge-dependent quark fields and operators
implies mixing with gauge-dependent and EoM-vanishing operators [Collins, "Renormalization"]

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Total: 14 operators (all flavor combinations)
Renormalization of cEDM: Position Space

Alternative: matching short-distance correlators at $X \approx \mu^{-1} \ll (\Lambda_{QCD})^{-1}$
("position space scheme"); extending work of [Gimenez et al (2004); Chetyrkin(2010)]

**Matching CPv operators**

\[ \mathcal{C} = \bar{q} \left[ \frac{1}{2}(\sigma_{\mu\nu} G^{\mu\nu}) \gamma_5 \right] q \]
\[ \mathcal{P} = \bar{q} \gamma_5 q \]

\[ \mathcal{C}^{MS} = Z_C [\mathcal{C}]^{\text{lat}} + Z_C / m^2 \mathcal{P} [m^2 \mathcal{P}]^{\text{lat}} \]
\[ \langle C(r)C(0) \rangle^{MS} = Z_C^2 \langle C(r)C(0) \rangle^{\text{lat}} \]
\[ + 2Z_C Z_C / m^2 \mathcal{P} \langle m^2 \mathcal{P}(r)C(0) \rangle^{\text{lat}} \]
\[ + Z_C^2 / m^2 \mathcal{P} \langle m^2 \mathcal{P}(r)m^2 \mathcal{P}(0) \rangle^{\text{lat}} \]

NLO($\alpha_s$) in MSbar
\[ \langle C(r)C(0) \rangle^{(1)} = \]

Also
\[ \langle C(r)\mathcal{P}(0) \rangle \]
\[ \langle \mathcal{P}(r)\mathcal{P}(0) \rangle \]

[M.Kellerstein, SNS, *in preparation*]

**Isoscalar cEDM (d=5) will also mix with G\tilde{G} (d=4) ⇒ "noisy" subtraction on a lattice**
Renormalization of cEDM: Position Space

Isovector cEDM: no G\bar{G} mixing

\begin{align*}
&\langle \mathcal{P}(r)\mathcal{P}(0) \rangle \\
&\approx r^{-6} \\
&\approx r^{-8} \\
&\approx r^{-10}
\end{align*}

LO scaling of correlators (dim.analysis)

\begin{align*}
&\langle \mathcal{P}(r)\mathcal{P}(0) \rangle \overset{\text{LO}}{\sim} r^{-6} \\
&\langle \mathcal{C}(r)\mathcal{P}(0) \rangle \overset{\text{LO}}{\sim} r^{-8} \\
&\langle \mathcal{C}(r)\mathcal{C}(0) \rangle \overset{\text{LO}}{\sim} r^{-10}
\end{align*}

- evident mixing from lattice data
- minimal displacement $|r|=2 \{1,1,1,1\}$ to avoid contact terms
- narrow window to avoid NP effects (pions):
  \[ |r| \lesssim (0.5 \text{ GeV})^{-1} \approx (3..4) a \]
- isoscalar cEDM will also require disconnected correlators and mix with (G\bar{G})
**EDM from θ-Term: Noise from Global Q_{top}**

Variance of lattice θ-induced nEDM signal \( \sim (\text{Volume})^{4d} \):

\[ d_N \sim \langle \mathbf{Q} \cdot (N J_{\mu} \tilde{N}) \rangle \]

Top. charge \( Q \sim \int_{V_4} (G \tilde{G}) \), with \( \langle |Q|^2 \rangle \sim V_4 \)

Constrain Q sum to the fiducial volume

- in time around current, \( |t_Q - t_J| < \Delta t \) [E.Shintani et al (2015); B.Yoon et al (2019)]
- in time around source, \( |t_Q - t_{\text{source}}| < \Delta t \) [J. Dragos, talk on Tue]
- 4-d sphere around sink, \( |x_Q - x_{\text{sink}}| < R \) [K.-F.Liu et al, (2017)]:

Proper treatment of nucleon parity mixing is critical for correct determination of \( F_3 \)

**⟹ nucleon must "settle" in the new \( \theta \neq 0 \) vacuum**

\[ N^{(+)} \rightarrow \tilde{N}^{(+)} \approx N^{(+)} + i\alpha N^{(-)} \]

\[ N^{(-)} \rightarrow \tilde{N}^{(-)} \approx N^{(-)} - i\alpha N^{(+)} \]

**⟹ constrain time and space differently :**

4d "cylinder" \( V_Q : |\mathbf{z}| < r_Q, \quad -\Delta t_Q < z_0 < T + \Delta t_Q \)
\( \theta \)-EDM at \( m_\pi = 330 \) MeV: Parity Mixing

- \( N_f=2+1 \) Domain Wall (RBC/UKQCD) \( 24^3 \times 64 \) \( a = 0.114 \) fm
- 1400 configs * (64sloppy+1exact) samples \( \rightarrow 89.6k \) stat.
- Top charge with 5-loop improved \( G\tilde{G} \) [P. de Forcrand et al '97] on Wilson-flowed \( (t=8a^2) \) gauge links [M.Luscher, 1006.4518]

Parity mixing angle \( \alpha_N \) as a function of \( r_Q, \Delta t_Q \)

- Convergence at \( r_Q \approx 16a, \Delta t_Q \approx 8a \)
θ_{QCD}-EDM at m_π=330 MeV: EDM Form Factor

\[ r_Q = 8a \]
θ_{QCD}-EDM at Physical point: EDM Form Factor

EDFF $F_3$ from constrained $Q$ sum (the most aggressive $Q$ cuts)

33k lattice samples, ~ 30 M core-hours on Argonne BlueGene/Q
connected diagrams only
result compatible with zero, $|F_{3n}| \leq 0.05$ constraint

From LO ChPT, $d_n m_q \sim (m\pi)^2 : |F_{3n}|_{phys} \approx 0.01$
Need x30..100 more statistics
EDM from (rev.) Feynman-Hellman Theorem

- FH relates perturbation's matrix element to energy shift

\[
\frac{\partial E_\lambda}{\partial \lambda} = \langle \phi_\lambda | \frac{\partial \hat{H}_\lambda}{\partial \lambda} | \phi_\lambda \rangle
\]

(used successfully to compute \(g_A\) to sub-% precision) and other forward hadron matrix elements

Nucleon EDM from FH:
- theta-term is a small perturbation:

\[
\delta \mathcal{L} = \frac{\theta g^2}{32\pi^2} \int dt \int d^3xG^a_{\mu\nu}\tilde{G}^a_{\mu\nu}
\]

\[
m'_N = m_N - (d^\theta_N \theta) \bar{\Sigma} \cdot \vec{E}
\]

- which leads to a mass shift of a polarized nucleon in background electric field

- FH \(\rightarrow\) relation between EDM and matrix element of local top charge density

\[
d^\theta_N \propto \langle N^\uparrow | \int d^3x \, G^a_{\mu\nu}\tilde{G}^a_{\mu\nu} | N^\uparrow \rangle \epsilon_z
\]

[Chang et al (CalLat), Nature 558:91 (2018)]
EDM = Top. charge Density in Polarized Nucleon

\[ d_{N}^{\theta} \propto \left\langle N_{\uparrow} \right| \int d^{3}x \, G_{\mu\nu}^{a} \tilde{G}_{\mu\nu}^{a} \left| N_{\uparrow} \right\rangle \mathcal{E}_{z} \]

in CP-even vacuum, both spin- and charge-polarization are required for \( \left\langle G\tilde{G} \right\rangle \neq 0 \)
(equivalent to background E \parallel B fields)

**permanent EDM** =
**correlation** of spin and charge =
- *charge polarization* of a spin-polarized state in CPv (\( \theta \))-vacuum, **OR**
- *spin polarization* of a charge-polarized state in CPv (\( \theta \))-vacuum, **OR**
- "topological" polarization of a charge- and spin-polarized state in CP-even vacuum

**NOTE**: without background E field, \( \left\langle N|G\tilde{G}|N \right\rangle \) is zero due to parity

\[
\mathcal{P}\left(\left| N_{\uparrow}^{(+)} \right\rangle \right) = +\left| N_{\uparrow}^{(+)} \right\rangle
\]
\[
\mathcal{P}(G\tilde{G}) = -GG
\]

\[
\implies \left\langle N_{\uparrow}^{(+)}|G\tilde{G}|N_{\uparrow}^{(+)} \right\rangle = 0
\]

background E polarizes state into a mixed-parity state:

\[
\left| N \right\rangle \mathcal{E}_{z} = \left| N^{(+)} \right\rangle + O(\mathcal{E}_{z}) \left| N^{(-)} \right\rangle
\]
(P-mixed state in CP-even vacuum)
Background Electric Field

Accessing magnetic and electric moments at $Q^2=0$
Imag.Minkowski/Real Euc. electric field on a lattice
[W.Detmold et al (2009)] : calculation of hadron polarizabilities

$U_\mu \rightarrow e^{iqA_\mu} U_\mu$

$A_z(z, t) = n \mathcal{E}_{\text{min}} \cdot t$

$A_t(z, t = L_t - 1) = -n \mathcal{E}_{\text{min}} \cdot L_t z$

Full flux through the "side" of the periodic box

$q\Phi = 2\pi \cdot n$

Constant Electric field has to be quantized,

$\mathcal{E}_{\text{min}} = \frac{1}{|q_d|} \frac{2\pi}{L_x L_t}$

Unambiguous determination of EDM from the energy shift
Straightforward for neutron with $Q=0$

Electric field on a $24^3 \times 64$ lattice

$\mathcal{E} = \frac{6\pi}{L_x L_t} \approx 0.037 \text{ GeV}^2$

$\approx 186 \text{ MV/fm}$
Gradient flow: covariant 4D-diffusion of quantum fields with "G.F." time $t_{GF}$:

$$\frac{d}{dt_{GF}} B_\mu(t_{GF}) = D_\mu G_{\mu\nu}(t_{GF}), \quad B_\mu(0) = A_\mu$$

Tree-level:

$$B_\mu(x, t_{GF}) \propto \int d^4 y \exp \left[ - \frac{(x - y)^2}{4t_{GF}} \right] A_\mu(y)$$

Gradient-flowed topological charge:

$$\tilde{Q}(t_{GF}) = \left. \int d^4 x \frac{g^2}{32\pi^2} \left[ G_{\mu\nu} \tilde{G}_{\mu\nu} \right] \right|_{t_{GF}}$$

- effective scale $\Lambda_{UV} \to (t_{GF})^{-1/2}$
- smooth fields (reduce $|G_{\mu\nu}|$)
  $\iff$ continuous "cooling"
- remove $G_{\mu\nu}$ dislocations
  $\Rightarrow$ dynamical separation of top. sectors
- diffusion of top. charge density

$Q = \int q(x)$

$Q = \tilde{Q}$

$Q = \tilde{Q}$

$Q = \tilde{Q}$
Gradient-Flowed Topological Charge Density

\[ q(x) = \frac{g^2}{32\pi^2} G_{\mu\nu}^a \tilde{G}_{\mu\nu}^a \]
\[ \approx \frac{1}{16\pi^2} \frac{1}{a^4} \text{Tr} [G_{\mu\nu}^\text{lat} \tilde{G}_{\mu\nu}^\text{lat}] \]
\[ \propto (\mathbf{E} \cdot \mathbf{H})_{\text{color}} \]

Gradient flow:
- effective scale \((\Lambda_{\text{UV}})^{-1} \rightarrow (t_{\text{GF}})^{1/2}\)
- make fields smooth (reduce \(|G_{\mu\nu}|\))
- remove dislocations \(\Rightarrow\) dynamical separation of topological sectors
- [M.Luscher, JHEP08:071; 1006.4518]
- 4D-diffusion (including time) of \(q(x)\)
  \[ \langle q(x)q(0) \rangle \sim \exp[-(x-y)^2/8t_{\text{GF}}] \]

24\(^3\) x 64 lattice, \(m_\pi \approx 340\) MeV
**Tunneling Between Topology Sectors**

\[ q(x) = \frac{g^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^a_{\mu\nu} \]

\[ \approx \frac{1}{16\pi^2} \frac{1}{a^4} \text{Tr} [G^\text{lat}_{\mu\nu} \tilde{G}^\text{lat}_{\mu\nu}] \]

\[ \propto (E \cdot H)_{\text{color}} \]

**Instantons and Anti-Instantons:**
Quantum tunneling of gluon field between topological sectors

\[ |\text{vac}\rangle_\theta = \sum_Q e^{i\theta Q} |Q\rangle \]

6 s video = 5 fm / c = 1.7 \times 10^{-23} \text{ s real time}

[Lattice QCD at the physical point]
Gradient-flowed topological charge density is nonlocal:

$$\tilde{q}(x) \propto \int d^D y \exp \left[ -\frac{(x - y)^2}{2r_Q^2(t_{GF})} \right] q(y)$$

Empirically for \( r, \sqrt{t_{GF}} \gg m_{\eta'}^{-1} \),

$$\langle \tilde{q}(r)\tilde{q}(0) \rangle \propto \exp \left[ -\frac{r^2}{4r_Q^2(t_{GF})} \right]$$

**Gradient flow leads to diffusion of \( q(x) \) in Euclidean (lattice) time:**

\[ \implies \text{complications for matrix element analysis} \]
Matrix Elements of Gradient-Flowed $Q(t_{GF})$

Two effects observed:
1. Convergence to ground state matrix el.
2. Diffusion of top.charge for $t_{sep} \lesssim 7a$

Analysis of $(\tau_Q, t_{GF})$ required to detangle

$\langle N | G\tilde{G} | N \rangle$,
$\langle N | G\tilde{G} | N \rangle_{exc}$,
$\langle \text{vac} | G\tilde{G} | N \bar{N} \rangle$,
...

in agreement with FF. method

$2md_n = F_3(0) \approx 0.11 .. 0.13$

PRELIMINARY

from plateaus

$\langle N_{\uparrow}(t_{sep}) \tilde{Q}(\tau_Q) \bar{N}_{\uparrow}(0) \rangle_{\varepsilon_z} \propto \exp \left[ - \frac{(\tau_Q - \tau_Q')^2}{4t_{GF}^2} \right] \otimes \langle N_{\uparrow}(t_{sep}) \tilde{Q}(\tau_Q') \bar{N}_{\uparrow}(0) \rangle_{\varepsilon_z}$
Encouraging results for nucleon EDM induced by quark chromo-EDM physical-point

~20% stochastic uncertainty for quark cEDM-induced EDM
Renormalization & mixing subtractions are underway
Full flavor dependence will require disconnected diagrams & $\theta_{QCD}$-term

Clear signal for $\theta_{QCD}$-induced nEDM at $m_\pi = 330$ MeV

Variance-reduction for Q sampling is essential

Novel method to compute nEDM from local topological charge

Potential method for physical-point calculations
Computing $\theta$-nEDM at the Physical Point?

- chiral fermions, $m_\pi = 330$ MeV [this work]
  
  $|2m_n d_n| = |F_{3n}(0)| \approx 0.05 \cdot \theta$

- Wilson fermions, $m_\pi = 360$ MeV [Guo et al 2015] after correction
  
  $|2m_n d_n| = |F_{3n}(0)| \approx 0.06 \cdot \theta$

- best guess for the physical point with LO ChPT,
  
  $|d_n| \sim m_q \sim (m_\pi)^2 \Rightarrow$ phys.point

  $|F_{3n}(0)| \approx 0.01 \cdot \theta, \; |d_n| \approx 0.001 \cdot \theta$ e fm

\[
|F_{3n}^{phys}(0)| \sim O(10^{-2}) \cdot \theta, \quad |d_n| \sim O(10^{-3}) \text{ e fm } \theta
\]

$\theta_{QCD}$ from estimated $|F_{3n}(0)| \approx 10^{-2} \cdot \theta$:

- from neutron: $|d_n| \leq 2.9 \cdot 10^{-26}$ e·cm [Baker et al (2006)] : $|\theta_{QCD}| \leq 2.9 \cdot 10^{-10}$

- from $^{199}$Hg: $|d_n| \leq 1.6 \cdot 10^{-26}$ e·cm [Graner et al (2016)] : $|\theta_{QCD}| \leq 1.6 \cdot 10^{-10}$
Physical point: $\theta_{QCD}$-induced Parity Mixing $\alpha_N$

Parity-mixing angle from constrained $Q$ sum

Reassuring results for noise reduction at the physical point

- time region 
  \[ \Delta t_Q \gtrsim 8a \approx 1.2 \text{ fm} \]
- spatial region 
  \[ r_Q \gtrsim 20a \approx 2.3 \text{ fm} \]
Physical point: $\theta_{QCD}$-induced $\text{EDFF } F_3$

EDFF $F_3$ from constrained $Q$ sum (the most aggressive $Q$ cuts)

- 33k lattice samples, ~30 M core-hours on Argonne BlueGene/Q
- connected diagrams only
- result compatible with zero, $|F_{3n}| \leq 0.05$ constraint

Need x30..100 more statistics to constrain $|F_{3n}| \approx 0$.$\theta$-$nEDM$ remains difficult at the physical point
Exp.constraint \( d_n \lesssim 1.6 \cdot 10^{-26} \text{ e} \cdot \text{cm} \)

Expected in 10yr \( d_n \lesssim 5 \cdot 10^{-28} \text{ e} \cdot \text{cm} \)

**Contributions to the neutron EDM in flavor-conserving 2HDM**

[S.Inoue, M.Ramsey-Musolf, Y.Zhang, PRD89:115023(2014)]

- **Assuming** opposite signs of EDM and CEDM contributions -> cancellation
- **Assuming** "ballpark" estimate for CEDM matrix elements similar to EDM

\[ q_{CEDM-induced} \text{ Nucleon EDMs induced by qCEDM, EDM, 3G} \]

\[ == \text{sensitivity of } n\text{EDM to CPv mechanisms in SM extensions} \]
Quark EDMs

Quark → nucleon EDM: "tensor charge"

\[ d_N = g_T^u d_u + g_T^d d_d + g_T^s d_s \]

\[ \langle N | \bar{q} \sigma^{\mu \nu} q | N \rangle = d_T^q \bar{u} \sigma^{\mu \nu} u \]

Constraints on split-SUSY model

[Y.Aoki et al, FLAG 2019 review 1902.08191]

[Bhattacharya et al, PRL115: 212002 (2019)]
Theta-nEDM with Clover fermions

[B. Yoon et al, LATTICE 2019]

\[ \Theta_{nEDM} \text{ with Clover fermions} \]

\[ \left[ 8t_{WF} \right]^{1/2} = 0.34 \text{ fm} \]

\[ F_{3,n}^0 / 2M_N (\text{fm}) \]

\[ Q^2 (\text{GeV}^2) \]

\[ R_T \text{-convergence need further exploration at the physical point} \]

\[ \text{Local } Q_{\text{top}} \]

\[ \text{Preliminary!!!} \]

\[ \text{Dragos, et. al., 2019} \]

\[ 410 \text{ MeV, 0.09 fm} \]

\[ 340 \text{ MeV, 0.11 fm} \]

\[ 139 \text{ MeV, 0.11 fm} \]