Extracting GPDs from DVCS data: Border and skewness functions at LO

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JLab Theory seminar
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Outline

- Introduction
- PARTONS project
- Global analysis of DVCS
- Summary
Deeply Virtual Compton Scattering (DVCS)

Chiral-even GPDs:
(helicity of parton conserved)

<table>
<thead>
<tr>
<th>$H^{q,g}(x, \xi, t)$</th>
<th>$E^{q,g}(x, \xi, t)$</th>
<th>for sum over parton helicities</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{H}^{q,g}(x, \xi, t)$</td>
<td>$\tilde{E}^{q,g}(x, \xi, t)$</td>
<td>for difference over parton helicities</td>
</tr>
</tbody>
</table>

nucleon helicity conserved

nucleon helicity changed

factorization for $|t|/Q^2 \ll 1$
GPDs accessible in various production channels and observables → experimental filters

- DVCS: Deeply Virtual Compton Scattering
- TCS: Timelike Compton Scattering
- HEMP: Hard Exclusive Meson Production

more production channels sensitive to GPDs exist!
Introduction

- Reduction to PDFs:
  \[
  H^q(x, 0, 0) \equiv q(x)
  \]
  \[
  \tilde{H}^q(x, 0, 0) \equiv \Delta q(x)
  \]
  \[
  H_T^q(x, 0, 0) \equiv h_1(x)
  \]

- Reduction to Elastic Form Factors (EFFs):
  \[
  \int_{-1}^{1} H^q(x, \xi, t) \equiv F_1^q(t)
  \]
  \[
  \int_{-1}^{1} \tilde{H}^q(x, \xi, t) \equiv g_A^q(t)
  \]
  \[
  \int_{-1}^{1} E^q(x, \xi, t) \equiv F_2^q(t)
  \]
  \[
  \int_{-1}^{1} \tilde{E}^q(x, \xi, t) \equiv g_P^q(t)
  \]

No corresponding relations exist for other GPDs.
Polynomiality - non-trivial consequence of Lorentz invariance:

\[ \int_{-1}^{1} x^n H^q(x, \xi, t) = h_0^{q,n}(t) + \xi^2 h_2^{q,n}(t) + \ldots + \text{mod}(n, 2)\xi^{n+1} h_{n+1}^{q,n}(t) \]

\[ \int_{-1}^{1} x^n \tilde{H}^q(x, \xi, t) = \tilde{h}_0^{q,n}(t) + \xi^2 \tilde{h}_2^{q,n}(t) + \ldots + \text{mod}(n + 1, 2)\xi^n \tilde{h}_n^{q,n}(t) \]

*strong constraint on GPD parameterizations*
Introduction

- Nucleon tomography

\[ q(x, b_{\perp}) = \int \frac{d^2 \Delta}{4\pi^2} e^{-i b_{\perp} \cdot \Delta} H^q(x, 0, t = -\Delta^2) \]

- Study of long. polarization with GPD $\tilde{H}$
- Study of distortion in transv. polarized nucleon with GPD $E$

- Impact parameter $b_{\perp}$ defined w.r.t. center of momentum, such as $\sum x b_{\perp} = 0$
Inequalities:

\[
|\Delta q(x, b_\perp)| \leq q(x, b_\perp)
\]

\[
\frac{b_\perp^2}{m^2} \left( \frac{\partial}{\partial b_\perp^2} e(x, b_\perp) \right)^2 \leq (q(x, b_\perp) + \Delta q(x, b_\perp)) \times (q(x, b_\perp) - \Delta q(x, b_\perp))
\]

to avoid violation of the positivity in the impact parameter space
Energy momentum tensor in terms of form factors:

\[
\langle p', s' | \hat{T}^{\mu\nu} | p, s \rangle = \bar{u}(p', s') \left[ \frac{P^\mu P^\nu}{M} A(t) + \frac{\Delta^\mu \Delta^\nu - \eta^{\mu\nu} \Delta^2}{M} C(t) + M \eta^{\mu\nu} \bar{C}(t) + \frac{P^\mu i\sigma^{\nu\lambda} \Delta_\lambda}{4M} [A(t) + B(t) + D(t)] + \frac{P^\nu i\sigma^{\mu\lambda} \Delta_\lambda}{4M} [A(t) + B(t) - D(t)] \right] u(p, s)
\]

Access to total angular momentum and forces acting on quarks

\[
A^q(0) + B^q(0) = \int_{-1}^{1} x \left[ H^q(x, \xi, 0) + E^q(x, \xi, 0) \right] = 2J^q
\]

Ji’s sum rule
Introduction

\[ \text{TMD}(x, k_\perp) \quad \text{SIDIS and DY} \]

\[ \int d^2 k_\perp \]

\[ \text{PDF}(x) \quad \text{DIS, SIDS and pp} \]

\[ \xi = t = 0 \]

\[ \text{GTMD}(x, \xi, k_\perp, \Delta) \]

\[ \xi = \Delta = 0 \]

\[ \int d^2 k_\perp \]

\[ \text{GPD}(x, \xi, t = \Delta^2) \quad \text{exclusive} \]

\[ \int dx \]

\[ \text{EFF}(t) \quad \text{elastic} \]
H. Moutarde, P. S., J. Wagner "Border and skewness functions from a leading order fit to DVCS data" arXiv:1807.07620 [hep-ph]

Goal: global extraction of Compton Form Factors (CFFs) from DVCS data using LO/LT formalism

Analysis done within PARTONS project
PARTONS project

- PARTONS - platform to study GPDs
- Come with number of available physics developments implemented
- Addition of new developments as easy as possible
- To support effort of GPD community
- Can be used by both theorists and experimentalists


http://partons.cea.fr
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More info in:  
  
http://partons.cea.fr

\[ H^u @ x = 0.2, t = -0.1 \text{ GeV}^2, \mu_F^2 = \mu_R^2 = 2 \text{ GeV}^2 \]

- GK11
- MPSSW13
- VGG
- Vinnikov
Compton Form Factors

- imaginary part
  \[ \text{Im} G(\xi, t) = \pi G^{(+)}(\xi, \xi, t) = \pi \sum_q e_q^2 G^{(+)}(\xi, \xi, t) \]
  \[ G^{(+)}(x, \xi, t) = G^q(x, \xi, t) \mp G^q(-x, \xi, t) \]
  \[ G^{(+)}(\xi, \xi, t) = G^{\text{val}}(\xi, \xi, t) + 2G^{\text{sea}}(\xi, \xi, t) \]

- real part
  \[ \text{Re} G(\xi, t) = \text{P.V.} \int_0^1 G^{(+)}(x, \xi, t) \left( \frac{1}{\xi - x} \mp \frac{1}{\xi + x} \right) dx \]
  \[ \text{Re} G(\xi, t) = \text{P.V.} \int_0^1 G^{(+)}(x, x, t) \left( \frac{1}{\xi - x} \mp \frac{1}{\xi + x} \right) dx + C_G(t) \]
  \[ C_H(t) = -C_E(t) \quad C_{\tilde{H}}(t) = C_{\bar{E}}(t) = 0 \]
Relation between subtraction constant and D-term:

\[ C^q_G(t) = 2 \int_{-1}^{1} \frac{D^q(z, t)}{1 - z} \, dz \equiv 4D^q(t) \]

where

\[ z = \frac{x}{\xi} \]

Decomposition into Gegenbauer polynomials:

\[ D^q(z, t) = (1 - z^2) \sum_{i=0}^{\infty} d^q_i(t) C_{2i+1}^{3/2}(z) \]

Connection to EMT FF:

\[ D^q(t) = \sum_{i=\text{odd}}^{\infty} d^q_i(t) \quad d^q_1(t) = 5C^q(t) \]
Comparing CFFs evaluated with two methods

\[ C_G^q(t) = \int_0^1 \left( G^{q(+)}(x, \xi, t) - G^{q(+)}(x, x, t) \right) \left( \frac{1}{\xi - x} - \frac{1}{\xi + x} \right) dx \]

for \( \xi = 0 \)

\[ C_G^q(t) = 2 \int_0^1 \left( G^{q(+)}(x, x, t) - G^{q(+)}(x, 0, t) \right) \frac{1}{x} dx \]

but

\[ C_{G,j}^q(t) = 2 \int_0^1 \left( G^{q(+)}(x, x, t) - G^{q(+)}(x, 0, t) \right) x^j dx \]

well defined for odd positive \( j \)

divergent integral!
Subtraction constant as analytic continuation of Mellin moments to $j = -1$

$$C^q_G(t) = C^q_{G, -1}(t) = 2 \int_0^1 \left( G^{q(+)}(x, x, t) - G^{q(+)}(x, 0, t) \right) \frac{1}{x} \, dx$$

Analytic regularization prescription

$$\int_0^1 \frac{f(x)}{x^{a+1}} = \int_0^1 \frac{f(x) - f(0) - xf'(0) - \cdots}{x^{a+1}} + f(0) \int_0^1 \frac{dx}{x^{a+1}} + f'(0) \int_0^1 \frac{dx}{x^a} + \cdots =$$

$$\int_0^1 \frac{f(x) - f(0) - xf'(0) - \cdots}{x^{a+1}} - \frac{f(0)}{a} - \frac{f'(0)}{a - 1} + \cdots$$

applicable if $f(x)$ analytic and not-vanishing at $x = 0$
Ansatz for H and $\tilde{H}$

\[ G^q(x, 0, t) = \text{pdf}_G^q(x) \exp(f_G^q(x)t) \]

\[ f_G^q(x) = A_G^q \log(1/x) + B_G^q (1 - x)^2 + C_G^q (1 - x)x \]

- modify "classical" $\log(1/x)$ term by $B_G^q(1-x)^2$ in low-x and by $C_G^q(1-x)x$ in high-x regions
- polynomials found in analysis of EFF data $\rightarrow$ good description of data
- allows to use the analytic regularization prescription
- finite proton size at $x \rightarrow 1$
Ansatz for $H$ and $\tilde{H}$

\[ G^q(x, x, t) = G^q(x, 0, t) \ g^q_G(x, x, t) \]

\[ g^q_G(x, x, t) = \frac{a^q_G}{(1 - x^2)^2} \ (1 + t(1 - x)(b^q_G + c^q_G \log(1 + x))) \]

- at $x \to 0$ constant skewness effect
- at $x \to 1$ reproduce power behavior predicted for GPDs in Phys. Rev. D69, 051501 (2004)
- $t$-dependence similar to DD-models with $(1-x)$ to avoid any $t$-dep. at $x = 1$
"trouble" with analytic regularization

\[
\int_0^1 f(x) x^{a+1} = \int_0^1 \frac{f(x) - f(0) - x f'(0) - \ldots}{x^{a+1}} \frac{f(0)}{a} - \frac{f'(0)}{a - 1} + \ldots
\]

where in our case

\[
a = \delta + A^q_G t
\]

\[
f(x) = \frac{G^q(x, x, t) - G^q(x, 0, t)}{x^{-a}} = \frac{G^q(x, 0, t) (g^q_G(x, t) - 1)}{x^{-a}}
\]

compensating terms infinite for \( t \equiv t_0^\infty = -\delta/A^q_G \) and \( t \equiv t_1^\infty = (1 - \delta)/A^q_G \)

unless \( f(0) = 0 \) at \( t_0^\infty \) and \( f'(0) = 0 \) at \( t_1^\infty \), condition provided by:

\[
b^q_G = \frac{A^q_G (a^q_G - 1)}{a^q_G \delta} \quad \quad c^q_G = \frac{(a^q_G - 1)}{p_0 (\delta - 1) a^q_G \delta} (p_0 (2 B^q_G - C^q_G) (\delta - 1) + A^q_G p_0 (\delta - 1 - \alpha) + A^q_G p_1)
\]

where \( \delta, \alpha, p_0, p_1 \) are PDF parameterization parameters
Ansatz for $E$ and $\tilde{E}$

- for GPD $E$

$$E^{q_{\text{val}}}(x, 0, t) = e^{q_{\text{val}}}(x) \exp(f_E^{q_{\text{val}}}(x)t)$$

$$e^{q_{\text{val}}}(x) = \kappa_q N_{q_{\text{val}}} x^{-\alpha_{q_{\text{val}}}} (1 - x)^{\beta_{q_{\text{val}}}} (1 + \gamma_{q_{\text{val}}}/\sqrt{x})$$

$$E^{q_{\text{val}}}(x, x, t) = E^{q_{\text{val}}}(x, 0, t) g_E^{q_{\text{val}}}(x, t)$$

$$g_E^{q_{\text{val}}}(x, t) = \frac{a_E^{q_{\text{val}}}}{(1 - x^2)^3}$$


- for GPD $\tilde{E}$

$$\tilde{E}(\xi, t) = N_{E}\tilde{E}_{\text{GK}}(\xi, t)$$

- CFF from GK GPD model

Steps of analysis:

Step 1
Analysis of PDFs

Step 2
Analysis of EFF data

Step 3
Analysis of DVCS data

Effectively we combine (semi-)inclusive, pp, elastic and exclusive data in a single analysis
Ansatz:

$$pdf_G(x, Q^2) = x^\alpha \sum_{i=0}^{4} g(p_i, q_i, Q^2) x^i$$

$$g(p, q, Q^2) = p + q \log \frac{Q^2}{Q_0}$$

13 parameters:

$$\delta_p, \delta_q, \alpha, p_i, q_i$$ where $$i = 0, 1, \ldots, 4$$

constrained by NNPDF3.0 and NNPDFpol11 sets (per each flavor and each PDF replica)
Elastic FF data

Free parameters for valance quarks and GPDs H and E constrained by EFF data

\[ \int_{-1}^{1} H^q(x, \xi, t) \equiv F_1^q(t) \]

\[ \int_{-1}^{1} E^q(x, \xi, t) \equiv F_2^q(t) \]

From Dirac and Pauli partonic FFs to Sachs nucleon FFs

\[ F^p_i = e_u F^u_i + e_d F^d_i \quad i = 1, 2 \]

\[ F^n_i = e_u F^d_i + e_d F^u_i \]

\[ G^i_M = F^i_1 + F^i_2 \quad i = p, n \]

\[ G^i_E = F^i_1 + \frac{t}{4m^2} F^i_2 \]

Observables

\[ G^i_{M,N}(t) = \frac{G^i_M(t)}{\mu_i G_D(t)} \quad i = p, n \]

\[ R^i(t) = \frac{\mu_i G^i_E(t)}{G^i_M(t)} \]

\[ r_{nE}^2(t) = 6 \frac{dG^i_M(t)}{dt} \bigg|_{t=0} \]

for the selection of observables and experimental data we follow

Performance:

\[
\chi^2/\text{ndf} = \frac{129.6}{(178 - 9)} \approx 0.77
\]

Replication of experimental data to estimate corresponding uncertainties:

\[
v_i \pm \Delta_i^{\text{tot}} \xrightarrow{\text{replica } j} \text{rnd}_j(v_i, \Delta_i^{\text{tot}}) = \Delta_i^{\text{tot}}
\]

\[
\Delta_i^{\text{tot}} = \sqrt{(\Delta_i^{\text{stat}})^2 + (\Delta_i^{\text{sys}})^2}
\]

Fitted values:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mean</th>
<th>Data unc.</th>
<th>Unpol. PDF unc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{H/E}^{\text{val}}$</td>
<td>0.99</td>
<td>0.01</td>
<td>0.08</td>
</tr>
<tr>
<td>$B_{H}^{\text{val}}$</td>
<td>−0.50</td>
<td>0.02</td>
<td>0.14</td>
</tr>
<tr>
<td>$A_{A_{H/E}}^{\text{val}}$</td>
<td>0.70</td>
<td>0.02</td>
<td>0.08</td>
</tr>
<tr>
<td>$B_{E}^{\text{val}}$</td>
<td>0.47</td>
<td>0.07</td>
<td>0.24</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.69</td>
<td>0.01</td>
<td>0.03</td>
</tr>
<tr>
<td>$B_{E}^{\text{val}}$</td>
<td>−0.69</td>
<td>0.04</td>
<td>0.18</td>
</tr>
<tr>
<td>$C_{E}^{\text{val}}$</td>
<td>−0.92</td>
<td>0.04</td>
<td>0.09</td>
</tr>
<tr>
<td>$B_{E}^{\text{val}}$</td>
<td>−0.54</td>
<td>0.06</td>
<td>0.20</td>
</tr>
<tr>
<td>$C_{E}^{\text{val}}$</td>
<td>−0.73</td>
<td>0.06</td>
<td>0.22</td>
</tr>
</tbody>
</table>
All DVCS proton data used in the fit, except:
• HERA data
• Hall A cross sections

Kinematic cuts:

\[ Q^2 > 1.5 \text{ GeV}^2 \]
\[ -t/Q^2 < 0.25 \]

Angles:
Performance:

\[ \chi^2 / \text{ndf} = 2346.3 / (2600 - 13) \approx 0.91 \]

Fitted values:

<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>( a^\text{val}_H )</td>
<td>0.81</td>
<td>0.04</td>
<td>0.17</td>
<td>0.02</td>
<td>&lt; 0.01</td>
</tr>
<tr>
<td>( a^\text{err}_H )</td>
<td>0.99</td>
<td>0.01</td>
<td>0.02</td>
<td>&lt; 0.01</td>
<td>&lt; 0.01</td>
</tr>
<tr>
<td>( b^\text{val}_H )</td>
<td>1.03</td>
<td>0.04</td>
<td>0.30</td>
<td>0.24</td>
<td>0.01</td>
</tr>
<tr>
<td>( N_E )</td>
<td>-0.46</td>
<td>0.10</td>
<td>0.09</td>
<td>0.06</td>
<td>0.01</td>
</tr>
<tr>
<td>( A^\text{val}_H )</td>
<td>2.56</td>
<td>0.23</td>
<td>0.30</td>
<td>0.09</td>
<td>0.03</td>
</tr>
<tr>
<td>( B^\text{val}_H )</td>
<td>-5</td>
<td></td>
<td></td>
<td>at the limit</td>
<td></td>
</tr>
<tr>
<td>( C^\text{val}_H )</td>
<td>34</td>
<td>27</td>
<td>49</td>
<td>14</td>
<td>3</td>
</tr>
<tr>
<td>( A^\text{val}_H )</td>
<td>0.77</td>
<td>0.12</td>
<td>0.30</td>
<td>0.23</td>
<td>0.07</td>
</tr>
<tr>
<td>( B^\text{val}_H )</td>
<td>-0.02</td>
<td>0.26</td>
<td>0.75</td>
<td>0.24</td>
<td>0.15</td>
</tr>
<tr>
<td>( C^\text{val}_H )</td>
<td>-0.92</td>
<td>0.07</td>
<td>0.44</td>
<td>0.24</td>
<td>0.04</td>
</tr>
<tr>
<td>( A^\text{val}_H )</td>
<td>0.64</td>
<td>0.24</td>
<td>0.30</td>
<td>0.28</td>
<td>0.05</td>
</tr>
<tr>
<td>( B^\text{val}_H )</td>
<td>-1.19</td>
<td>0.45</td>
<td>0.91</td>
<td>0.98</td>
<td>0.22</td>
</tr>
<tr>
<td>( C^\text{val}_H )</td>
<td>-0.55</td>
<td>0.24</td>
<td>0.26</td>
<td>0.27</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Replication of experimental data to estimate corresponding uncertainties:

\[ v_i \pm \Delta_i^{\text{tot}} \xrightarrow{\text{replica } j} \left( \text{rnd}_j(v_i, \Delta_i^{\text{tot}}) \pm \Delta_i^{\text{tot}} \right) \times \text{rnd}_j(1, \Delta_i^{\text{norm}}) \quad \Delta_i^{\text{tot}} = \sqrt{(\Delta_i^{\text{stat}})^2 + (\Delta_i^{\text{sys}})^2} \]
Results

CLAS data:

\[ x_{\text{Bj}} = 0.244, \ t = -0.15 \text{ GeV}^2, \ Q^2 = 1.79 \text{ GeV}^2 \]

\[ x_{\text{Bj}} = 0.257, \ t = -0.23 \text{ GeV}^2, \ Q^2 = 2.02 \text{ GeV}^2 \]
HERMES data:

\[ A_{C, \cos \theta} \]

\[ A_{UT, \sin(g_{\phi}) \cos \phi} \]

JHEP 06, 066 (2008)

\( t = -0.12 \text{ GeV}^2, Q^2 = 2.5 \text{ GeV}^2 \)
Results

Hall A data:

\begin{align*}
\sigma^\text{GK model} \quad \sigma^\text{VGG model}
\end{align*}


\[ x_{\text{Bj}} = 0.392, \ t = -0.233 \ \text{GeV}^2, \ Q^2 = 2.054 \ \text{GeV}^2 \]
Results

COMPASS and HERA:

arXiv: hep-ex/1802.02739

$Q^2 = 1.8 \text{ GeV}^2$
Results

Compton Form Factors:

\[ t = -0.3 \text{ GeV}^2, \quad Q^2 = 2 \text{ GeV}^2 \]
Results

Compton Form Factors:

\[ t = -0.3 \text{ GeV}^2, \quad Q^2 = 2 \text{ GeV}^2 \]
Results

Subtraction constant:

\[ Q^2 = 2 \text{ GeV}^2 \]

\[ t = 0 \]
Results

Nucleon tomography:

\[ Q^2 = 2 \text{ GeV}^2 \]

\[ \Delta u_{val} \]

\[ b_{\perp} \text{ [fm]} \]

\[ x \]

no uncertainties!
Results

Compton Form Factors:

$$\langle b_{\perp}^2 \rangle_q(x) = \frac{\int d^2 b_{\perp} \; b_{\perp}^2 q(x, b_{\perp})}{\int d^2 b_{\perp} \; q(x, b_{\perp})}$$

---

$$Q^2 = 2 \text{ GeV}^2$$
Results

Compton Form Factors:

\[ \langle d_{\perp}^2 \rangle_q(x) = \frac{\langle b_{\perp}^2 \rangle_q(x)}{(1-x)^2} \]

\[ Q^2 = 2 \text{ GeV}^2 \]
SUMMARY

Fits to DVCS data
- New parameterizations of border and skewness function proposed
  → basic properties of GPD as building blocks
  → small number of parameters
  → encoded access to nucleon tomography and subtraction constant
- Successful to fit EFF and DVCS data
  → replica method for a careful propagation of uncertainties

What next?
- Neural network parameterization of CFFs
- Include other channels and more observables
- Include new and already existing theory developments
- Make consistent analysis of all those ingredients → PARTONS
Layered structure:

- one layer = collection of objects designed for common purpose
- one module = one physical development
- operations on modules provided by Services, e.g. for GPD Layer

```cpp
GPDResult computeGPDModel(const GPDKinematic& gpdKinematic, GPDModule* pGPDModule) const;
GPDResult computeGPDModelRestrictedByGPDType(const GPDKinematic& gpdKinematic, GPDModule* pGPDModule, GPDType::Type gpdType) const;
GPDResult computeGPDModelWithEvolution(const GPDKinematic& gpdKinematic, GPDModule* pGPDModule, GPDEvolutionModule* pEvolQCDModule) const;
...
```

- what can be automated is automated
- features improving calculation speed
  e.g. CFF Layer Service stores the last calculated values
Existing modules:

- GPD: GK11, VGG, Vinnikov, MPSSW13, MMS13
- Evolution: Vinnikov code
- CFF (DVCS only): LO, NLO (gluons and light or light + heavy quarks)
- Cross Section (DVCS only): VGG, BMJ, GV
- Running coupling: 4-loop PDG expression, constant value

\[ H^u @ x = 0.2, \; t = -0.1 \; \text{GeV}^2, \; \mu_F^2 = \mu_R^2 = 2 \; \text{GeV}^2 \]