Quark Structure of Hadrons, Nuclei, and Neutron Stars
Overview

- Motivation
- Framework: QMC Model
- Finite Nuclei
- Neutron Stars
  - NS Phenomenology
  - Mass and Radius
  - Gravitational Waves
  - Isovector-Scalar Effects
  - Delta Isobars
Motivation

- Hadrons are composite objects
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- There could be quark effects on the physics of nuclei
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- There could be quark effects on the physics of nuclei
- There could be quark effects on the physics of neutron stars
QMC Model
The Quark-Meson Coupling model describes the baryons as **bags** of three quarks that couple directly to meson fields (P.A.M Guichon, J.R. Stone A.W.Thomas (2018))
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\begin{figure}
\centering
\includegraphics[width=\textwidth]{quark_meson_coupling_diagram.png}
\end{figure}
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• The Quark-Meson Coupling model describes the baryons as \textbf{bags} of three quarks that couple \textbf{directly} to meson fields \cite{Guichon2018}.
So on the quark level we have the following lagrangian

\[ \mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m_q + g_\sigma^q \hat{\sigma} + g_\omega^q \hat{\omega} + g_\rho^q \hat{\rho} \cdot \tau + g_\delta^q \hat{\delta} \cdot \tau)\psi + \mathcal{B} \]
• We solve the equations for the bag model

\[(i \gamma \cdot \hat{\nabla} - m_q) \psi = 0, \quad \forall r < R_B\]

\[(1 + i \gamma \cdot \hat{r}) \psi \bigg|_{r=R_B} = 0\]
The lowest positive energy mode with spin $m$

$$
\phi_m(\vec{r}) = N \left( j_0 \left( \frac{x r}{R_B} \right) \right) \times \frac{\chi m}{\sqrt{4\pi}}
$$

Where

$$
\beta = \sqrt{\frac{\Omega - m_q R_B}{\Omega + m_q R_B}}
$$

$$
\Omega = \sqrt{x^2 + (m_q R_B)^2}
$$
Now in our case, the quarks couple to the mesons.

The solution is the same only with effective mass and momentum

\[ m_q^* = m_q - g^q_\sigma \sigma + g^q_\delta \delta \times I^q \]

\[ \vec{k}^*_q = (g^q_\omega \omega(\vec{R}) + g^q_\rho \rho(\vec{R}) \times I^q) \hat{\nu} \cosh \xi \]

And

\[ \Omega = \sqrt{x^2 + (m_q^* R_B)^2} \quad \beta = \sqrt{\frac{\Omega - m_q^* R_B}{\Omega + m_q^* R_B}} \]

Now depend on the meson fields
Ok, now, the **quarks** are sources of meson fields:

\[
(\partial^2 + m^2_\sigma) \hat{\sigma} = g^q_\sigma \bar{\psi}_q \psi_q
\]

Or the **baryons** are sources of meson fields:

\[
(\partial^2 + m^2_\sigma) \langle A \mid \hat{\sigma} \mid A \rangle = g^q_\sigma \langle A \mid \bar{\psi}_q \psi_q \mid A \rangle
\]

Which at the **baryonic** level becomes field dependent

\[
g^q_\sigma \langle A \mid \bar{\psi}_q \psi_q \mid A \rangle \rightarrow g_\sigma(\sigma, \delta) \bar{\Psi} \Psi
\]
• If we have Baryons as sources for Meson fields we can write:

\[ \mathcal{L} = \bar{\Psi} (i \slashed{\partial} - M_B) \Psi + g_\sigma (\sigma, \delta) \bar{\Psi} \Psi + \cdots \]

• So we have an effective Lagrangian, describing the physics of Baryons, but including effects of their quark internal structure!!!

• All there is left to do is solve the many body problem for this system...

\[ \mathcal{E} = \sum_i \mathcal{E}_i + \sum_{i,j} \left( \mathcal{E}_{ij} + \mathcal{E}_{ji} + \cdots \right) \]

\[ \mu_i = \sqrt{k_F^2 + m_i^2} + \sum_{j,l} \left( \mathcal{E}_{ijl} + \mathcal{E}_{jl} + \cdots \right) \]
We can see the system as baryons with effective mass.

\[ M_B^* (\sigma, \delta) = M_B - g_\sigma (\sigma, \delta) \sigma + g_\delta (\sigma, \delta) \delta \times I^q \]

Where the meson fields are the respective mean field values.

In order to connect the quark-level theory to the nuclear-level theory we fit this values to the mass derived from the MIT bag model solution:

\[ M_B^* = \sum_f \frac{n_f \Omega_0 - z_0}{R_B} + BV_B + \Delta_{EM} \]
Finite Nuclei

- Binding energies for heavy nuclei

Finite Nuclei

- Binding energies for heavy nuclei
- Deformation

Finite Nuclei

- Binding energies for heavy nuclei
- Deformation
- Neutron Skin Thickness

Neutron Stars
Why Neutron Stars?

The diagram illustrates a phase diagram involving temperature, net baryon density, and early universe conditions. It shows transitions from hadrons to quarks and gluons, with critical points and deconfinement and chiral transition regions. The diagram also highlights neutron stars and the RHIC, LHC, and FAIR SIS 300 experiments.
Why Neutron Stars?

- Expected density around $\sim 1 \text{ fm}^{-3}$.
  (If the star’s mass is large!!!)
Why Neutron Stars?

- Expected density around $\sim 1\text{ fm}^{-3}$.
  (If the star’s mass is large!!)

- If you take the baryon to be a spherical object of avg radius $\sim 0.8\text{ fm}$

Then the volume of the baryon is around $2.1\text{ fm}^3$. 
Why Neutron Stars?

- How does matter look like at such high densities?

- QCD at **low densities** has only two flavours of quark (u,d)
Why Neutron Stars?

- But if **density increases** it should generate strangeness
Why Neutron Stars?

- But if density increases it should generate strangeness

![Diagram showing the phase transition between hadrons, quarks, and gluons with Neutron stars at a critical point.](image)
Why Neutron Stars?

- And at high enough densities the baryons deconfine
Mass - Radius
Mass

- Measuring masses is hard! Most data points have large errors
- Suspected to have large systematic uncertainties
- Each measurement is done differently
Mass-Radius

The graph represents the relationship between mass (in $M_{\odot}$) and radius (in km) for various neutron star models. The graph includes models labeled PSR J0348+0432, PSR J1614-2230, and PSR J1946+3417, among others. The black hole region and the causality limit are also indicated. The models vary in color and style, allowing for a comparison of their mass-radius profiles.
Mass-Radius

- Hyperon puzzle
- Quark puzzle
- Delta puzzle
Mass-Radius

- Hyperon puzzle
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Mass-Radius

- Hyperon puzzle
- Quark puzzle
- Delta puzzle

J.R. Stone, Nuclear Physics A 792 (2007)
Gravitational Waves
• Abbot et al (2017) measured a GW event of NS-NS merger
• From that they deduced the EOS to 90% confidence.
• Predictions go from 7 to 15 km

Özel & Freire (2016)
• Predictions go from 7 to 15 km
• GW170817 restricts that to 90% confidence
NICER: The Neutron star Interior Composition Explorer

“NICER observations will achieve an accuracy of $\approx 2\%$ in the measurement of radius (Gendreau, Arzoumanian, and Okajima, 2012; Bogdanov, 2013). In practice, the measurement will be limited by uncertainties in these two requirements. The uncertainty in the mass measurement of NICER’s primary target, the bright pulsar PSR J0437−4715, is $\sim 5\%$ (Reardonet al., 2016).”

-Watts et al. (2016)
Ligo is still measuring!
September 30th:
QMC Applied to NS
\[ \mathcal{E} = \sum_{i} + \sum_{i,j} (\text{Diagram}) \]

- Minimize energy density (\(\beta\)-equilibrium)
- Get Population chart
- Get Energy and Pressure for each baryon density (EOS)
\[ \mathcal{E} = \sum_{i} + \sum_{i,j} \left( \text{Crust} \right) \]

- Minimize energy density (\(\beta\)-equilibrium)
- Get Population chart
- Get Energy and Pressure for each baryon density (EOS)
\[ \rho_0 = 0.16 \pm 0.01 \text{fm}^{-3} \]
\[ S = 30 \pm 0.2 \text{MeV} \]
\[ \varepsilon = -16.0 \pm 1 \text{MeV} \]
Crust!

$\rho_0 = 0.16 \pm 0.01 \text{fm}^{-3}$
$S = 30 \pm 0.2 \text{MeV}$
$\varepsilon = -16.0 \pm 1 \text{MeV}$
\[ \frac{8\pi G}{c^4} T_{\mu\nu} = G_{\mu\nu} \]

**Ideal Fluid**

\[ T^0_0 = \varepsilon \]
\[ T^i_i = -P \]

\[ \frac{dP}{dr} = - \frac{Gm}{r^2} \rho \left( 1 + \frac{P}{\rho c^2} \right) \left( 1 + \frac{4\pi r^3 P}{mc^2} \right) \left( 1 - \frac{2Gm}{rc^2} \right)^{-1} \]
$G_5 = 3 fm$

$M(M_\odot)$ vs. $R(km)$

Equation of state does produce stars with masses of up to 1.9 solar masses for all values of delta coupling.
• Equation of state does produce stars with masses of up to 1.9 solar masses for all values of delta coupling

• Variations on saturation density give rise to small variations in mass-radius
• Why wasn’t it there in the first place?
  - It’s mass is large and coupling is small
  - Wasn’t likely to change the maximum mass
  - We only had mass to think about!

Mass-Radius

Standard Fit Only

GW170817
90% confidence

We find that the delta coupling changes the radius very significantly!

With the upcoming NICER mission such a result is quite important.

Effects of variations in baryon density are much smaller than delta effects
But couldn’t it be that such results are inconsistent?

E.g. we use the same crust EoS all throughout this study

Doesn’t the crust contribute much more to the radius than the core does?
Mass-Radius

Mass-Radius

A.M Kalaitzis, TFM, A.W.Thomas (2019)
Crust consists of a **low percentage** of the radius for stars with $M>1.4$.

We also expect that, IF the model we used for the crust turns out to be incorrect, variations would probably be much smaller than this.
Tidal Deformability

• Tidal deformability (or equivalently, the Love number)

\[ \Lambda^{\text{Tid}} = \frac{2}{3} k_2^{\text{Tid}} \beta^{-5} \]

• Give the ratio between Quadrupole moment and tidal potential induced by the companion

\[ \Lambda^{\text{Tid}} \propto \frac{Q^{\text{Tid}}}{\mathcal{E}^{\text{Tid}}} \]
• Delta meson changes little the Tidal Deformability

• Still all values of $\delta$ coupling fit to GW’s band

• Again the case with double delta coupling is in most tension with GW constraints
Moment of Inertia

\[ \frac{1}{MR^2} = (0.237 \pm 0.008)(1 + 2.844\beta + 18.91\beta^4) \]
- Delta doesn’t change at all the moment of inertia for most (M,R) points.

- All parametrizations agree with band from Zhao and Lattimer (2018)
Steiner et al. (2013)

• For binary tidal deformability we also have QMC compatible with the GW measurement

• Same pattern with delta coupling
Crust

A.M Kalaitzis, TFM, A.W.Thomas (2019)
- Moment of inertia contribution to the crust is small
- Surprisingly, so is the $\Lambda$ contribution
- Both below 10% for the QMC model
Delta Isobars

Chemical Potentials

\[ \mu(\text{MeV}) \]

- \( \eta \)
- \( \rho \)
- \( \Lambda \)
- \( \Xi^0 \)
- \( \Xi^- \)
- \( \Sigma^+ \)
- \( \Sigma^0 \)
- \( \Sigma^- \)
- \( \Delta^- \)

\[ \rho/\rho_0 \]

TFM, A.W.Thomas, P.A.M Guichon (2019)
Delta Isobars

TFM, A.W.Thomas, P.A.M Guichon (2019)
• The QMC model predicts no Delta isobars in the core of neutron stars

• This can be seen as a natural $\Delta$NN three body repulsion
Summary and Conclusions
Summary

- QMC agrees with GW constraints for all studied values of $\delta$ coupling
- QMC agrees with existent measurements for radius (preferring larger radius than most models)
- Measurements of the radius will give us strength of $\delta$ coupling
- Values for GW quantities depend little on the crust EOS
- QMC proposes a solution for both the "hyperon puzzle" and the "Delta puzzle"