

F Hautmann

TMDs from low to high energies  
and  
the parton branching method

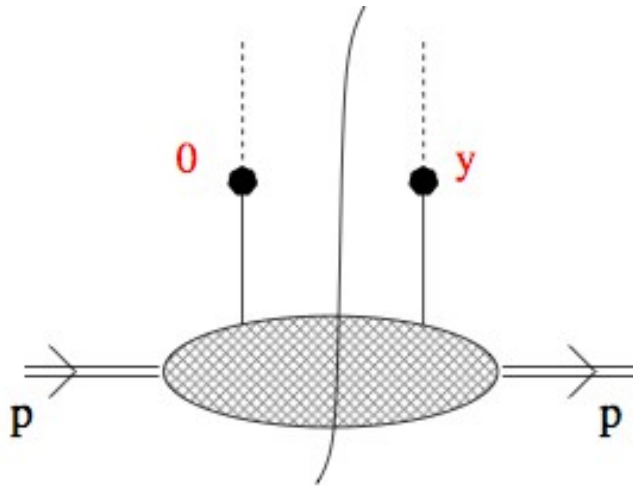
Theory Center Seminar, Jefferson Laboratory

October 2018

# Overview

## TRANSVERSE MOMENTUM DEPENDENT (TMD) PARTON DISTRIBUTION FUNCTIONS

- Parton correlation functions at non-lightlike distances:



$$p = (p^+, m^2 / 2 p^+, 0_\perp)$$

$$\tilde{f}(y) = \langle P | \bar{\psi}(y) V_y^\dagger(n) \gamma^+ V_0(n) \psi(0) | P \rangle, \quad y = (0, y^-, y_\perp)$$

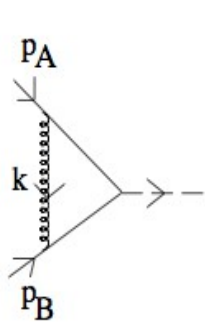
$$V_y(n) = \mathcal{P} \exp \left( i g_s \int_0^\infty d\tau \, n \cdot A(y + \tau n) \right)$$

- TMD pdfs:

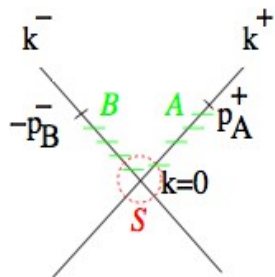
$$f(x, k_\perp) = \int \frac{dy^-}{2\pi} \frac{d^{d-2} y_\perp}{(2\pi)^{d-2}} e^{-ixp^+ y^- + ik_\perp \cdot y_\perp} \tilde{f}(y)$$

# Evolution equations for TMD parton distribution functions

low  $q_T : q_T \ll Q$



(a)

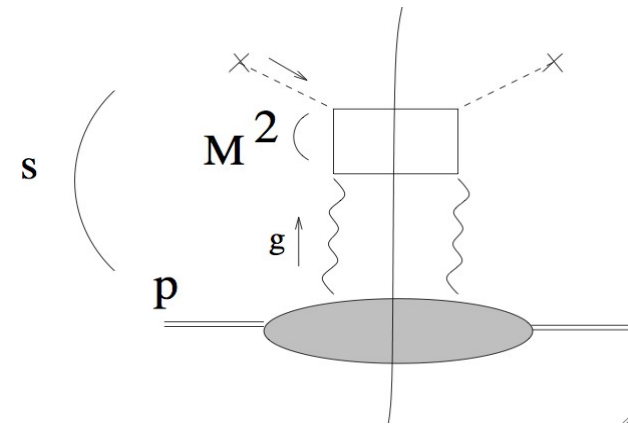


(b)

$$\alpha_s^n \ln^m Q/q_T$$

CSS evolution equation

high  $\sqrt{s} : \sqrt{s} \gg M$



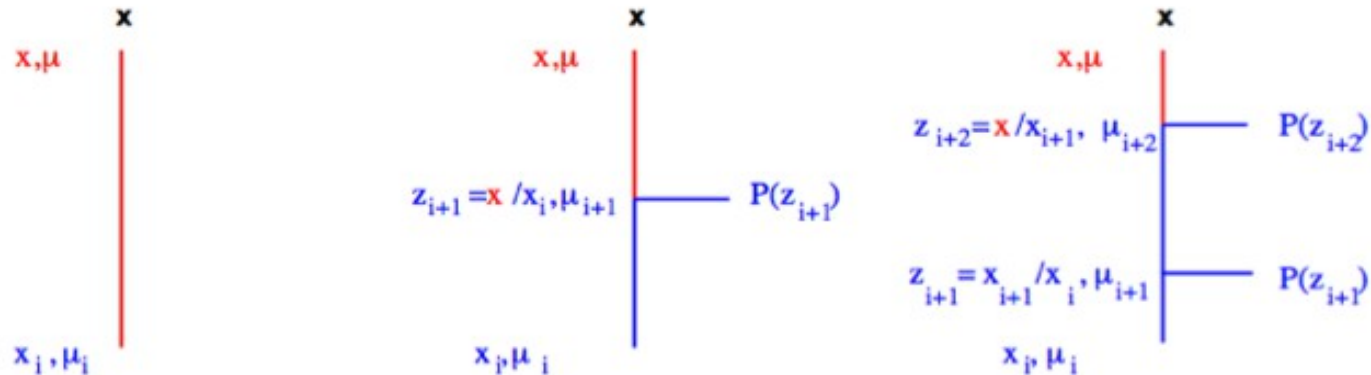
$$(\alpha_s \ln \sqrt{s}/M)^n$$

CCFM evolution equation

R. Angeles-Martinez et al., “Transverse momentum dependent (TMD) parton distribution functions: status and prospects”, Acta Phys. Polon. B46 (2015) 2501

# Parton Branching (PB) approach

Jung, Lelek, Radescu, Zlebcik & H, “Collinear and TMD quark and gluon densities from parton branching”, JHEP 1801 (2018) 070



PB evolution equation motivated by

- applicability over large kinematic range from low to high transverse momenta
- applicability to exclusive final states and Monte Carlo event generators

# TMD distributions (unpolarized and polarized)

TABLE I

(Colour on-line) Quark TMD pdfs: columns represent quark polarization, rows represent hadron polarization. Distributions encircled by a dashed line are the ones which survive integration over transverse momentum. The shades of the boxes (light gray (blue) *versus* medium gray (pink)) indicate structures that are *T*-even or *T*-odd, respectively. *T*-even and *T*-odd structures involve, respectively, an even or odd number of spin-flips.

QUARKS	<i>unpolarized</i>	<i>chiral</i>	<i>transverse</i>
U	$f_1$		$h_1^\perp$
L		$g_{1L}$	$h_{1L}^\perp$
T	$f_{1T}^\perp$	$g_{1T}$	$h_{1T}^\perp, h_{1T}^\perp$

TABLE II

(Colour on-line) Gluon TMD pdfs: columns represent gluon polarization, rows represent hadron polarization. Distributions encircled by a dashed line are the ones which survive integration over transverse momentum. The shades of the boxes (light gray (blue) *versus* medium gray (pink)) indicate structures that are *T*-even or *T*-odd, respectively. *T*-even and *T*-odd structures involve, respectively, an even or odd number of spin-flips. Linearly polarized gluons represent a double spin-flip structure.

GLUONS	<i>unpolarized</i>	<i>circular</i>	<i>linear</i>
U	$f_1^g$		$h_1^{\perp g}$
L		$g_{1L}^g$	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	$g_{1T}^g$	$h_{1T}^g, h_{1T}^{\perp g}$

R. Angeles-Martinez et al., “Transverse momentum dependent (TMD) parton distribution functions: status and prospects”, Acta Phys. Polon. B46 (2015) 2501

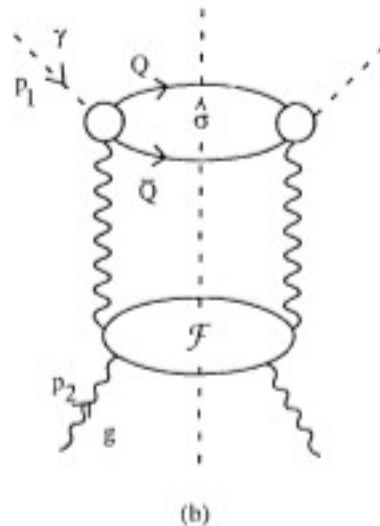
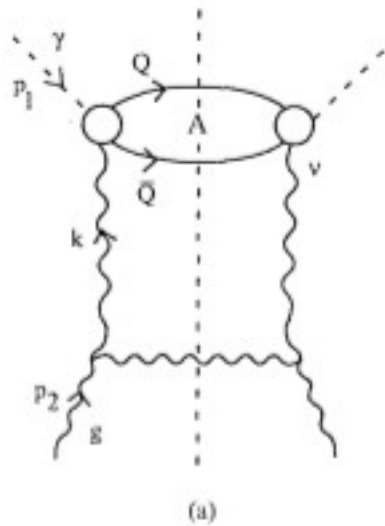
# Outline of this talk

- TMDs at high  $\sqrt{s}$  and at low  $q_T$
- The parton branching (PB) method
- New results and applications

# I. INTRODUCTION

## TMDs at high energies

Ex.: heavy flavor electroproduction for  $s \gg M^2 \gg \Lambda_{\text{QCD}}^2$



$$\gamma + h \rightarrow Q + \bar{Q} + X$$

$$4M^2 \sigma(x, M^2) = \int d^2\mathbf{k}_\perp \int_x^1 \frac{dz}{z} \hat{\sigma}_{\gamma g}(x/z, \mathbf{k}_\perp^2/M^2, \alpha_s(M^2)) \mathcal{A}_{g/h}(z, \mathbf{k}_\perp)$$

where TMD gluon distribution is given by  
Balitsky-Fadin-Kuraev-Lipatov (BFKL) evolution:

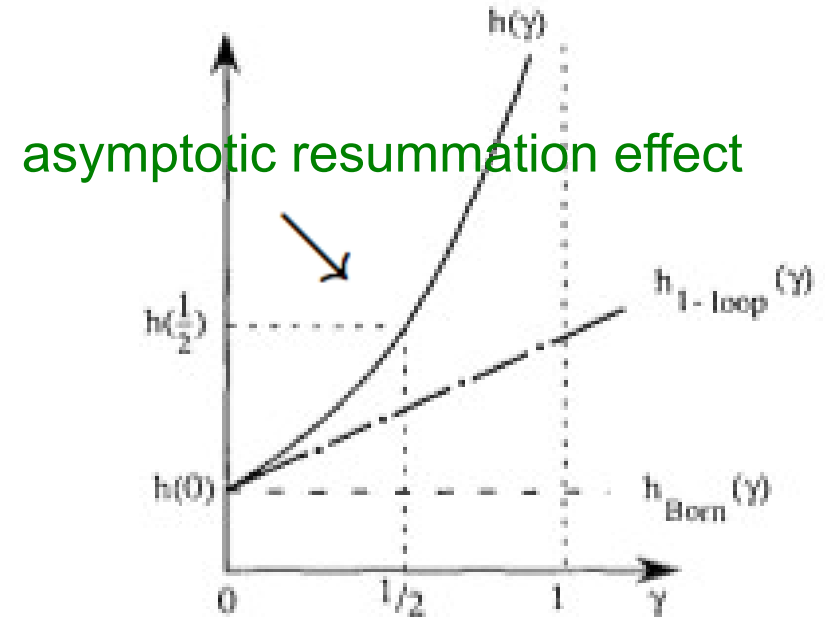
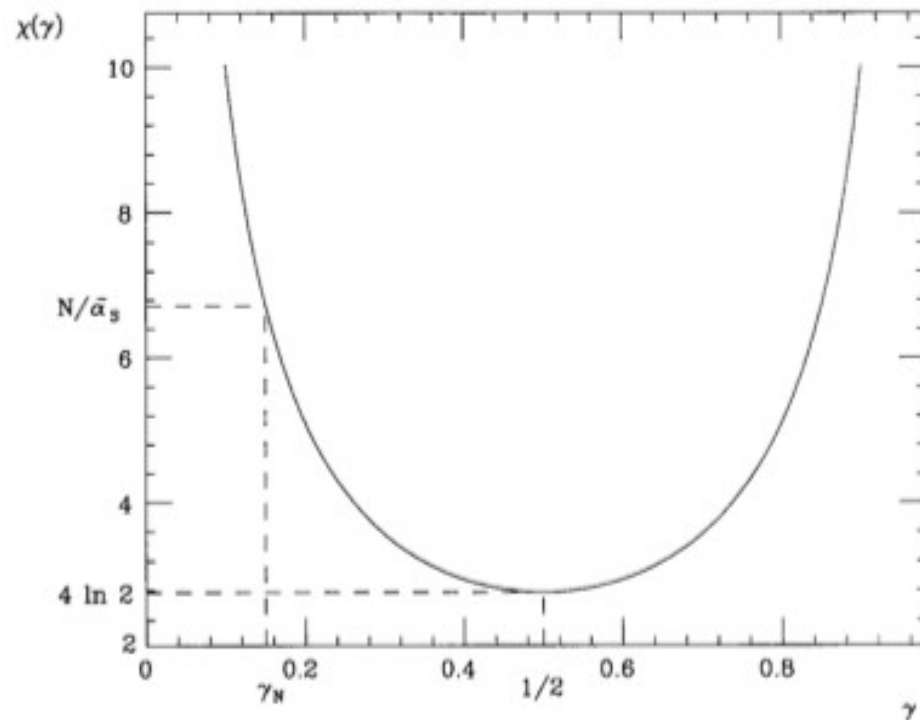
$$\mathcal{A}_{g/h}(x, \mathbf{k}_\perp) \sim \frac{1}{2\pi} e^{-\lambda \ln x} (\mathbf{k}_\perp^2)^{\gamma-1}, \quad \lambda = 4 C_A \frac{\alpha_s}{\pi} \ln 2$$

$$\gamma = \frac{1}{2}$$

# TMDs at high energies

$$\Rightarrow 4M^2 \sigma(x, M^2) \sim x^{-\lambda} (M^2)^{\frac{1}{2}} h(1/2) ,$$

$$\text{where } h(1/2) = \frac{1}{2} \int_0^\infty \frac{d\mathbf{k}_\perp^2}{\mathbf{k}_\perp^2} \left( \frac{\mathbf{k}_\perp^2}{M^2} \right)^{\frac{1}{2}} \int_0^1 \frac{dx}{x} \hat{\sigma}_{\gamma g}(x, \mathbf{k}_\perp^2/M^2, \alpha_s)$$

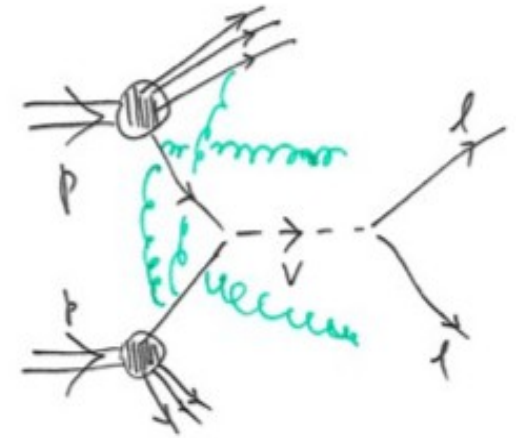


realistic effects at EIC, LHeC, VHEeP?

- NB:
- incorporate sub-asymptotic, finite- $x$  terms  $\rightarrow$  CCFM evolution
  - dense-medium modifications in nucleons and nuclei  $\rightarrow$  nonlinear evolution



# TMDs for low $q_T$



Ex.: Drell-Yan production  $q_T$  spectra for  $Q \gg q_T$

$$\frac{d\sigma}{d^2\mathbf{q}_T dQ^2 dy} = \sum_{i,j} \frac{\sigma^{(0)}}{s} H(\alpha_s) \int \frac{d^2\mathbf{b}}{(2\pi)^2} e^{i\mathbf{q}_T \cdot \mathbf{b}} \mathcal{A}_i(x_1, \mathbf{b}, \mu, \zeta) \mathcal{A}_j(x_2, \mathbf{b}, \mu, \zeta) + \{\mathbf{q}_T\text{-finite}\} + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}^2}{Q^2}\right)$$

where  $\frac{\partial \ln \mathcal{A}}{\partial \ln \sqrt{\zeta}} = K(\mathbf{b}, \mu)$  Collins-Soper-Sterman (CSS) evolution

and  $\frac{d \ln \mathcal{A}}{d \ln \mu} = \gamma_f(\alpha_s(\mu), \zeta/\mu^2)$  ,  $\frac{dK}{d \ln \mu} = -\gamma_K(\alpha_s(\mu))$  RG evolution

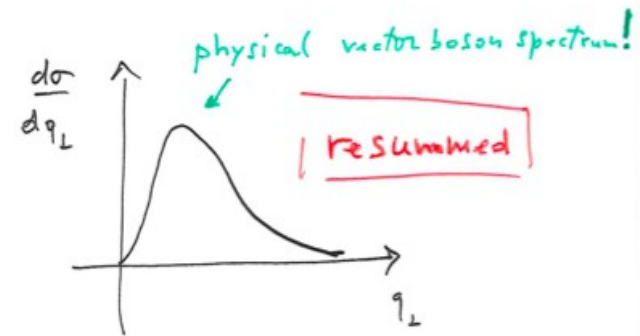
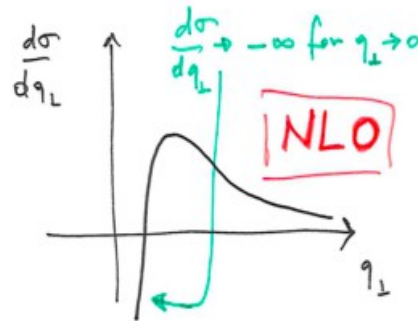
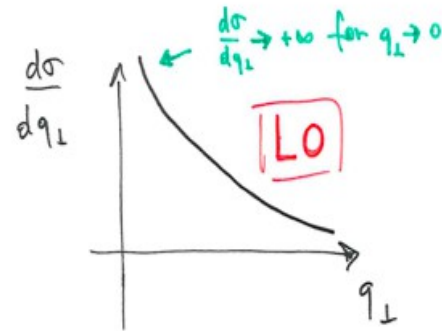
cuspidal anomalous dimension

$$\Rightarrow -\gamma_K = \frac{\partial}{\partial \ln \sqrt{\zeta}} \gamma_f \quad \text{i.e.} \quad \gamma_f(\alpha_s(\mu), \zeta/\mu^2) = \gamma_f(\alpha_s(\mu), 1) - \frac{1}{2} \gamma_K \ln \zeta$$

- Soft Collinear Effective Theory (SCET) provides alternative approach leading to same results

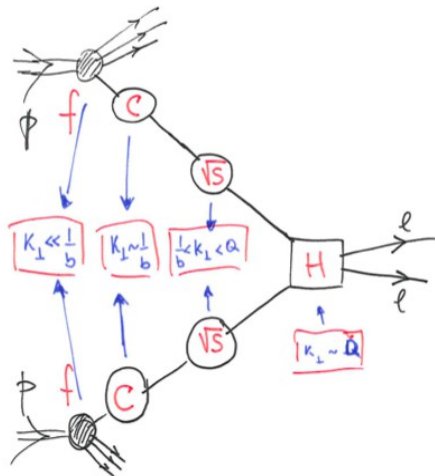
# TMDs for low $q_T$

- OUTCOME:  $\sum \alpha_s^m \ln^k Q^2/q_\perp^2$  TO ALL ORDERS IN  $\alpha_s$



NOTE: SHOWER MONTE CARLO GENERATORS DO THIS "EFFECTIVELY"

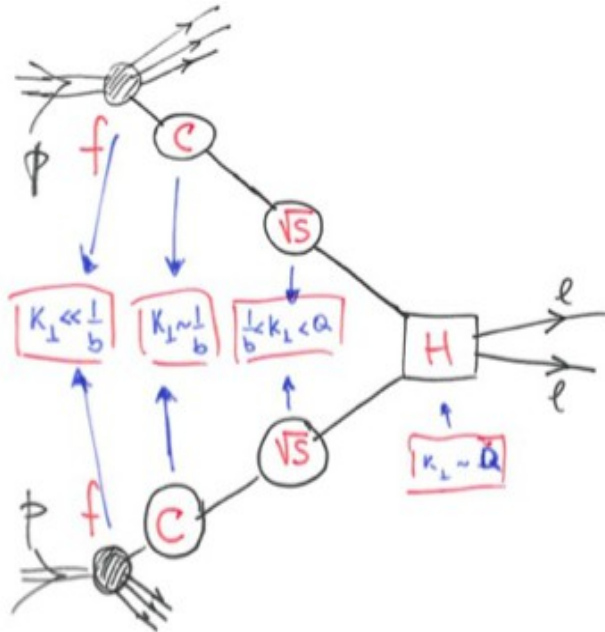
"Parton-like" formulation by decomposing the TMD pdfs in terms of ordinary pdfs ("OPE"):



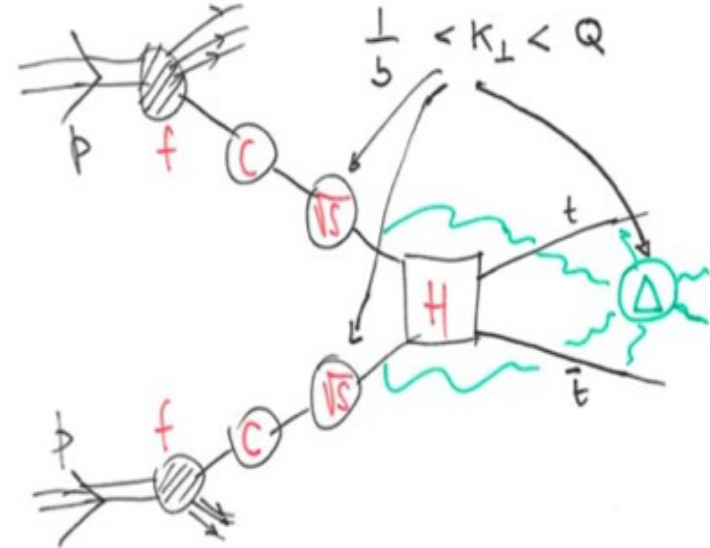
$$\mathcal{A}_i(\mathbf{b}, \mu) \sim \sum_k \underset{\substack{\text{Sudakov} \\ \text{form factor}}}{S_i} \otimes \underset{\substack{\text{evolution} \\ \text{coefficients}}}{C_{ik}} \otimes \underset{\text{pdfs}}{f_k}$$

# From color-neutral to color-charged final states

Color neutral:



Color charged:



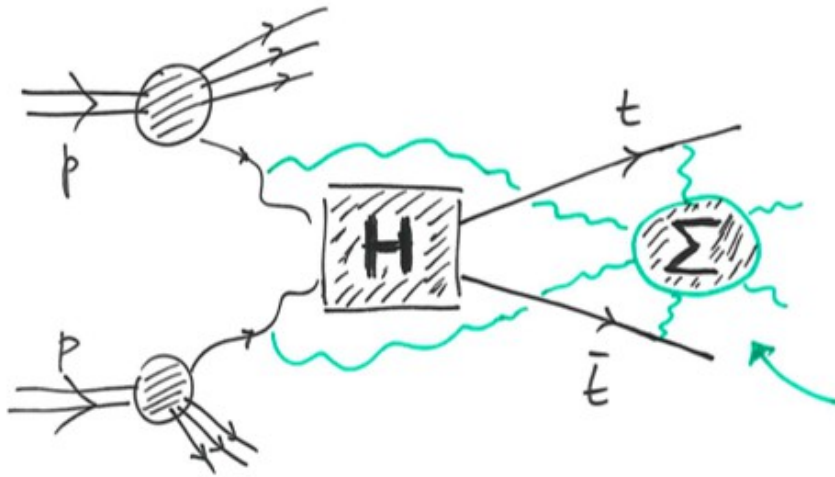
- New long-time correlations in color-charged case:

$$\left(\frac{d\sigma}{d^4q}\right)_{t\bar{t}} = \sum_{ija_1a_2} \int d^2\mathbf{b} e^{i\mathbf{q}_T \cdot \mathbf{b}} \int dz_1 \int dz_2 S(Q, \mathbf{b}) f_{a_1} \otimes [\text{Tr}(H\Delta)C_1C_2]_{ija_1a_2} \otimes f_{a_2}$$

- Generate azimuthal correlations
- Observable for  $\Delta p_\perp$  high compared to  $\Lambda_{\text{QCD}}$

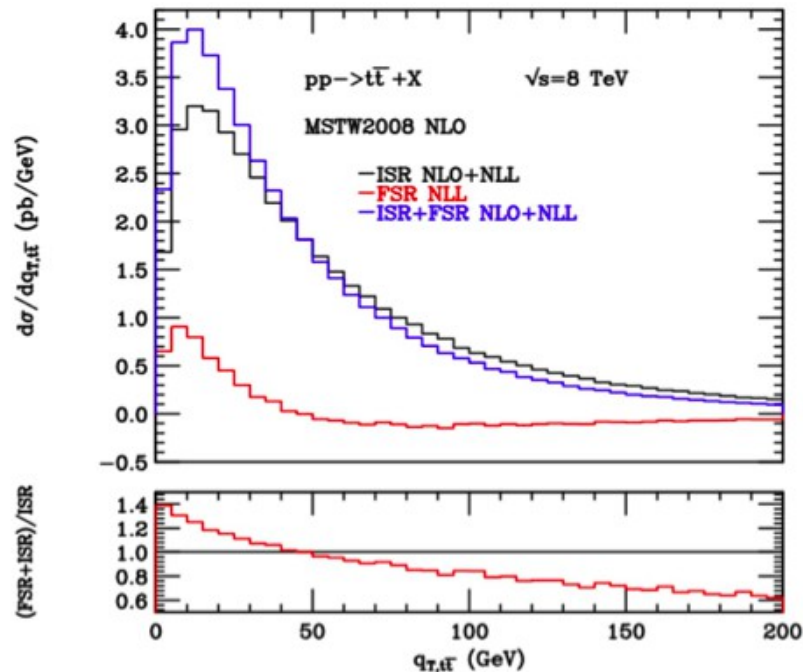
soft gluons coupling  
initial and final states

# Color correlations in jet and heavy-flavor production



- Initial state / final state soft-gluon correlations  
→ new “color entanglement” effects?

- A recent quantitative estimate of the size of color correlations for the top quark pair spectrum at the LHC:



Catani, Grazzini, Sargsyan  
JHEP 1706 (2017) 017

# II. The Parton Branching (PB) method

## MOTIVATION

- Provide evolution equation connected in a controllable way with DGLAP evolution of collinear parton distributions
- Applicable over broad kinematic range from low to high transverse momenta, for inclusive as well as non-inclusive observables
- Implementable in Monte Carlo event generators

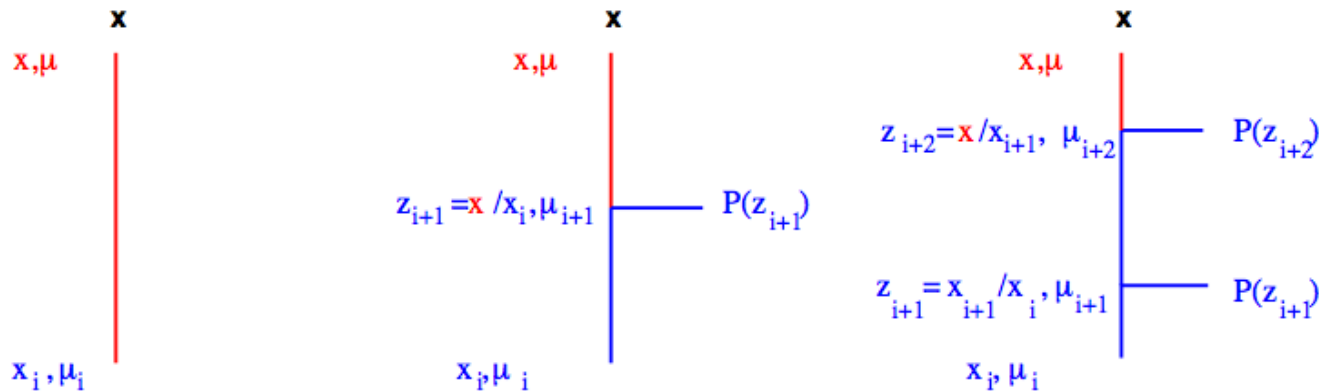
# Parton Branching (PB) method: collinear PDFs

## QCD evolution and soft-gluon resolution scale

[Jung, Lelek, Radescu, Zlebcik & H, PLB772 (2017) 446 + in progress]

$$\tilde{f}_a(x, \mu^2) = S_a(\mu^2) \tilde{f}_a(x, \mu_0^2) + \sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \frac{S_a(\mu'^2)}{S_a(\mu'^2)} \int_x^{z_M} dz P_{ab}^{(R)}(\alpha_s(\mu'^2), z) \tilde{f}_b(x/z, \mu'^2)$$

$$\text{where } S_a(z_M, \mu^2, \mu_0^2) = \exp \left( - \sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \int_0^{z_M} dz z P_{ba}^{(R)}(\alpha_s(\mu'^2), z) \right)$$



- ▷ soft-gluon resolution parameter  $z_M$  separates resolvable and nonresolvable branchings
- ▷ no-branching probability  $S$ ; real-emission probability  $P^{(R)}$

- Equivalent to DGLAP evolution equation for  $z_M \rightarrow 1$



# Non-resolvable emissions and unitarity method

- Introduce resolution scale  $z_M$ , where  $1 - z_M \sim \mathcal{O}(\Lambda_{\text{QCD}}/\mu)$ .
- Classify singular behavior of splitting kernels  $P_{ab}(z, \alpha_s)$  in non-resolvable region  $1 > z > z_M$ :

$$P_{ab}(\alpha_s, z) = D_{ab}(\alpha_s) \delta(1 - z) + K_{ab}(\alpha_s) \frac{1}{(1 - z)_+} + R_{ab}(\alpha_s, z)$$

$$\text{where } \int_0^1 \frac{1}{(1 - z)_+} \varphi(z) dz = \int_0^1 \frac{1}{1 - z} [\varphi(z) - \varphi(1)] dz$$

and  $R_{ab}(\alpha_s, z)$  contains logarithmic and analytic contributions for  $z \rightarrow 1$

- Expand plus-distributions in non-resolvable region and use sum rule  $\sum_c \int_0^1 z P_{ca}(\alpha_s, z) dz = 0$  (for any  $a$ ) to eliminate  $D$ -terms in favor of  $K$ - and  $R$ -terms

$\Rightarrow$  real-emission probabilities exponentiate into Sudakov form factors

# Parton Branching (PB) method: TMD PDFs

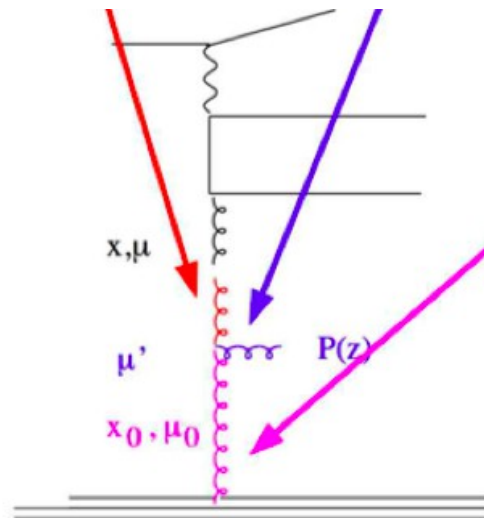
$$\begin{aligned}\tilde{\mathcal{A}}_a(x, \mathbf{k}, \mu^2) &= S_a(\mu^2) \tilde{\mathcal{A}}_a(x, \mathbf{k}, \mu_0^2) + \sum_b \int \frac{d^2 \mathbf{q}'}{\pi \mathbf{q}'^2} \frac{S_a(\mu^2)}{S_a(\mathbf{q}'^2)} \Theta(\mu^2 - \mathbf{q}'^2) \Theta(\mathbf{q}'^2 - \mu_0^2) \\ &\times \int_x^{z_M} dz P_{ab}^{(R)}(\alpha_s(\mathbf{q}'^2), z) \tilde{\mathcal{A}}_b(x/z, \mathbf{k} + (1-z)\mathbf{q}', \mathbf{q}'^2)\end{aligned}$$

Solve iteratively :  $\tilde{\mathcal{A}}_a^{(0)}(x, \mathbf{k}, \mu^2) = S_a(\mu^2) \tilde{\mathcal{A}}_a(x, \mathbf{k}, \mu_0^2)$  ,

$$\begin{aligned}\tilde{\mathcal{A}}_a^{(1)}(x, \mathbf{k}, \mu^2) &= \sum_b \int \frac{d^2 \mathbf{q}'}{\pi \mathbf{q}'^2} \Theta(\mu^2 - \mathbf{q}'^2) \Theta(\mathbf{q}'^2 - \mu_0^2) \\ &\times \frac{S_a(\mu^2)}{S_a(\mathbf{q}'^2)} \int_x^{z_M} dz P_{ab}^{(R)}(\alpha_s(\mathbf{q}'^2), z) \tilde{\mathcal{A}}_b(x/z, \mathbf{k} + (1-z)\mathbf{q}', \mu_0^2) S_b(\mathbf{q}'^2)\end{aligned}$$

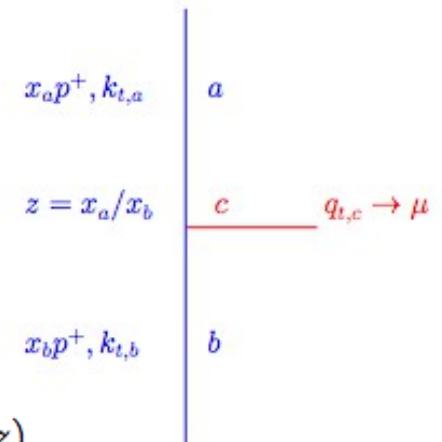
*Jung, Lelek,  
Radescu, Zlebcik & H,  
JHEP 01 (2018) 070*

- A new evolution equation!



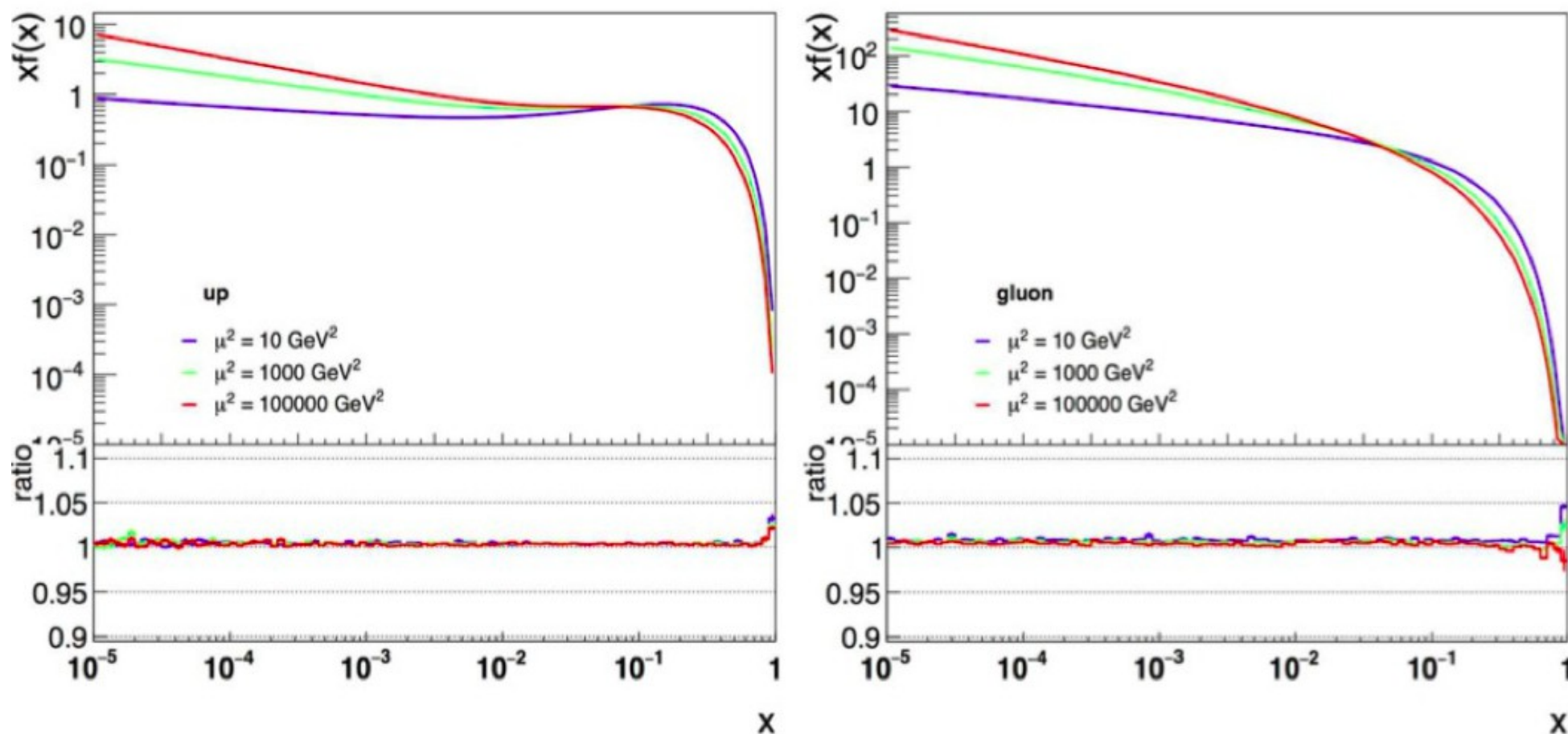
↖ NB: angular ordering

$$\mu = |\mathbf{q}_c|/(1-z)$$



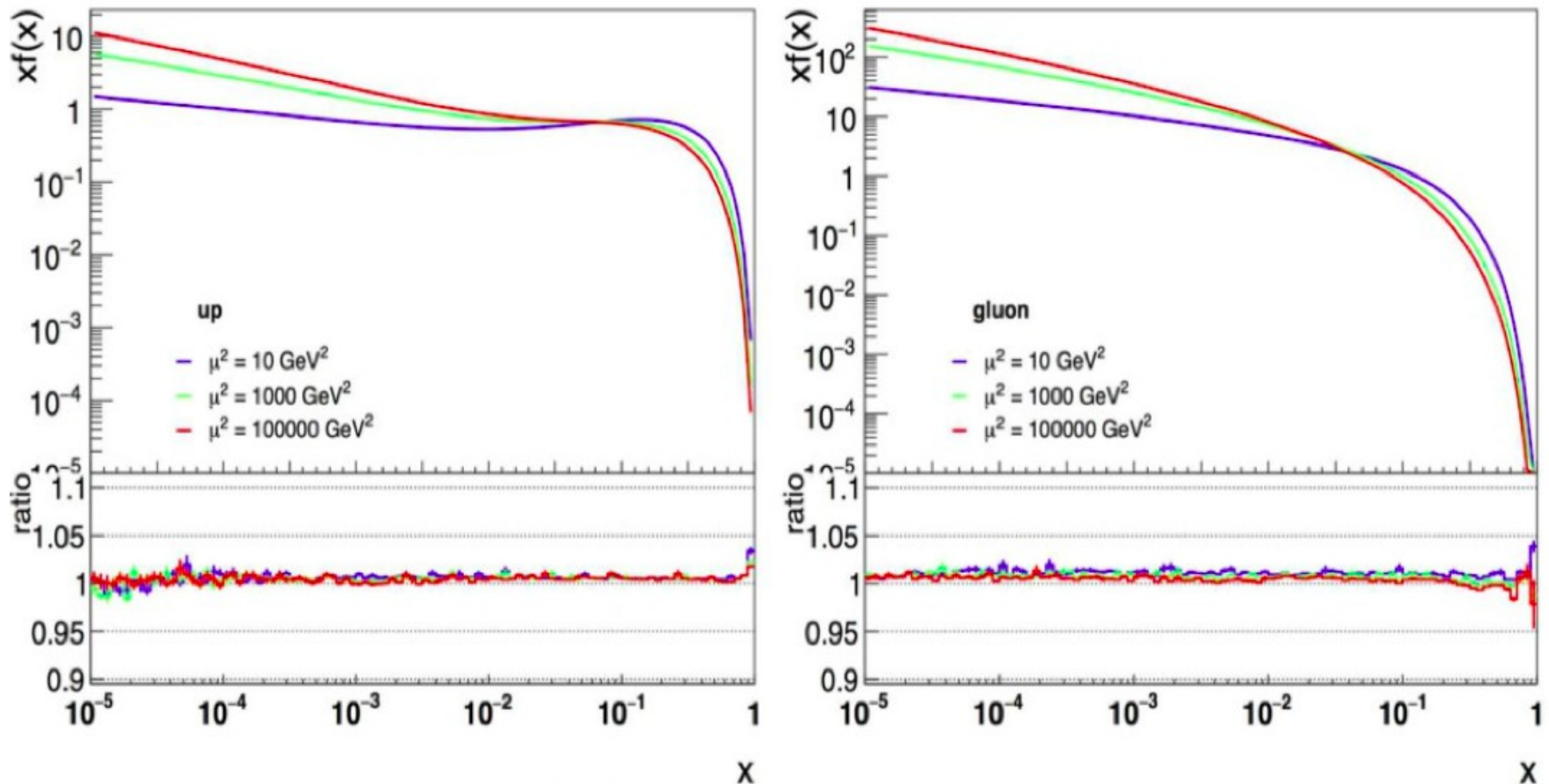


# Validation of the method with semi-analytic result from QCDNUM at LO



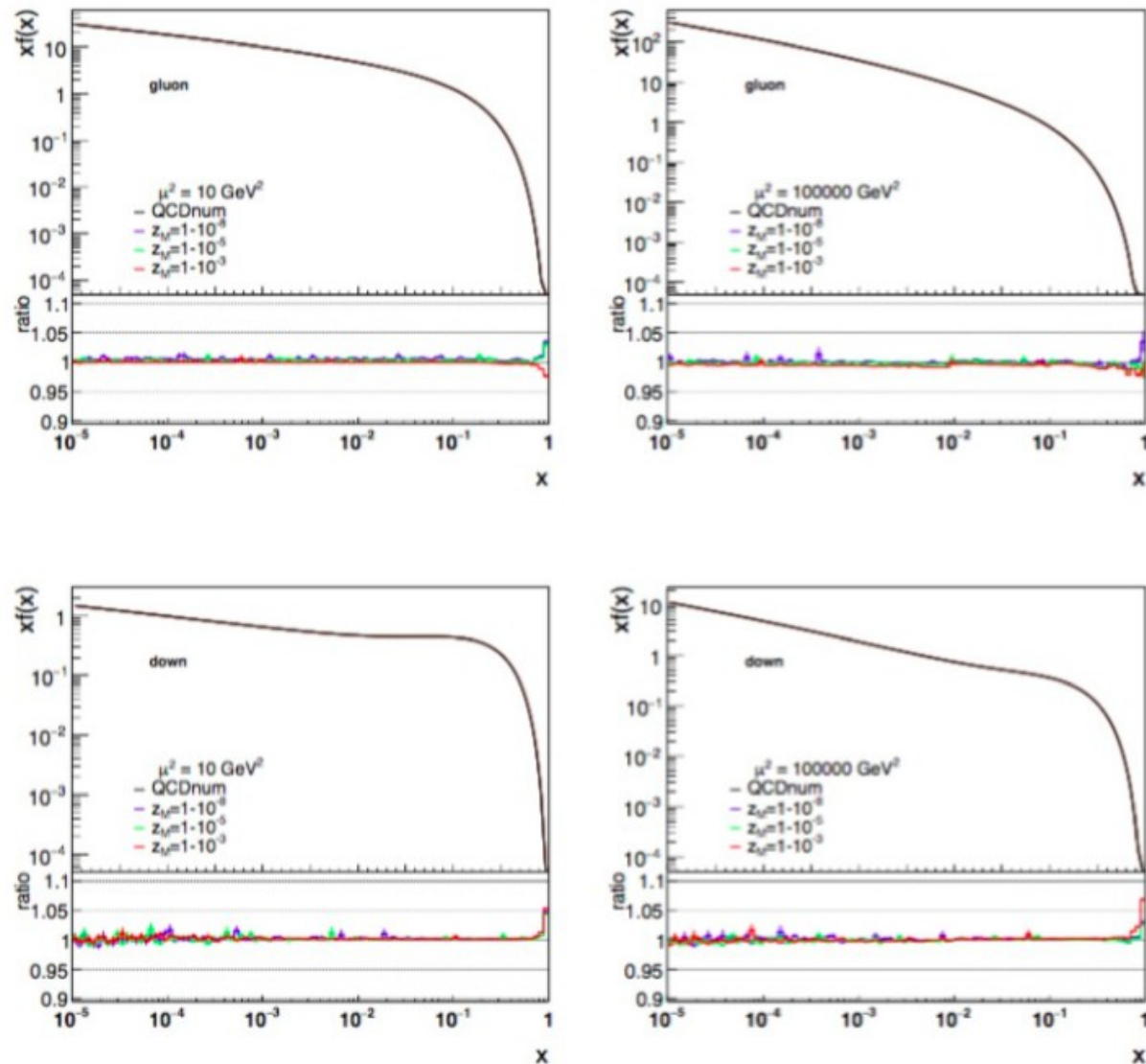
Agreement to better than 1 % over several orders of magnitude in  $x$  and  $\mu$

# Validation of the method with semi-analytic result from QCDNUM at NLO

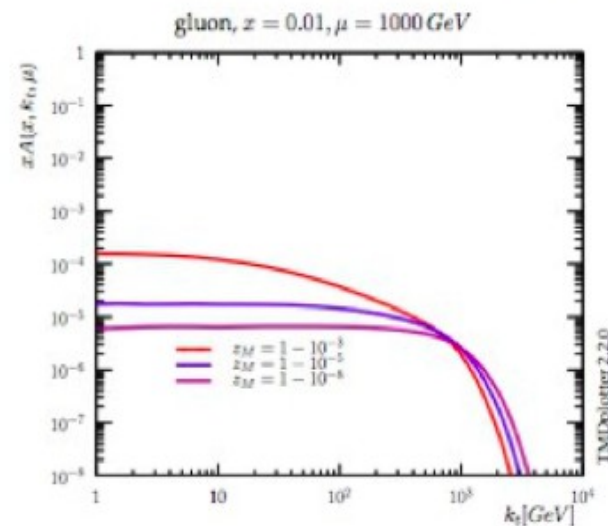
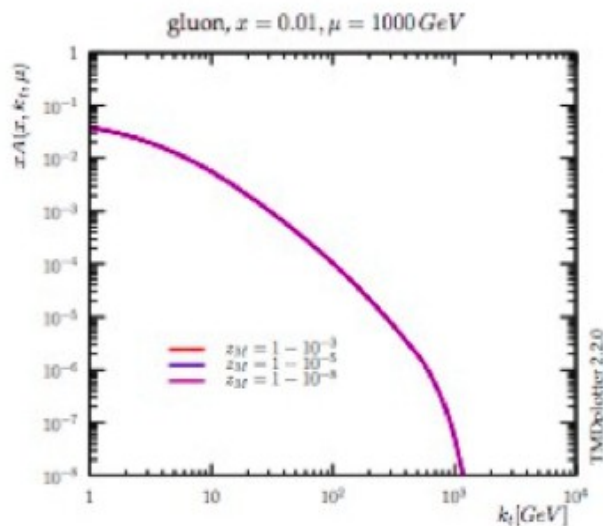
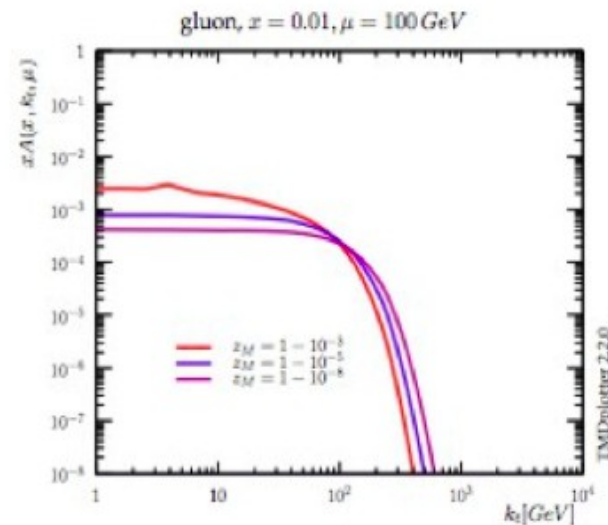
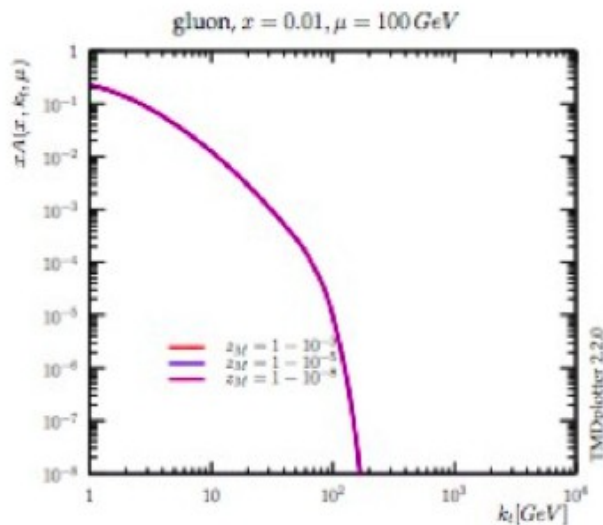


Very good agreement at NLO over all  $x$  and  $\mu$ .  
NB: the same approach is designed to work at NNLO.

# Stability with respect to resolution scale $z_M$



# TMDs and soft gluon resolution effects



angular ordering

transverse momentum ordering

Well-defined TMDs require appropriate ordering condition



# PB method in xFitter

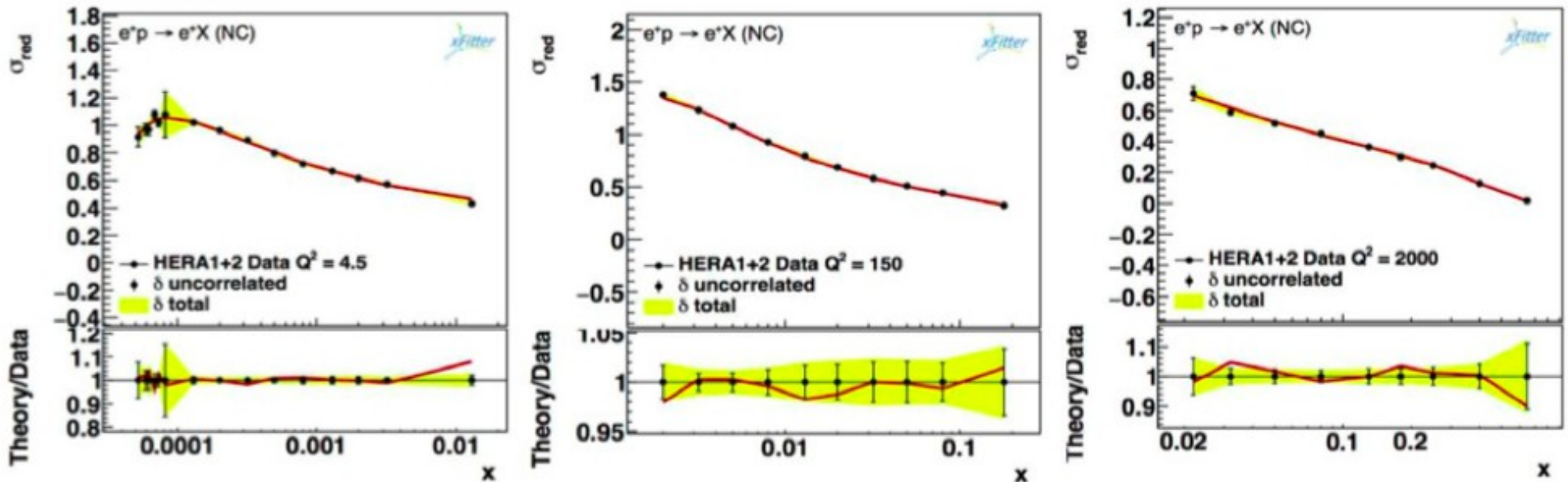
- Determine starting distribution

A Bermudez et al, arXiv:1804.11152

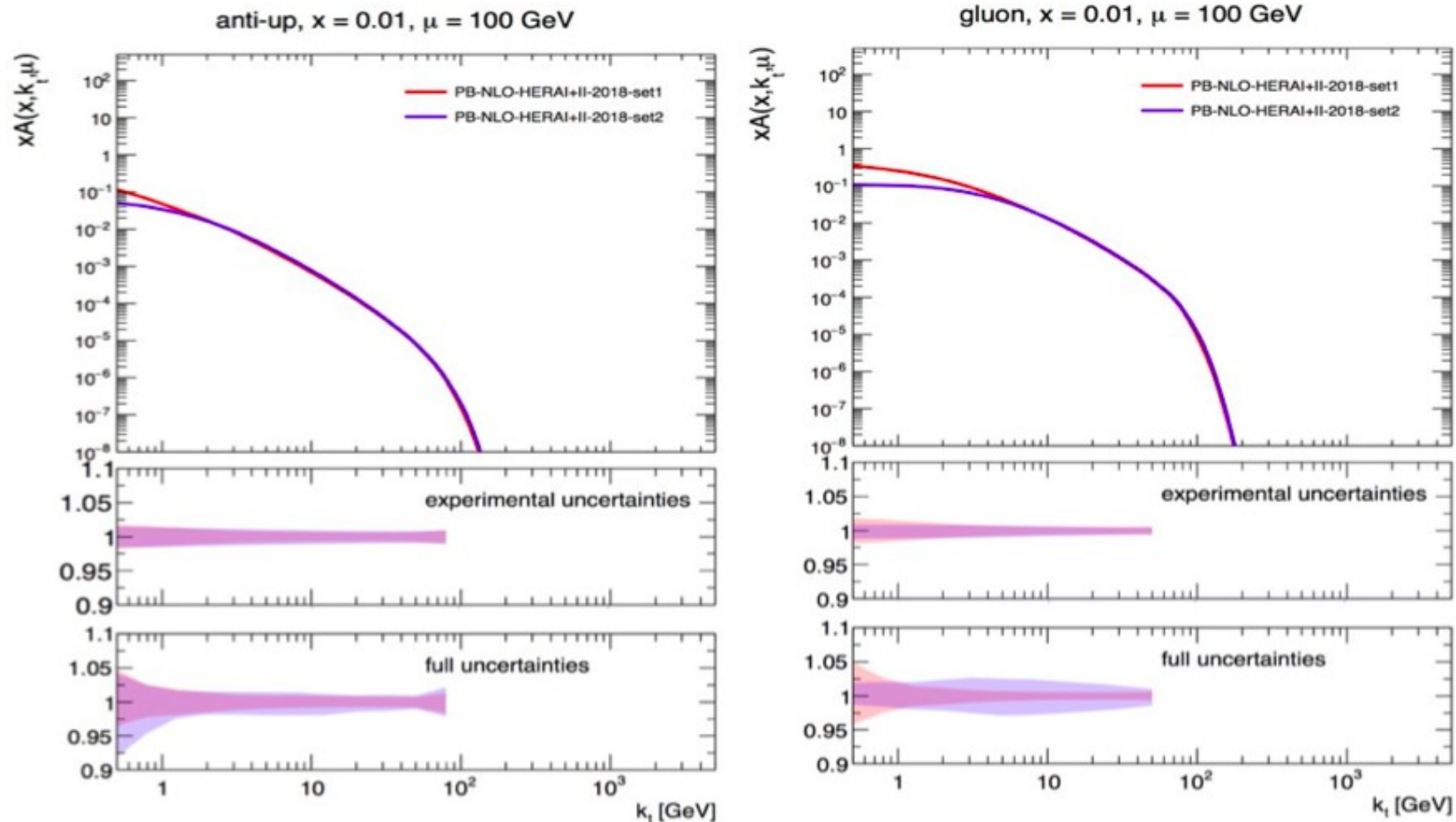
A. Lelek et al REF 2016

$$\begin{aligned}
 x f_a(x, \mu^2) &= x \int dx' \int dx'' \mathcal{A}_{0,b}(x') \tilde{\mathcal{A}}_a^b(x'', \mu^2) \delta(x'x'' - x) \\
 &= \int dx' \mathcal{A}_{0,b}(x') \cdot \frac{x}{x'} \tilde{\mathcal{A}}_a^b\left(\frac{x}{x'}, \mu^2\right)
 \end{aligned}$$

- fit to HERA data (using xFitter) with  $Q^2 \geq 3.5 \text{ GeV}^2$  gives  $\chi^2/ndf \sim 1.2$



# TMD distributions from fits to precision HERA data



A Bermudez et al, arXiv:1804.11152

- NLO determination of TMDs with uncertainties

# Where to find TMDs? TMDlib and TMDplotter

- TMDlib proposed in 2014 as part of the REF Workshop and developed since

- A library of parameterizations and fits of TMDs (LHAPDF-style)

<http://tmdlib.hepforge.org>

<http://tmdplotter.desy.de>

- Also contains collinear (integrated) pdfs

Eur. Phys. J. C (2014) 74:3220  
DOI 10.1140/epjc/s10052-014-3220-9

THE EUROPEAN  
PHYSICAL JOURNAL C

Special Article - Tools for Experiment and Theory

## TMDlib and TMDplotter: library and plotting tools for transverse-momentum-dependent parton distributions

F. Hautmann<sup>1,2</sup>, H. Jung<sup>3,4</sup>, M. Krämer<sup>3</sup>, P. J. Mulders<sup>5,6</sup>, E. R. Nocera<sup>7</sup>, T. C. Rogers<sup>8,9</sup>, A. Signori<sup>5,6,a</sup>

<sup>1</sup> Rutherford Appleton Laboratory, Oxford, UK

<sup>2</sup> Department of Theoretical Physics, University of Oxford, Oxford, UK

<sup>3</sup> DESY, Hamburg, Germany

<sup>4</sup> University of Antwerp, Antwerp, Belgium

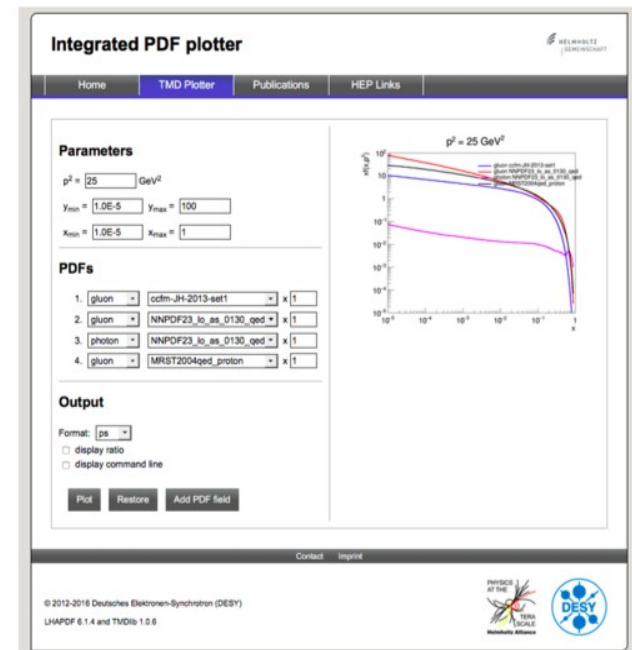
<sup>5</sup> Department of Physics and Astronomy, VU University Amsterdam, Amsterdam, The Netherlands

<sup>6</sup> Nikhef, Amsterdam, The Netherlands

<sup>7</sup> Università degli Studi di Genova, INFN, Genoa, Italy


<sup>8</sup> C.N. Yang Institute for Theoretical Physics, Stony Brook University, Stony Brook, USA

<sup>9</sup> Department of Physics, Southern Methodist University, Dallas, TX 75275, USA



# Next REF Workshop: Cracow, 19-22 November 2018

<https://indico.cern.ch/event/696311>



## Workshop on Resummation, Evolution, Factorization 2018

19-22 November 2018  
Other Institutes  
Europe/Warsaw timezone

Overview

Timetable

Participant List

Venue

Travel

Contact

✉ [krzysztof.kutak@ifj.edu.pl](mailto:krzysztof.kutak@ifj.edu.pl)

✉ [jolanta.mosurek@ifj.edu.pl](mailto:jolanta.mosurek@ifj.edu.pl)

**REF 2018** is the 5th workshop in the series of workshops on Resummation, Evolution, Factorization. The workshop wishes to bring together experts of different communities specialized in: nuclear structure; transverse momentum dependend distributions; small-x physics; effective field theories.

Previous meetings

- [13-16 November 2017 Madrid \(Spain\)](#)
- [7-10 November 2016 Antwerp \(Belgium\)](#)
- [2-5 November 2015 DESY Hamburg \(Germany\)](#)
- [8-11 December 2014 Antwerp \(Belgium\)](#)

**Scientific committee:**

Elke Aschenauer	Daniel Boer
Igor Cherednikov	Markus Diehl
Didar Dobur	David Dudal
Miguel García Echevarría	
Laurent Favart	Francesco Hautmann
Hannes Jung	Fabio Maltoni
Piet Mulders	Gunar Schnell
Andrea Signori	Pierre Van Mechelen



# III. New results and applications

## ONGOING WORK:

- Drell-Yan  $p_T$  spectrum from convolution of two transverse momentum dependent distributions
- Comparison of parton branching results with analytic TMD resummation (Collins-Soper-Sterman method)
- First implementation for jets (using NLO matrix elements for color-charged final states)

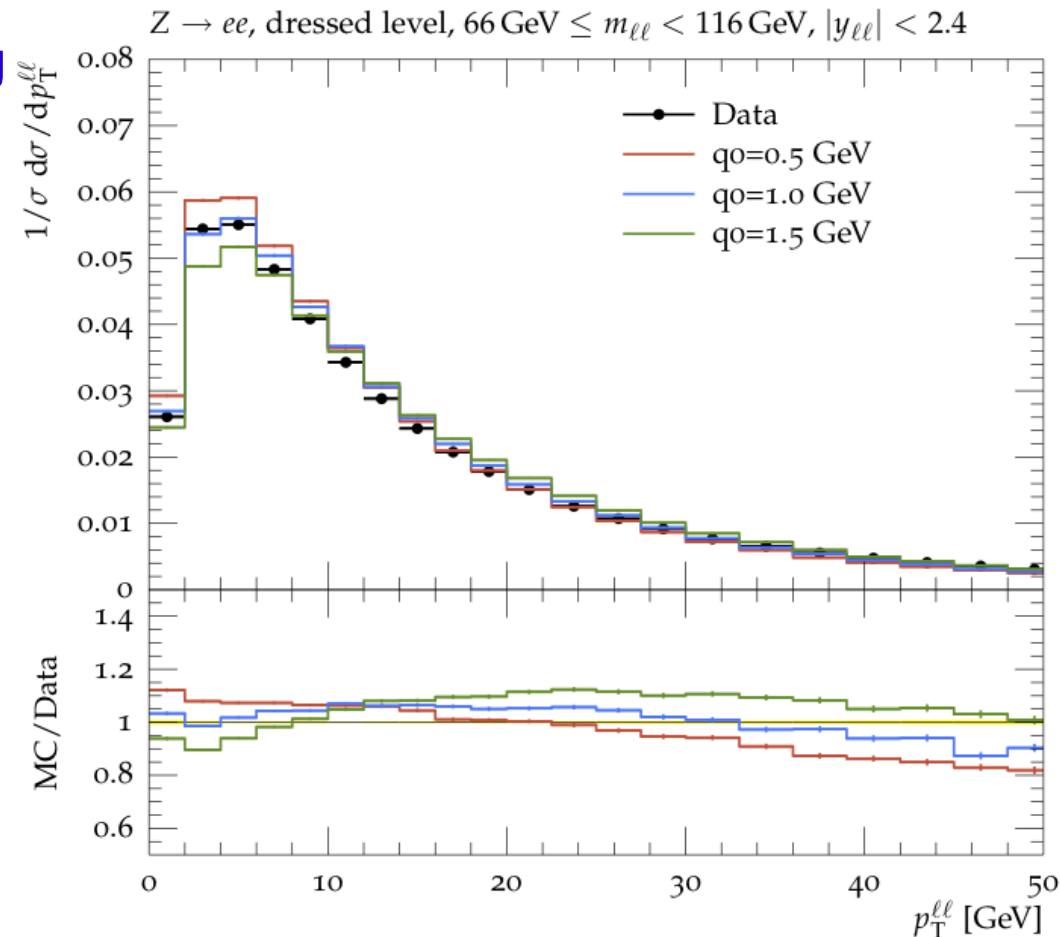
# Application of PB method to Z-boson transverse momentum spectrum in Drell-Yan production

- Parton branching TMD defined by using angular ordering
- Scale in running coupling also by angular ordering

$$\alpha_s(\mu^2(1-z)^2)$$

- mu-dependent soft-gluon resolution scale parameter  $z_M$

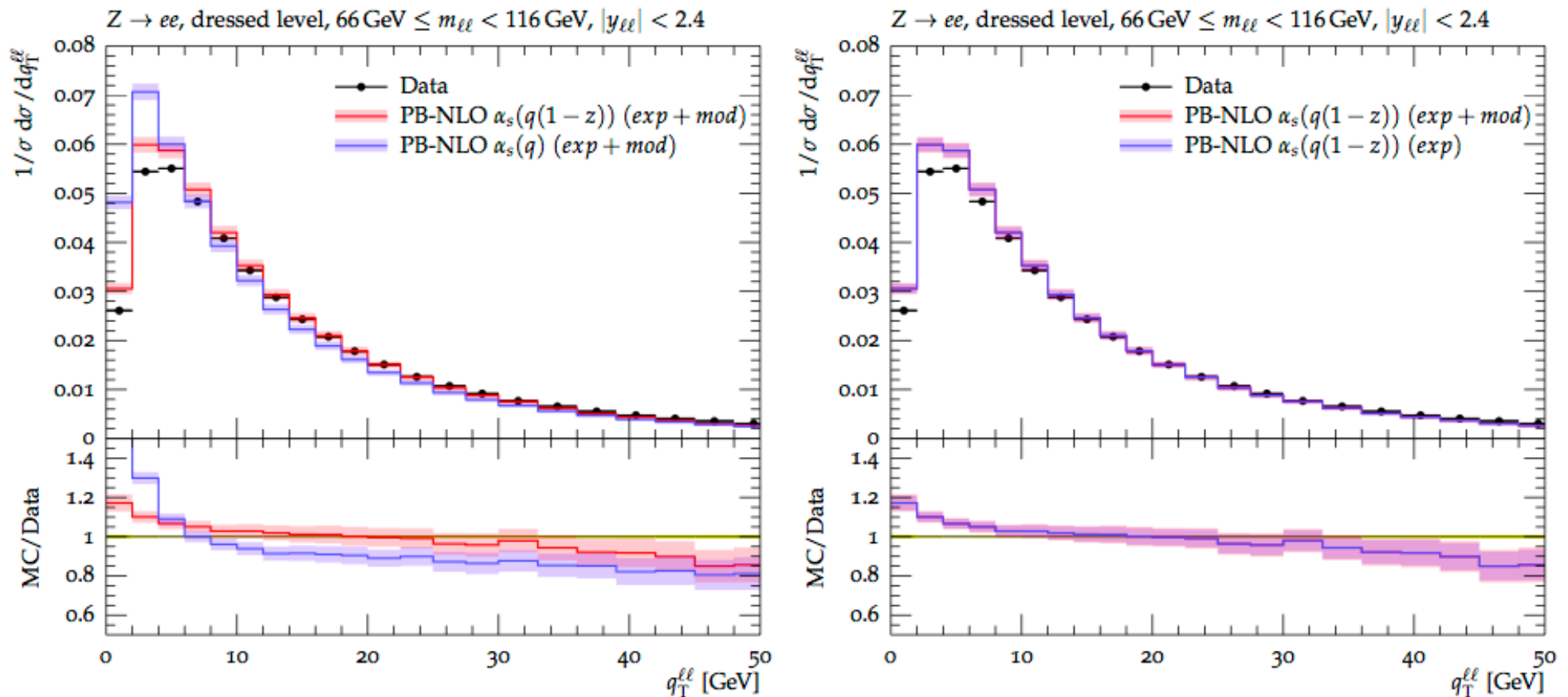
$$z_M(\mu) = 1 - q_0/\mu$$



LHC Electroweak WG Meeting, CERN, June 2018

# Z-boson transverse momentum spectrum: soft-gluon angular ordering effects

Zlebcik, Radescu, Lelek, Jung & H,  
JHEP 1801 (2018) 070;  
A Bermudez Martinez et al.,  
arXiv:1804.11152 [hep-ph]



ATLAS data, EPJC 76 (2016) 291

# Comparison with CSS (Collins-Soper-Sterman) resummation

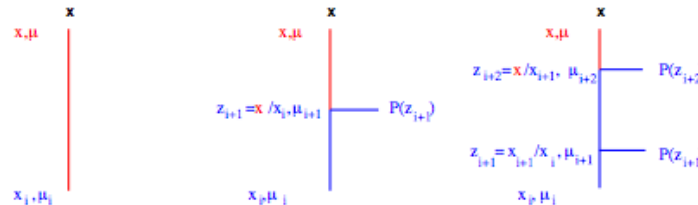
◇ The resummed DY differential cross section is given by

$$\frac{d\sigma}{d^2\mathbf{q}dQ^2dy} = \sum_{q,\bar{q}} \frac{\sigma^{(0)}}{s} H(\alpha_S) \int \frac{d^2\mathbf{b}}{(2\pi)^2} e^{i\mathbf{q}\cdot\mathbf{b}} \mathcal{A}_q(x_1, \mathbf{b}, Q) \mathcal{A}_{\bar{q}}(x_2, \mathbf{b}, Q) + \mathcal{O}\left(\frac{|\mathbf{q}|}{Q}\right) \quad \text{where}$$

$$\begin{aligned} \mathcal{A}_i(x, \mathbf{b}, Q) &= \exp \left\{ \frac{1}{2} \int_{c_0/b^2}^{Q^2} \frac{d\mu'^2}{\mu'^2} \left[ A_i(\alpha_S(\mu'^2)) \ln \left( \frac{Q^2}{\mu'^2} \right) + B_i(\alpha_S(\mu'^2)) \right] \right\} G_i^{(\text{NP})}(x, \mathbf{b}) \\ &\times \sum_j \int_x^1 \frac{dz}{z} C_{ij} \left( z, \alpha_S \left( \frac{c_0}{\mathbf{b}^2} \right) \right) f_j \left( \frac{x}{z}, \frac{c_0}{\mathbf{b}^2} \right) \end{aligned}$$

and the coefficients  $H, A, B, C$  have power series expansions in  $\alpha_S$ .

◇ The parton branching TMD is expressed in terms of real-emission  $P^{(R)}$ :



- ▷ via momentum sum rules, use unitarity to relate  $P^{(R)}$  to virtual emission
- ▷ identify the coefficients in the two formulations, order by order in  $\alpha_S$ , at LL, NLL, ...

# Comparison with CSS (Collins-Soper-Sterman) resummation

More precisely:

▷ The parton branching TMD contains Sudakov form factor in terms of

$$P_{ab}^{(R)}(\alpha_S, z) = K_{ab}(\alpha_S) \frac{1}{1-z} + R_{ab}(\alpha_S, z) \quad \text{where}$$

$$K_{ab}(\alpha_S) = \delta_{ab} k_a(\alpha_S), \quad k_a(\alpha_S) = \sum_{n=1}^{\infty} \left( \frac{\alpha_S}{2\pi} \right)^n k_a^{(n-1)}, \quad R_{ab}(\alpha_S, z) = \sum_{n=1}^{\infty} \left( \frac{\alpha_S}{2\pi} \right)^n R_{ab}^{(n-1)}(z)$$

▷ Via momentum sum rules, use unitarity to re-express this in terms of

$$P^{(V)} = P - P^{(R)}, \quad \text{where}$$

$$P_{ab}(\alpha_S, z) = D_{ab}(\alpha_S) \delta(1-z) + K_{ab}(\alpha_S) \frac{1}{(1-z)_+} + R_{ab}(\alpha_S, z)$$

is full splitting function (at LO, NLO, etc.)

$$\text{with } D_{ab}(\alpha_S) = \delta_{ab} d_a(\alpha_S), \quad d_a(\alpha_S) = \sum_{n=1}^{\infty} \left( \frac{\alpha_S}{2\pi} \right)^n d_a^{(n-1)}$$

▷ Identify  $d_a(\alpha_S)$  and  $k_a(\alpha_S)$  with resummation formula coefficients (LL, NLL, . . .)

# Comparison with CSS (Collins-Soper-Sterman) resummation

- $d_a(\alpha_s)$  and  $k_a(\alpha_s)$  perturbative coefficients

one – loop :

$$d_q^{(0)} = \frac{3}{2} C_F \quad , \quad k_q^{(0)} = 2 C_F$$

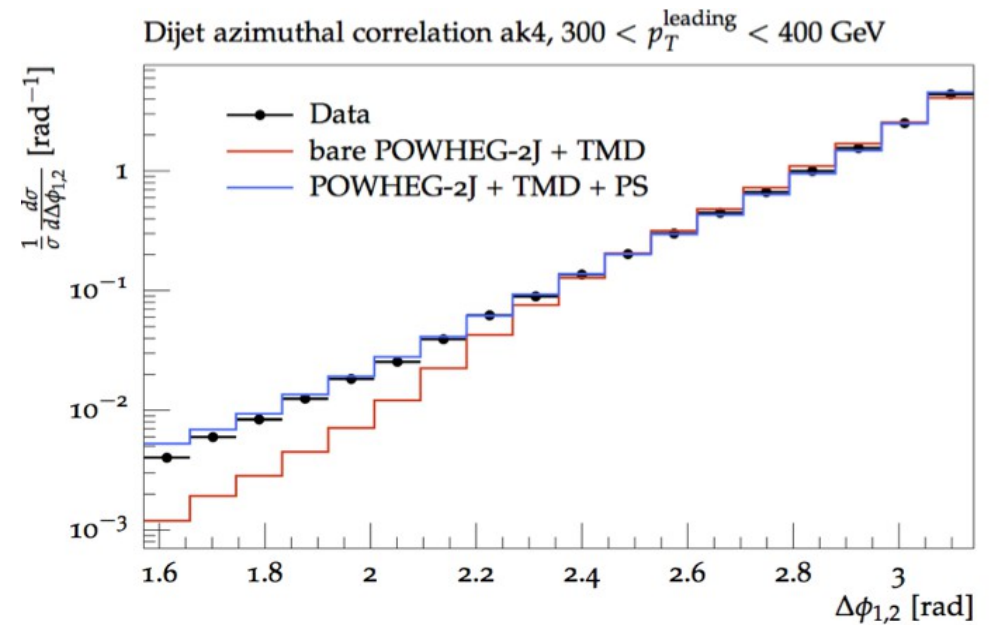
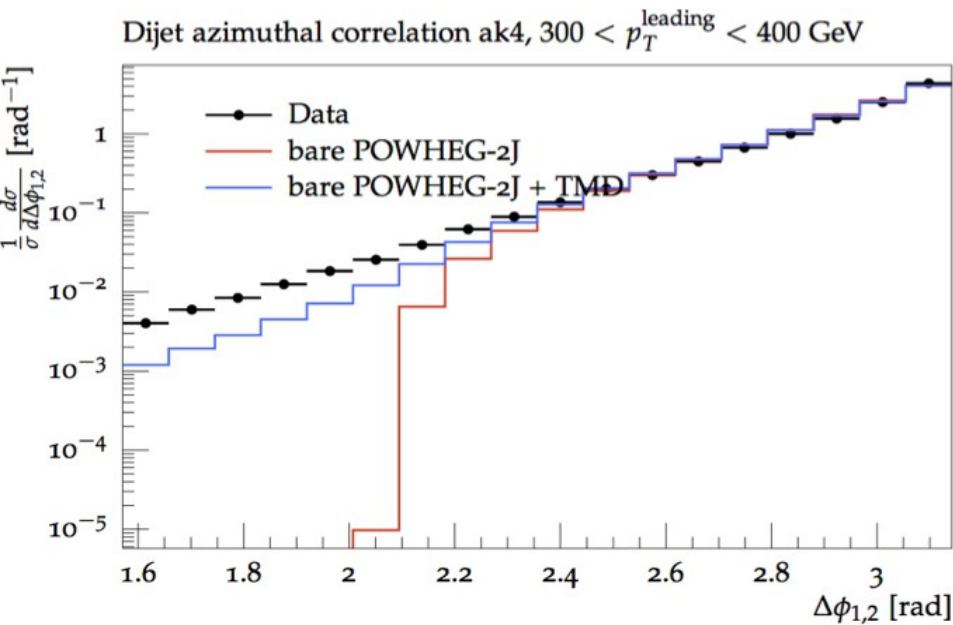
two – loop :

$$d_q^{(1)} = C_F^2 \left( \frac{3}{8} - \frac{\pi^2}{2} + 6 \zeta(3) \right) + C_F C_A \left( \frac{17}{24} + \frac{11\pi^2}{18} - 3 \zeta(3) \right) - C_F T_R N_f \left( \frac{1}{6} + \frac{2\pi^2}{9} \right) ,$$

$$k_q^{(1)} = 2 C_F \Gamma \quad , \quad \text{where } \Gamma = C_A \left( \frac{67}{18} - \frac{\pi^2}{6} \right) - T_R N_f \frac{10}{9}$$

- The k and d coefficients of the PB formalism match, order by order, the A and B coefficients of the CSS formalism

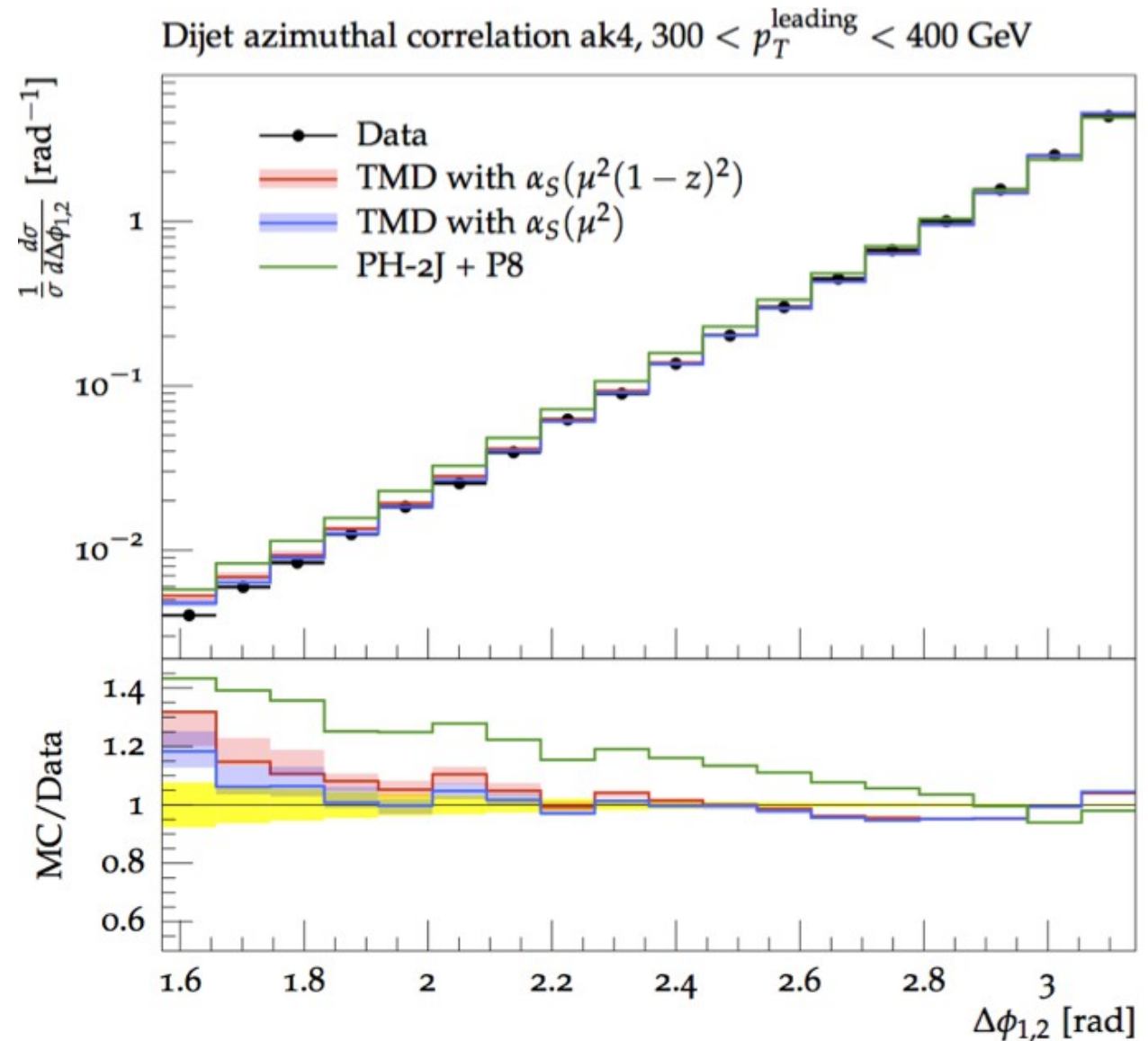
# Di-jets from PB method: towards NLO-matched parton-shower Monte Carlo generators with TMDs



- Events by NLO POWHEG 2 jets
- Parton branching TMD (with angular ordering)
- TMD parton shower

# Di-jets from PB method: towards NLO-matched parton-shower Monte Carlo generators with TMDs

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# Conclusions

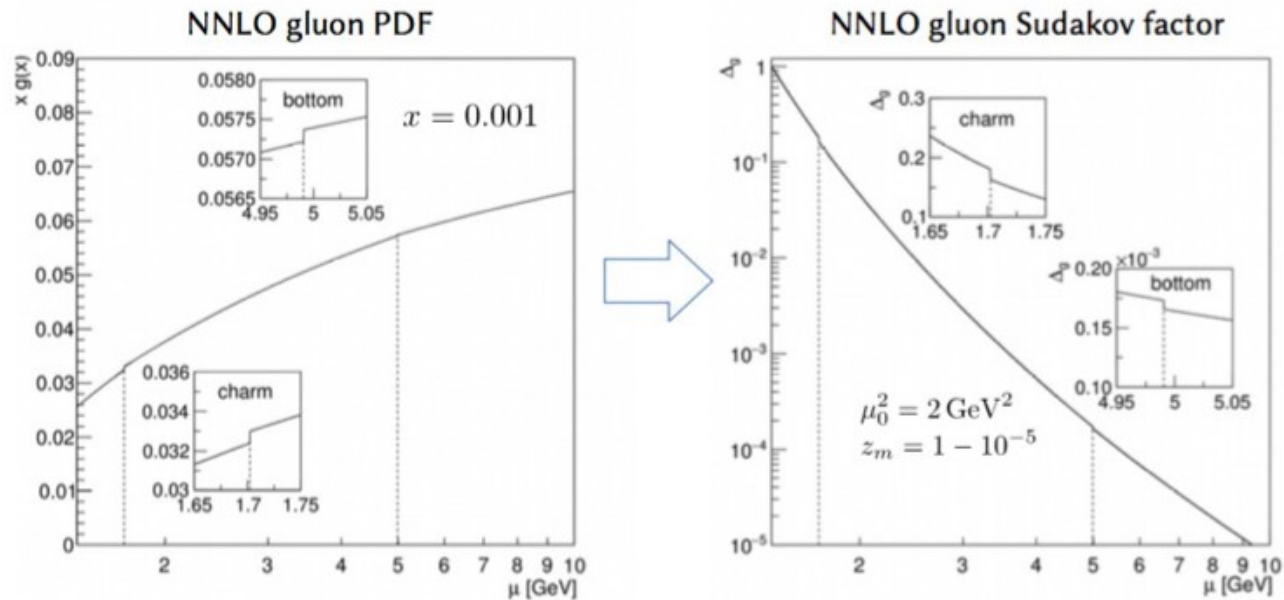
- PB method to take into account simultaneously soft-gluon emission at  $z \rightarrow 1$  and transverse momentum  $q_T$  recoils in the parton branchings along the QCD cascade
- potentially relevant for calculations both in collinear factorization and in TMD factorization
  - cf. parton shower calculations and analytic resummation
- terms in powers of  $\ln(1 - zM)$  can be related to large- $x$  resummation? → relevant to near-threshold, rare processes to be investigated at high luminosity
- systematic studies of ordering effects and color coherence
  - helpful to analyze long-time color correlations?

# EXTRA SLIDES

# PB method at NNLO

- In NNLO VFNS discontinuities both in  $\alpha_S$  and PDFs
- These discontinuities ensure continuity of observables, e.g.  $F_2$

Discontinuities in the quark and gluon Sudakov factors

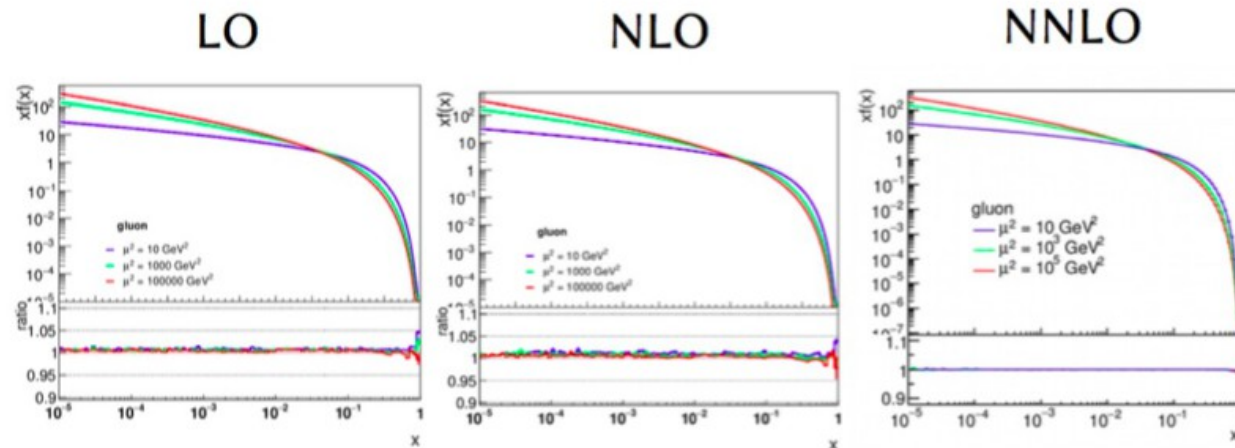


R. Zlebcik, talk at REF 2017, November 2017

Workshop REF2017, Universidad Complutense Madrid, 13-16 November 2017

# PB method at NNLO

## The Monte Carlo solution vs QCDNUM



The Monte Carlo evolution implemented up to NNLO and cross-checked against the semi-analytical solution of DGLAP

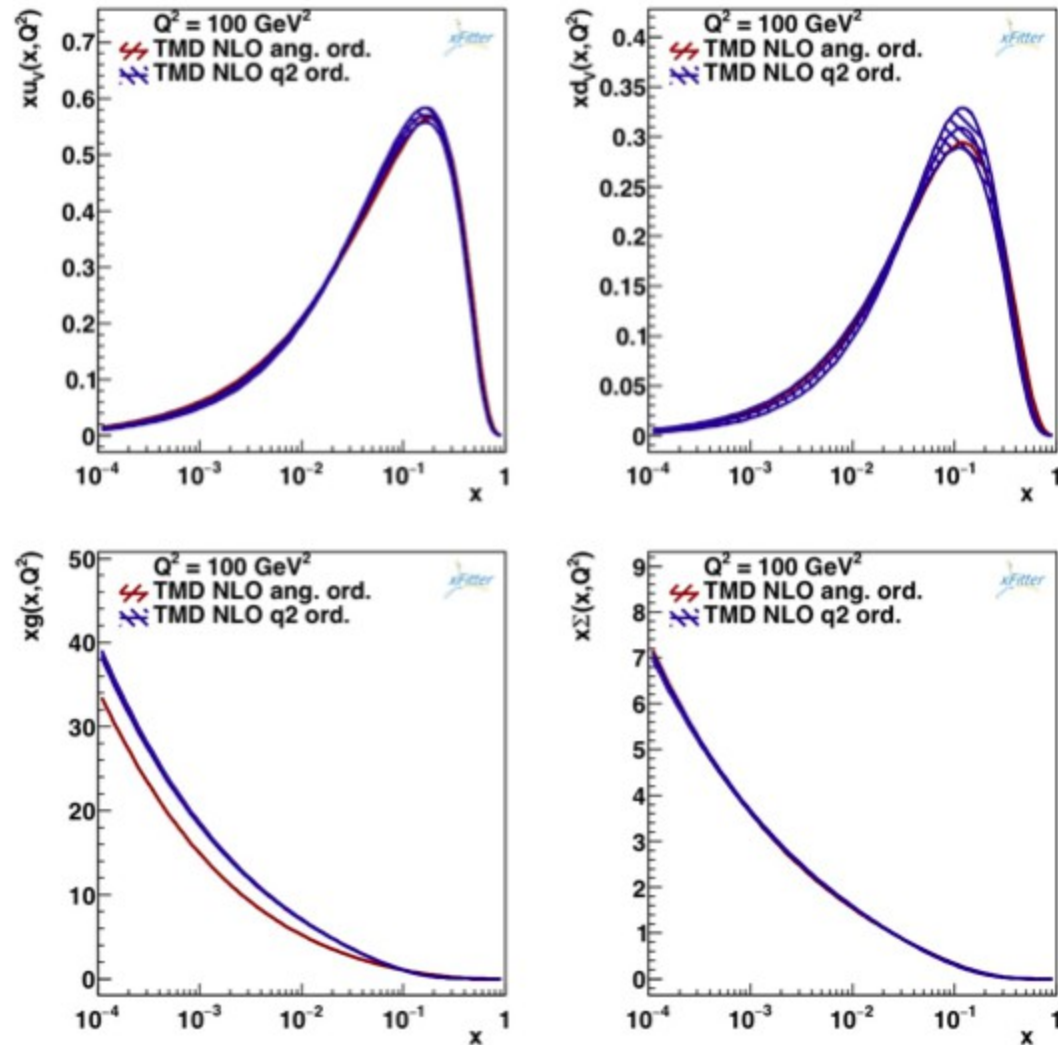
The solution's uncertainties are mainly statistical  
(~ number of generated MC evolutions)

8

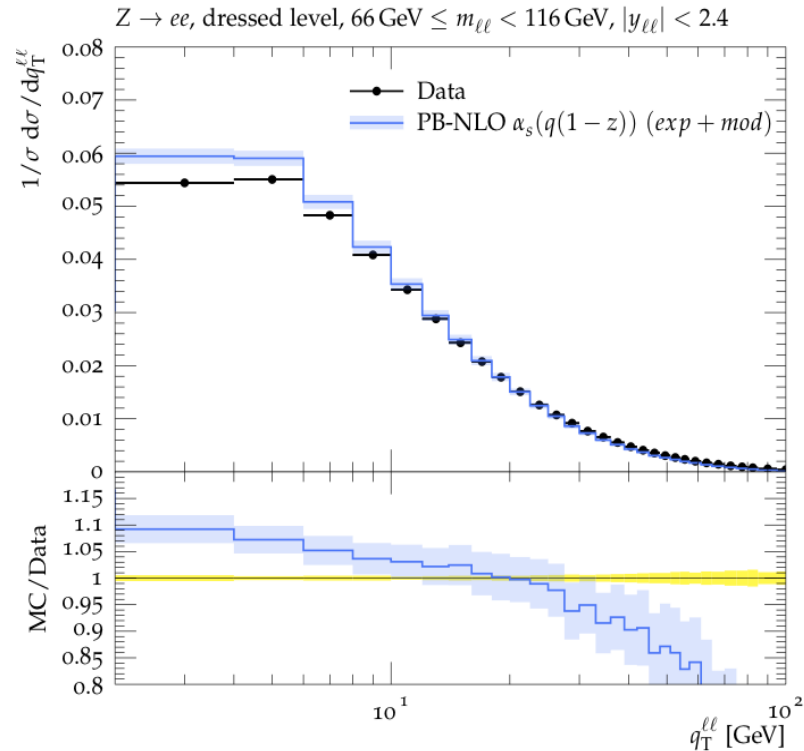
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# Effects of coupling's scale and angular ordering in integrated parton distributions



# Z-boson pT spectrum including TMD uncertainties



- Cf. predictions from fixed-order + resummed calculations  
Bizon et al.,  
arXiv:1805.05916

