JLab

Theory Center Seminar

27 April, 2020

Two-photon exchange in lepton-proton scattering.

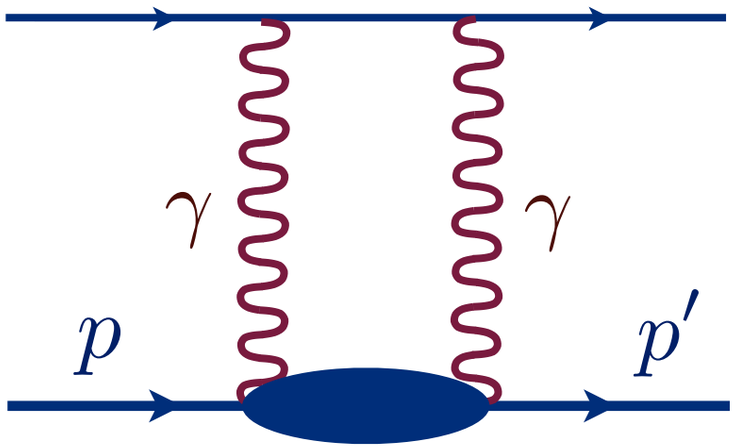
Low-energy nucleon structure

University of Kentucky

Oleksandr Tomalak

Outline

Experimental motivation



1) electron-proton scattering:

near-forward and dispersive calculations

2) muon-proton scattering:

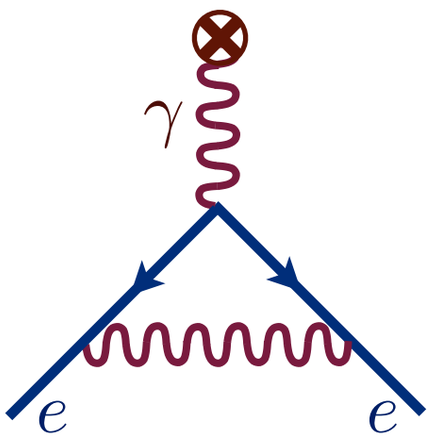
near-forward and dispersive calculations

3) 1S HFS in muonic hydrogen. 2γ in atomic physics

uncertain corrections, relation to electronic hydrogen

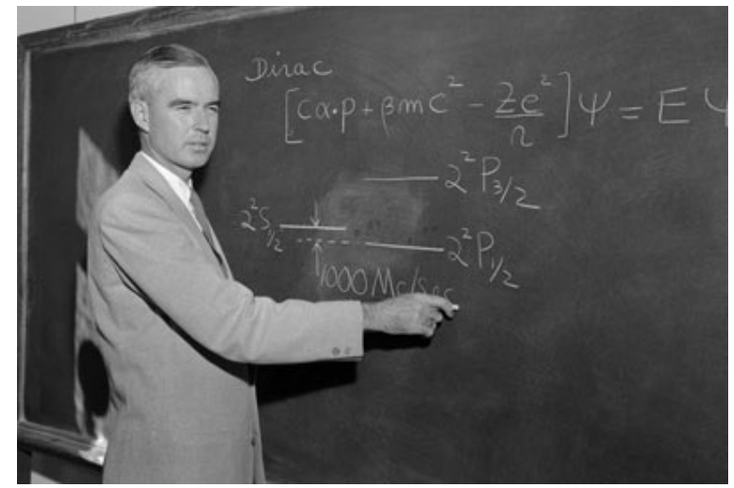
Muon discrepancies: new physics?

anomalous magnetic moment



Polykarp Kusch (1947)

Lamb shift

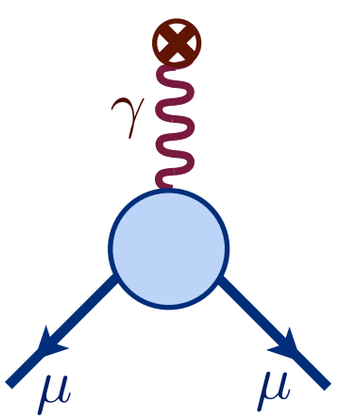


W. Lamb and R. Retherford (1947)

formulation of QED as first successful QFT

J. Schwinger, F. Dyson, R. Feynman, S. Tomonaga (1947-1949)

anomalous magnetic moment



3.6 σ
theory vs exp.

μH Lamb shift

proton size discrepancy

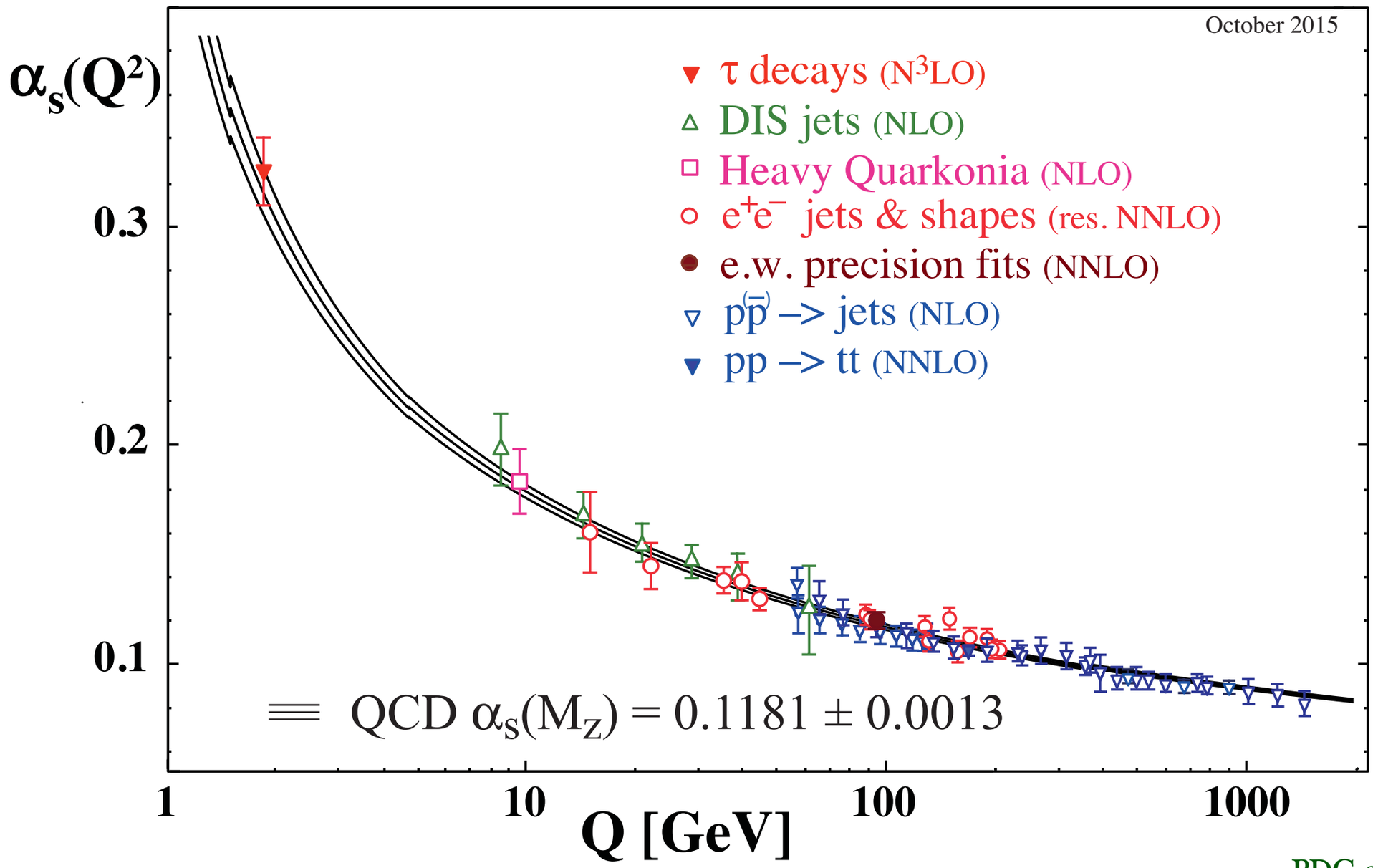
initially 5-6σ

eH, ep vs μH

- hadronic uncertainty is dominant in theory

QCD running coupling

- asymptotic freedom at high energies

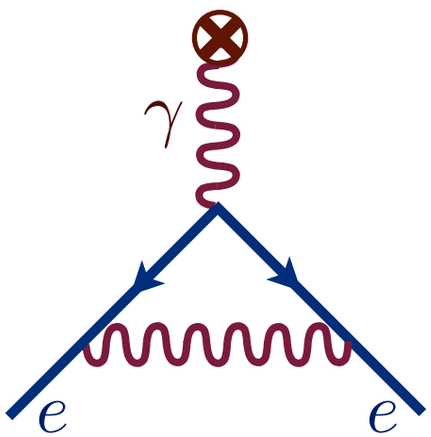


PDG 2015

- EFTs, lattice QCD and phenomenology at low energies

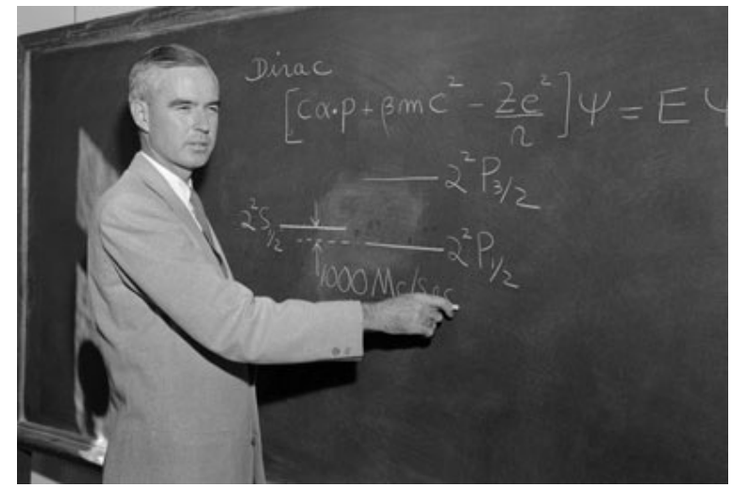
Muon discrepancies: new physics?

anomalous magnetic moment



Polykarp Kusch (1947)

Lamb shift

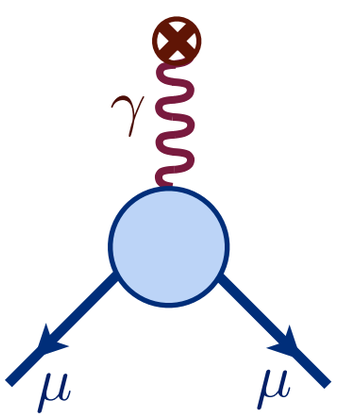


W. Lamb and R. Retherford (1947)

formulation of QED as first successful QFT

J. Schwinger, F. Dyson, R. Feynman, S. Tomonaga (1947-1949)

anomalous magnetic moment



3.6 σ
theory vs exp.

μH Lamb shift

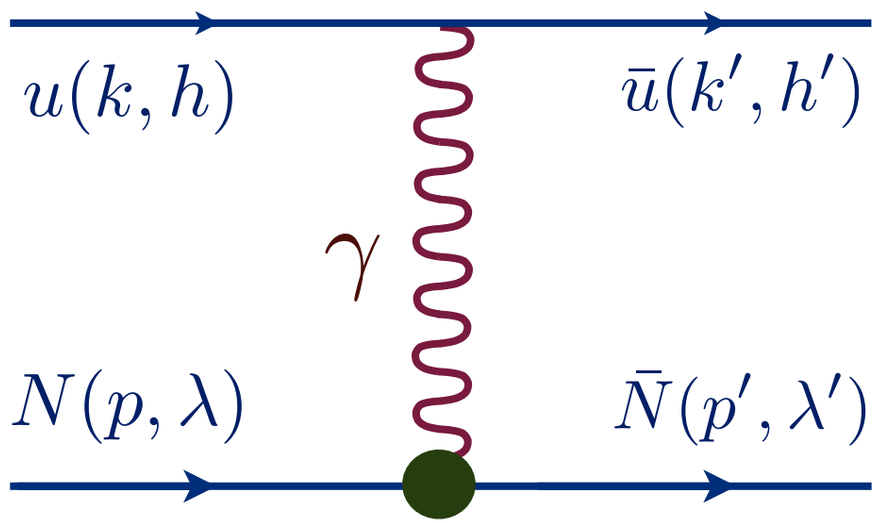
proton size discrepancy

initially 5-6σ

eH, ep vs μH

- hadronic uncertainty is dominant in theory

Tool to explore the proton structure



photon-proton vertex

$$\Gamma^\mu(Q^2) = \gamma^\mu F_D(Q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2M} F_P(Q^2)$$

Dirac and Pauli form factors

lepton energy ω

momentum transfer $Q^2 = -(k-k')^2$

1ɣ amplitude

$$\mathrm{T} = \frac{e^2}{Q^2}\left(\bar{u}\left(k',h'\right)\gamma_\mu u\left(k,h\right)\right)\cdot\left(\bar{N}\left(p',\lambda'\right)\Gamma^\mu(Q^2)N\left(p,\lambda\right)\right)$$

Form factors measurement

Sachs electric and magnetic form factors

$$G_E = F_D - \tau F_P \qquad G_M = F_D + F_P$$

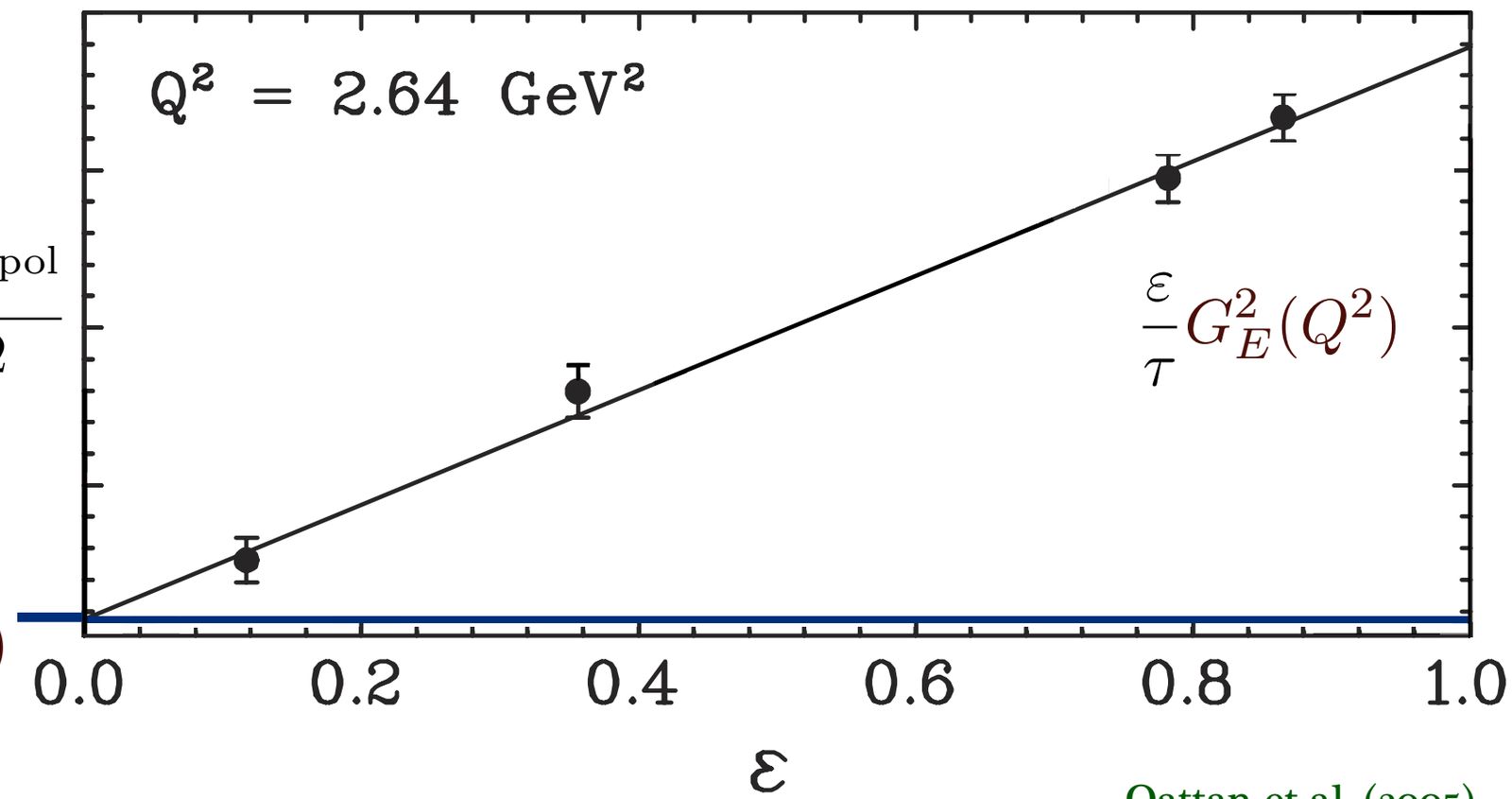
Rosenbluth separation

$$\frac{\mathrm{d}\sigma^{\mathrm{unpol}}}{\mathrm{d}\Omega} \sim G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2)$$

$$\tau = \frac{Q^2}{4M^2}$$

$\tau \quad \varepsilon$ kinematical variables

$\varepsilon \leftrightarrow \theta_{\mathrm{lab}}$



Qattan et al. (2005)

- Rosenbluth slope is sensitive to corrections beyond 1γ

Form factors measurement

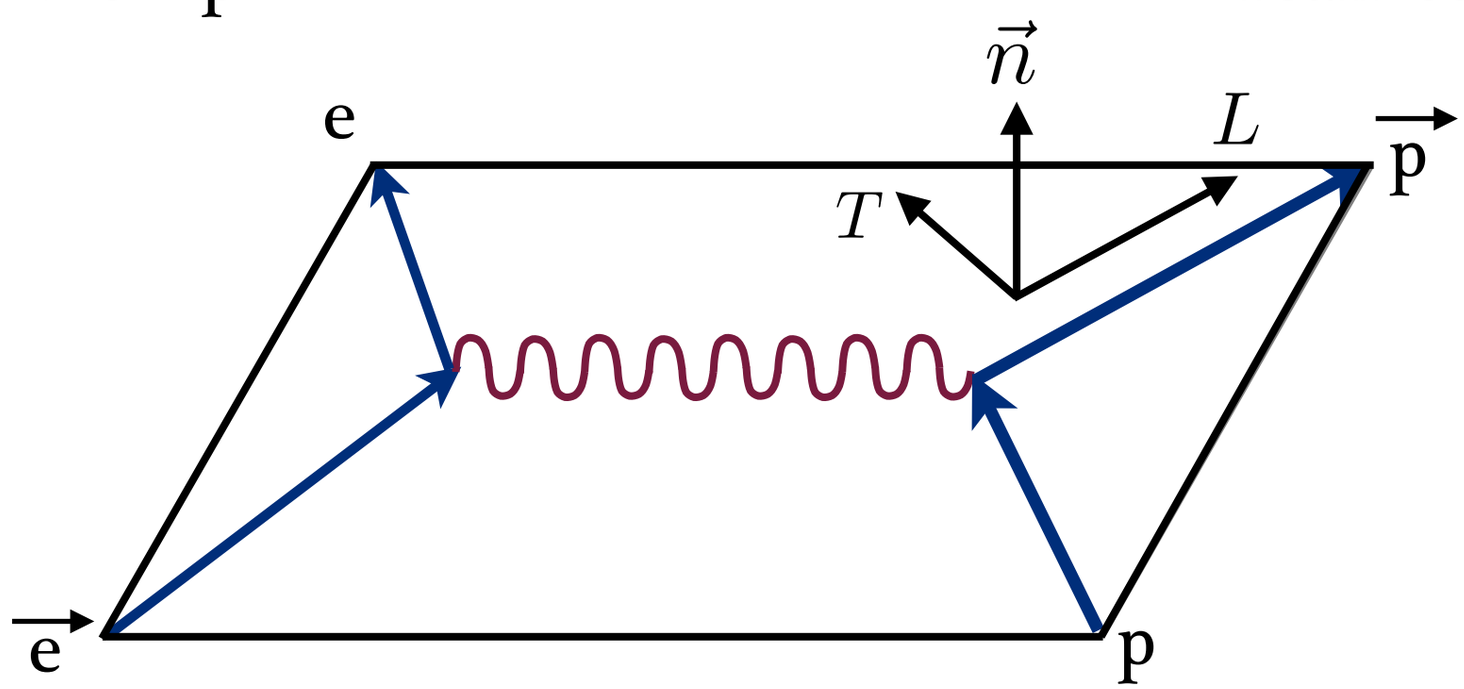
Sachs electric and magnetic form factors

$$G_E = F_D - \tau F_P \qquad G_M = F_D + F_P$$

polarization transfer

$\vec{e}$ + p -> e + $\vec{p}$

realized in 2000



$$\mathrm{P_T} \sim G_E(Q^2)G_M(Q^2)$$

$$\mathrm{P_L} \sim G_M^2(Q^2)$$

$$\frac{\mathrm{P_T}}{\mathrm{P_L}} \sim \frac{G_E(Q^2)}{G_M(Q^2)}$$

Proton form factors puzzle

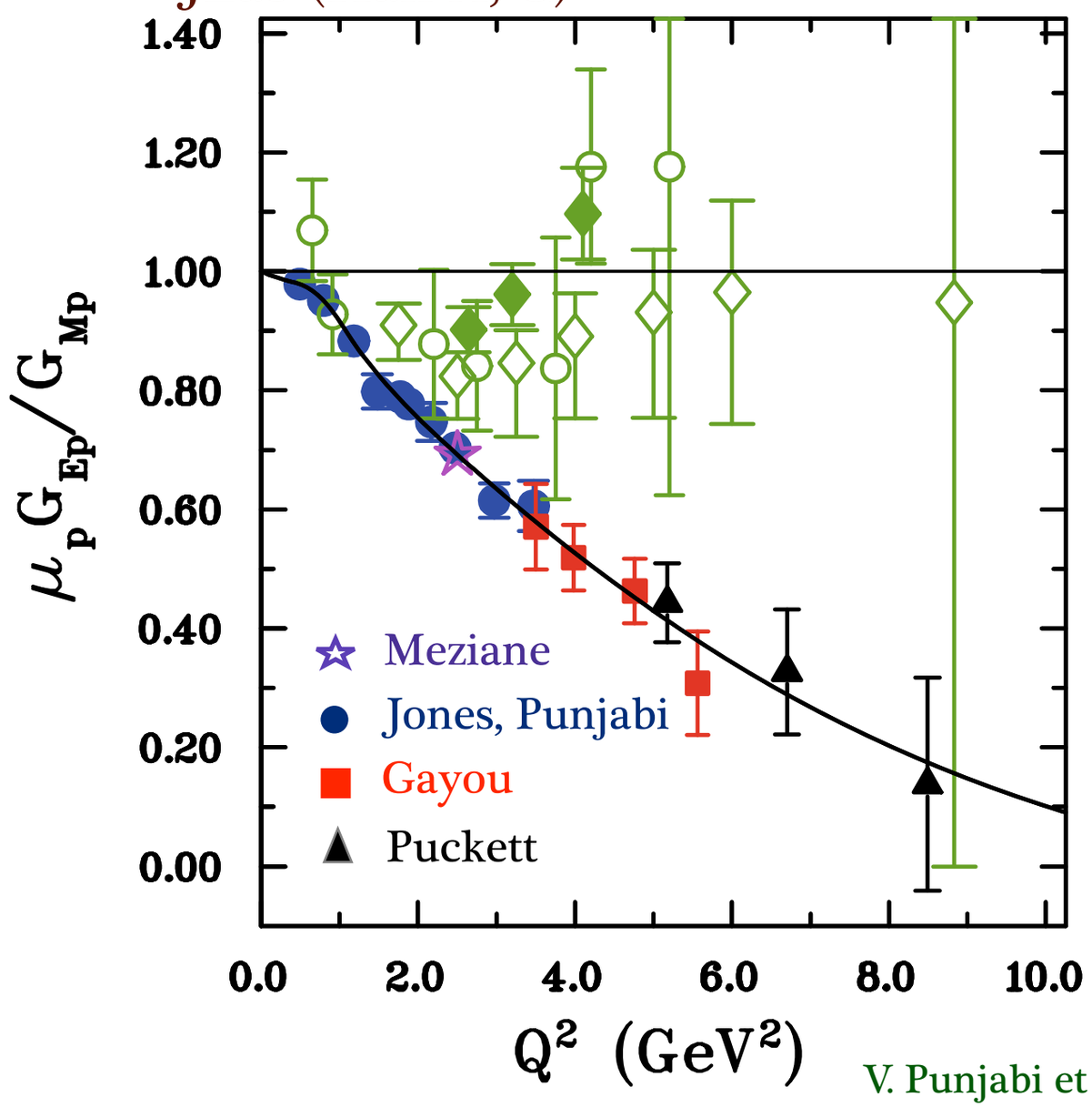
Polarization transfer

JLab (Hall A, C)

vs.

Rosenbluth separation

SLAC, JLab (Hall A, C)



V. Punjabi et al. (2015)

Proton form factors puzzle

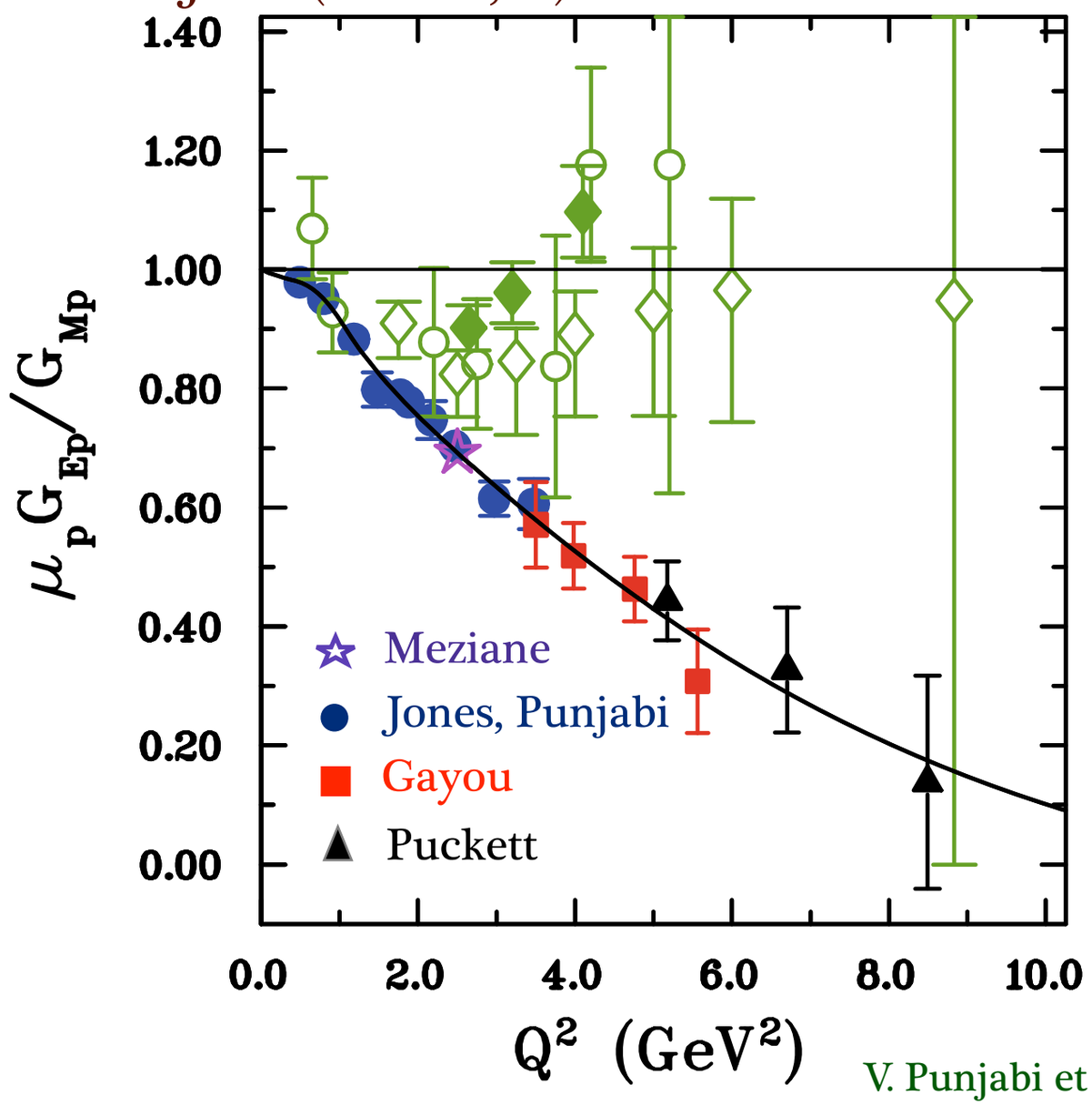
Polarization transfer

JLab (Hall A, C)

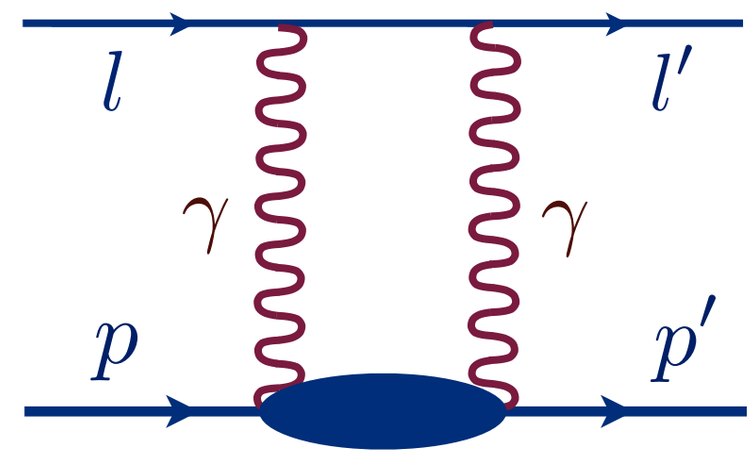
vs.

Rosenbluth separation

SLAC, JLab (Hall A, C)



V. Punjabi et al. (2015)



possible explanation
two-photon exchange

Proton form factors puzzle

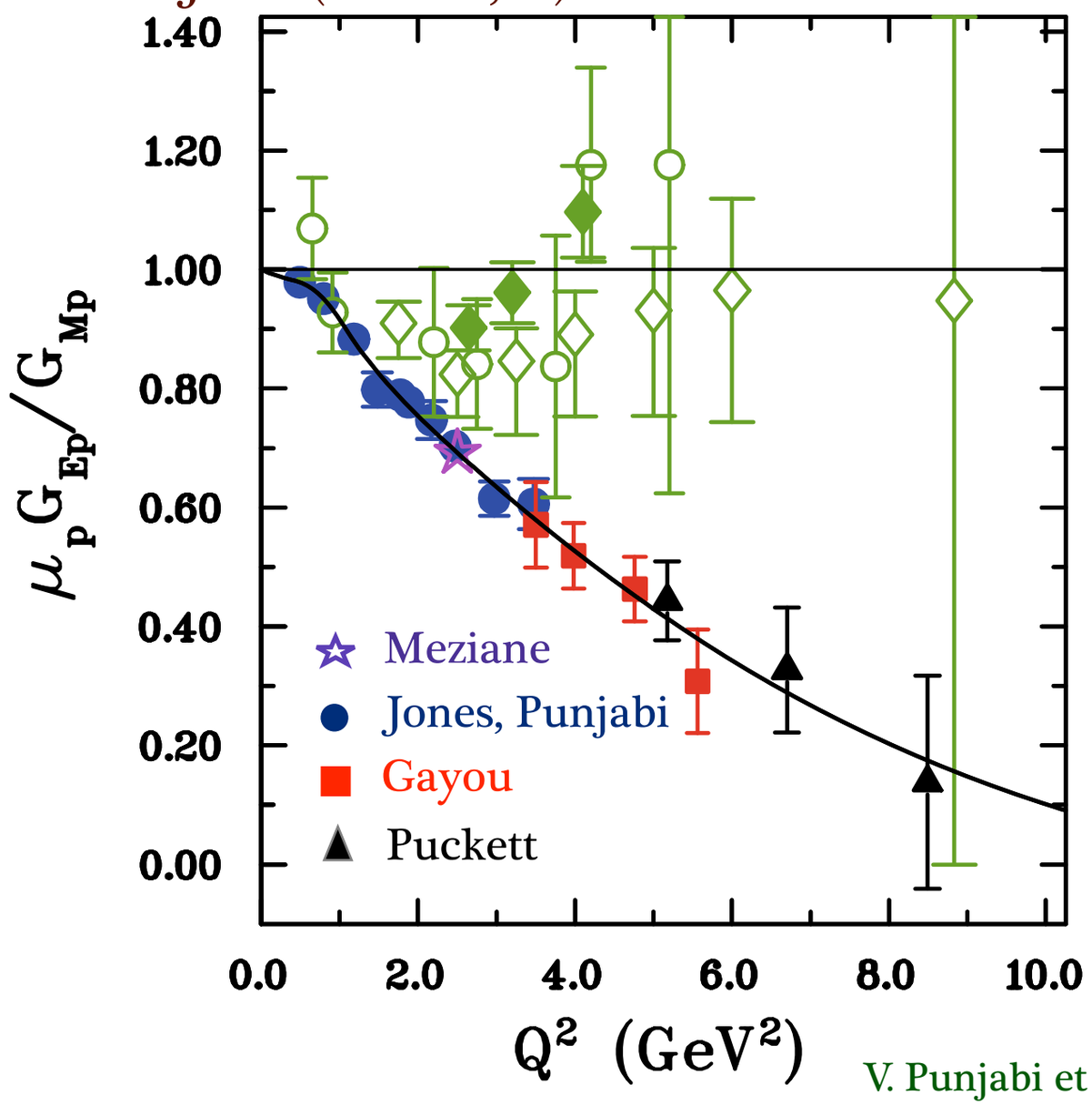
Polarization transfer

JLab (Hall A, C)

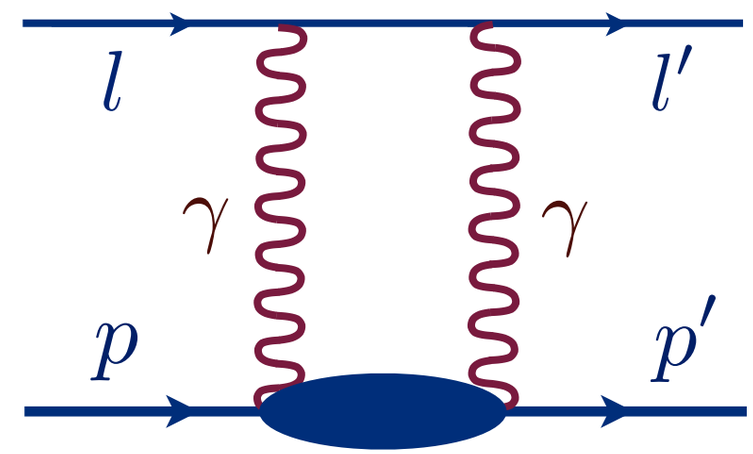
vs.

Rosenbluth separation

SLAC, JLab (Hall A, C)



V. Punjabi et al. (2015)



possible explanation
two-photon exchange

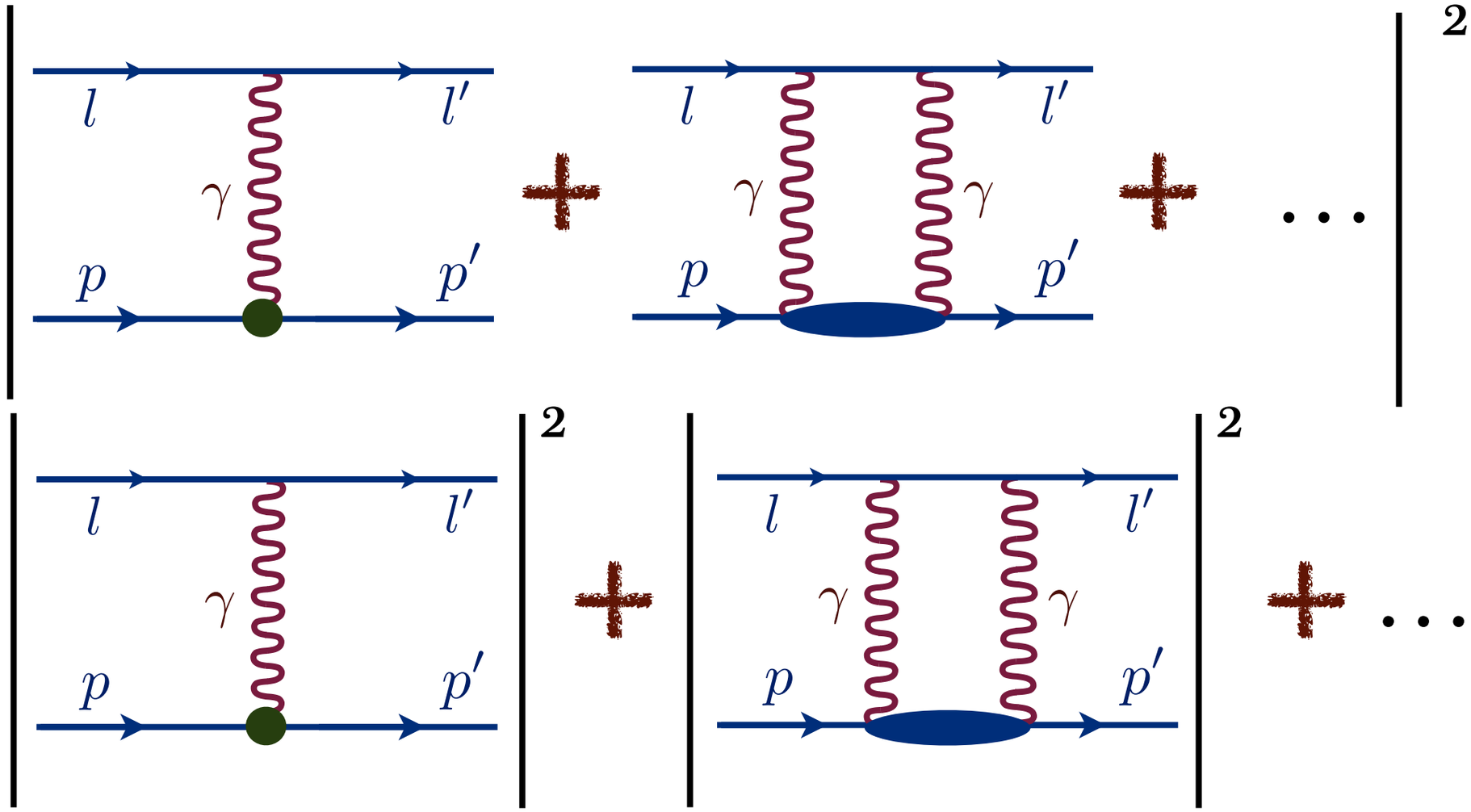
↓

new 2γ measurements

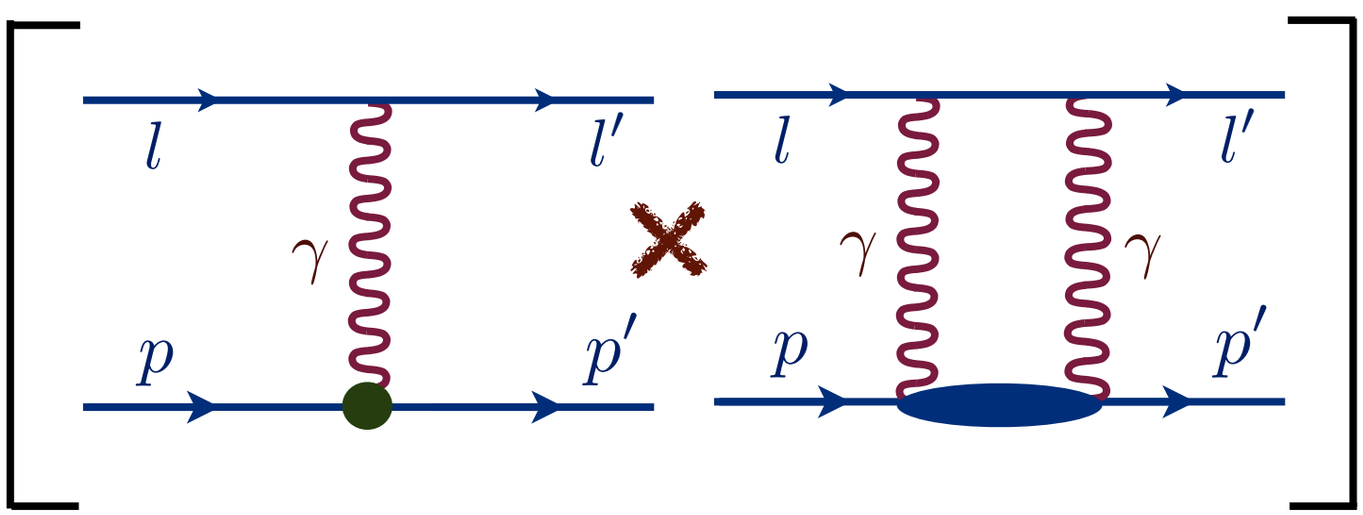
- discrepancy motivates study of 2γ

2γ from experiment

$d\sigma \sim$

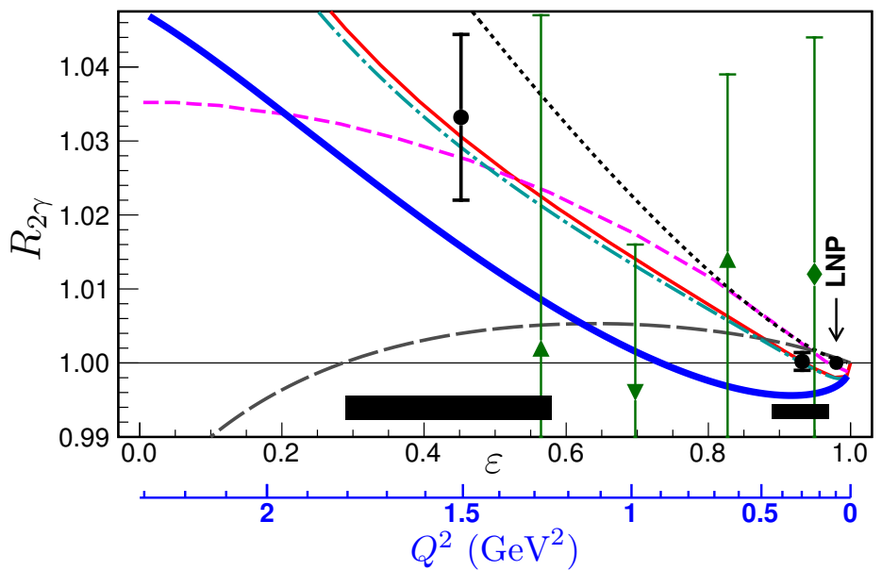


2Re

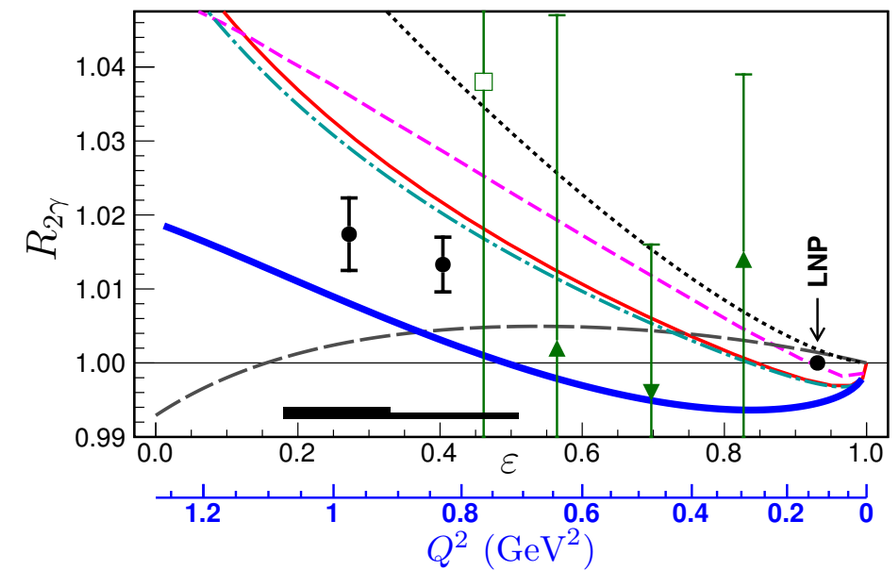


$$\mathrm{R} = \frac{\sigma(e^+p)}{\sigma(e^-p)}$$

VEPP-3 and CLAS data



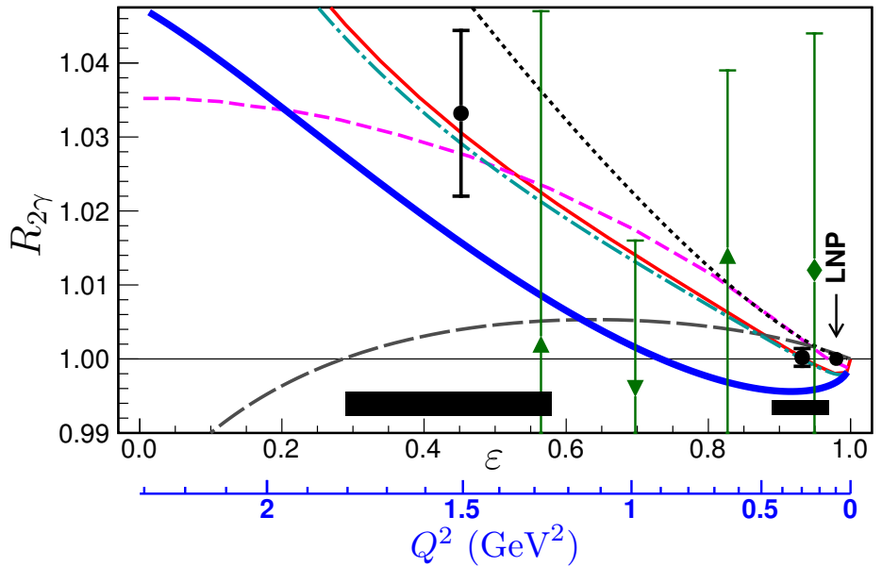
$\omega = 1.6$ GeV



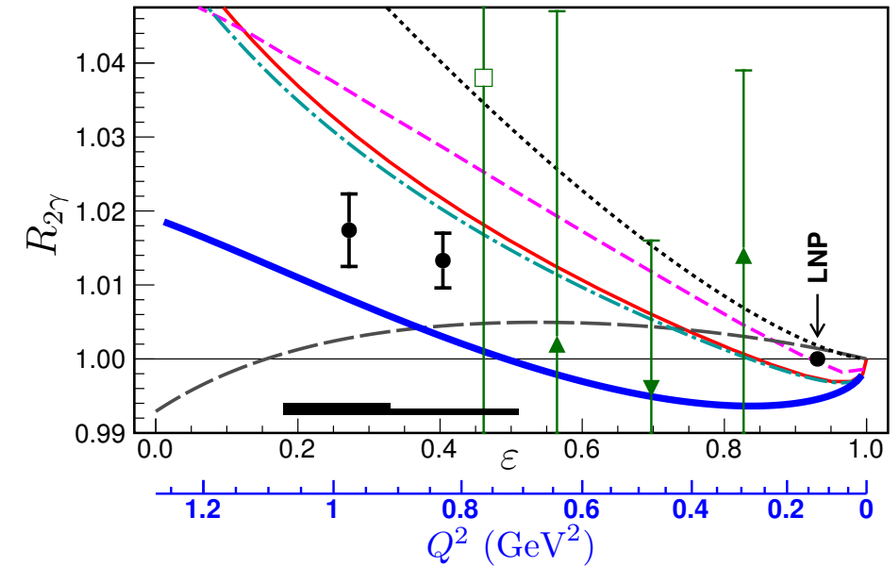
$\omega = 1$ GeV

VEPP-3@Novosibirsk: I. A. Rachel et al. (2015)

VEPP-3 and CLAS data

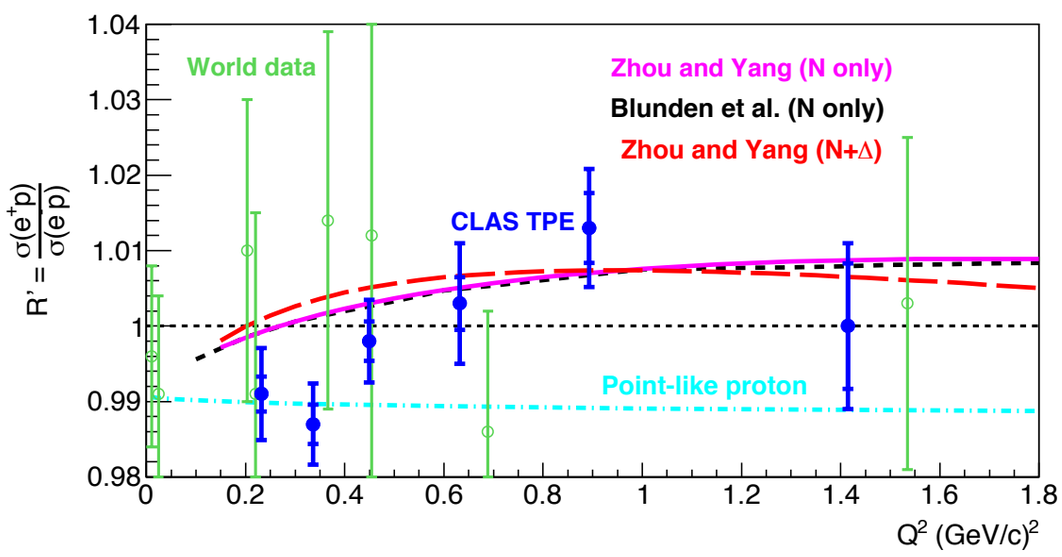


$\omega = 1.6$ GeV

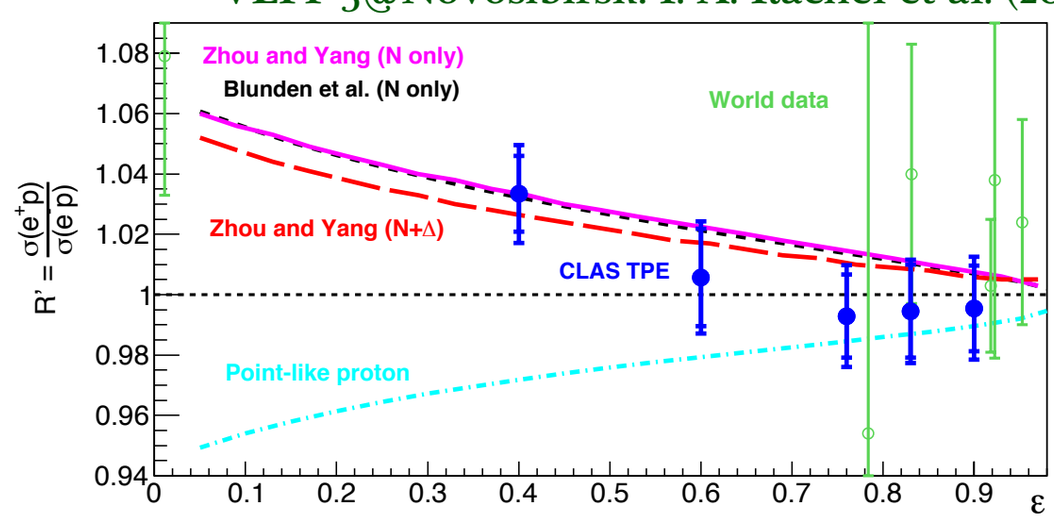


$\omega = 1$ GeV

VEPP-3@Novosibirsk: I. A. Rachel et al. (2015)



$\varepsilon = 0.88$

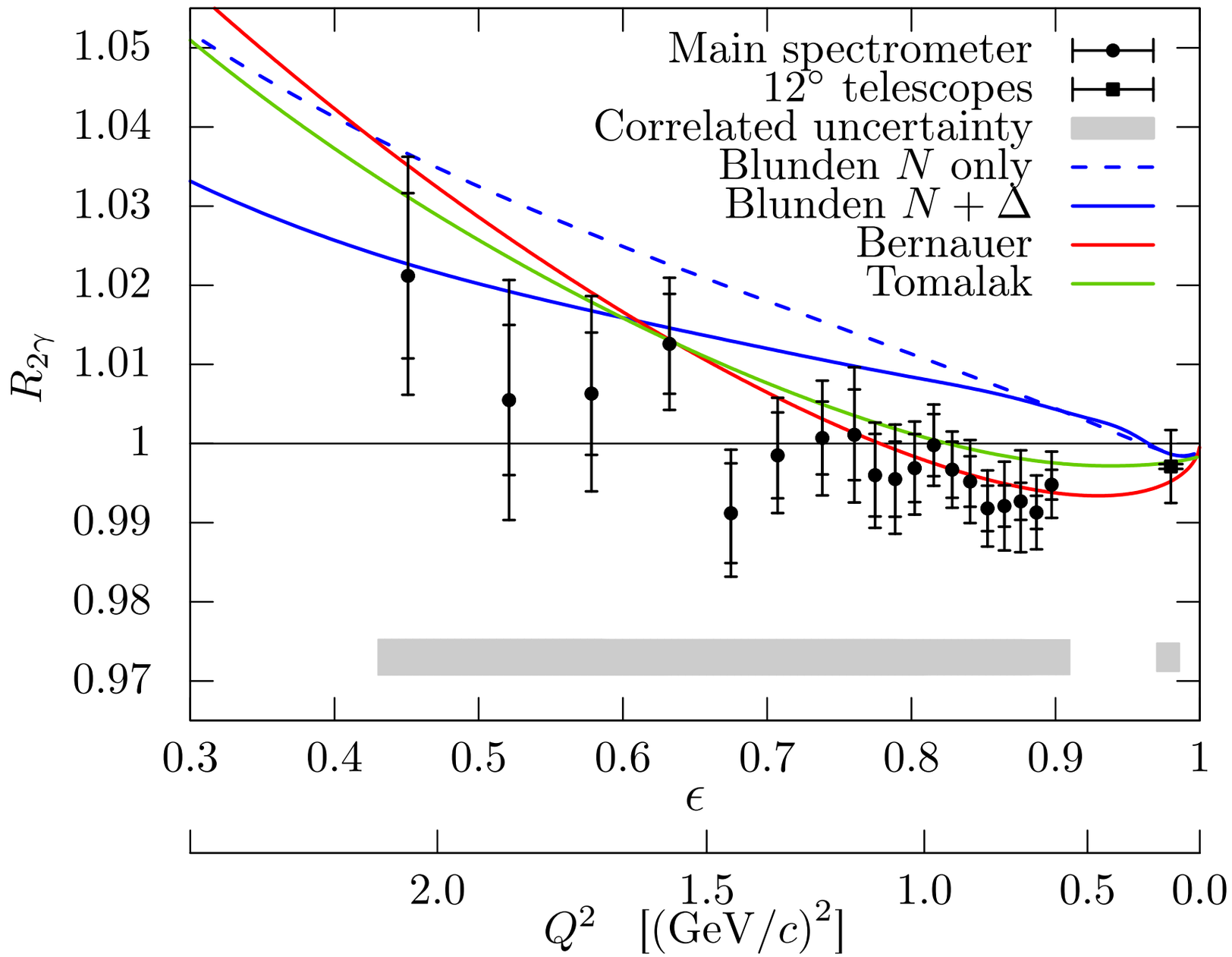


$Q^2 = 1.45$ GeV^2

CLAS@JLab: D. Adikaram et al. (2015)

- 2γ effect within 2-3σ
- reasonable agreement theory vs. experiment

OLYMPUS data



$\omega = 2$ GeV

OLYMPUS@DESY: B. Henderson et al. (2017)

- in agreement with phenomenology
- radiative corrections are important

Proton form factors puzzle

- problem is not completely solved !!!

- interest in high-Q^2 measurements of R

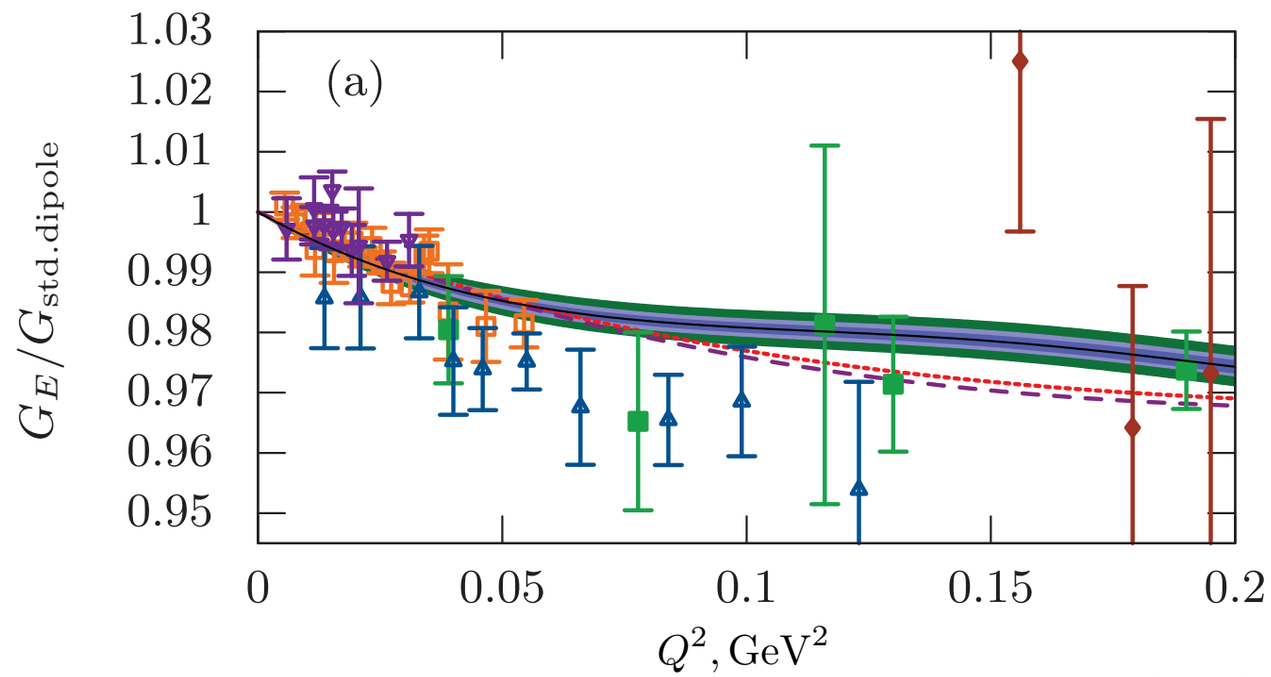
Proton radius

electric charge radius

$$< r_E^2 > \equiv -6 \left. \frac{\mathrm{d}G_E(Q^2)}{\mathrm{d}Q^2} \right|_{Q^2=0}$$

- ep elastic scattering

$$r_E = 0.879 \pm 0.008 \text{ fm}$$



A1@MAMI: J. C. Bernauer et al. (2014)

$0.003 \text{ GeV}^2 < Q^2 < 1 \text{ GeV}^2$

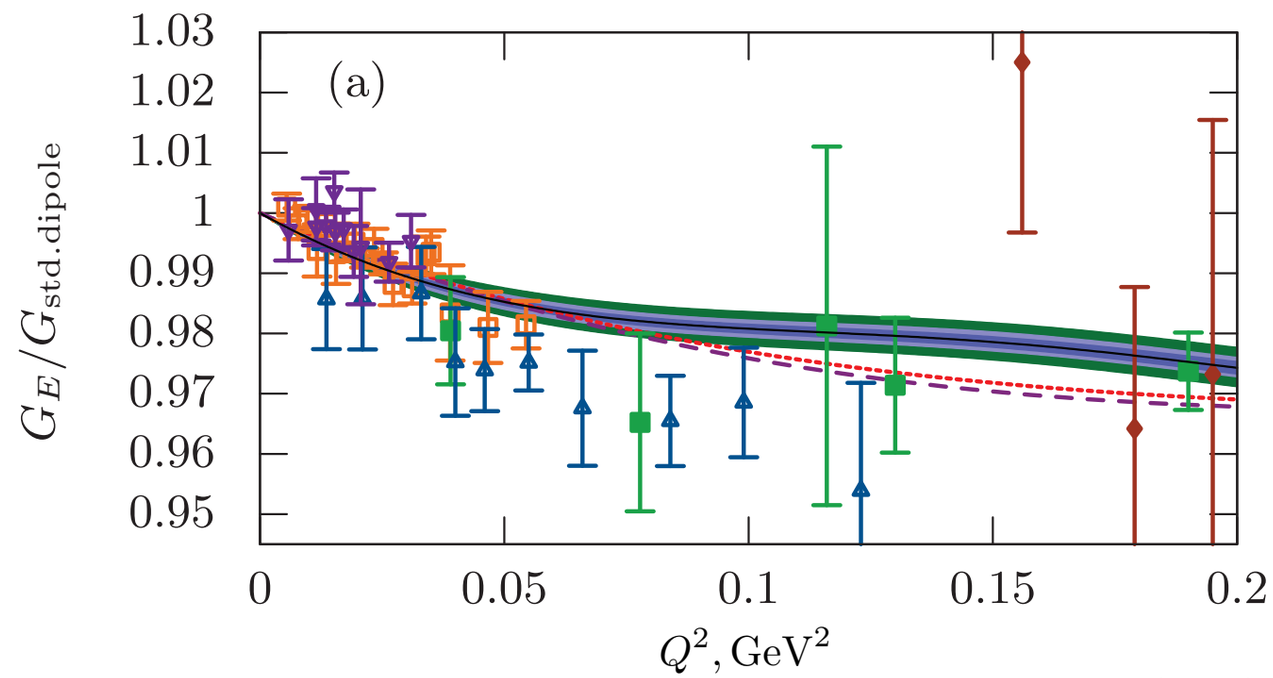
Proton radius

electric charge radius

$$< r_E^2 > \equiv -6 \left. \frac{\mathrm{d}G_E(Q^2)}{\mathrm{d}Q^2} \right|_{Q^2=0}$$

- ep elastic scattering

$r_E = 0.879 \pm 0.008$ fm



A1@MAMI: J. C. Bernauer et al. (2014)

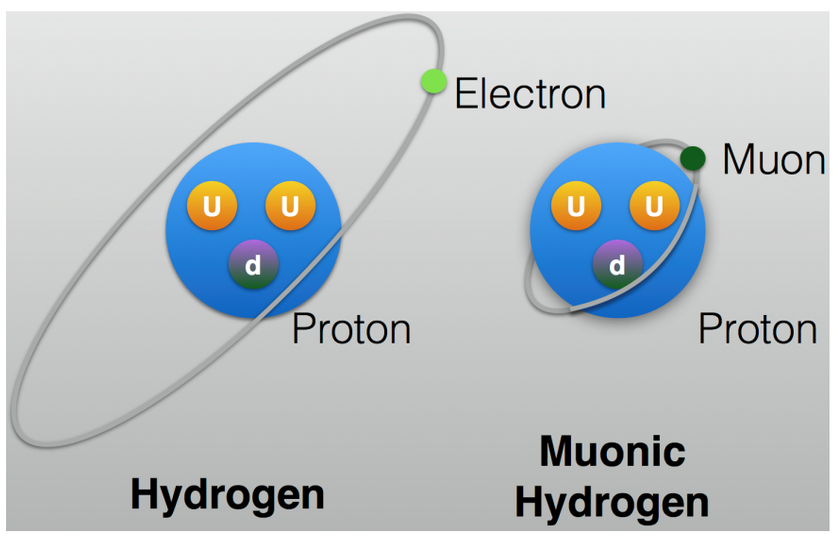
- atomic spectroscopy

$$\Delta E_{\mathrm{nS}} \sim m_r^3 < r_E^2 >$$

H, D spectroscopy

$r_E = 0.8758 \pm 0.0077$ fm

CODATA 2010



μH Lamb shift

$r_E = 0.8409 \pm 0.0004$ fm

CREMA (2010, 2013)

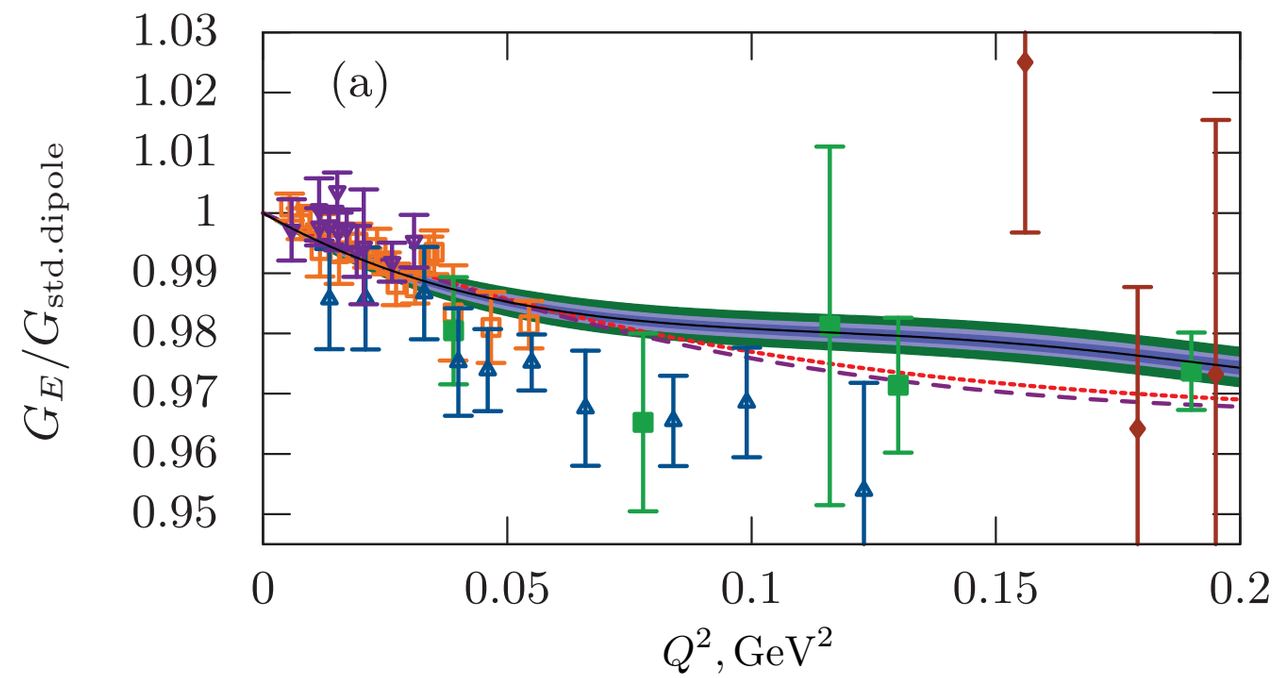
Proton radius puzzle

electric charge radius

$$< r_E^2 > \equiv -6 \left. \frac{\mathrm{d}G_E(Q^2)}{\mathrm{d}Q^2} \right|_{Q^2=0}$$

- ep elastic scattering

$r_E = 0.879 \pm 0.008$ fm



A1@MAMI: J. C. Bernauer et al. (2014)

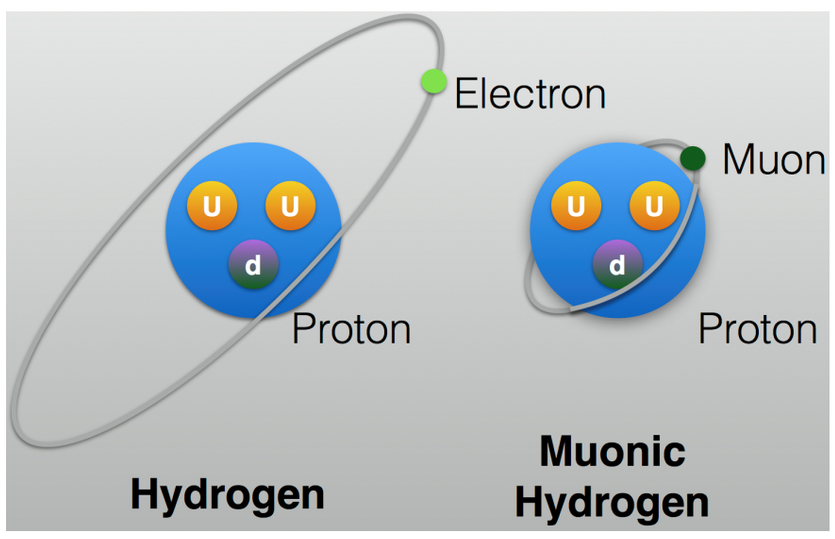
- atomic spectroscopy

$$\Delta E_{\mathrm{nS}} \sim m_r^3 < r_E^2 >$$

H, D spectroscopy

$r_E = 0.8758 \pm 0.0077$ fm

CODATA 2010



μH Lamb shift

$r_E = 0.8409 \pm 0.0004$ fm

CREMA (2010, 2013)

5.6σ difference !

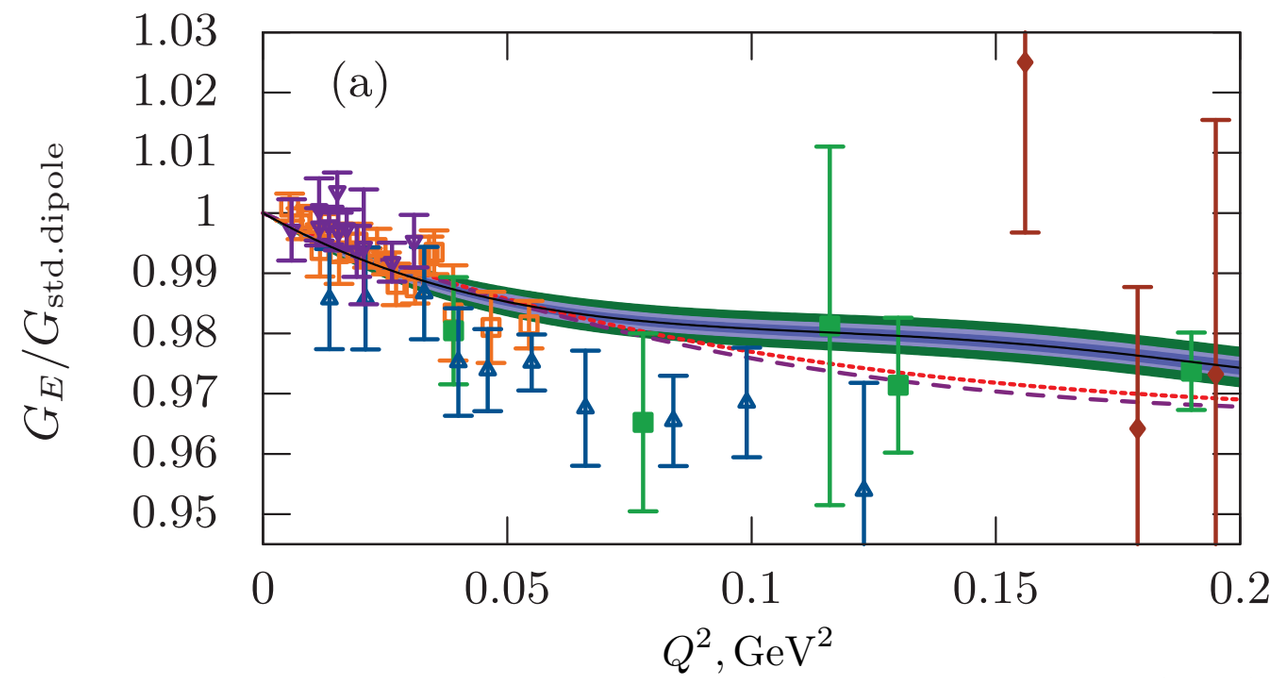
Proton radius puzzle

electric charge radius

$$< r_E^2 > \equiv -6 \left. \frac{\mathrm{d}G_E(Q^2)}{\mathrm{d}Q^2} \right|_{Q^2=0}$$

- ep elastic scattering

$r_E = 0.879 \pm 0.008$ fm ?



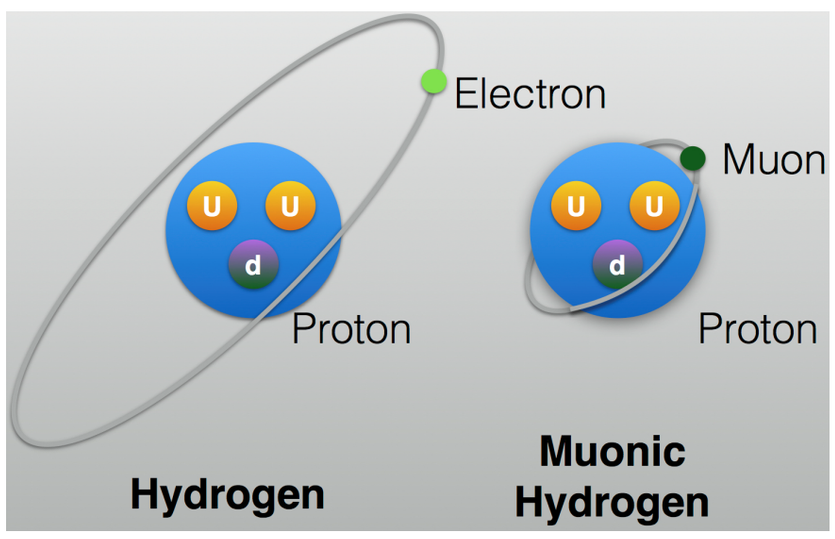
A1@MAMI: J. C. Bernauer et al. (2014)

- atomic spectroscopy $\Delta E_{\mathrm{nS}} \sim m_r^3 < r_E^2 >$

H, D spectroscopy

$r_E = 0.8758 \pm 0.0077$ fm

CODATA 2010



μH Lamb shift

$r_E = 0.8409 \pm 0.0004$ fm

CREMA (2010, 2013)

5.6σ difference !

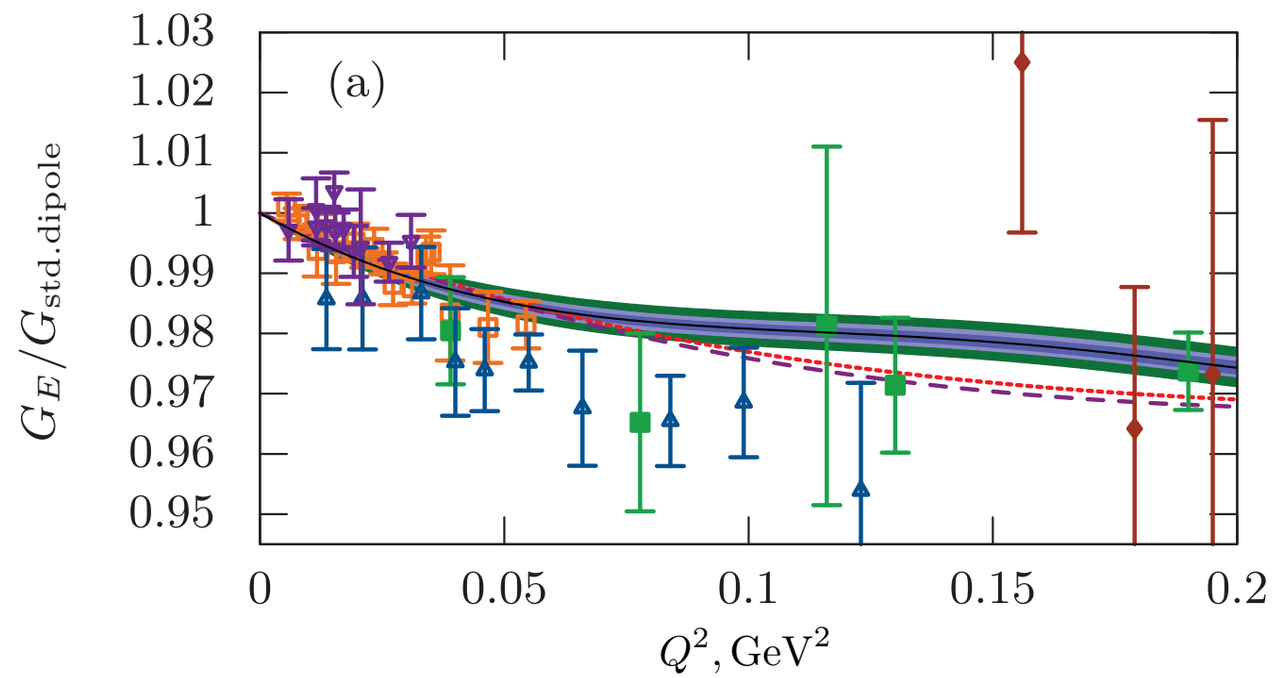
Proton radius puzzle

electric charge radius

$$< r_E^2 > \equiv -6 \left. \frac{\mathrm{d}G_E(Q^2)}{\mathrm{d}Q^2} \right|_{Q^2=0}$$

- ep elastic scattering

$r_E = 0.879 \pm 0.008$ fm ?



A1@MAMI: J. C. Bernauer et al. (2014)

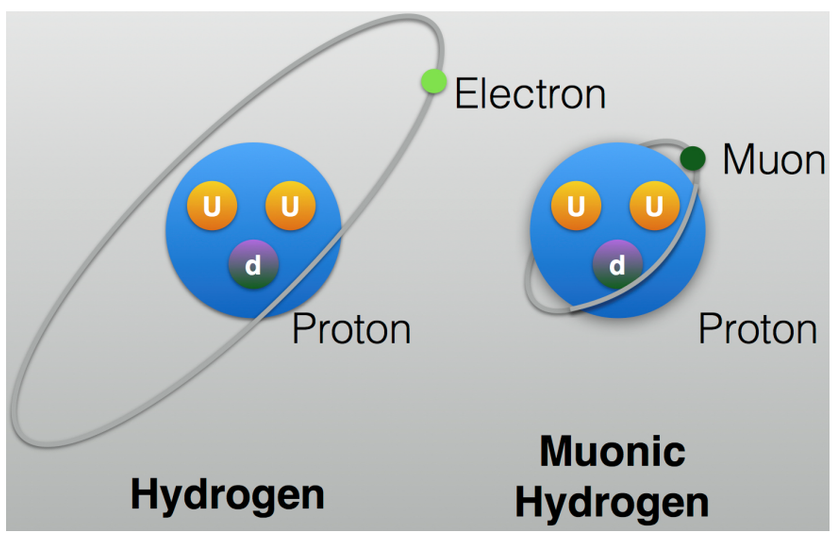
- atomic spectroscopy

$$\Delta E_{\mathrm{nS}} \sim m_r^3 < r_E^2 >$$

H, D spectroscopy

$r_E = 0.8758 \pm 0.0077$ fm

CODATA 2010



μH Lamb shift

$r_E = 0.8409 \pm 0.0004$ fm

CREMA (2010, 2013)

eH 2S-4P (Garching, 2017)

5.6σ difference !

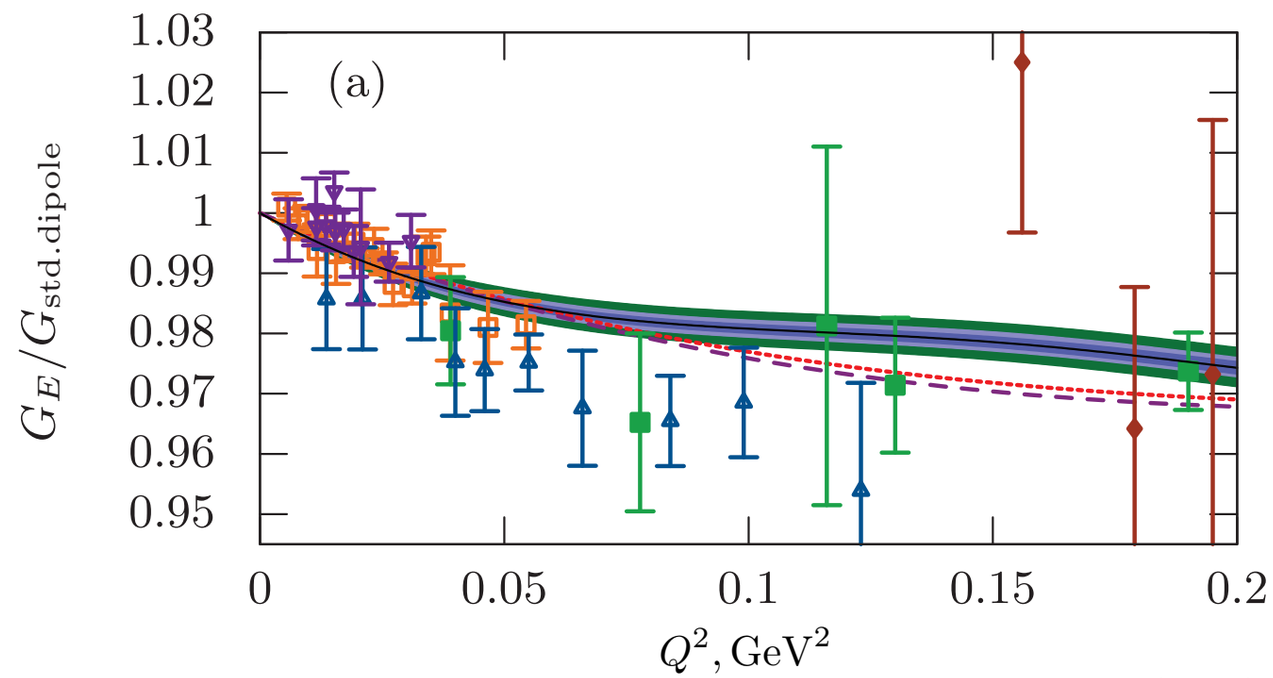
Proton radius puzzle

electric charge radius

$$< r_E^2 > \equiv -6 \left. \frac{\mathrm{d}G_E(Q^2)}{\mathrm{d}Q^2} \right|_{Q^2=0}$$

- ep elastic scattering

$r_E = 0.879 \pm 0.008$ fm ?



A1@MAMI: J. C. Bernauer et al. (2014)

- atomic spectroscopy

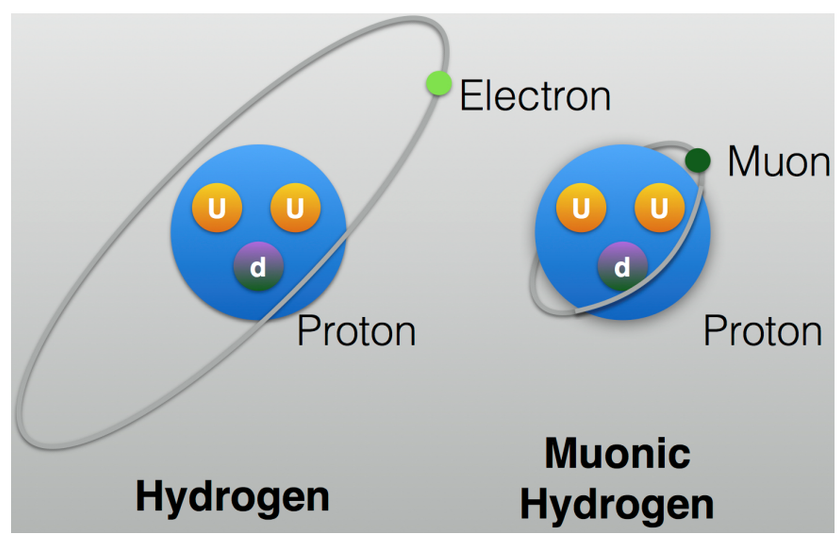
$$\Delta E_{\mathrm{nS}} \sim m_r^3 < r_E^2 >$$

H, D spectroscopy

$r_E = 0.8758 \pm 0.0077$ fm

CODATA 2010

eH 1S-3S (LKB, Paris, 2018)



μH Lamb shift

$r_E = 0.8409 \pm 0.0004$ fm

CREMA (2010, 2013)

eH 2S-4P (Garching, 2017)

5.6σ difference !

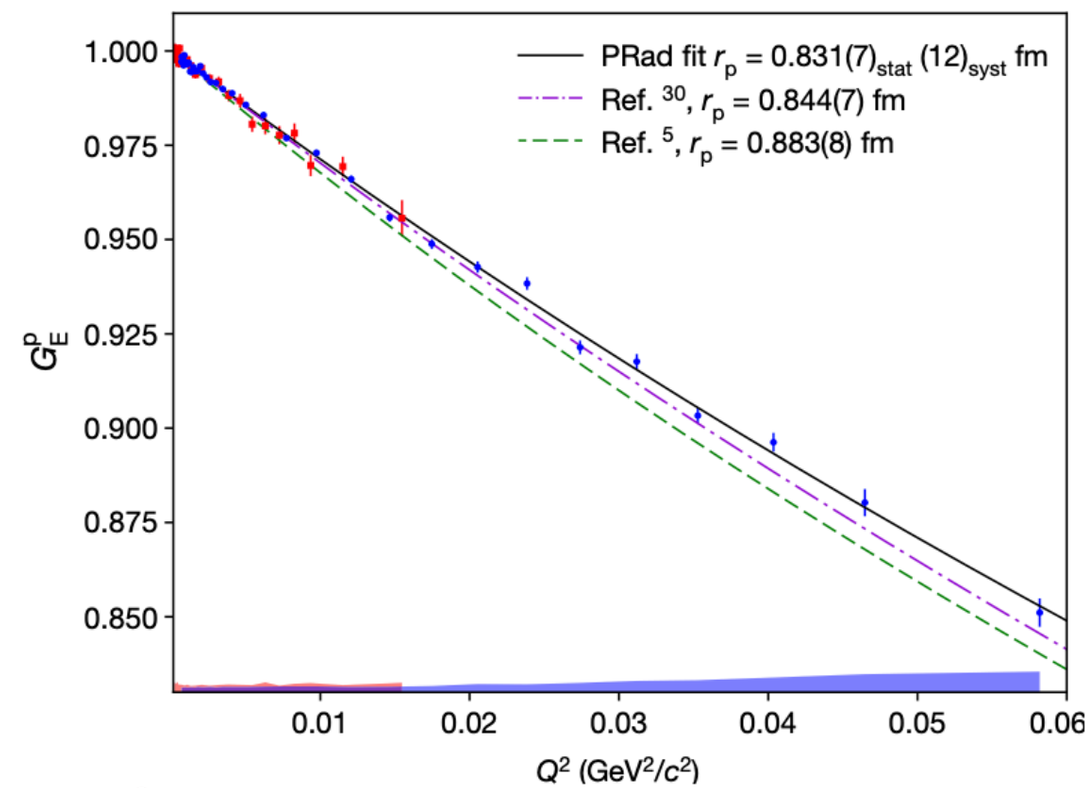
Proton radius puzzle

electric charge radius

$$< r_E^2 > \equiv -6 \left. \frac{\mathrm{d}G_E(Q^2)}{\mathrm{d}Q^2} \right|_{Q^2=0}$$

- ep elastic scattering

$r_E = 0.879 \pm 0.008$ fm ?



PRad (JLab, 2019)

- atomic spectroscopy

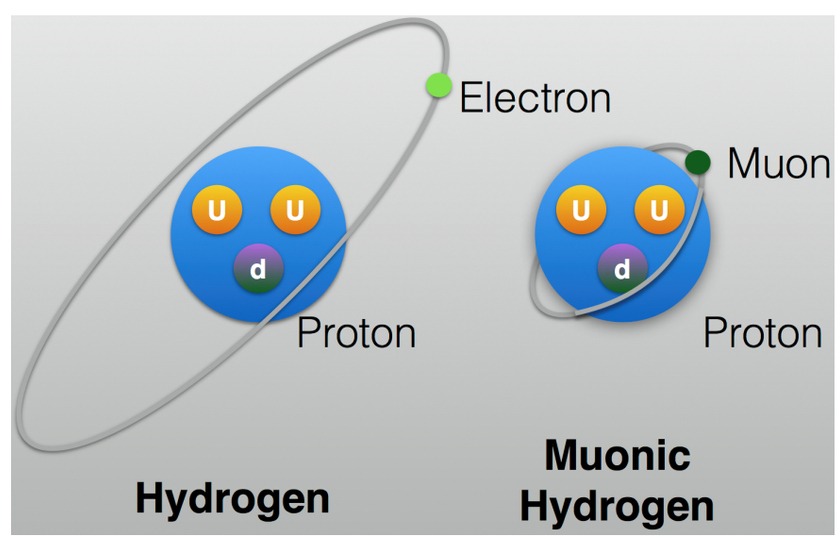
$$\Delta E_{\mathrm{nS}} \sim m_r^3 < r_E^2 >$$

H, D spectroscopy

$r_E = 0.8758 \pm 0.0077$ fm

CODATA 2010

eH 1S-3S (LKB, Paris, 2018)



5.6σ difference !

μH, μD Lamb shift

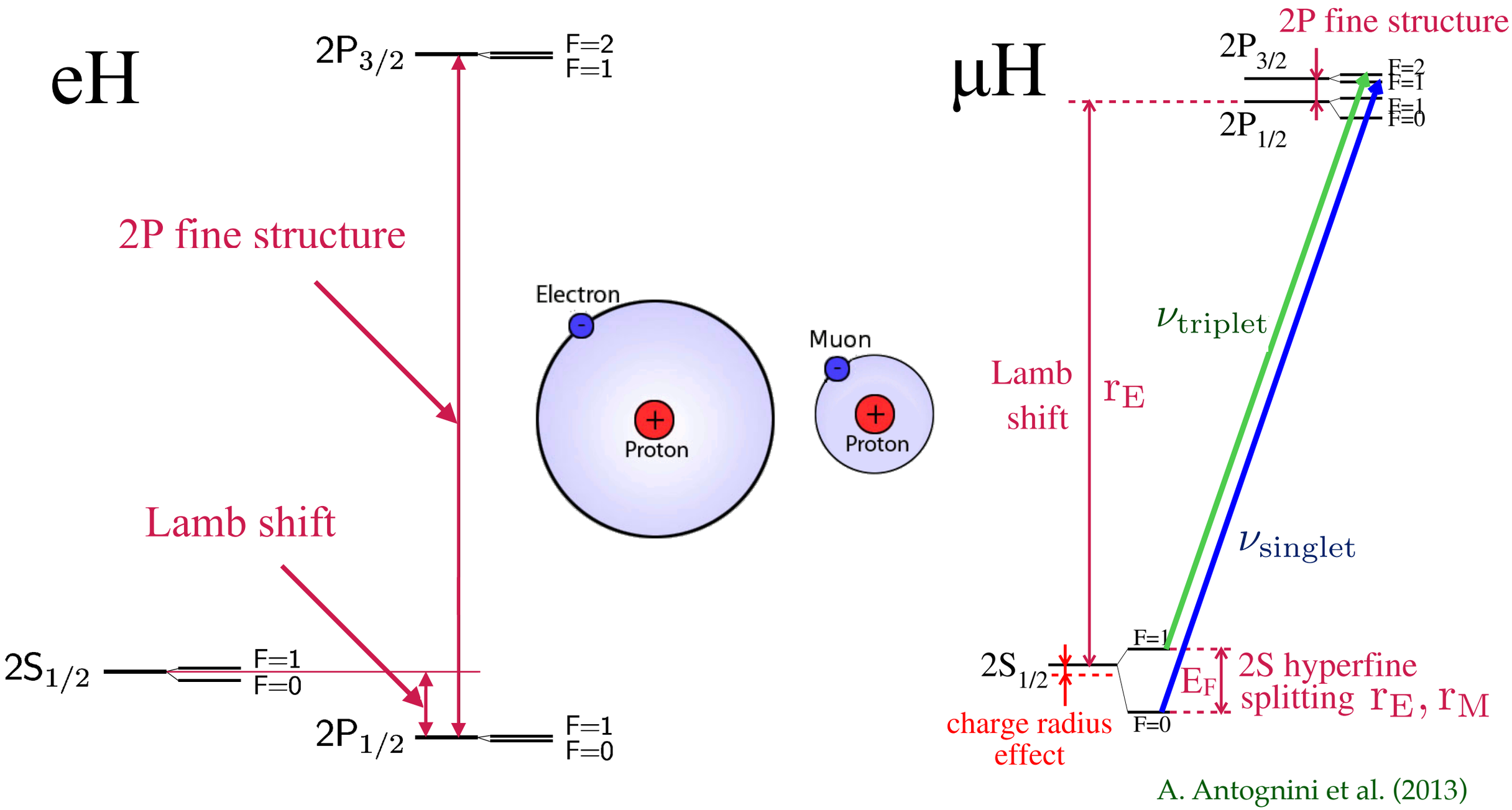
$r_E = 0.8409 \pm 0.0004$ fm

CREMA (2010, 2013)

eH 2S-4P (Garching, 2017)

eH 2S-2P (York U., 2019)

Lamb shift and hyperfine splitting in H

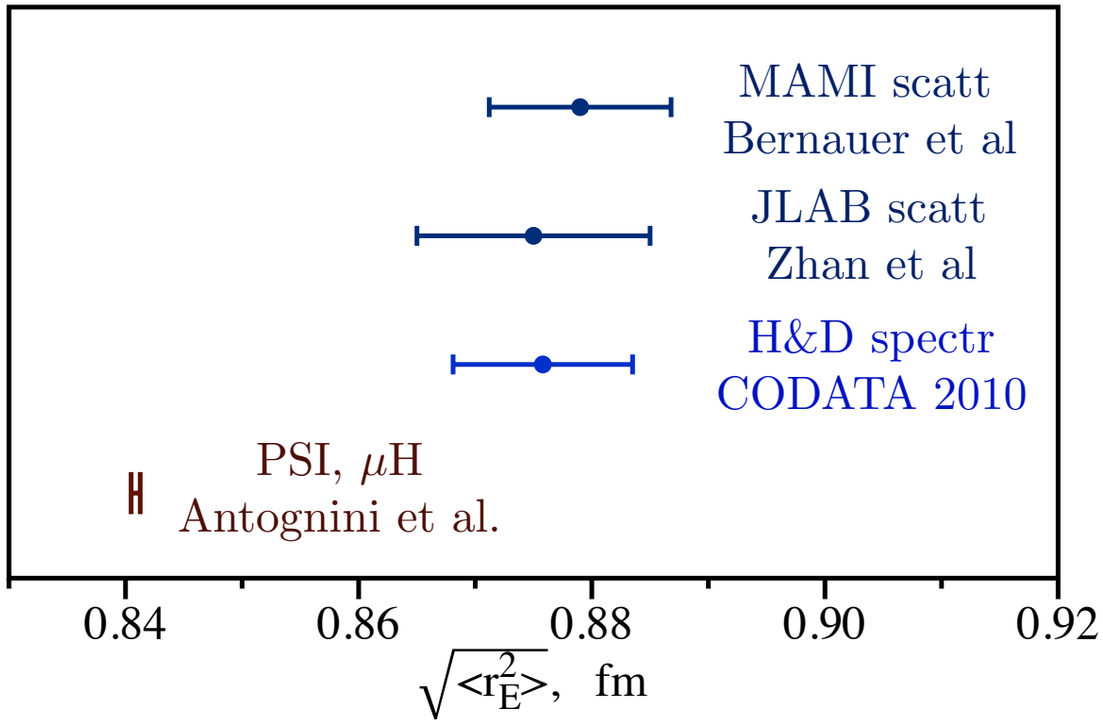


A. Antognini et al. (2013)

- 1S HFS in μH with 1 ppm accuracy at PSI, J-PARC, RIKEN-RAL

R. Pohl et al. (2016)

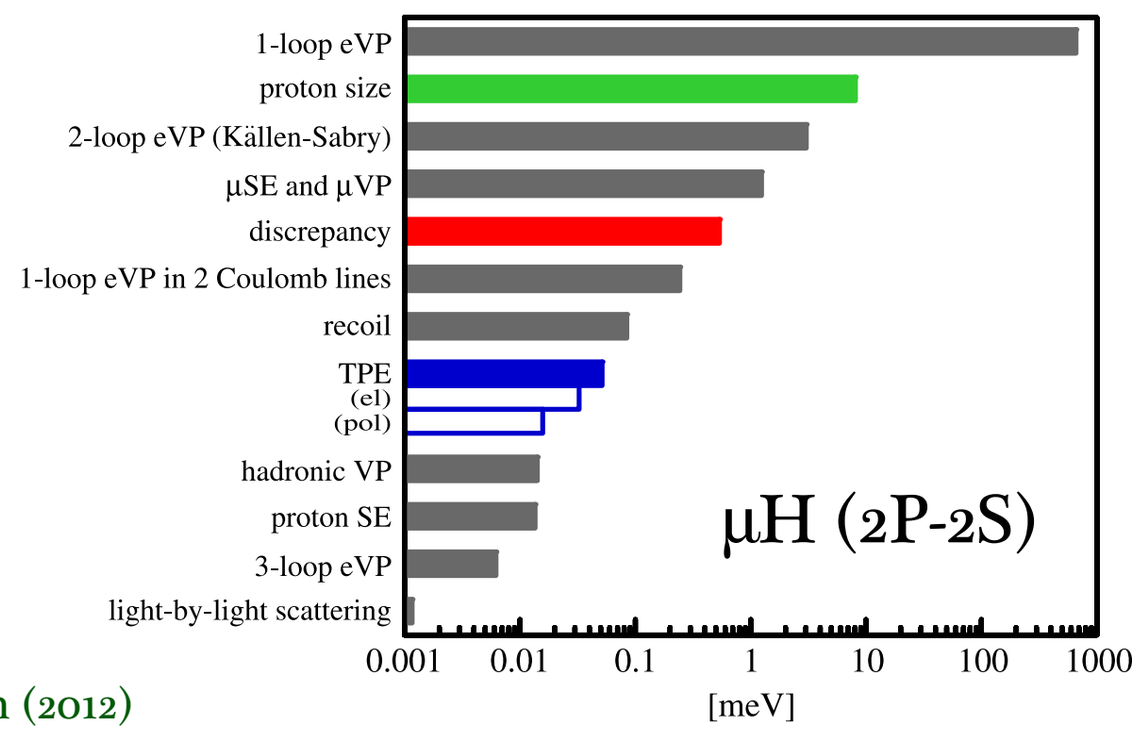
μH Lamb shift and 2γ



2P-2S transition in μH

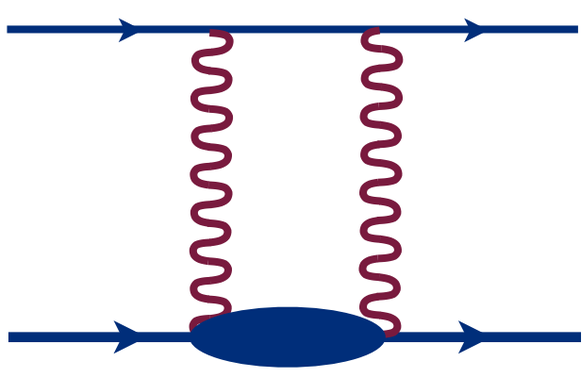
- discrepancy: 310 μeV

- μH uncertainty: 2.5 μeV



A. Antognini et al. (2013)

2γ hadronic correction



$$\Delta E^{2\gamma}_{2P-2S} = 33 \pm 2\ \mu\text{eV}$$

C. Carlson, M. Vanderhaeghen (2011) + M. Birse, J. McGovern (2012)

- important to reduce ambiguities of 2γ

Elastic electron-proton scattering

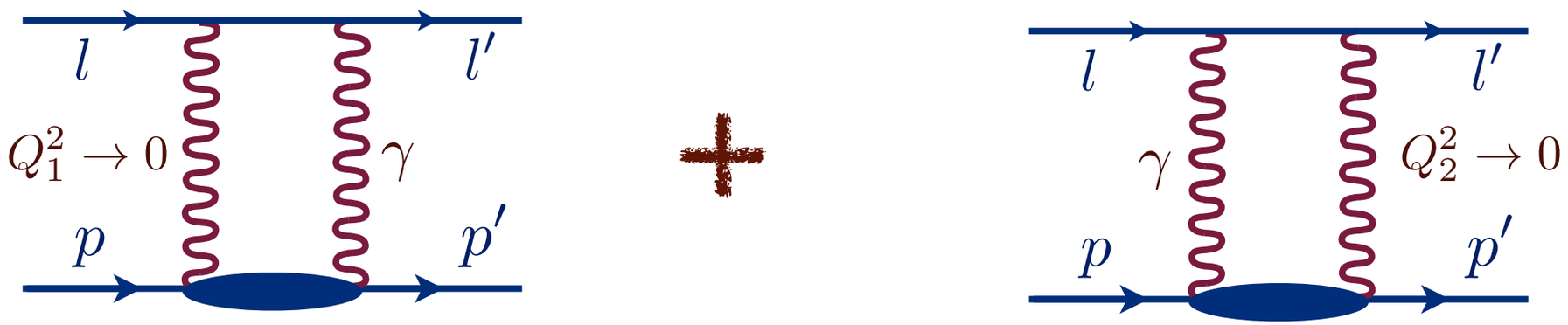
and two-photon exchange

Scattering experiments and 2γ

- 2γ is not among standard radiative corrections

$$\sigma^{\rm exp} \equiv \sigma_{1\gamma}(1 + \delta_{\rm rad} + \delta_{\rm soft} + \delta_{2\gamma})$$

- soft-photon contribution is included



L.C. Maximon and J. A. Tjon (2000)

- hard-photon contribution modelled by Feshbach correction

- charge radius insensitive to 2γ model

- magnetic radius depends on 2γ model

A1@MAMI: J. C. Bernauer et al. (2014)

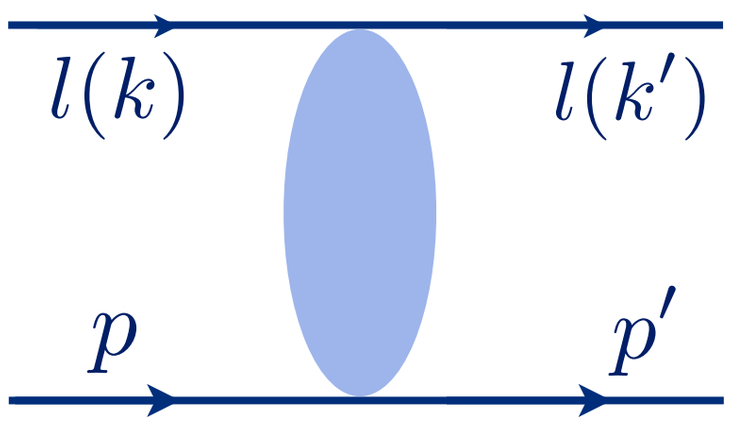
Elastic lepton-proton scattering and 2γ

momentum transfer

$$Q^2 = -(k-k')^2$$

crossing-symmetric variable

$$\nu = \frac{(k, p+p')}{2}$$



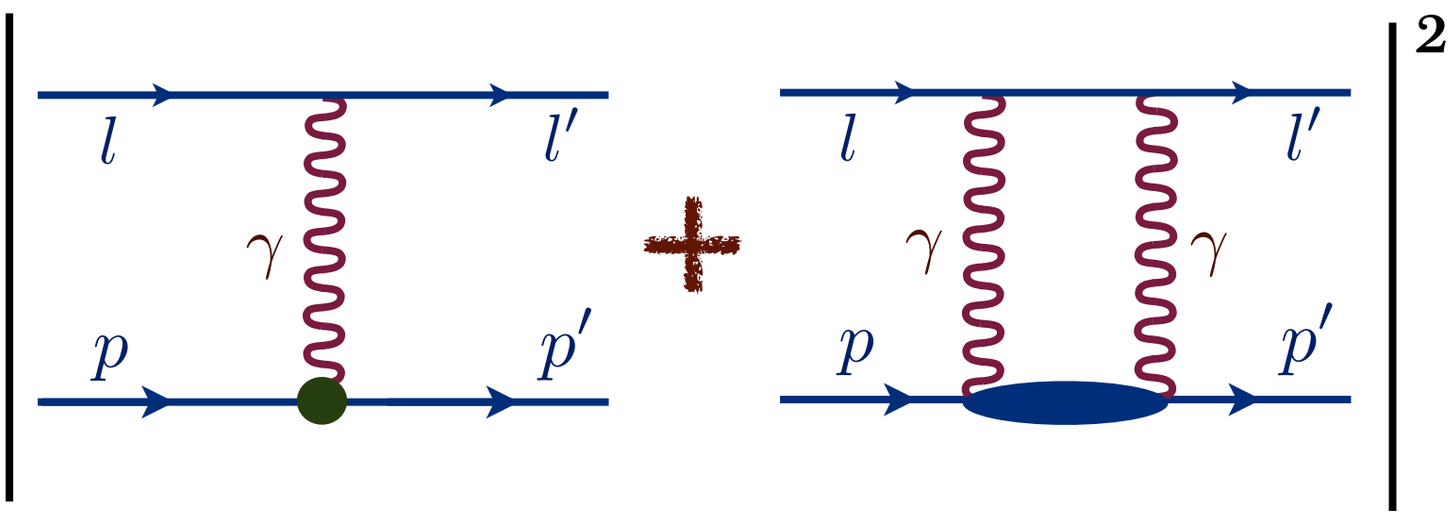
photon polarization parameter

$$\varepsilon$$

forward scattering

$$\varepsilon \to 1$$

- leading 2γ contribution: interference term

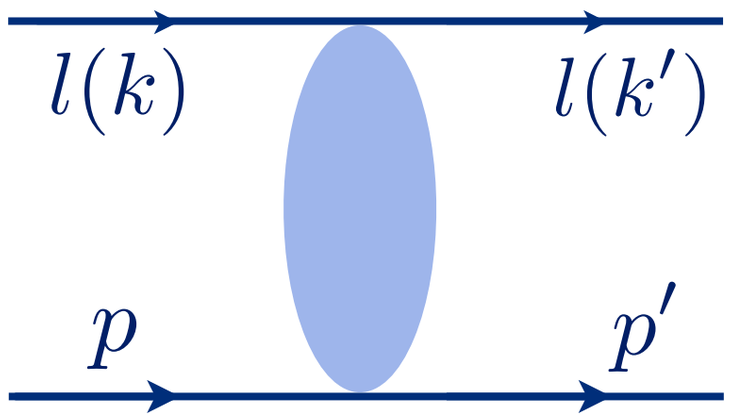


$$\delta_{2\gamma} = \frac{2\sum_{\text{spin}} T^{1\gamma} \Re T^{2\gamma}}{\sum_{\text{spin}} |T^{1\gamma}|^2}$$

- 2γ correction to cross section is given by amplitudes real parts

Elastic lepton-proton scattering and 2γ

$$K = \frac{k + k'}{2}$$



$$P = \frac{p + p'}{2}$$

- electron-proton scattering: 3 structure amplitudes

$$\mathrm{T}^{\mathrm{non-flip}} = \frac{e^2}{Q^2} \bar{l} \gamma_\mu l \cdot \bar{N} \left(\mathcal{G}_M(\nu, Q^2) \gamma^\mu - \mathcal{F}_2(\nu, Q^2) \frac{P^\mu}{M} + \mathcal{F}_3(\nu, Q^2) \frac{\hat{K} P^\mu}{M^2} \right) N$$

P.A.M. Guichon and M. Vanderhaeghen (2003)

- real parts of all 2γ amplitudes extracted at one point Q^2=2.5 GeV^2

J. Guttmann, N. Kivel, M. Meziane and M. Vanderhaeghen (2011)

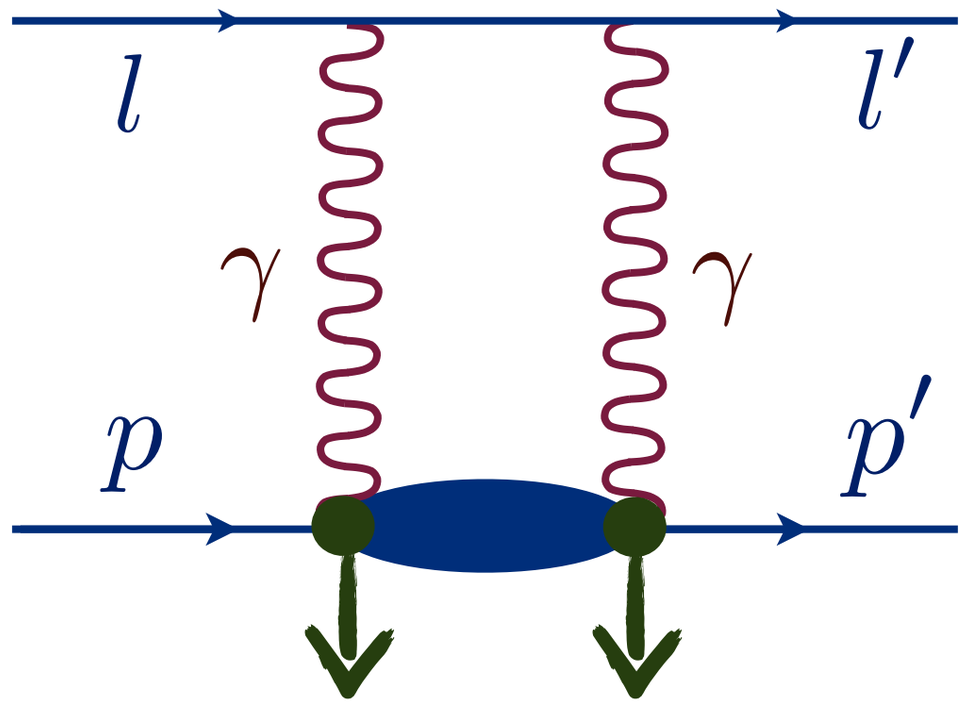
Dm. Borisyuk and A. Kobushkin (2011)

I. A. Qattan (2017-2018)

- 2γ correction to cross section is given by amplitudes real parts

non-forward scattering

at low momentum transfer



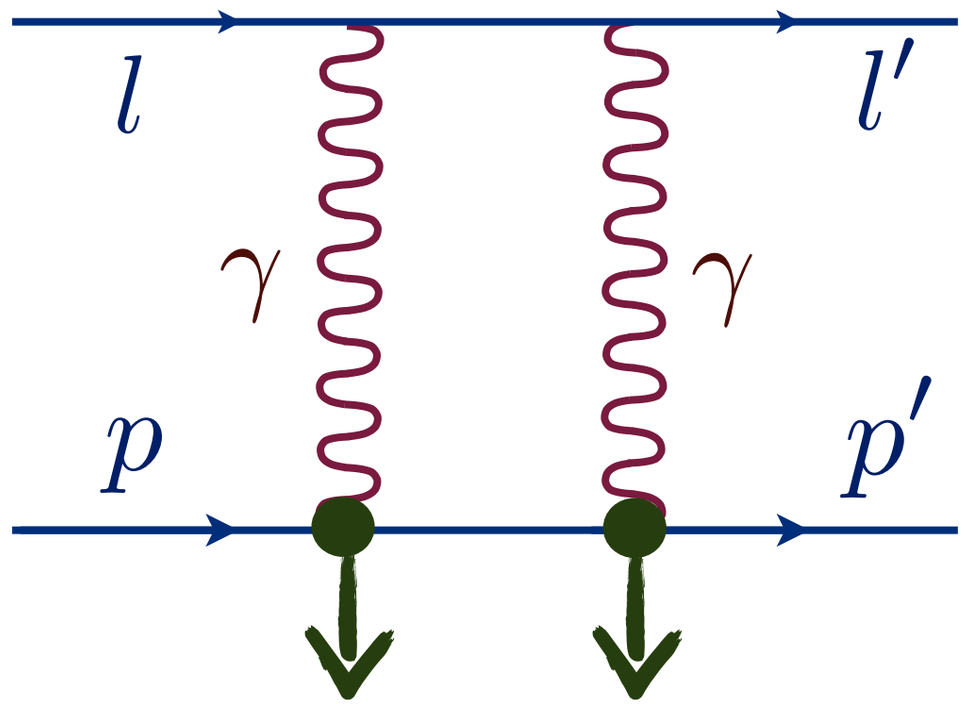
photoproduction vertex or Compton tensor

box diagram

assumption about the vertex

non-forward scattering

proton state



Dirac and Pauli form factors

box diagram

$$\Gamma^\mu(Q^2) = \gamma^\mu F_D(Q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2M} F_P(Q^2)$$

ep scattering: P. G. Blunden, W. Melnitchouk and J. A. Tjon (2003)

assumption about the vertex

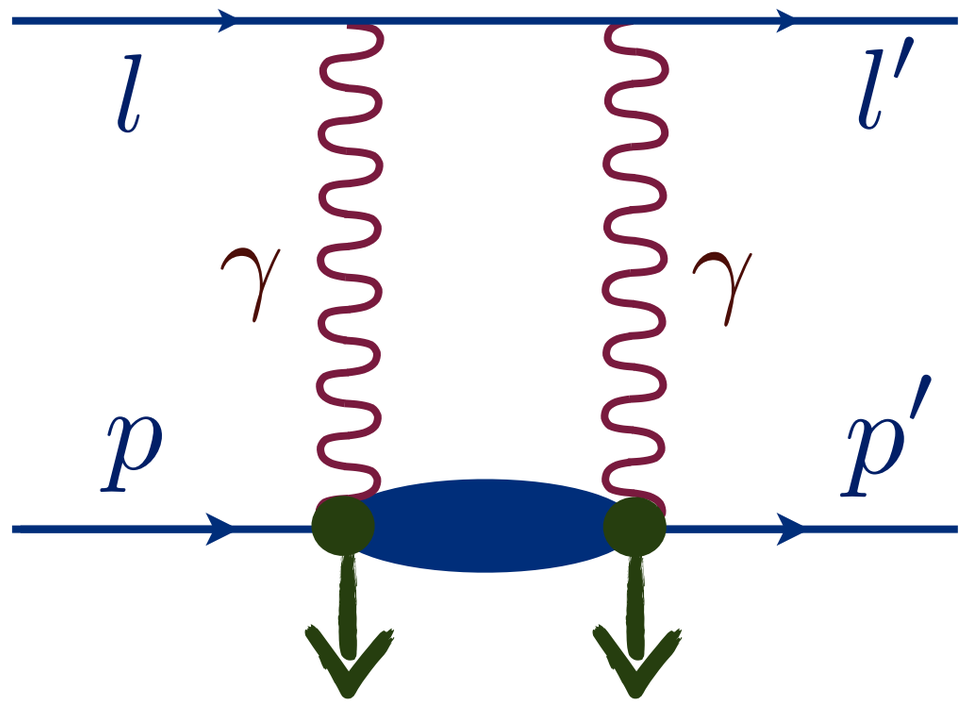
violation of unitarity for resonances !

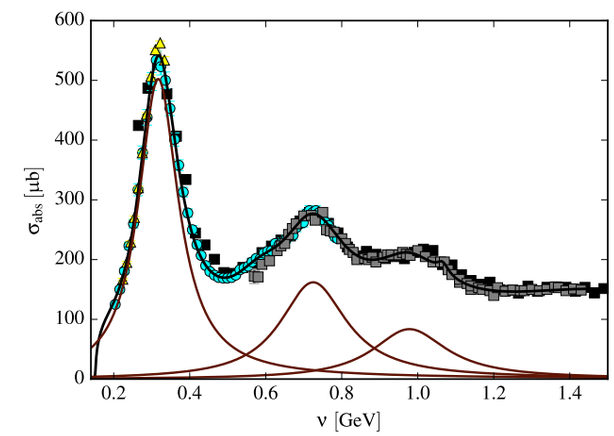
non-forward scattering

inelastic states

works at small

scattering angles





forward doubly-virtual Compton tensor

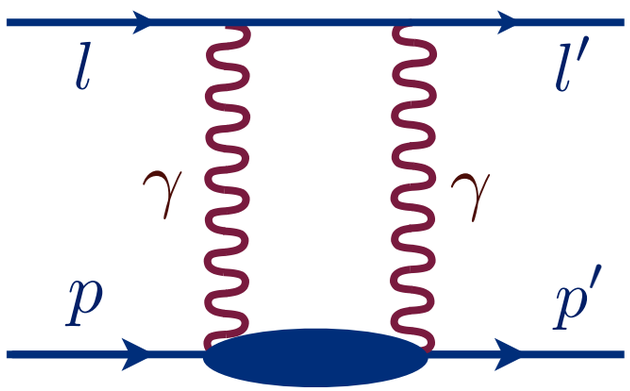
box diagram

unpolarized proton structure

M. E. Christy, P. E. Bosted (2010)

proton + inelastic = total

Low-Q^2 inelastic 2γ correction (e^-p)



- 2γ blob: near-forward virtual Compton scattering

Feshbach inelastic elastic

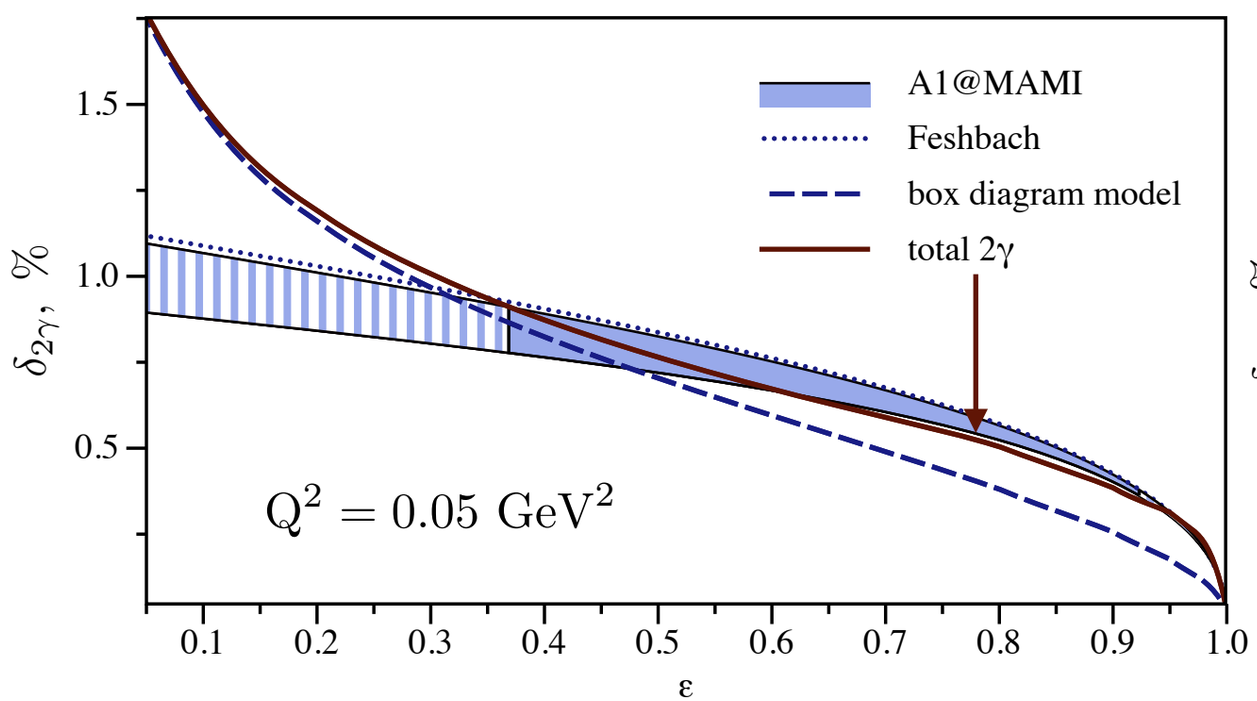
$$\text{ep}: \quad \delta_{2\gamma} \sim a\,\sqrt{Q^2} + b\,Q^2 \ln Q^2 + c\,Q^2 \ln^2 Q^2$$

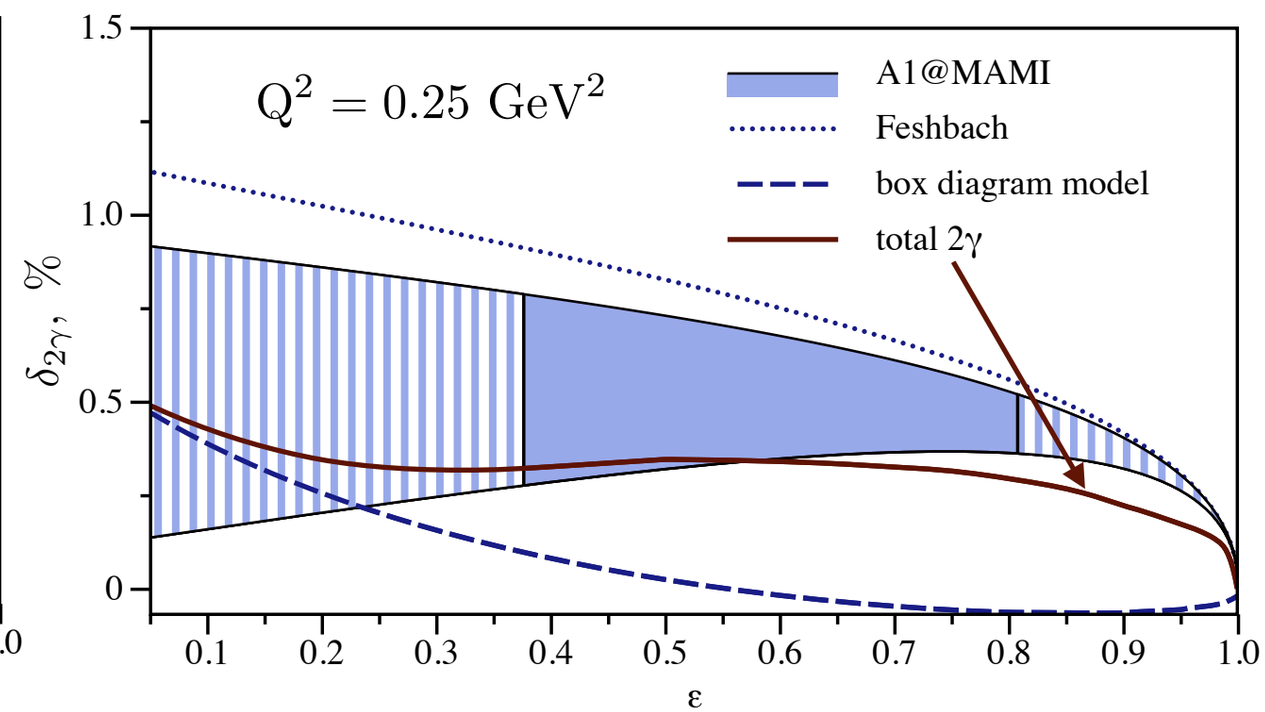
R. W. Brown (1970)

unpolarized proton structure

$$\delta_{2\gamma} = \int \mathrm{d}\nu_\gamma \mathrm{d}Q^2 (w_1(\nu_\gamma, Q^2) \cdot F_1(\nu_\gamma, Q^2) + w_2(\nu_\gamma, Q^2) \cdot F_2(\nu_\gamma, Q^2))$$

M. E. Christy, P. E. Bosted (2010)



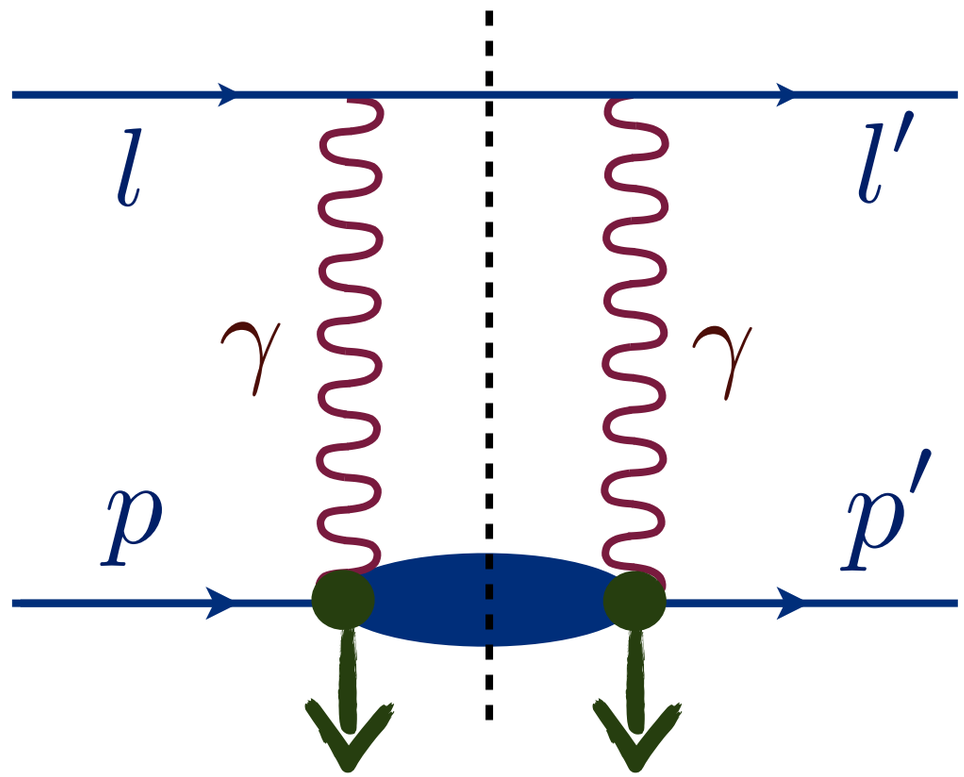


O. T. and M. Vanderhaeghen (2016)

- 2γ at large ε agrees with empirical fit

non-forward scattering

at low momentum transfer



photoproduction vertex or Compton tensor

box diagram

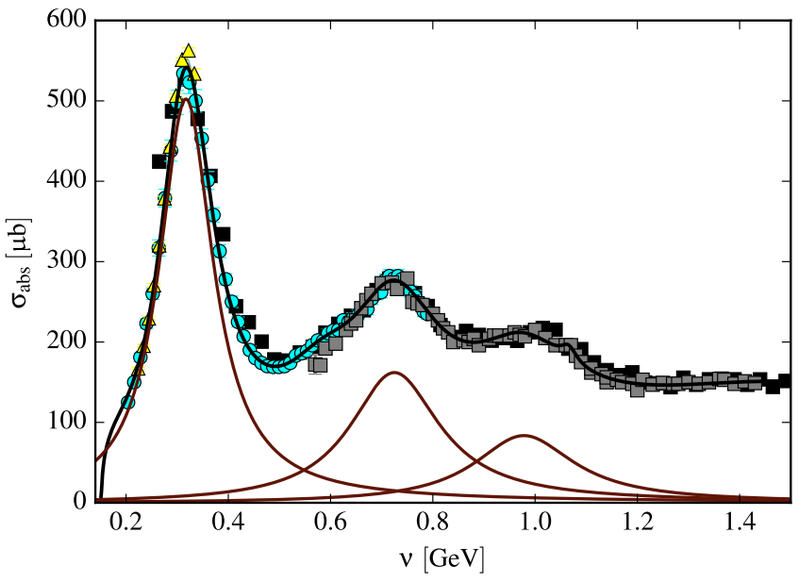
dispersion relations

assumption about the vertex

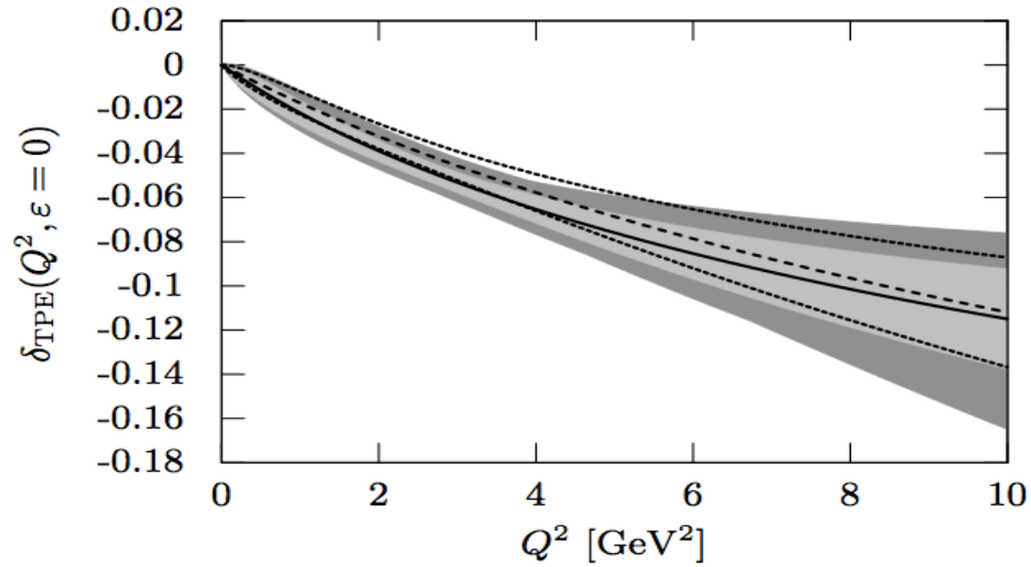
based on on-shell information

Fixed-Q^2 dispersion relation framework

on-shell 1γ amplitudes



2γ prediction



experimental data

elastic and πN

cross section correction

unitarity

$$\Re\mathcal{F}(\nu) = \frac{2\nu}{\pi}\mathcal{P}\int_{\nu_{min}}^{\infty}\frac{\Im\mathcal{F}(\nu' + i0)}{\nu'^2 - \nu^2}\mathrm{d}\nu'$$

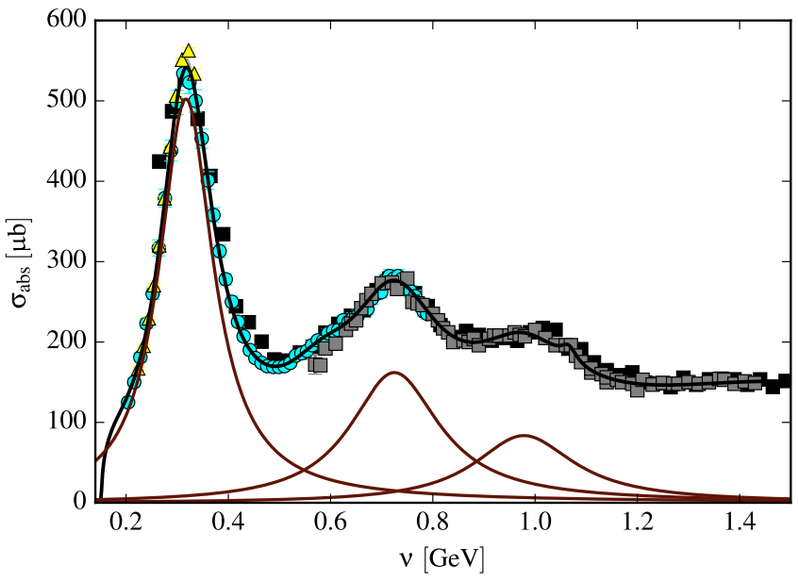
2γ imaginary parts

disp. rel.

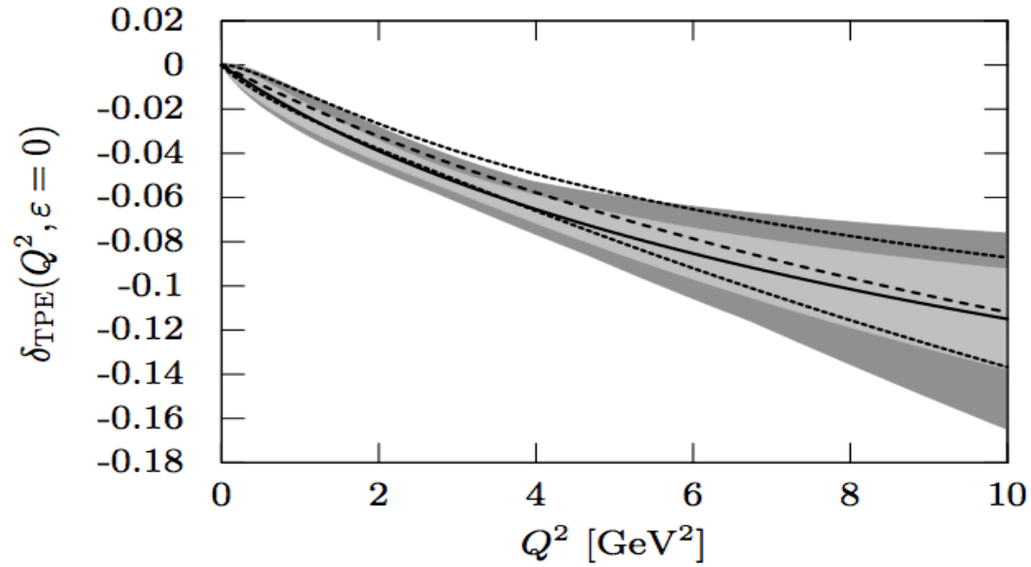
2γ real parts

Fixed-Q^2 dispersion relation framework

on-shell 1γ amplitudes



2γ prediction



experimental data

elastic and πN

cross section correction

unitarity

$$\Re\mathcal{F}(\nu) = \frac{2\nu}{\pi}\mathcal{P}\int_{\nu_{min}}^{\infty}\frac{\Im\mathcal{F}(\nu'+i0)}{\nu'^2-\nu^2}\mathrm{d}\nu'$$

2γ imaginary parts

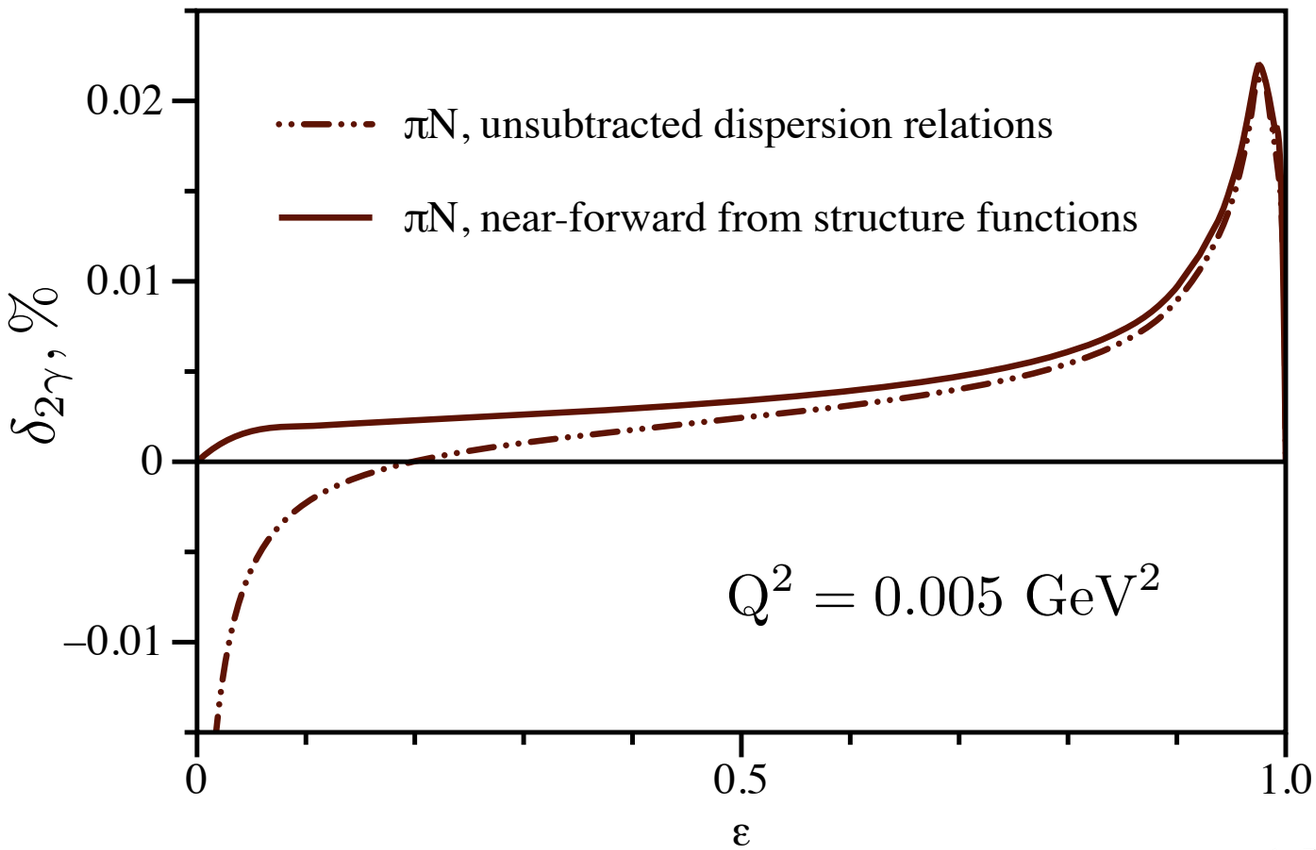
disp. rel.

2γ real parts

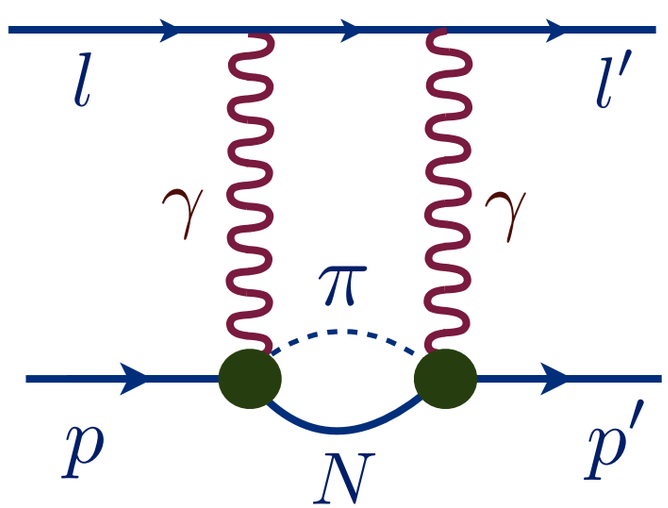
Dm. Borisyuk and A. Kobushkin (2008-15)

O. T. and M. Vanderhaeghen (2014-17)

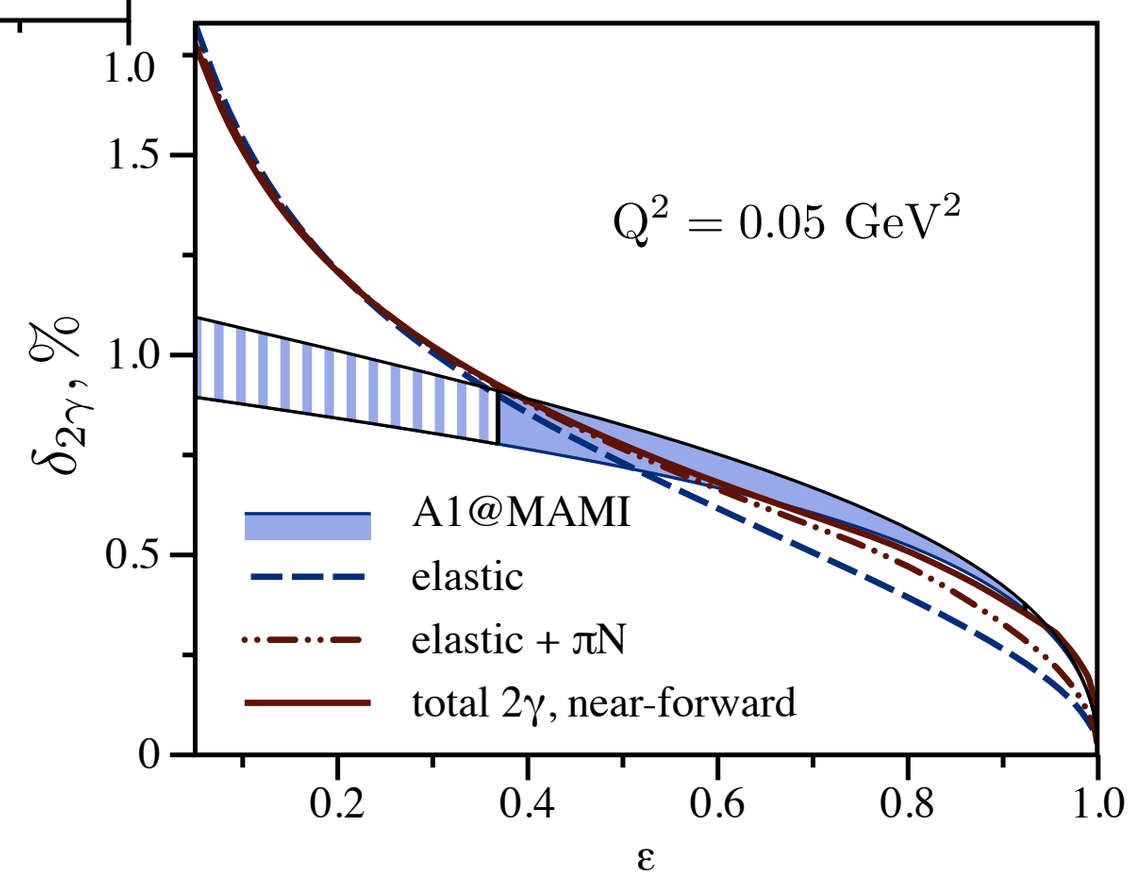
πN in dispersive framework (e-p)



- dispersion relations agree with near-forward at large ε

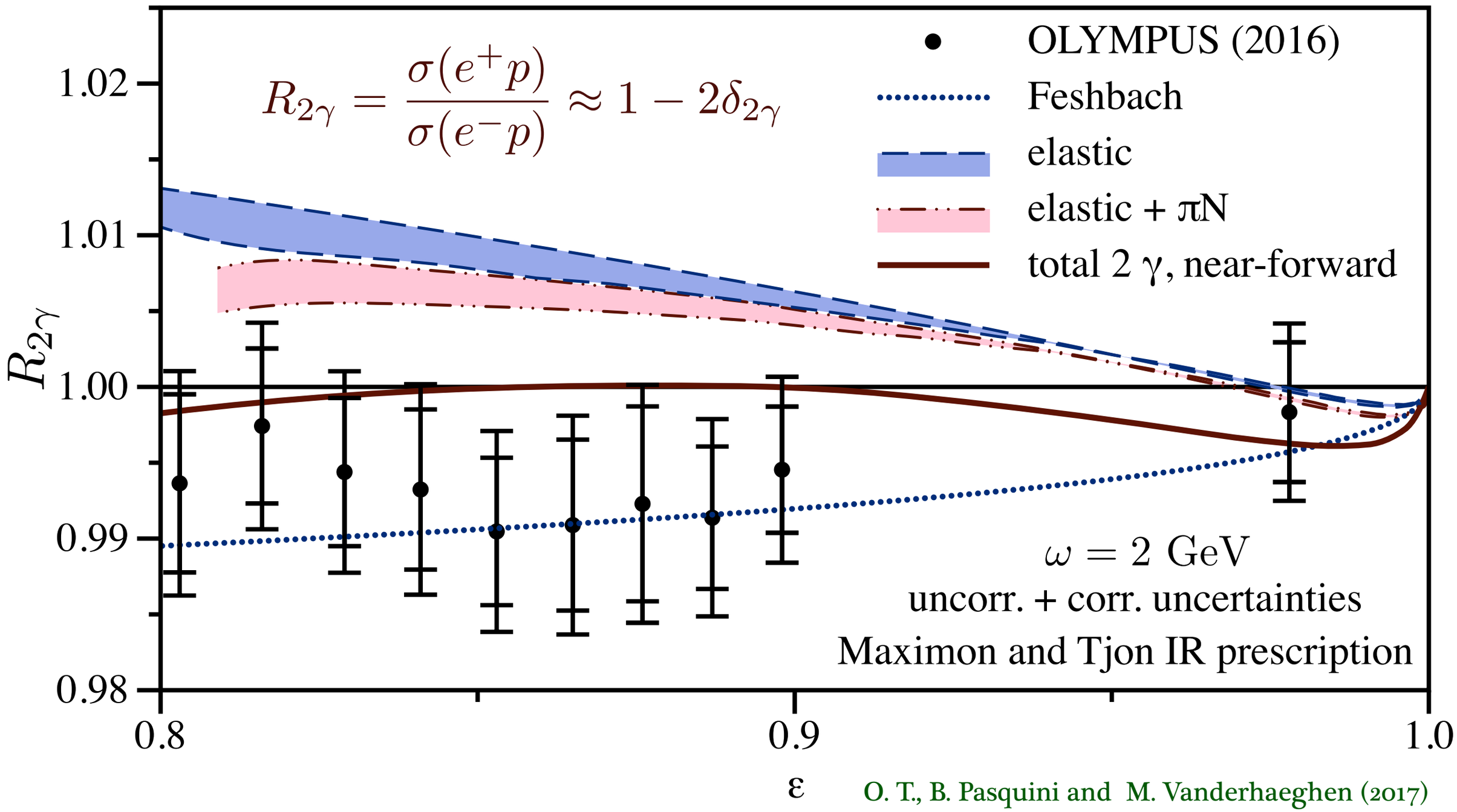


- πN is dominant inelastic 2γ



O. T., B. Pasquini and M. Vanderhaeghen (2017)

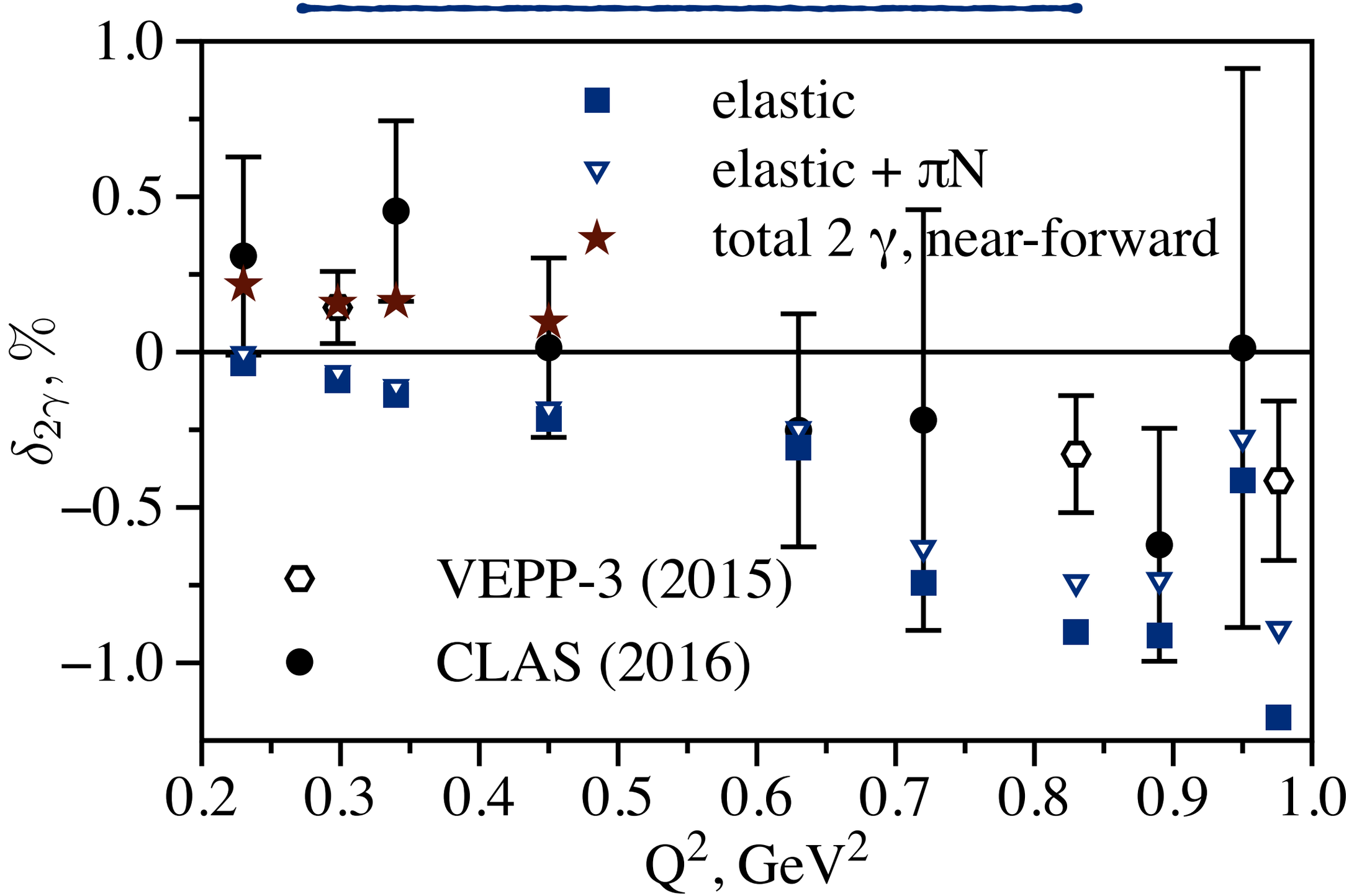
Comparison with data



O. T., B. Pasquini and M. Vanderhaeghen (2017)

- near-forward 2γ agree with data
- multi-particle 2γ, e.g. ππN, is important

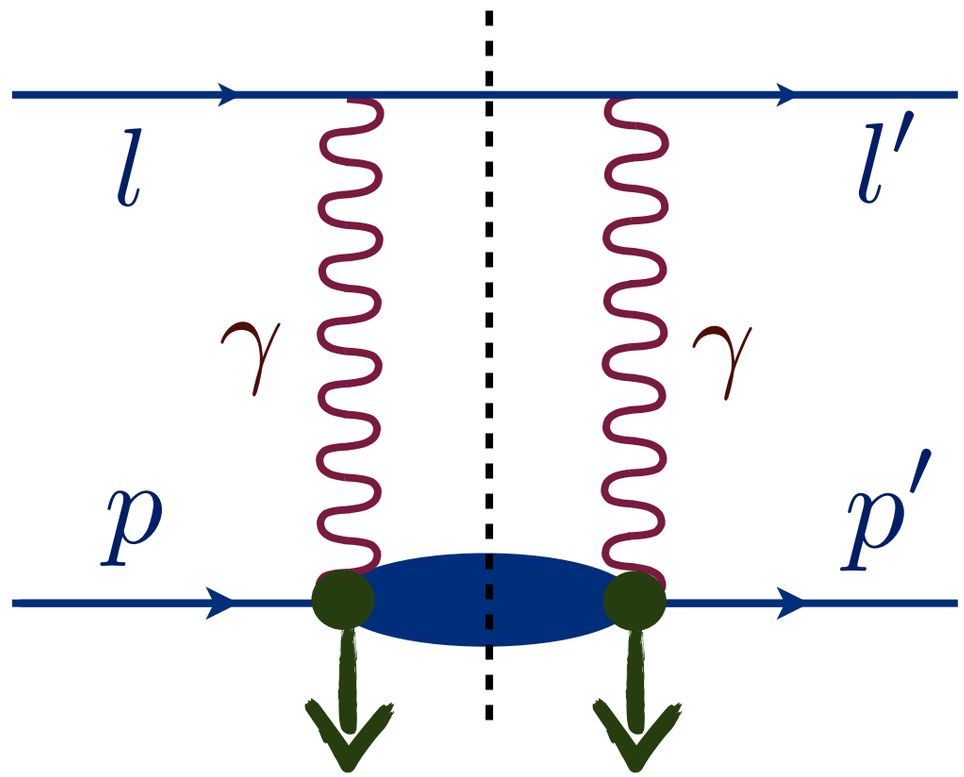
Comparison with data



O. T., B. Pasquini and M. Vanderhaeghen (2017)

- dispersion relations agree with CLAS data

non-forward scattering



photoproduction vertex or Compton tensor

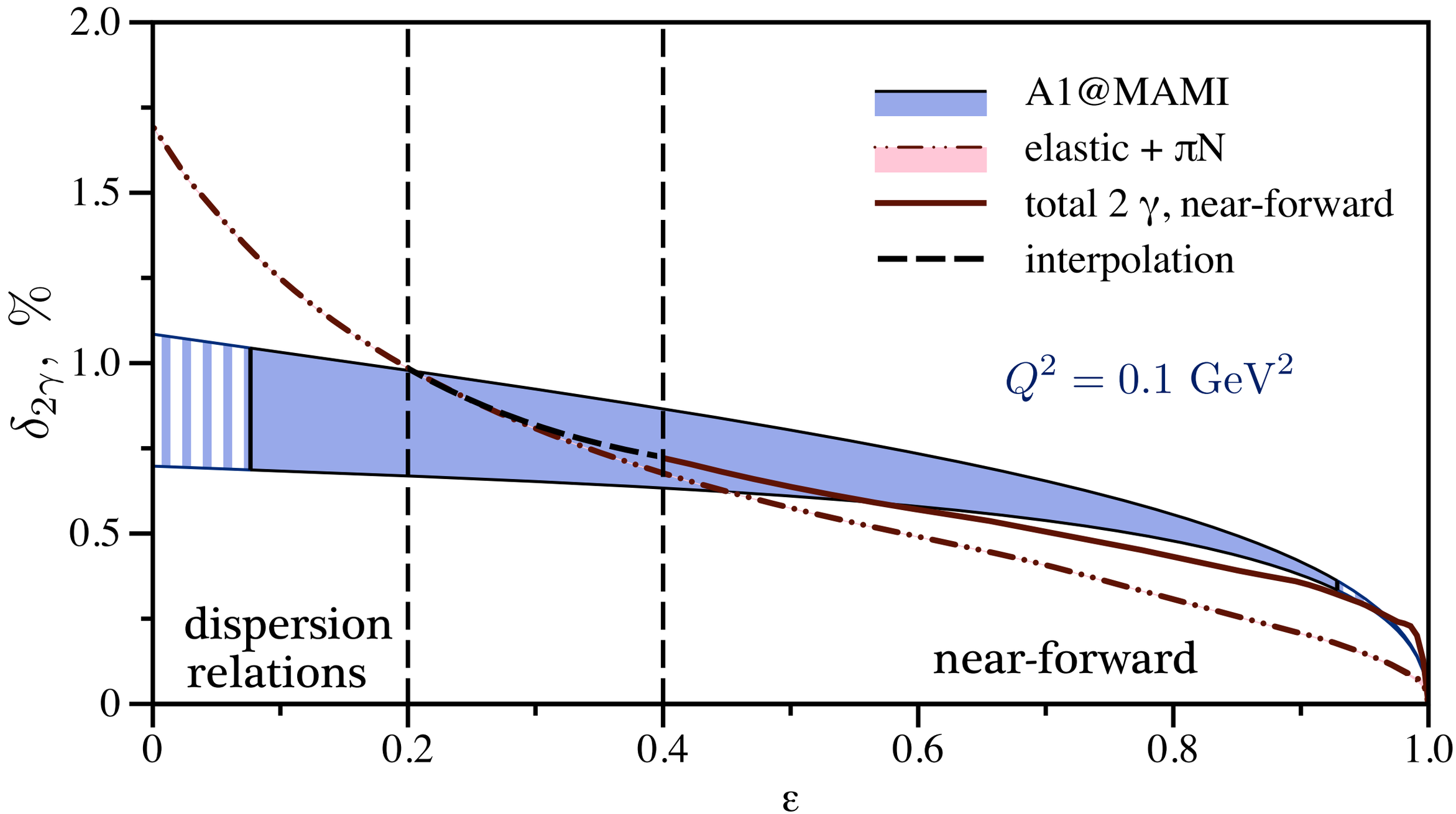
box diagram

dispersion relations

valid at small scattering

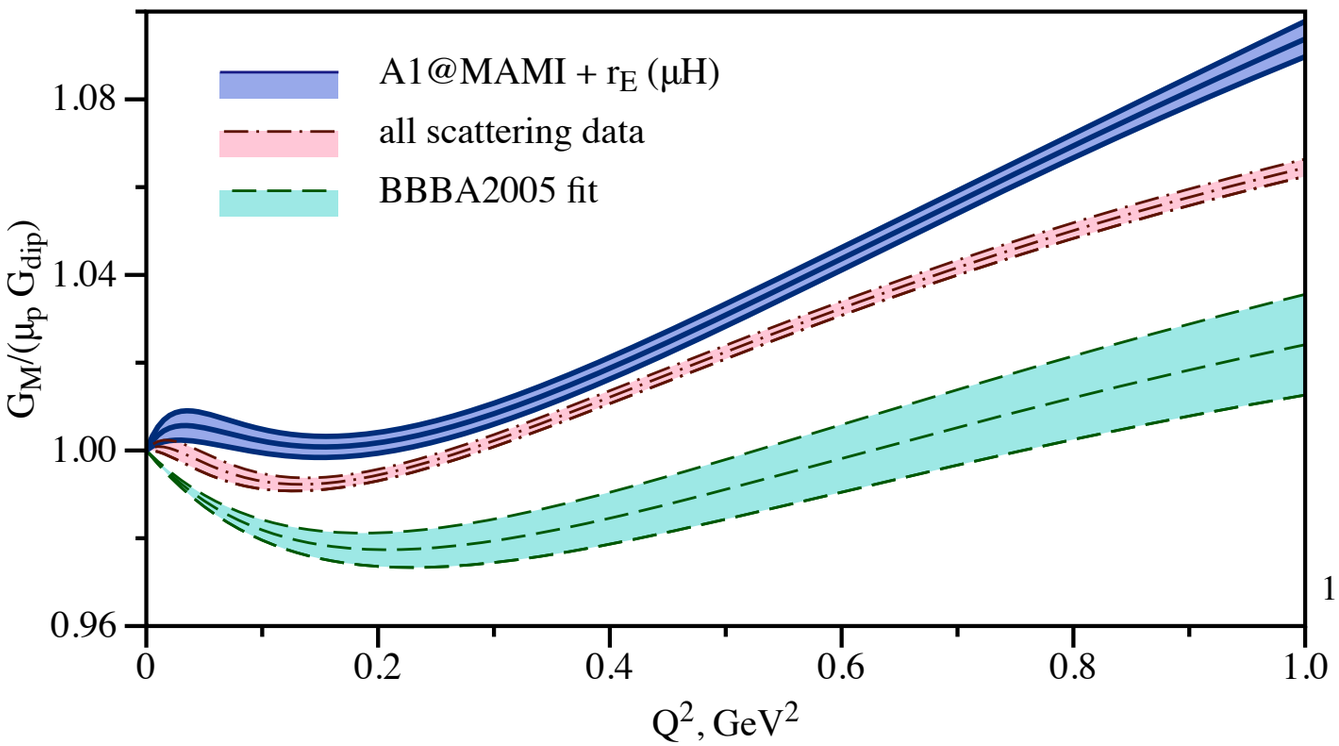
based on on-shell information

Our best 2γ knowledge



- small Q^2: near-forward at large ε, all inelastic states
- $Q^2 \lesssim 1$ GeV^2: elastic+πN within dispersion relations
- intermediate range: interpolation

Applications to nucleon form factors



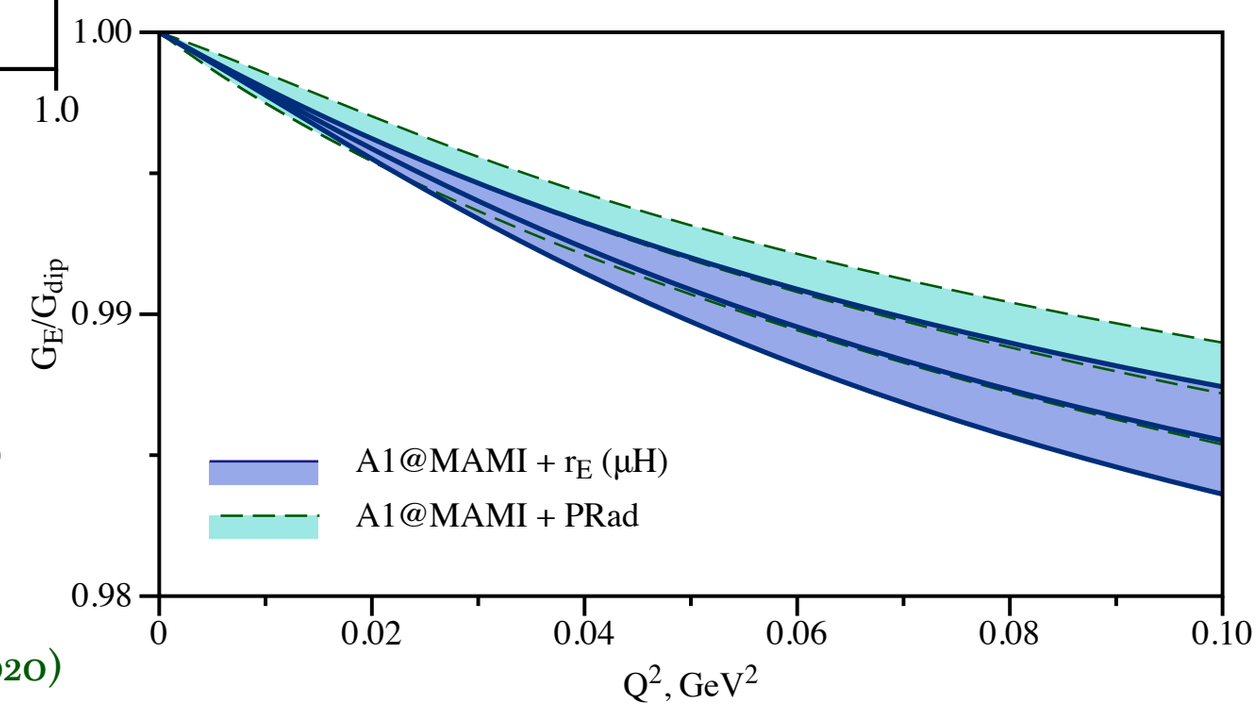
BBBA: fit used in neutrino physics

z expansion fit

$$z(Q^2) = \frac{\sqrt{t_{cut} + Q^2} - \sqrt{t_{cut} - t_0}}{\sqrt{t_{cut} + Q^2} + \sqrt{t_{cut} - t_0}}$$

with 4-5 independent coefficients

$$G(Q^2) = \sum_{k=0}^{k_{max}} a_k z(Q^2)^k$$

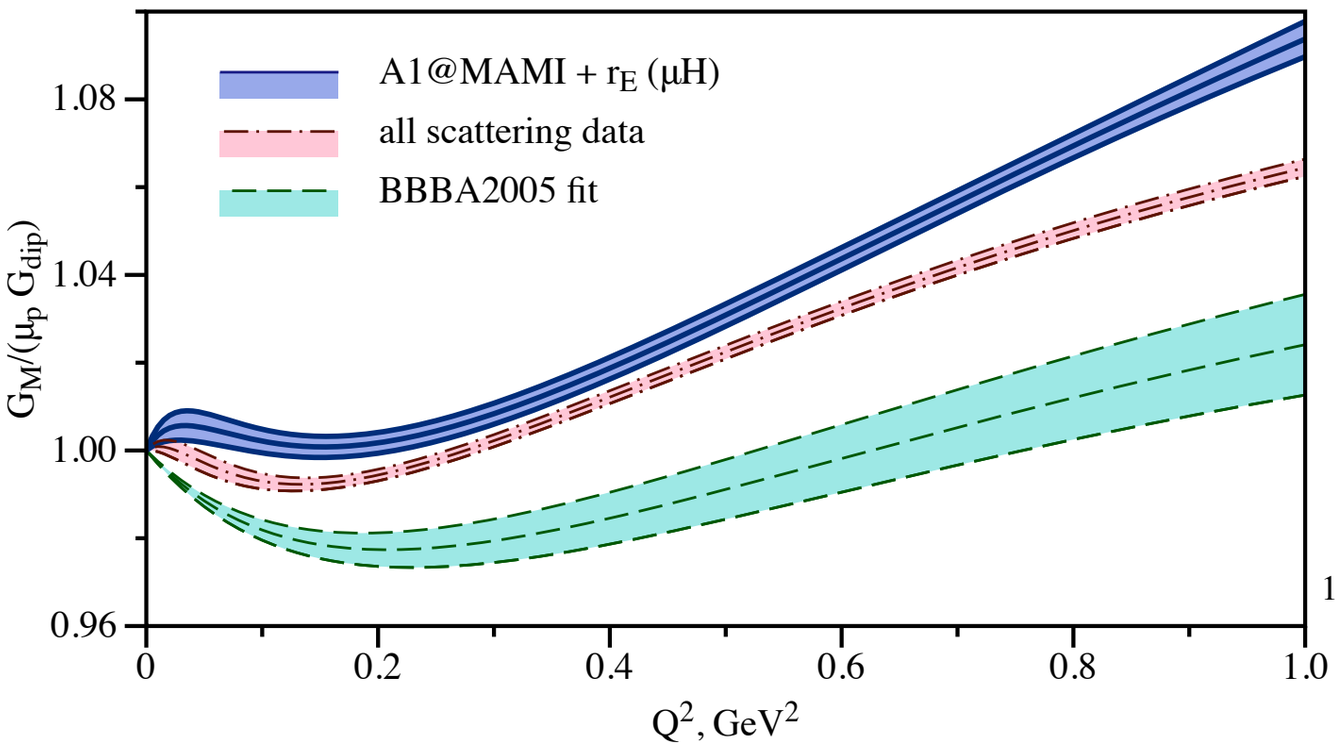


PRad data or μH charge radius ?

K. Borah, G. Lee, R. J. Hill and O. T. (2020)

- first model-independent fits presenting covariance matrix
- 2γ provides nontrivial hadronic radiative correction
- proton charge radius as a constraint

Applications to nucleon form factors



z expansion fit

$$z(Q^2) = \frac{\sqrt{t_{cut} + Q^2} - \sqrt{t_{cut} - t_0}}{\sqrt{t_{cut} + Q^2} + \sqrt{t_{cut} - t_0}}$$

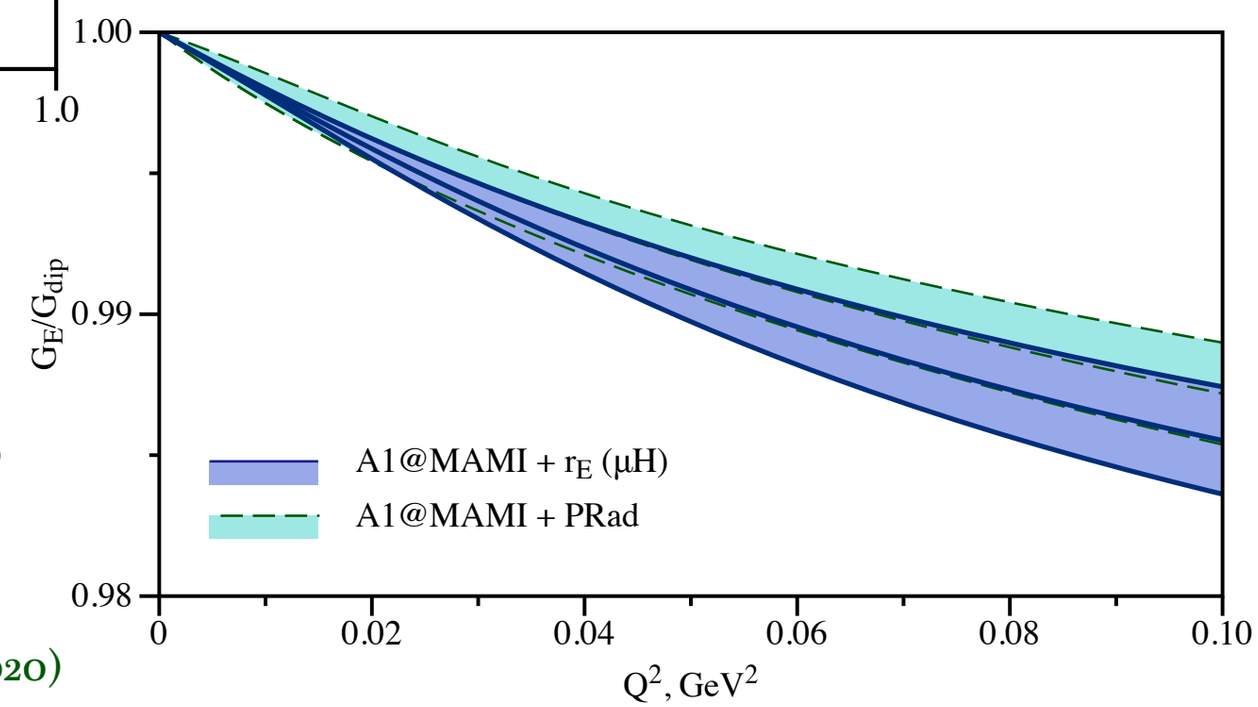
with 4-5 independent coefficients

$$G(Q^2) = \sum_{k=0}^{k_{max}} a_k z(Q^2)^k$$

BBBA: fit used in neutrino physics

PRad data or μH charge radius ?

K. Borah, G. Lee, R. J. Hill and O. T. (2020)



- inclusion of PRad data is consistent with μH constant
- significant difference in magnetic form factor propagates to observables

Elastic muon-proton scattering

and two-photon exchange

Elastic muon-proton scattering

- charge radius extractions:

eH, eD spectroscopy	ep scattering
μH, μD spectroscopy	μp scattering ????

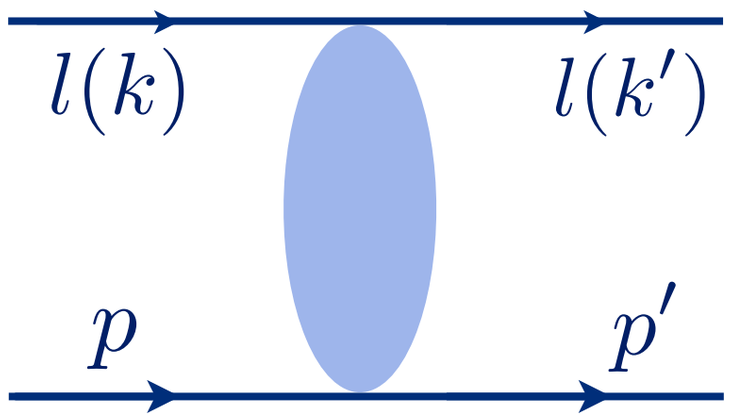
- μp elastic scattering is planned by MUSE@PSI(ongoing)
  measure with both electron/muon charges

- three nominal beam energies: 115, 153, 210 MeV, $Q^2 < 0.1$ GeV^2

- 2γ correction in MUSE ?

Elastic lepton-proton scattering and 2γ

$$K = \frac{k + k'}{2}$$



$$P = \frac{p + p'}{2}$$

- electron-proton scattering: 3 structure amplitudes

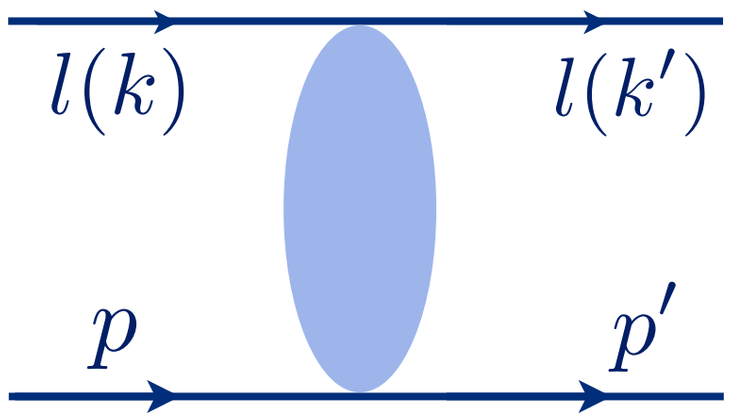
$$\mathrm{T}^{\mathrm{non-flip}} = \frac{e^2}{Q^2} \bar{l} \gamma_\mu l \cdot \bar{N} \left(\mathcal{G}_M(\nu, Q^2) \gamma^\mu - \mathcal{F}_2(\nu, Q^2) \frac{P^\mu}{M} + \mathcal{F}_3(\nu, Q^2) \frac{\hat{K} P^\mu}{M^2} \right) N$$

P.A.M. Guichon and M. Vanderhaeghen (2003)

- 2γ correction to cross section is given by amplitudes real parts

Elastic lepton-proton scattering and 2γ

$$K = \frac{k + k'}{2}$$



$$P = \frac{p + p'}{2}$$

- electron-proton scattering: 3 structure amplitudes

$$\mathrm{T}^{\mathrm{non-flip}} = \frac{e^2}{Q^2}\bar{l}\gamma_\mu l \cdot \bar{N}\left(\mathcal{G}_M(\nu, Q^2)\gamma^\mu - \mathcal{F}_2(\nu, Q^2)\frac{P^\mu}{M} + \mathcal{F}_3(\nu, Q^2)\frac{\hat{K}P^\mu}{M^2}\right)N$$

P.A.M. Guichon and M. Vanderhaeghen (2003)

- muon-proton scattering: add helicity-flip amplitudes

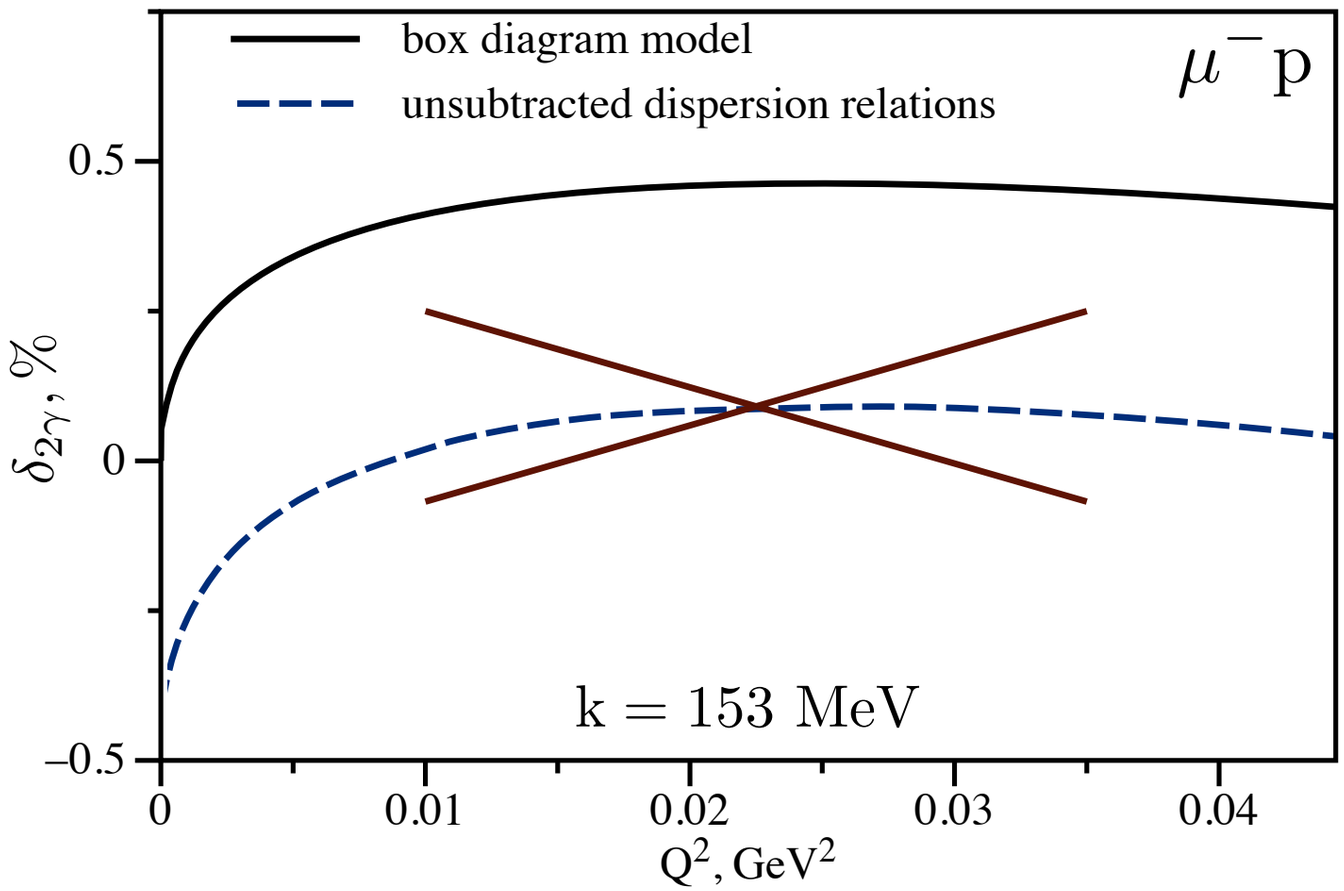
$$\mathrm{T}^{\mathrm{flip}} = \frac{e^2}{Q^2}\frac{m}{M}\bar{l}l \cdot \bar{N}\left(\mathcal{F}_4(\nu, Q^2) + \mathcal{F}_5(\nu, Q^2)\frac{\hat{K}}{M}\right)N + \frac{e^2}{Q^2}\frac{m}{M}\mathcal{F}_6(\nu, Q^2)\bar{l}\gamma_5 l \cdot \bar{N}\gamma_5 N$$

M. Gorchtein, P.A.M. Guichon and M. Vanderhaeghen (2004)

- 2γ correction to cross section is given by amplitudes real parts

Dispersion relation approach?

- proton state contribution to 2γ:



unsubtracted approach violates low-Q^2 behavior

$\delta_{2\gamma} \to 0$

O. T. and M. Vanderhaeghen (2018)

- problematic amplitude: $\mathcal{F}_4$

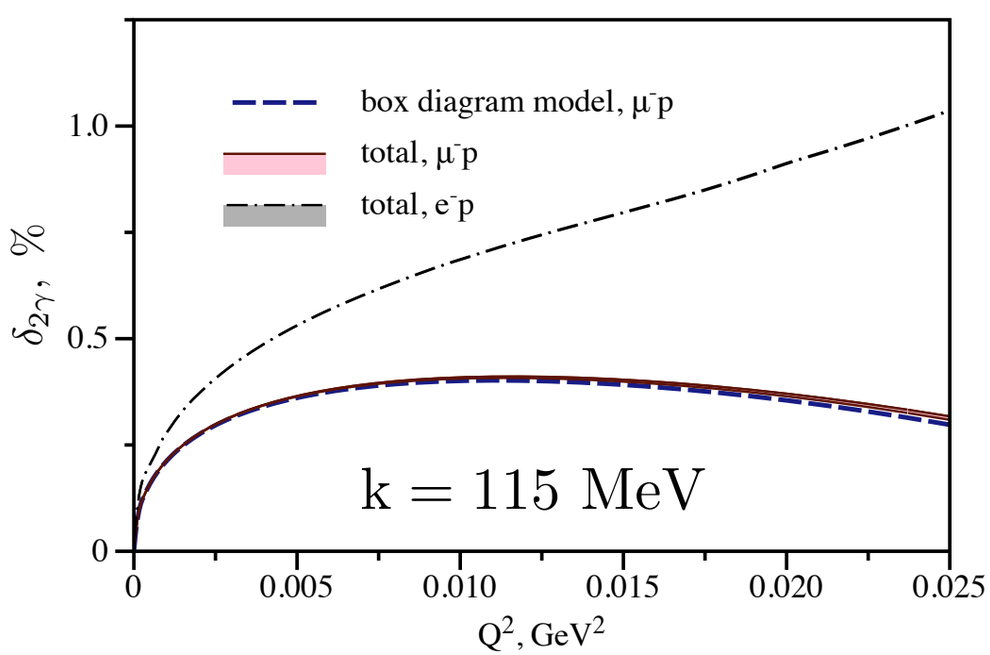
$\bar{l}l \cdot \bar{N}N$

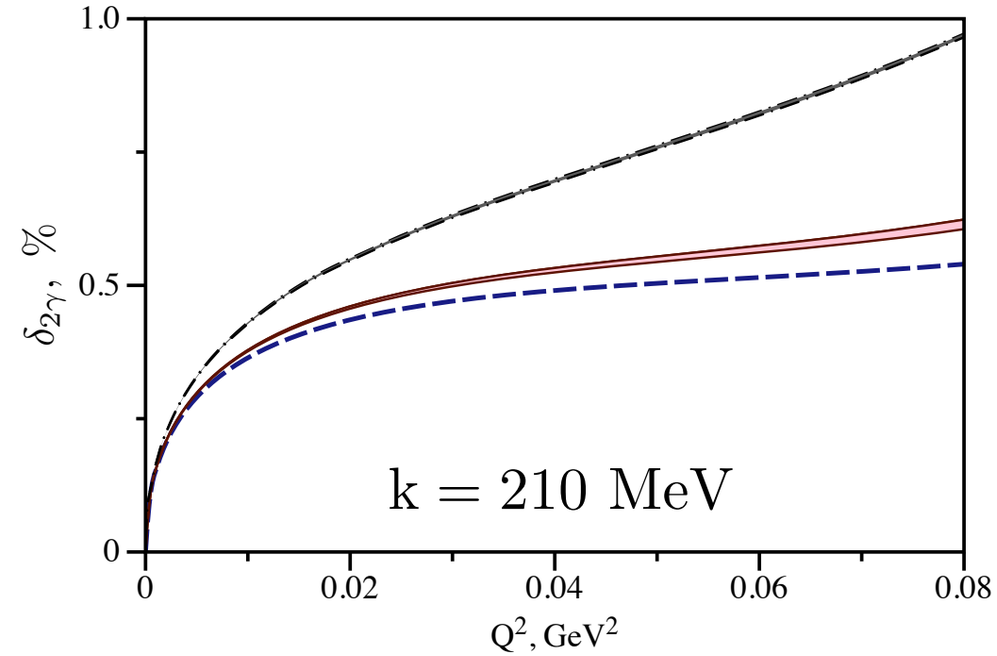
structure

- dispersion relation approach requires a subtraction

MUSE@PSI (2018-19) estimates (μ^-p)

- proton box diagram model + inelastic 2ɣ





O. T. and M. Vanderhaeghen (2014, 2016)

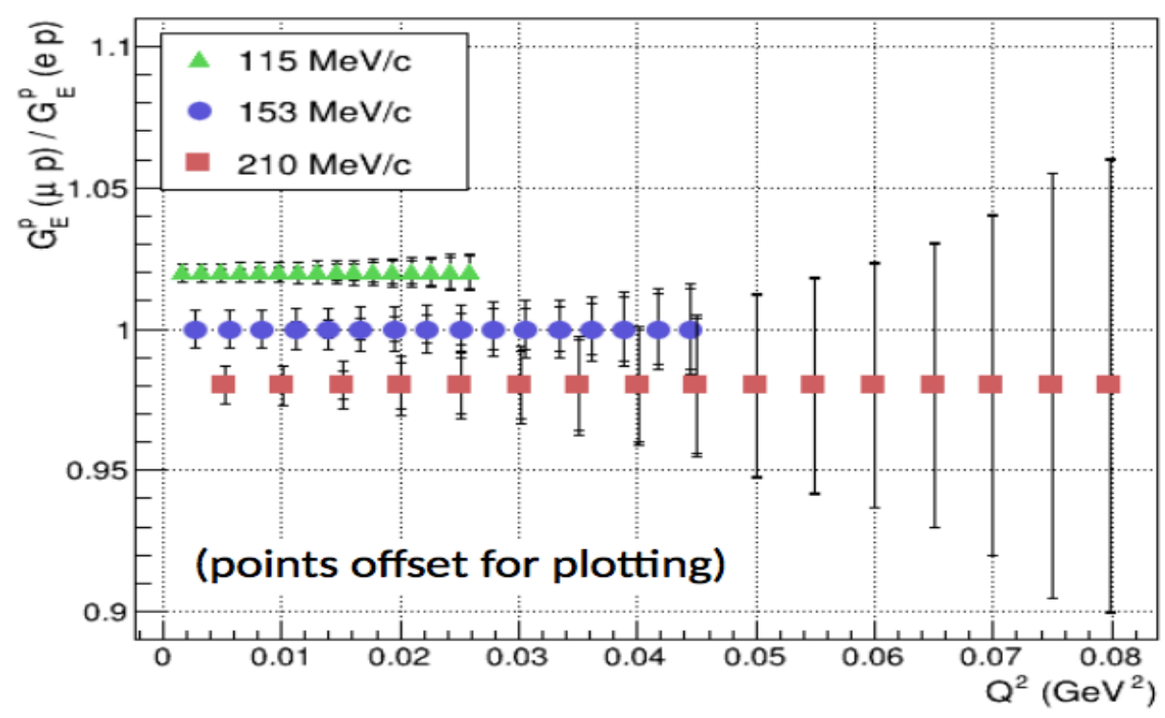
- expected muon over electron ratio

small inelastic 2ɣ

↓

small 2ɣ uncertainty

- MUSE can test r_E in one charge channel



K. Mesick talk (PAVI 2014), MUSE TDR (2016)

COMPASS proton radius experiment

- elastic μp scattering at SPS with 100 GeV beam

- measure $G_E^2 + \tau G_M^2$ at forward angles

- data taking in 2022

COMPASS proton radius experiment

- elastic μp scattering at SPS with 100 GeV beam

- measure $G_E^2 + \tau G_M^2$ at forward angles

- data taking in 2022

2γ corrections?

- Feshbach correction (+ recoil)

$$\delta_{2\gamma} = \frac{\alpha \pi Q}{2\omega}\left(1 + \frac{m}{M}\right)$$ → 2-3 orders below MUSE

- inelastic states: kinematically enhanced

- sub per mille level of 2γ in COMPASS kinematics

COMPASS proton radius experiment

- elastic μp scattering at SPS with 100 GeV beam

- measure $G_E^2 + \tau G_M^2$ at forward angles

- data taking in 2022

2γ corrections?

- Feshbach correction (+ recoil)

$$\delta_{2\gamma} = \frac{\alpha \pi Q}{2\omega}\left(1 + \frac{m}{M}\right)$$ → 2-3 orders below MUSE

- inelastic states: kinematically enhanced

- sub per mille level of 2γ in COMPASS kinematics

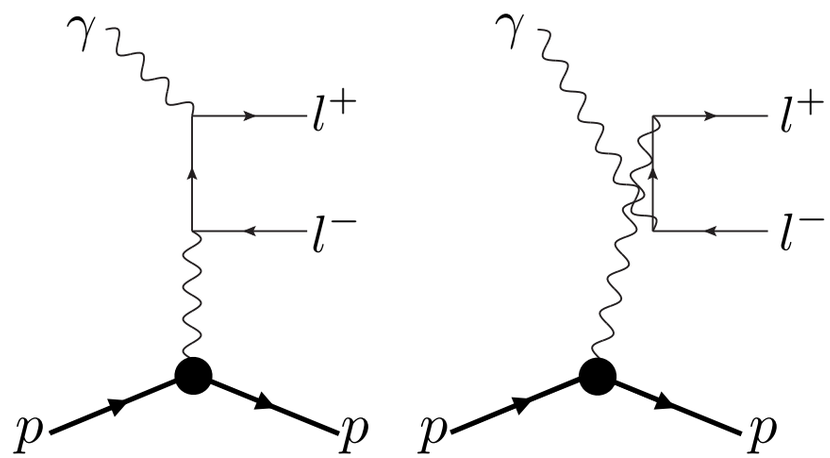
Lepton-pair production

- universality test by lepton-pair photoproduction:

$\gamma p \to l^+ l^- p$@MAMI

$\omega = 0.5 - 1.5$ GeV

$Q^2 = 0.0018 - 0.042$ GeV^2



$$\frac{\sigma(e^+e^-) + \sigma(\mu^+\mu^-)}{\sigma(e^+e^-)}$$

below and above muon threshold

normalisation and proton structure errors are suppressed

Vl. Pauk and M. Vanderhaeghen (2015)

radiative corrections are presented in analytic form

M. Heller, O. T., M. Vanderhaeghen and Sh. Wu (2018-2019)

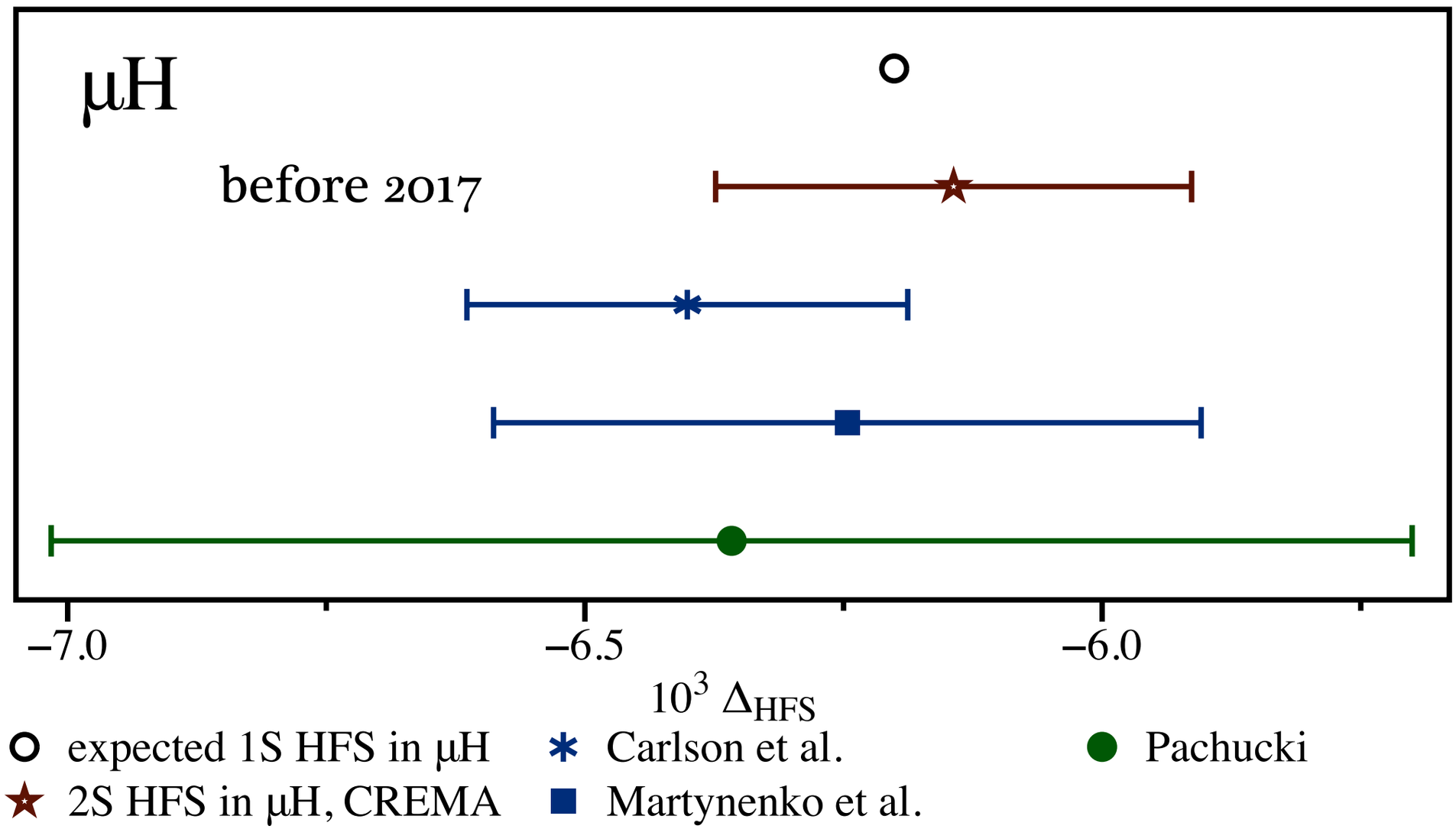
- one-loop QED is calculated
- 2γ vanishes averaging over lepton angles

Hyperfine splitting in ordinary and muonic hydrogen

2γ correction to μH HFS

$$\delta E^{2\gamma}_{\mathrm{HFS}} = \Delta_{\mathrm{HFS}} E_{\mathrm{F}}$$

$$E_{\mathrm{F}} = \frac{8\alpha^4}{3} \frac{M^2 m^2}{(M+m)^3} \frac{\mu_P}{n^3}$$



- reduction of uncertainty is needed

Hyperfine splitting correction

- traditional decomposition of 2γ:

$$\Delta_{\mathrm{HFS}} = \Delta_{\mathrm{Z}} + \Delta^{\mathrm{R}} + \Delta^{\mathrm{pol}}$$

Zemach term
$\mathrm{G_E},\ \mathrm{G_M}$

recoil correction
$\mathrm{G_E},\ \mathrm{G_M}$

polarizability
$\mathrm{F}_2,\ g_1,\ g_2$

- leading correction:

$$\Delta_{\mathrm{Z}} = \frac{8\alpha m_r}{\pi} \int_0^\infty \frac{\mathrm{d}Q}{Q^2} \left(\frac{\mathrm{G_M}\left(Q^2\right) \mathrm{G_E}\left(Q^2\right)}{\mu_{\mathrm{P}}} - 1 \right)$$

- uncertainty budget:

> 100 ppm | < 10 ppm | 100 ppm

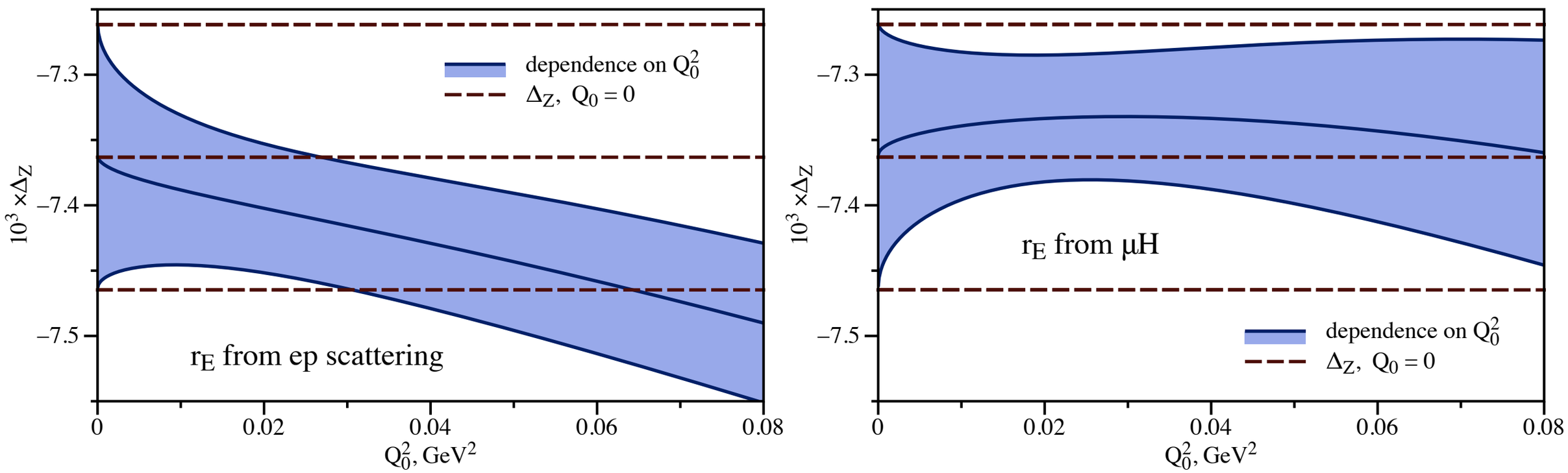
Zemach correction in μH

- Zemach correction expanding form factors:

$$\Delta_Z = \frac{8\alpha m_r}{\pi}\int_{Q_0}^{\infty}\frac{dQ}{Q^2}\left(\frac{G_M(Q^2)\,G_E(Q^2)}{\mu_P}-1\right)+\frac{4\alpha m_r Q_0}{3\pi}\left(-r_E^2-r_M^2+\frac{r_E^2 r_M^2}{18}Q_0^2\right)$$

extraction of radii by Karshenboim (2014)

- dependence on splitting: consistency check



O. T. (2017)

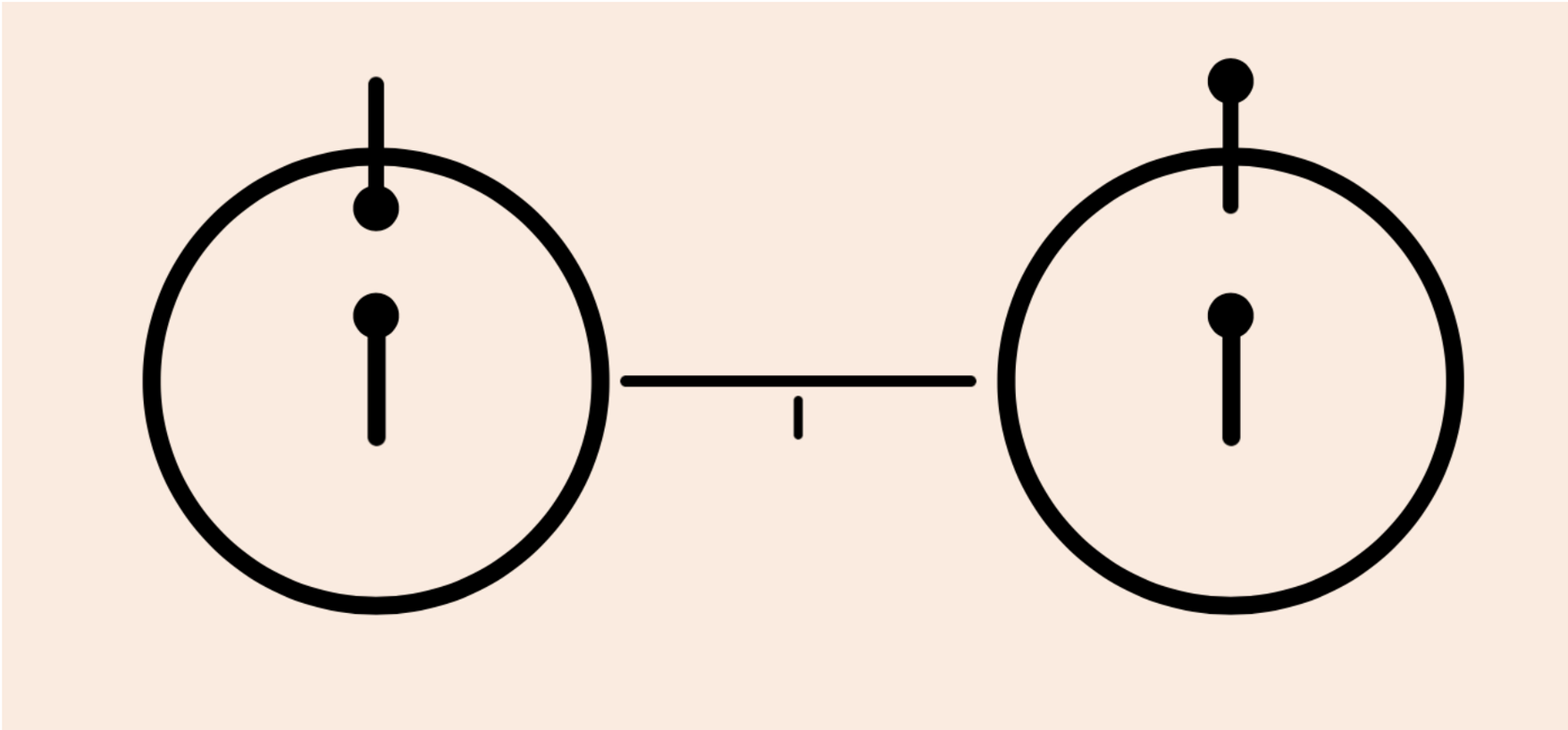
- 95 ppm change for μH and ep radii with $Q_0 = 0.2$ GeV

- 1.5-2 times more precise
- magnetic radius is equally important

2γ correction in eH 1S HFS

- measurements of 1S HFS in eH (21 cm line):

$$\nu_{\mathrm{HFS}}(\mathrm{H}) = 1420.4057517667(9)\ \mathrm{MHz}$$ 1970th



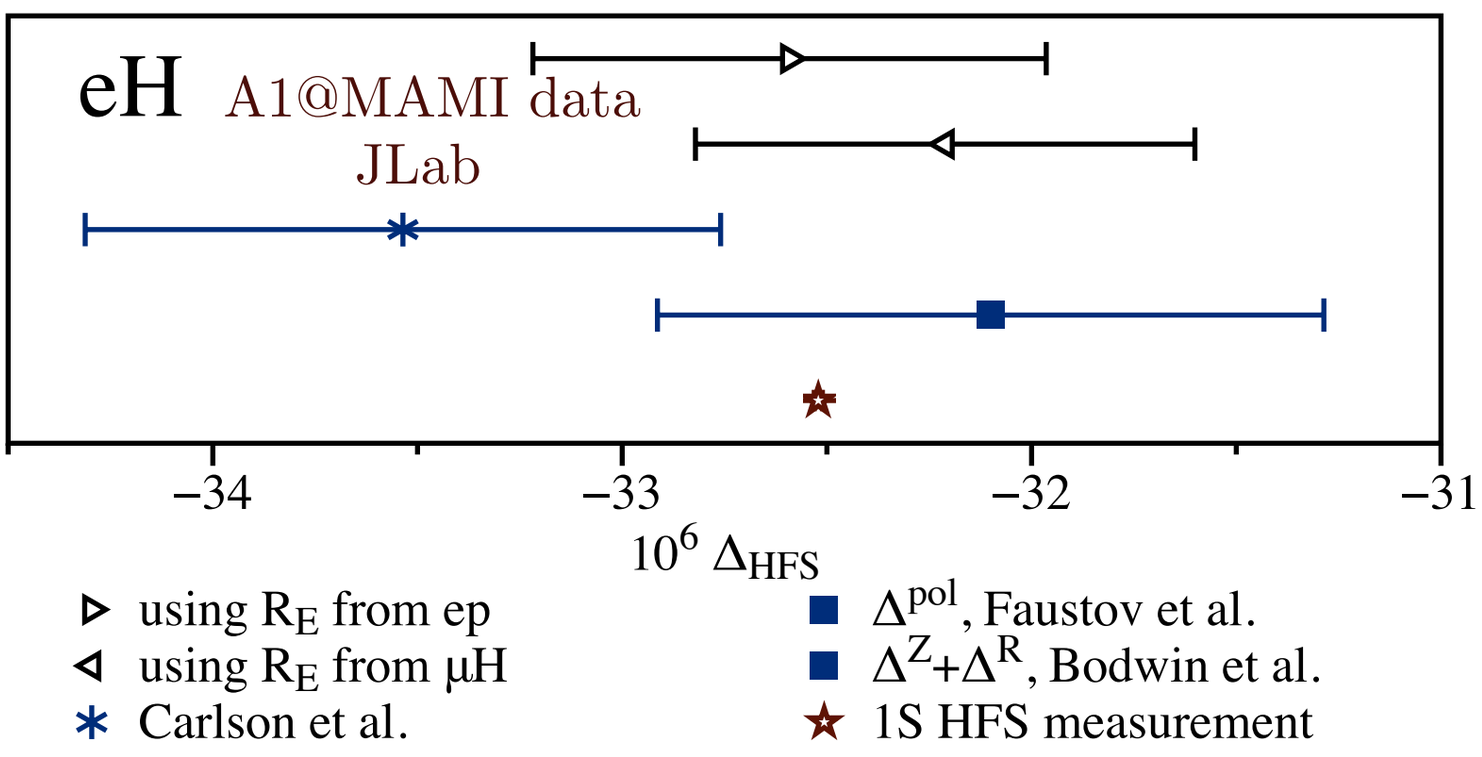
Pioneer plaque, from Wikipedia

2γ correction in eH 1S HFS

- measurements of 1S HFS in eH (21 cm line):

$$\nu_{\mathrm{HFS}}(\mathrm{H}) = 1420.4057517667(9)\ \mathrm{MHz}$$ 1970th

- accuracy 10^{-12} - precise extraction of 2γ



higher order corrections

M. I. Eides et al. (2008)
A. P. Martynenko et al.

error $\alpha\Delta_{\mathrm{HFS}}$

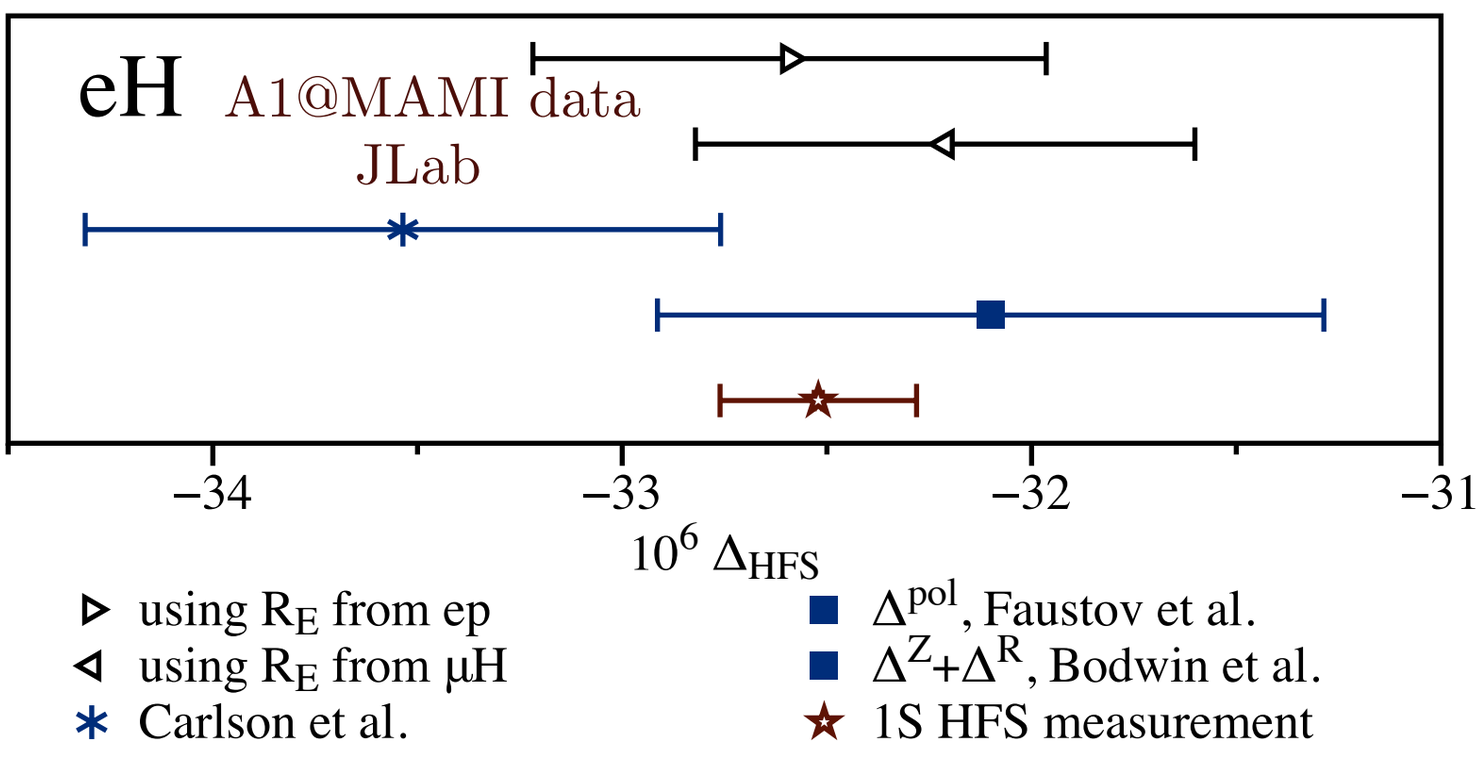
- dispersive evaluation and phenomenological extractions agree

2γ correction in eH 1S HFS

- measurements of 1S HFS in eH (21 cm line):

$$\nu_{\mathrm{HFS}}(\mathrm{H}) = 1420.4057517667(9)\ \mathrm{MHz}$$ 1970th

- accuracy 10^{-12} - precise extraction of 2γ



higher order corrections

M. I. Eides et al. (2008)
A. P. Martynenko et al.

error $\alpha\Delta_{\mathrm{HFS}}$

- dispersive evaluation and phenomenological extractions agree

Hyperfine splitting correction

$$\delta E^{2\gamma}_{\mathrm{HFS}} = \Delta_{\mathrm{HFS}} E_{\mathrm{F}}$$

- traditional decomposition of 2γ:

$$\Delta_{\mathrm{HFS}} = \Delta_{\mathrm{Z}} + \Delta^{\mathrm{R}} + \Delta^{\mathrm{pol}}$$

Zemach term
$G_E,\ G_M$

recoil correction
$G_E,\ G_M$

polarizability
F_P, g_1, g_2

- leading correction:

$$\Delta_{\mathrm{Z}} = \frac{8\alpha m_r}{\pi} \int_0^\infty \frac{\mathrm{d}Q}{Q^2} \left(\frac{G_M\left(Q^2\right) G_E\left(Q^2\right)}{\mu_P} - 1 \right) \qquad \Delta_{\mathrm{pol}} \sim m_r$$

- uncertainty budget:

> 100 ppm | < 10 ppm | 100 ppm

Hyperfine splitting correction

$$\delta E^{2\gamma}_{\mathrm{HFS}} = \Delta_{\mathrm{HFS}} E_{\mathrm{F}}$$

- traditional decomposition of 2γ:

$$\Delta_{\mathrm{HFS}} = \Delta_{\mathrm{Z}} + \Delta^{\mathrm{R}} + \Delta^{\mathrm{pol}}$$

Zemach term
$G_E,\ G_M$

recoil correction
$G_E,\ G_M$

polarizability
F_P, g_1, g_2

- leading correction:

$$\Delta_{\mathrm{Z}} = \frac{8\alpha m_r}{\pi} \int_0^\infty \frac{\mathrm{d}Q}{Q^2} \left(\frac{G_M\left(Q^2\right) G_E\left(Q^2\right)}{\mu_P} - 1 \right) \qquad \Delta_{\mathrm{pol}} \sim m_r$$

- uncertainty budget:

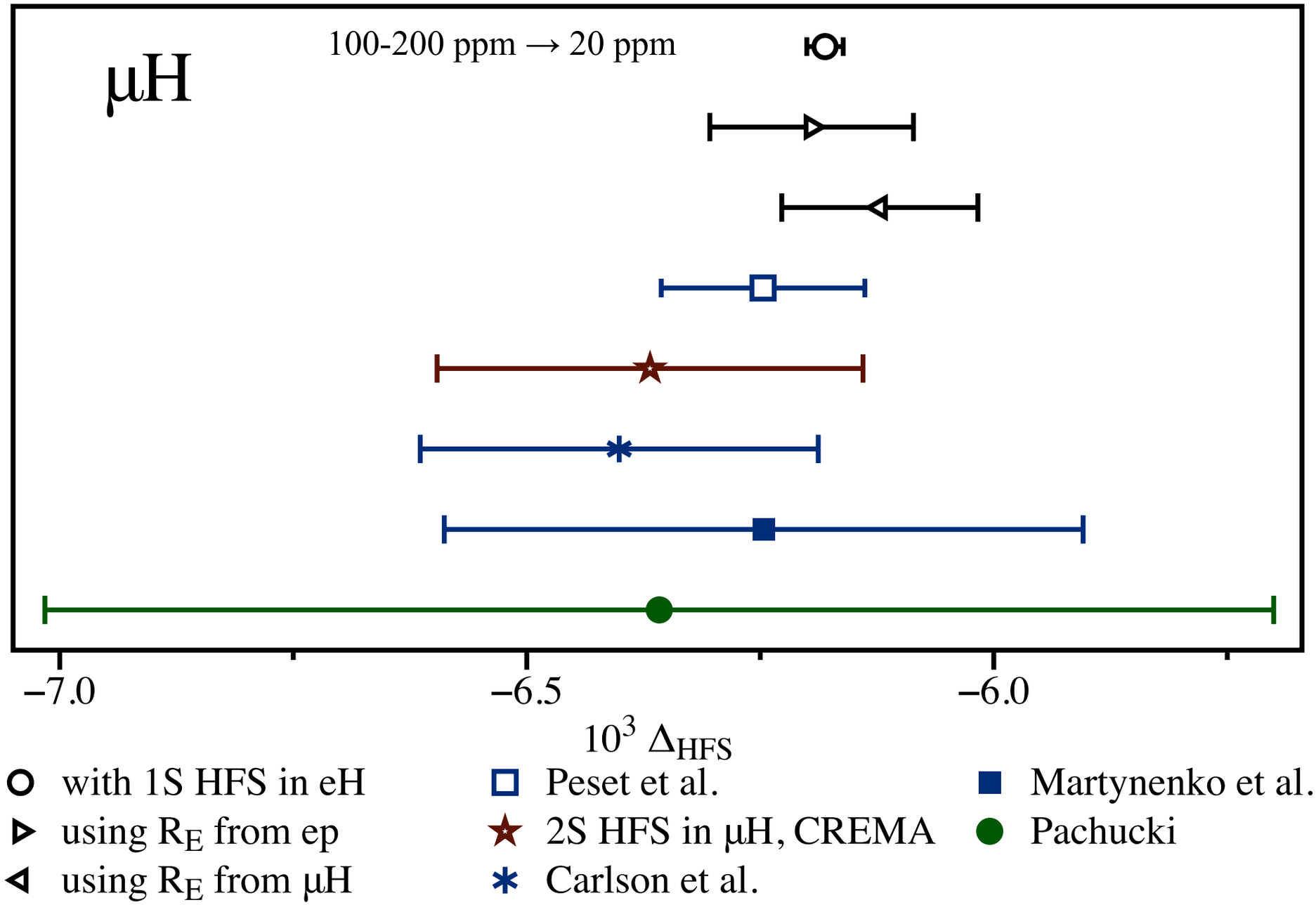
> 100 ppm < 10 ppm 100 ppm

- μH splitting from eH:

$$\Delta_{\mathrm{HFS}}(\mu\mathrm{H}) = \frac{m_r(m_\mu)}{m_r(m_e)} \Delta_{\mathrm{HFS}}\,(\mathrm{eH}) + \Delta^{\mathrm{th}}_{\mathrm{HFS}}\,(m_\mu) - \frac{m_r(m_\mu)}{m_r(m_e)} \Delta^{\mathrm{th}}_{\mathrm{HFS}}\,(m_e)$$

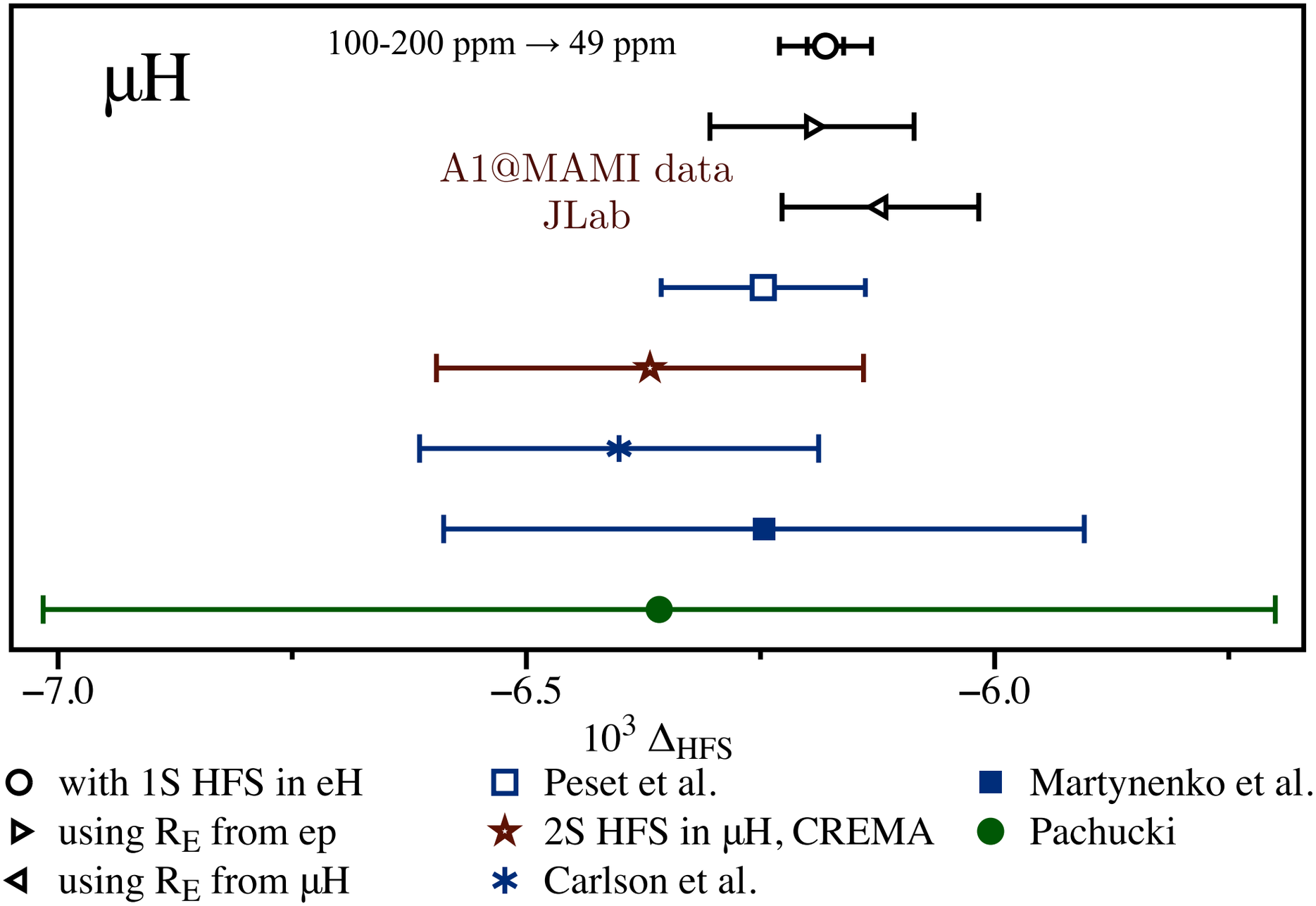
- Zemach correction vanishes and polarizability term is almost 0

2γ correction in μH from eH HFS



- error of 2γ is significantly reduced

2γ correction in μH from eH HFS



$$E_{1S}^{HFS} = 182.625 \pm 0.012 \text{ meV}$$

O. T. (2018, 2019)

- precise S-level HFS prediction

1S-2S splitting in hydrogen

1S-2S transition in hydrogen and 2γ

- measurements of 1S-2S transition in eH with 4×10^{-15} accuracy:

$$\nu_{1S-2S}(\mathrm{H}) = 2466061413187018(11)\ \mathrm{Hz}$$ 2010th

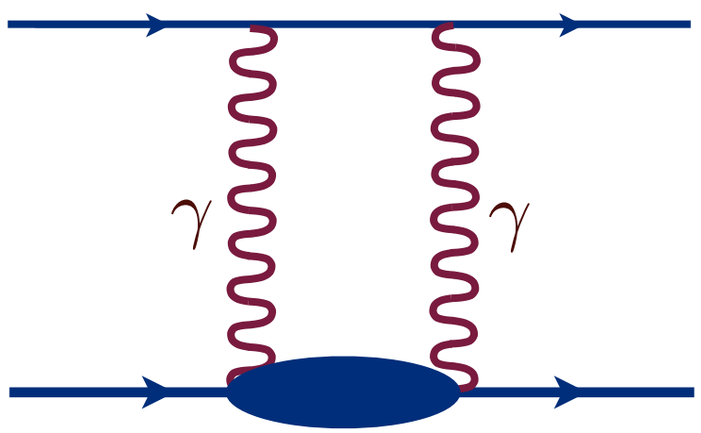
A. Matveev et al. (2013)

- more precise than recent Lamb shift measurement (error: 3.2 kHz)

N. Bezginov et al. (2019)

$$\nu_{\mathrm{nS}} = -\frac{R_\infty}{n^2} + \frac{L_{1S}(r_E)}{n^3}$$

- main input to determine Rydberg constant



1S-2S transition in hydrogen and 2γ

- measurements of 1S-2S transition in eH with 4×10^{-15} accuracy:

$$\nu_{1S-2S}(\mathrm{H}) = 2466061413187018(11)\ \mathrm{Hz}$$ 2010th

A. Matveev et al. (2013)

- more precise than recent Lamb shift measurement (error: 3.2 kHz)

N. Bezginov et al. (2019)

$$\nu_{nS}^{2\gamma} = \nu_{\mathrm{Born}} + \nu_{\mathrm{subt}} + \nu_{\mathrm{inel}}$$

Born

$G_E,\ G_M$

rE from μH: -39.9(6.8) Hz

average rE: -44.1(9.6) Hz

subtraction

T_1

O. T. and M. Vanderhaeghen (2015)

18.5(4.4) Hz

inelastic

F_1, F_2

-83.6(5.7) Hz

1S-2S transition in hydrogen and 2γ

- measurements of 1S-2S transition in eH with 4×10^{-15} accuracy:

$$\nu_{1S-2S}(\mathrm{H}) = 2466061413187018(11)\ \mathrm{Hz}$$ 2010th

A. Matveev et al. (2013)

- more precise than recent Lamb shift measurement (error: 3.2 kHz)

N. Bezginov et al. (2019)

$$\nu_{nS}^{2\gamma} = \nu_{\mathrm{Born}} + \nu_{\mathrm{subt}} + \nu_{\mathrm{inel}}$$

Born
$G_E,\ G_M$

rE from μH: -39.9(6.8) Hz

average rE: -44.1(9.6) Hz

subtraction
T_1

O. T. and M. Vanderhaeghen (2015)

18.5(4.4) Hz

inelastic
F_1, F_2

-83.6(5.7) Hz

- 2γ shifts 1S level by -109.2(12.0) Hz

O. T. (2019)

- 2γ uncertainty is at the level of experimental precision

Lamb shift from 2γ on nucleons

2γ to Lamb shift on neutrons

- forward 2γ on nucleons contributes to Lamb shift

$$\nu_{\mathrm{nS}}^{2\gamma} = \nu_{\mathrm{Born}} + \nu_{\mathrm{subt}} + \nu_{\mathrm{inel}}$$

Born
$G_E,\ G_M$

subtraction
T_1
O. T. and M. Vanderhaeghen (2015)

inelastic
F_1, F_2

- neutron magnetic form factor determines Born contribution

$\nu_{1S}^{2\gamma}$	ep, Hz	en, Hz
Born	-39.9(6.8)	9.4(0.6)
subtraction	18.5(4.4)	28.8(10.2)
inelastic	-83.6(5.7)	-81.7(8.4)
total	-109.2(12.0)	-43.5(13.2)

- 2γ on nucleons in electronic and muonic nuclei

O. T. (2019)

- neutral neutron vs charged proton differs in Born contribution

HFS from 2γ on nucleons

2γ to HFS on neutrons

$$\delta E^{2\gamma}_{HFS} \sim \vec{s}_l < \sum \mu_N \vec{s}_N r^N_{2\gamma} >$$

- traditional decomposition of 2γ:

$$r^N_{2\gamma} = r_Z + r^R + r^{pol}$$

Zemach term
$G_E,\ G_M$

recoil correction
$G_E,\ G_M$

polarizability
F_P, g_1, g_2

- leading correction:

$$r_Z = -\frac{4}{\pi}\int_0^\infty \frac{dQ}{Q^2}\frac{G_M(Q^2)\,G_E(Q^2)}{\mu_n}$$

- Zemach radius expanding form factors:

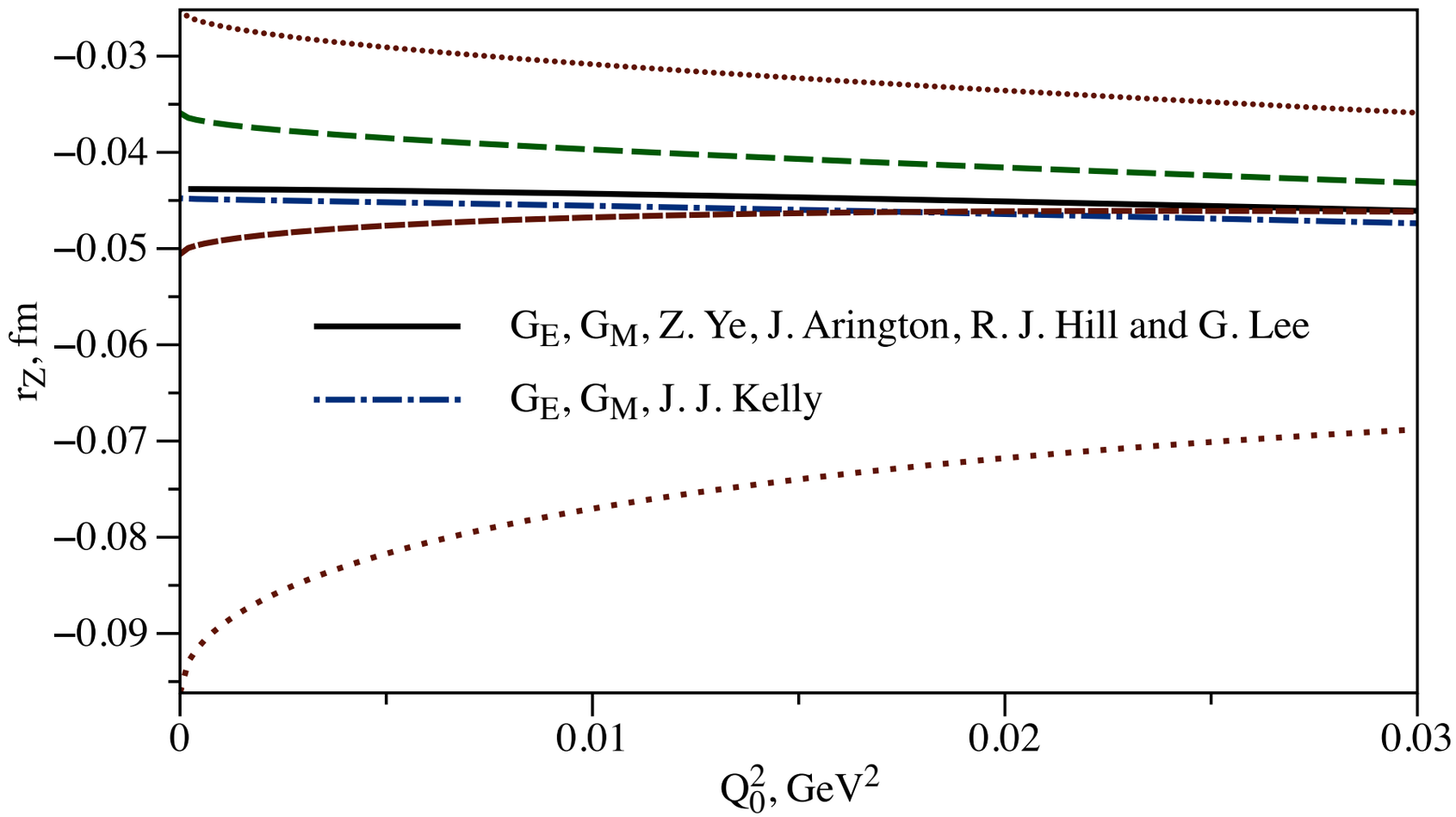
$$r_Z = -\frac{4}{\pi}\int_{Q_0}^\infty \frac{dQ}{Q^2}\frac{G_M(Q^2)\,G_E(Q^2)}{\mu_n} + \frac{2Q_0 r_E^2}{3\pi}\left(1 - \frac{r_M^2}{18}Q_0^2\right)$$

extraction of radii by S. G. Karshenboim (2014)
calculation of r_Z for proton by O. T. (2017)

- dependence on splitting value: consistency check

O. T. (2017)

2γ to HFS on neutrons



- Zemach radius expanding form factors:

$$r_Z = -\frac{4}{\pi}\int_{Q_0}^{\infty}\frac{dQ}{Q^2}\frac{G_M\left(Q^2\right)G_E\left(Q^2\right)}{\mu_n} + \frac{2Q_0 r_E^2}{3\pi}\left(1-\frac{r_M^2}{18}Q_0^2\right)$$

extraction of radii by S. G. Karshenboim (2014)

calculation of r_Z for proton by O. T. (2017)

- dependence on splitting value: consistency check

O. T. (2017, 2019)

- two form factor parametrizations are valid for r_Z calculation

2γ to HFS on neutrons vs protons

- polarizability for neutron is estimated from MAID solution assuming a similar to proton saturation as for GDH sum rule

$\nu_{\mathrm{HFS}}^{2\gamma}$	en, fm	μn, fm
Zemach	-0.0449(13)	-0.0449(13)
recoil	0.328(2)	0.0823(8)
polarizability	0.064(38)	0.065(39)
total	0.347(38)	0.102(39)

- proton effective radii are the same for e and μ:

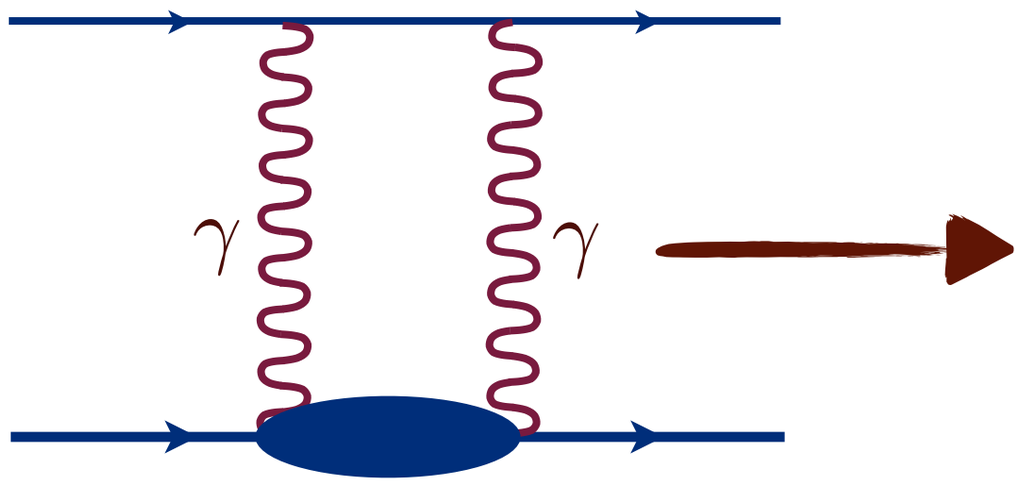
$\nu_{\mathrm{HFS}}^{2\gamma}$	ep, fm	μp, fm
Zemach	1.055(13)	1.055(13)
recoil	-0.1411(14)	-0.1203(8)
polarizability	-0.051(13)	-0.052(13)
total	0.863(20)	0.883(19)

- lepton mass scale contributes to recoil correction

O. T. (2019)

- effective radii e vs μ are distinct for neutron

Conclusions



largest theoretical uncertainty
in low-energy proton structure

- total 2γ estimate in ep and μp: proton+inelastic states

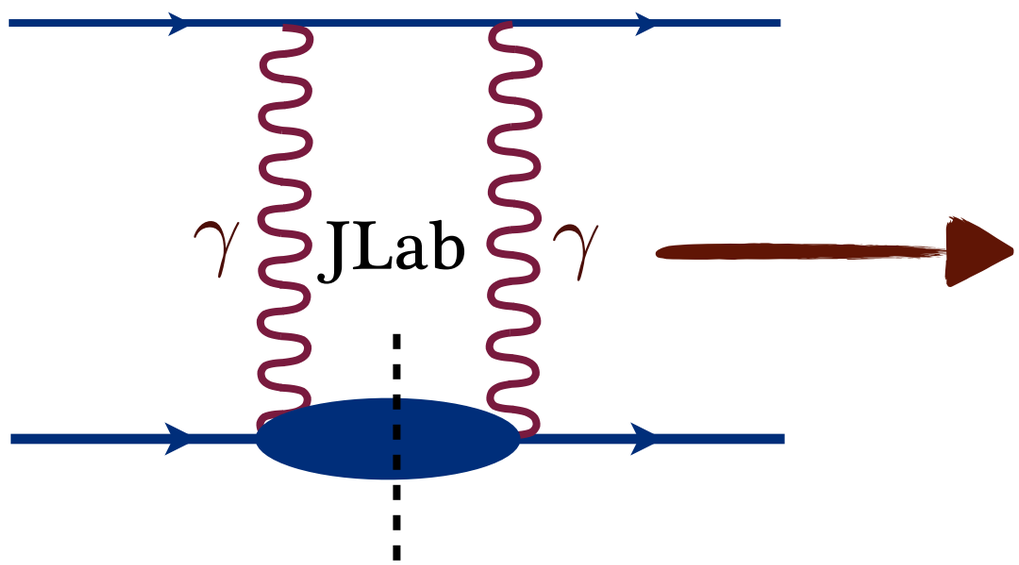
- dispersive framework in ep: elastic and πN
  input for nucleon form factor fits

- dispersive framework in μp: elastic; expect MUSE data

- precise 2γ in μH from 1S HFS in eH

- 2γ correction to 1S-2S transition in eH, 2γ on neutrons

Conclusions



largest theoretical uncertainty
in low-energy proton structure

- total 2γ estimate in ep and μp: proton+inelastic states

- dispersive framework in ep: elastic and πN
  input for nucleon form factor fits

- dispersive framework in μp: elastic; expect MUSE data

- precise 2γ in μH from 1S HFS in eH

- 2γ correction to 1S-2S transition in eH, 2γ on neutrons

Thanks for your attention !!!

Future projects

- ep elastic scattering at low Q^2:

ProRad@PRAE

$\omega = 30 - 70$ MeV

$Q^2 = 10^{-5} - 10^{-4}$ GeV^2

ep scattering@MAMI

$\omega = 500$ and 720 MeV

$Q^2 = 0.002 - 0.02$ GeV^2

Ultra-Low Q^2@Tohoku

$\omega = 20 - 60$ MeV

$Q^2 = 0.0003 - 0.008$ GeV^2

ISR@MAMI

$\omega = 0.2 - 0.5$ GeV

$Q^2 = 0.001 - 0.17$ GeV$^2 \rightarrow 0.0002$ GeV^2

arXiv: 1905.11182

MAGIX@MESA

magnetic form factor

Future projects

- μp elastic scattering at low Q^2:

MUSE@PSI

$\omega = 115 - 210$ MeV

$Q^2 = 0.0016 - 0.08$ GeV^2

e and μ

COMPASS@CERN

$\omega = 100$ GeV

$Q^2 = 0.001 - 0.2$ GeV^2

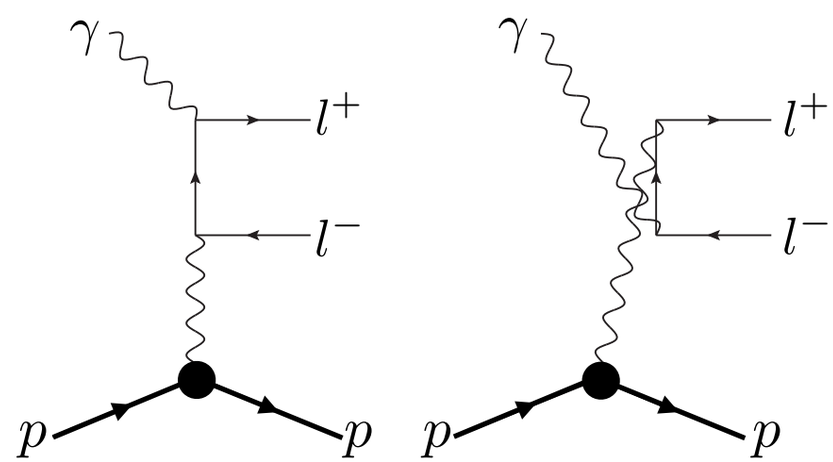
small structure effects

- universality test by lepton-pair photoproduction:

γp -> l^+l^-p@MAMI

$\omega = 0.5 - 1.5$ GeV

$Q^2 = 0.0018 - 0.042$ GeV^2

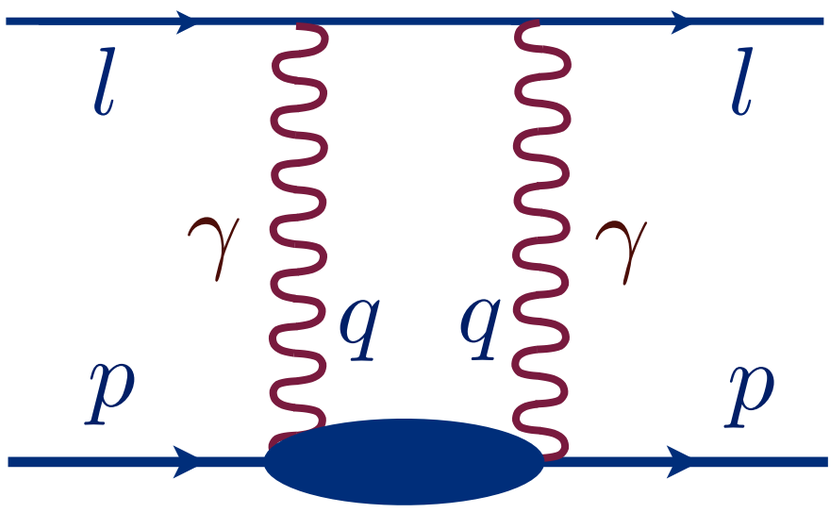


$$\frac{\sigma(e^+e^-) + \sigma(\mu^+\mu^-)}{\sigma(e^+e^-)}$$

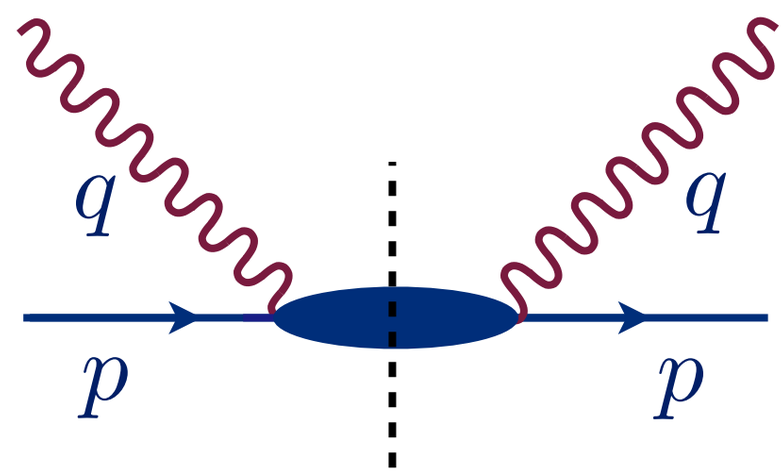
below and above muon threshold

normalisation and proton structure errors are suppressed

Lamb shift 2γ correction. Forward VVCS



2γ blob - forward virtual Compton scattering



Optical theorem

$\mathrm{Im}\ \mathrm{T}_1 \sim F_1$ $\mathrm{Im}\ \mathrm{T}_2 \sim F_2$ $\mathrm{Im}\ \mathrm{S}_1 \sim g_1$ $\mathrm{Im}\ \mathrm{S}_2 \sim g_2$

Fixed-Q^2 dispersion relations

Disp. rel. for amplitude T_1 requires subtraction function

$$\mathrm{T}_1^{\mathrm{subt}}(0,Q^2) \equiv \mathrm{T}_1(0,Q^2) - \mathrm{T}_1^{\mathrm{Born}}(0,Q^2)$$

Unsubtracted disp. rel. works for $\mathrm{T}_2,\ \mathrm{S}_1,\ \mathrm{S}_2$

Empirical estimate of subtraction function

Subtract Regge behavior + DR + resonance region and DIS data

G. Gasser, H. Leutwyler et al. (1974, 2015) M. Gorchtein et al. (2013) I. Caprini (2016)

expected low-Q^2 behavior

$$\mathrm{T}_1^{\mathrm{subtr}}(0, Q^2) = \beta(Q^2)Q^2$$

$\beta(0) = \beta_{\mathrm{M}}$ - magnetic polarizability

Empirical estimate of subtraction function

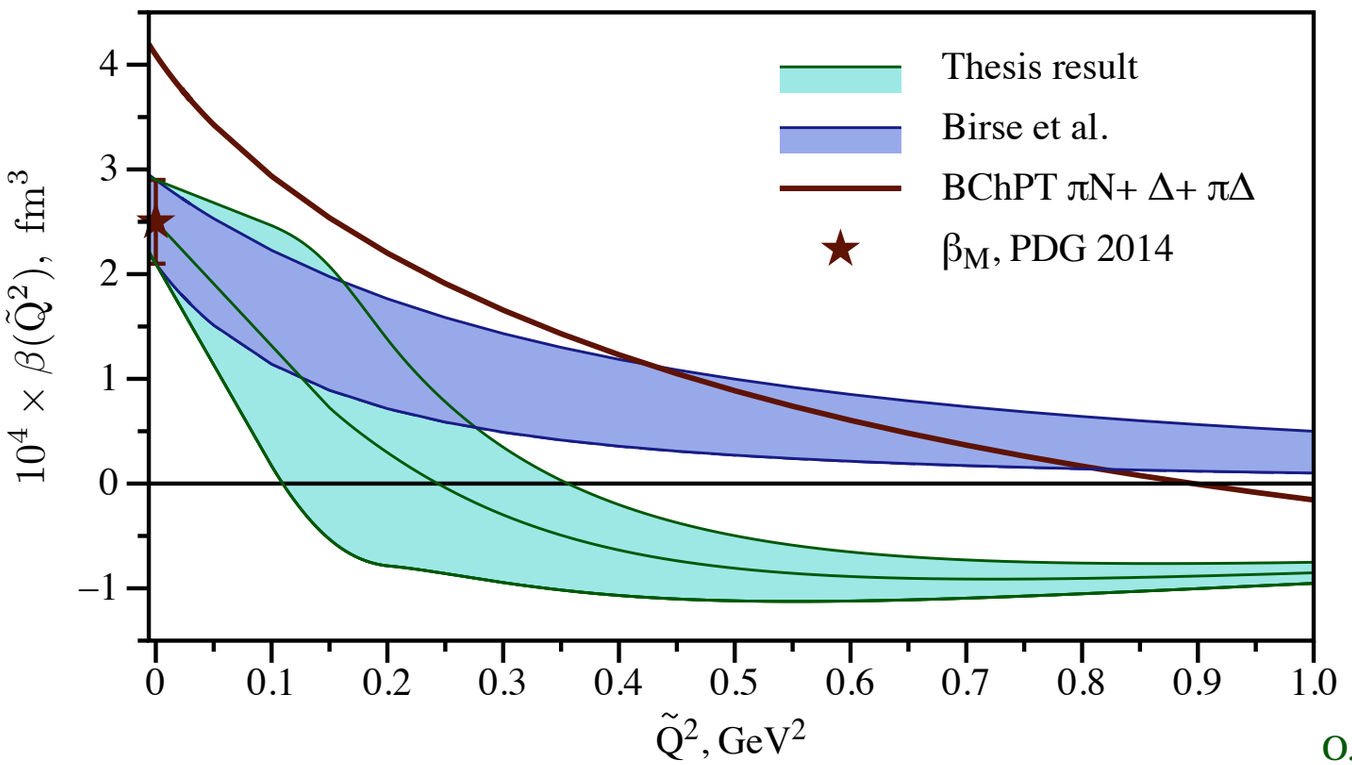
Subtract Regge behavior + DR + resonance region and DIS data

G. Gasser, H. Leutwyler et al. (1974, 2015) M. Gorchtein et al. (2013) I. Caprini (2016)

expected low-Q^2 behavior

$T_1^{\rm subtr}(0, Q^2) = \beta(Q^2)Q^2$ $\beta(0) = \beta_{\rm M}$ - magnetic polarizability

data-based result vs. theoretical predictions



O. T. and M. Vanderhaeghen (2016)

Lamb shift correction

$\Delta E_{2S}^{\rm subt}(\mu H) \approx 2.3 \pm 1.3\ \mu{\rm eV}$

slightly smaller than Birse et al.

$\Delta E_{2S}^{\rm subt}(\mu H) \approx 4.2 \pm 1.0\ \mu{\rm eV}$