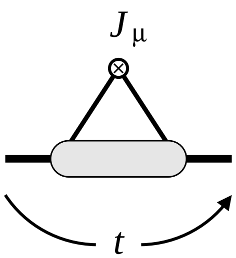
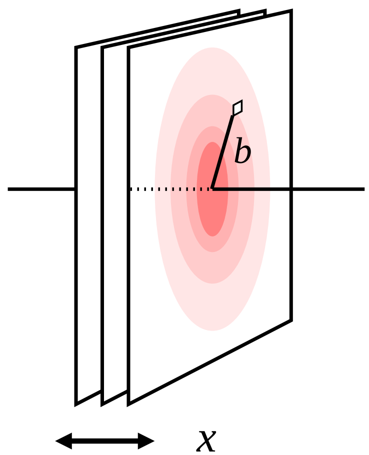
Exploring hadron structure with EFT-based methods

C. Weiss (JLab), JLab Theory Cake Seminar, 06-Feb-2019

based on work with J. M. Alarcon, C. Granados, A. Hiller Blin, M. Strikman, D. Higinbotham, Zhihong Ye





Systematic methods:
Chiral effective field theory
Analytic amplitude methods
$1/N_c$ expansion of QCD

- Hadron structure in QCD

  Operators and matrix elements

- Nucleon form factors $\qquad |t| \lesssim 1$ GeV^2

  New method – dispersively improved χEFT

  Vector FFs and proton radius extraction

  Scalar FF and radius

  Extensions: Baryon resonances, 3π unitarity

- Transverse densities and GPDs

  Peripheral structure $b \sim 1/M_\pi$

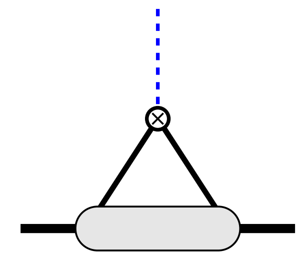
  Light-front formulation of χEFT

  x-dependent distributions

Hadron structure: Operators

- Local operators

$\bar{\psi}\gamma^\mu\psi$	vector	elastic $eh/\mu h$ scattering, e^+e^- annihilation
$\bar{\psi}\gamma^\mu\gamma^5\psi$	axial	weak interactions, PV, νh scattering
$\bar{\psi}\sigma^{\mu\nu}\psi$	tensor	BSM processes
$m\bar{\psi}\psi$, $F^2_{\mu\nu}$	scalar	quark mass dep, trace anomaly, τ decays
EM tensor		gravity; mass, momentum, spin, forces

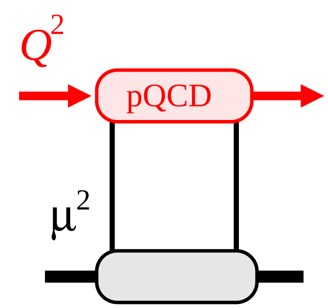


- Non-local operators

  $\bar{\psi}(0)\ldots\psi(z)$, $z^2 = 0$, light-cone operators twist-2, 3...

  Factorization of high-momentum transfer processes:
  DIS, inclusive, exclusive, semi-inclusive

  Scale μ^2, dependence $\mu\,(d/d\mu)$ from RNG equation



- Hadronic matrix elements

$\langle h'\vert\mathcal{O}\vert h\rangle$	transition, expectation value
$\langle hh'\vert\mathcal{O}\vert 0\rangle$	creation/annihilation, incl. multihadron states, resonances

Hadron structure: Methods

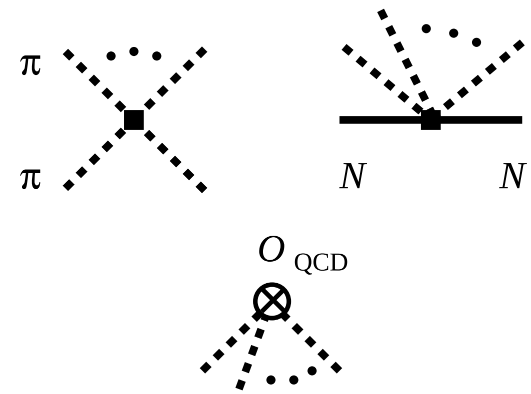
- Chiral effective field theory

  Describes dynamics at $k_\pi \sim M_\pi \ll \Lambda_\chi \sim 1$ GeV

  Constructed/solved in expansion in $\{k_\pi, M_\pi\}/\Lambda_\chi$

  Includes baryons N, Δ as dynamical fields

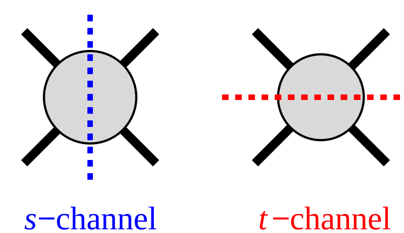
  Permits matching with QCD operators



- Analytic amplitude methods

  Dispersion relations   Amp $= \int$ Singularities

  Unitarity, N/D method

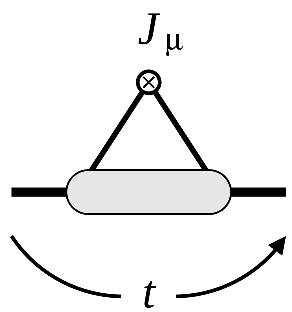


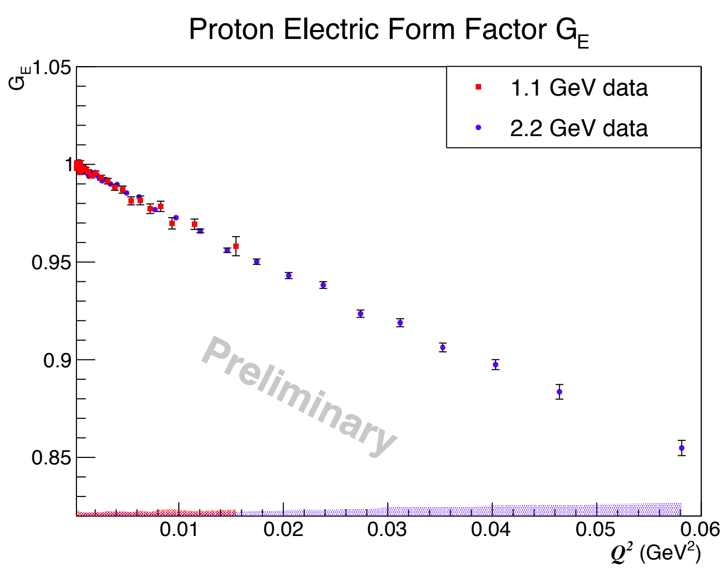
- $1/N_c$ expansion of QCD

  Parametric classification of meson/baryon matrix elements, spin-flavor components

Systematic methods: Parametric expansion, defined accuracy, uncertainty estimates
Reduce complexity, relate structures, provide insight
Predictive, but require dynamical input!

Nucleon FFs: Electromagnetic FFs





JLab PRad preliminary Gasparian et al. 2018

- Current matrix element parametrized by invariant form factors

  $\langle N'|J_\mu|N\rangle \rightarrow F_1(t), F_2(t)$ Dirac/Pauli

  Much interest in low-$|t|$ FFs!

- Next-gen elastic scattering experiments

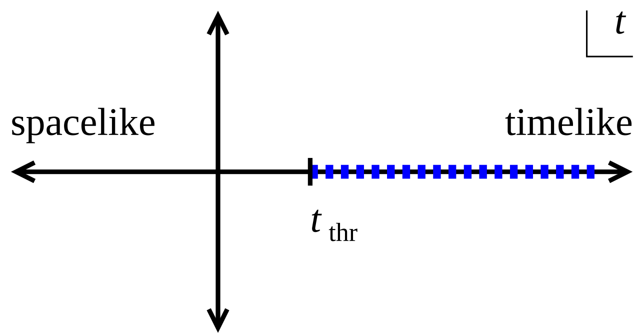
  Mainz MAMI ep
  JLab PRad ep down to $Q^2 \sim 10^{-4}$ GeV^2
  PSI MUSE $e^+p/e^-p/\mu^+p/\mu^-p$

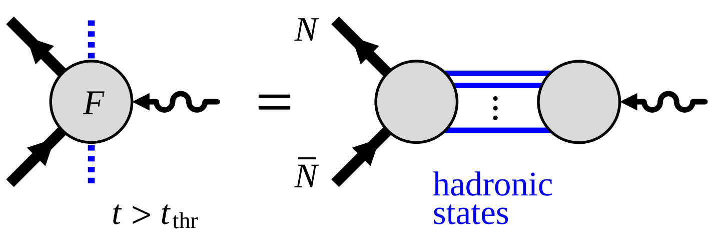
- Proton radius puzzle

  Discrepancies between charge radius from electronic hydrogen, muonic hydrogen electron scattering
  Reviews Pohl 2013, Carlson 2015, Miller 2018

- Transverse densities $\leftrightarrow$ GPDs

Nucleon FFs: Dispersive representation





Isovector: $\pi\pi$ (incl. ρ), 4π, $K\bar{K}$, ...
Isoscalar: 3π (incl. ω), $K\bar{K}$ (incl. ϕ), ...

- Dispersive representation

$$F_i(t) = \int_{t_{\rm thr}}^{\infty} \frac{dt'}{\pi} \frac{\mathrm{Im}\, F_i(t')}{t' - t - i0}$$

Expresses analytic structure of $F_i(t)$

- Spectral functions $\mathrm{Im}\, F_i(t)$

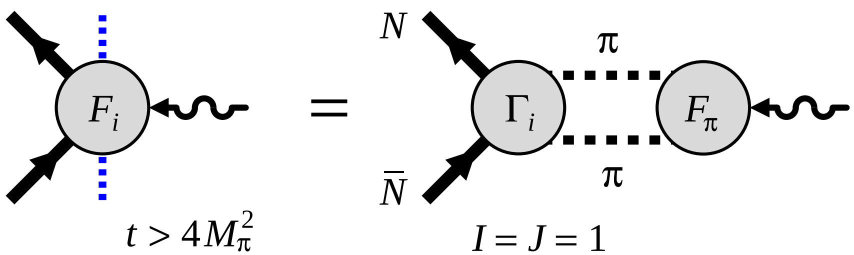
Current $\rightarrow$ hadronic states $\rightarrow$ $N\bar{N}$

Processes in unphysical region $t < 4M_N^2$

Spectral functions to be provided by theory
Frazer, Fulco 1960; Höhler et al 1975+...

- $\pi\pi$ cut — can we use χEFT?
Gasser, Sainio, Svarc 1988; Bernard, Kaiser, Meissner 1996; Becher, Leutwyler 1999; Kubis, Meissner 2003; Kaiser 03...

Nucleon FFs: Spectral functions on $\pi\pi$ cut



$$\mathrm{Im} F_i(t) = \frac{k_{\mathrm{cm}}^3}{\sqrt{t}} \, \Gamma_i(t) \;\; F_\pi^*(t)$$

$$= \frac{k_{\mathrm{cm}}^3}{\sqrt{t}} \underbrace{\frac{\Gamma_i(t)}{F_\pi(t)}}_{\chi\text{EFT}} \underbrace{|F_\pi(t)|^2}_{\text{Data}}$$

- Elastic unitarity relation

  $F_\pi(t)$ timelike pion FF, $\Gamma_i(t)$ partial-wave amplitude $\pi\pi \to N\bar{N}$

  Amplitudes have same phase from $\pi\pi$ rescattering — Watson's theorem

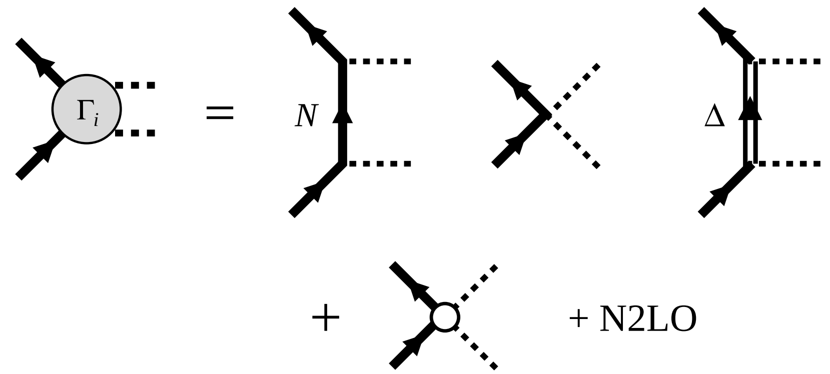
- Factorized representation (N/D method)

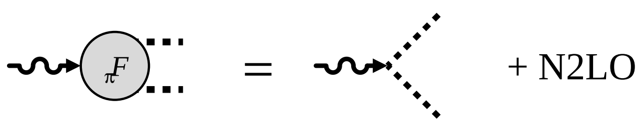
  Γ_i/F_π free of $\pi\pi$ rescattering $\rightarrow$ calculate in χEFT, well convergent

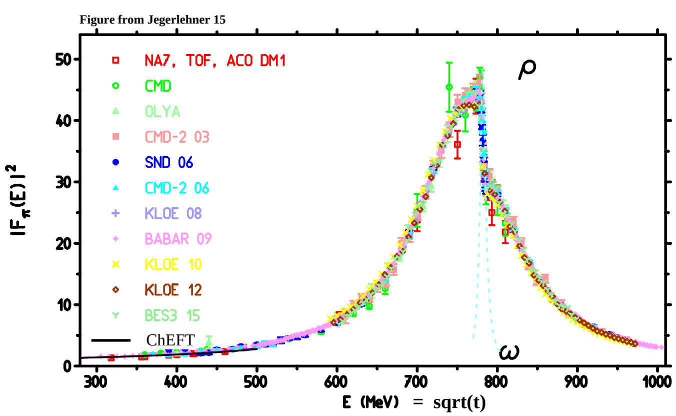
  $|F_\pi|^2$ includes $\pi\pi$ rescattering, ρ resonance $\rightarrow$ take from e^+e^- data, LQCD

- New χEFT-based approach
  Alarcon, Hiller Blin, Vicente Vacas, Weiss, NPA 964, 18 (2017);
  Alarcon, Weiss, PRC **96,** 055206 (2017) PRC **97,** 055203 (2018)
  Similar method: Granados, Leupold, Perotti 2017

Nucleon FFs: Chiral EFT







- Relativistic χEFT

  Expansion in $\{M_\pi, k_\pi\}/\Lambda_\chi$

  Include Δ isobar

- Calculation of $\Gamma_i(t)/F_\pi(t)$

  LO: Born terms + Weinberg-Tomozawa

  NLO: Contact term in $\Gamma_i(t)$

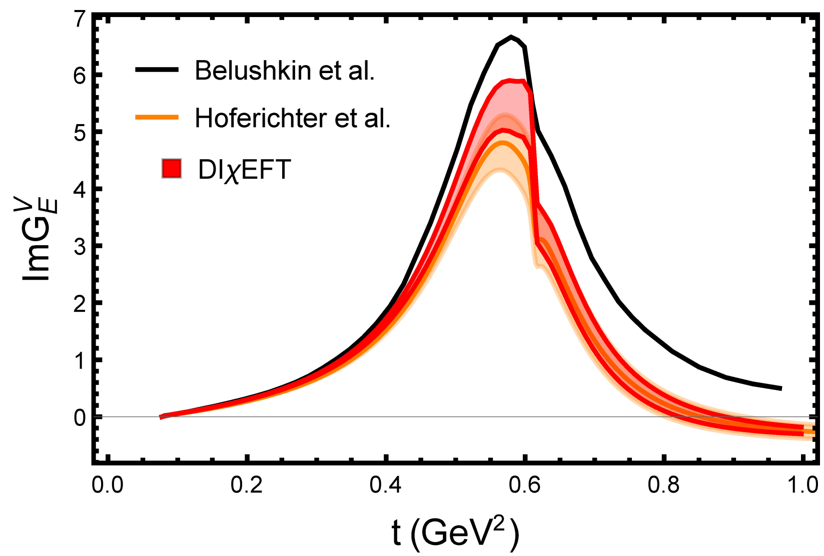
  N2LO: Contact term and pion loops

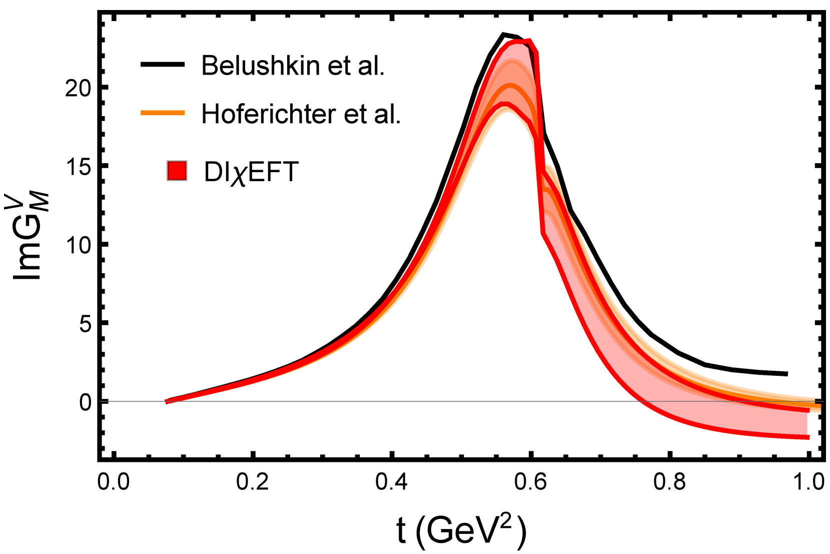
  Good convergence

- Pion timelike FF $|F_\pi(t)|^2$

  Measured accurately in $e^+e^- \to \pi^+\pi^-$

Nucleon FFs: Spectral functions





- Spectral functions on $\pi\pi$ cut

  Include ρ resonance through $|F_\pi(t)|^2$

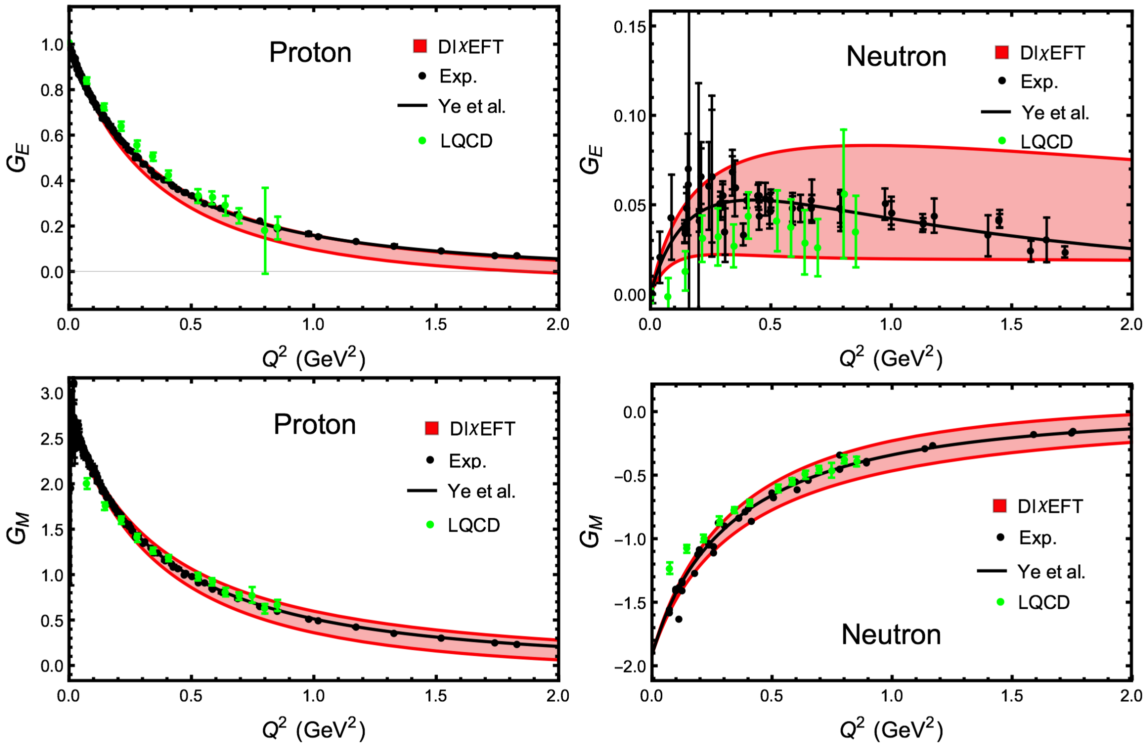
  Good agreement with Roy-Steiner analysis
  Hoferichter et al 2017

- Qualitative improvement compared to traditional χEFT

  $\pi\pi$ rescattering effects included

Alarcon, Weiss, PLB 784, 373 (2018)

Nucleon FFs: Spacelike FF predictions



$$G_i(t) = \int_{4M_\pi^2}^{\infty} \frac{dt'}{\pi} \, \frac{\mathrm{Im}\, G_i(t')}{t' - t - i0}$$

Alarcon, Weiss, PLB 784, 373 (2018)
Uncertainty bands: PDG range of nucleon radii

- Form factors evaluated using DR

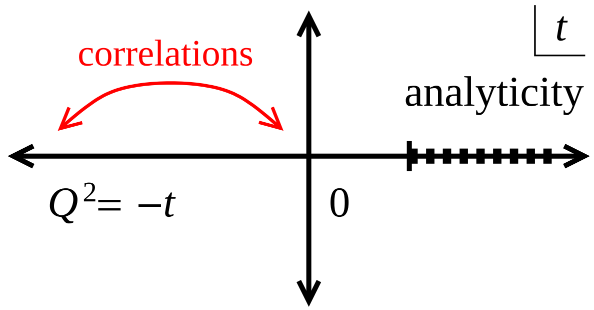
  $\pi\pi$ isovector spectral function calculated in DIχEFT

  High-mass states described by effective pole, strength fixed by sum rules (charges, radii)

- Excellent description of data up to $|t| \sim 1$ GeV^2

  Not fit, but dynamical prediction. Theoretical uncertainty estimates

Nucleon FFs: Proton radius extraction



- Proton radius from electron scattering

  Data at $Q^2 > 0 \quad \leftrightarrow \quad$ Slope at $Q^2 = 0$

  Several methods, extensive literature
  Bernauer et al 10+, Lee et al 15, Griffioen et al 16, Higinbotham et al 16, Horbatsch et al 17. Review Yan et al 18

- Analyticity implies correlations

  Use data at "larger" $Q^2 \sim$ few 0.1 GeV^2 to constrain slope at $Q^2 = 0$

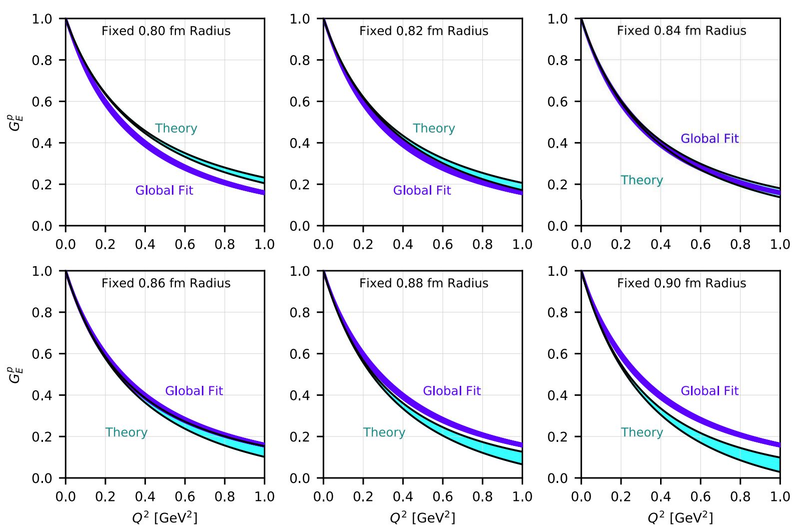
  Complement "extrapolation" methods

- DIχEFT-based extraction

  Used parametrization w. LECs $\leftrightarrow$ radii

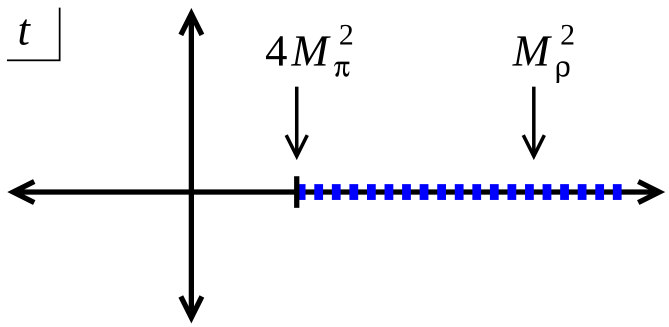
  Obtained $r_p = 0.844(7)$ fm

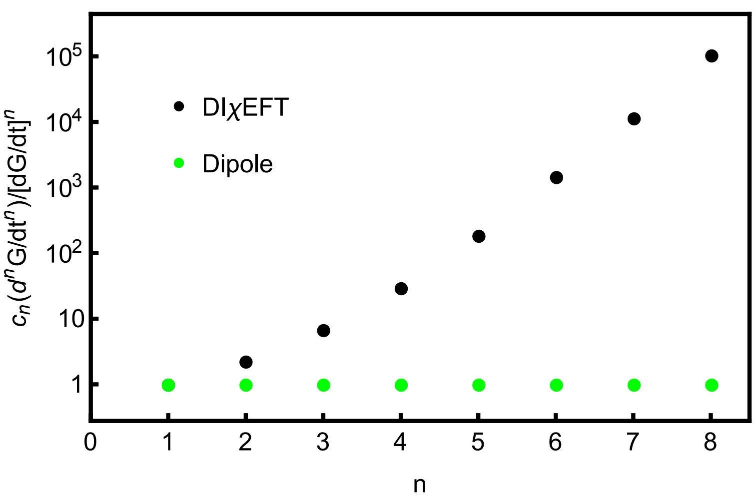
  Quantified thy and exp uncertainties



Alarcon, Higinbotham, Weiss, Ye, arXiv:1809.06373
Global FF fit adapted from Ye et al 2017

Nucleon FFs: Higher derivatives





Alarcon, Weiss, PRC **97,** 055203 (2018)

- Form factor derivatives from DR

$$\left.\frac{d^n G_i^V(t)}{dt^n}\right|_{t=0} = \int_{4M_\pi^2}^{\infty} \frac{dt'}{\pi} \frac{\mathrm{Im}\, G_i^V(t')}{t'^{n+1}}$$

- Two dynamical scales

  $4M_\pi^2$ two-pion threshold

  M_ρ^2 maximum of spectral function

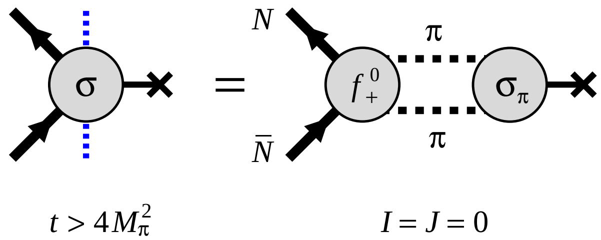
  Relative weight depends on n

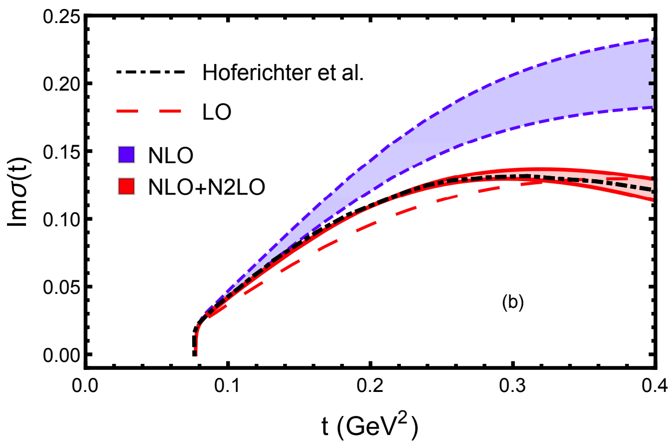
- Unnatural size of higher derivatives

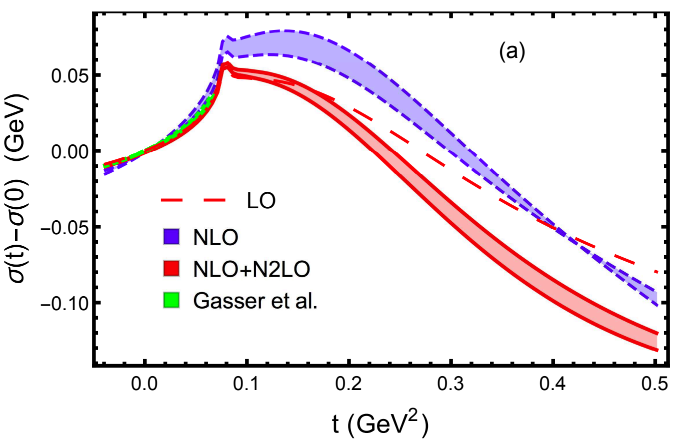
  Model-independent prediction

  Could be tested in polynomial fits
  JLab PRad first study: Yan et al 18

Nucleon FFs: Scalar FF







Alarcon, Weiss, PRC **96,** 055206 (2017)

- Scalar QCD operator

  $O_\sigma = \hat{m}\bar{\psi}\psi$ scale-independent

  $\langle N'|O_\sigma|N\rangle \to \sigma(t)$ scalar nucleon FF

- Scalar nucleon FF from DIχEFT

  Im $\sigma(t)$ from unitarity + ChEFT + $|\sigma_\pi(t)|^2$

  $\sigma_\pi(t)$ from empirical dispersion analysis
  Colangelo et al 04; Celis, Cirigliano, Passemar 14

$$\sigma(t) = \sigma(0) + \frac{t}{\pi}\int_{4M_\pi^2}^{\infty} dt' \, \frac{\text{Im}\,\sigma(t')}{t'(t'-t)}$$

- Scalar nucleon radius predicted

  $\langle r^2\rangle_\sigma = 1.03\text{–}1.13\ \text{fm}^2$ [with $\sigma(0) = 59$ MeV]

  $\gg \langle r^2\rangle_1$ vector radius

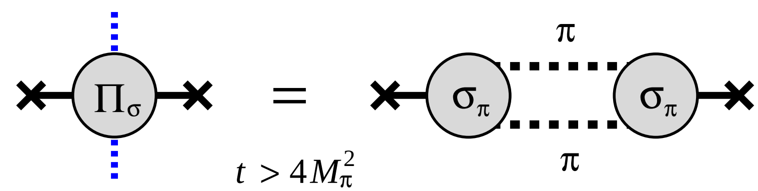
Nucleon FFs: Pion timelike FF from correlator

- DIχEFT requires only modulus $|\sigma_\pi(t)|^2$, not complex $\sigma_\pi(t)$

- Modulus can be extracted from vacuum correlator

$$\Pi_\sigma(t) \;\equiv\; i \int d^4x\; e^{iqx}\; \langle 0|\, \mathrm{T}\, O_\sigma(x) O_\sigma(0)\, |0\rangle, \qquad t \equiv q^2$$

$$\mathrm{Im}\; \Pi_\sigma(t) = \frac{k_{\mathrm{cm}}}{8\pi\sqrt{t}}\, |\sigma_\pi(t)|^2 + \ldots$$



$$\langle 0|\, \mathrm{T}\, O_\sigma(x) O_\sigma(0)\, |0\rangle \;\sim\; \int_{4M_\pi^2}^{\infty} dt\; \frac{\sqrt{t}\, K_1(\sqrt{t}|x|)}{4\pi^2 |x|}\; \mathrm{Im}\,\Pi_\sigma(t), \qquad |x| \equiv \sqrt{-x^2} \to \infty$$

Pion timelike FF governs asymptotic behavior of correlator at large Euclidean distances

- Possible combination DIχEFT $\leftrightarrow$ Euclidean methods, LQCD

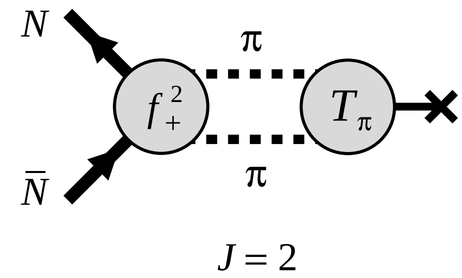
  Alt approach: Timelike pion FF from Lüscher method. H. Meyer 2011

Nucleon FFs: Applications and extensions

- Nucleon FFs of other QCD operators

  Spin-1 tensor operator

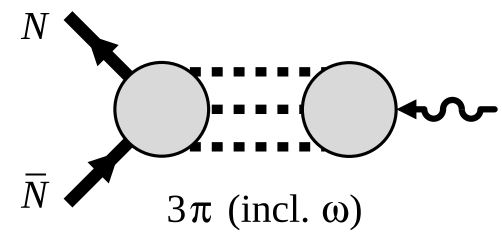
  Spin-2 energy-momentum tensor:
  Mass, momentum, spin, forces (D-term)



- Nucleon FFs with 3π cut

  Isoscalar-vector FF, isovector-axial FF

  Use methods of 3-body unitarity, presently being
  developed for amplitude analysis and LQCD
  Szczepaniak et al, Jackura, Pilloni, Döring et al



- Resonance transition FFs, e.g. $N \to \Delta$

  S-matrix theory: Stable particle transition ME $\langle \pi N | \mathcal{O} | N \rangle$,
  pole at $s_{\pi N} = M_\Delta^2$ complex, residue factorizes

  Can be implemented in χEFT — Ledwig et al 10

  LQCD results for Δ FFs — Alexandrou et al 08; Aubin, Orginos, Pascalutsa, Vanderhaegehen 08

Nucleon FFs: 1/N expansion of QCD

- Study scaling behavior of non-perturbative QCD quantities with N_c: Meson and baryon masses, current matrix elements, hadronic couplings, ...
  'tHooft 73, Witten 79

  N_c scaling can be established on general grounds

  Parametric classification, hierarchy of structures, qualitative insight

  Very successful phenomenology

  $N_c \to \infty$ corresponds to semiclassical limit of QCD

- Relations between N and Δ

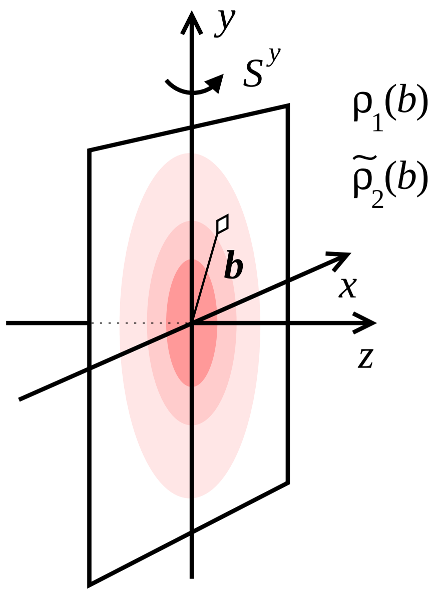
  $M_N, M_\Delta = \mathcal{O}(N_c), \; M_\Delta - M_N = \mathcal{O}(N_c^{-1})$ — N, Δ almost degenerate

  $g_{\pi N\Delta} = \frac{3}{2} g_{\pi NN}$ — Pion couplings simply related

  $\langle B'|J^\mu|B\rangle =$ common function — N and Δ current MEs related

- χEFT results have correct N_c-scaling if Δ isobar included as dynamical DoF
  Cohen, Broniowski 90's. Alarcon, Weiss 2018; Strikman CW 2004/2010; Granados, CW 2013+
  EFT with combined chiral and $1/N_c$ expansion: Goity, Calle Cordon, Fernando

Transverse densities: Concepts



- Transverse densities

Soper 76, Burkardt 00, Miller 07

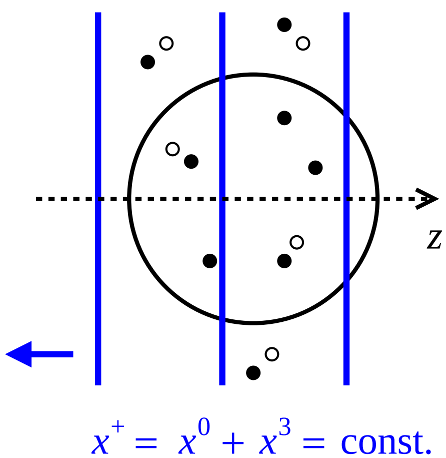
$$F_{1,2}(t = -\boldsymbol{\Delta}_T^2) \;=\; \int d^2b\; e^{i\boldsymbol{\Delta}_T \boldsymbol{b}}\; \rho_{1,2}(b)$$

Transverse charge/magnetization density

- Structure at fixed light-front time $x^+ = x^0 + x^3$

Frame-independent densities

Spatial representation appropriate for relativistic systems

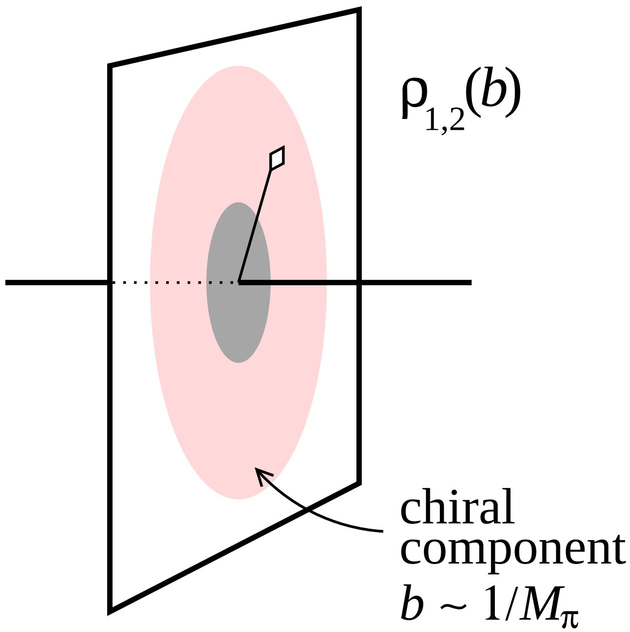


- Connection with GPDs/QCD

$$\rho_1(b) \;=\; \textstyle\sum_q e_q \displaystyle\int_0^1 dx\; [q - \bar{q}](x, b)$$

$\tilde{\rho}_2(b)$ spin-dep distortion

Transverse densities: Chiral periphery



Strikman, Weiss 2004/2010+

- Peripheral densities $b = \mathcal{O}(M_\pi^{-1})$

  Governed by chiral dynamics, model-independent

  Calculable using χEFT + dispersion theory

- Interest for low-energy structure

  Transverse distance as parameter

  Proper definition of mesonic component

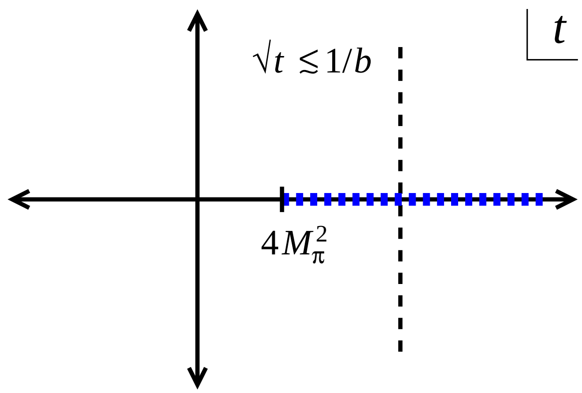
  Space-time picture of chiral dynamics

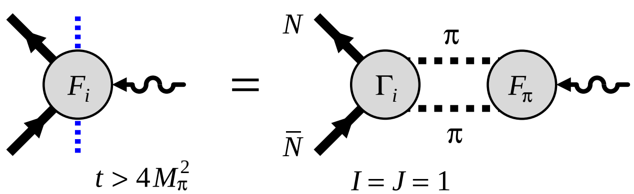
- Interest for high-energy processes

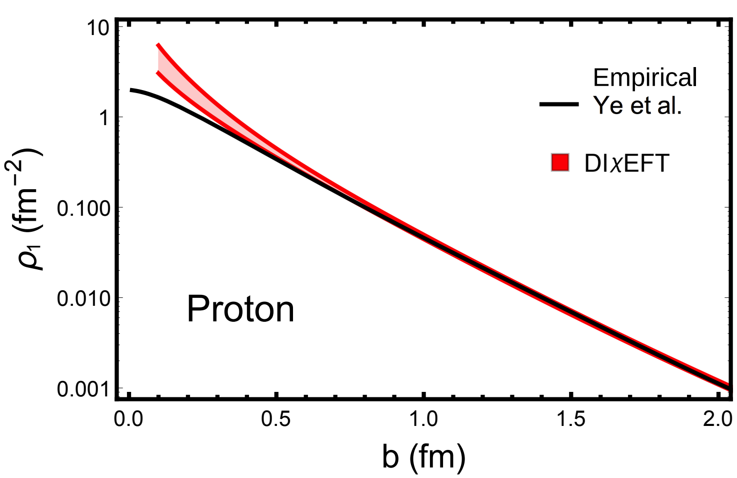
  Peripheral quark/gluon structure, GPDs

  Peripheral hard processes   JLab12, EIC, RHIC/LHC

Transverse densities: Dispersive representation







Alarcon, Weiss 2019. Includes uncertainties

- Dispersive representation

$$\rho(b) = \int_{4M_\pi^2}^{\infty} \frac{dt}{2\pi} K_0(\sqrt{t}b) \frac{\mathrm{Im}\, F(t)}{\pi}$$

$K_0 \sim e^{-b\sqrt{t}}$ exponential suppression of large t

Distance b selects masses $\sqrt{t} \sim 1/b$: Filter

- Peripheral densities from $\pi\pi$ cut

Calculated with DIχEFT spectral functions

Describe empirical densities at $b \gtrsim 1$ fm

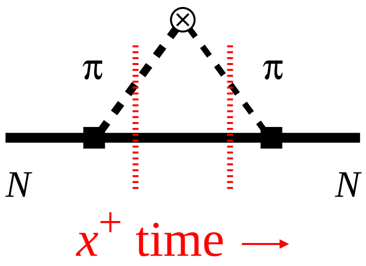
- Asymptotic behavior

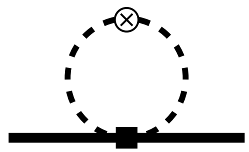
$$\rho_1^V, \widetilde{\rho}_2^V(b) = e^{-2M_\pi b} \times \mathcal{F}(M_N, M_\pi; b)$$

Rich structure, relation $\rho_1^V \leftrightarrow \widetilde{\rho}_2^V$

Granados, Weiss JHEP 1401, 092 (2014)

Transverse densities: Light-front formulation





$$\psi^{\pi N}_{L=0,1}(y, \boldsymbol{r}_T) \;=\; \underbrace{\frac{\langle \pi N | \mathcal{L}_\chi | N \rangle}{\mathcal{M}^2_{\pi N} - \mathcal{M}^2_N}}_{\text{invariant mass denominator}}$$

$$\rho^V_1(b) = \int_0^1 dy \; \left[|\psi_0|^2 + |\psi_1|^2 \right]_{r_T = b/\bar{y}}$$

$+$ contact term

$$\widetilde{\rho}^V_2(b) = \;\ldots\quad \psi_0^* \psi_1 + \psi_1^* \psi_0$$

- Evolution in LF time $x^+ = x^0 + x^3$

- Wave function of chiral πN system

  Describes transition $N \rightarrow N\pi$ in χEFT.
  Universal, frame-independent also in $\bar{u} - \bar{d}$ etc.

  Pion momentum fraction $y \sim M_\pi / M_N$,
  transverse distance $r_T \sim M_\pi^{-1}$

  Orbital angular momentum $L_z = 0, 1$

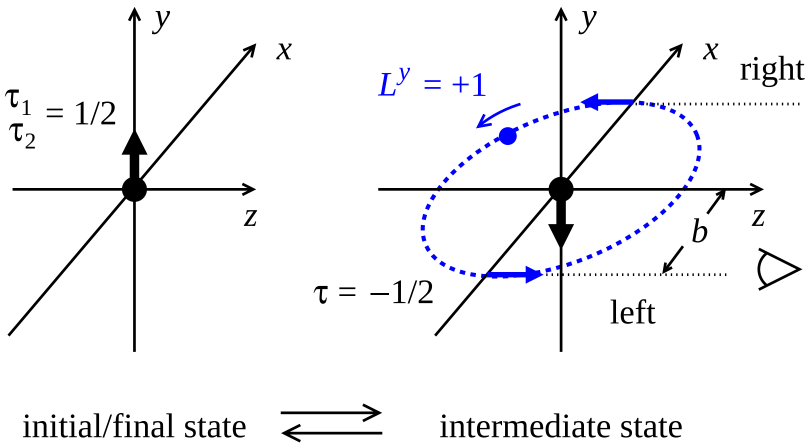
- Densities as wave function overlap

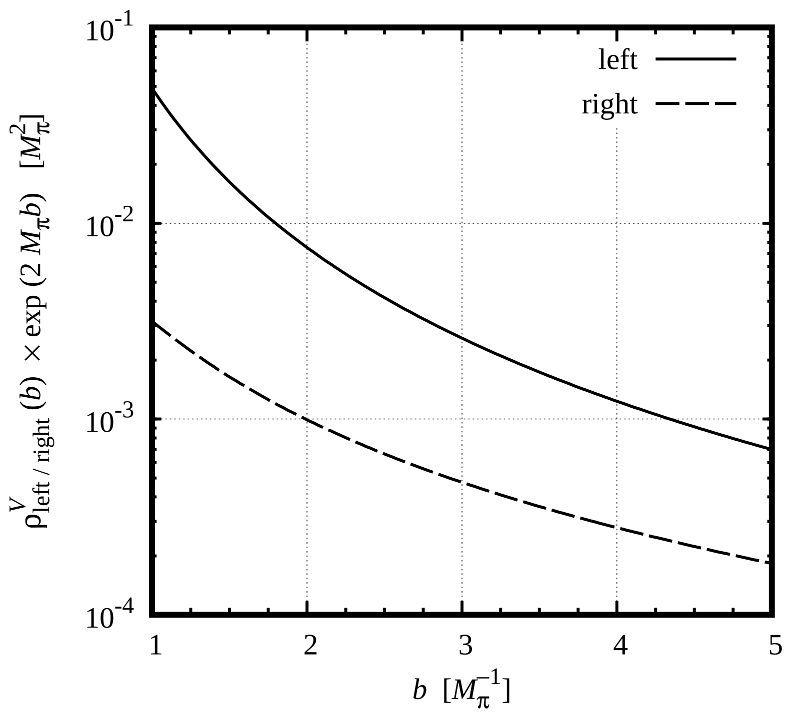
  Inequality $|\rho^V_2| < \rho^V_1$

  Contact terms $\delta(y)$ represent high-mass intermed states; coefficient $(1 - g_A^2)$

  Equivalent to invariant formulation
  Granados, CW 13. See also Ji, Melnitchouk et al. 09+

Transverse densities: Mechanical picture





- χEFT process as time sequence

  Rest frame, nucleon polarized in y–direction

  Bare N fluctuates into πN system via χEFT interaction

  Peripheral densities result from J^+ current carried by orbiting pion

- Explains peripheral densities

  $$\rho_1, \widetilde{\rho}_2 \;=\; \langle J^+\rangle_{\text{right}} \;\pm\; \langle J^+\rangle_{\text{left}}$$

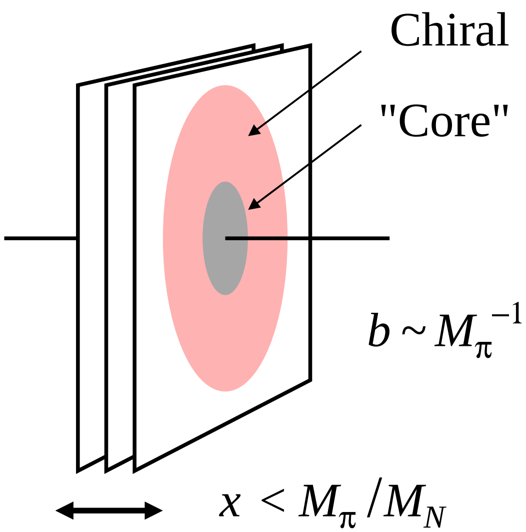
  $\langle J^+\rangle_{\text{left}} \gg \langle J^+\rangle_{\text{right}}$ large asymmetry

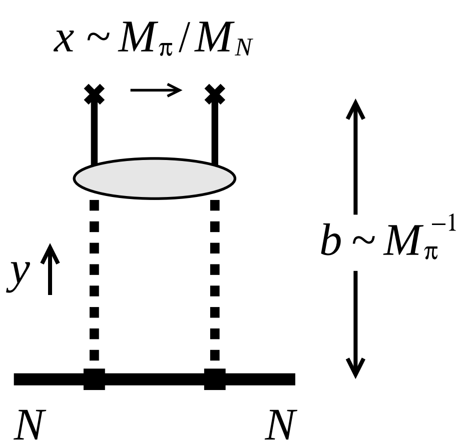
  Pion motion relativistic $k_\pi \sim M_\pi$

- Quantitative picture based on χEFT

  Granados, Weiss JHEP 1401, 092 (2014); JHEP 1507, 170 (2015)
  Extended to Δ, EM tensor FFs

Transverse densities: x dependence





- Transverse spatial distribution (GPD)

  $f(x, b)$ longitudinal momentum
  transverse position

- Chiral component

  $b \sim M_\pi^{-1}$ transverse distance

  $x \sim M_\pi / M_N$ mom fraction of soft pion

- Calculable model-independently

  Pion distribution in nucleon from χEFT

  Quark/gluon distn in pion from measurements

- Observables and measurements

  t-slope of hard exclusive processes $x \ll 0.1$

  Pion knockout in peripheral processes at EIC

Strikman, Weiss, PRD 69, 054012 (2004); PRD 80, 114029 (2009)

Summary

- Systematic methods for hadron structure in QCD:
  Chiral EFT, analytic amplitude methods, $1/N_c$ expansion

- DIχEFT new method for calculating $\pi\pi$ spectral functions of nucleon FFs

  Includes $\pi\pi$ rescattering in t-channel through unitarity + N/D + timelike pion FF

  Overcomes main limitation of traditional χEFT

- Numerous applications and extensions

  Electromagnetic FFs, peripheral transverse densities,

  Low-Q^2 ep scattering analysis, proton radius extraction

  Scalar FF and radius

  Energy-momentum tensor, other QCD operators, 3π cut, resonance FFs

- Chiral dynamics in peripheral partonic structure

  Transverse densities — mechanical picture, theoretical insight

  x-dependent distributions — experimental probes