Exploring hadron structure with EFT-based methods

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• Hadron structure in QCD
  Operators and matrix elements

• Nucleon form factors $|t| \lesssim 1 \text{ GeV}^2$
  New method – dispersively improved $\chi$EFT
  Vector FFs and proton radius extraction
  Scalar FF and radius
  Extensions: Baryon resonances, $3\pi$ unitarity

• Transverse densities and GPDs
  Peripheral structure $b \sim 1/M_\pi$
  Light-front formulation of $\chi$EFT
  $x$-dependent distributions

Systematic methods:
Chiral effective field theory
Analytic amplitude methods
$1/N_c$ expansion of QCD
Hadron structure: Operators

- **Local operators**
  - $\bar{\psi} \gamma^\mu \psi$: vector elastic $eh/\mu h$ scattering, $e^+e^-$ annihilation
  - $\bar{\psi} \gamma^\mu \gamma^5 \psi$: axial weak interactions, PV, $\nu h$ scattering
  - $\bar{\psi} \sigma^{\mu\nu} \psi$: tensor BSM processes
  - $m\bar{\psi}\psi$, $F_{\mu\nu}^2$: scalar quark mass dep, trace anomaly, $\tau$ decays
  - EM tensor: gravity; mass, momentum, spin, forces

- **Non-local operators**
  - $\bar{\psi}(0)\ldots\psi(z)$, $z^2 = 0$, light-cone operators twist-2, 3...

  Factorization of high-momentum transfer processes:
  - DIS, inclusive, exclusive, semi-inclusive

  Scale $\mu^2$, dependence $\mu (d/d\mu)$ from RNG equation

- **Hadronic matrix elements**
  - $\langle h'|\mathcal{O}|h \rangle$: transition, expectation value
  - $\langle hh'|\mathcal{O}|0 \rangle$: creation/annihilation, incl. multihadron states, resonances
Hadron structure: Methods

- **Chiral effective field theory**
  
  Describes dynamics at \( k_\pi \sim M_\pi \ll \Lambda_\chi \sim 1 \text{ GeV} \)
  
  Constructed/solved in expansion in \( \{k_\pi, M_\pi\}/\Lambda_\chi \)
  
  Includes baryons \( N, \Delta \) as dynamical fields
  
  Permits matching with QCD operators

- **Analytic amplitude methods**
  
  Dispersion relations \( \text{Amp} = \int \text{Singularities} \)
  
  Unitarity, \( N/D \) method

- **\( 1/N_c \) expansion of QCD**
  
  Parametric classification of meson/baryon matrix elements, spin-flavor components

*Systematic methods: Parametric expansion, defined accuracy, uncertainty estimates*

*Reduce complexity, relate structures, provide insight*

*Predictive, but require dynamical input!*
Nucleon FFs: Electromagnetic FFs

- Current matrix element parametrized by invariant form factors
  \[ \langle N' | J_\mu | N \rangle \rightarrow F_1(t), F_2(t) \]  
  Dirac/Pauli

  Much interest in low-|t| FFs!

- Next-gen elastic scattering experiments
  - Mainz MAMI ep
  - JLab PRad ep down to \( Q^2 \sim 10^{-4} \text{ GeV}^2 \)
  - PSI MUSE \( e^+p/e^-p/\mu^+p/\mu^-p \)

- Proton radius puzzle
  Discrepancies between charge radius from electronic hydrogen, muonic hydrogen electron scattering
  Reviews Pohl 2013, Carlson 2015, Miller 2018

- Transverse densities ⇔ GPDs
Nucleon FFs: Dispersive representation

- **Dispersive representation**

\[
F_i(t) = \int_{t_{thr}}^{\infty} \frac{dt'}{\pi} \frac{\text{Im} F_i(t')}{t' - t - i0}
\]

Expresses analytic structure of \( F_i(t) \)

- **Spectral functions** \( \text{Im} F_i(t) \)

Current \( \rightarrow \) hadronic states \( \rightarrow N \bar{N} \)

Processes in unphysical region \( t < 4M_N^2 \)

Spectral functions to be provided by theory
Frazer, Fulco 1960; Höhler et al 1975+

- \( \pi \pi \) cut — can we use \( \chi \)EFT?
Gasser, Sainio, Svarc 1988; Bernard, Kaiser, Meissner 1996;
Becher, Leutwyler 1999; Kubis, Meissner 2003; Kaiser 03...

**Isovector:** \( \pi \pi \) (incl. \( \rho \)), \( 4\pi \), \( K\bar{K} \), ...

**Isoscalar:** \( 3\pi \) (incl. \( \omega \)), \( K\bar{K} \) (incl. \( \phi \)), ...
Nucleon FFs: Spectral functions on $\pi\pi$ cut

\[ \text{Im} F_i(t) = \frac{k_{cm}^3}{\sqrt{t}} \Gamma_i(t) F_\pi^*(t) = \frac{k_{cm}^3}{\sqrt{t}} \frac{\Gamma_i(t)}{F_\pi(t)} |F_\pi(t)|^2 \]

\( \chi\text{EFT} \) \hspace{1cm} \text{Data}

- Elastic unitarity relation
  \( F_\pi(t) \) timelike pion FF, \( \Gamma_i(t) \) partial-wave amplitude $\pi\pi \rightarrow N\bar{N}$
  Amplitudes have same phase from $\pi\pi$ rescattering — Watson’s theorem

- Factorized representation (N/D method)
  \( \Gamma_i/F_\pi \) free of $\pi\pi$ rescattering
  $|F_\pi|^2$ includes $\pi\pi$ rescattering, $\rho$ resonance
  \( \rightarrow \) calculate in \( \chi\text{EFT} \), well convergent
  \( \rightarrow \) take from $e^+e^-$ data, LQCD

- New $\chi\text{EFT}$-based approach
  Alarcon, Hiller Blin, Vicente Vacas, Weiss, NPA 964, 18 (2017);
  Similar method: Granados, Leupold, Perotti 2017
Nucleon FFs: Chiral EFT

\[ \Gamma_i = N \times \Delta + \text{N2LO} \]

\[ F = \text{N2LO} \]

- Relativistic \( \chi \)EFT
  - Expansion in \( \{ M_\pi, k_\pi \} / \Lambda_\chi \)
  - Include \( \Delta \) isobar

- Calculation of \( \Gamma_i(t)/F_\pi(t) \)
  - LO: Born terms + Weinberg-Tomozawa
  - NLO: Contact term in \( \Gamma_i(t) \)
  - N2LO: Contact term and pion loops
    - Good convergence

- Pion timelike FF \( |F_\pi(t)|^2 \)
  - Measured accurately in \( e^+e^- \rightarrow \pi^+\pi^- \)
• Spectral functions on $\pi\pi$ cut
  
  Include $\rho$ resonance through $|F_\pi(t)|^2$

  Good agreement with Roy-Steiner analysis
  Hoferichter et al 2017

• Qualitative improvement compared to traditional $\chi$EFT
  
  $\pi\pi$ rescattering effects included
Nucleon FFs: Spacelike FF predictions

\[ G_i(t) = \int_{4M_R^2}^{\infty} \frac{dt'}{\pi} \frac{\text{Im} G_i(t')}{t' - t - i0} \]

- Form factors evaluated using DR
  \( \pi \pi \) isovector spectral function calculated in DI\( \chi \)EFT
  High-mass states described by effective pole, strength fixed by sum rules (charges, radii)

- Excellent description of data up to \(|t| \sim 1 \text{ GeV}^2\)
  Not fit, but dynamical prediction. Theoretical uncertainty estimates

Alarcon, Weiss, PLB 784, 373 (2018)
Uncertainty bands: PDG range of nucleon radii
Nucleon FFs: Proton radius extraction

- Proton radius from electron scattering

\[ Q^2 = t \]

Data at \( Q^2 > 0 \) ↔ Slope at \( Q^2 = 0 \)

Several methods, extensive literature
Bernauer et al 10+, Lee et al 15, Griffioen et al 16, Higinbotham et al 16, Horbatsch et al 17. Review Yan et al 18

- Analyticity implies correlations

Use data at “larger” \( Q^2 \sim \) few 0.1 GeV\(^2\) to constrain slope at \( Q^2 = 0 \)

Complement “extrapolation” methods

- DI\(\chi\)EFT-based extraction

Used parametrization w. LECs ↔ radii

Obtained \( r_p = 0.844(7) \) fm

Quantified thy and exp uncertainties

Alarcon, Higinbotham, Weiss, Ye, arXiv:1809.06373
Global FF fit adapted from Ye et al 2017
Form factor derivatives from DR

\[
\frac{d^n G^V_i(t)}{dt^n} \bigg|_{t=0} = \int_{4M^2_{\pi}}^{\infty} \frac{dt'}{\pi} \frac{\text{Im} G^V_i(t')}{t'^{n+1}}
\]

Two dynamical scales

- \(4M^2_{\pi}\) two-pion threshold
- \(M^2_{\rho}\) maximum of spectral function

Relative weight depends on \(n\)

Unnatural size of higher derivatives

Model-independent prediction

Could be tested in polynomial fits

JLab PRad first study: Yan et al 18

Alarcon, Weiss, PRC 97, 055203 (2018)
Nucleon FFs: Scalar FF

- Scalar QCD operator
  \[ O_\sigma = \hat{m} \bar{\psi} \psi \]
  scale-independent
  \[ \langle N' | O_\sigma | N \rangle \rightarrow \sigma(t) \]
  scalar nucleon FF

- Scalar nucleon FF from DIχEFT
  \[ \text{Im } \sigma(t) \text{ from unitarity + ChEFT + } |\sigma_\pi(t)|^2 \]
  \[ \sigma_\pi(t) \text{ from empirical dispersion analysis} \]
  Colangelo et al 04; Celis, Cirigliano, Passemard 14

\[ \sigma(t) = \sigma(0) + \frac{t}{\pi} \int_{4M_\pi^2}^{\infty} dt' \frac{\text{Im } \sigma(t')}{t'(t' - t)} \]

- Scalar nucleon radius predicted
  \[ \langle r^2 \rangle_\sigma = 1.03 - 1.13 \text{ fm}^2 \]
  [with \( \sigma(0) = 59 \text{ MeV} \)]
  \[ \gg \langle r^2 \rangle_1 \text{ vector radius} \]
Nucleon FFs: Pion timelike FF from correlator

- DIχEFT requires only modulus $|\sigma_\pi(t)|^2$, not complex $\sigma_\pi(t)$

- Modulus can be extracted from vacuum correlator

$$\Pi_\sigma(t) \equiv i \int d^4x \ e^{iqx} \langle 0 | T O_\sigma(x) O_\sigma(0) | 0 \rangle, \quad t \equiv q^2$$

$$\text{Im} \ \Pi_\sigma(t) = \frac{k_{cm}}{8\pi \sqrt{t}} |\sigma_\pi(t)|^2 + \ldots$$

$$\langle 0 | T O_\sigma(x) O_\sigma(0) | 0 \rangle \sim \int_{4M_\pi^2}^{\infty} dt \ \frac{\sqrt{t} K_1(\sqrt{t}|x|) \text{Im} \Pi_\sigma(t)}{4\pi^2 |x|}$$

Pion timelike FF governs asymptotic behavior of correlator at large Euclidean distances

- Possible combination DIχEFT ↔ Euclidean methods, LQCD

Alt approach: Timelike pion FF from Lüscher method. H. Meyer 2011
Nucleon FFs: Applications and extensions

• Nucleon FFs of other QCD operators
  
  Spin-1 tensor operator
  
  Spin-2 energy-momentum tensor:
  Mass, momentum, spin, forces (D-term)

• Nucleon FFs with $3\pi$ cut
  
  Isoscalar-vector FF, isovector-axial FF
  
  Use methods of 3-body unitarity, presently being developed for amplitude analysis and LQCD
  
  Szczepaniak et al, Jackura, Pilloni, Döring et al

• Resonance transition FFs, e.g. $N \rightarrow \Delta$
  
  S-matrix theory: Stable particle transition ME $\langle \pi N | O | N \rangle$,
  pole at $s_{\pi N} = M^2_{\Delta}$ complex, residue factorizes
  
  Can be implemented in $\chi$EFT
  
  LQCD results for $\Delta$ FFs
  
  Alexandrou et al 08; Aubin, Orginos, Pascalutsa, Vanderhaeghen 08
Nucleon FFs: 1/N expansion of QCD

- Study scaling behavior of non-perturbative QCD quantities with $N_c$:
  Meson and baryon masses, current matrix elements, hadronic couplings, ...

  'tHooft 73, Witten 79

  $N_c$ scaling can be established on general grounds

  Parametric classification, hierarchy of structures, qualitative insight

  Very successful phenomenology

  $N_c \to \infty$ corresponds to semiclassical limit of QCD

- Relations between $N$ and $\Delta$

  \[ M_N, M_\Delta = O(N_c), \quad M_\Delta - M_N = O(N_c^{-1}) \]

  $N, \Delta$ almost degenerate

  \[ g_{\pi NN} = \frac{3}{2} g_{\pi NN} \]

  Pion couplings simply related

  \[ \langle B' | J^\mu | B \rangle = \text{common function} \]

  $N$ and $\Delta$ current MEs related

- $\chi$EFT results have correct $N_c$-scaling if $\Delta$ isobar included as dynamical DoF

  Cohen, Broniowski 90’s, Alarcon, Weiss 2018; Strikman CW 2004/2010; Granados, CW 2013+

  EFT with combined chiral and $1/N_c$ expansion: Goity, Calle Cordon, Fernando
Transverse densities: Concepts

- **Transverse densities**

\[ F_{1,2}(t = -\Delta^2_T) = \int d^2b \ e^{i\Delta T^ b} \rho_{1,2}(b) \]

Transverse charge/magnetization density

- **Structure at fixed light-front time** \( x^+ = x^0 + x^3 \)

Frame-independent densities

Spatial representation appropriate for relativistic systems

- **Connection with GPDs/QCD**

\[ \rho_1(b) = \sum_q e_q \int_0^1 dx \ [q - \bar{q}](x, b) \]

\[ \tilde{\rho}_2(b) \] spin-dep distortion
Transverse densities: Chiral periphery

• Peripheral densities \( b = \mathcal{O}(M_\pi^{-1}) \):
  - Governed by chiral dynamics, model-independent
  - Calculable using \( \chi \)EFT + dispersion theory

• Interest for low-energy structure
  - Transverse distance as parameter
  - Proper definition of mesonic component
  - Space-time picture of chiral dynamics

• Interest for high-energy processes
  - Peripheral quark/gluon structure, GPDs
  - Peripheral hard processes: JLab12, EIC, RHIC/LHC

\( \rho_{1,2}(b) \)

chiral component

\( b \sim 1/M_\pi \)

Strikman, Weiss 2004/2010+
Transverse densities: Dispersive representation

\[ \rho(b) = \int_{4M_i^2}^{\infty} \frac{dt}{2\pi} K_0(\sqrt{tb}) \frac{\text{Im} F(t)}{\pi} \]

\[ K_0 \sim e^{-b\sqrt{t}} \] exponential suppression of large \( t \)

Distance \( b \) selects masses \( \sqrt{t} \sim 1/b \): Filter

- Dispersive representation

- Peripheral densities from \( \pi\pi \) cut

Calculated with DI\( \chi \)EFT spectral functions

Describe empirical densities at \( b \gtrsim 1 \) fm

- Asymptotic behavior

\[ \rho_V^1, \tilde{\rho}_V^2(b) = e^{-2M_\pi b} \times \mathcal{F}(M_N, M_\pi; b) \]

Rich structure, relation \( \rho_V^1 \leftrightarrow \tilde{\rho}_V^2 \)

Granados, Weiss JHEP 1401, 092 (2014)
Transverse densities: Light-front formulation

\[ N\pi \quad x^+ \text{ time } \rightarrow \quad N \]

- Evolution in LF time \( x^+ = x^0 + x^3 \)

- Wave function of chiral \( \pi N \) system

  Describes transition \( N \rightarrow N\pi \) in \( \chi \)EFT. Universal, frame-independent also in \( \bar{u} - \bar{d} \) etc.

\[
\psi_{L=0,1}^{\pi N}(y, r_T) = \frac{\langle \pi N | \mathcal{L}_\chi | N \rangle}{\mathcal{M}_{\pi N}^2 - \mathcal{M}_N^2}
\]

invariant mass denominator

- Densities as wave function overlap

  Inequality \( |\rho_2^V| < \rho_1^V \)

  Contact terms \( \delta(y) \) represent high-mass intermed states; coefficient \( (1 - g_A^2) \)

  Equivalent to invariant formulation

Granados, CW 13. See also Ji, Melnitchouk et al. 09+
Transverse densities: Mechanical picture

- $\chi$EFT process as time sequence
  Rest frame, nucleon polarized in $y$–direction
  Bare $N$ fluctuates into $\pi N$ system via $\chi$EFT interaction
  Peripheral densities result from $J^+$ current carried by orbiting pion

- Explains peripheral densities
  $\rho_1, \tilde{\rho}_2 = \langle J^+ \rangle_{\text{right}} \pm \langle J^+ \rangle_{\text{left}}$
  $\langle J^+ \rangle_{\text{left}} \gg \langle J^+ \rangle_{\text{right}}$ large asymmetry
  Pion motion relativistic $k_\pi \sim M_\pi$

- Quantitative picture based on $\chi$EFT
  Extended to $\Delta$, EM tensor FFs
Transverse densities: \( x \) dependence

- Transverse spatial distribution (GPD)
  \[ f(x, b) \]
  longitudinal momentum
  transverse position

- Chiral component
  \[ b \sim M_\pi^{-1} \]
  transverse distance
  \[ x \sim M_\pi / M_N \]
  mom fraction of soft pion

- Calculable model-independently
  Pion distribution in nucleon from \( \chi \)EFT
  Quark/gluon distn in pion from measurements

- Observables and measurements
  \( t \)-slope of hard exclusive processes \( x \ll 0.1 \)
  Pion knockout in peripheral processes at EIC

Strikman, Weiss, PRD 69, 054012 (2004); PRD 80, 114029 (2009)
Summary

• Systematic methods for hadron structure in QCD:
  Chiral EFT, analytic amplitude methods, $1/N_c$ expansion

• DIχEFT new method for calculating $\pi\pi$ spectral functions of nucleon FFs
  Includes $\pi\pi$ rescattering in $t$-channel through unitarity + N/D + timelike pion FF
  Overcomes main limitation of traditional χEFT

• Numerous applications and extensions
  Electromagnetic FFs, peripheral transverse densities,
  Low-$Q^2$ $ep$ scattering analysis, proton radius extraction
  Scalar FF and radius
  Energy-momentum tensor, other QCD operators, $3\pi$ cut, resonance FFs

• Chiral dynamics in peripheral partonic structure
  Transverse densities — mechanical picture, theoretical insight
  $x$-dependent distributions — experimental probes