Partonic angular momentum in the nucleon's chiral periphery

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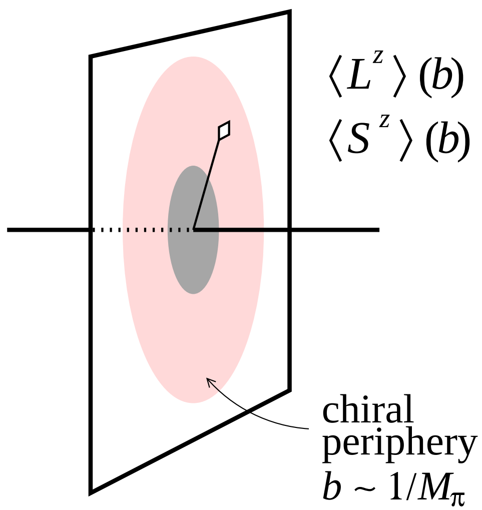
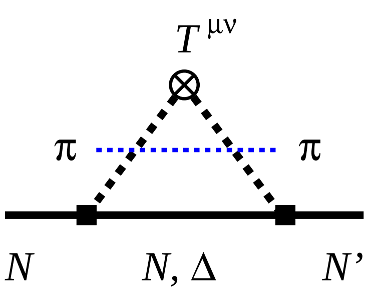




Granados, CW, arXiv:1905.02742

- Angular momentum in QCD

  Spin and orbital AM operators

  Energy-momentum tensor and form factors

  Light-front AM densities

- Angular momentum in chiral periphery

  Transverse densities at $b \sim 1/M_\pi$

  Dispersive representation and χEFT calculation

  Spin and orbital AM in chiral periphery

  Mechanical picture

- Connections and extensions

  Electromagnetic densities

  $\pi\pi$ rescattering through unitarity with J. M. Alarcon

AM in QCD: Objectives

Composition of nucleon spin from quark and gluon spin and orbital angular momentum

Local densities of angular momenta and mechanical interpretation $\leftarrow$

Measurement of angular momenta in deep-inelastic processes

Calculations using nonperturbative methods: EFT, LQCD $\leftarrow$

Challenges

Gauge invariance: Redundant DOF, equivalence classes, different form of operators

Non-uniqueness of EM tensor: Terms conserved without equations of motion, improvement

Interactions: Partonic operator beyond twist-2

Rotational invariance: Not manifest in light-front quantization $\leftrightarrow$ high-energy processes

Measurement vs. interpretation: May favor/require different choice of operators

No attempt to review history of subject here. For review see Leader, Lorcé Phys. Rept. **541** (2014) 163 2014
Some important works: Jaffe, Manohar 1990; Ji 1996; Polyakov 2003; Bakker, Leader, Trueman 2004; Burkardt 2005;
Chen, Lu, Sun, Wang, Goldman 2008; Wakamatsu 2010; Hatta 2011; Lorce, Pasquini 2011; Lorce 2013; Leader; Lorcé 2014

AM in QCD: Operators

- Invariance of action → conserved local currents → global charges

  Space-time translations → EM tensor $T^{\mu\nu}(x)$ → total mom $P^i = \int d^3x\, T^{0i}(x)$

  Rotations → AM tensor $J^{\mu\alpha\beta}(x)$ → total AM $J^i = \frac{1}{2}\epsilon^{ijk} \int d^3x\, J^{0jk}(x)$

- Angular momentum tensor

  Lorcé, Mantovani, Pasquini 2017

  $J^{\mu\alpha\beta} = S_q^{\mu\alpha\beta} + L_q^{\mu\alpha\beta} + J_g^{\mu\alpha\beta}$ total AM, "kinetic" definition

  $S_q^{\mu\alpha\beta} = \frac{1}{2}\epsilon^{\mu\alpha\beta\gamma} \sum_f \bar{\psi}_f \gamma_\gamma \gamma^5 \psi_f$ quark spin, cf. axial current

  $L_q^{\mu\alpha\beta} = x^\alpha T_q^{\mu\beta} - x^\beta T_q^{\mu\alpha}$ quark orbital AM

  $J_g^{\mu\alpha\beta} = x^\alpha T_g^{\mu\beta} - x^\beta T_g^{\mu\alpha}$ gluon total AM

  Gauge-invariant local operators

  Individual terms scale-dependent, total scale-independent

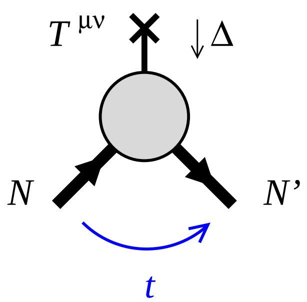
  Gluon AM cannot be split in spin and orbital in gauge-invariant manner

  $T_q^{\mu\nu}$ not symmetric in kinetic definition; alt definition with Belinfante-improved symmetric $T_q^{\mu\nu}$ and no explicit quark spin

  Lorcé, Leader 2014; Lorcé 2015

→

AM in QCD: EM tensor form factors



$$\langle N' | T^{\mu\nu} | N \rangle = \bar{u}_2 \left[\gamma^{\{\mu} p^{\nu\}} A - \frac{p^{\{\mu} \sigma^{\nu\}\alpha} \Delta_\alpha}{2M_N} B + \frac{\Delta^\mu \Delta^\nu - \Delta^2 g^{\mu\nu}}{M_N} C - \frac{p^{[\mu} \sigma^{\nu]\alpha} \Delta_\alpha}{2M_N} D + M_N g^{\mu\nu} \tilde{C} \right] u_1,$$

- Nucleon matrix element of EM tensor — Bakker, Leader, Trueman 04

  $A(t), B(t)$... invariant functions of $t \equiv \Delta^2$, cf. vector/axial form factors

  $A = A_q + A_g$, individually scale-dependent, total scale-independent

  Sum rules $A(0) = 1, B(0) = 0$

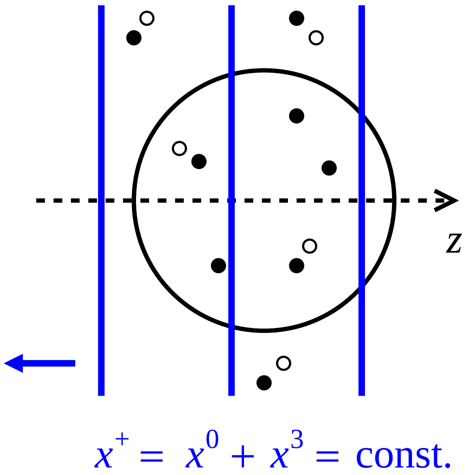
- Relation to GPDs

  $[A_q + B_q](t) = \int_{-1}^{1} dx\, x\, [H_q + E_q](x, \xi, t)$ second moment of GPDs — Ji 96

  $C_q(t) = \int_{-1}^{1} d\alpha\, \alpha\, D_q(\alpha, t)$ normalization of D-term — Polyakov, CW 99; Polyakov et al. 2000

$\rightarrow$

AM in QCD: EM tensor transverse densities



- Light-front view — Soper 1976, Burkardt 2000, Miller 2007

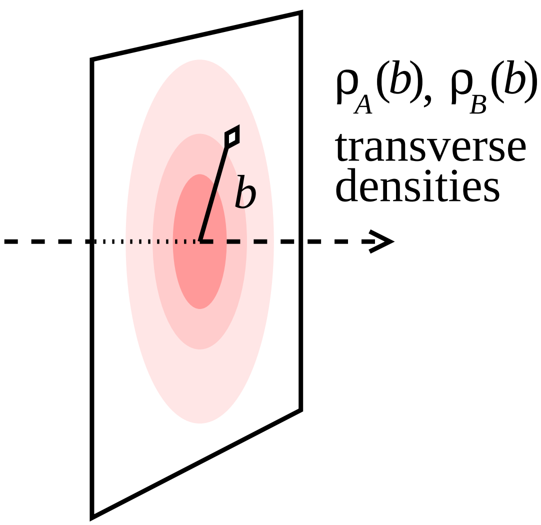
  Structure at fixed LF time $x^+ = x^0 + x^3$

  Densities boost-invariant, frame-independent

  Separate hadron structure $\leftrightarrow$ vacuum fluctuations

  Dynamical models: LF quantization, wave function

  Connection with parton picture, QCD operators



- Transverse densities

$$A(t = -\boldsymbol{\Delta}_T^2) \; = \int d^2b \, e^{i\boldsymbol{\Delta_T b}} \, \rho_A(b) \qquad \text{[same } B, D\text{]}$$

  Density of momentum etc. at transverse position $\boldsymbol{b}$

$$A(0) \; = \int d^2b \, \rho_A(b) \quad \text{total momentum}$$

AM in QCD: AM transverse densities

- Transverse density of orbital AM in LF quantization

$$\langle \boldsymbol{T}^{+T} \rangle (\boldsymbol{\Delta}_T) \equiv \langle p^+, \boldsymbol{\Delta}_T/2, \sigma | T^{+T} | p^+, -\boldsymbol{\Delta}_T/2, \sigma \rangle$$

matrix element, σ LF helicity

$$\boldsymbol{T}^{+T}(\boldsymbol{b}) \equiv \int \frac{d^2\Delta_T}{(2\pi)^2} \, e^{-i\boldsymbol{\Delta}_T \cdot \boldsymbol{b}} \, \langle \boldsymbol{T}^{+T} \rangle (\boldsymbol{\Delta}_T)$$

T^{+T} transverse density

$$\langle L^z \rangle (b) \equiv [\boldsymbol{b} \times \boldsymbol{T}^{+T}(\boldsymbol{b})]^z / (2p^+)$$

orbital AM density

$$= -\frac{\sigma}{2} \left(b \, \frac{d}{db} \right) [\rho_A + \rho_B + \rho_D](b)$$

expressed through EM form factors

Adhikari, Burkardt 2016; Lorcé, Mantovani, Pasquini 2017; Granados, CW 2019

- Transverse density of quark spin

$$\langle S^z \rangle (b) \equiv \sigma \; \rho_S(b)$$

ρ_S transverse density of nucleon axial FF $G_A(t)$

- AM sum rule

$$\int d^2 b \, [\langle S^z \rangle \; + \; \langle L^z \rangle](b) \; = \; \sigma \; = \; S^z(\text{rest rame}) \; = \; \pm 1/2.$$

AM in QCD: AM transverse densities II

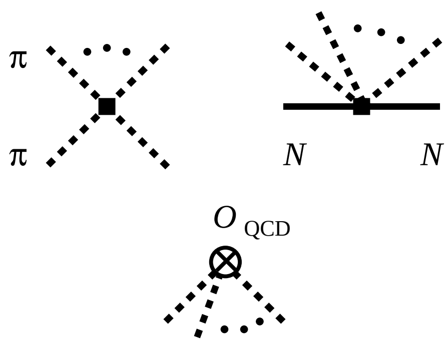
- "Dual role" of AM transverse densities $\langle L^z \rangle(b)$, $\langle S^z \rangle(b)$

  Calculated through the invariant form factors $A, B, D(t)$ and $G_A(t)$ without reference to LF quantization, using a variety of methods: Chiral EFT, dispersion theory, Euclidean correlators and LQCD

  Interpreted in context of LF quantization: Mechanical picture, partonic interpretation

- "New quantities" for nucleon structure studies!

Chiral periphery: Effective dynamics



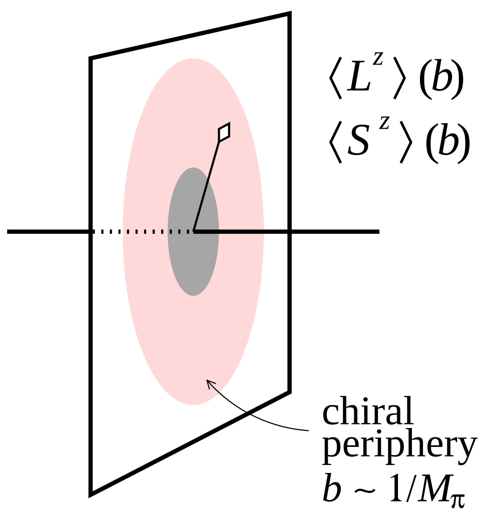
- Large-distance dynamics emerging from QCD

  Spontaneous breaking of chiral symmetry

  Pion as Goldstone boson, almost massless $M_\pi \ll \Lambda_\chi$, weakly coupled for $p_\pi = \mathcal{O}(M_\pi)$, form of interactions determined by underlying chiral symmetry

  Dynamics constructed and solved using EFT methods
  Gasser, Leutwyler 1983; Weinberg 1990

  Coupling to QCD operators



- Peripheral transverse densities

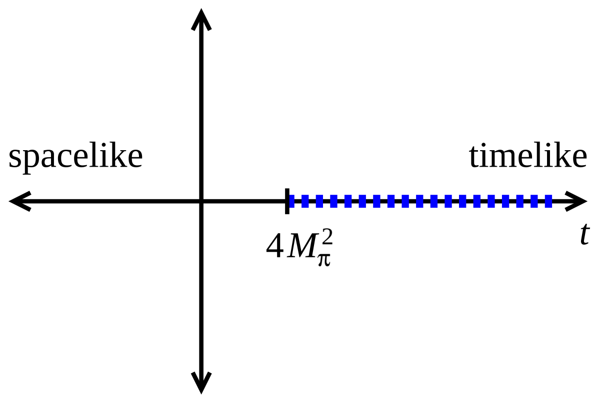
  Use distance as parameter $b = \mathcal{O}(M_\pi^{-1})$

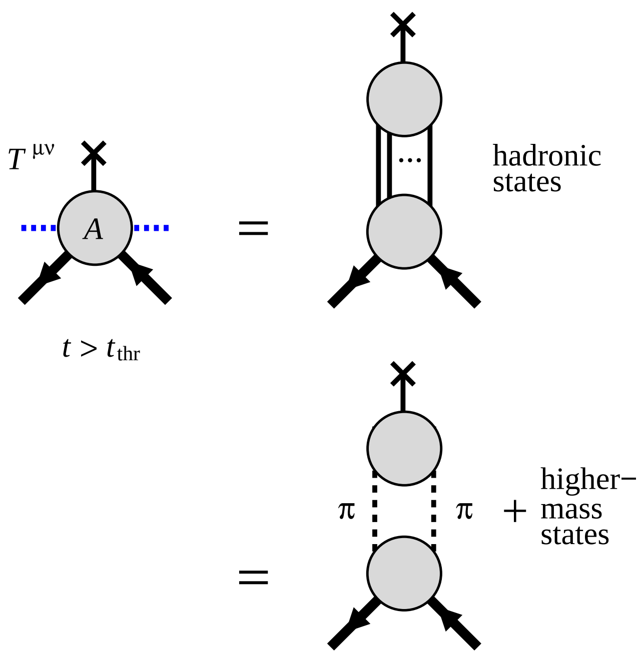
  Calculate densities in χEFT: systematic, model-indep., actual large-distance dynamics of QCD

  Mechanical picture, new insight

  Method developed for electromagnetic densities
  Granados, Weiss JHEP 1401, 092 (2014), JHEP 1507, 170 (2015), JHEP 1606 (2016) 075

Chiral periphery: Dispersive representation





- Dispersive representation of form factors

$$A(t) \;=\; \int_{t_{\text{thr}}}^{\infty} \frac{dt'}{\pi} \, \frac{\text{Im}\, A(t')}{t' - t - i0} \qquad \text{[same } B, D\text{]}$$

Process operator $\rightarrow$ hadronic states $\rightarrow N\bar{N}$
in unphysical region $t < 4M_N^2$

- Dispersive representation of densities

$$\rho_A(b) \;=\; \int_{t_{\text{thr}}}^{\infty} \frac{dt}{2\pi^2} \, K_0(\sqrt{t}b) \; \text{Im}\, A(t)$$

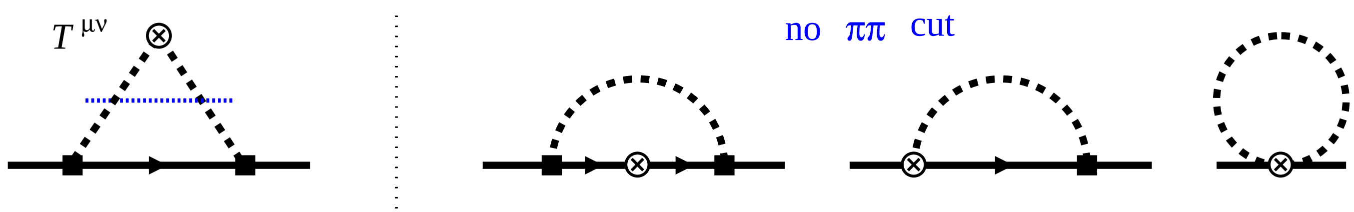
$K_0 \sim e^{-b\sqrt{t}}$ exponential suppression of large t

Large $b \leftrightarrow$ small t'
Strikman, CW 2010; Miller, Strikman, CW, 2011

- Densities at $b = \mathcal{O}(M_\pi^{-1})$ from $\pi\pi$ cut

→

Chiral periphery: EM tensor $\pi\pi$ cut



- EM tensor in χEFT from Noether theorem

  Current made from π and N fields, πN interactions in Lagrangian

  Need only $\pi\pi$ cut of form factors, generated by pionic current

  $$T^{\mu\nu}[\pi] \; = \; \sum_a \left(\partial^\mu \pi^a \partial^\nu \pi^a - \tfrac{1}{2} g^{\mu\nu} \partial^\rho \pi^a \partial_\rho \pi^a + \tfrac{1}{2} g^{\mu\nu} M_\pi^2 \pi^a \pi^a \right) + \text{terms } \pi^4 \ldots$$

  Symmetric tensor, uniquely determined by chiral symmetry, not to be improved
  Voloshin, Dolgov 82; Leutwyler, Shifman 89

- Nucleon form factors and densities

$A(t), B(t)$	$\pi\pi$ cut	$\rho_A(b), \rho_B(b)$ leading at $b = \mathcal{O}(M_\pi^{-1})$
$D(t)$	no $\pi\pi$ cut	$\rho_D(b)$ suppressed
$G_A(t)$	no $\pi\pi$ cut	$\rho_S(b)$ suppressed

Chiral periphery: AM densities in QCD

- The EM tensor from Noether's theorem in χEFT corresponds to the total quark + gluon EM tensor in QCD

- The χEFT results imply that at $b = \mathcal{O}(M_\pi^{-1})$ the orbital + gluon AM density is leading, the quark spin density is suppressed

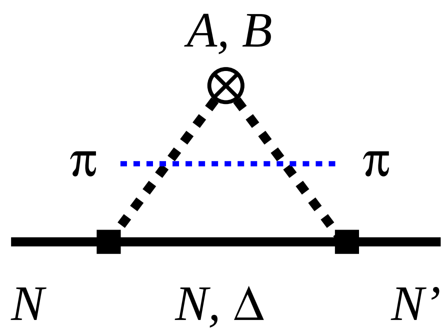
- These properties follow from (i) the specific form of the pion EM tensor dictated by chiral invariance; (ii) the dominance of the $\pi\pi$ cut at peripheral distances. Qualitative conclusions, robust.

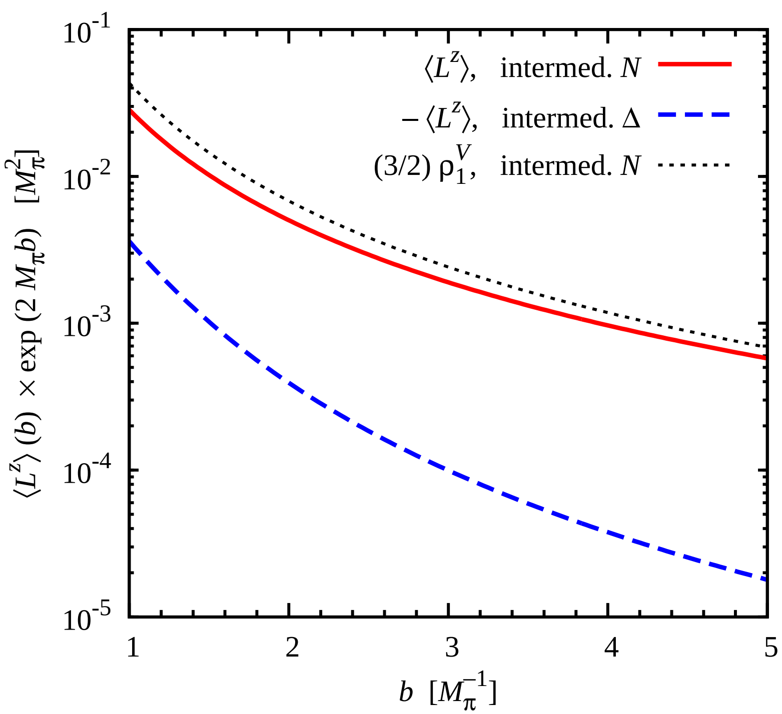
- The conclusions do not depend on the choice of QCD EM tensor

  $T^{\{\mu\nu\}}[\text{Belinfante}] \; = \; T^{\{\mu\nu\}}[\text{kinetic}]$ leading,

  $T^{[\mu\nu]}[\text{Belinfante}] \; \neq \; T^{[\mu\nu]}[\text{kinetic}]$ suppressed

Chiral periphery: AM densities





- Calculated $\mathrm{Im} A, B(t)$ in χEFT, $\rho_{A,B}(b)$ and $\langle L^z \rangle$ from dispersion relatn

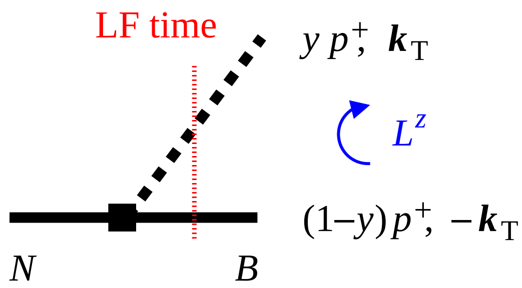
- Densities decay exponentially $\langle L^z \rangle \sim \exp(-2 M_\pi b) P(M_\pi, M_N, b)$

- $\langle L^z \rangle$ similar to isovector charge density

- N and Δ intermediate states produce opposite sign, cancel in large-N_c limit

Granados, CW, arXiv:1905.02742

Chiral periphery: Light-front formulation



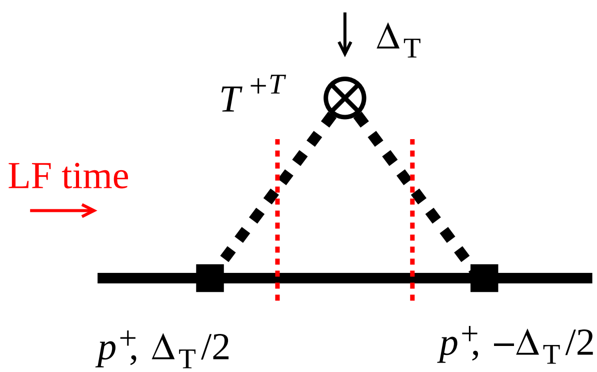
- χEFT in LF formulation

  Chiral processes as sequence in LF time

  Chiral LF wave function $N \rightarrow \pi B$ $(B = N, \Delta)$

  $$\Psi_{N\rightarrow\pi B}(y, \boldsymbol{k}_T, \sigma_B|\sigma) \;=\; \frac{\langle \pi B|\mathcal{L}_\chi|N\rangle}{M^2_{\pi B} - M^2_N}$$

  LF helicity states, πB configurations with $L^z \neq 0$



- AM density in LF formulation

  First-quantized representation, WF overlap

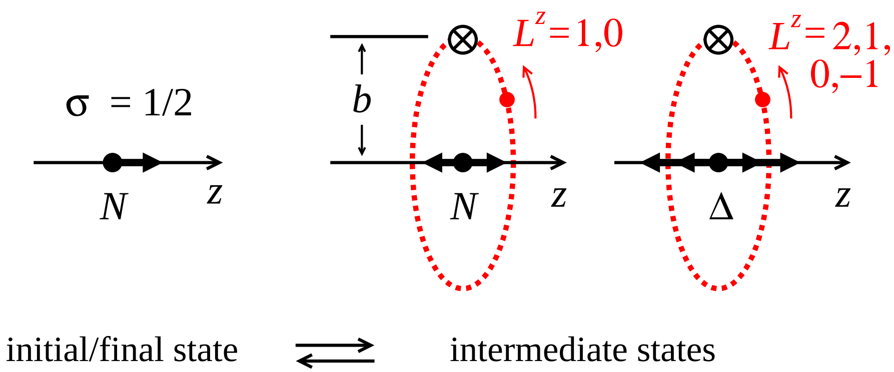
  Operator is quantum-mechanical L^z

  Equivalent to invariant formulation
  Granados, CW, arXiv:1905.02742

$$\langle L^z\rangle(b) = \sum_{B=N,\Delta} \frac{C_B}{4\pi} \int \frac{dy}{y\bar{y}^2} \sum_{\sigma_B} \Phi^*_{N\rightarrow\pi B}(y, \boldsymbol{r}_T, \sigma_B|\sigma) \left[\boldsymbol{r}_T \times (-i)\frac{\partial}{\partial \boldsymbol{r}_T} \right] \Phi_{N\rightarrow\pi B}(y, \boldsymbol{r}_T, \sigma_B|\sigma)$$

[Φ coordinate-space WF, $\boldsymbol{r}_T = \boldsymbol{b}/\bar{y}$, $\sigma = +\frac{1}{2}$].

Chiral periphery: Mechanical picture



Original nucleon with $\sigma = \frac{1}{2}$

Transition to πB state with $L^z \leftrightarrow \sigma_B$

Operator measures density of $L^z \neq 0$

- Useful representation

  Intuitive understanding

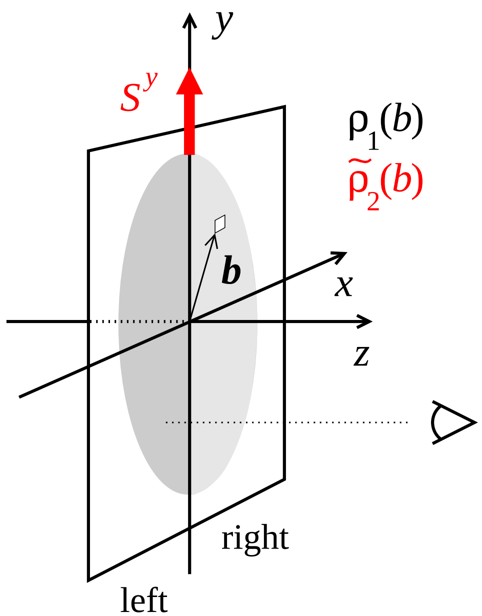
  Relation of peripheral AM density to other densities

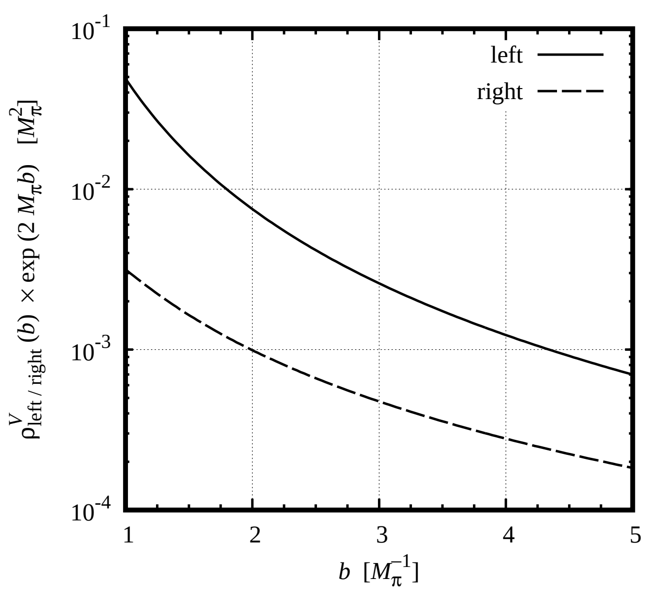
  Weights of N and Δ intermediate states, $N_c \to \infty$ limit

  Positivity conditions from quadratic form

- Based on "true" large-distance dynamics of QCD encoded in χEFT

Extensions: Electromagnetic densities





- Transverse charge/magnetization densities
  Soper 1976, Burkardt 2000, Miller 2007

$$\langle N' | J^\mu | N \rangle \;\rightarrow\; F_1, F_2(t) \rightarrow \rho_1, \rho_2(b)$$

- Interpretation in transverse polarized state

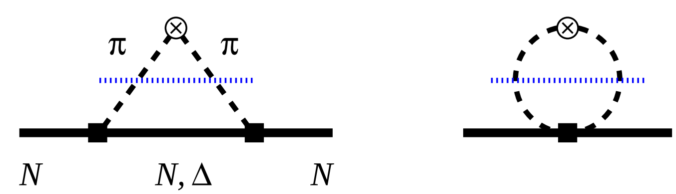
$$\langle J^+(\boldsymbol{b}) \rangle_{y-\mathrm{pol}} \;=\; \rho_1(b) + \;(2S^y)\; \cos\phi\; \widetilde{\rho}_2(b)$$

$$\rho_1, \widetilde{\rho}_2 = \langle J^+ \rangle_{\mathrm{right}} \pm \langle J^+ \rangle_{\mathrm{left}}$$ left-right asymmetry

- Peripheral densities calculated in χEFT

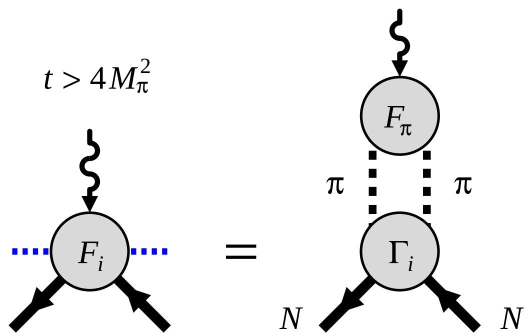
  Large left-right asymmetry

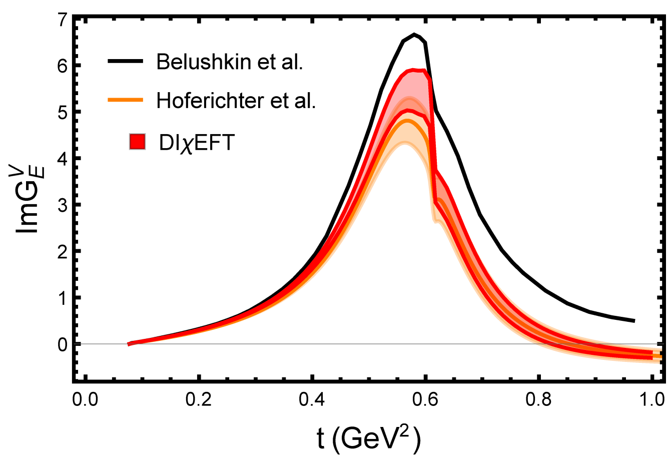
  Mechanical picture with transversely orbiting pion

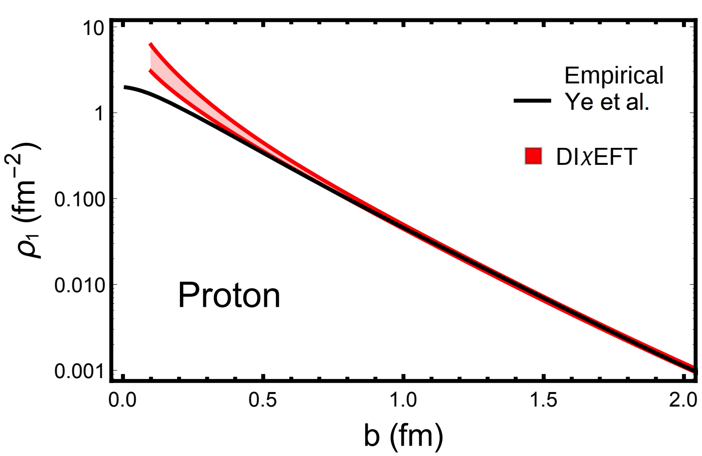


Granados, CW JHEP 1401, 092 (2014); 1507, 170 (2015); 1606 (2016) 075

Extensions: $\pi\pi$ rescattering through unitarity







- $\mathrm{Im}\, F_{1,2}(t)$ on $\pi\pi$ cut strongly affected by $\pi\pi$ rescattering, ρ resonance

  Traditional χEFT calculations poorly convergent

- Method for including $\pi\pi$ rescattering

  Elastic unitarity relation + N/D representation

  $\pi\pi \to N\bar{N}$ coupling from χEFT

  $\pi\pi$ rescattering from timelike pion form factor measured in e^+e^- annihilation

  Realistic spectral functions up to $t \sim 1$ GeV^2
  Alarcon, Hiller Blin, Vicente Vacas, Weiss, NPA 964, 18 (2017);
  Alarcon, Weiss, PRC 96, 055206 (2017) PRC 97, 055203 (2018)
  Similar method: Granados, Leupold, Perotti 2017

- Realistic densities down to $b \sim 0.5$ fm

  Other application: Proton radius extraction
  Alarcon, Higinbotham, Weiss, Ye PRC 99, 044303 (2019)

Extensions: Other structures

- EM tensor and AM densities with $\pi\pi$ rescattering

  Requires pion form factors as input — D-term: Pasquini, Polyakov, Vanderhaeghen 2017

- Form factors and densities with 3π cut

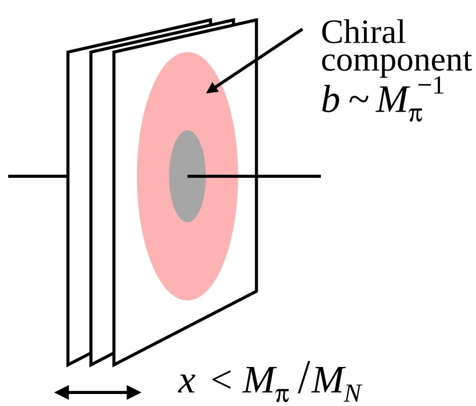
  Isoscalar electromagnetic densities, isovector EM tensor densities, isoscalar spin density

  Use methods of 3-body unitarity, presently being developed for amplitude analysis and LQCD
  Szczepaniak et al, Jackura, Pilloni, Döring et al

- Peripheral x-dependent GPDs

  Calculated in χEFT — Strikman, CW 2009; Granados, CW 2015

  Can be probed in peripheral high-energy processes at EIC — Strikman, CW 2004



Summary

- Definition of AM operators in QCD well understood, incl. local densities. Can be used for nucleon structure studies!

- Transverse AM densities at $b = \mathcal{O}(M_\pi^{-1})$ calculated in χEFT

  Two-pion cut of invariant form factors + dispersion relation

  Light-front time-ordered formulation

- In periphery the symmetric EM tensor dominates, antisymmetric suppressed. In terms of QCD DOF, orbital + gluon AM dominates, quark spin suppressed

- In periphery the field-theoretical AM density coincides with the quantum-mechanical AM density of the soft pions in the chiral processes

- Peripheral nucleon structure as new field of study

  "Deconstruct" nucleon in systematic approximation

  Manifestation of chiral symmetry breaking in QCD

  Peripheral high-energy processes at EIC