Precise determination of proton magnetic radius from electron scattering data

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Radius extraction using theory-based method: Dispersively improved chiral EFT

- Combines dispersion theory (analyticity, sum rules) and $\chi$EFT (dynamics, controlled accuracy)
- Correlates values of radii with FF behavior at larger $Q^2 \lesssim 1\text{ GeV}^2$
- Enables reliable determination of magnetic radius

**Method:** J. M. Alarcon, C. Weiss, PLB 784 (2018) 373; PRC 97, 055203 (2018);

**Radius extraction:** J. M. Alarcon, D. Higinbotham, C. Weiss, PRC 102 (2020) 035203
Motivation: Analyticity in radius extraction

- Challenges in proton radius extraction
  
  Derivative at $Q^2 = 0$ from data at finite $Q^2 > 0$
  
  Extrapolation $Q^2 \to 0$: Stability, functional bias?  
  Barcus, Higinbotham, this session
  
  Magnetic radius: Contribution of $G_M^p$ to cross section $\propto \tau/\epsilon$, vanishes for $Q^2 \to 0$

- Analyticity
  
  FFs analytic functions of $t = -Q^2$
  
  Singularities at $t > 0$: Hadronic exchanges
  
  Correlates functional behavior of FF at $Q^2 > 0$ with derivative at $Q^2 = 0$
  
  Predicts size of higher derivatives
  
  Global properties: Sum rules
  
  *Use in radius extraction!*
**DIχEFT: Dispersively improved chiral EFT**

- **Dispersive representation**

\[
F_i(t) = \int_{t_{\text{thr}}}^{\infty} \frac{dt'}{\pi} \frac{\text{Im} F_i(t')}{t' - t - i0}
\]

Expresses analytic structure

\[\text{Im} F_i\] spectral function, constructed theoretically

- **Spectral function in ππ region**

Elastic unitarity relation
Frazer, Fulco 1960; Höhler et al 1975+

Factorize \( \pi\pi \) rescattering using N/D method

\[
\Gamma_i/F_{\pi}: \pi\pi-NN \text{ coupling, calculated in } \chi\text{EFT}
\]

good convergence

\[
|F_{\pi}|^2: \pi\pi \text{ rescattering, taken from } e^+e^- \text{ data}
\]

Presently implemented LO + NLO + partial N2LO
**DIχEFT: Sum rules and parameters**

- Spectral function in high-mass region
  
  Parameterized by effective pole

  Sufficient for low-$Q^2$ form factors, uncertainty quantified
  
  Alarcon, Weiss PLB 784 (2018) 373

- Sum rules and parameters

  Sum rules for $F(0), F'(0) = \text{charges, radii}$

  Express $\chi$EFT LEC in terms of radii

  Radii appear directly as parameters of spectral functions, control behavior

\[
\begin{align*}
\frac{1}{\pi} \int_{t_{\text{thr}}}^{\infty} dt \frac{\text{Im} F_1(t)}{t} &= Q \\
\frac{1}{\pi} \int_{t_{\text{thr}}}^{\infty} dt \frac{\text{Im} F_1(t)}{t^2} &= \frac{1}{6} \langle r^2 \rangle_1 \\
\frac{1}{\pi} \int_{t_{\text{thr}}}^{\infty} dt \frac{\text{Im} F_2(t)}{t} &= \kappa \\
\frac{1}{\pi} \int_{t_{\text{thr}}}^{\infty} dt \frac{\text{Im} F_2(t)}{t^2} &= \frac{1}{6} \kappa \langle r^2 \rangle_2
\end{align*}
\]
• Spectral functions in $\pi\pi$ region

Band shows variation with radii (PDG range)

Good agreement with Roy-Steiner results

Hoferichter et al 2017
DIXEFT: Form factors

- Form factors from dispersion integral
  \[ G_{E,M}(t) = \int_0^\infty \frac{dt'}{\pi} \frac{\text{Im} G_{E,M}(t')}{t' - t - i0} \]

- Family of FFs depending on radii
  Each member respects analyticity, sum rules
  Each has intrinsic theoretical uncertainty

- Radius correlated with finite-\(Q^2\) behavior
  Provided by analyticity
  *Use for radius extraction!*

\(G_M\) similar, dependence on \(r_M\)

Alarcon, Higinbotham, Weiss, Ye PRC 99 (2019) 044303
Empirical FF: Global fit Ye et al 2017
Magnetic radius extraction: Procedure

- Use DIχEFT $G_{E,M}^p(Q^2)$ with params $r_E^p, r_M^p$

- Fit Mainz A1 cross section data

  $E = 0.18–0.855$ GeV, $Q^2 = 0.003–1.0$ GeV$^2$

  Fit original cross secns with floating normalizations

  Alt: Fit reanalyzed cross secns of Lee Arrington Hill 2015

  with recalc uncertainties: Same radii, lower $\chi^2$

- Impact on magnetic radius

  Sensitivity of cross section to $G_M^p$

  Dependence of DIχEFT $G_M^p$ on $r_M^p$

  Theoretical uncertainty from high-mass pole

  Use data up to $Q^2 \approx 0.5$ GeV$^2$
Magnetic radius extraction: Results

- Extracted radii

\[ r^D_E = 0.842 \pm 0.002 \text{ (fit 1}\sigma) \pm^{0.005}_{-0.002} \text{ (theory full-range) fm} \]

\[ r^D_M = 0.850 \pm 0.001 \text{ (fit 1}\sigma) \pm^{0.009}_{-0.004} \text{ (theory full-range) fm} \]

Magnetic radius has smaller fit uncertainty, larger theory uncertainty.

Magnetic radius needs theory-based extraction method.

Consistent with results of empirical dispersive fits

Lorenz, Hammer, Meissner 2012

Alarcon, Higinbotham, Weiss, PRC 102 (2020) 035203
Summary

- DI$\chi$EFT describes nucleon FFs combining dispersion theory and $\chi$EFT
  
  Includes $\pi\pi$ rescattering and $\rho$ resonance through unitarity
  Enables predictive calculations, controlled theoretical accuracy
  Excellent agreement with empirical FFs up to $Q^2 \sim 1 \text{ GeV}^2$ and beyond

- DI$\chi$EFT enables theory-based radius extraction
  
  Correlates $Q^2 = 0$ derivatives with finite-$Q^2$ behavior through analyticity + sum rules
  Employs radii directly as parameters $\leftrightarrow$ LECs
  Enables reliable determination of magnetic radius from finite-$Q^2$ data

- Other DI$\chi$EFT applications
  
  Nucleon transverse charge/magnetization densities
  Alarcon, Weiss, in progress. APS DNP presentation KC.2 (Saturday 8:30 CDT)

  Nucleon scalar FF
  Alarcon, Weiss, PRC 96, 055206 (2017)
Supplement: DIχEFT form factors

\[ G_{E,M}(t) = \int \frac{dt'}{4M^2} \frac{\text{Im} G_{E,M}(t')}{\pi} \frac{t' - t - i0}{t'^2} \]

- DIχEFT form factors
  - Evaluated using dispersion integral with spectral functions
  - Band shows variation with radii (PDG range).
  - Also quantified uncertainty from high-mass states
  - Excellent agreement with data. Not fit, but prediction based on dynamics