# Precise determination of proton magnetic radius from electron scattering data

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## Radius extraction using theory-based method: Dispersively improved chiral EFT

- Combines dispersion theory (analyticity, sum rules) and $\chi$EFT (dynamics, controled accuracy)
- Correlates values of radii with FF behavior at larger $Q^2 \lesssim 1$ GeV$^2$
- Enables reliable determination of magnetic radius

**Method:** J. M. Alarcon, C. Weiss, PLB 784 (2018) 373; PRC **97,** 055203 (2018);
J. M. Alarcon, A. N. Hiller Blin, M. Vicente Vacas, C. Weiss, NPA **964**, 18 (2017)

**Radius extraction:** J. M. Alarcon, D. Higinbotham, C. Weiss, PRC 102 (2020) 035203 ←
See also: J. M. Alarcon, D. Higinbotham, C. Weiss, Z. Ye, PRC 99 (2019) 044303

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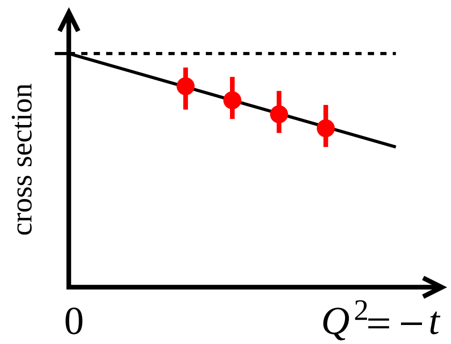
# Motivation: Analyticity in radius extraction



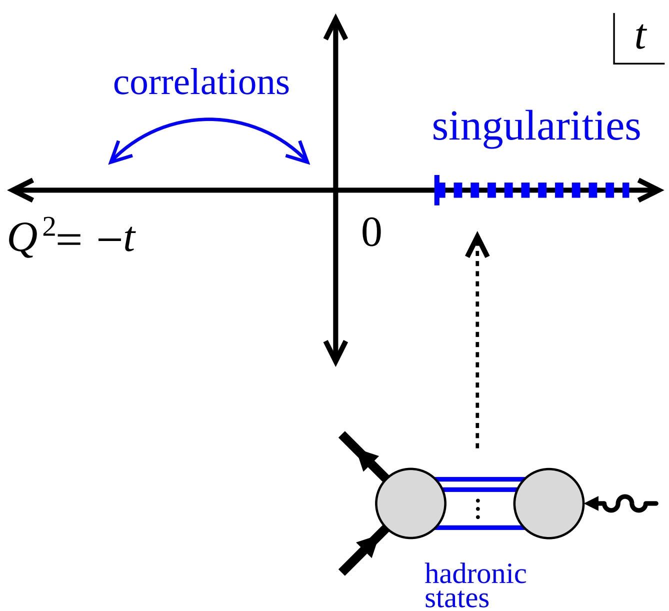



- Challenges in proton radius extraction

  Derivative at $Q^2 = 0$ from data at finite $Q^2 > 0$

  Extrapolation $Q^2 \to 0$: Stability, functional bias?
  Barcus, Higinbotham, this session

  Magnetic radius: Contribution of $G_M^p$ to cross section $\propto \tau/\epsilon$, vanishes for $Q^2 \to 0$




- Analyticity

  FFs analytic functions of $t = -Q^2$

  Singularities at $t > 0$: Hadronic exchanges

  Correlates functional behavior of FF at $Q^2 > 0$ with derivative at $Q^2 = 0$

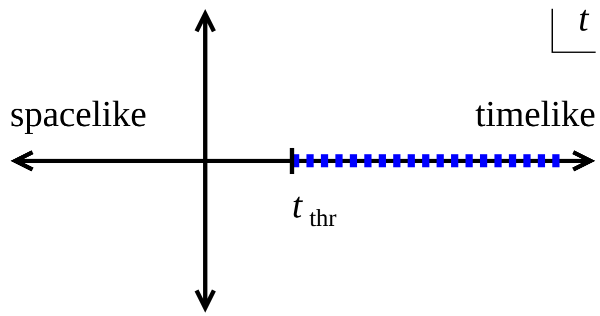
  Predicts size of higher derivatives

  Global properties: Sum rules

  *Use in radius extraction!*

# DI$\chi$EFT: Dispersively improved chiral EFT



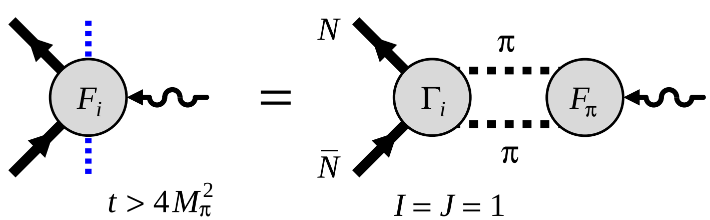



- Dispersive representation

$$F_i(t) = \int_{t_{\rm thr}}^{\infty} \frac{dt'}{\pi} \frac{\mathrm{Im}\, F_i(t')}{t' - t - i0}$$

Expresses analytic structure

$\mathrm{Im}\, F_i$ spectral function, constructed theoretically




$$\mathrm{Im} F_i(t) = \frac{k_{\rm cm}^3}{\sqrt{t}}\, \Gamma_i(t) \;\; F_\pi^*(t)$$

$$= \frac{k_{\rm cm}^3}{\sqrt{t}} \underbrace{\frac{\Gamma_i(t)}{F_\pi(t)}}_{\chi\text{EFT}} \underbrace{|F_\pi(t)|^2}_{\text{Data}}$$

- Spectral function in $\pi\pi$ region

Elastic unitarity relation
Frazer, Fulco 1960; Höhler et al 1975+

Factorize $\pi\pi$ rescattering using N/D method

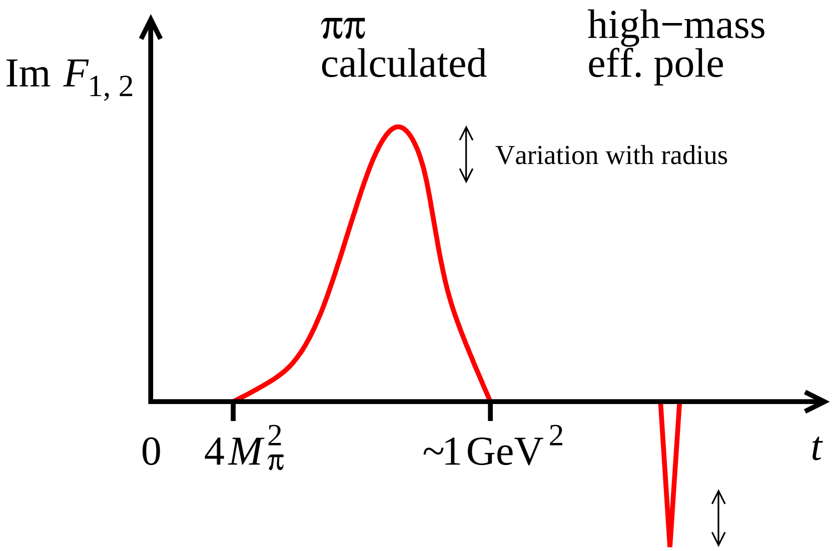
$\Gamma_i/F_\pi$: $\pi\pi$–$NN$ coupling, calculated in $\chi$EFT
good convergence

$|F_\pi|^2$: $\pi\pi$ rescattering, taken from $e^+e^-$ data

Presently implemented LO + NLO + partial N2LO
Alarcon, Weiss, PLB 784 (2018) 373; PRC 97 (2018) 055203

# DI$\chi$EFT: Sum rules and parameters






$$\frac{1}{\pi}\int_{t_{\rm thr}}^{\infty} dt\, \frac{{\rm Im}F_1(t)}{t} = Q$$

$$\frac{1}{\pi}\int_{t_{\rm thr}}^{\infty} dt\, \frac{{\rm Im}F_1(t)}{t^2} = \frac{1}{6}\langle r^2\rangle_1$$

$$\frac{1}{\pi}\int_{t_{\rm thr}}^{\infty} dt\, \frac{{\rm Im}F_2(t)}{t} = \kappa$$

$$\frac{1}{\pi}\int_{t_{\rm thr}}^{\infty} dt\, \frac{{\rm Im}F_2(t)}{t^2} = \frac{1}{6}\kappa\langle r^2\rangle_2$$

- Spectral function in high-mass region

  Parameterized by effective pole

  Sufficient for low-$Q^2$ form factors, uncertainty quantified
  Alarcon, Weiss PLB 784 (2018) 373

- Sum rules and parameters

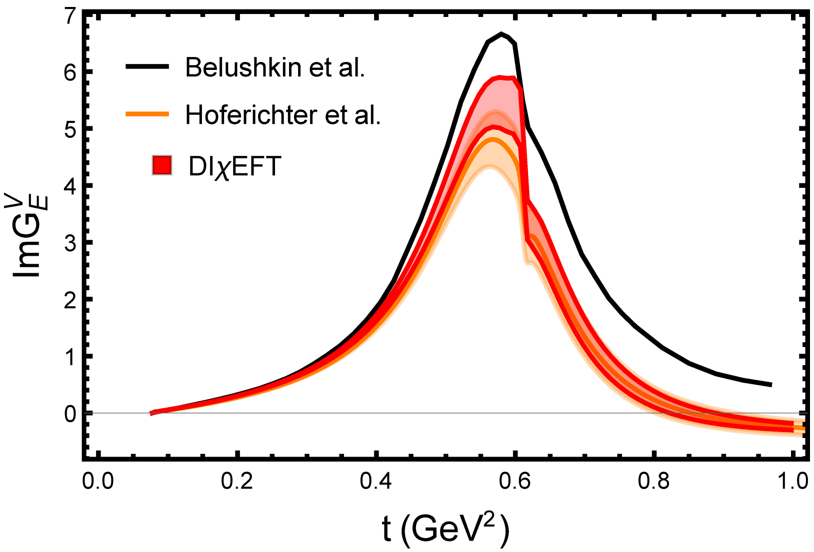
  Sum rules for $F(0), F'(0)$ = charges, radii

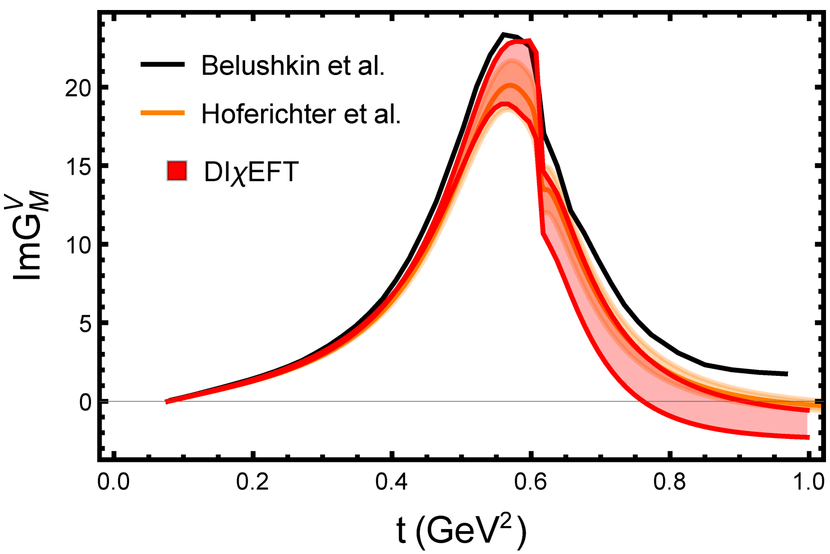
  Express $\chi$EFT LEC in terms of radii

  Radii appear directly as parameters of spectral functions, control behavior

# DI$\chi$EFT: Spectral functions





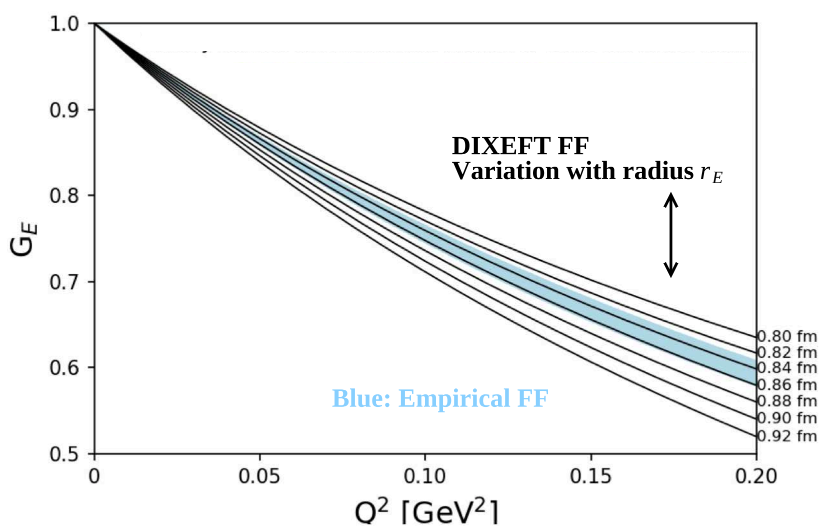

- Spectral functions in $\pi\pi$ region

  Band shows variation with radii (PDG range)

  Good agreement with Roy-Steiner results
  Hoferichter et al 2017

Alarcon, Weiss, PLB 784 (2018) 373
Bands: Variation with nucleon radii (PDG range)

# DI$\chi$EFT: Form factors




$G_M$ similar, dependence on $r_M$

Alarcon, Higinbotham, Weiss, Ye PRC 99 (2019) 044303
Empirical FF: Global fit Ye et al 2017

- Form factors from dispersion integral

$$G_{E,M}(t) = \int_{4M_\pi^2}^{\infty} \frac{dt'}{\pi} \frac{\operatorname{Im} G_{E,M}(t')}{t' - t - i0}$$

- Family of FFs depending on radii

  Each member respects analyticity, sum rules

  Each has intrinsic theoretical uncertainty

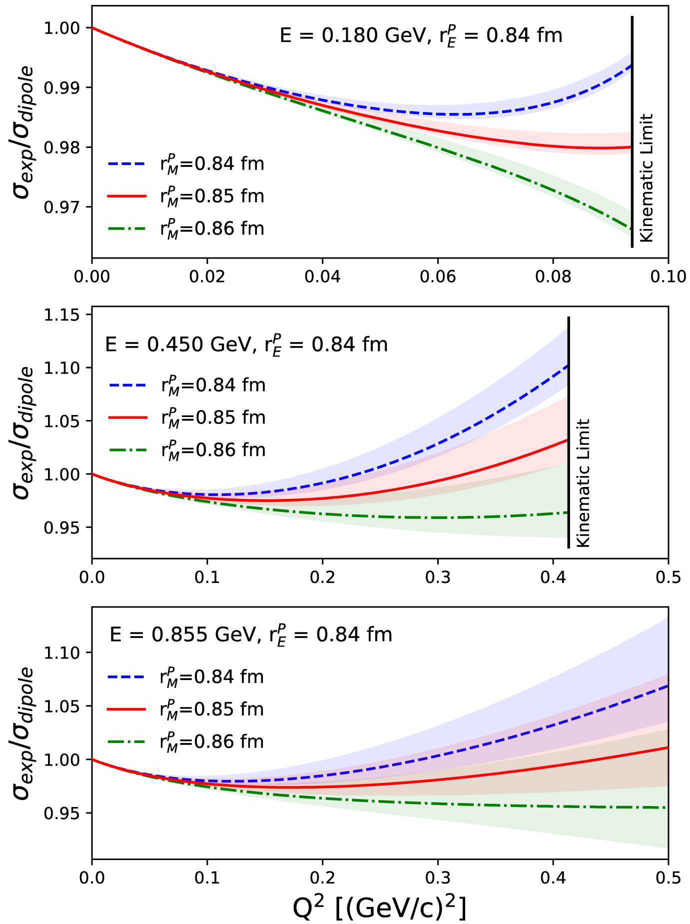
- Radius correlated with finite-$Q^2$ behavior

  Provided by analyticity

  *Use for radius extraction!*

# Magnetic radius extraction: Procedure






$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Mott}} \frac{\epsilon\,[G_E^p]^2 + \tau[G_M^p]^2}{\epsilon(1+\tau)}$$

- Use DIχEFT $G^p_{E,M}(Q^2)$ with params $r^p_E$, $r^p_M$
- Fit Mainz A1 cross section data

  $E = 0.18$–$0.855$ GeV, $Q^2 = 0.003$–$1.0$ GeV$^2$

  Fit original cross secns with floating normalizations

  Alt: Fit reanalyzed cross secns of Lee Arrington Hill 2015 with recalc uncertainties: Same radii, lower $\chi^2$

- Impact on magnetic radius

  Sensitivity of cross section to $G^p_M$

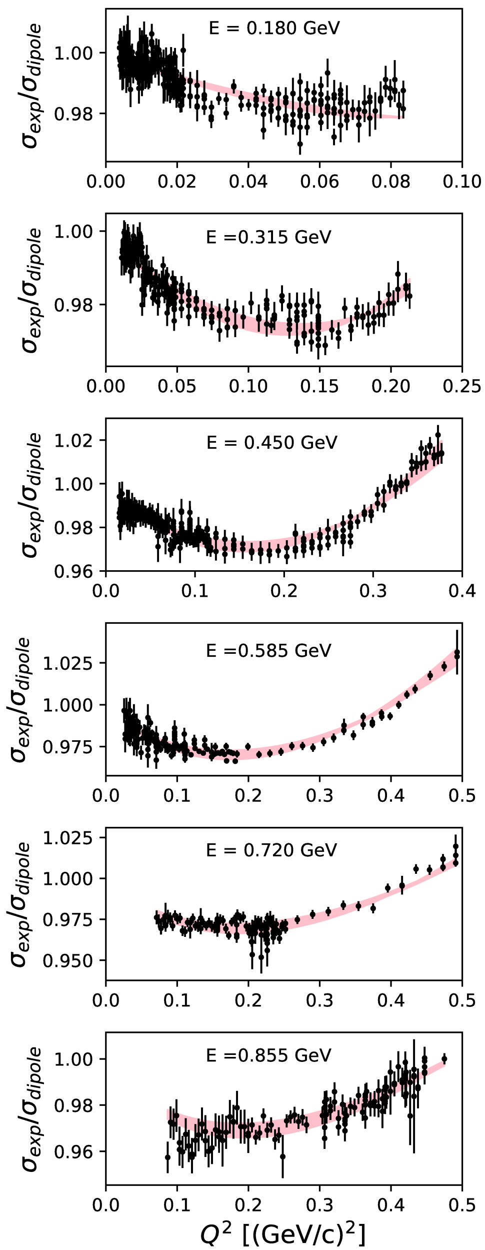
  Dependence of DIχEFT $G^p_M$ on $r^p_M$

  Theoretical uncertainty from high-mass pole

  Use data up to $Q^2 \approx 0.5$ GeV$^2$

Alarcon, Higinbotham, Weiss, PRC 102 (2020) 035203

# Magnetic radius extraction: Results




- Extracted radii

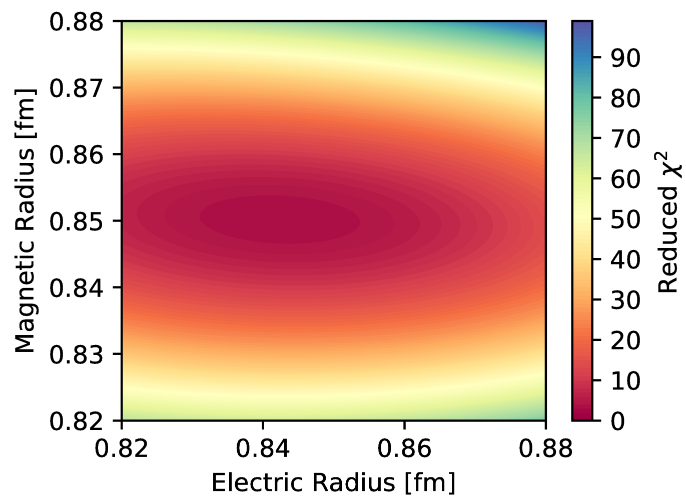
$r_E^p$ = 0.842 ±0.002 (fit 1$\sigma$) $^{+0.005}_{-0.002}$ (theory full-range) fm

$r_M^p$ = 0.850 ±0.001 (fit 1$\sigma$) $^{+0.009}_{-0.004}$ (theory full-range) fm

Magnetic radius has smaller fit uncertainty, larger theory unc

Magnetic radius needs theory-based extraction method

Consistent with results of empirical dispersive fits
Lorenz, Hammer, Meissner 2012

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# Summary

- DI$\chi$EFT describes nucleon FFs combining dispersion theory and $\chi$EFT

  Includes $\pi\pi$ rescattering and $\rho$ resonance through unitarity

  Enables predictive calculations, controlled theoretical accuracy

  Excellent agreement with empirical FFs up to $Q^2 \sim 1$ GeV$^2$ and beyond

- DI$\chi$EFT enables theory-based radius extraction

  Correlates $Q^2 = 0$ derivatives with finite-$Q^2$ behavior through analyticity + sum rules

  Employs radii directly as parameters $\leftrightarrow$ LECs

  Enables reliable determination of magnetic radius from finite-$Q^2$ data

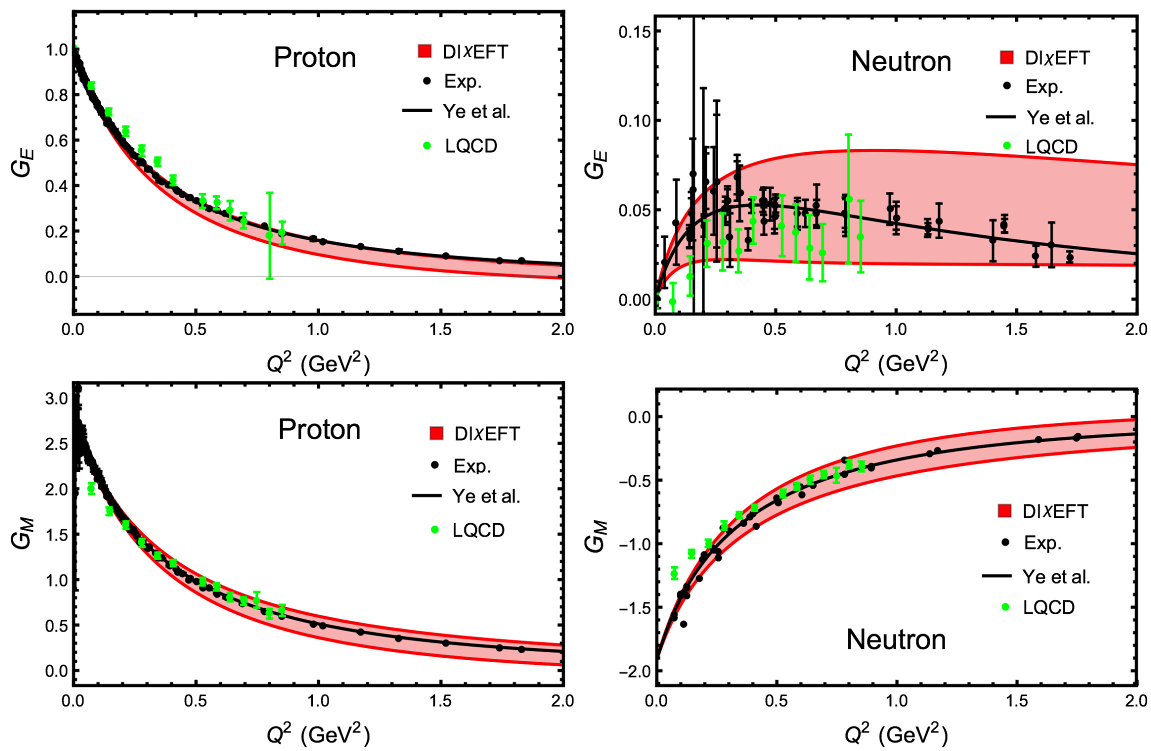
- Other DI$\chi$EFT applications

  Nucleon transverse charge/magnetization densities
  Alarcon, Weiss, in progress. APS DNP presentation KC.2 (Saturday 8:30 CDT)

  Nucleon scalar FF
  Alarcon, Weiss, PRC **96,** 055206 (2017)

# Supplement: DI$\chi$EFT form factors






$$G_{E,M}(t) = \int_{4M_\pi^2}^{\infty} \frac{dt'}{\pi} \frac{\mathrm{Im}\, G_{E,M}(t')}{t' - t - i0}$$

Alarcon, Weiss, PLB 784 (2018) 373
Bands: Variation with nucleon radii (PDG range)

- DI$\chi$EFT form factors

  Evaluated using dispersion integral with spectral functions

  Band shows variation with radii (PDG range).
  Also quantified uncertainty from high-mass states

  Excellent agreement with data. Not fit, but prediction based on dynamics