Nucleon EM form factors and radius extraction with dispersively improved chiral EFT

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Summary: New method for calculating/analyzing nucleon FFs combining dispersion theory and χEFT

Implements analyticity in momentum transfer t

Includes $\pi\pi$ rescattering and ρ resonance through unitarity

Enables predictive calculations, controlled theoretical accuracy

Uses radii as parameters $\leftrightarrow$ LECs, ideally suited for radius extraction

Outline

Motivation: Analyticity in radius extraction

Method: Dispersive representation, elastic unitarity, χEFT input, radii as parameters

Results: Spectral functions, form factors, uncertainties

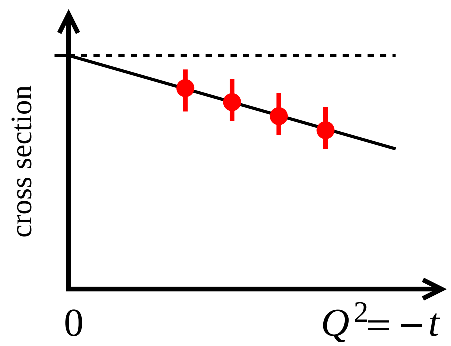
Applications: Proton radius extraction from ep scattering, electric and magnetic

Applications: μp scattering $\leftarrow$

Method: J. M. Alarcon, C. Weiss, PLB 784 (2018) 373; PRC **97,** 055203 (2018);
J. M. Alarcon, A. N. Hiller Blin, M. Vicente Vacas, C. Weiss, NPA **964**, 18 (2017)

Applications: J. M. Alarcon, D. Higinbotham, C. Weiss, Z. Ye, PRC 99 (2019) 044303;
J. M. Alarcon, D. Higinbotham, C. Weiss, PRC 102 (2020) 035203

Motivation: Analyticity



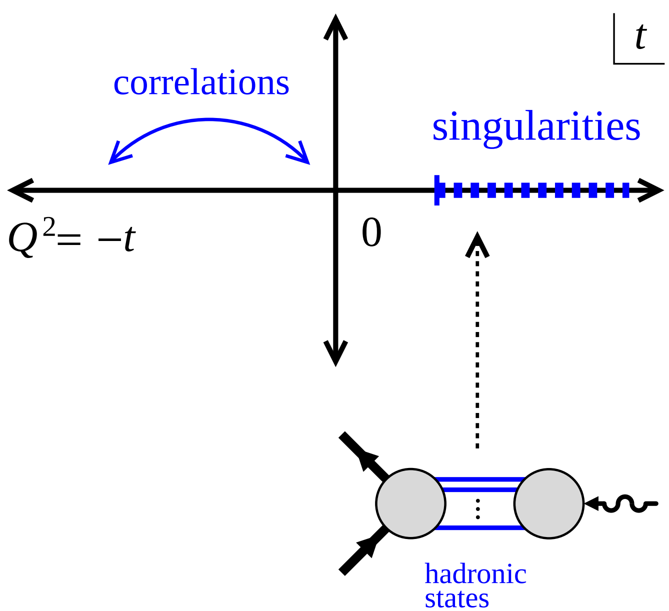
- Challenges in proton radius extraction

  Derivative at $Q^2 = 0 \quad \leftrightarrow \quad$ data at finite $Q^2 > 0$

  Extrapolation $Q^2 \to 0$: Stability, functional bias

  Higher derivatives? $\to$ Dynamical scales?

  Magnetic radius: Contribution of $G_M^p \propto \tau/\epsilon$, vanishes for $Q^2 \to 0$



- Analyticity

  FFs analytic functions of $t = -Q^2$

  Singularities: Branch cuts at $t > 0$.
  Hadronic exchanges

  Positions of singularities: Hadron masses ✓

  Strengh of singularities: Couplings, phase space ?

  Implications for FF behavior at $t < 0$
  and radius extraction!

Motivation: Analyticity

- Implications of analyticity

  Correlates functional behavior of FF at $Q^2 > 0$ with derivatives $Q^2 = 0$

  Predicts size of higher derivatives from dynamical scales (in singularities), stable results

  Provides sum rules relating integrals over singularities with local derivatives $F'(0)$ etc.

  These features can be implemented/exploited without complete knowledge of the functions!
  We can/should use them for radius extraction!

- New method: DIχEFT

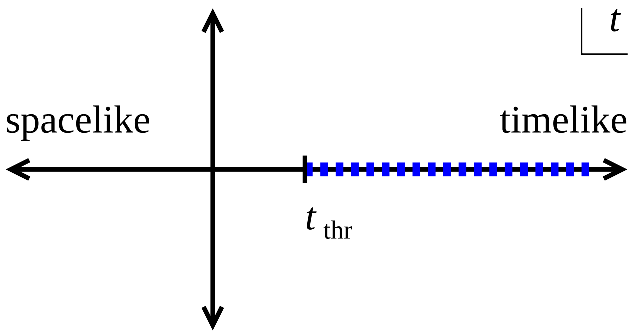
  Implement analyticity through dispersion relation

  Correlate radii with behavior of FFs at finite $Q^2 > 0$ through sum rules (radii as parameters)

  Uniquely suited for radius extraction!

  [Other approaches based on analyticity: z-expansion – only analyticity, no dynamical input
  Traditional dispersion theory with $\pi\pi$ continuum – complete predictions, no flexibility
  Höhler et al. 1975+; Belushkin, Hammer, Meissner 2007; Lorenz, Hammer, Meissner 2012

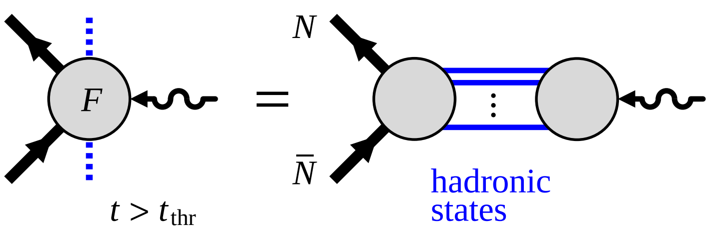
Method: Dispersive representation



- Dispersive representation

$$F_i(t) \;=\; \int\limits_{t_{\mathrm{thr}}}^{\infty} \frac{dt'}{\pi}\,\frac{\mathrm{Im}\,F_i(t')}{t'-t-i0}$$

Expresses analytic structure of $F_i(t)$



- Spectral functions $\mathrm{Im}\,F_i(t)$

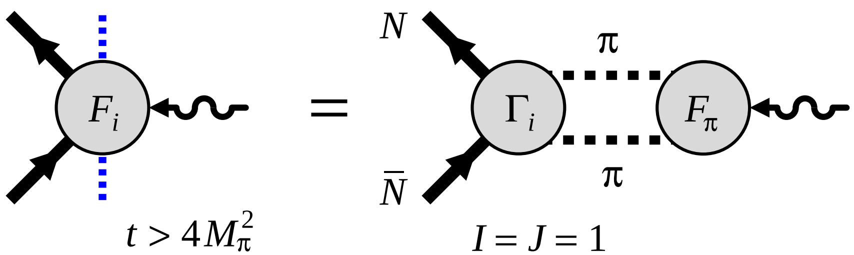
Current $\longrightarrow$ hadronic states $\longrightarrow$ $N\bar{N}$

Processes in unphysical region $t < 4M_N^2$

Spectral functions to be provided by theory
Frazer, Fulco 1960; Höhler et al 1975+

Isovector: $\pi\pi$ (incl. ρ), 4π, $K\bar{K}$, ...
Isoscalar: 3π (incl. ω), $K\bar{K}$ (incl. ϕ), ...

Method: Spectral functions on $\pi\pi$ cut



$$\mathrm{Im} F_i(t) = \frac{k_{\mathrm{cm}}^3}{\sqrt{t}} \, \Gamma_i(t) \;\; F_\pi^*(t)$$

$$= \frac{k_{\mathrm{cm}}^3}{\sqrt{t}} \, \underbrace{\frac{\Gamma_i(t)}{F_\pi(t)}}_{\chi\text{EFT}} \;\; \underbrace{|F_\pi(t)|^2}_{\text{Data}}$$

- Elastic unitarity relation

  $F_\pi(t)$ timelike pion FF, $\Gamma_i(t)$ partial-wave amplitude $\pi\pi \to N\bar{N}$

  Amplitudes have same phase from $\pi\pi$ rescattering — Watson's theorem

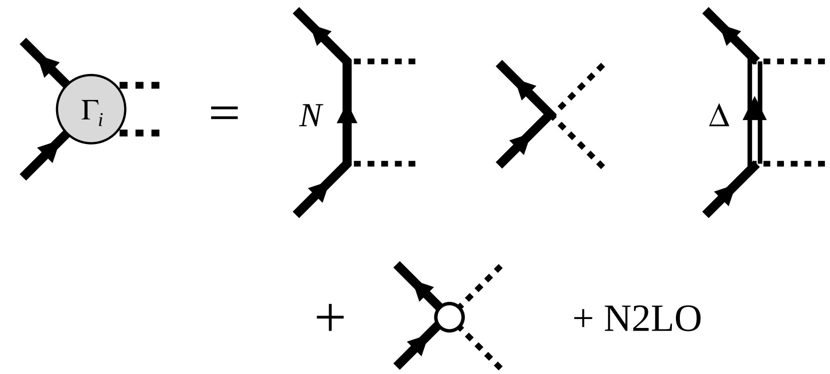
- Factorized representation (N/D method)

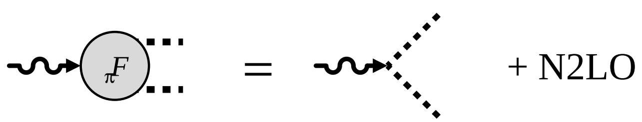
  Γ_i/F_π free of $\pi\pi$ rescattering → calculate in χEFT, well convergent

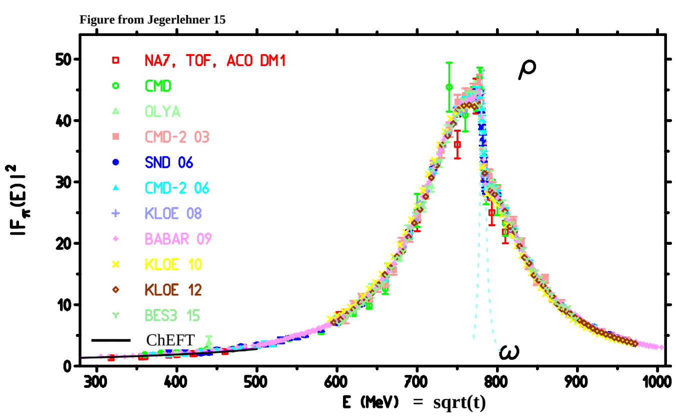
  $|F_\pi|^2$ includes $\pi\pi$ rescattering, ρ resonance → take from e^+e^- data, LQCD

- New χEFT-based approach

Method: Chiral EFT







- Relativistic χEFT

  Expansion in $\{M_\pi, k_\pi\}/\Lambda_\chi$

  Include Δ isobar

- Calculation of $\Gamma_i(t)/F_\pi(t)$

  LO: Born terms + Weinberg-Tomozawa

  NLO: Contact term in $\Gamma_i(t)$

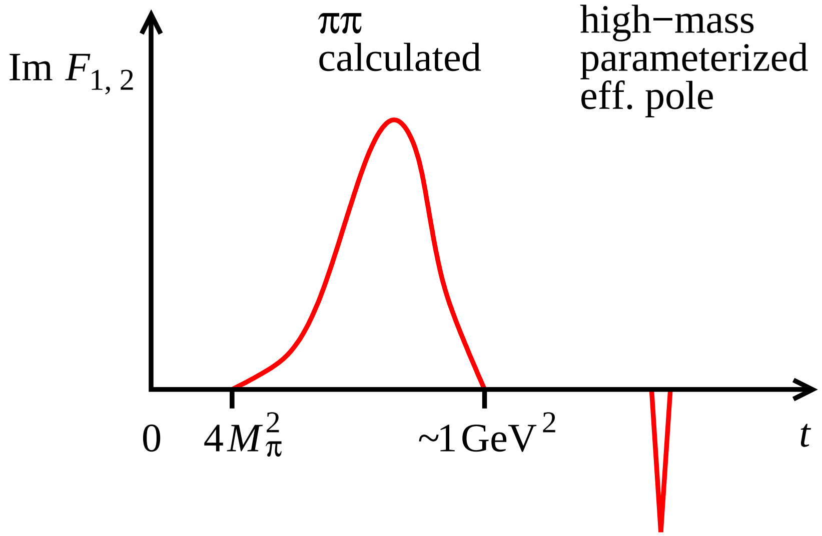
  N2LO: Contact term and pion loops
  Presently: Use Partial N2LO, unknown LEC

  Good convergence

- Pion timelike FF $|F_\pi(t)|^2$

  Measured accurately in $e^+e^- \to \pi^+\pi^-$

Method: Sum rules and parameters



$$\frac{1}{\pi}\int_{t_{\rm thr}}^{\infty} dt\, \frac{\mathrm{Im} F_1(t)}{t} = Q$$

$$\frac{1}{\pi}\int_{t_{\rm thr}}^{\infty} dt\, \frac{\mathrm{Im} F_1(t)}{t^2} = \frac{1}{6}\langle r^2\rangle_1$$

$$\frac{1}{\pi}\int_{t_{\rm thr}}^{\infty} dt\, \frac{\mathrm{Im} F_2(t)}{t} = \kappa$$

$$\frac{1}{\pi}\int_{t_{\rm thr}}^{\infty} dt\, \frac{\mathrm{Im} F_2(t)}{t^2} = \frac{1}{6}\kappa\langle r^2\rangle_2$$

- Spectral functions

  $\pi\pi$ continuum calculated:
  Elastic unitarity + χEFT

  High-mass states parameterized:
  Effective pole

  Sufficient for low-Q^2 form factors,
  uncertainty quantified

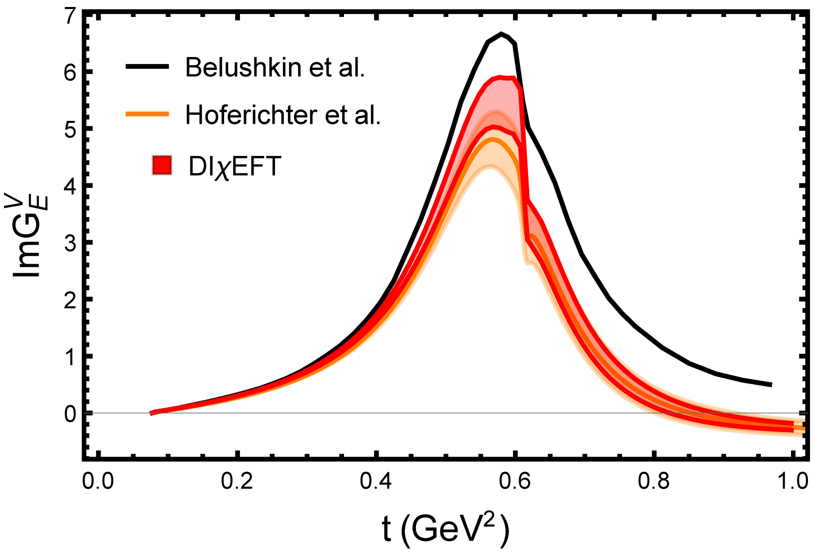
- Sum rules and parameters

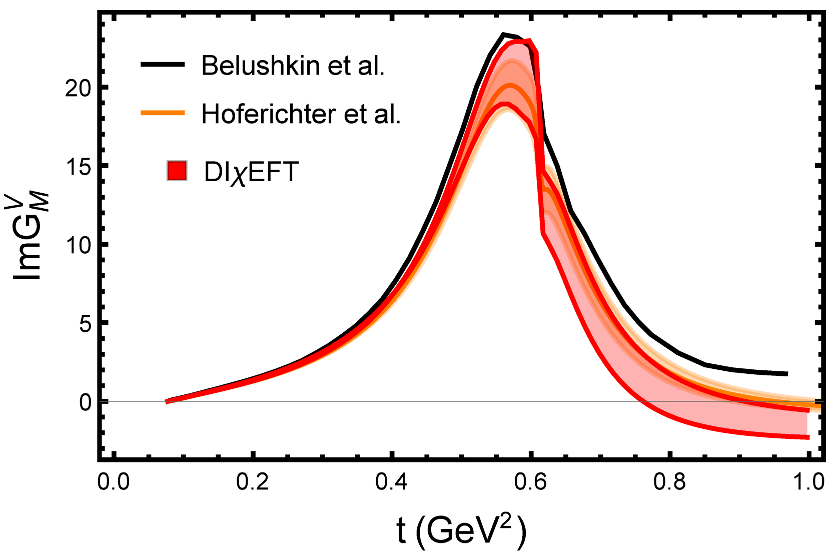
  Sum rules for $F(0)$, $F'(0)$ = charges, radii

  Express χEFT LEC in terms of radii

  Radii appear directly as parameters,
  control global behavior of FF!

Results: Spectral functions





- DIχEFT spectral functions in $\pi\pi$ region

  Band shows uncertainty from radii

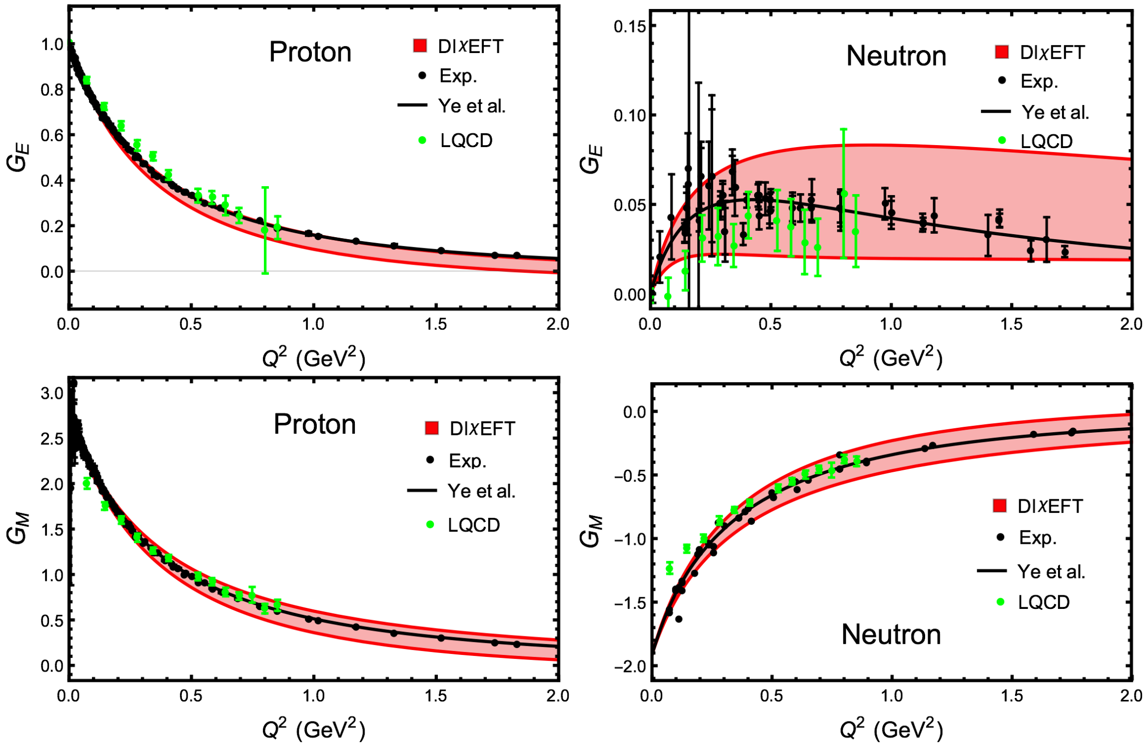
  Good agreement with Roy-Steiner results
  Hoferichter et al 2017

- Qualitative improvement compared to traditional χEFT

  $\pi\pi$ rescattering included through $|F_\pi(t)|^2$

Alarcon, Weiss, PLB 784 (2018) 373
Uncertainty bands: PDG range of nucleon radii

Results: Form factors



$$G_i(t) = \int_{4M_\pi^2}^{\infty} \frac{dt'}{\pi} \frac{\operatorname{Im} G_i(t')}{t' - t - i0}$$

Alarcon, Weiss, PLB 784 (2018) 373
Uncertainty bands: PDG range of nucleon radii

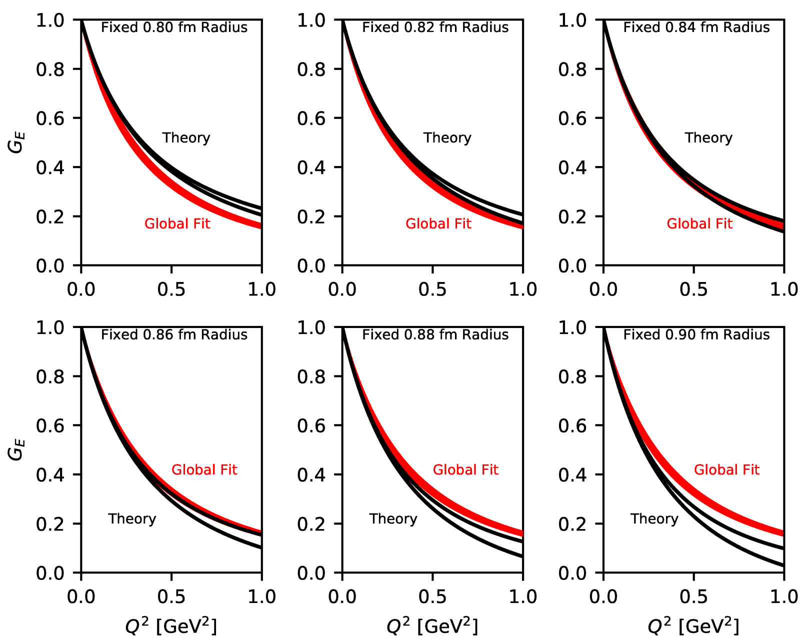
- DIχEFT form factors

  Evaluated using dispersion integral with spectral functions

  Band shows uncertainty from radii. Also quantified uncertainty from high-mass states

  Excellent agreement with data. Not fit, but prediction based on dynamics

Applications: Proton radius extraction



Alarcon, Higinbotham, Weiss, Ye PRC 99 (2019) 044303
Global FF fit from Ye et al 2017

- DIχEFT provides family of FFs $G_{E,M}(Q^2)$ depending on radii as parameters

  Each member respects analyticity, sum rules

  Each has intrinsic theoretical uncertainty

- Radius correlated with finite-Q^2 behavior

  Provided by analyticity

  Use for radius extraction

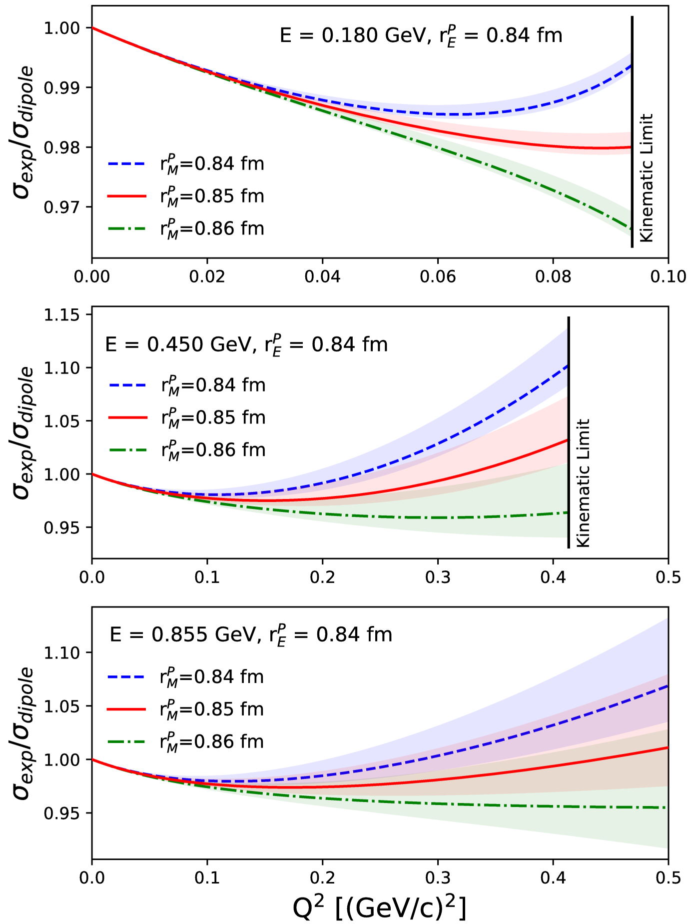
- Extracted r_E^p from FF fit

  Compared DIχEFT FFs with global fit $G_E^p(Q^2)$

  Obtained $r_E^p = 0.844(7)$ fm (cf. muonic hydrogen)

  r_E^p constrained by data up to $Q^2 \sim 0.5$ GeV^2

Applications: Proton radius extraction



$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Mott}} \frac{\epsilon\,[G_E^p]^2 + \tau[G_M^p]^2}{\epsilon(1+\tau)}$$

- Extracted r_E^p and r_M^p from cross section fit

  Mainz A1 data E=0.18–0.855 GeV, Q^2=0.003–1.0 GeV^2

  Used DIχEFT $G^p_{E,M}(Q^2)$ dep on r_E^p, r_M^p

  Fitted cross sections with floating normalizations

  Quantified fit and theoretical uncertainties

  r_E^p = 0.842 $\pm$0.002 (fit 1σ) $^{+0.005}_{-0.002}$ (theory full-range) fm

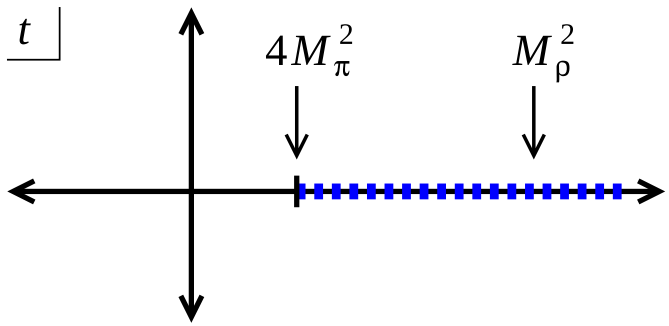
  r_M^p = 0.850 $\pm$0.001 (fit 1σ) $^{+0.009}_{-0.004}$ (theory full-range) fm

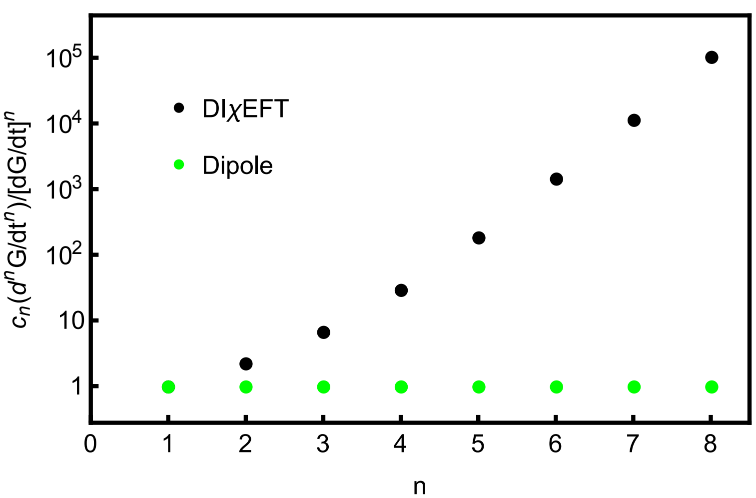
- DIχEFT enables accurate r_M^p extraction

  Sensitivity to G_M^p only at finite Q^2

Alarcon, Higinbotham, Weiss, PRC 102 (2020) 035203

Applications: Higher derivatives





Alarcon, Weiss, PRC **97,** 055203 (2018)

- Form factor derivatives from DR

$$\left.\frac{d^n G_i^V(t)}{dt^n}\right|_{t=0} = \int_{4M_\pi^2}^{\infty} \frac{dt'}{\pi} \frac{\mathrm{Im}\, G_i^V(t')}{t'^{n+1}}$$

- Two dynamical scales

  $4M_\pi^2$ two-pion threshold

  M_ρ^2 maximum of spectral function

  Relative weight depends on n

- Unnatural size of higher derivatives

  Model-independent prediction

  Could be tested in polynomial fits

Applications: Muon scattering

- Same framework can be used to simulate/analyze μp elastic scattering

- Implemented so far

  Code for evaluating μp elastic cross section in terms of DIχEFT FFs: Jupyter notebook

  Studied sensitivity of cross section to radii in MUSE kinematics
  $\rightarrow$ J. Alarcon

  Two-photon exchange corrections can be included
  Use results of Tomalak, Vanderhaeghen 2016-18

- Further steps: Discussion

Summary

- DIχEFT describes nucleon FFs combining dispersion theory and χEFT

  Includes $\pi\pi$ rescattering and ρ resonance through unitarity

  Enables predictive calculations, controlled theoretical accuracy

  Excellent agreement with empirical FFs up to $Q^2 \sim 1$ GeV^2 and beyond

- DIχEFT enables theory-based radius extraction

  Correlates $Q^2 = 0$ derivatives with finite-Q^2 behavior through analyticity + sum rules

  Employs radii directly as parameters $\leftrightarrow$ LECs

  Extractions performed: FF and cross section fits, electric and magnetic radius

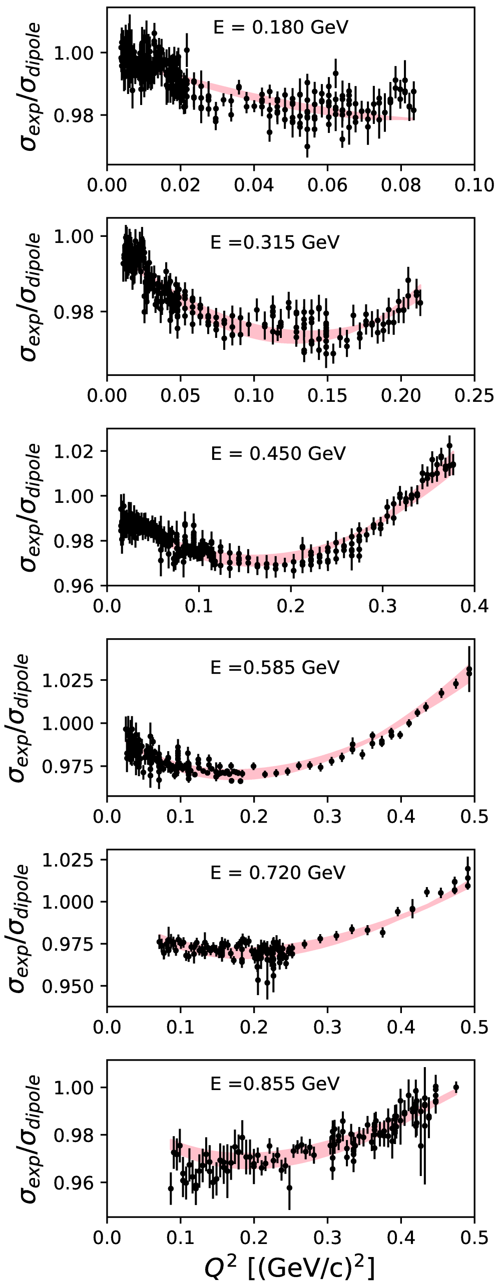
- Other applications and extensions (not covered here)

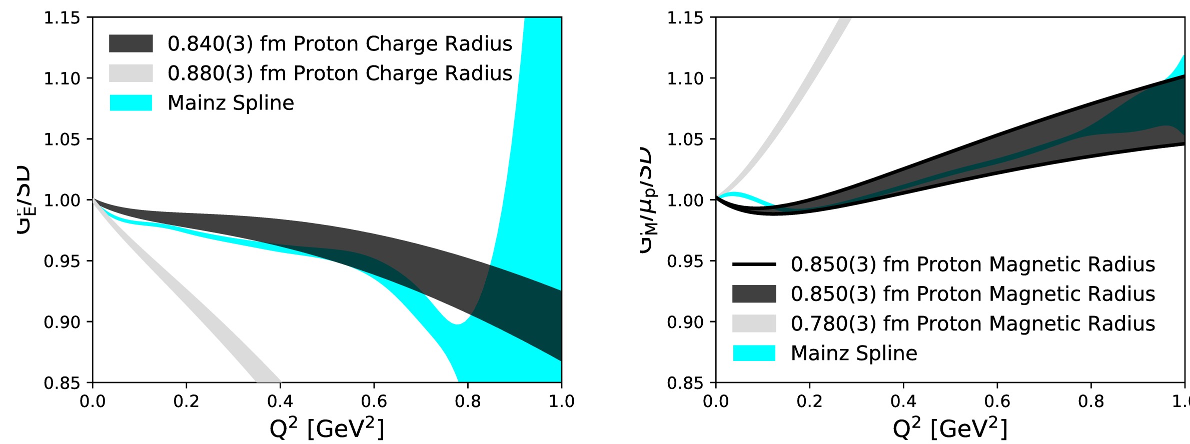
  Nucleon transverse charge/magnetization densities
  Alarcon, Hiller Blin, Vicente Vacas, Weiss, NPA **96,** 18 (2017); Alarcon, Weiss, in progress

  Nucleon scalar FF
  Alarcon, Weiss, PRC **96,** 055206 (2017)

  Resonance transition FFs $N \to \Delta$ planned

Supplement: Radius extraction





Left: Mainz A1 data and DIχEFT fit. The DIχEFT predictions are shown at the radii corresponding to the best fit. The band shows the fit uncertainty.

Above: Comparison of DIχEFT parameterization with the Mainz spline fit. The DIχEFT predictions are shown for two values of the radii within the range of the Mainz fit.

J. M. Alarcon, D. Higinbotham, C. Weiss, PRC 102 (2020) 035203