Instanton vacuum, chiral symmetry breaking, and the gluonic structure of hadrons

C. Weiss (Jefferson Lab), Simons Center Summer Seminar, 19-July-2021

Based on work with: D.I. Diakonov, V.Yu. Petrov, M.V. Polyakov, P.V. Pobylitsa

Essential work in this field by: E.V. Shuryak, I. Zahed, J. Verbaarschot, M. Nowak, T. Schäfer, others (see refs)

This seminar
Attempt high-level overview; can provide further explanations in discussion
Emphasize concepts and connections

Physical picture
Topological gauge fields
Chiral symmetry breaking ↔ fermionic zero modes

Instanton vacuum
Variational approximation
Effective theory of massive fermions
Correlation functions and nucleon state

Gluonic operators
Effective operators
$\tilde{F}F$ and axial anomaly ↔ topology
$\tilde{F}FF$ dim-6 operator ↔ CP-violation
Twist-2 quark/gluon ops ↔ partonic structure
$F^2$ and scale anomaly ↔ nucleon mass
Physical picture: Topological fluctuations

Vacuum fluctuations of gauge fields

Average size $\bar{\rho} \approx 0.3$ fm, separation $\bar{R} \approx 1$ fm

Packing fraction $\pi^2 \bar{\rho}^4 / \bar{R}^4 \approx 0.1 \rightarrow$ small parameter

Topologically charged: $\frac{1}{32\pi^2} \int_{\bar{R}} d^4 x \tilde{F}F(x) \approx \pm 1$

$\leftrightarrow$ Instantons: Classical solutions, self-dual

Strong fields: $(F^2)^{1/4} \approx (32\pi^2 / \pi^2 \bar{\rho}^4)^{1/4} \approx 1.5$GeV

Evidence

Indirect: Correlation functions, hadron structure
[Shuryak 1982; Diakonov, Petrov 1984; Shuryak, Schafer 1993 \rightarrow see later refs]

Direct: LQCD “cooling” of field configurations
Topological fields $\rightarrow$ fermionic zero modes

$$(i\hat{\nabla}_{\text{top}} + im)\Phi_{\pm} = \lambda \Phi_{\pm}, \quad \lambda = im \rightarrow 0$$

Interactions flip chirality ($\leftrightarrow$ perturbative int)

Chiral symmetry breaking

Mechanical: Quarks experience chirality flip at finite rate during propagation

Mathematical: Finite density of Dirac eigenvalues near $\lambda = 0 \leftrightarrow$ chiral condensate [Banks, Casher 1980]

Consequences

Order parameter: $\langle \bar{\psi}\psi \rangle \equiv \langle \bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L \rangle \neq 0$

Massless bosons: $\langle \bar{\psi}_L^a \psi_R^b \rangle \sim U^{ab}(x)$, phase fluctuations in flavor space

Hadron mass generation, interactions
Physical picture: Instanton vacuum

- Abstract from physical picture

- Construct description of QCD vacuum based on topological fluctuations

- Study chiral symmetry breaking and hadronic correlation functions

- Use diluteness \( (\pi^2 \frac{\bar{\rho}^4}{\bar{R}^4}) \ll 1 \) for systematic construction
Instanton vacuum: Integration over modes

Separate modes

$k > \bar{\rho}^{-1}$: Integrate perturbatively:
Renormalization, $\bar{\rho}^{-2} \gg \Lambda_{QCD}^2$

$k < \bar{\rho}^{-1}$: Integrate nonperturbatively:
Instantons + massive fermions

Integrate over modes

\[
\int [DA]_{\text{low}} \int [DA]_{\text{high}} \exp(-S_{YM}) \times \text{fermions} \ldots
\]

\[
A(x) = \sum_I A_I(x|z_I, \rho_I, O_I) + \sum_{\bar{I}} A_{\bar{I}}(x|\ldots)
\]

\[
\rightarrow \int \prod_{I, \bar{I}} dz_I d\rho_I dO_I d_0(\rho_I) \ldots
\]

\[
d_0(\rho) = C \rho^{-5} (\rho \Lambda_{QCD})^b \times \text{NLO}, \quad b = \frac{11}{3} N_c
\]
Instanton interaction

Needed to stabilize medium, suppress large \( \rho \)
[other ideas: Shuryak 1995]

Generic description sufficient: \( \langle U_{\bar{I}}(\bar{I}) \rangle \)
[Diakonov, Petrov 1984; ...]

All dimensions from \( \Lambda_{\text{QCD}} \), no external scales

Variational approximation

Interacting instanton ensemble as variational approximation to QCD partition function (Feynman variational principle)
[Diakonov, Petrov 1984]

Model dependence (gauge choice, instanton superposition, interactions) properly “contained” in variational approximation

Independent instantons with effective size distribution \( d_{\text{eff}}(\rho) \):
Includes interaction effects (“mean field”), suppresses large \( \rho \)
[Beyond mean field: Numerical simulations of interfacing ensemble. Shuryak et al.]

Robust description achieved, only generic properties used

Preserves renormalization properties of QCD:
All dynamical scales “emerge” from \( \Lambda_{\text{QCD}} \) via \( \bar{\rho}, N/V \)
Fermions in 1-instanton background

Spectrum contains zero + non-zero modes

\[ \int dO_I \bar{\psi} | \Phi_I ) ( \Phi_I | \psi = \text{det}_{ab} \bar{\psi}_L^a \psi_R^b \]

'tHooft vertex

Chiral symmetry breaking in instanton ensemble

Construct fermion Green function in multi-instanton background, average over instanton ensemble

[Diakonov, Petrov 1986; Pobylitsa 1989; Nowak, Verbaarschot, Zahed 1989; ...]

\( N_c \rightarrow \infty \): Saddle point approxn in functional integral

\[ Z = \int D\bar{\psi} D\psi \int \prod_{I,\bar{I}} d\bar{z}_I d\rho_I dO_I d_{\text{eff}}(\rho_I) \exp \left\{ -S_I[\bar{\psi}, \psi] \right\} \]

average of fermion action over instanton ensemble

\[ \rightarrow \int D\bar{\psi} D\psi \exp \left\{ \int \bar{\psi} i \hat{D} \psi + \lambda \det \bar{\psi} \frac{1 + \gamma_5}{2} \psi + \lambda \det \bar{\psi} \frac{1 - \gamma_5}{2} \psi + \mathcal{O}(m_f) \right\} \]

effective fermion action with multifermonic interactions

Saddle point \( \lambda = \text{function}(N/V, \bar{\rho}) \) sets scale, defines dynamical quark mass ("gap equation")
Instanton vacuum: Bosonization

Bosonization of fermionic integral

Auxiliary integration variable \( U^{ab} \sim \bar{\psi}^a_L \psi^b_R \), no kinetic term (Hubbard-Stratonovich transformation)

\[ Z = \int D\bar{\psi} D\psi \exp \left\{ \int \bar{\psi} i \gamma^\mu \partial_{\mu} \psi + \lambda \det \bar{\psi} \frac{1 + \gamma_5}{2} \psi + \lambda \det \bar{\psi} \frac{1 - \gamma_5}{2} \psi \right\} \]

Effective theory with multifermionic interaction

\[ \rightarrow \int D U \int D\bar{\psi} D\psi \exp \left\{ \int \bar{\psi} i \gamma^\mu \partial_{\mu} \psi - M \bar{\psi} U \frac{1 + \gamma_5}{2} \psi - M \bar{\psi} U^\dagger \frac{1 - \gamma_5}{2} \psi \right\} \]

Yukawa-type interaction with meson field

Dynamical content of effective theory

Degree of freedom: Massive quarks + Goldstone bosons

Dynamical quark mass \( M \) active for \( k \lesssim \bar{\rho}^{-1} \approx 0.6 \text{ GeV} \)

\( M(0) \approx 0.3\text{-}0.4 \text{ GeV} \) typical “constituent quark” mass

Quark-pion coupling \( g_{\pi qq} = M/F_\pi \approx 4 \)

Strongly coupled system, solved nonperturbatively
**Instanton vacuum: Correlation functions**

### 1/N_c expansion

Saddle point approximation to functional integral

### Meson correlators

Saddle point \( U = 1 \): Vacuum fields

Pseudoscalar: Pion pole (isovector), \( \eta' \) mass (isoscalar)

### Baryon correlators

Source with \( N_c \) quark fields

Saddle point \( U \neq 1 \): Classical pion field (“soliton”)

[Diakonov, Petrov, Pobylitsa 1988]

Fixed time: Quarks moving in self-consistent pion field with single-particle Hamiltonian \( H = \alpha p + \beta M U_{\text{class}} \)

[Kahana, Ripka 1984]

Realization of generic relativistic mean-field picture of nucleon at large \( N_c \) [Witten 83]. Unifies “quark model” and “skyrmion”

Numerous results: Masses, form factors, \( N \rightarrow \Delta, \text{SU(3)} \)

[Review: Christov et al 1995]
Instanton vacuum: Polarized antiquarks in nucleon

Predicts large flavor asymmetry of polarized antiquarks in nucleon at scale $\mu^2 = \bar{\rho}^{-2}$

$$| \Delta \bar{u} - \Delta \bar{d} | > | \bar{u} - \bar{d} |$$

in $1/N_c$ expansion

$\Delta \bar{u} - \Delta \bar{d}$ from spin-flavor structure of chiral mean field, robust feature

Seen in RHIC $W^\pm$ production data

[Adamczyk et al (STAR) 2014]
• Effective theory derived from integration over QCD gauge fields; known connection between effective and fundamental degrees of freedom

• Can evaluate gluonic operators and their hadronic correlation functions

• Use same approximations as in derivation of effective theory: Diluteness \( \left( \frac{\pi^2 \bar{\rho}^4}{\bar{R}^4} \right) \ll 1 \), saddle point approximation \( 1/N_c \)
Gluon operators: Effective operators

Gluon operator

\[ \mathcal{F}[A] \] local QCD gluon operator

Normalized at scale \( \mu = \rho^{-1} \)

Gluon operator in instanton vacuum correlation functions

\[ \mathcal{F}[A] \rightarrow \sum_{I+\bar{I}} \mathcal{F}[A_I] + \mathcal{O}(\rho^4/R^4) \]

\[ \langle \ldots \mathcal{F}[A] \ldots \rangle_{\text{inst}} \rightarrow \langle \ldots \mathcal{F}''[\bar{\psi},\psi] \ldots \rangle_{\text{eff}} \]

\[ "\mathcal{F}''[\bar{\psi},\psi] = N \int d\bar{z}_I dO_I d\rho_I d_{\text{eff}}(\rho_I) \mathcal{F}[A_I] \bar{\psi} | \Phi_I(\Phi_I|\psi \]

Construction possible in saddle-point approximation (large \( N_c \))

Advantages of effective operator representation: Universal, relations between various hadronic matrix elements, insight into origin of gluonic structure

[Diakonov, Polyakov, Weiss, 1995]
Gluon operators: \( \tilde{F}F \)

\( \tilde{F}F \) in QCD: Axial anomaly

\[
\langle N' | \tilde{F}F(0) | N \rangle = A_{\tilde{F}F}(t) \bar{u}'i\gamma_5 u
\]

Form of nucleon matrix element, \( t = (p' - p)^2 \)

\[
\partial^\mu J_5^\mu(x) = \frac{N_f}{16\pi^2} \tilde{F}F(x) + \mathcal{O}(m_f)
\]

Anomalous divergence of flavor-singlet axial current, QCD operator relation, from renormalization

\[
A_{\tilde{F}F}(0) = 32\pi^2 g_A(0)/N_f
\]

Matrix element of \( \tilde{F}F \) determined by axial coupling \( g_A(0) \)

\( \tilde{F}F \) in instanton vacuum: Effective operators

\[
''\tilde{F}F'' = 32\pi^2 \lambda \left[ \det \bar{\psi} \frac{1 + \gamma_5}{2} \psi - \det \bar{\psi} \frac{1 - \gamma_5}{2} \psi \right]
\]

Difference of \( I \) and \( \bar{I} \) 'tHooft vertices, \( \lambda \) scale from saddle point

\[
J_5^\mu(x)_{\text{eff}} = \bar{\psi}\gamma_\mu\gamma_5\psi, \quad \partial^\mu J_5^\mu(x)_{\text{eff}} = 2N_f \lambda \left[ \det \bar{\psi} \frac{1 + \gamma_5}{2} \psi - \det \bar{\psi} \frac{1 - \gamma_5}{2} \psi \right]
\]

Axial current and its divergence in effective theory (Noether theorem)

Effective operators obey same relation as QCD operators in axial anomaly!

Nucleon matrix element of ''\( \tilde{F}F'' \) given by \( g_A(0) \) calculated in effective theory

[Diakonov, Polyakov, Weiss, 1995. See also Nowak, Verbaarschot, Zahed 1989]

\( U(1)_A \) symmetry breaking realized differently in QCD and effective theory:

Anomalous breaking from high-momentum modes ↔ explicit breaking from dynamical scale
Gluon operators: $\tilde{F}FF$

**Dimension-6 gluon operator**

$$f^{abc}\tilde{F}^a_{\mu\nu}F^b_{\mu\rho}F^c_{\nu\rho}(x)$$  
Dimension-6 CP-odd gluon operator, essentially non-abelian structure

Appears in scenario of hadronic CP violation [Weinberg 89]

Need estimates of hadronic matrix elements!  
[Bigi, Uraltsev 1991, Hatta 2020]

**Instanton vacuum estimate**

Operators $\tilde{F}FF$ and $\tilde{F}F$ proportional in field of single $I(\bar{I})$, effective operators simply related  
[Weiss 2021]

Operators $\tilde{F}FF$ and $\tilde{F}F$ proportional in field of single $I(\bar{I})$, effective operators simply related  
[Weiss 2021]

- Nucleon matrix element of $\tilde{F}FF$ inferred from $\tilde{F}F \leftrightarrow g_A^{(0)}$
- Large numerical value due to localization of instanton field

Further conclusion (paradox): Neutron EDM induced by $\tilde{F}FF$ is proportional to that induced by $\tilde{F}F$ and therefore chirally suppressed  
[Chiral behavior of neutron EDM: Crewther, DiVecchia, Veneziano, Witten 1979]
Gluon operators: Parton distributions

\[ \bar{\psi} \gamma_{\mu_1} \nabla_{\mu_2} \ldots \nabla_{\mu_n} \psi - \text{traces} \]

\[ F_\alpha \{ \mu_1 D_{\mu_2} \ldots D_{\mu_{n-1}} F_{\mu_n} \}_\alpha - \text{traces} \]

Leading-twist QCD operators

Twist-2 spin-n quark/gluon operators, scale-dep.

Matrix elements = moments of parton distributions

Measured in deep-inelastic processes in scaling limit

Matrix elements from instanton vacuum

Quark/antiquark distributions leading

Gluon distribution suppressed \[ \{ \text{in instanton packing fraction} \] \]

Parton picture with quarks/antiquarks only, no gluons

\[ \int dx \ x [q + \bar{q}](x) = 1: \text{Momentum sum rule saturated by quarks + antiquarks} \]

Twist-2 and leading order in packing fraction: Instantons entirely subsumed in interactions in effective theory, not manifest in parton content

[Diakonov, Petrov, Pobylitsa, Polyakov, Weiss, 1996]
Gluon operators: Higher twist

Higher-twist QCD operators

Power corrections $1/Q^2$ to DIS processes
[Shuryak Vainshtein 1982; Jaffe, Soldate 1982; Ellis, Fumanski, Petronzio 1982]

Quark-gluon correlations (also quark-quark)

Matrix elements from instanton vacuum

Hierarchy of matrix elements: Twist-3 $\ll$ Twist-4
[Balla, Polyakov, Weiss 1998]

Twist-3: $\langle \bar{\psi} \tilde{F}_{\mu\nu} \gamma_\rho \psi \rangle \sim \bar{R}^{-2}$

Twist-4: $\langle \bar{\psi} \tilde{F}_{\mu\nu} \gamma_\nu \psi \rangle \sim \bar{\rho}^{-2}$

Instanton calculation: $-0.22$ GeV$^2$

DIS data analysis: $-0.20 \pm 0.14$ GeV$^2$
[Sidorov, Weiss 2006]

[Also: Quark-gluon vs. quark-quark correlations in unpolarized structure functions $F_2, F_L(x, Q^2)$
[Dressler, Maul, Weiss 2000]
Gluon operators: $F^2$ and scale anomaly

\[ \Theta_{\mu\mu} = \frac{\beta(g^2)}{4g^2} F^2 \approx -\frac{b}{32\pi^2} F^2 \]

\[ \langle N' | F^2 | N \rangle = A_{F^2}(t) \ m_{N'} \bar{u} u \]

\[ A_{F^2}(0) = -\frac{32\pi^2}{b} \]

\[ P(N) \propto \left( \frac{N}{\langle N \rangle} \right)^{-bN/4} \exp \frac{bN}{4} \]

\[ \langle N^2 \rangle - \langle N \rangle^2 = \frac{4}{b} \langle N \rangle \text{ etc.} \]

Scale invariance breaking in QCD

Scale invariance broken by quantum fluctuations:

UV cutoff → RNG equation → $\Lambda_{\text{QCD}}$ scale

Trace anomaly: $\Theta_{\mu\mu} \neq 0$, generator of scale transformations

Implications: LETs for correlation functions of $F^2$, constraints on hadronic matrix elements of $F^2$

Realization in instanton vacuum

Grand canonical ensemble with fluctuating $N = N_+ + N_-$

Number distribution $P(N)$ governed by $b$ (beta function)

Implemented naturally in variational approach

Reproduces LETs for $F^2$, nucleon matrix of $F^2$ at $t = 0$

Challenge: Matrix elements at nonzero momentum transfer, form factors ↔ hadron structure

Further topics

Transverse momentum and parton correlations from instantons

Intrinsic transverse momentum of sea quarks in nucleon extends up to scale $\bar{\rho}^{-1}$

$p_T(\text{sea}) \gg p_T(\text{valence})$

Parton short-range correlations $\rightarrow$ hadron rapidity correlations in DIS

[Schweitzer, Strikman, Weiss 2012]

High-momentum transfer processes mediated by instantons

Meson form factors at $Q^2 \gtrsim \bar{\rho}^{-2}$

Nonperturbative mechanism, alternative to pQCD factorization

[Diakononv, Petrov 86; Faccioli, Schwenk, Shuryak 03; Shuryak, Zahed 2020]
Summary

• Topological fluctuations of gauge fields play essential role in low-energy QCD: Fermionic zero modes → chiral symmetry breaking → hadron structure/interactions

• Instanton vacuum provides successful effective description:
  Packing fraction \((\pi^2 \rho^4 / \bar{R}^4)\) as small parameter
  All scales generated dynamically from \(\Lambda_{\text{QCD}}\)
  Analytic \((1/N_c)\) and numerical implementations

• Gluon operators can be evaluated systematically:
  Axial and scale anomalies realized, renormalization properties
  Parametric picture of nucleon’s “gluonic content”

• Essential tool for studying questions of hadron physics:
  Partonic content, “origin of mass”, spin effects, quark/gluon correlations,
  high-momentum transfer processes, CP violation, ...