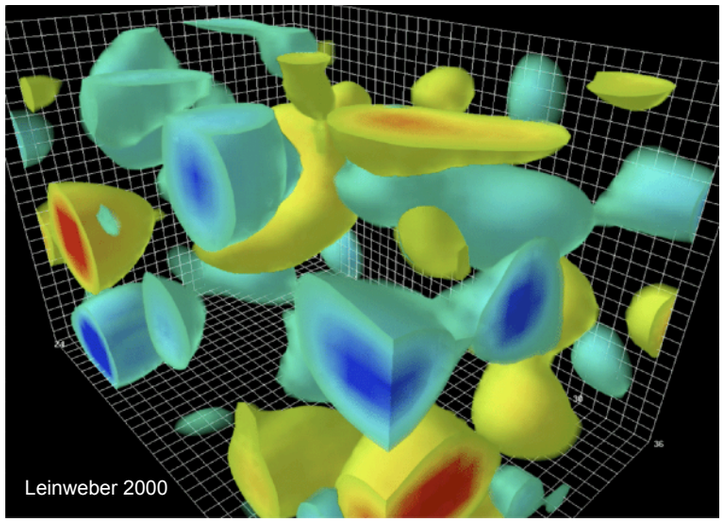
Instanton vacuum, chiral symmetry breaking, and the gluonic structure of hadrons

C. Weiss (Jefferson Lab), Simons Center Summer Seminar, 19-July-2021

Jefferson Lab



Based on work with: D.I. Diakonov, V.Yu. Petrov, M.V. Polyakov, P.V. Pobylitsa

Essential work in this field by:
E.V. Shuryak, I. Zahed, J. Verbaarschot, M. Nowak, T. Schäfer, others (see refs)

This seminar

Attempt high-level overview; can provide further explanations in discussion

Emphasize concepts and connections

Physical picture

Topological gauge fields

Chiral symmetry breaking ↔ fermionic zero modes

Instanton vacuum

Variational approximation

Effective theory of massive fermions

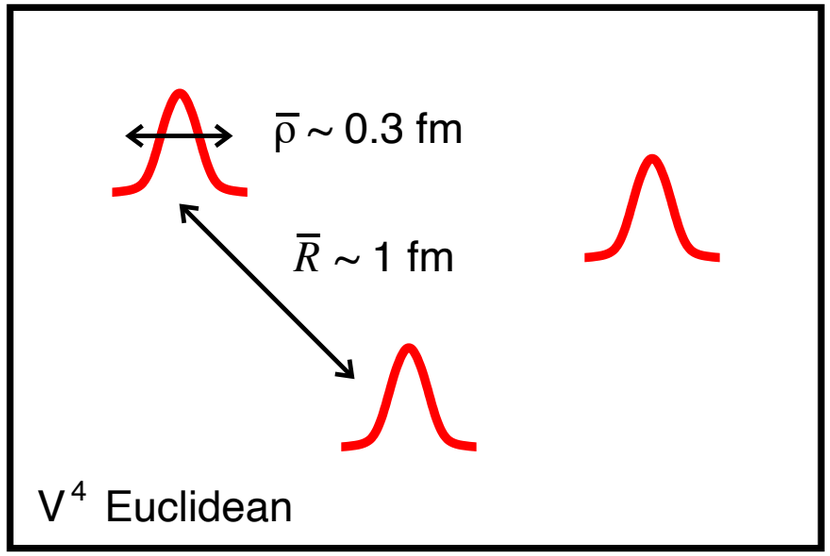
Correlation functions and nucleon state

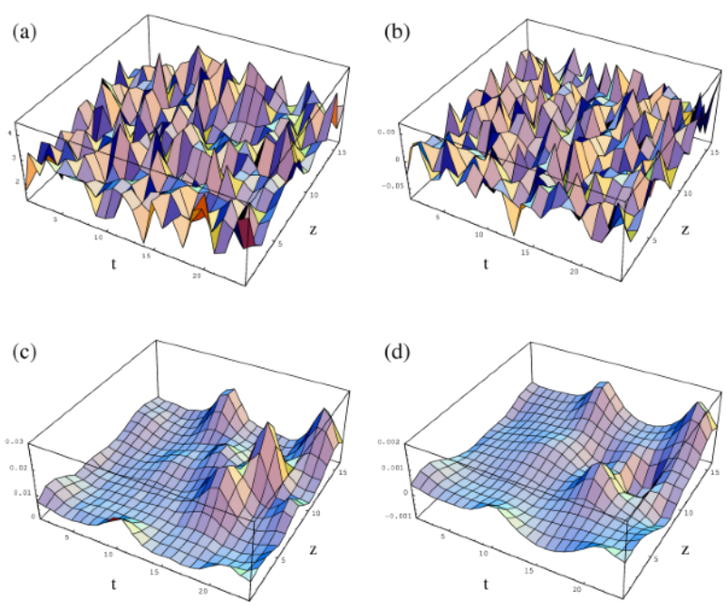
Gluonic operators

Effective operators

$\tilde{F}F$ and axial anomaly	↔ topology
$\tilde{F}FF$ dim-6 operator	↔ CP-violation
Twist-2 quark/gluon ops	↔ partonic structure
F^2 and scale anomaly	↔ nucleon mass

Physical picture: Topological fluctuations





Chu et al, PRL 70 (1993) 225; PRD 49 (1994) 6039

Vacuum fluctuations of gauge fields

Average size $\bar{\rho} \approx 0.3$ fm, separation $\bar{R} \approx 1$ fm

Packing fraction $\pi^2\bar{\rho}^4/\bar{R}^4 \approx 0.1 \rightarrow$ small parameter

Topologically charged: $\frac{1}{32\pi^2}\int_{\bar{R}} d^4x\, \tilde{F}F(x) \approx \pm 1$

$\leftrightarrow$ Instantons: Classical solutions, self-dual

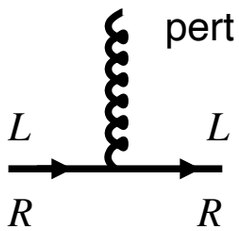
Strong fields: $(F^2)^{1/4} \approx (32\pi^2/\pi^2\bar{\rho}^4)^{1/4} \approx 1.5$ GeV

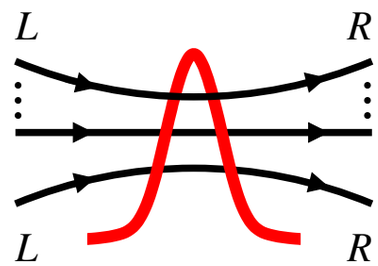
Evidence

Indirect: Correlation functions, hadron structure
[Shuryak 1982; Diakonov, Petrov 1984; Shuryak, Schafer 1993 → see later refs]

Direct: LQCD “cooling” of field configurations
[Polikarpov, Veselov 1988; Campostrini et al. 1990; Chu, Negele et al 1993; DeGrand et al 1997; de Forcrand et al 1997, …, Athenodorou et al 2018]

Physical picture: Chiral symmetry breaking

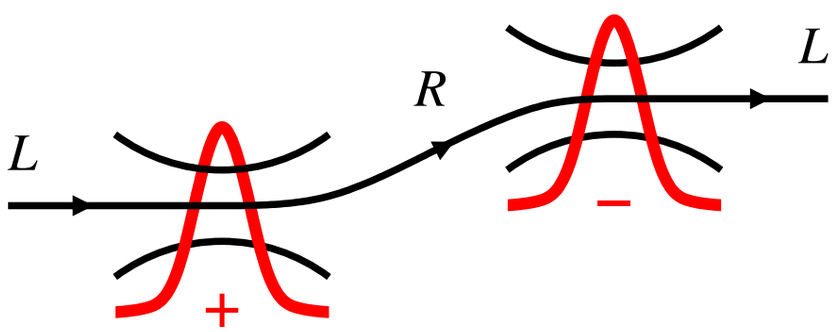




Topological fields → fermionic zero modes

$$(i\hat{\nabla}_{\text{top}} + im)\Phi_{\pm} = \lambda\Phi_{\pm}, \quad \lambda = im \to 0$$

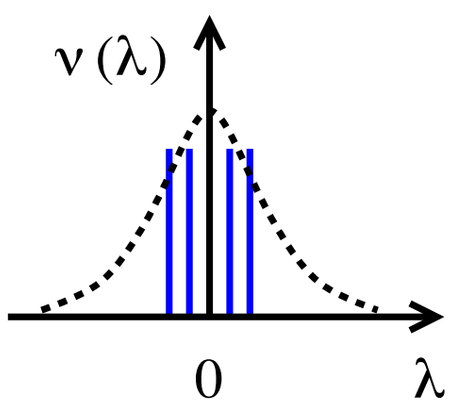
Interactions flip chirality (↔ perturbative int)



Chiral symmetry breaking

Mechanical: Quarks experience chirality flip at finite rate during propagation

Mathematical: Finite density of Dirac eigenvalues near $\lambda = 0 \leftrightarrow$ chiral condensate [Banks, Casher 1980]



Consequences

Order parameter: $\langle\bar{\psi}\psi\rangle \equiv \langle\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L\rangle \neq 0$

Massless bosons: $\langle\bar{\psi}_L^a\psi_R^b\rangle \sim U^{ab}(x)$,
phase fluctuations in flavor space

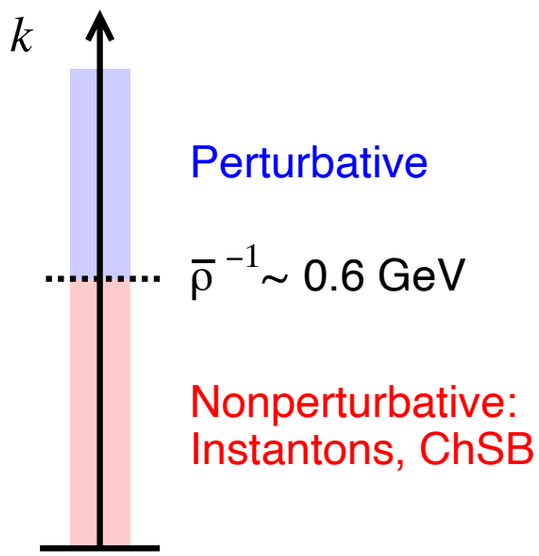
Hadron mass generation, interactions

Physical picture: Instanton vacuum

- Abstract from physical picture
- Construct description of QCD vacuum based on topological fluctuations
- Study chiral symmetry breaking and hadronic correlation functions

- Use diluteness $(\pi^2\bar{\rho}^4/\bar{R}^4) \ll 1$ for systematic construction

Instanton vacuum: Integration over modes



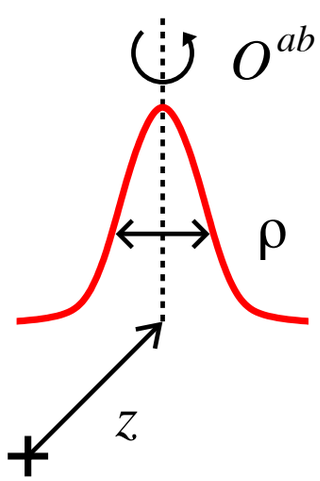
Separate modes

$k > \bar{\rho}^{-1}$: Integrate perturbatively:
Renormalization, $\bar{\rho}^{-2} \gg \Lambda^2_{\rm QCD}$

$k < \bar{\rho}^{-1}$: Integrate nonperturbatively:
Instantons + massive fermions

Integrate over modes

$$\underbrace{\int [DA]_{\rm low}} \int [DA]_{\rm high} \, \exp(-S_{\rm YM}) \;\; [\times \text{ fermions}] \; \ldots$$



$$A(x) \;=\; \sum\nolimits_I A_I(x\,|\,z_I, \rho_I, O_I) \;+\; \sum\nolimits_{\bar{I}} A_{\bar{I}}(x\,|\,\ldots)$$

superposition of $I\,(\bar{I})$ (singular gauge)

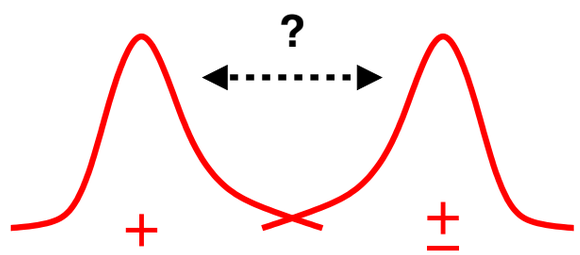
$$\rightarrow \int \prod_{I,\bar{I}} dz_I \, d\rho_I \, dO_I \; d_0(\rho_I) \; \ldots$$

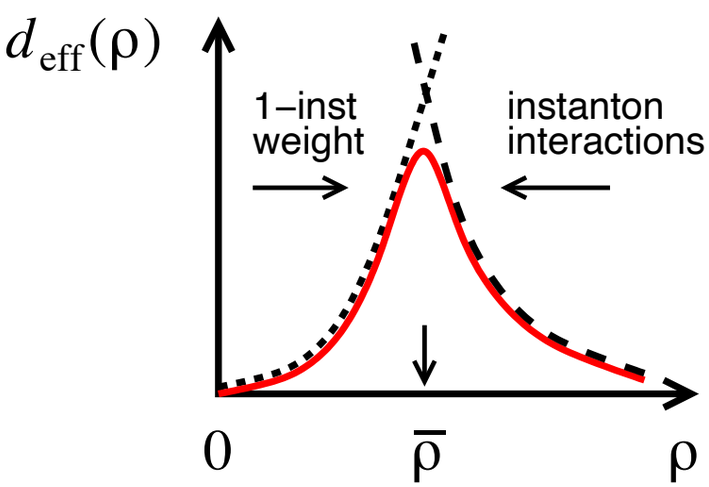
integration over collective coordinates

$$d_0(\rho) \;=\; C\,\rho^{-5}\,(\rho\Lambda_{\rm QCD})^b \; [\times \text{ NLO}], \;\; b = \frac{11}{3} N_c$$

weight from high-mom integration [‘tHooft 1976]

Instanton vacuum: Variational approach





$$Z_{\rm int} = \int \prod_{I,\bar{I}} dz_I \, d\rho_I \, dO_I \; d_{\rm eff}(\rho_I)$$

$$d_{\rm eff}(\rho) \;\sim\; d_0(\rho) \;\exp(-\alpha\rho^2)$$

Instanton interaction

Needed to stabilize medium, suppress large ρ
[other ideas: Shuryak 1995}

Generic description sufficient: $\langle U_{II\,(I\bar{I})} \rangle$
[Diakonov, Petrov 1984; ….]

All dimensions from $\Lambda_{\rm QCD}$, no external scales

Variational approximation

Interacting instanton ensemble as variational approximation to QCD partition function (Feynman variational principle)
[Diakonov, Petrov 1984]

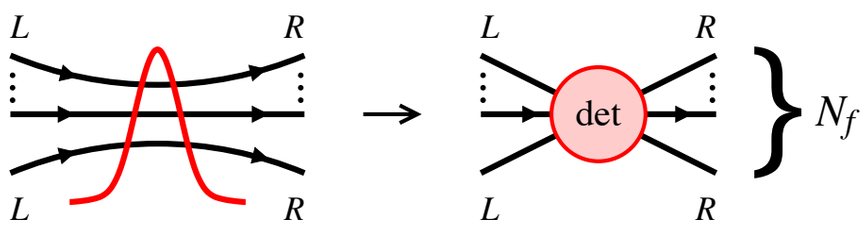
Model dependence (gauge choice, instanton superposition, interactions) properly “contained” in variational approximation

Independent instantons with effective size distribution $d_{\rm eff}(\rho)$:
Includes interaction effects (“mean field”), suppresses large ρ
[Beyond mean field: Numerical simulations of interfacing ensemble. Shuryak et al.]

Robust description achieved, only generic properties used

Preserves renormalization properties of QCD:
All dynamical scales “emerge” from $\Lambda_{\rm QCD}$ via $\bar{\rho}, N/V$

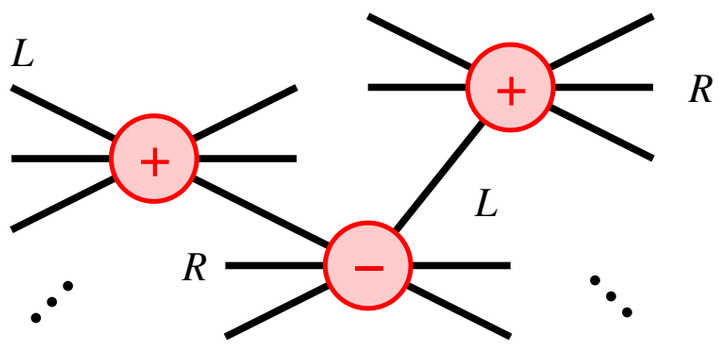
Instanton vacuum: Fermions and ChSB



Fermions in 1-instanton background

Spectrum contains zero + non-zero modes

$$\int dO_I \, \bar{\psi}|\Phi_I)(\Phi_I|\psi \;=\; \det_{ab} \bar{\psi}^a_L \psi^b_R$$ 'tHooft vertex



Chiral symmetry breaking in instanton ensemble

Construct fermion Green function in multi-instanton background, average over instanton ensemble

[Diakonov, Petrov 1986; Pobylitsa 1989; Nowak, Verbaarschot, Zahed 1989; …]

$N_c \to \infty$: Saddle point approxn in functional integral

$$Z = \int D\bar{\psi} D\psi \int \prod_{I,\bar{I}} dz_I \, d\rho_I \, dO_I \; d_{\text{eff}}(\rho_I) \; \exp\left\{-S_I[\bar{\psi},\psi]\right\}$$

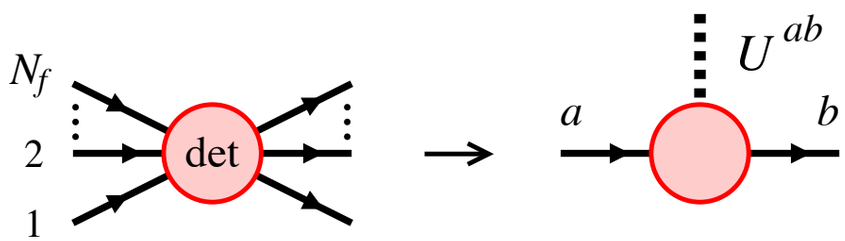
average of fermion action over instanton ensemble

$$\to \int D\bar{\psi} D\psi \; \exp\left\{ \int \bar{\psi} i\hat{\partial}\psi + \lambda \det \bar{\psi} \frac{1+\gamma_5}{2}\psi + \lambda \det \bar{\psi} \frac{1-\gamma_5}{2}\psi + \mathcal{O}(m_f) \right\}$$

effective fermion action with multifermionic interactions

Saddle point $\lambda =$ function$(N/V, \bar{\rho})$ sets scale, defines dynamical quark mass ("gap equation")

Instanton vacuum: Bosonization



Bosonization of fermionic integral

Auxiliary integration variable $U^{ab} \sim \bar{\psi}^a_L \psi^b_R$,
no kinetic term (Hubbard-Stratonovich transformation)

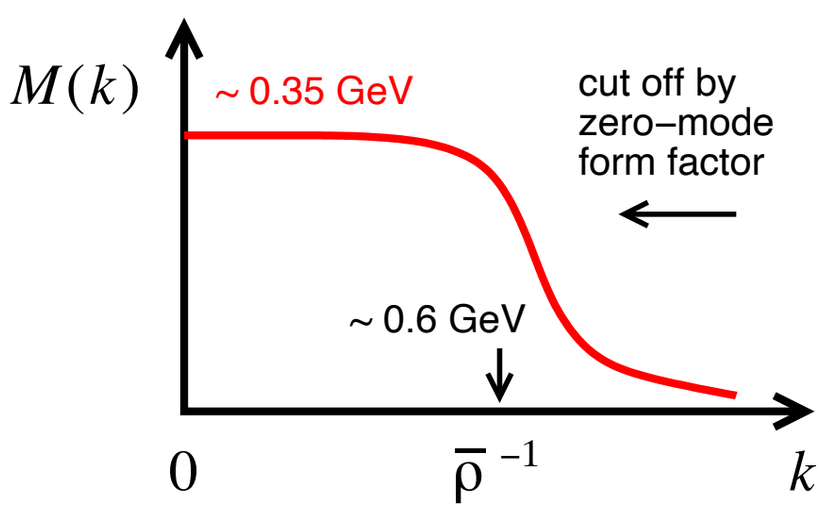
[Diakonov, Petrov 1986; Kacir, Prakash, Zahed 1995; …]

$$Z = \int D\bar{\psi} D\psi \ \exp\left\{ \int \bar{\psi} i\hat{\partial}\psi + \lambda \det \bar{\psi} \frac{1+\gamma_5}{2}\psi + \lambda \det \bar{\psi} \frac{1-\gamma_5}{2}\psi \right\}$$

Effective theory with multifermionic interaction

$$\to \int DU \int D\bar{\psi} D\psi \ \exp\left\{ \int \bar{\psi} i\hat{\partial}\psi - M\bar{\psi}U\frac{1+\gamma_5}{2}\psi - M\bar{\psi}U^\dagger\frac{1-\gamma_5}{2}\psi \right\}$$

Yukawa-type interaction with meson field



Dynamical content of effective theory

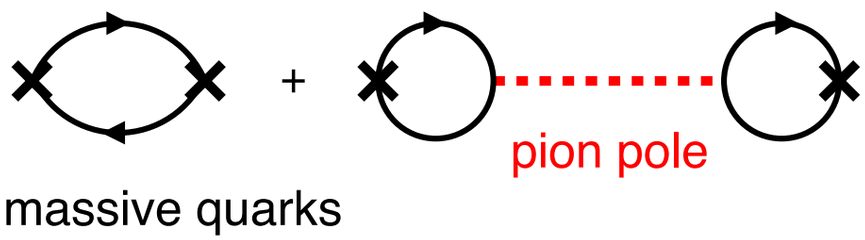
Degree of freedom: Massive quarks + Goldstone bosons

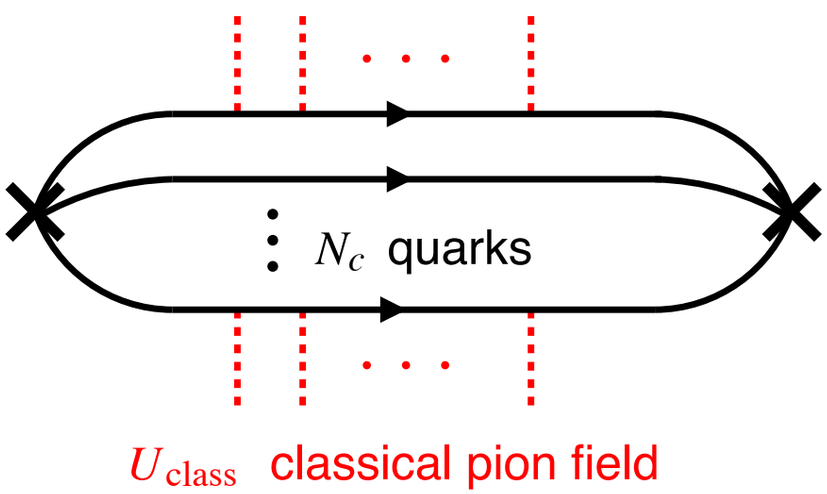
Dynamical quark mass M active for $k \lesssim \bar{\rho}^{-1} \approx$ 0.6 GeV

$M(0) \approx$ 0.3-0.4 GeV typical “constituent quark” mass

Quark-pion coupling $g_{\pi qq} = M/F_\pi \approx 4$
Strongly coupled system, solved nonperturbatively

Instanton vacuum: Correlation functions





1/Nc expansion

Saddle point approximation to functional integral

Meson correlators

Saddle point $U = 1$: Vacuum fields

Pseudoscalar: Pion pole (isovector), η' mass (isoscalar)

Baryon correlators

Source with N_c quark fields

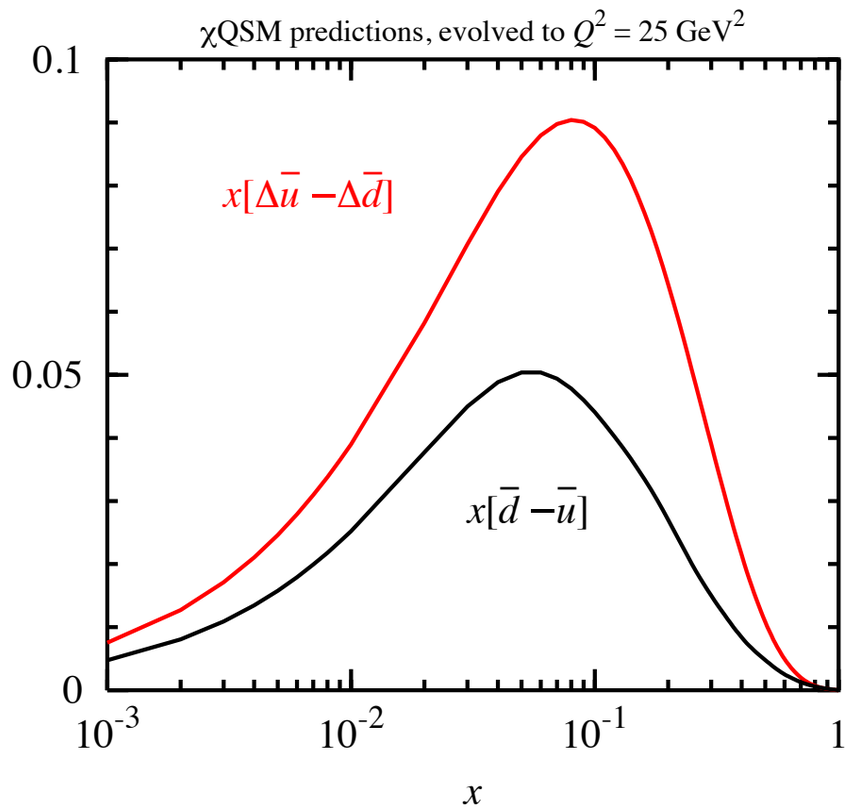
Saddle point $U \neq 1$: Classical pion field ("soliton")
[Diakonov, Petrov, Pobylitsa 1988]

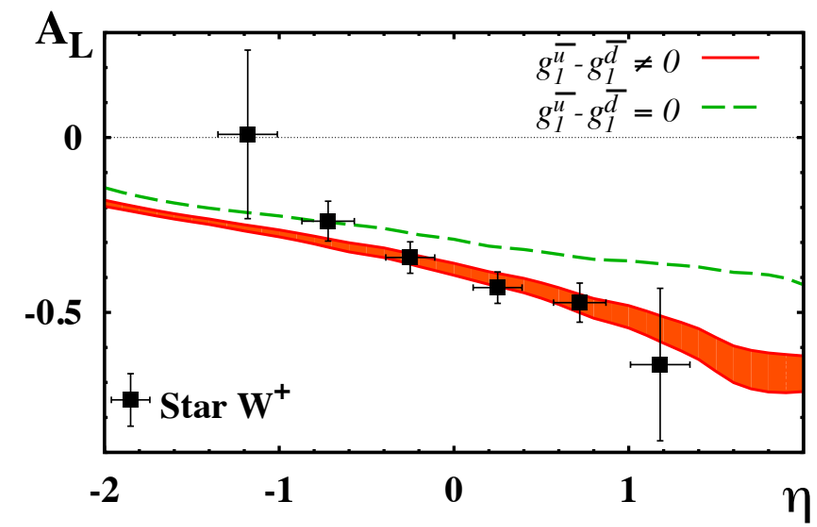
Fixed time: Quarks moving in self-consistent pion field with single-particle Hamiltonian $H = \alpha \mathbf{p} + \beta M U_{\text{class}}$
[Kahana, Ripka1984]

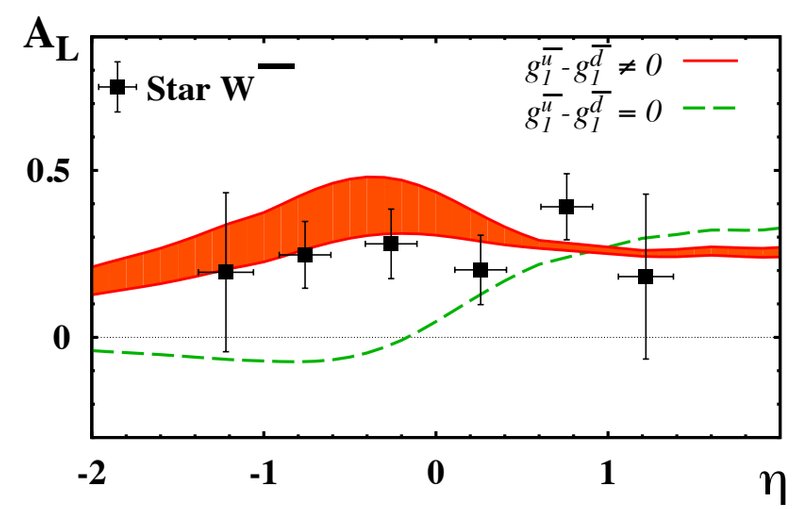
Realization of generic relativistic mean-field picture of nucleon at large N_c [Witten 83]. Unifies "quark model" and "skyrmion"

Numerous results: Masses, form factors, $N \to \Delta$, SU(3)
[Review: Christov et al 1995]

Instanton vacuum: Polarized antiquarks in nucleon







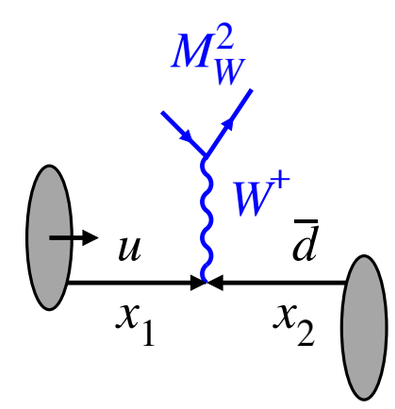
Predicts large flavor asymmetry of polarized antiquarks in nucleon at scale $\mu^2 = \bar{\rho}^{-2}$
[Diakonov, Petrov, Pobylitsa, Polyakov, Weiss, 1996]

$|\Delta\bar{u} - \Delta\bar{d}| > |\bar{u} - \bar{d}|$ in $1/N_c$ expansion

$\Delta\bar{u} - \Delta\bar{d}$ from spin-flavor structure of chiral mean field, robust feature

Seen in RHIC $W^{\pm}$ production data
[Adamczyk et al (STAR) 2014]

[Figs from Mini-Review: Polyakov, Schweitzer, Vogelsang, Weiss, in preparation]



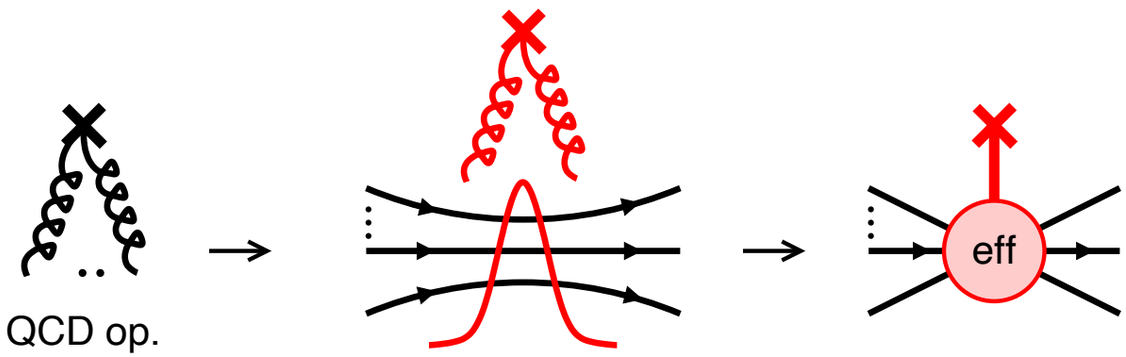
Instanton vacuum: Gluonic operators

- Effective theory derived from integration over QCD gauge fields; known connection between effective and fundamental degrees of freedom

- Can evaluate gluonic operators and their hadronic correlation functions

- Use same approximations as in derivation of effective theory: Diluteness $(\pi^2\bar{\rho}^4/\bar{R}^4) \ll 1$, saddle point approximation $1/N_c$

Gluon operators: Effective operators



Gluon operator

$\mathscr{F}[A]$ local QCD gluon operator

Normalized at scale $\mu = \bar{\rho}^{-1}$

Gluon operator in instanton vacuum correlation functions

$$\mathscr{F}[A] \to \sum_{I+\bar{I}} \mathscr{F}[A_I] \; + \; \mathscr{O}(\rho^4/R^4)$$

Evaluated in gluon field of single $I(\bar{I})$

$$\langle \ldots \mathscr{F}[A] \ldots \rangle_{\text{inst}} \; \to \; \langle \ldots {}''\mathscr{F}''[\bar{\psi}, \psi] \ldots \rangle_{\text{eff}}$$

Converted to effective fermionic operator in effective theory of massive fermions

$$''\mathscr{F}''[\bar{\psi}, \psi] \; = \; N \int dz_I \, dO_I \, d\rho_I \; d_{\text{eff}}(\rho_I) \;\; \mathscr{F}[A_I] \;\; \bar{\psi} \,|\, \Phi_I)(\Phi_I \,|\, \psi$$

Coupling through zero modes ↔ ‘tHooft vertex

Construction possible in saddle-point approximation (large N_c)

[Diakonov, Polyakov, Weiss, 1995]

Advantages of effective operator representation: Universal, relations between various hadronic matrix elements, insight into origin of gluonic structure

Gluon operators: $\tilde{F}F$

$\tilde{F}F$ in QCD: Axial anomaly

$$\langle N' | \tilde{F}F(0) | N \rangle = A_{\tilde{F}F}(t)\, \bar{u}' i\gamma_5 u$$

Form of nucleon matrix element, $t = (p' - p)^2$

$$\partial^\mu J_{5\mu}(x) = \frac{N_f}{16\pi^2} \tilde{F}F(x) + \mathcal{O}(m_f)$$

Anomalous divergence of flavor-singlet axial current, QCD operator relation, from renormalization

$$A_{\tilde{F}F}(0) = 32\pi^2\, g_A^{(0)}/N_f$$

Matrix element of $\tilde{F}F$ determined by axial coupling $g_A^{(0)}$

$\tilde{F}F$ in instanton vacuum: Effective operators

$$''\tilde{F}F'' = 32\pi^2 \lambda \left[\det \bar{\psi} \frac{1+\gamma_5}{2} \psi - \det \bar{\psi} \frac{1-\gamma_5}{2} \psi\right]$$

Difference of I and $\bar{I}$ 'tHooft vertices, λ scale from saddle point

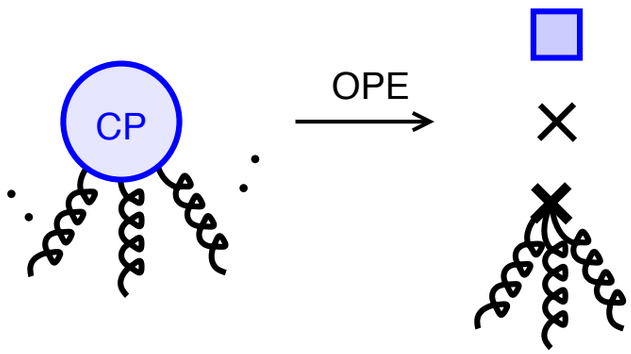
$$J_{5\mu}(x)_{\text{eff}} = \bar{\psi}\gamma_\mu\gamma_5\psi, \quad \partial^\mu J_{5\mu}(x)_{\text{eff}} = 2N_f \lambda \left[\det \bar{\psi} \frac{1+\gamma_5}{2} \psi - \det \bar{\psi} \frac{1-\gamma_5}{2} \psi\right]$$

Axial current and its divergence in effective theory (Noether theorem)

Effective operators obey same relation as QCD operators in axial anomaly!
Nucleon matrix element of $''\tilde{F}F''$ given by $g_A^{(0)}$ calculated in effective theory
[Diakonov, Polyakov, Weiss, 1995. See also Nowak, Verbaarschot, Zahed 1989]

$U(1)_A$ symmetry breaking realized differently in QCD and effective theory:
Anomalous breaking from high-momentum modes ↔ explicit breaking from dynamical scale

Gluon operators: $\tilde{F}FF$



Dimension-6 gluon operator

$f^{abc}\tilde{F}^a_{\mu\nu}F^b_{\mu\rho}F^c_{\nu\rho}(x)$ Dimension-6 CP-odd gluon operator, essentially non-abelian structure

Appears in scenario of hadronic CP violation [Weinberg 89]

Need estimates of hadronic matrix elements!
[Bigi, Uraltsev 1991, Hatta 2020]

Instanton vacuum estimate

$$\frac{\int d^4x\, \tilde{F}FF(x)_{I(\bar{I})}}{\int d^4x\, \tilde{F}F(x)_{I(\bar{I})}} = -\frac{12}{5\bar{\rho}^2}$$

Operators $\tilde{F}FF$ and $\tilde{F}F$ proportional in field of single $I(\bar{I})$, effective operators simply related
[Weiss 2021]

$$\frac{A_{\tilde{F}FF}(0)}{32\pi^2} = \left(-\frac{12}{5\bar{\rho}^2}\right)\frac{g_A^{(0)}}{N_f}$$

Nucleon matrix element of $\tilde{F}FF$ inferred from $\tilde{F}F \leftrightarrow g_A^{(0)}$

$$\frac{12}{5\bar{\rho}^2} = 0.86\,\text{GeV}^2 = (0.22\,\text{fm})^{-2}$$

Large numerical value due to localization of instanton field

Further conclusion (paradox): Neutron EDM induced by $\tilde{F}FF$ is proportional to that induced by $\tilde{F}F$ and therefore chirally suppressed [Chiral behavior of neutron EDM: Crewther, DiVecchia, Veneziano, Witten 1979]

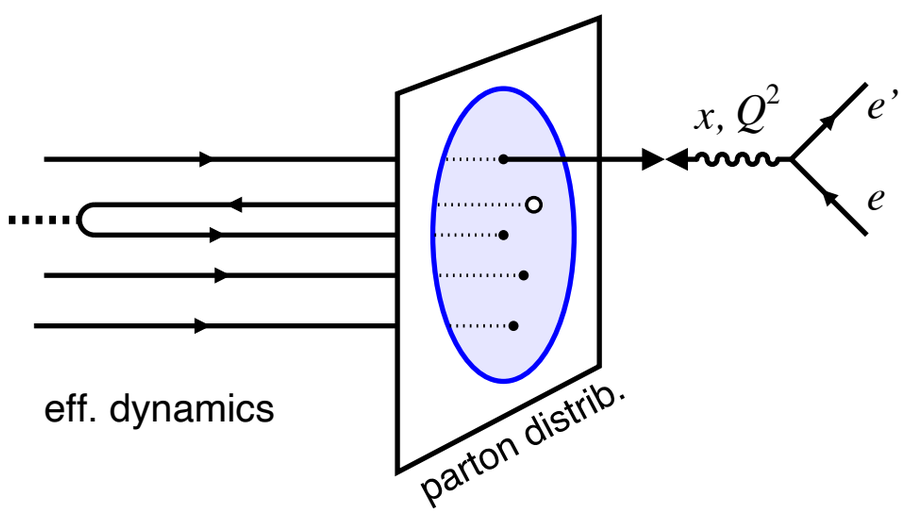
Gluon operators: Parton distributions

$\bar{\psi}\gamma_{\{\mu_1}\nabla_{\mu_2}\ldots\nabla_{\mu_n\}}\psi$ − traces

$F_{\alpha\{\mu_1}D_{\mu_2}\ldots D_{\mu_{n-1}}F_{\mu_n\}\alpha}$ − traces

$$A_q^{(n)} = \mathcal{O}(1)$$

$$A_g^{(n)} = \mathcal{O}[(\bar{\rho}/\bar{R})^4]$$



Leading-twist QCD operators

Twist-2 spin-n quark/gluon operators, scale-dep.

Matrix elements = moments of parton distributions

Measured in deep-inelastic processes in scaling limit

Matrix elements from instanton vacuum

Quark/antiquark distributions leading
Gluon distribution suppressed
} in instanton packing fraction

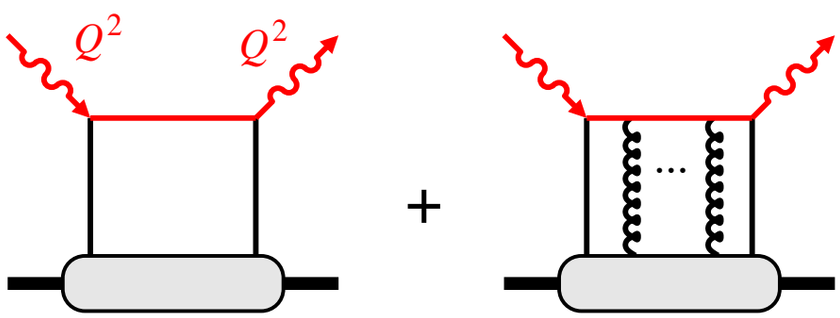
Parton picture with quarks/antiquarks only, no gluons

$\int dx\, x\, [q + \bar{q}](x)$ = 1: Momentum sum rule saturated by quarks + antiquarks

Twist-2 and leading order in packing fraction: Instantons entirely subsumed in interactions in effective theory, not manifest in parton content

[Diakonov, Petrov, Pobylitsa, Polyakov, Weiss, 1996]

Gluon operators: Higher twist



$\bar{\psi}\ldots\psi$ $\qquad$ $\bar{\psi}\ldots F\ldots\psi$

Twist-3: $\langle\bar{\psi}\tilde{F}_{\mu\nu}\gamma_\rho\psi\rangle \sim \bar{R}^{-2}$

Twist-4: $\langle\bar{\psi}\tilde{F}_{\mu\nu}\gamma_\nu\psi\rangle \sim \bar{\rho}^{-2}$

Instanton calculation: -0.22 GeV^2
DIS data analysis: -0.20 ± 0.14 GeV^2
[Sidorov, Weiss 2006]

Higher-twist QCD operators

Power corrections $1/Q^2$ to DIS processes
[Shuryak Vainshtein 1982; Jaffe, Soldate 1982; Ellis, Fumanski, Petronzio 1982]

Quark-gluon correlations (also quark-quark)

Matrix elements from instanton vacuum

Hierarchy of matrix elements: Twist-3 $\ll$ Twist-4
[Balla, Polyakov, Weiss 1998]

Seen in pattern of power corrections to polarized nucleon structure functions $g_1, g_2(x, Q^2)$

Prediction of instanton vacuum, confirmed by experiment and LQCD calculations

Instantons visible in pattern of power corrections to nucleon structure functions

[Also: Quark-gluon vs. quark-quark correlations in unpolarized structure functions $F_2, F_L(x, Q^2)$
[Dressler, Maul, Weiss 2000]

Gluon operators: F^2 and scale anomaly

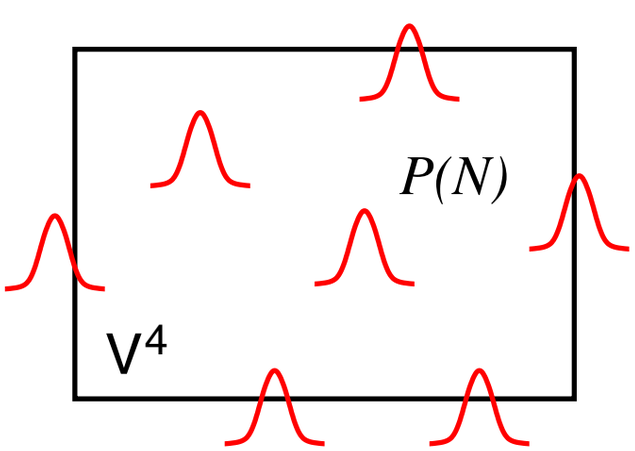
$$\Theta_{\mu\mu} = \frac{\beta(g^2)}{4g^2} F^2 \approx -\frac{b}{32\pi^2} F^2$$

$$\langle N' | F^2 | N \rangle = A_{F^2}(t)\, m_N \bar{u}' u$$

$$A_{F^2}(0) = -\frac{32\pi^2}{b}$$

$$P(N) \propto \left(\frac{N}{\langle N \rangle}\right)^{-\frac{bN}{4}} \exp \frac{bN}{4}$$

$$\langle N^2 \rangle - \langle N \rangle^2 = \frac{4}{b} \langle N \rangle \text{ etc.}$$



Scale invariance breaking in QCD

Scale invariance broken by quantum fluctuations:
UV cutoff → RNG equation → Λ_{QCD} scale

Trace anomaly: $\Theta_{\mu\mu} \neq 0$, generator of scale transformations

Implications: LETs for correlation functions of F^2,
constraints on hadronic matrix elements of F^2
[Shifman, Vainshtein, Zakharov 1978; Novikov, Shifman, Vainshtein, Zakharov 1981]

Realization in instanton vacuum

Grand canonical ensemble with fluctuating $N = N_+ + N_-$

Number distribution $P(N)$ governed by b (beta function)
[Diakonov, Polyakov, Weiss, 1995]

Implemented naturally in variational approach

Reproduces LETs for F^2, nucleon matrix of F^2 at $t = 0$

Challenge: Matrix elements at nonzero momentum transfer,
form factors ↔ hadron structure
[Kacir, Prakash, Zahed, 1995. Review Zahed 2021]

Further topics

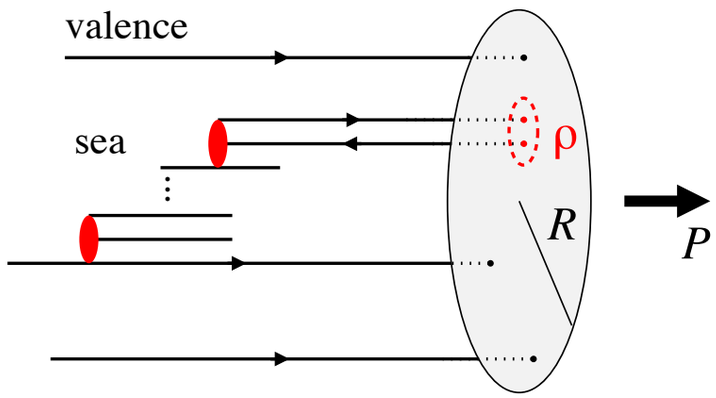
Transverse momentum and parton correlations from instantons

Intrinsic transverse momentum of sea quarks in nucleon extends up to scale $\bar{\rho}^{-1}$

$p_T(\text{sea}) \gg p_T(\text{valence})$

Parton short-range correlations
→ hadron rapidity correlations in DIS

[Schweitzer, Strikman, Weiss 2012]

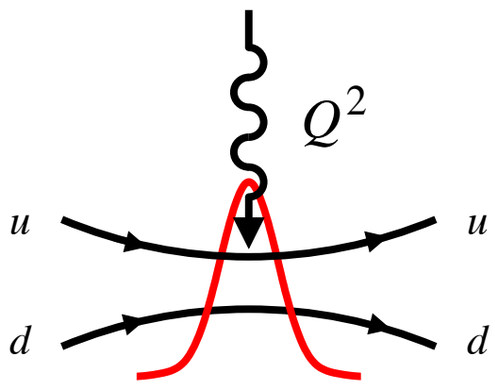


High-momentum transfer processes mediated by instantons

Meson form factors at $Q^2 \gtrsim \bar{\rho}^{-2}$

Nonperturbative mechanism,
alternative to pQCD factorization

[Diakononv, Petrov 86; Faccioli, Schwenk, Shuryak 03; Shuryak, Zahed 2020]



Summary

- Topological fluctuations of gauge fields play essential role in low-energy QCD:
  Fermionic zero modes → chiral symmetry breaking → hadron structure/interactions

- Instanton vacuum provides successful effective description:
  Packing fraction $(\pi^2\bar{\rho}^4/\bar{R}^4)$ as small parameter
  All scales generated dynamically from Λ_{QCD}
  Analytic $(1/N_c)$ and numerical implementations

- Gluon operators can be evaluated systematically:
  Axial and scale anomalies realized, renormalization properties
  Parametric picture of nucleon's "gluonic content"

- Essential tool for studying questions of hadron physics:
  Partonic content, "origin of mass", spin effects, quark/gluon correlations,
  high-momentum transfer processes, CP violation, …