GPDs and gluonic structure from the instanton vacuum

C. Weiss (JLab), GPDs for Nucleon Tomography in the EIC Era, BNL, 19-Jan-2024

Introduction

Hadron structure ↔ ChSB ↔ topological fields

Program

Instanton vacuum

Effective dynamics from chiral symmetry breaking

Hadronic correlation functions

Effective operators from QCD operators

→ Chiral symmetry breaking
  Mass generation
  Long-range hadron structure

→ Hadronic matrix elements of QCD operators \( \langle N' | \hat{O}_{\text{QCD}} | N \rangle \)
  GPDs Twist-2, 3
  Gluonic structure

Diakonov, Polyakov, Weiss, NPB 461, 539 (1996) [INSPIRE]
J-Y Kim, Weiss, PLB 848 (2024) 138387 [INSPIRE]

→ Hadronic matrix elements of QCD operators

Gluonic structure

\( F^2, F \bar{F} \) → trace anomaly, U(1) anomaly

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Introduction: Matrix elements of QCD operators

QCD operators from factorization

Processes: DIS, hard exclusive, heavy quarkonium production, ...

Local spin-n or light-ray operators

Involve QCD gauge potential or fields

Scale-dependent

Need hadronic matrix elements!

[Typical scale $\mu \sim 1$ GeV]

Hadron structure from chiral symmetry breaking

Chiral symmetry breaking governs long-range hadron structure:
Pion as massless excitation, chiral interactions. Systematic construction in Chiral EFT

Chiral symmetry breaking associated with nucleon mass generation:
 Constituent quark picture, supported by extensive phenomenology, connected with parton picture

Can we use this knowledge to compute/estimate matrix elements of QCD operators?
**Introduction: Effective operators**

- **QCD** → **Effective dynamics from ChSB**

\[ \hat{\mathcal{O}}_{\text{QCD}} \quad \rightarrow \quad \hat{\mathcal{O}}_{\text{eff}} \quad \text{Need “effective operators” representing QCD operators in effective dynamics!} \]

\[ \vec{V}^\alpha, T^{\alpha\beta} \quad \rightarrow \quad \vec{V}_{\text{eff}}^\alpha, T_{\text{eff}}^{\alpha\beta} \quad \text{Conserved currents (vector current, EM tensor): Effective operators from global symmetries} \checkmark \]

\[ \bar{\psi} \gamma^{\{\alpha_1} \nabla^{\alpha_2} \cdots \nabla^{\alpha_n\}} \psi \]  
\[ F^{\{\alpha_1\beta} \cdots F_{\beta\alpha_n\}} \quad \rightarrow \quad ? \quad \text{Twist-2 QCD operator: Effective operators determined by dynamics!} \]

\[ \bar{\psi} \gamma^{[\alpha_5} \nabla^{\beta]} \psi \]  
\[ = -\frac{1}{4} \epsilon^{\alpha\beta\gamma\delta} \partial_\gamma [\bar{\psi} \gamma^\delta \tau \psi] \quad \rightarrow \quad ? \quad \text{Twist-3 QCD operator (spin-orbit correlations): Relations from QCD equations of motion, should be satisfied by effective operators} \]

**Effective operators need to be derived from QCD, consistently with effective dynamics!**
Topological fluctuations of QCD gauge fields

Topological charge \( \frac{1}{32\pi^2} \int_{\bar{R}} d^4x \bar{F}F(x) \approx \pm 1 \)

Instantons: Classical solutions of YM equations, self-dual fields \( \bar{F} = \pm F \), localized

Typical size \( \bar{\rho} \approx 0.3 \text{ fm} \), separation \( \bar{R} \approx 1 \text{ fm} \)

Strong fields: \( (F^2)^{1/2} \approx (32\pi^2 / \pi^2 \bar{\rho}^4)^{1/2} \approx 2 \text{ GeV}^2 \), semiclassical

Induce zero mode of fermion field \( i\gamma \nabla_{\text{top}} \Phi_{\pm} = 0 \)

Definite chirality \( \gamma_5 \Phi_{\pm} = \pm \Phi_{\pm} \)

\( \rightarrow \) Chiral symmetry breaking in QCD

Evidence direct and indirect:

LQCD cooling: Polikarpov, Veselov 1988; Campostrini et al. 1990; Chu, Negele et al 1993; DeGrand et al 1997; de Forcrand et al 1997, ..., Athenodorou et al 2018

Correlation functions: Shuryak 1982; Diakonov, Petrov 1984; Shuryak, Schafer 1993, ...
Introduction: Program

• Construct description of QCD vacuum based on topological gauge fields: Instanton vacuum

• Derive effective dynamics from chiral symmetry breaking → hadron structure

• Derive effective operators resulting from QCD operators in same scheme → Twist-2,3 GPDs, gluonic structure

Use systematic parametric approximation: Packing fraction, $1/N_c$

Obtain effective operators consistent with effective dynamics

Preserve operator relations from QCD equation of motion, trace and U(1) anomaly
Instanton vacuum: Gauge fields

Separation of modes

\[ k > \bar{\rho}^{-1} : \text{Integrate perturbatively:} \]
\[ \text{Renormalization, } \bar{\rho}^{-2} \gg \Lambda_{\text{QCD}}^2 \]

\[ k < \bar{\rho}^{-1} : \text{Integrate nonperturbatively:} \]
\[ \text{Instantons + massive fermions} \]

Instanton ensemble

\[ A(x) = \sum_{I} A_I(x | z_I, \rho_I, O_I) + \sum_{I} A_I(\ldots) \]
\[ \text{gauge potential } \rightarrow \text{classical top. fields} \]

\[ \int [DA] \rightarrow \int \prod_{I,J} dz_I d\rho_I dO_I d_0(\rho_I) \]
\[ \text{functional integral } \rightarrow \text{collective coordinates} \]

Stable system emerges due to instanton interactions
Implementations: Variational principle Diakonov, Petrov 1984; numerical simulations Shuryak 1988+

Small parameter: Packing fraction \[ \pi^2 \bar{\rho}^4 / \bar{R}^4 \approx 0.1 \]

All dynamical scales “emerge” from \( \Lambda_{\text{QCD}} \) via instanton density
Preserves renormalization properties of QCD
Instanton vacuum: Fermions and ChSB

Fermions in 1-instanton background

Zero mode induces multifermion vertex ('tHooft)

\[ \int dO_I \bar{\psi} | \Phi_I \rangle | \psi \rangle \propto \det_{ab} \bar{\psi}_L^a \psi_R^b \times \text{form factor} \]

Chiral symmetry breaking in instanton medium

Fermions hop between zero modes — chirality flip
Diakonov, Petrov 1986; Pobylitsa 1989; Nowak, Verbaarschot, Zahed 1989; …

Dynamical quark mass \( M \approx 0.3\text{-}0.4 \text{ GeV}, \) active for momenta \( k \lesssim \bar{\rho}^{-1} \approx 0.6 \text{ GeV} \)

Ground state determined in \( 1/N_c \) expansion, saddle point approximation

Effective dynamics of massive quarks with multi-fermion interactions
Instanton vacuum: Bosonization

Auxiliary field $U^{ij} \sim \bar{\psi}^i_L \psi^j_R$

[Diaconov, Petrov 1986; Kacir, Prakash, Zahed 1995; ...]

\[
\int DU \int D\bar{\psi} D\psi \exp \int d^4x \; \bar{\psi}(x) \left[ i\hat{\partial} \psi - M U^\gamma_5(x) \right] \psi(x)
\]

Yukawa-type interaction of massive quarks with chiral field

Form of interaction constrained by chiral invariance

Effective dynamics of massive quarks and Goldstone bosons with chiral interactions

Quark-pion coupling $g_{\pi qq} = M/F_\pi \approx 4$, strongly coupled system!
Solved nonperturbatively using $1/N_c$ expansion — saddle point approximation

\[
U(x) = e^{i\pi \pi(x)/F_\pi}
\]
**Meson correlators**

- Saddle point $U = 1$: Vacuum fields
- Pseudoscalar: Pion pole (isovector), $\eta'$ mass (isoscalar)

**Baryon correlators**

- Saddle point $U \neq 1$: Classical pion field (“soliton”)  
  [Diakonov, Petrov, Pobylitsa 1988]
- Fixed time: Quarks moving in self-consistent pion field with single-particle Hamiltonian $H = \alpha p + \beta MU_{\text{class}}$  
  [Kahana, Ripka 1984]
- Realization of general relativistic mean-field picture at large $N_c$. Unifies “quark model” and “chiral soliton”  
  [Witten 1983]
Instanton vacuum: Nucleon structure

3-point functions include disconnected quark diagrams... connected by chiral field!

Example: Axial and vector and current matrix elements for SU(2) flavor

[Review: Christov et al 1995. Many more results: Sigma term, N → Δ, SU(3), ...]

Reasonable description of nucleon static properties, current matrix elements

→ Talk H-C Kim
QCD operators: Effective operators

Gluon operator

$\mathcal{F}[A]$  local QCD gluon operator

Normalized at scale $\mu = \bar{\rho}^{-1}$

Gluon operator in instanton vacuum correlation functions

$\mathcal{F}[A] \rightarrow \sum_{I+\bar{I}} \mathcal{F}[A_I] + \mathcal{O}(\rho^4/R^4)$

Evaluated in gluon field of single $I(\bar{I})$

$\langle \ldots \mathcal{F}[A] \ldots \rangle_{\text{inst}} \rightarrow \langle \ldots \mathcal{F}_{\text{eff}}[\bar{\psi}, \psi] \ldots \rangle_{\text{eff}}$

Converted to effective fermionic operator in effective theory of massive fermions

$\mathcal{F}_{\text{eff}}[\bar{\psi}, \psi] = N \int d\bar{z}_I dO_I d\rho_I d_{\text{eff}}(\rho_I) \mathcal{F}[A_I] \bar{\psi} | \Phi_I)(\Phi_I | \psi$

Coupling through zero modes $\leftrightarrow 't\text{Hooft vertex}$

Construction possible in saddle-point approximation $1/N_c$

[Diakonov, Polyakov, Weiss, 1995]

Advantages of effective operator representation:
Universal, relations between hadronic matrix elements, insight into origin of gluonic structure
QCD operators: Twist-2

\[ O^q_{\alpha_1...\alpha_n} = \bar{\psi} \gamma_{\alpha_1} \nabla_{\alpha_2} \ldots \nabla_{\alpha_n} \psi - \text{traces} \]

\[ O^g_{\alpha_1...\alpha_n} = F_{\beta} \{ \alpha_1 \nabla_{\alpha_2} \ldots \nabla_{\alpha_{n-1}} F_{\alpha_n} \} \beta - \text{traces} \]

Twist-2 spin-n QCD operators, scale \( \mu = \tilde{\rho}^{-1} \)

Matrix elements = moments of PDFs/GPDs

\[ (O^q_{\text{eff}})_{\alpha_1...\alpha_n} = \bar{\psi} \gamma_{\alpha_1} \partial_{\alpha_2} \ldots \partial_{\alpha_n} \psi + \mathcal{O}(\rho^4/R^4) \]

Quark/antiquark operators \( \mathcal{O}(1) \)

\[ (O^g_{\text{eff}})_{\alpha_1...\alpha_n} = 0 + \mathcal{O}(\rho^4/R^4) \]

Gluon operators suppressed in packing fraction

\[ \int dx \ x [q + \bar{q}](x) = 1: \text{Momentum sum rule} \]
saturated by quarks + antiquarks…. consistent

Nucleon’s antiquark content \( \mathcal{O}(1) \), rich spin-flavor dependence

[Diakonov, Petrov, Pobylitsa, Polyakov, Weiss, 1996]

Twist-2: Instanton field subsumed in massive quarks/antiquarks, no direct effect
QCD operators: Twist-3 non-forward

\[ O^{\alpha \beta}(x) = \bar{\psi}(x) \gamma^{[\alpha} \nabla^{\beta]} \tau \psi(x) \]

\[ = -\frac{1}{4} \epsilon^{\alpha \beta \gamma \delta} \partial_\gamma [\bar{\psi}(x) \gamma_5 \gamma_5 \psi(x)] \]

\[ \xrightarrow{\text{effective theory}} \]

\[ O^{\alpha \beta}_{\text{eff}}(x) = \bar{\psi}(x) \left( \gamma^{[\alpha} \partial^{\beta]} \tau + i \frac{1}{4} M \sigma^{\alpha \beta} [U^{\gamma_5}(x), \tau] \right) \psi(x) \]

\[ = -\frac{1}{4} \epsilon^{\alpha \beta \gamma \delta} \partial_\gamma [\bar{\psi}(x) \gamma_5 \gamma_5 \psi(x)]_{\text{eff}} \]

Twist-3 QCD operator, quark spin density in EMT
\( \nabla \equiv \partial - iA \) contains gauge potential

Relation from QCD equations of motion \( \hat{\nabla} \psi = 0 \)

Effective operator in chiral theory (bosonized).
Instantons induce “potential” term containing interaction of massive quarks with chiral field

Same operator relation as in QCD, now from effective equations of motion \( [\hat{\partial} - M U^{\gamma_5}] \psi = 0 \)

J-Y Kim, Weiss 2023

Similar effect in QCD operator with \( \gamma^\alpha \gamma_5 \) describing quark spin-orbit correlations

Lorce 2014

Twist-3 non-forward: Instanton field induces spin-flavor dependent chiral interactions, \( \mathcal{O}(1) \) dynamical effect on quark spin density and spin-orbit correlations
QCD operators: $F^2$ and $F\tilde{F}$

$\langle N' | F^2(0) | N \rangle = C(t) m_N \bar{u}'u$

Nucleon form factor of $F^2$

$C(0) = -\frac{32\pi^2}{b}$

Relation from QCD trace anomaly $T_{\alpha\alpha}(x) = -\frac{b}{32\pi^2} F^2(x)$

$b = \frac{11}{3} N_c - \frac{2}{3} N_f$ beta function coefficient

[Shifman, Vainshtein, Zakharov 1978; Novikov SVZ 1981]

Realized in instanton vacuum! Effective operator $F^2 \propto N_+ + N_- \equiv N$ instanton number.

Fluctuations in ensemble controlled by trace anomaly $\langle N^2 \rangle - \langle N \rangle^2 = 4/b \langle N \rangle$

[Diakonov, Polyakov, Weiss, 1995]

$\langle N' | F\tilde{F}(0) | N \rangle = \tilde{C}(t) m_N \bar{u}'i\gamma_5u$

Nucleon form factor of $F\tilde{F}$

$\tilde{C}(0) = \frac{32\pi^2 g_A^{(0)}}{N_f}$

Relation from axial anomaly $\partial_\alpha J_{5\alpha}(x) = \frac{N_f}{16\pi^2} F\tilde{F}(x) + \mathcal{O}(m_f)$

Realized in instanton vacuum! Effective operator $F\tilde{F} \propto N_+ - N_-$ topological charge fluctuations

Instanton vacuum realizes low-energy theorems from trace and axial anomaly. Framework suitable for analyzing hadronic form factors of $F^2, F\tilde{F}$

[See also Zahed 2021+]
Summary

• Topological fluctuations of gauge fields play essential role in low-energy QCD:
  Fermionic zero modes $\rightarrow$ chiral symmetry breaking $\rightarrow$ hadron structure/interactions

• Instanton vacuum provides effective description: Packing fraction as small parameter, all scales generated dynamically from $\Lambda_{\text{QCD}}$

• Instanton vacuum enables systematic derivation of effective dynamics from chiral symmetry breaking and effective operators representing QCD operators, including gluon operators: Equation-of-motion relations, scale and axial anomaly

• Twist-2: Instanton field subsumed in massive quarks/antiquarks, no direct effect

• Twist-3 non-forward: Instanton field produces spin-flavor dependent chiral interactions, $\mathcal{O}(1)$ effect

• Essential tool for exploring GPDs and gluonic structure of nucleon.
  In progress: Spin-orbit correlations, chiral-odd structures, nucleon form factors $F^2, F\tilde{F}$

• Other applications (not covered here): Twist-3,4 operators from DIS power corrections; higher-dim operators from BSM physics

• Beyond instantons: Include non-topological fluctuations of gauge fields ("$I\bar{I}$ molecules"), important for Wilson loops, coupling to heavy quarks  [Shuryak, Zahed 2023]
Supplemental material
QCD operators: Antiquark distributions in nucleon

Unpolarized flavor asymmetry $\bar{d} - \bar{u}$ from chiral soliton (← instanton vacuum)
[Pobylitsa, Polyakov, Goeke, Watabe, Weiss, 1998]

FNAL E866 Drell-Yan data + analysis

Polarized flavor asymmetry $\Delta \bar{u} - \Delta \bar{d}$ predicted by chiral soliton (← instanton vacuum)
[Diakonov, Petrov, Pobylitsa, Polyakov, Weiss, 1996]

RHIC $W^\pm$ production data
[Adamczyk et al (STAR) 2014]
QCD operators: $\tilde{F}FF$

**Dimension-6 gluon operator**

\[ f^{abc} \tilde{F}^a_{\mu\nu} F^b_{\mu\rho} F^c_{\nu\rho}(x) \]

Dimension-6 CP-odd gluon operator, essentially non-abelian structure

Appears in scenario of hadronic CP violation [Weinberg 89]

Need estimates of hadronic matrix elements!

[Bigi, Uraltsev 1991, Hatta 2020]

**Instanton vacuum estimate**

Operators $\tilde{F}FF$ and $\tilde{F}F$ proportional in field of single $I(\bar{I})$, effective operators simply related

[Weiss 2021]

Operators $\tilde{F}FF$ and $\tilde{F}F$ proportional in field of single $I(\bar{I})$, effective operators simply related

Nucleon matrix element of $\tilde{F}FF$ inferred from $\tilde{F}F \leftrightarrow g_A^{(0)}$

Large numerical value due to localization of instanton field

Further conclusion (paradox): Neutron EDM induced by $\tilde{F}FF$ is proportional to that induced by $\tilde{F}F$ and therefore chirally suppressed [Chiral behavior of neutron EDM: Crewther, DiVecchia, Veneziano, Witten 1979]