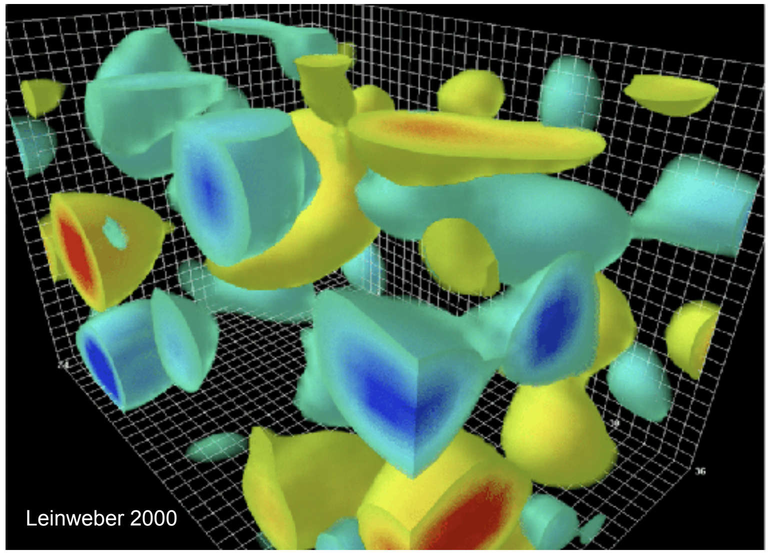
Trace anomaly and gravitational form factors in the QCD instanton vacuum

C. Weiss (JLab), Chiral Dynamics 2024, Ruhr-University Bochum, 27-Aug-2024

Jefferson Lab



Scale symmetry breaking in QCD

Energy-momentum tensor and trace anomaly

Hadron mass decomposition

Instanton vacuum

Chiral symmetry breaking from topological fields

Trace anomaly from density fluctuations

Pion gravitational form factors

Pion mass decomposition

Pion gravitational form factors at $Q^2 > 0$

Further applications

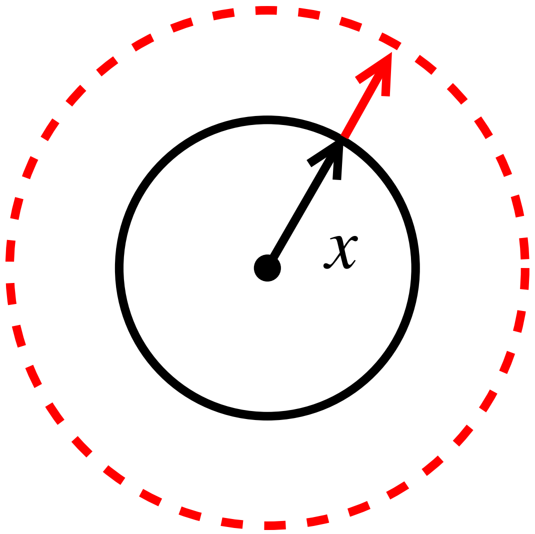
Quark-gluon operators, GPDs, nucleon…

Based on

D. Diakonov, M. Polyakov, C. Weiss, NPB 461, 539 (1996) [INSPIRE]

W.-Y. Liu, E. Shuryak, C. Weiss, I. Zahed, arXiv:2405.14026

J.-Y. Kim, C. Weiss, PLB 848, 138387 (2024) [INSPIRE]

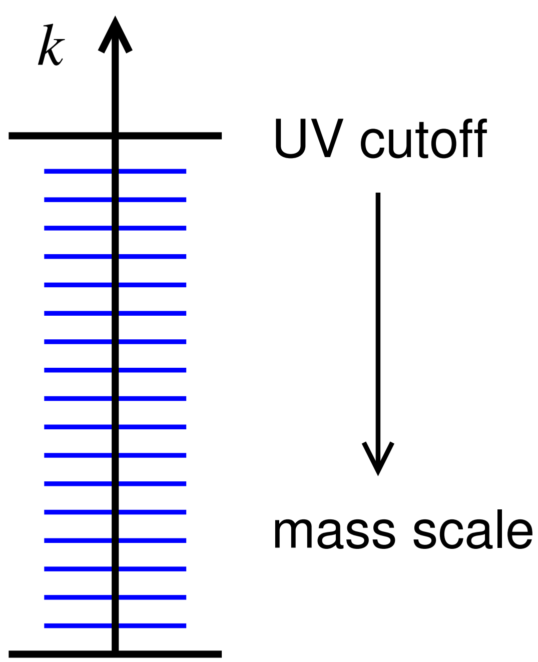


Classical QCD scale invariant (m = 0)

Space-time dilatation $x^\mu \to \lambda x^\mu, \quad A_\mu \to \lambda^{-1} A_\mu$

Conserved current $J^\mu = T^{\mu\nu} x_\nu \qquad \partial_\mu J^\mu = T^\mu{}_\mu = \mathcal{O}(m)$

$$T^{\mu\nu} = -F^{\mu\lambda} F_\lambda{}^\nu + \tfrac{1}{4} g^{\mu\nu} F^2 + \bar\psi \gamma^{\{\mu} i \overleftrightarrow{\nabla}^{\nu\}} \psi$$ EM tensor



Scale invariance broken by quantum fluctuations

UV cutoff provides mass scale

$$T^\mu{}_\mu = \underbrace{\frac{\beta(g)}{4g^4}}_{\frac{b}{32\pi^2} + \mathcal{O}(g^2)} F^2 + m\bar\psi\psi$$ Trace anomaly

$$b = \tfrac{11}{3} N_c - \tfrac{2}{3} N_f$$

Operator relation → hadronic matrix elements

Hadron mass arises from gluon fields and quark scalar density

Crewther 72; Chanowitz, Ellis 72; Nielsen 77; Adler, Collins, Duncan 77; Collins, Duncan, Joglekar 77

Pion matrix element

$$\langle \pi(p') \,|\, T^{\mu\nu} \,|\, \pi(p) \rangle \;=\; 2P^\mu P^\nu A(t) \;+\; \tfrac{1}{2}(\Delta^\mu \Delta^\nu - \Delta^2 g^{\mu\nu}) D(t)$$

from relativistic covariance,
EMT conservation $\partial_\mu T^{\mu\nu} = 0$
$P = (p' + p)/2, \quad \Delta = p' - p$

$A(0) = 1$ from T^{00} = energy density

$$\langle \pi(p) \,|\, T^\mu{}_\mu \,|\, \pi(p) \rangle \;=\; 2M_\pi^2$$

forward matrix element of EMT trace

Pion mass decomposition

$$-\frac{b}{32\pi^2} \frac{\langle \pi \,|\, F^2 \,|\, \pi \rangle}{2M_\pi} + \frac{\langle \pi \,|\, m\bar\psi\psi \,|\, \pi \rangle}{2M_\pi} \;=\; M_\pi$$

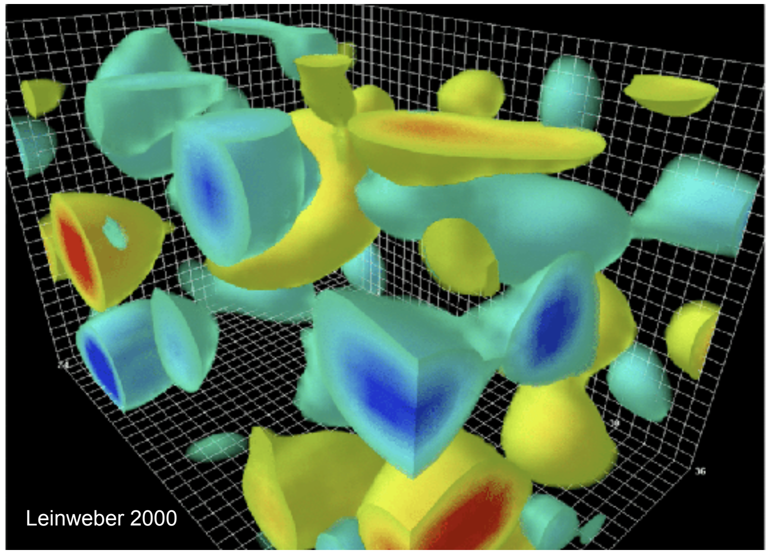
Pion mass arises from gluon fields and scalar quark density

Remarkable properties:

$\langle \pi \,|\, F^2 \,|\, \pi \rangle$ “knows about” QCD beta function

$\langle \pi \,|\, F^2 \,|\, \pi \rangle$ vanishes in chiral limit

} How does this happen dynamically?

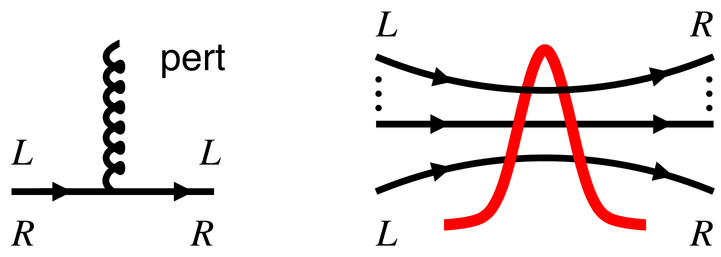


QCD vacuum populated by topological gauge fields: Tunneling trajectories, top. charge ± 1

Instantons: Classical solutions of YM equations, self-dual fields $\tilde{F} = \pm F$, localized

Typical size $\bar{\rho} \approx$ 0.3 fm, separation $\bar{R} \approx$ 1 fm

Strong fields: $(F^2)^{1/2} \approx (32\pi^2/\pi^2\bar{\rho}^4)^{1/2} \sim 2\,\text{GeV}^2$, can be described semiclassically



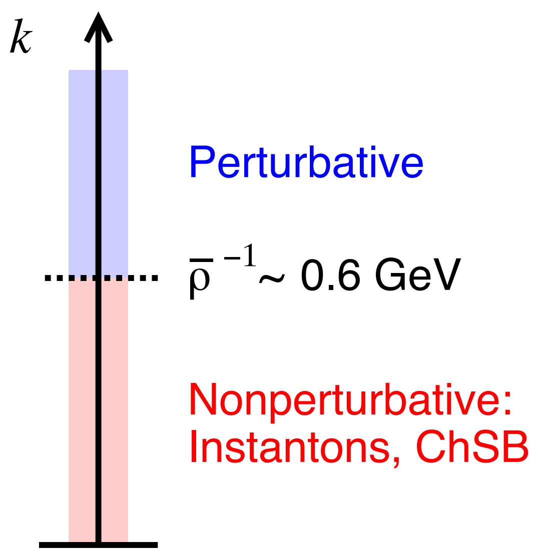
Induce zero mode of fermion field $i\gamma\nabla_\pm\Phi_\pm = 0$
Definite chirality $\gamma_5\Phi_\pm = \pm\,\Phi_\pm$

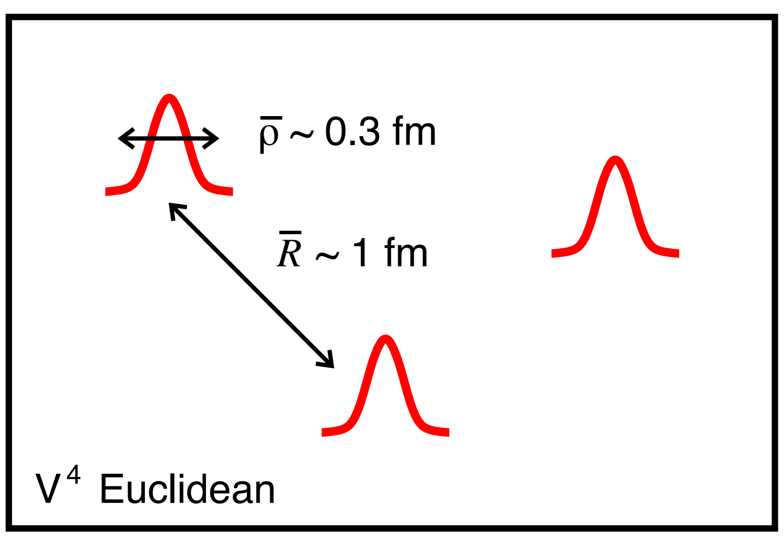
→ Chiral symmetry breaking in QCD

LQCD cooling: Polikarpov, Veselov 1988; Campostrini et al. 1990; Chu, Negele et al 1993; DeGrand et al 1997; de Forcrand et al 1997, …, Athenodorou et al 2018

Correlation functions: Shuryak 1982; Diakonov, Petrov 1984; Shuryak, Schafer 1993, …

Instanton vacuum: Approximate realization





Separation of modes

$k > \bar{\rho}^{-1}$: Integrate perturbatively:
Renormalization, $\bar{\rho}^{-2} \gg \Lambda^2_{\mathrm{QCD}}$

$k < \bar{\rho}^{-1}$: Integrate nonperturbatively:
Instantons + massive fermions

Instanton ensemble

$$A(x) = \sum_I A_I(x \,|\, z_I, \rho_I, O_I) + \sum_{\bar{I}} A_{\bar{I}}(\,.\,.\,)$$

gauge potential → classical top. fields

$$\int [DA] \to \int \prod_{I,\bar{I}} dz_I \, d\rho_I \, dO_I \; d_0(\rho_I)$$

functional integral → collective coordinates

Stable system emerges due to instanton interactions
Implementations: Variational principle Diakonov, Petrov 1984; numerical simulations Shuryak 1988+

Small parameter: Packing fraction $\pi^2 \bar{\rho}^4 / \bar{R}^4 \approx$ 0.1

All dynamical scales “emerge” from Λ_{QCD} via instanton density
Preserves renormalization properties of QCD

Instanton vacuum: Trace anomaly

$$\sum_N P(N) \left\langle \ \ldots \ \right\rangle_N$$

Instanton number fluctuates: Grand canonical ensemble

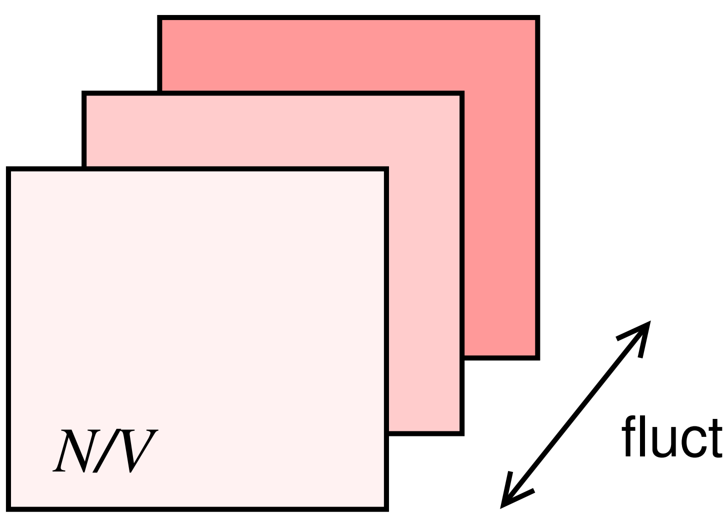
$$P(N) = \left(\frac{N}{\bar{N}}\right)^{-\frac{bN}{4}} e^{\frac{bN}{4}}$$

Instanton number distribution obtained from renormalization properties of gluodynamics $dZ_{\text{ren}}/d(1/g^2)$

$$\frac{\overline{(N-\bar{N})^2}}{\bar{N}} = \frac{4}{b}$$

Variance of instanton number fluctuations controlled by QCD beta function. “Vacuum compressibility”

Trace anomaly encoded in instanton number fluctuations



Fluctuations suppressed in $1/N_c$:
Instanton density $N/V \sim N_c$
Variance of fluctuations $1/b \sim 1/N_c$

Diakonov, Polyakov, CW 96

Results: Pion mass decomposition

$$\frac{\langle \pi | F^2 | \pi \rangle}{\langle \pi | \pi \rangle} = \lim_{T\to\infty} \frac{\langle J_\pi(T)\, F^2(0)\, J_\pi(-T)\rangle_{\text{conn}}}{\langle J_\pi(T)\, J_\pi(-T)\rangle}$$

Correlation function

$$= \lim_{T\to\infty} \frac{32\pi^2}{2V_3 T} \underbrace{\frac{\overline{(N-\bar{N})^2}}{\bar{N}}}_{4/b} \left. \frac{N \dfrac{d}{dN} \langle J_\pi(T)\, J_\pi(-T)\rangle_N}{\langle J_\pi(T)\, J_\pi(-T)\rangle_N} \right|_{N=\bar{N}}$$

Connected part given by fluctuations of instanton density

$$M_\pi = \text{function}(N/V, m) \propto m^{1/2} \left(\frac{N}{V}\right)^{1/8} \left[1 + \mathcal{O}(m)\right]$$

Pion mass as function of instanton density (and quark mass)

$$-\frac{b}{32\pi^2} \frac{\langle \pi | F^2 | \pi \rangle}{2M_\pi} = \frac{M_\pi}{2} \left[1 + \mathcal{O}(m)\right]$$

Trace anomaly contributes half of pion mass!

Liu, Shuryak, CW, Zahed, arXiv:2405.14026

Pion decouples from vacuum gluon condensate $\langle F^2 \rangle \sim \bar{N}$, couples only to fluctuations $(N - \bar{N})^2$

Pion expectation value $\langle \pi | F^2 | \pi \rangle < 0$: Reduction of gluon field compared to vacuum

Results: Pion mass decomposition

$$\frac{\langle \pi | m\bar{\psi}\psi | \pi \rangle}{2M_\pi} = m\frac{dE_\pi}{dm} = \frac{M_\pi}{2}$$

Quark contribution to pion mass from Feynman-Hellman theorem. Instanton result agrees with ChEFT

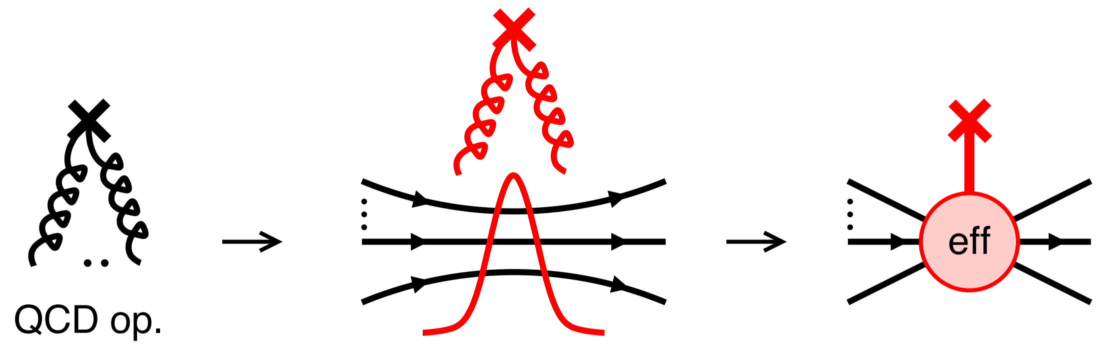
$$\frac{\langle \pi | T^\mu{}_\mu | \pi \rangle}{2M_\pi} = \left.\frac{M_\pi}{2}\right|_{\text{gluon}} + \left.\frac{M_\pi}{2}\right|_{\text{quark}}$$

Pion mass decomposition, up to chiral corrections $\mathcal{O}(m)$

Pion mass arises half from gluon trace anomaly, half from quark mass

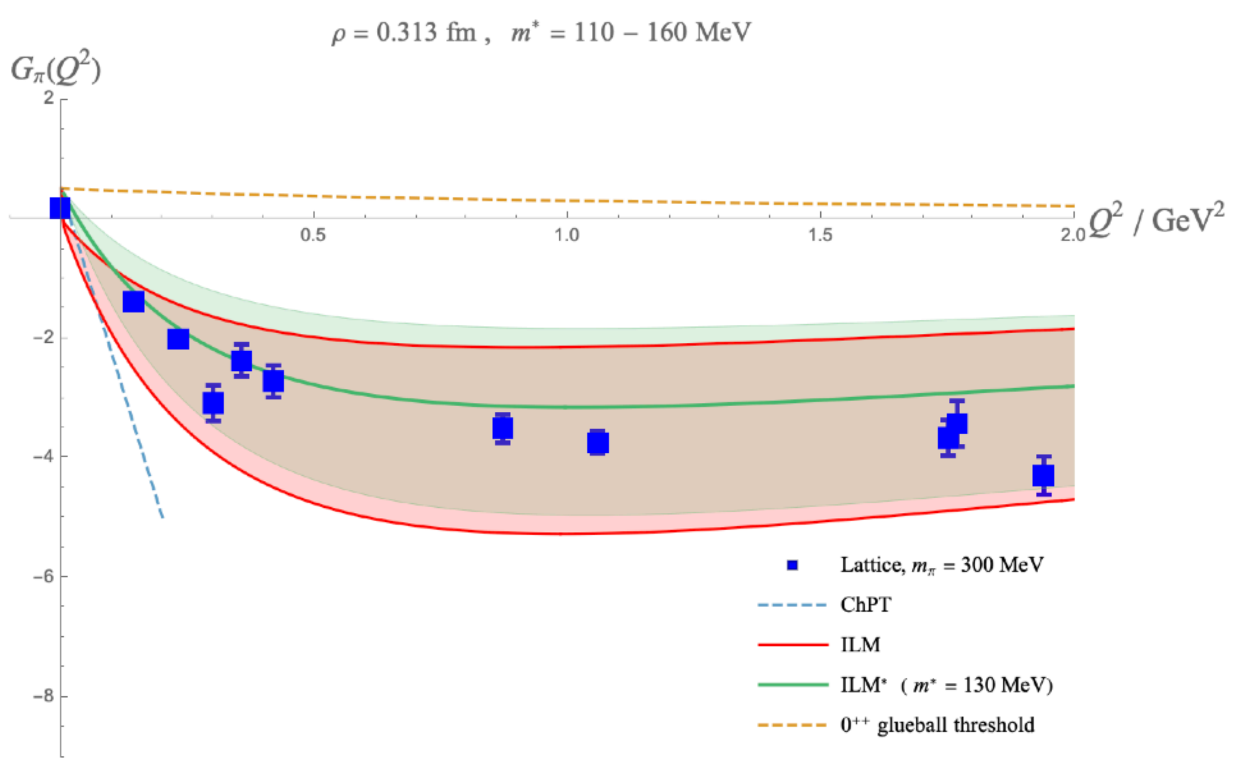
Instanton vacuum provides dynamical realization and interpretation:
All scales from instanton density N/V; trace anomaly from fluctuations

Results: Pion gravitational form factors



Instanton vacuum permits calculation of hadronic matrix elements of gluon operators

QCD gluon operator evaluated in classical gauge field of instantons, coupled to quarks through zero modes: Effective quark operator
Diakonov, Polyakov, CW 1996



Liu, Shuryak, CW, Zahed, arXiv:2405.14026

Systematic expansion in packing fraction

Twist-2 gluons	$\sim (\rho/R)^4$	suppressed
Twist-4 gluons	$\sim (\rho/R)^0$	leading

Example: Pion scalar gluon form factor $G_\pi(Q^2)$

Momentum transfer $Q^2 \sim \rho^{-2}$

Good agreement with LQCD results
χQCD Collab: B. Wang et al., Phys.Rev.D 109 (2024) 094504

Instanton vacuum: Further applications

Nucleon matrix elements

Large-N_c limit: Mean-field solution, classical chiral field (“soliton”)
Diakonov, Petrov, Pobylitsa 88

Extensive work on form factors, partonic structure, energy-momentum tensor
Bochum group 1990’s - 2000’s

Generalized parton distributions

Large instanton effects in nonforward twist-3 quark-gluon matrix elements, spin-orbit correlations
J.-Y. Kim, Weiss, PLB 848, 138387 (2024); J.-Y. Kim, H.-Y. Won, H.-Ch. Kim, Weiss, arXiv:2403.07186

Spin structure functions

Higher-twist quark-gluon correlations from instantons
Balla, Polyakov, Weiss 1998

BSM physics

Hadronic matrix elements of higher-dimensional quark/gluon operators describing BSM processes

Example: Neutron EDM from CP violation by dimension-6 Weinberg operator $FF\tilde{F}$
Weiss, Phys.Lett.B 819 (2021) 136447

Summary

- Instanton vacuum describes chiral symmetry breaking by topological gauge fields:
  Packing fraction as small parameter, all scales generated dynamically from Λ_{QCD}

- Trace anomaly encoded in instanton number fluctuations with variance $1/b$

- Pion couples to instanton number fluctuations as Goldstone mode, decouples in chiral limit

- Trace anomaly accounts for half of pion mass, sigma term for other half

- Predictions for pion gluonic form factors at $Q^2 > 0$ in good agreement with LQCD

- Many applications to pion/nucleon gluonic structure, quark-gluon correlations

Beyond topological gauge fields: Include incomplete tunneling trajectories (“$I\bar{I}$ molecules”), important for Wilson loops, coupling to heavy quarks, twist-2 gluon operators

Shuryak, Zahed 2023