Tomography of pions and protons via transverse momentum dependent distributions

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What do we know about structures?

• Most well-known structure is through longitudinal structure of hadrons, particularly protons

Other structures?

• To give deeper insights into color confined systems, we shouldn’t limit ourselves to proton structures

• Pions are also important because of their Goldstone-boson nature while also being made up of quarks and gluons
Available datasets for pion structures

• Much less available data than in the proton case

• Still valuable to study
Available datasets for pion structures

- Much less available data than in the proton case
- Still valuable to study
Pion PDFs in JAM

Drell-Yan (DY)

Leading Neutron (LN)

Threshold resummation in DY

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3D structures of hadrons

• Even more challenging is the 3d structure through GPDs and TMDs
Unpolarized TMD PDF

\[ \tilde{f}_{q/N}(x, b_T) = \int \frac{db^{-}}{4\pi} e^{-ixP^{+}b_-} \text{Tr} \left[ \langle N | \bar{\psi}_q(b)\gamma^+ \mathcal{W}(b, 0)\psi_q(0) | N \rangle \right] \]

\[ b \equiv (b^-, 0^+, b_T) \]

- \( b_T \) is the Fourier conjugate to the intrinsic transverse momentum of quarks in the hadron, \( k_T \)
- We can learn about the coordinate space correlations of quark fields in hadrons
- Modification needed for UV and rapidity divergences; acquire regulators: \( \tilde{f}_{q/N}(x, b_T) \rightarrow \tilde{f}_{q/N}(x, b_T; \mu, \zeta) \)

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Factorization for low-$q_T$ Drell-Yan

- Like collinear observable, a **hard part** with two functions that describe structure of beam and target
- So called “$W$”-term, valid only at low-$q_T$

\[
\frac{d^3\sigma}{d\tau dY dq_T^2} = \frac{4\pi^2\alpha^2}{9\tau S^2} \sum_q H_{q\bar{q}}(Q^2, \mu) \int d^2b_T e^{i b_T \cdot q_T} \times \tilde{f}_q(x, b_T, \mu, Q^2) \tilde{f}_{\bar{q}/A}(x_A, b_T, \mu, Q^2),
\]
TMD PDF within the $b_*$ prescription

\[ b_*(b_T) \equiv \frac{b_T}{\sqrt{1 + b_T^2/b_{\text{max}}^2}}. \]

\[ \tilde{f}_{q/N(A)}(x, b_T, \mu_Q, Q^2) = (C \otimes f)_{q/N(A)}(x; b_*) \]

\[ \times \exp \left\{ -g_{q/N(A)}(x, b_T) - g_K(b_T) \ln \frac{Q}{Q_0} - S(b_*, Q_0, Q, \mu_Q) \right\} \]

- Low-$b_T$: perturbative
- High-$b_T$: non-perturbative

Relates the TMD at small-$b_T$ to the collinear PDF
\[ \Rightarrow \text{TMD is sensitive to collinear PDFs} \]

$g_{q/N(A)}$: intrinsic non-perturbative structure of the TMD

$g_K$: universal non-perturbative Collins-Soper kernel

Controls the perturbative evolution of the TMD
A few details

• Nuclear TMD model linear combination of bound protons and neutrons
  • Include an additional $A$-dependent nuclear parameter

• We use the MAP collaboration’s parametrization for non-perturbative TMDs
  • Only tested parametrization flexible enough to capture features of $Q$ bins

• Perform a **simultaneous global analysis** of pion TMD and collinear PDFs, with proton (nuclear) TMDs
  • Include both $q_T$-dependent and collinear pion data and fixed-target $pA$ data
Data and theory agreement

- Fit both $pA$ and $\pi A$ DY data and achieve good agreement to both

<table>
<thead>
<tr>
<th>Process</th>
<th>Experiment</th>
<th>$\sqrt{s}$ GeV</th>
<th>$\chi^2$/np</th>
<th>$Z$-score</th>
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</thead>
<tbody>
<tr>
<td>$q_T$-integ. DY $\pi W \rightarrow \mu^+\mu^- X$</td>
<td>E615 [37]</td>
<td>21.8</td>
<td>0.86</td>
<td>0.76</td>
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<td></td>
<td>NA10 [38]</td>
<td>19.1</td>
<td>0.54</td>
<td>2.27</td>
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<td></td>
<td>NA10 [38]</td>
<td>23.2</td>
<td>0.91</td>
<td>0.18</td>
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<td>Leading neutron $ep \rightarrow e' n X$</td>
<td>H1 [73]</td>
<td>318.7</td>
<td>0.36</td>
<td>4.61</td>
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<td>ZEUS [74]</td>
<td>300.3</td>
<td>1.48</td>
<td>2.16</td>
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<td>$q_T$-dep. $pA$ DY $pA \rightarrow \mu^+\mu^- X$</td>
<td>E288 [67]</td>
<td>19.4</td>
<td>0.93</td>
<td>0.25</td>
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<tr>
<td></td>
<td>E288 [67]</td>
<td>23.8</td>
<td>1.33</td>
<td>1.54</td>
</tr>
<tr>
<td></td>
<td>E288 [67]</td>
<td>24.7</td>
<td>0.95</td>
<td>0.23</td>
</tr>
<tr>
<td></td>
<td>E605 [68]</td>
<td>38.8</td>
<td>1.07</td>
<td>0.39</td>
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<td>E772 [69]</td>
<td>38.8</td>
<td>2.41</td>
<td>5.74</td>
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<td></td>
<td>E866 $(Fe/Be)$ [70]</td>
<td>38.8</td>
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<td>E866 $(W/Be)$ [70]</td>
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<td>0.11</td>
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<td>1.61</td>
<td>2.58</td>
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<tr>
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<td>E537 [71]</td>
<td>15.3</td>
<td>1.11</td>
<td>0.57</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>1.15</td>
<td>2.55</td>
<td></td>
</tr>
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</table>
• The small-$q_T$ data do not constrain much the PDFs
Resulting TMD PDFs of proton and pion

\[ \tilde{f}_{q/N}(b_T|x; Q, Q^2) = \frac{\tilde{f}_{q/N}(x, b_T; Q, Q^2)}{\int d^2 b_T \tilde{f}_{q/N}(x, b_T; Q, Q^2)} \]

- Broadening appearing as \( x \) increases
- Up quark in pion is narrower than up quark in proton

\[ b_T \tilde{f}_{u/p}(b_T|x) \]

\[ x = 0.60 \]
\[ x = 0.54 \]
\[ x = 0.48 \]
\[ x = 0.42 \]
\[ x = 0.36 \]
\[ x = 0.30 \]

\[ b_T (\text{fm}) \]

\[ b_T (\text{fm}) \]
Resulting average $b_T$

$$\langle b_T | x \rangle_{q/N} = \int d^2 b_T b_T \tilde{f}_{q/N}(b_T | x; Q, Q^2)$$

- Average transverse spatial correlation of the up quark in proton is $\sim 1.2$ times bigger than that of pion
- Pion’s $\langle b_T | x \rangle$ is $5.3 - 7.5\sigma$ smaller than proton in this range
- Decreases as $x$ decreases
Possible explanation

• At large $x$, we are in a valence region, where only the valence quarks are populating the momentum dependence of the hadron
Possible explanation

• At small $x$, sea quarks and potential $q\bar{q}$ bound states allowing only for a smaller bound system
Outlook

• Future studies needed for theoretical explanations of these phenomena

• Lattice QCD can in principle calculate any hadronic state – look to kaons, rho mesons, etc.

• Future tagged experiments such as at EIC and JLab 22 GeV can provide measurements for neutrons, pions, and kaons

• We should study other ways to formulate the TMD such as: Qiu-Zhang method, the $\zeta$-prescription, or the hadron structure oriented approach
Backup
Small $b_T$ operator product expansion

• At small $b_T$, the TMDPDF can be described in terms of its OPE:

$$f_{f/h}(x, b_T; \mu, \zeta_F) = \sum_j \int_x^1 \frac{d\xi}{\xi} \tilde{C}_{f/j}(x/\xi, b_T; \zeta_F, \mu) f_{j/h}(\xi; \mu) + \mathcal{O}((\Lambda_{QCD} b_T)^a)$$

• where $\tilde{C}$ are the Wilson coefficients, and $f_{j/h}$ is the collinear PDF

• Breaks down when $b_T$ gets large
$b_*$ prescription

- A common approach to regulating large $b_T$ behavior

$$b_*(b_T) \equiv \frac{b_T}{\sqrt{1 + b_T^2/b_{\text{max}}^2}}.$$  

- At small $b_T$, $b_*(b_T) = b_T$
- At large $b_T$, $b_*(b_T) = b_{\text{max}}$

Must choose an appropriate value; a transition from perturbative to non-perturbative physics
Introduction of non-perturbative functions

- Because $b_* \neq b_T$, have to non-perturbatively describe large $b_T$ behavior

\[
g_K(b_T; b_{\text{max}}) = -\tilde{K}(b_T, \mu) + \tilde{K}(b_*, \mu)
\]

 Completely general – independent of quark, hadron, PDF or FF

Non-perturbative function dependent in principle on flavor, hadron, etc.

\[
e^{-g_{j/H}(x, b_T; b_{\text{max}})} = \frac{\tilde{f}_{j/H}(x, b_T; \zeta, \mu)}{\tilde{f}_{j/H}(x, b_*; \zeta, \mu)} e^{g_K(b_T; b_{\text{max}}) \ln(\sqrt{\zeta}/Q_0)}.
\]
TMD factorization in Drell-Yan

- In small-$q_T$ region, use the Collins-Soper-Sterman (CSS) formalism and $b_*$ prescription

\[
\frac{d\sigma}{dQ^2 dy dq_T^2} = \frac{4\pi^2\alpha^2}{9Q^2s} \sum_{j,j_A,j_B} H_{jj}^{DY}(Q, \mu_Q, a_s(\mu_Q)) \int \frac{d^2b_T}{(2\pi)^2} e^{i q_T \cdot b_T} \]

Can these data constrain the pion collinear PDF?

Non-perturbative pieces

Non-perturbative piece of the CS kernel

Perturbative pieces
MAP parametrization

• A recent work from the MAP collaboration (arXiv:2206.07598) used a complicated form for the non-perturbative function

\[ f_{1NP}(x, b_T^2; \zeta, Q_0) = \frac{g_1(x) e^{-g_1(x) \frac{b_T^2}{4}} + \lambda^2 g_{1B}^2(x) \left[ 1 - g_{1B}(x) \frac{b_T^2}{4} \right] e^{-g_{1B}(x) \frac{b_T^2}{4}} + \lambda_2^2 g_{1C}(x) e^{-g_{1C}(x) \frac{b_T^2}{4}}}{g_1(x) + \lambda^2 g_{1B}^2(x) + \lambda_2^2 g_{1C}(x)} \left[ \frac{\zeta}{Q_0^2} \right]^{g_K(b_T^2)/2} \]  

(38)

\[ g_{\{1,1B,1C\}}(x) = N_{\{1,1B,1C\}} \frac{x^{\sigma_{\{1,2,3\}}} (1 - x)^{\alpha_{\{1,2,3\}}}}{\hat{x}^{\sigma_{\{1,2,3\}}} (1 - \hat{x})^{\alpha_{\{1,2,3\}}}} , \]

• 11 free parameters for each hadron! (flavor dependence not necessary) (12 if we include the nuclear TMD parameter)

\[ g_K(b_T^2) = -g_2^2 \frac{b_T^2}{2} \quad \text{Universal CS kernel} \]
Resulting $\chi^2$ for each parametrization

- Tried multiple parametrizations for non-perturbative TMD structures
- MAP parametrization is able to describe better all the datasets
Nuclear TMD PDFs – working hypothesis

• We must model the nuclear TMD PDF from proton

\[
\tilde{f}_{q/A}(x, b_T, \mu, \zeta) = \frac{Z}{A} \tilde{f}_{q/p/A}(x, b_T, \mu, \zeta) + \frac{A - Z}{A} \tilde{f}_{q/n/A}(x, b_T, \mu, \zeta)
\]

• Each object on the right side independently obeys the CSS equation
  • Assumption that the bound proton and bound neutron follow TMD factorization

• Make use of isospin symmetry in that \(u/p/A \leftrightarrow d/n/A\), etc.
Building of the nuclear TMD PDF

• Then taking into account the intrinsic non-perturbative, we model the flavor-dependent pieces of the TMD PDF as

\[
(C \otimes f)_{u/A}(x)e^{-g_{u/A}(x,b_T)} \rightarrow \frac{Z}{A}(C \otimes f)_{u/p/A}(x)e^{-g_{u/p/A}(x,b_T)}
\]

\[
+ \frac{A-Z}{A}(C \otimes f)_{d/p/A}(x)e^{-g_{d/p/A}(x,b_T)}
\]

and

\[
(C \otimes f)_{d/A}(x)e^{-g_{d/A}(x,b_T)} \rightarrow \frac{Z}{A}(C \otimes f)_{d/p/A}(x)e^{-g_{d/p/A}(x,b_T)}
\]

\[
+ \frac{A-Z}{A}(C \otimes f)_{u/p/A}(x)e^{-g_{u/p/A}(x,b_T)}.
\]
Nuclear TMD parametrization

• Specifically, we include a parametrization similar to Alrashed, et al., Phys. Rev. Lett 129, 242001 (2022).

\[ g_{q/N/A} = g_{q/N} \left( 1 - a_N \left( A^{1/3} - 1 \right) \right) \]

• Where \( a_N \) is an additional parameter to be fit
Datasets in the $q_T$-dependent analysis

<table>
<thead>
<tr>
<th>Expt.</th>
<th>$\sqrt{s}$ (GeV)</th>
<th>Reaction</th>
<th>Observable</th>
<th>$Q$ (GeV)</th>
<th>$x_F$ or $y$</th>
<th>$N_{pts.}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>E288 [39]</td>
<td>19.4</td>
<td>$p + Pt \rightarrow \ell^+ \ell^- X$</td>
<td>$E d^3 \sigma / d^3 q$</td>
<td>4 – 9</td>
<td>$y = 0.4$</td>
<td>38</td>
</tr>
<tr>
<td>E288 [39]</td>
<td>23.8</td>
<td>$p + Pt \rightarrow \ell^+ \ell^- X$</td>
<td>$E d^3 \sigma / d^3 q$</td>
<td>4 – 12</td>
<td>$y = 0.21$</td>
<td>48</td>
</tr>
<tr>
<td>E288 [39]</td>
<td>24.7</td>
<td>$p + Pt \rightarrow \ell^+ \ell^- X$</td>
<td>$E d^3 \sigma / d^3 q$</td>
<td>4 – 14</td>
<td>$y = 0.03$</td>
<td>74</td>
</tr>
<tr>
<td>E605 [40]</td>
<td>38.8</td>
<td>$p + Cu \rightarrow \ell^+ \ell^- X$</td>
<td>$E d^3 \sigma / d^3 q$</td>
<td>7 – 18</td>
<td>$x_F = 0.1$</td>
<td>49</td>
</tr>
<tr>
<td>E772 [41]</td>
<td>38.8</td>
<td>$p + D \rightarrow \ell^+ \ell^- X$</td>
<td>$E d^3 \sigma / d^3 q$</td>
<td>5 – 15</td>
<td>$0.1 \leq x_F \leq 0.3$</td>
<td>61</td>
</tr>
<tr>
<td>E866 [50]</td>
<td>38.8</td>
<td>$p + Fe \rightarrow \ell^+ \ell^- X$</td>
<td>$R_{FeBe}$</td>
<td>4 – 8</td>
<td>$0.13 \leq x_F \leq 0.93$</td>
<td>10</td>
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<tr>
<td>E866 [50]</td>
<td>38.8</td>
<td>$p + W \rightarrow \ell^+ \ell^- X$</td>
<td>$R_{WBe}$</td>
<td>4 – 8</td>
<td>$0.13 \leq x_F \leq 0.93$</td>
<td>10</td>
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<tr>
<td>E537 [38]</td>
<td>15.3</td>
<td>$\pi^- + W \rightarrow \ell^+ \ell^- X$</td>
<td>$d^2 \sigma / dx_F dq_T$</td>
<td>4 – 9</td>
<td>$0 &lt; x_F &lt; 0.8$</td>
<td>48</td>
</tr>
<tr>
<td>E615 [4]</td>
<td>21.8</td>
<td>$\pi^- + W \rightarrow \ell^+ \ell^- X$</td>
<td>$d^2 \sigma / dx_F dq_T^2$</td>
<td>4.05 – 8.55</td>
<td>$0 &lt; x_F &lt; 0.8$</td>
<td>45</td>
</tr>
</tbody>
</table>

- Total of 383 number of points
- All fixed target, low-energy data
- We perform a cut of $q_T^{\text{max}} < 0.25 \ Q$

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Transverse EMC effect

• Compare the average $b_T$ given $x$ for the up quark in the bound proton to that of the free proton
• Less than 1 by $\sim 5 - 10\%$ over the $x$ range