
# Tomography of pions and protons via transverse momentum dependent distributions

**Patrick Barry**, Leonard Gamberg, Wally Melnitchouk, Eric Moffat, Daniel Pitonyak, Alexei Prokudin, Nobuo Sato

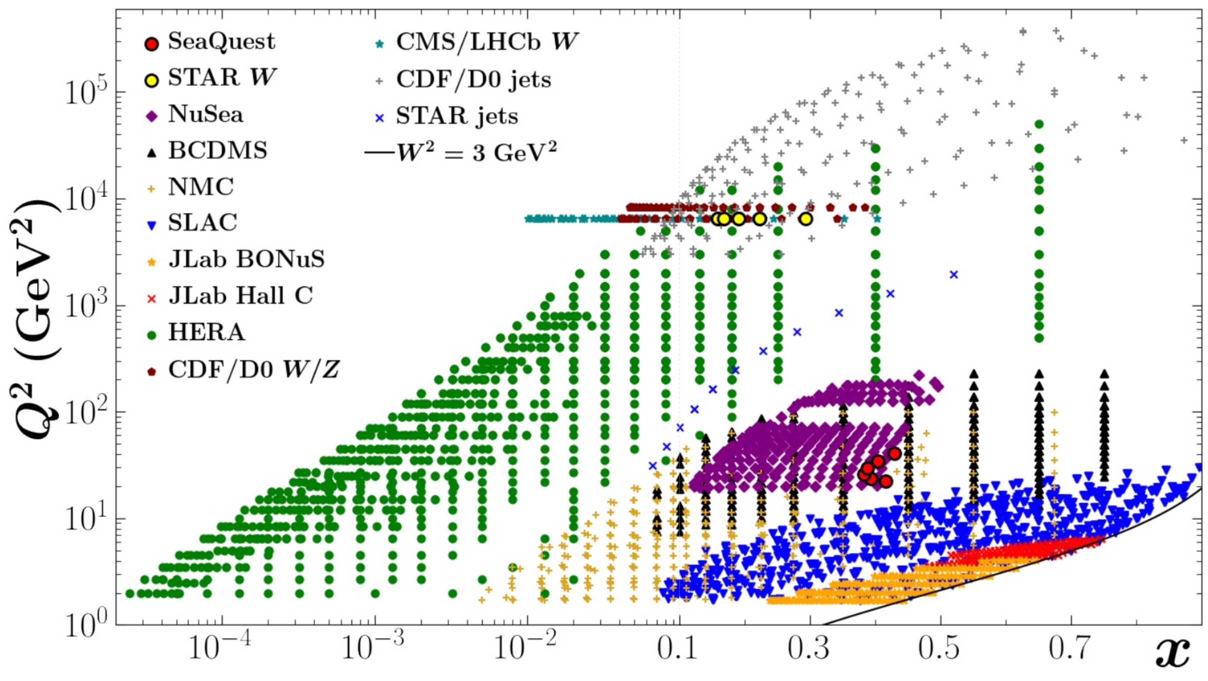
Based on: arXiv:2302.01192

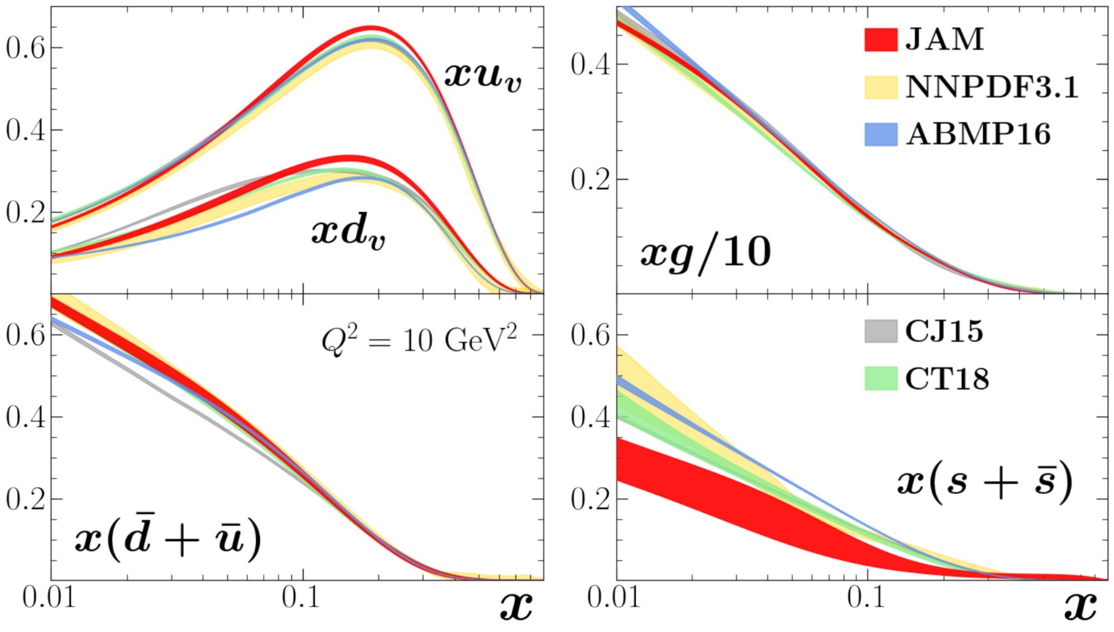
Jefferson Lab
Exploring the Nature of Matter

*P. C. Barry acknowledges financial support from The Gordon and Betty Moore Foundation and the American Physical Society to present this work at the GHP 2023 workshop.*

# What do we know about structures?

- Most well-known structure is through longitudinal structure of hadrons, particularly protons

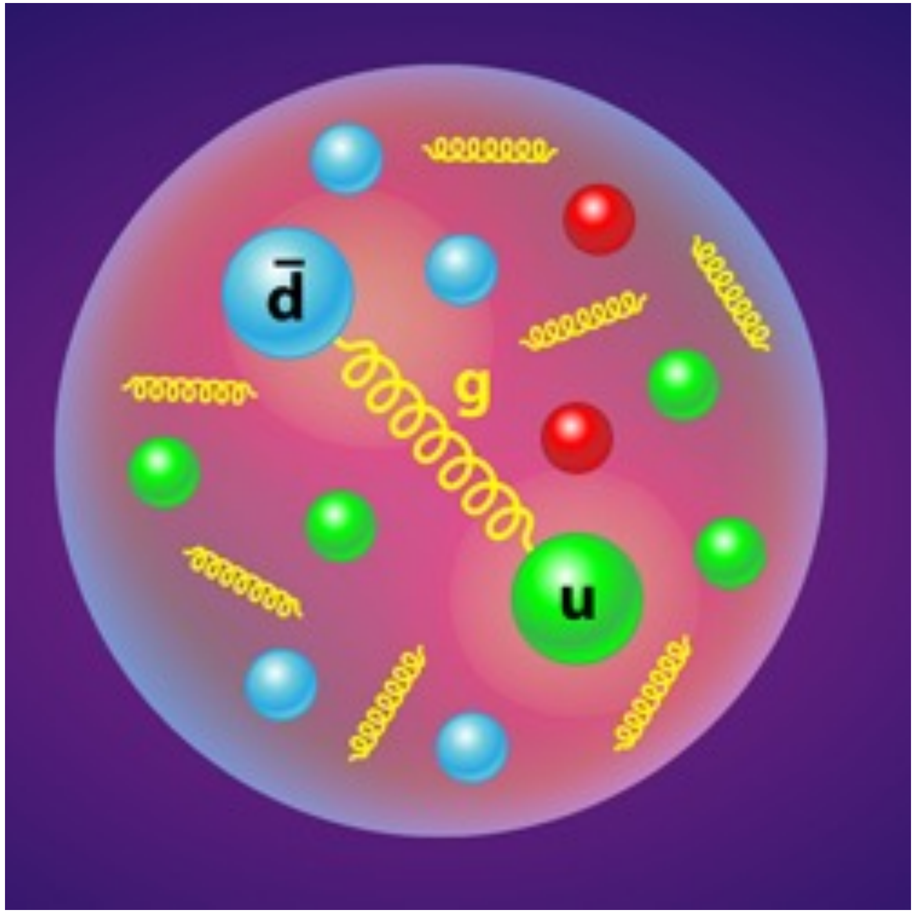



C. Cocuzza, et al., Phys. Rev. D **104**, 074031 (2021)

# Other structures?

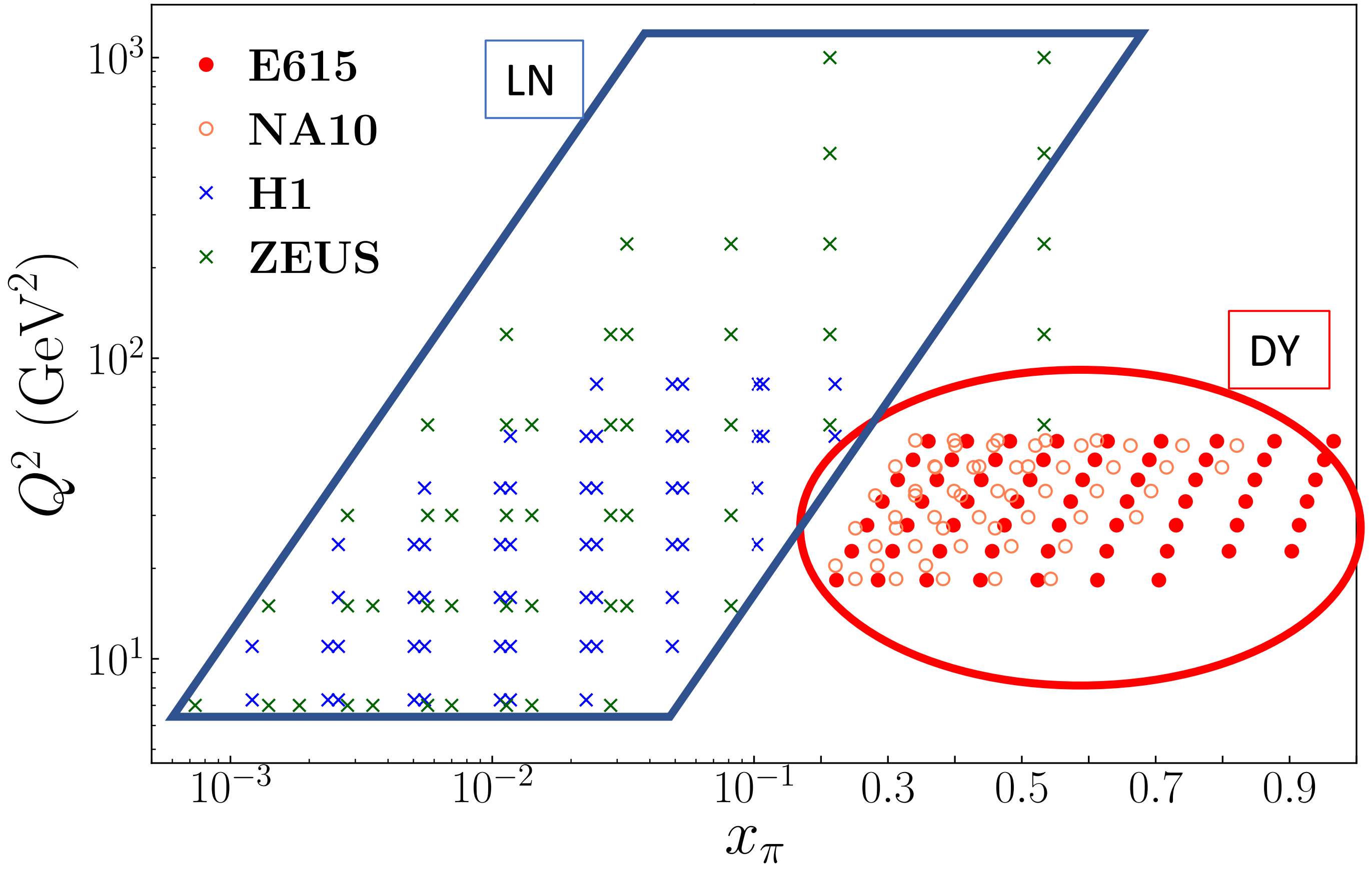
- To give deeper insights into color confined systems, we shouldn't limit ourselves to proton structures
- Pions are also important because of their Goldstone-boson nature while also being made up of quarks and gluons



# Available datasets for pion structures

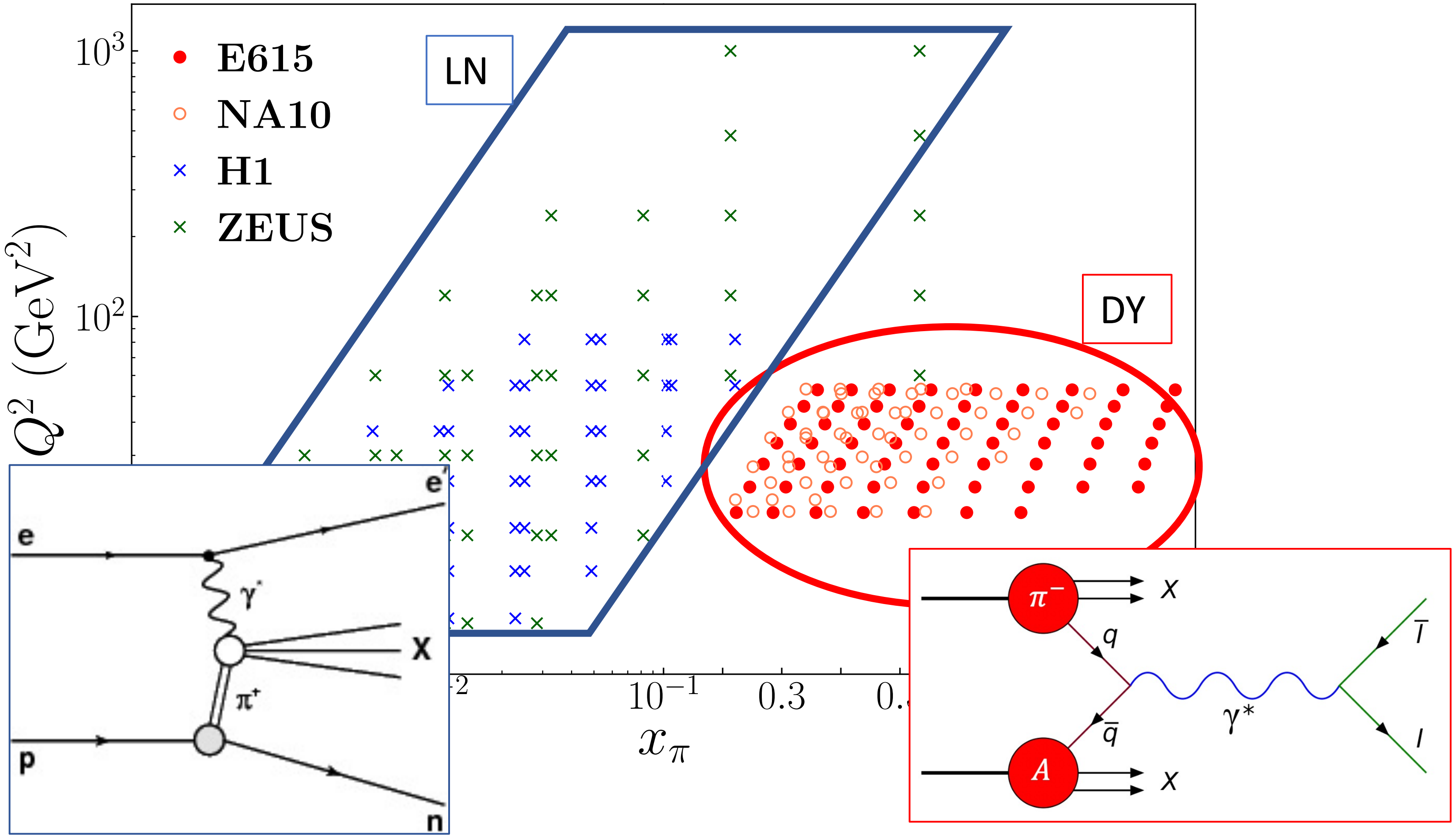
- Much less available data than in the proton case

- Still valuable to study



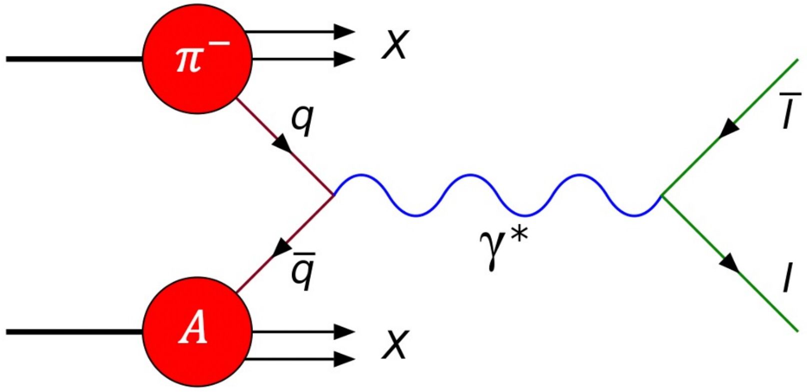
# Available datasets for pion structures

- Much less available data than in the proton case
- Still valuable to study

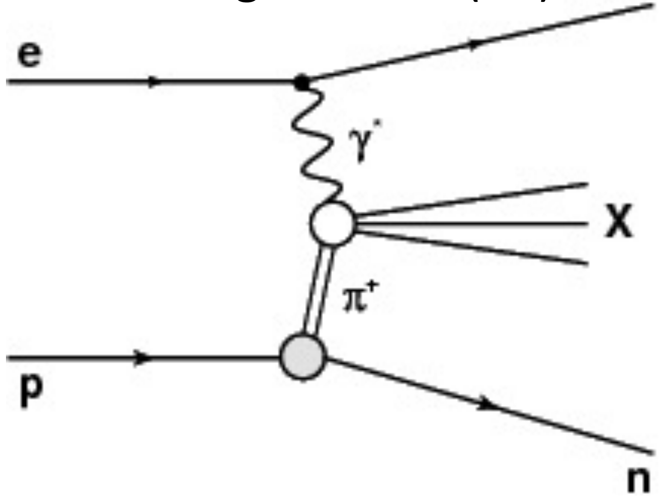


# Pion PDFs in JAM

Drell-Yan (DY)




Leading Neutron (LN)

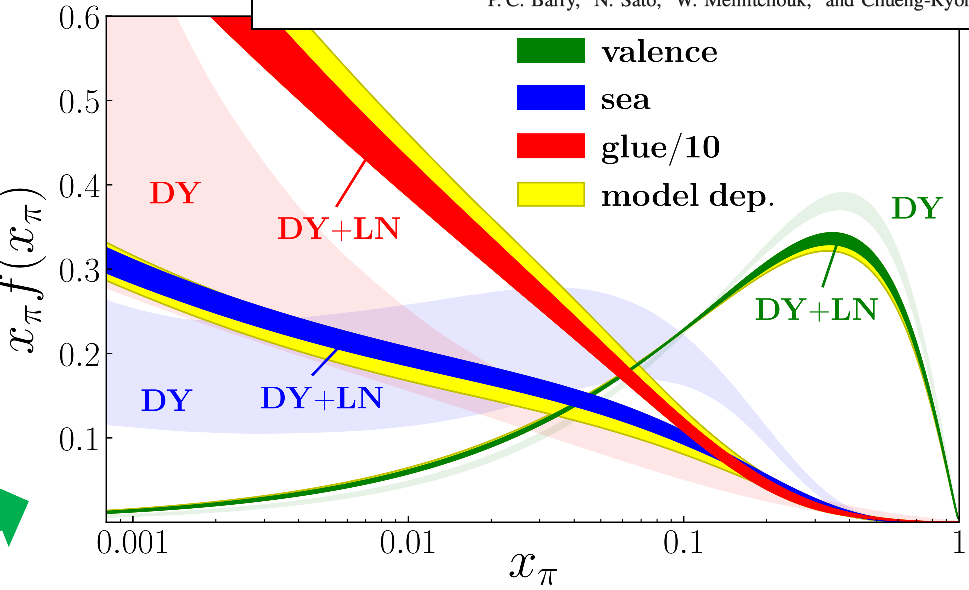



PHYSICAL REVIEW LETTERS **121,** 152001 (2018)

Featured in Physics

**First Monte Carlo Global QCD Analysis of Pion Parton Distributions**

P. C. Barry,[1] N. Sato,[2] W. Melnitchouk,[3] and Chueng-Ryong Ji[1]

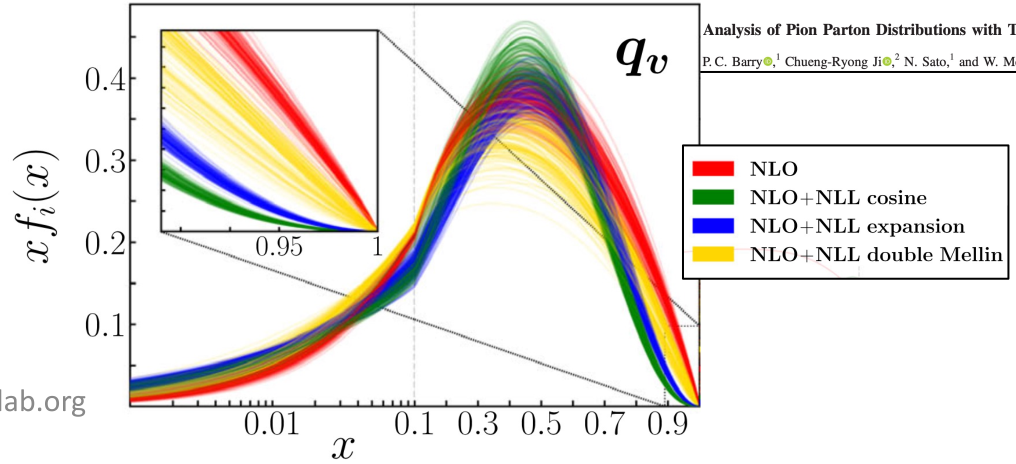



Threshold resummation in DY

PHYSICAL REVIEW LETTERS **127,** 232001 (2021)

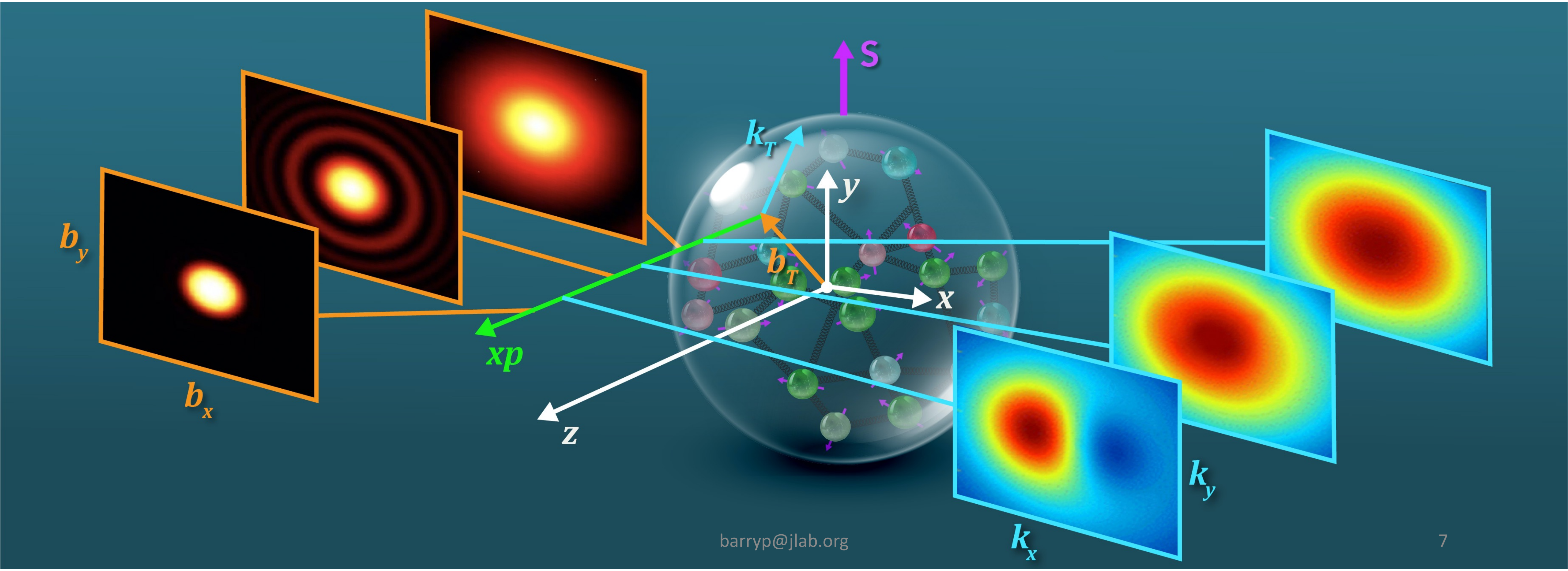
**Analysis of Pion Parton Distributions with Threshold Resummation**

P. C. Barry,[1] Chueng-Ryong Ji,[2] N. Sato,[1] and W. Melnitchouk[1]



# 3D structures of hadrons

- Even more challenging is the 3d structure through GPDs and TMDs



# Unpolarized TMD PDF

$$\tilde{f}_{q/\mathcal{N}}(x, b_T) = \int \frac{\mathrm{d}b^-}{4\pi} e^{-ixP^+b^-} \mathrm{Tr}\left[\langle \mathcal{N} \,|\, \bar{\psi}_q(b)\gamma^+ \mathcal{W}(b,0)\psi_q(0) \,|\, \mathcal{N} \rangle\right]$$

$$b \equiv (b^-, 0^+, \boldsymbol{b_T})$$

- $\boldsymbol{b_T}$ is the Fourier conjugate to the intrinsic transverse momentum of quarks in the hadron, $\boldsymbol{k_T}$
- We can learn about the coordinate space correlations of quark fields in hadrons
- Modification needed for UV and rapidity divergences; acquire regulators: $\tilde{f}_{q/\mathcal{N}}(x, b_T) \to \tilde{f}_{q/\mathcal{N}}(x, b_T; \mu, \zeta)$

# Factorization for low-$q_T$ Drell-Yan

- Like collinear observable, a hard part with two functions that describe structure of beam and target
- So called "$W$"-term, valid only at low-$q_T$

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}\tau \mathrm{d}Y \mathrm{d}q_T^2} = \frac{4\pi^2\alpha^2}{9\tau S^2} \sum_q H_{q\bar{q}}(Q^2,\mu) \int \mathrm{d}^2 b_T \, e^{i b_T \cdot q_T} \times \tilde{f}_{q/\pi}(x_\pi, b_T, \mu, Q^2) \, \tilde{f}_{\bar{q}/A}(x_A, b_T, \mu, Q^2) \, ,$$

# TMD PDF within the $b_*$ prescription

$$\mathbf{b}_*(\mathbf{b}_T) \equiv \frac{\mathbf{b}_T}{\sqrt{1 + b_T^2/b_{\rm max}^2}}.$$

Low-$b_T$: perturbative
high-$b_T$: non-perturbative

$$\tilde{f}_{q/\mathcal{N}(A)}(x, b_T, \mu_Q, Q^2) = (C \otimes f)_{q/\mathcal{N}(A)}(x; b_*)$$
$$\times \exp\left\{-g_{q/\mathcal{N}(A)}(x, b_T) - g_K(b_T) \ln\frac{Q}{Q_0} - S(b_*, Q_0, Q, \mu_Q)\right\}$$

Relates the TMD at small-$b_T$ to the **collinear** PDF
$\Rightarrow$ TMD is sensitive to collinear PDFs

$g_{q/\mathcal{N}(A)}$: intrinsic non-perturbative structure of the TMD
$g_K$: universal non-perturbative Collins-Soper kernel

Controls the perturbative evolution of the TMD

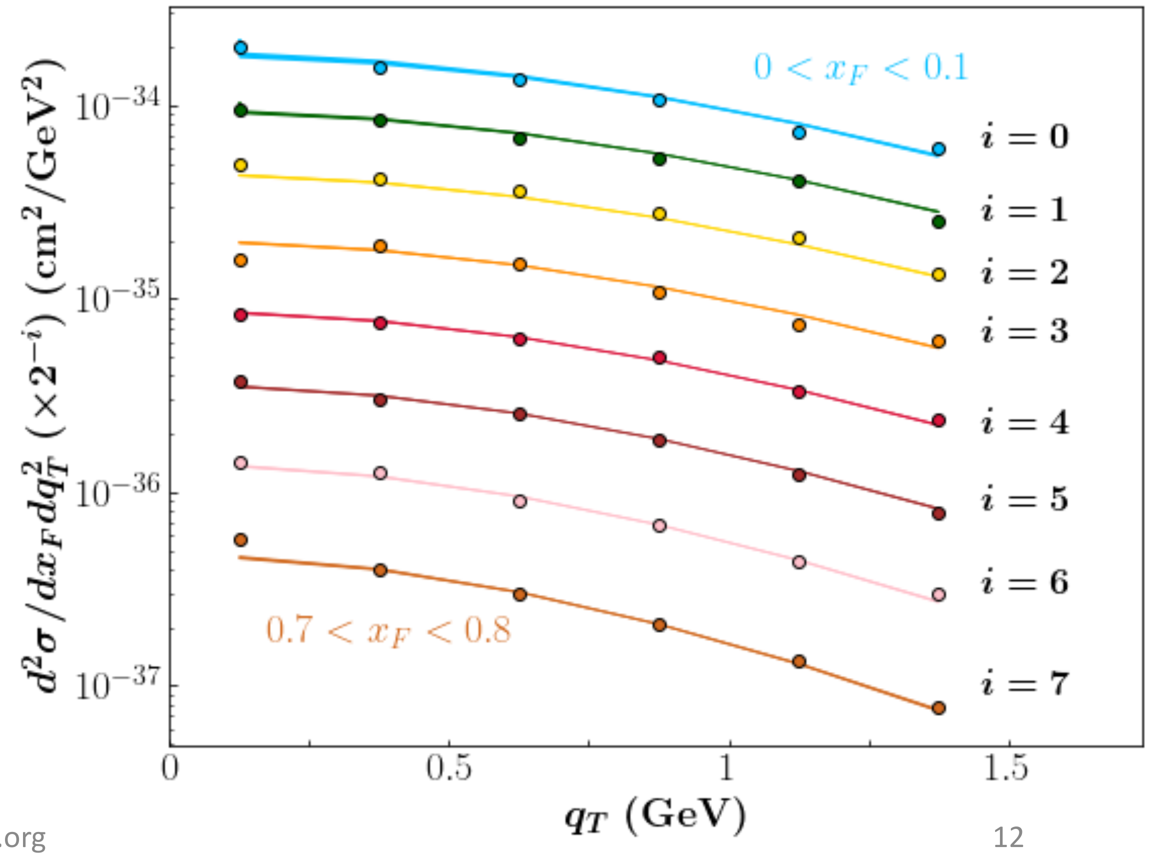
# A few details

- Nuclear TMD model linear combination of bound protons and neutrons
  - Include an additional $A$-dependent nuclear parameter
- We use the MAP collaboration's parametrization for non-perturbative TMDs
  - Only tested parametrization flexible enough to capture features of $Q$ bins
- Perform a **simultaneous global analysis** of pion TMD and collinear PDFs, with proton (nuclear) TMDs
  - Include both $q_T$-dependent and collinear pion data and fixed-target $pA$ data

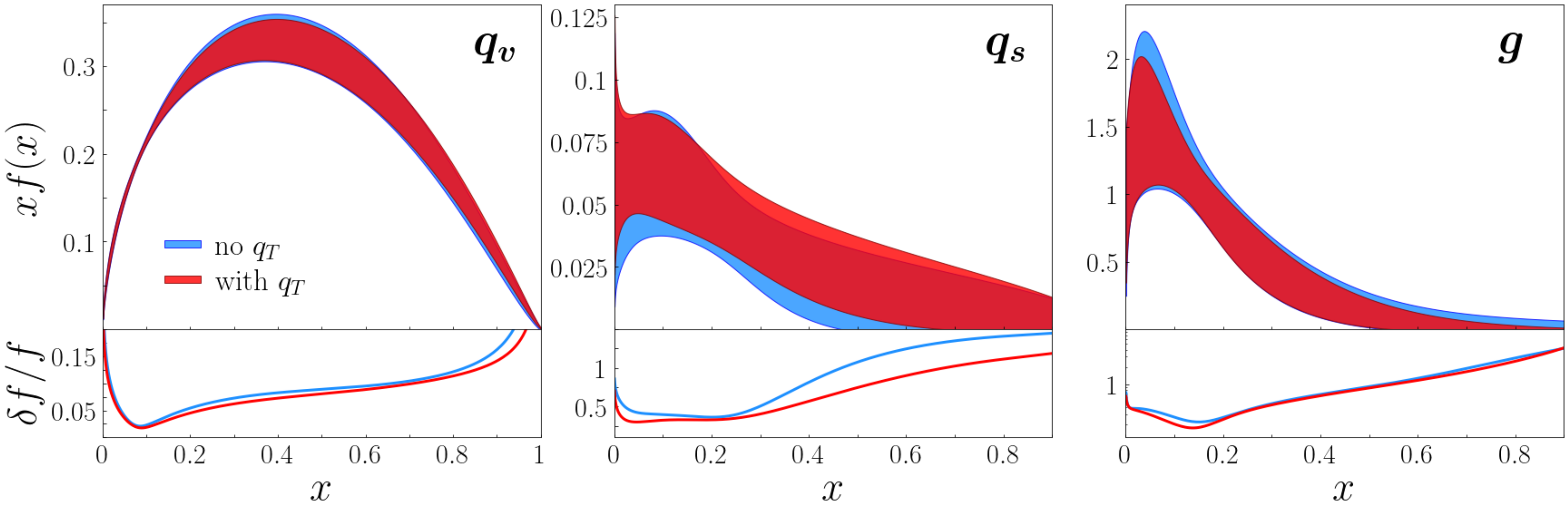
# Data and theory agreement

- Fit both $pA$ and $\pi A$ DY data and achieve good agreement to both

| Process | Experiment | $\sqrt{s}$ GeV | $\chi^2$/np | $Z$-score |
|---|---|---|---|---|
| $q_T$-integr. DY | E615 [37] | 21.8 | 0.86 | 0.76 |
| $\pi W \to \mu^+\mu^- X$ | NA10 [38] | 19.1 | 0.54 | 2.27 |
| | NA10 [38] | 23.2 | 0.91 | 0.18 |
| Leading neutron | H1 [73] | 318.7 | 0.36 | 4.61 |
| $ep \to e'nX$ | ZEUS [74] | 300.3 | 1.48 | 2.16 |
| $q_T$-dep. $pA$ DY | E288 [67] | 19.4 | 0.93 | 0.25 |
| $pA \to \mu^+\mu^- X$ | E288 [67] | 23.8 | 1.33 | 1.54 |
| | E288 [67] | 24.7 | 0.95 | 0.23 |
| | E605 [68] | 38.8 | 1.07 | 0.39 |
| | E772 [69] | 38.8 | 2.41 | 5.74 |
| | E866 ($Fe/Be$) [70] | 38.8 | 1.07 | 0.29 |
| | E866 ($W/Be$) [70] | 38.8 | 0.89 | 0.11 |
| $q_T$-dep. $\pi A$ DY | E615 [37] | 21.8 | 1.61 | 2.58 |
| $\pi W \to \mu^+\mu^- X$ | E537 [71] | 15.3 | 1.11 | 0.57 |
| Total | | | 1.15 | 2.55 |



# Extracted pion PDFs

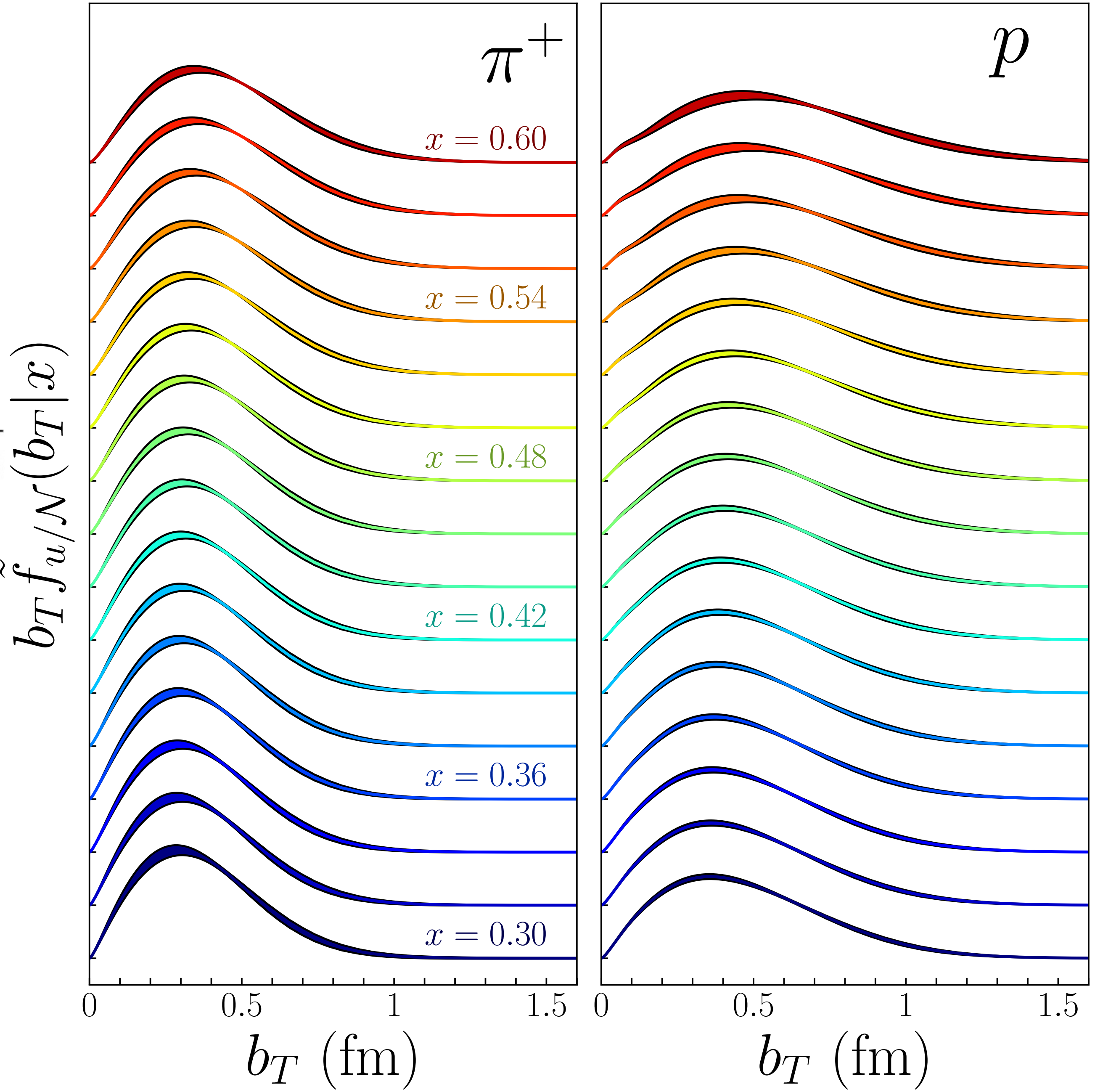



- The small-$q_T$ data do not constrain much the PDFs

# Resulting TMD PDFs of proton and pion

$$\tilde{f}_{q/\mathcal{N}}(b_T|x;Q,Q^2) \equiv \frac{\tilde{f}_{q/\mathcal{N}}(x,b_T;Q,Q^2)}{\int \mathrm{d}^2\boldsymbol{b}_T \tilde{f}_{q/\mathcal{N}}(x,b_T;Q,Q^2)}$$

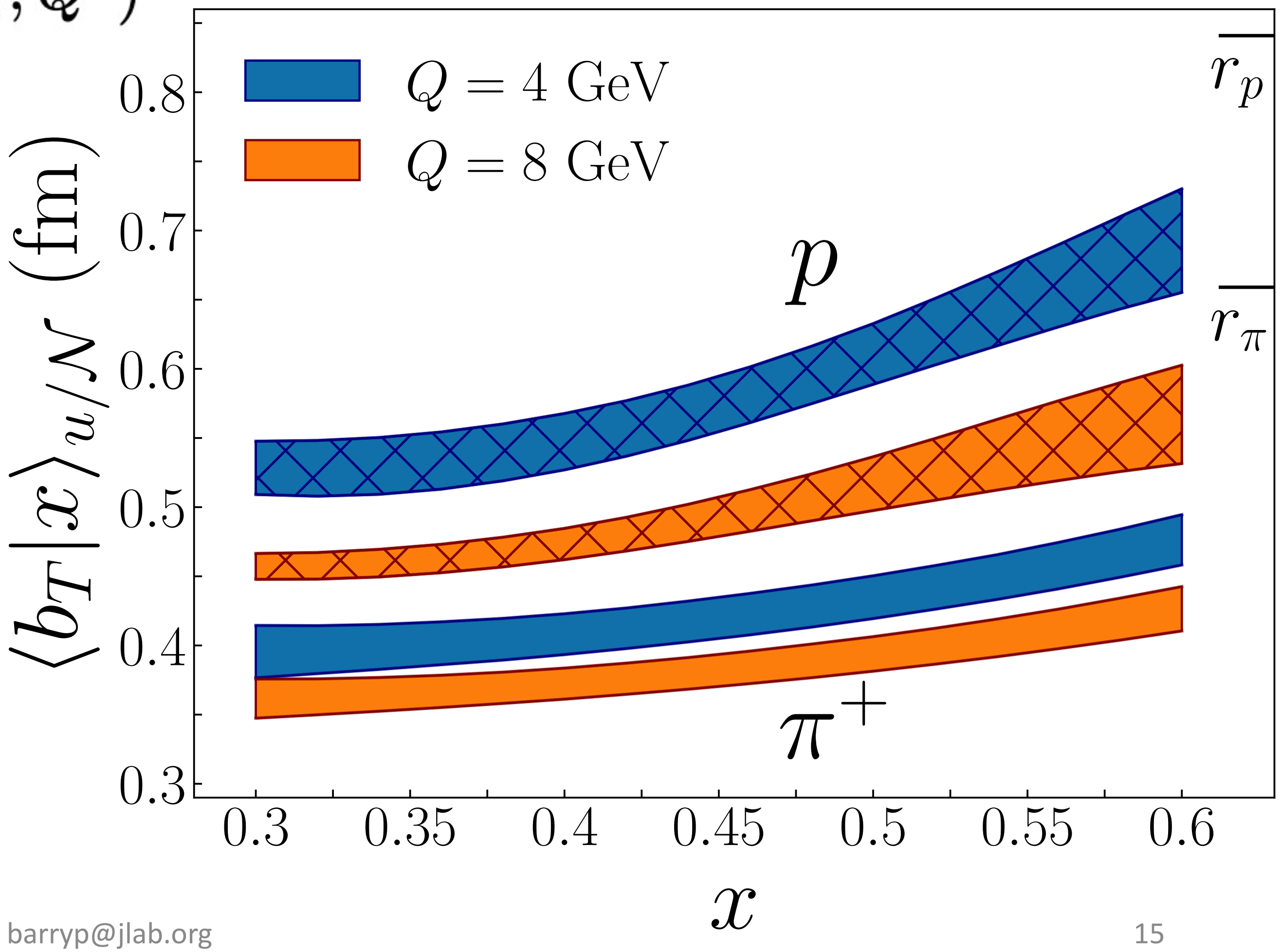
- Broadening appearing as $x$ increases
- Up quark in pion is narrower than up quark in proton



# Resulting average $b_T$

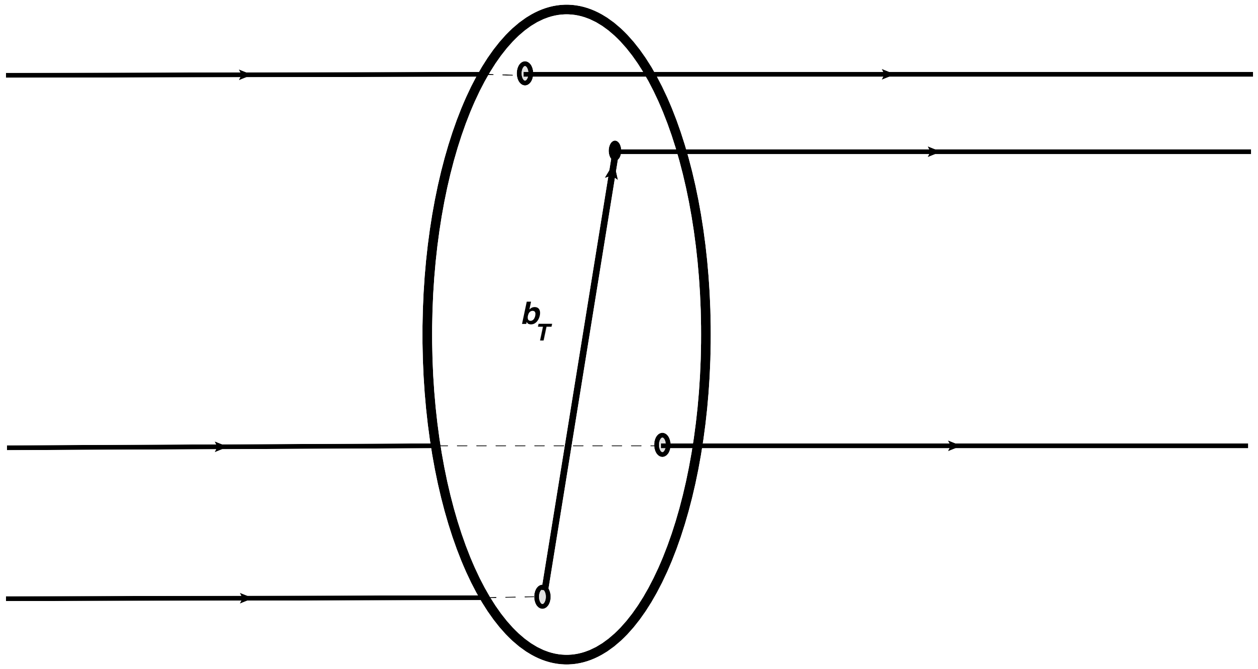
$$\langle b_T | x \rangle_{q/\mathcal{N}} = \int \mathrm{d}^2\boldsymbol{b}_T \, b_T \, \tilde{f}_{q/\mathcal{N}}(b_T | x; Q, Q^2)$$

- Average transverse spatial correlation of the up quark in proton is ~ 1.2 times bigger than that of pion
- Pion's $\langle b_T | x \rangle$ is $5.3 - 7.5\sigma$ smaller than proton in this range
- Decreases as $x$ decreases



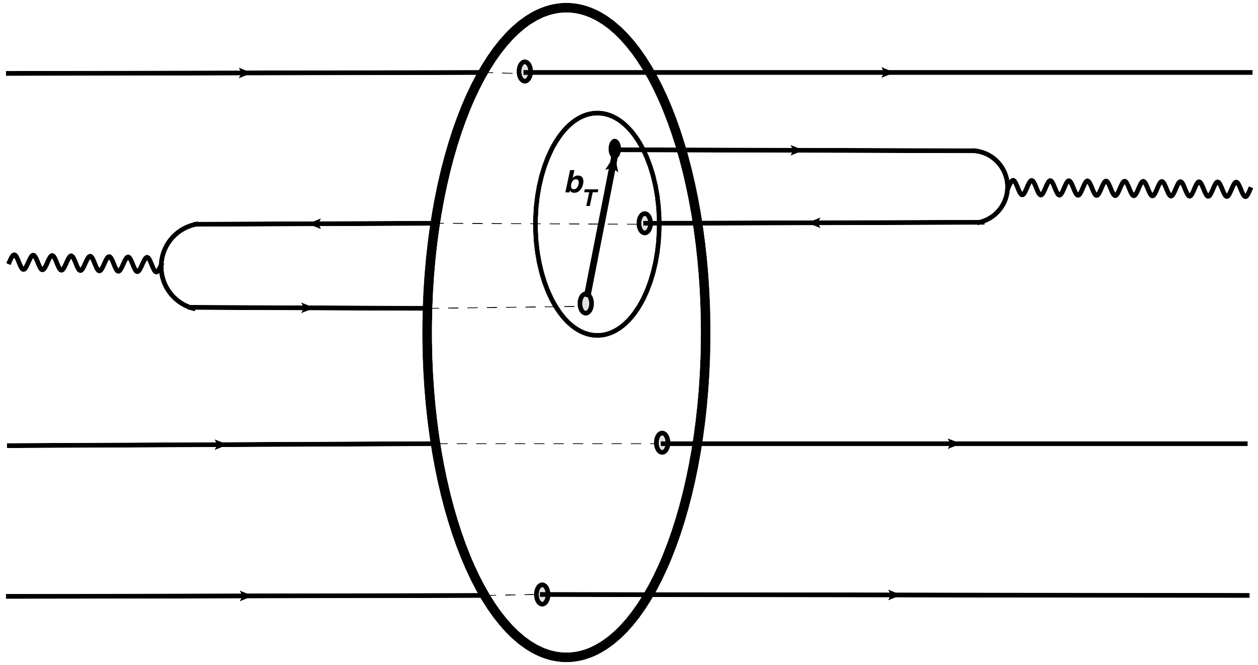
# Possible explanation

- At large $x$, we are in a valence region, where only the valence quarks are populating the momentum dependence of the hadron



# Possible explanation

- At small $x$, sea quarks and potential $q\bar{q}$ bound states allowing only for a smaller bound system



# Outlook

- Future studies needed for theoretical explanations of these phenomena
- Lattice QCD can in principle calculate any hadronic state – look to kaons, rho mesons, etc.
- Future tagged experiments such as at EIC and JLab 22 GeV can provide measurements for neutrons, pions, and kaons
- We should study other ways to formulate the TMD such as: Qiu-Zhang method, the $\zeta$-prescription, or the hadron structure oriented approach

# Backup

# Small $b_T$ operator product expansion

- At small $b_T$, the TMDPDF can be described in terms of its OPE:

$$\tilde{f}_{f/h}(x, b_T; \mu, \zeta_F) = \sum_j \int_x^1 \frac{d\xi}{\xi} \tilde{\mathcal{C}}_{f/j}(x/\xi, b_T; \zeta_F, \mu) f_{j/h}(\xi; \mu) + \mathcal{O}((\Lambda_{\mathrm{QCD}} b_T)^a)$$

- where $\tilde{C}$ are the Wilson coefficients, and $f_{j/h}$ is the collinear PDF
- Breaks down when $b_T$ gets large

# $b_*$ prescription

- A common approach to regulating large $b_T$ behavior

$$\mathbf{b}_*(\mathbf{b}_T) \equiv \frac{\mathbf{b}_T}{\sqrt{1 + b_T^2/b_{\max}^2}}.$$

Must choose an appropriate value; a transition from perturbative to non-perturbative physics

- At small $b_T$, $b_*(b_T) = b_T$
- At large $b_T$, $b_*(b_T) = b_{\max}$

# Introduction of non-perturbative functions

- Because $b_* \neq b_T$, have to non-perturbatively describe large $b_T$ behavior

Completely general – independent of quark, hadron, PDF or FF

$$g_K(b_T; b_{\max}) = -\tilde{K}(b_T, \mu) + \tilde{K}(b_*, \mu)$$

Non-perturbative function dependent in principle on flavor, hadron, etc.

$$e^{-g_{j/H}(x, \boldsymbol{b}_{\mathrm{T}}; b_{\max})} = \frac{\tilde{f}_{j/H}(x, \boldsymbol{b}_{\mathrm{T}}; \zeta, \mu)}{\tilde{f}_{j/H}(x, \boldsymbol{b}_*; \zeta, \mu)} e^{g_K(b_{\mathrm{T}}; b_{\max}) \ln(\sqrt{\zeta}/Q_0)}.$$

# TMD factorization in Drell-Yan

- In small-$q_{\mathrm{T}}$ region, use the Collins-Soper-Sterman (CSS) formalism and $b_*$ prescription

Can these data constrain the pion collinear PDF?

$$\frac{d\sigma}{dQ^2\,dy\,dq_{\mathrm{T}}^2} = \frac{4\pi^2\alpha^2}{9Q^2 s} \sum_{j,j_A,j_B} H_{j\bar{j}}^{\mathrm{DY}}(Q,\mu_Q,a_s(\mu_Q)) \int \frac{d^2\boldsymbol{b}_{\mathrm{T}}}{(2\pi)^2} e^{i\boldsymbol{q}_{\mathrm{T}}\cdot\boldsymbol{b}_{\mathrm{T}}}$$

$$\times\, e^{-g_{j/A}(x_A,b_{\mathrm{T}};b_{\max})} \int_{x_A}^{1} \frac{d\xi_A}{\xi_A} f_{j_A/A}(\xi_A;\mu_{b_*})\, \tilde{C}_{j/j_A}^{\mathrm{PDF}}\left(\frac{x_A}{\xi_A}, b_*; \mu_{b_*}^2, \mu_{b_*}, a_s(\mu_{b_*})\right)$$

$$\times\, e^{-g_{\bar{j}/B}(x_B,b_{\mathrm{T}};b_{\max})} \int_{x_B}^{1} \frac{d\xi_B}{\xi_B} f_{j_B/B}(\xi_B;\mu_{b_*})\, \tilde{C}_{\bar{j}/j_B}^{\mathrm{PDF}}\left(\frac{x_B}{\xi_B}, b_*; \mu_{b_*}^2, \mu_{b_*}, a_s(\mu_{b_*})\right)$$

$$\times \exp\left\{-g_K(b_{\mathrm{T}};b_{\max}) \ln\frac{Q^2}{Q_0^2} + \tilde{K}(b_*;\mu_{b_*}) \ln\frac{Q^2}{\mu_{b_*}^2} + \int_{\mu_{b_*}}^{\mu_Q} \frac{d\mu'}{\mu'}\left[2\gamma_j(a_s(\mu')) - \ln\frac{Q^2}{(\mu')^2}\gamma_K(a_s(\mu'))\right]\right\}$$

Perturbative pieces

Non-perturbative pieces

Non-perturbative piece of the CS kernel

# MAP parametrization

- A recent work from the MAP collaboration (arXiv:2206.07598) used a complicated form for the non-perturbative function

$$f_{1\,NP}(x, \boldsymbol{b}_T^2; \zeta, Q_0) = \frac{g_1(x)\, e^{-g_1(x)\frac{\boldsymbol{b}_T^2}{4}} + \lambda^2\, g_{1B}^2(x) \left[1 - g_{1B}(x)\frac{\boldsymbol{b}_T^2}{4}\right] e^{-g_{1B}(x)\frac{\boldsymbol{b}_T^2}{4}} + \lambda_2^2\, g_{1C}(x)\, e^{-g_{1C}(x)\frac{\boldsymbol{b}_T^2}{4}}}{g_1(x) + \lambda^2\, g_{1B}^2(x) + \lambda_2^2\, g_{1C}(x)} \left[\frac{\zeta}{Q_0^2}\right]^{g_K(\boldsymbol{b}_T^2)/2}, \quad (38)$$

$$g_{\{1,1B,1C\}}(x) = N_{\{1,1B,1C\}} \frac{x^{\sigma_{\{1,2,3\}}} (1-x)^{\alpha^2_{\{1,2,3\}}}}{\hat{x}^{\sigma_{\{1,2,3\}}} (1-\hat{x})^{\alpha^2_{\{1,2,3\}}}},$$

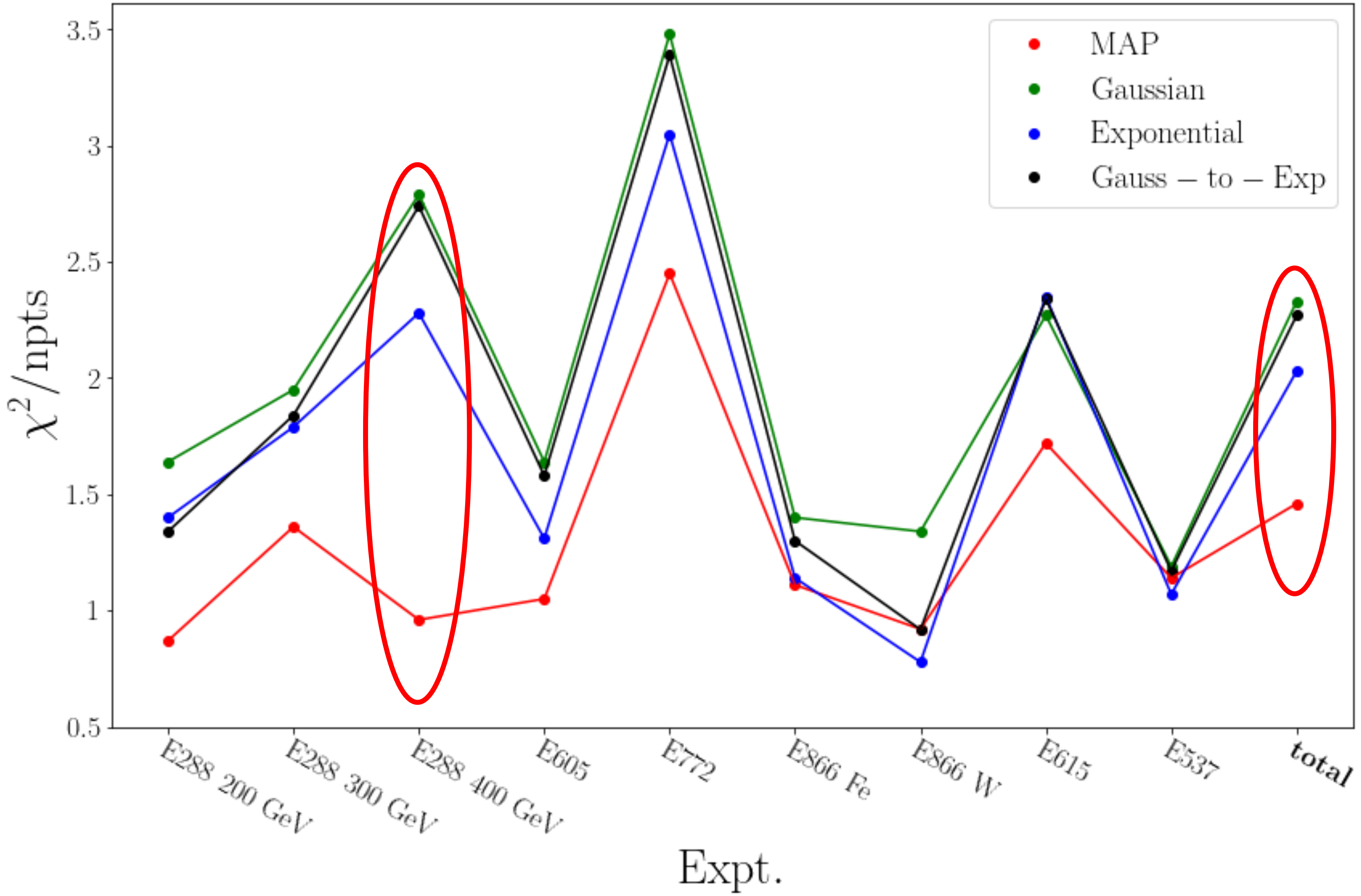
$$g_K(\boldsymbol{b}_T^2) = -g_2^2 \frac{\boldsymbol{b}_T^2}{2}$$

Universal CS kernel

- 11 free parameters for each hadron! (flavor dependence not necessary) (12 if we include the nuclear TMD parameter)

# Resulting $\chi^2$ for each parametrization

- Tried multiple parametrizations for non-perturbative TMD structures
- MAP parametrization is able to describe better all the datasets



# Nuclear TMD PDFs – working hypothesis

- We must model the nuclear TMD PDF from proton

$$\tilde{f}_{q/A}(x, b_T, \mu, \zeta) = \frac{Z}{A}\tilde{f}_{q/p/A}(x, b_T, \mu, \zeta) + \frac{A-Z}{A}\tilde{f}_{q/n/A}(x, b_T, \mu, \zeta)$$

- Each object on the right side independently obeys the CSS equation
  - **Assumption** that the bound proton and bound neutron follow TMD factorization
- Make use of isospin symmetry in that $u/p/A \leftrightarrow d/n/A$, etc.

# Building of the nuclear TMD PDF

- Then taking into account the intrinsic non-perturbative, we model the flavor-dependent pieces of the TMD PDF as

$$(C \otimes f)_{u/A}(x) e^{-g_{u/A}(x,b_T)} \rightarrow \frac{Z}{A}(C \otimes f)_{u/p/A}(x) e^{-g_{u/p/A}(x,b_T)} + \frac{A-Z}{A}(C \otimes f)_{d/p/A}(x) e^{-g_{d/p/A}(x,b_T)}$$

and

$$(C \otimes f)_{d/A}(x) e^{-g_{d/A}(x,b_T)} \rightarrow \frac{Z}{A}(C \otimes f)_{d/p/A}(x) e^{-g_{d/p/A}(x,b_T)} + \frac{A-Z}{A}(C \otimes f)_{u/p/A}(x) e^{-g_{u/p/A}(x,b_T)}.$$

# Nuclear TMD parametrization

- Specifically, we include a parametrization similar to Alrashed, et al., Phys. Rev. Lett **129**, 242001 (2022).

$$g_{q/\mathcal{N}/A} = g_{q/\mathcal{N}} \left(1 - a_{\mathcal{N}} \left(A^{1/3} - 1\right)\right)$$

- Where $a_{\mathcal{N}}$ is an additional parameter to be fit

# Datasets in the $q_T$-dependent analysis

| Expt. | $\sqrt{s}$ (GeV) | Reaction | Observable | $Q$ (GeV) | $x_F$ or $y$ | $N_{pts.}$ |
|---|---|---|---|---|---|---|
| E288 [39] | 19.4 | $p + Pt \to \ell^+\ell^- X$ | $E\mathrm{d}^3\sigma/\mathrm{d}^3\mathbf{q}$ | $4-9$ | $y = 0.4$ | 38 |
| E288 [39] | 23.8 | $p + Pt \to \ell^+\ell^- X$ | $E\mathrm{d}^3\sigma/\mathrm{d}^3\mathbf{q}$ | $4-12$ | $y = 0.21$ | 48 |
| E288 [39] | 24.7 | $p + Pt \to \ell^+\ell^- X$ | $E\mathrm{d}^3\sigma/\mathrm{d}^3\mathbf{q}$ | $4-14$ | $y = 0.03$ | 74 |
| E605 [40] | 38.8 | $p + Cu \to \ell^+\ell^- X$ | $E\mathrm{d}^3\sigma/\mathrm{d}^3\mathbf{q}$ | $7-18$ | $x_F = 0.1$ | 49 |
| E772 [41] | 38.8 | $p + D \to \ell^+\ell^- X$ | $E\mathrm{d}^3\sigma/\mathrm{d}^3\mathbf{q}$ | $5-15$ | $0.1 \le x_F \le 0.3$ | 61 |
| E866 [50] | 38.8 | $p + Fe \to \ell^+\ell^- X$ | $R_{FeBe}$ | $4-8$ | $0.13 \le x_F \le 0.93$ | 10 |
| E866 [50] | 38.8 | $p + W \to \ell^+\ell^- X$ | $R_{WBe}$ | $4-8$ | $0.13 \le x_F \le 0.93$ | 10 |
| E537 [38] | 15.3 | $\pi^- + W \to \ell^+\ell^- X$ | $\mathrm{d}^2\sigma/\mathrm{d}x_F\mathrm{d}q_T$ | $4-9$ | $0 < x_F < 0.8$ | 48 |
| E615 [4] | 21.8 | $\pi^- + W \to \ell^+\ell^- X$ | $\mathrm{d}^2\sigma/\mathrm{d}x_F\mathrm{d}q_T^2$ | $4.05-8.55$ | $0 < x_F < 0.8$ | 45 |

- Total of 383 number of points
- All fixed target, low-energy data
- We perform a cut of $q_T^{\max} < 0.25\,Q$

# Transverse EMC effect

- Compare the average $b_T$ given $x$ for the up quark in the bound proton to that of the free proton
- Less than 1 by $\sim 5-10\%$ over the $x$ range