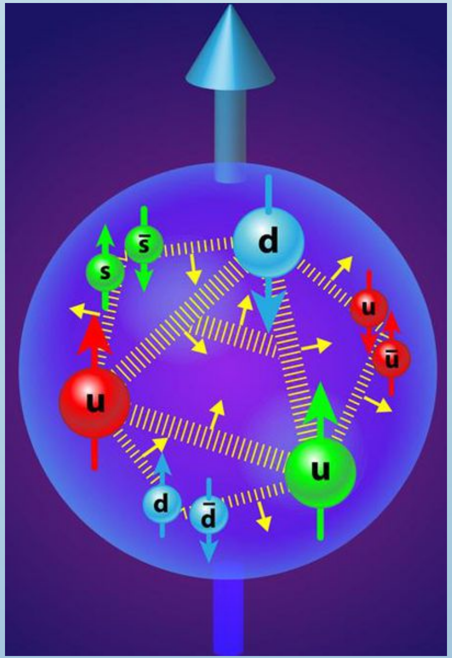
Simultaneous Global Analysis of Di-Hadron Fragmentation Functions and Transversity PDFs

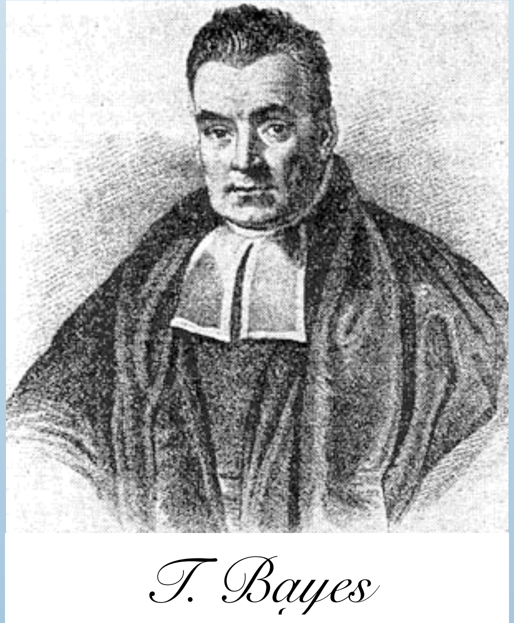
Christopher Cocuzza

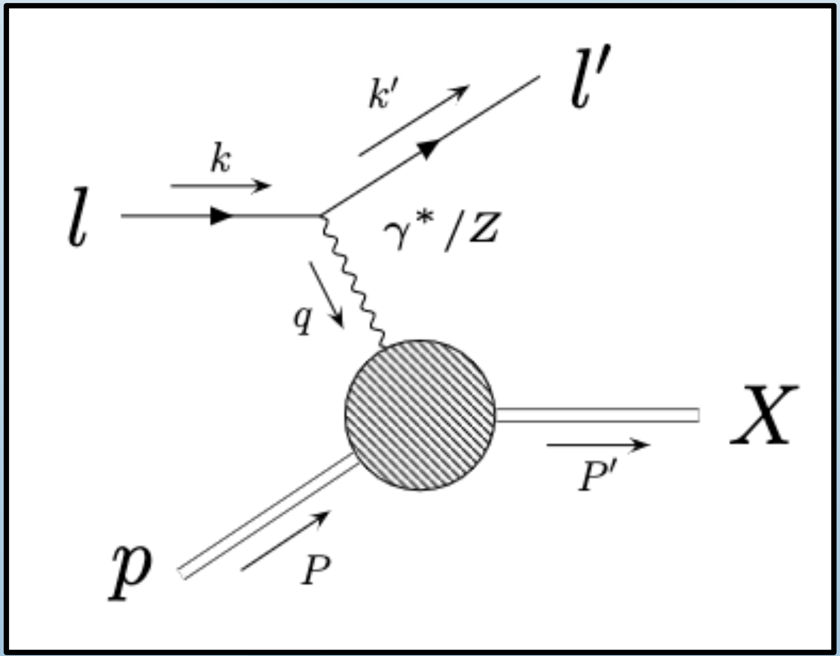
June 22, 2023

Center for Frontiers in Nuclear Science

1. Introduction
2. Extraction of DiFFs
3. Extraction of Transversity PDFs
4. Extraction of Tensor Charges
5. Conclusions and Outlook



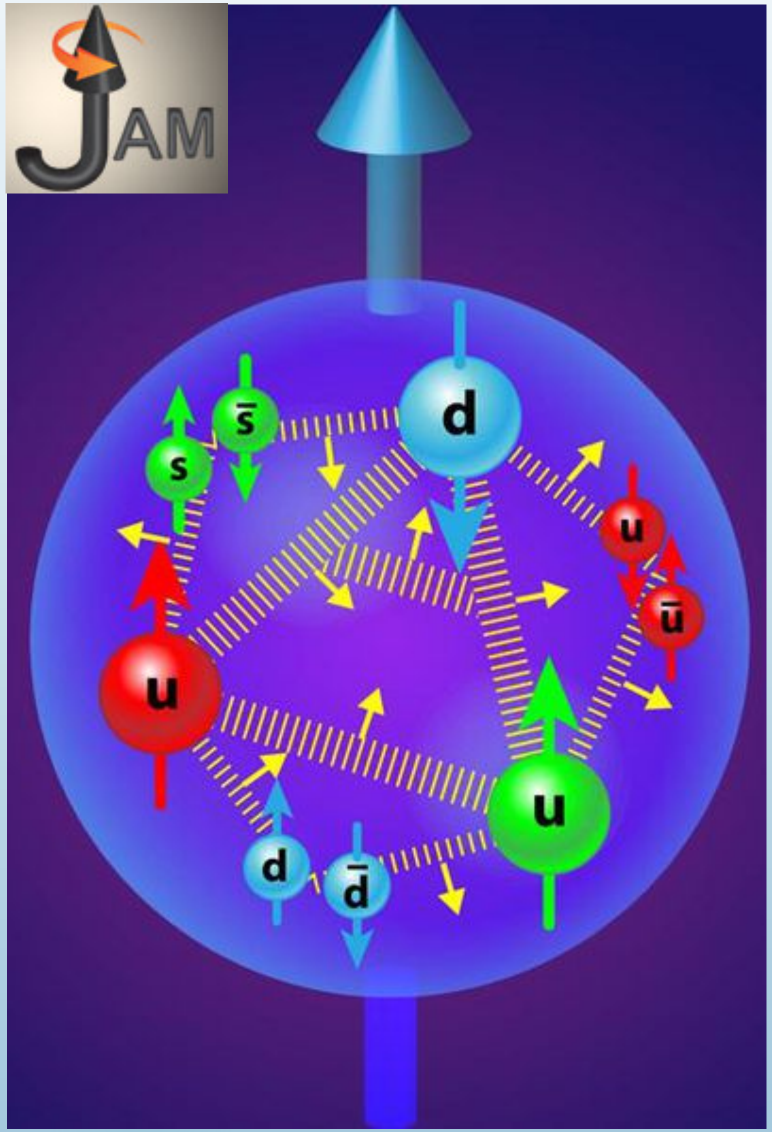




JAM Collaboration

3-dimensional structure of nucleons:

- <u>Parton distribution functions (PDFs)</u>
- <u>Fragmentation functions (FFs)</u>
- Transverse momentum dependent distributions (TMDs)
- Generalized parton distributions (GPDs)



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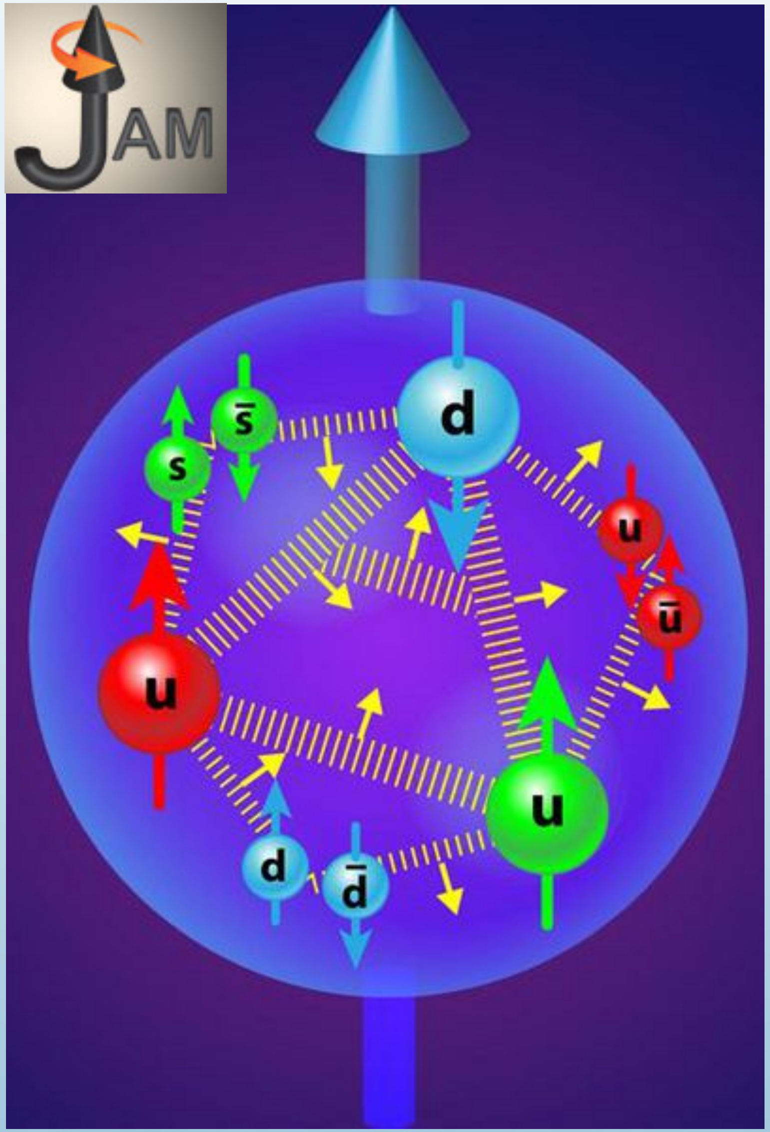
3-dimensional structure of nucleons:
- Parton distribution functions (PDFs)
- Fragmentation functions (FFs)
- Transverse momentum dependent distributions (TMDs)
- Generalized parton distributions (GPDs)

- Collinear factorization in perturbative QCD
- Simultaneous determinations of PDFs, FFs, etc.
- Monte Carlo methods for Bayesian inference



Introduction

Hadron Structure

Global QCD Analysis

Hadron Structure

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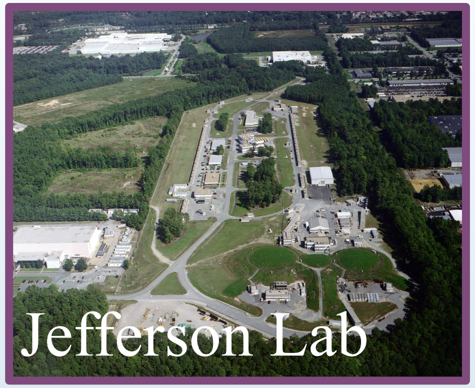
Global QCD Analysis



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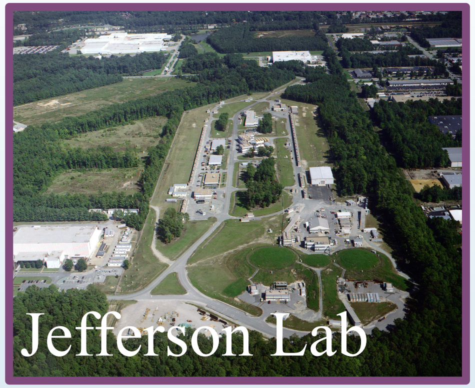
Global QCD Analysis

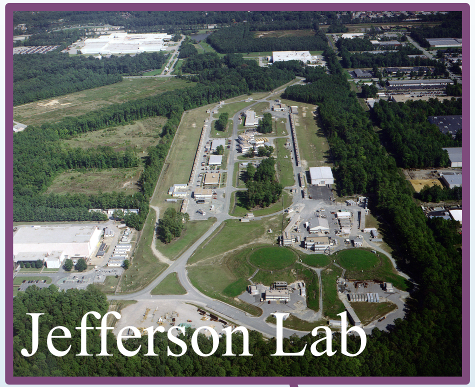


Hadron Structure

Global QCD Analysis

USQCD

Extended Twisted Mass Collaboration



Global QCD Analysis

Hadron Structure

USQCD

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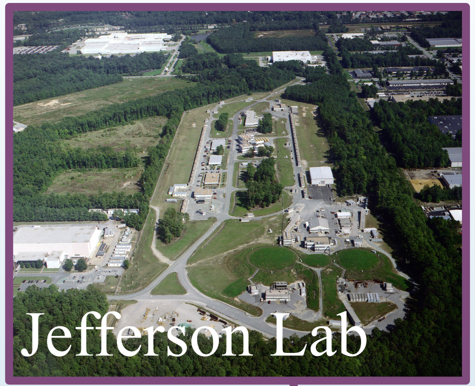


Hadron Structure

$$\frac{d}{d\,\ln(\mu^2)} f_i(x,\mu) = \sum_j \int_x^1 \frac{dz}{z} P_{ij}(z,\mu) f_j(\frac{x}{z},\mu)$$

Param. + Evolve + Factorization

$$\sigma = \sum_{i,j} H_{ij} \otimes f_i \otimes f_j$$

Global QCD Analysis

USQCD

Extended Twisted Mass Collaboration

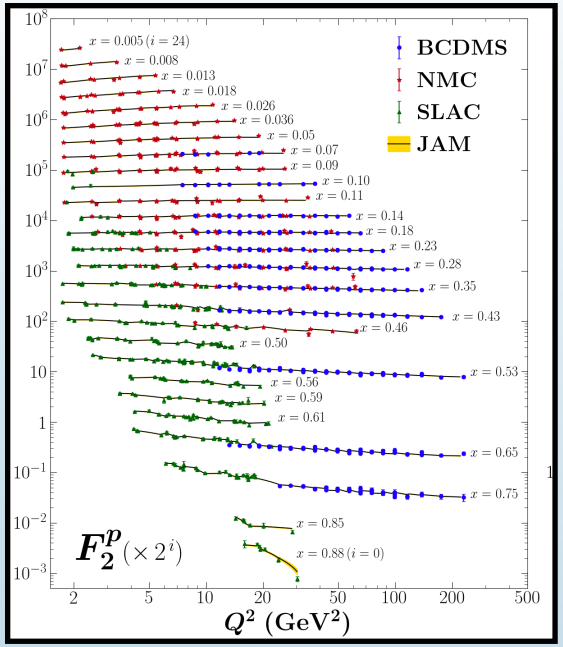
Introduction

$$\chi^2(\boldsymbol{a}) = \sum_{i,e} \left(\frac{d_{i,e} - \sum_k r_e^k \beta_{i,e}^k - T_{i,e}(\boldsymbol{a})/N_e}{\alpha_{i,e}} \right)^2 + \sum_k \left(r_e^k\right)^2 + \left(\frac{1 - N_e}{\delta N_e} \right)^2$$

χ^2 Minimization

$$\mathcal{L}\left(\boldsymbol{a}, \text{data}\right) = \exp\left(-\frac{1}{2}\chi^2\left(\boldsymbol{a}, \text{data}\right)\right)$$

$$\mathcal{P}(\boldsymbol{a}|\text{data}) \sim \mathcal{L}(\boldsymbol{a}, \text{data})\, \pi(\boldsymbol{a})$$



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Global QCD Analysis

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Extended Twisted Mass Collaboration

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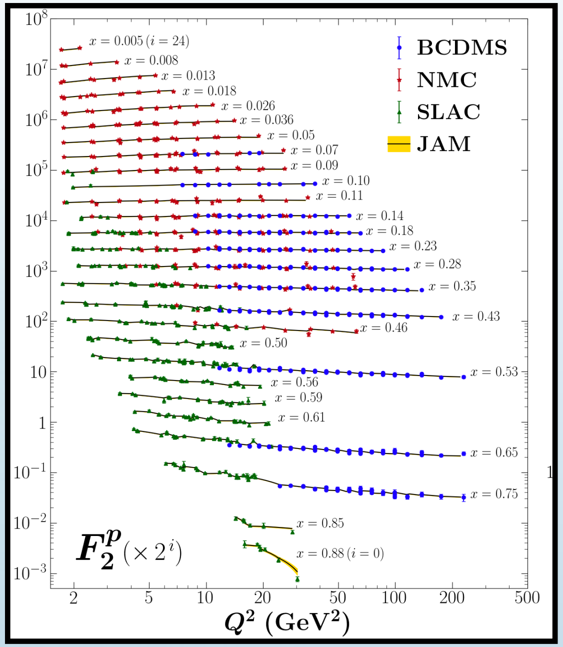

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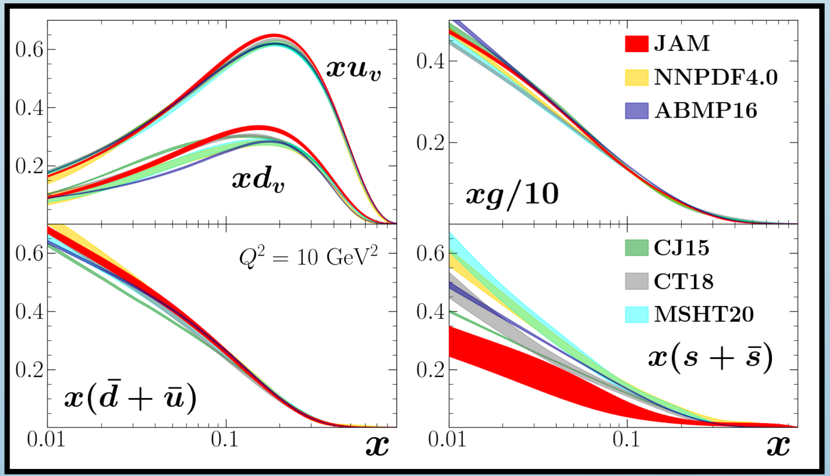

Data Resampling

$$\tilde{\sigma} = \sigma + N(0,1)\ \alpha$$

Approaches to Extract Transversity

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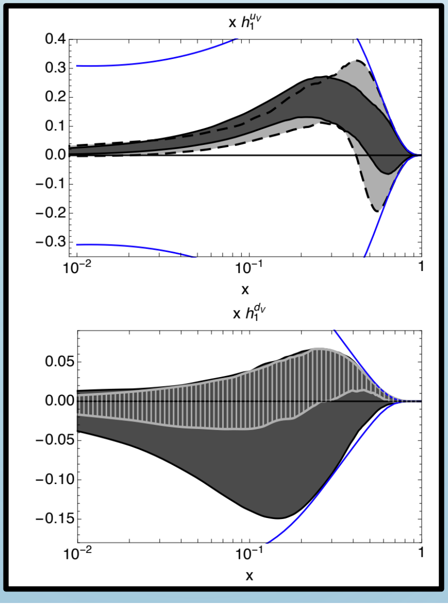
Dihadron Frag.

- Radici + Bacchetta (RB18)
- Benel + Courtoy + Ferro-Hernandez (2020)



M. Radici and A. Bacchetta,
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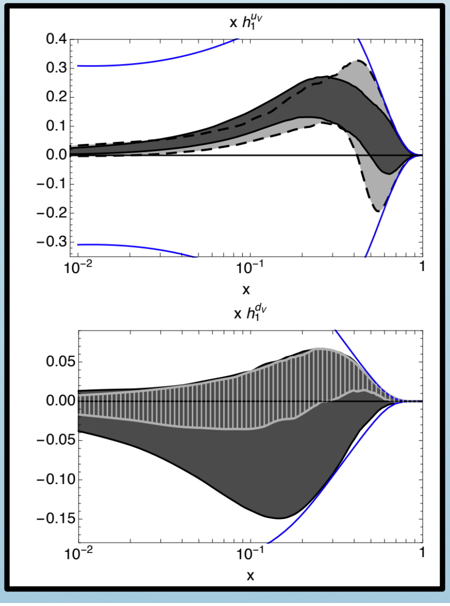

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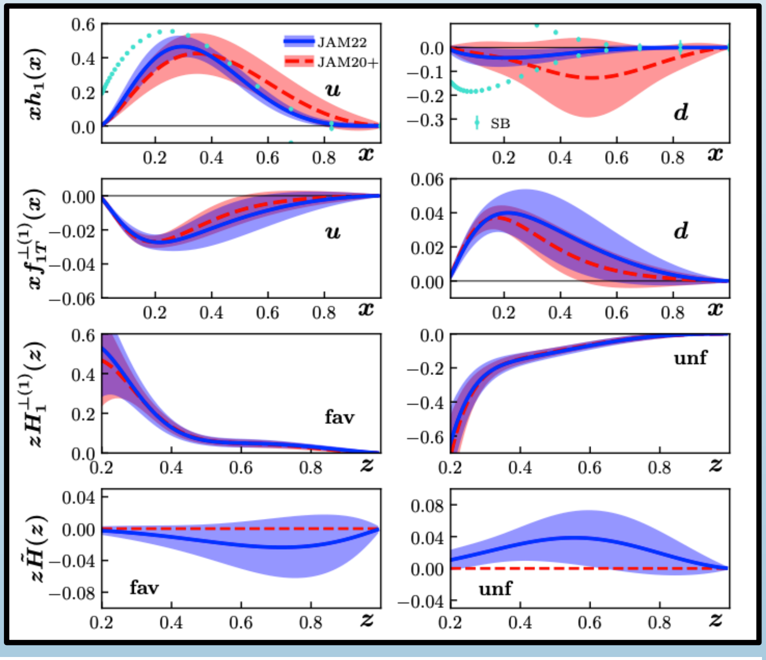
TMD + Collinear Twist-3

- JAM3D



L. Gamberg *et al.*, Phys. Rev. D **106**, no. 3, 034014 (2022)

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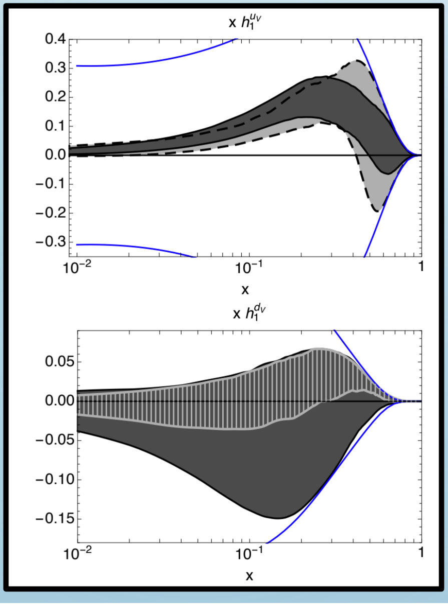

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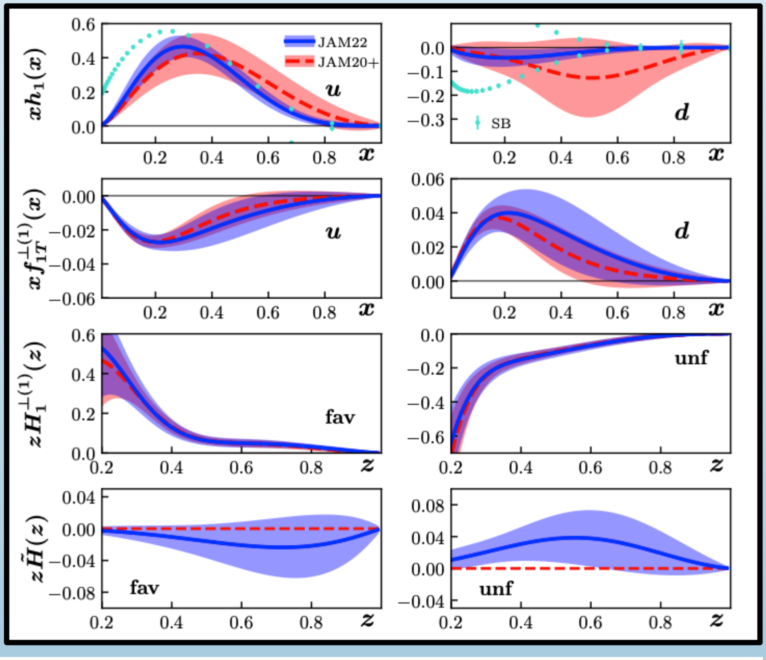
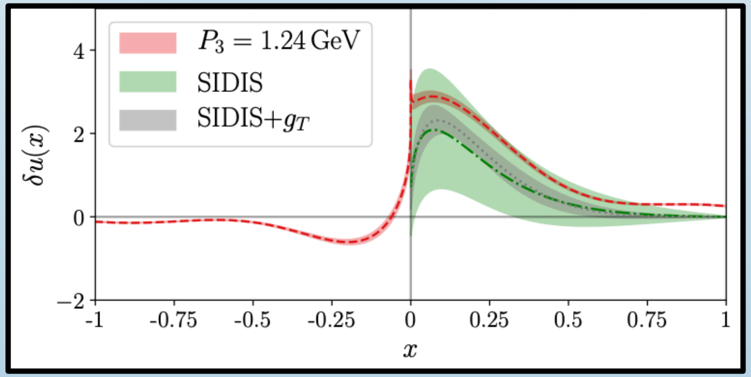


L. Gamberg *et al.*, Phys. Rev. D **106**, no. 3, 034014 (2022)

Lattice QCD

- ETMC Collaboration
- PNDME Collaboration
- LHPC Collaboration

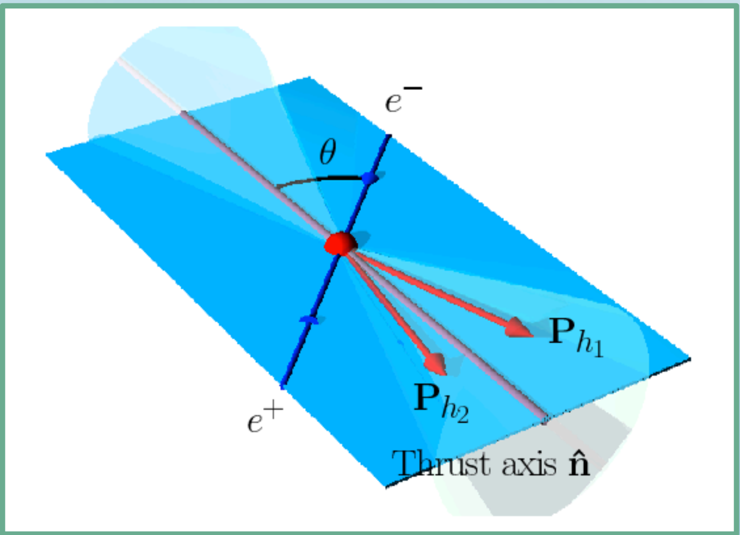


C. Alexandrou *et al.*, Phys. Rev. D **104**, no. 5, 054503 (2021)

JAM Global Analysis in the collinear DiFF Approach

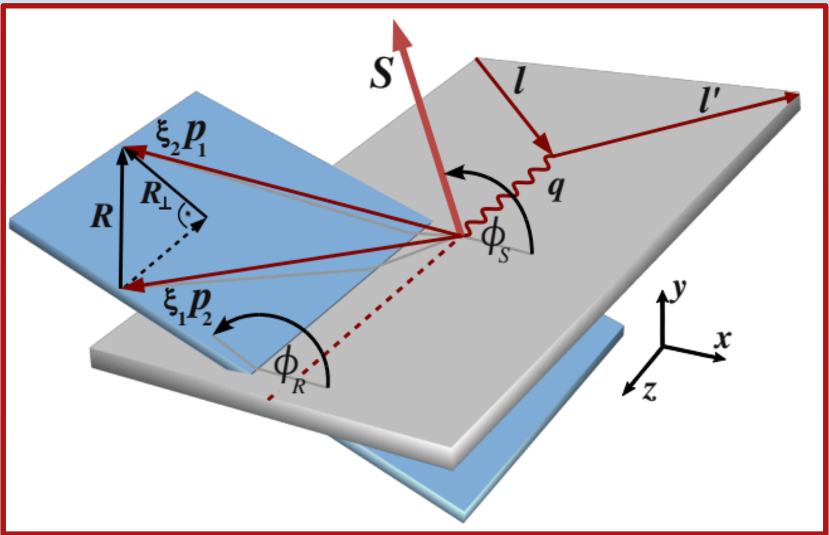
First *simultaneous* extraction of $\pi^+\pi^-$ DiFFs (D_1^q), IFFs ($H_1^{\sphericalangle,q}$), and transversity PDFs (h_1^q) at LO

Semi-Inclusive Annihilation



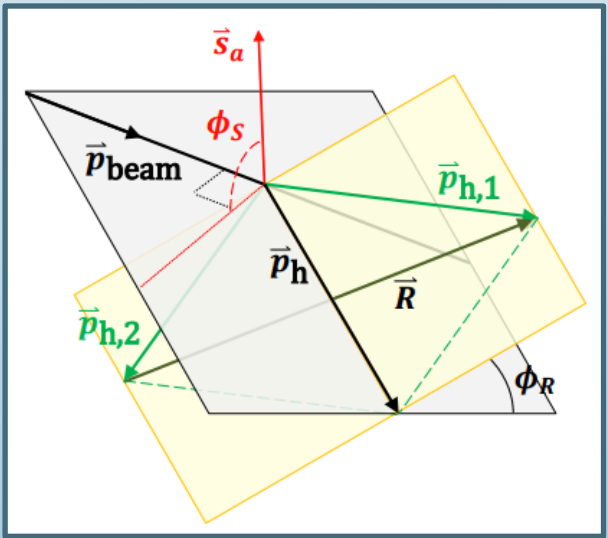
R. Seidl *et al.*, Phys. Rev. D **96**, no. 3, 032005 (2017)

Semi-Inclusive Deep Inelastic Scattering



C. Adolph *et al.*, Phys. Lett. B **713**, 10-16 (2012)

Proton-Proton Collisions



L. Adamczyk *et al.*, Phys. Rev. Lett. **115**, 242501 (2015)

Tensor Charges

$$\delta u \equiv \int_0^1 \mathrm{d}x (h_1^u - h_1^{\bar{u}}),$$

$$\delta d \equiv \int_0^1 \mathrm{d}x (h_1^d - h_1^{\bar{d}}),$$

$$g_T \equiv \delta u - \delta d,$$

Tensor Charges

Figure taken from D. Pitonyak

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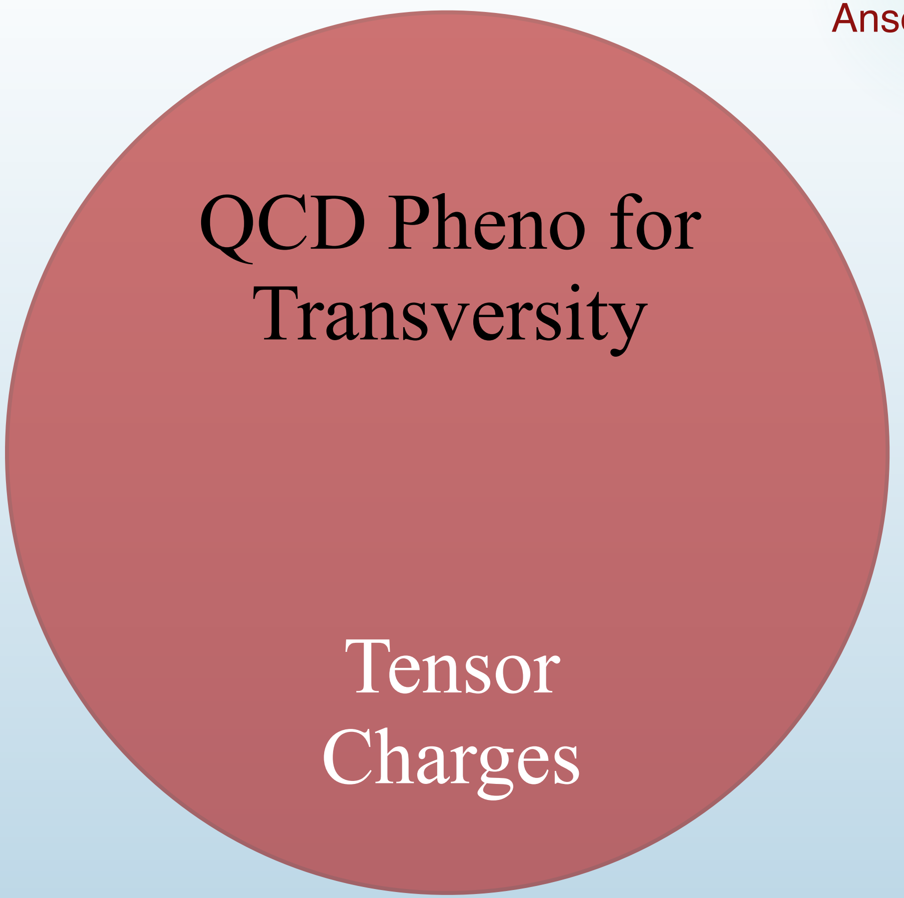

Anselmino, *et al.* (2007, 2009, 2013, 2015);
Goldstein, *et al.* (2014);
Kang, *et al.* (2016);
D'Alesio, *et al.* (2020);
Cammarota, *et al.* (2020);
Gamberg, *et al.* (2022)

Radici, *et al.* (2013, 2015, 2018);
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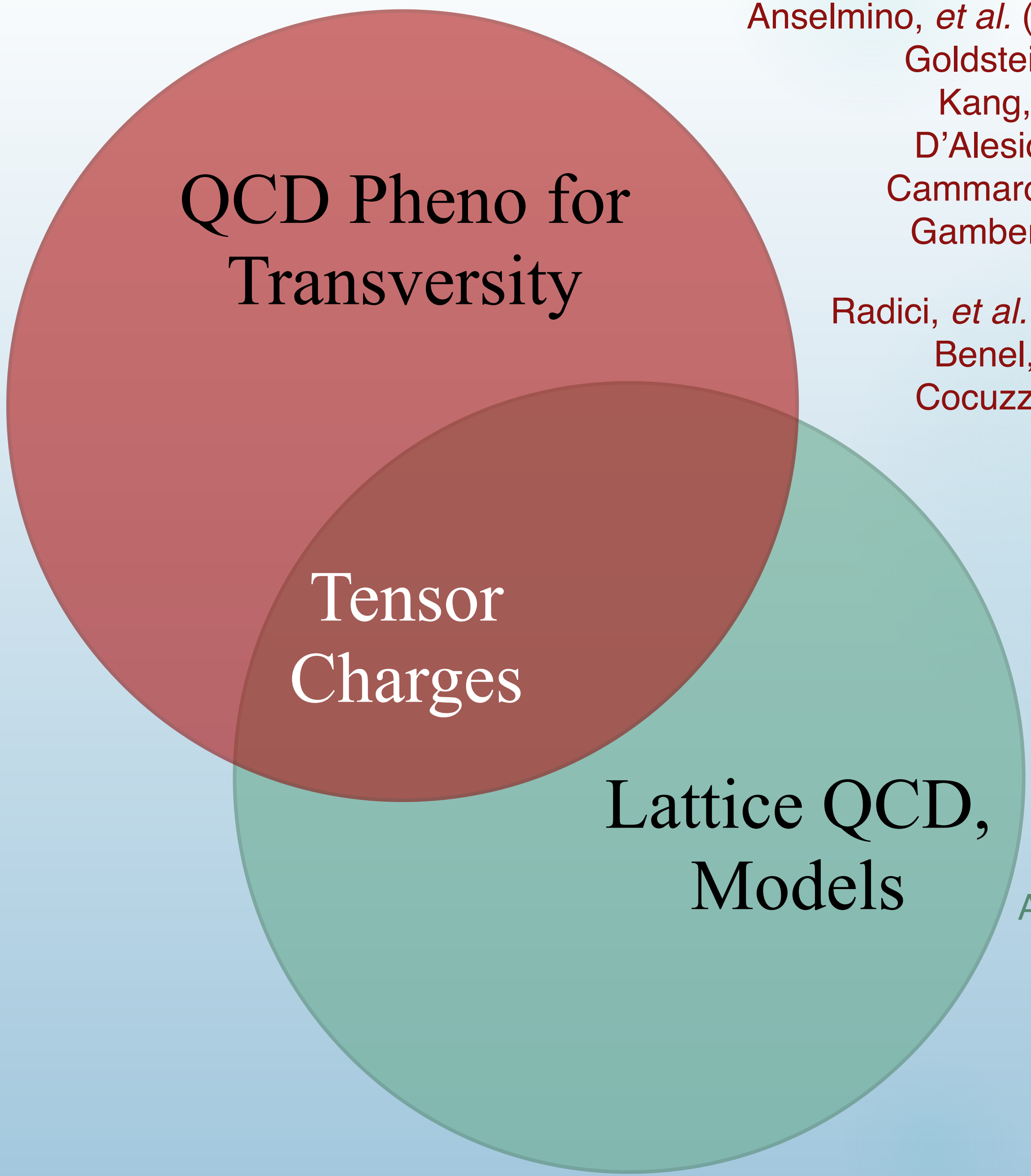

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Cocuzza, *et al.* (2023)

He, Ji (1995);
Barone, *et al.* (1997);
Schweitzer, *et al.* (2001);
Gamberg, Goldstein (2001);
Pasquini, *et al.* (2005);
Wakamatsu (2007);
Lorce (2009);
Gupta, *et al.* (2018);
Yamanaka, *et al.* (2018);
Hasan, *et al.* (2019);
Alexandrou, *et al.* (2019, 2023)
Yamanaka, *et al.* (2013);
Pitschmann, *et al.* (2015);
Xu, *et al.* (2015);
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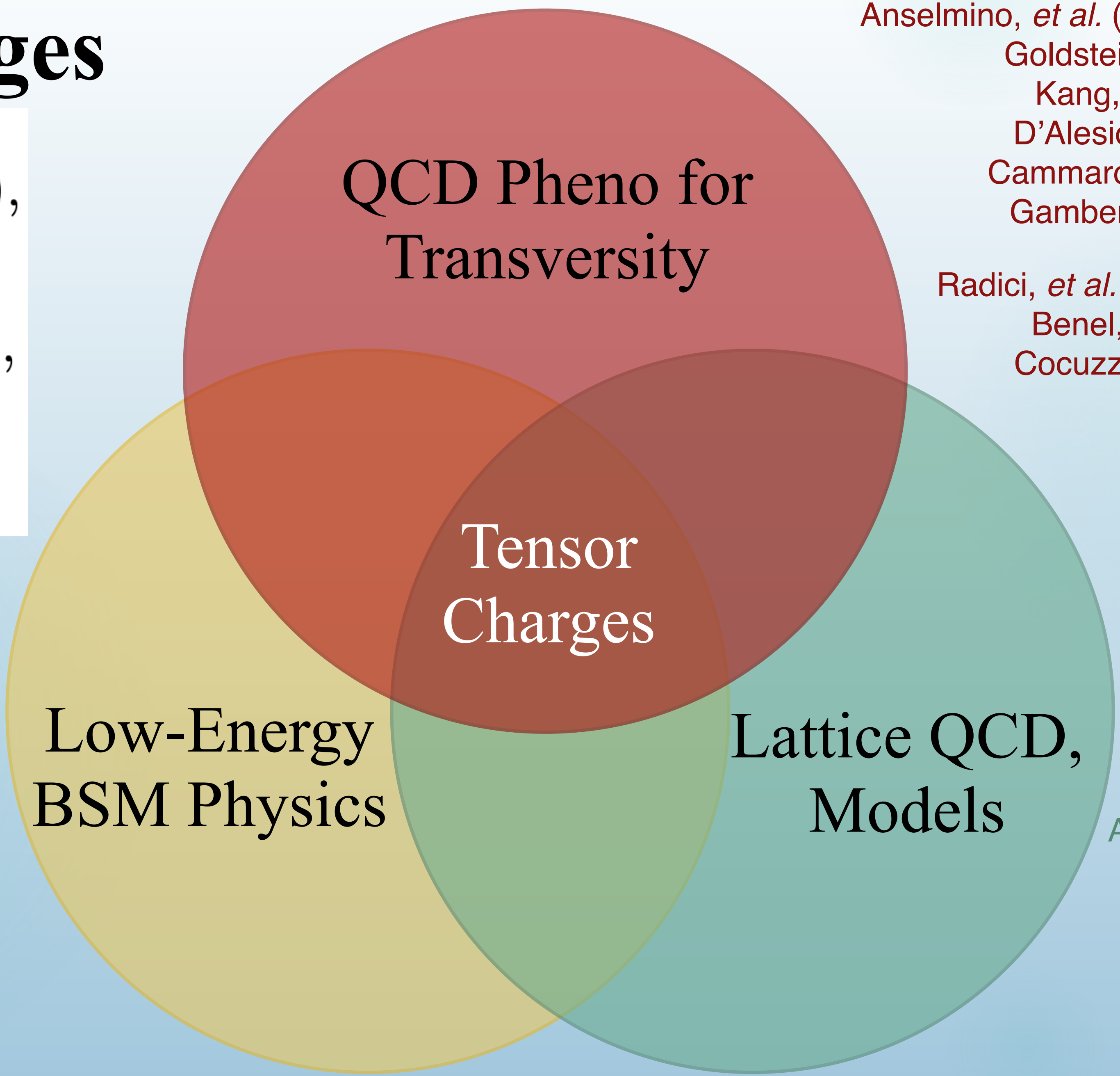

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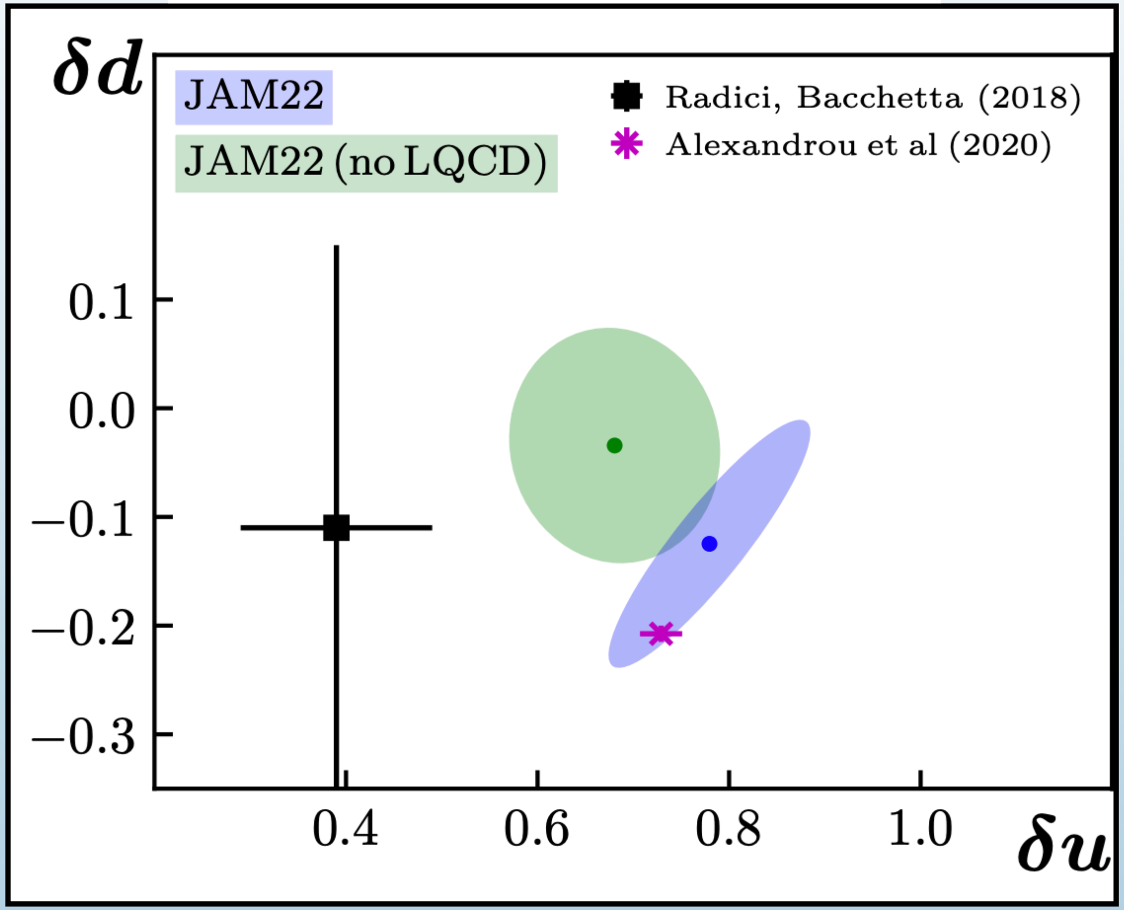
Herczeg (2001);
Erler, Ramsey-Musolf (2005);
Pospelov, Ritz (2005);
Severijns, *et al.* (2006);
Cirigliano, *et al.* (2013);
Courtoy, *et al.* (2015);
Yamanaka, *et al.* (2017);
Liu, *et al.* (2018);
Gonzalez-Alonso, *et al.* (2019)

Figure taken from D. Pitonyak

The Transverse Spin Puzzle?

L. Gamberg *et al.*, Phys. Rev. D **106**, no. 3, 034014 (2022)



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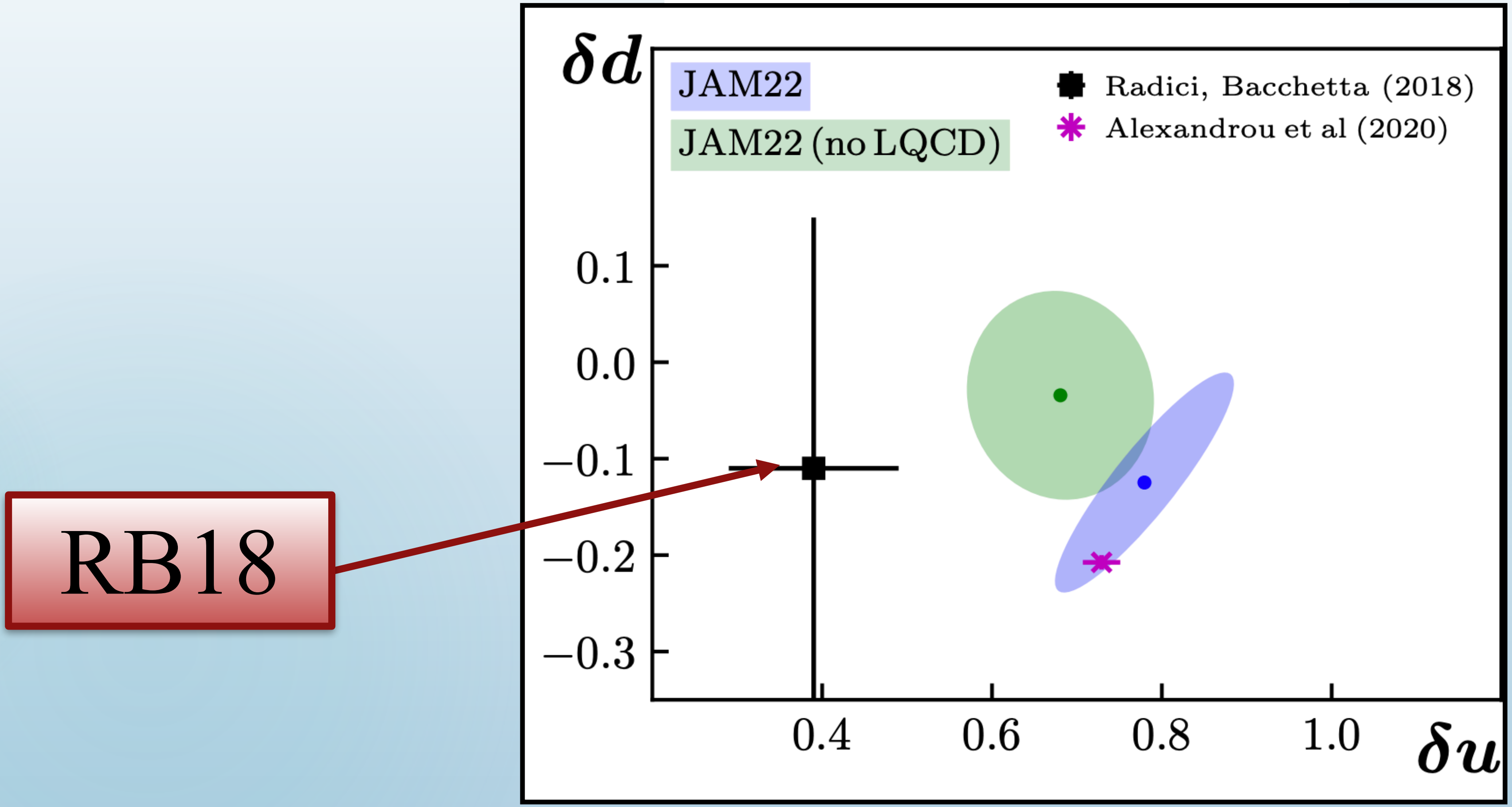

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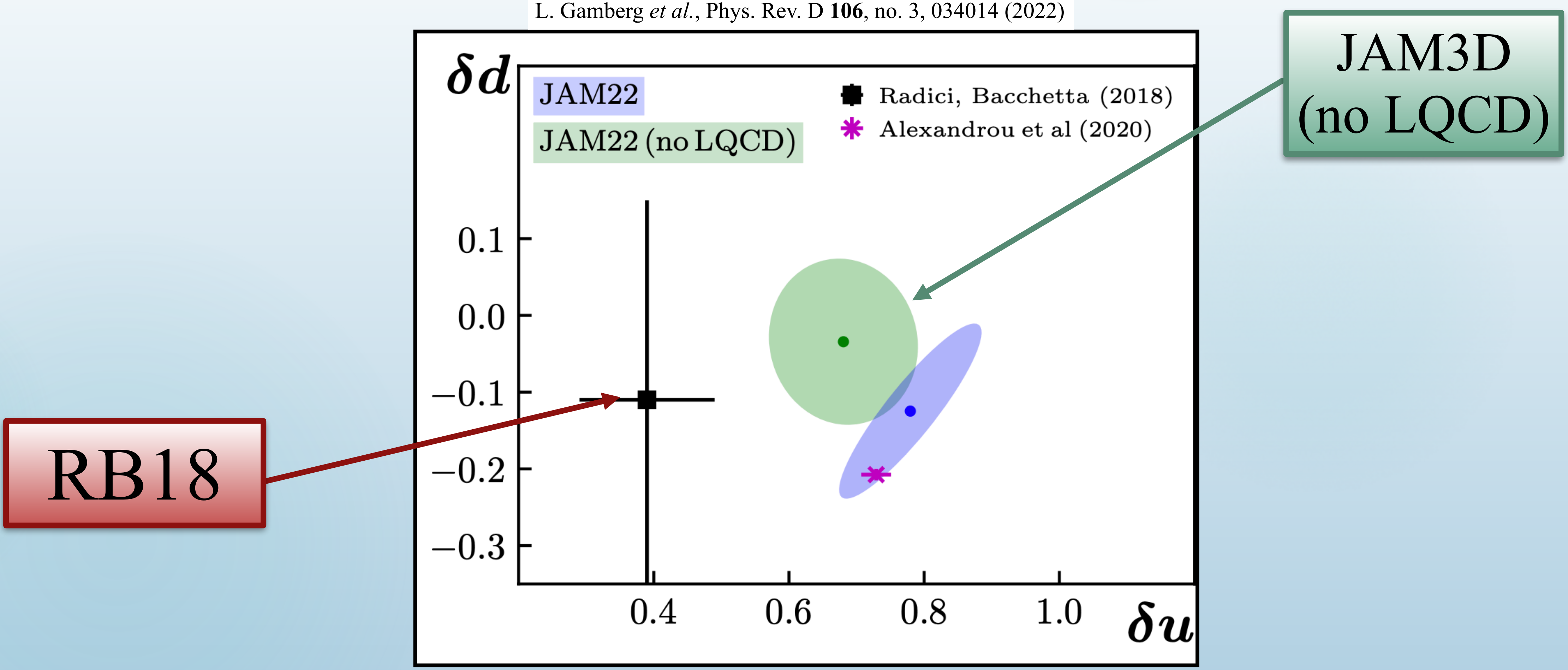

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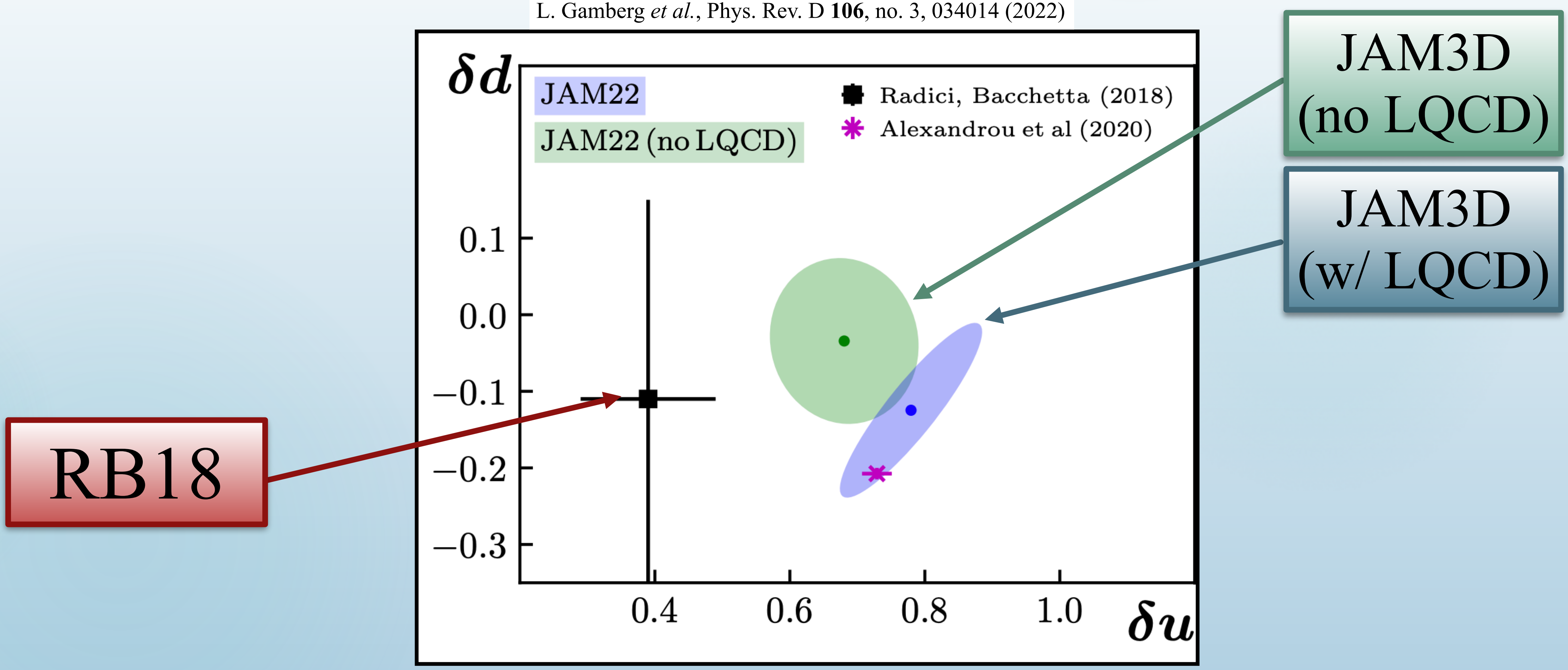

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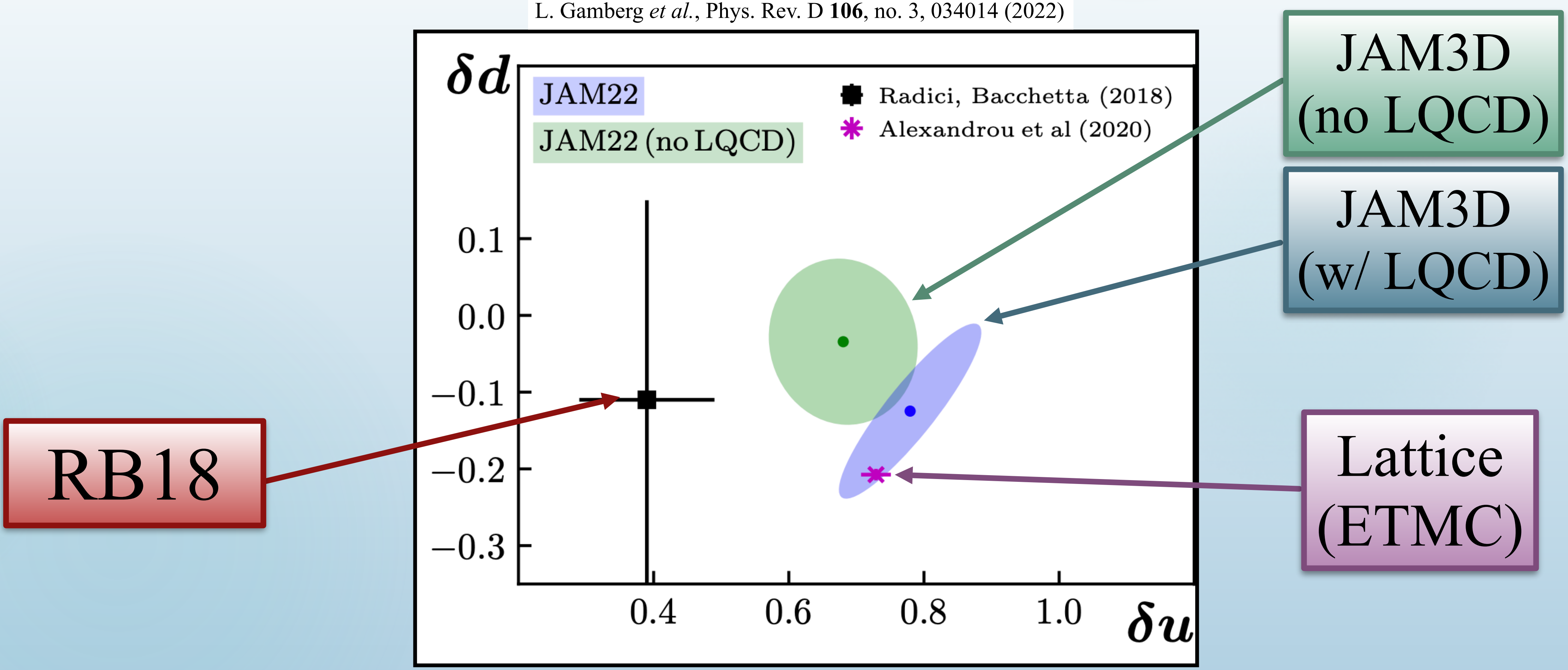

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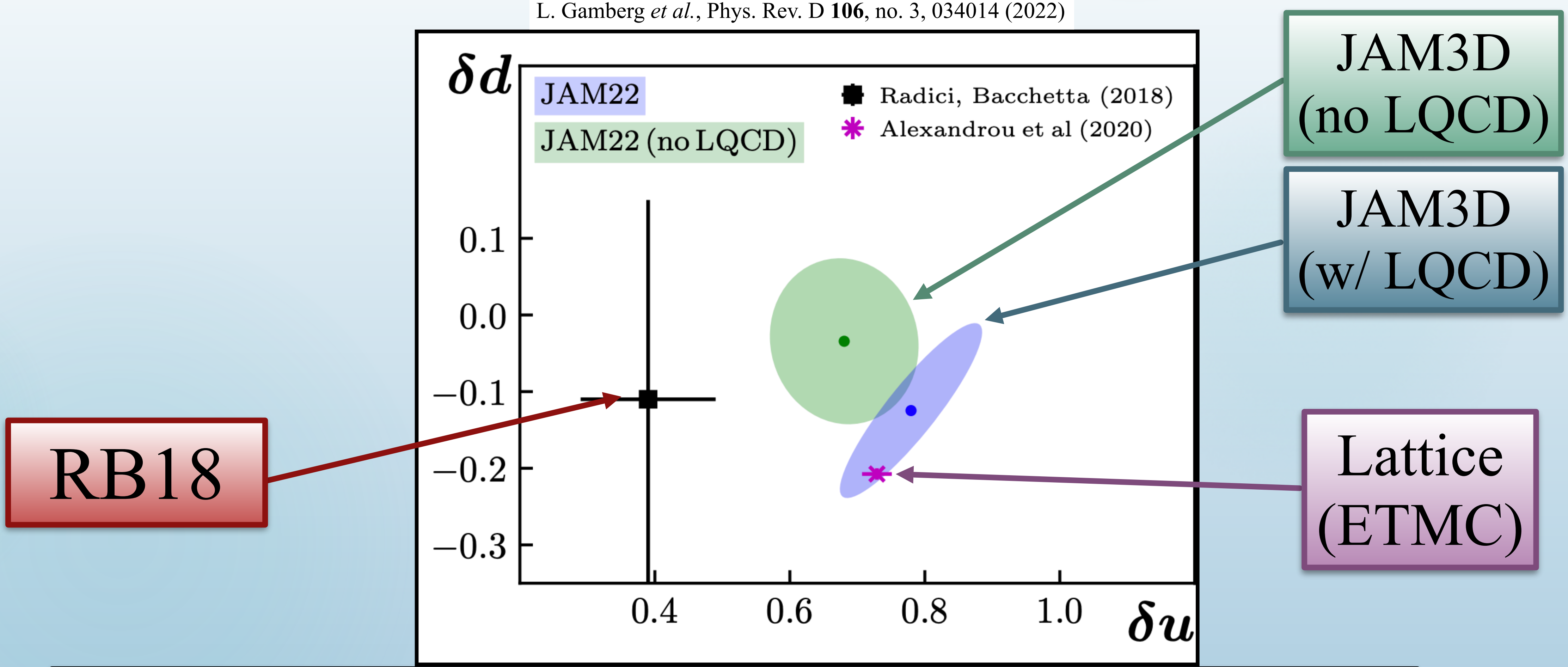
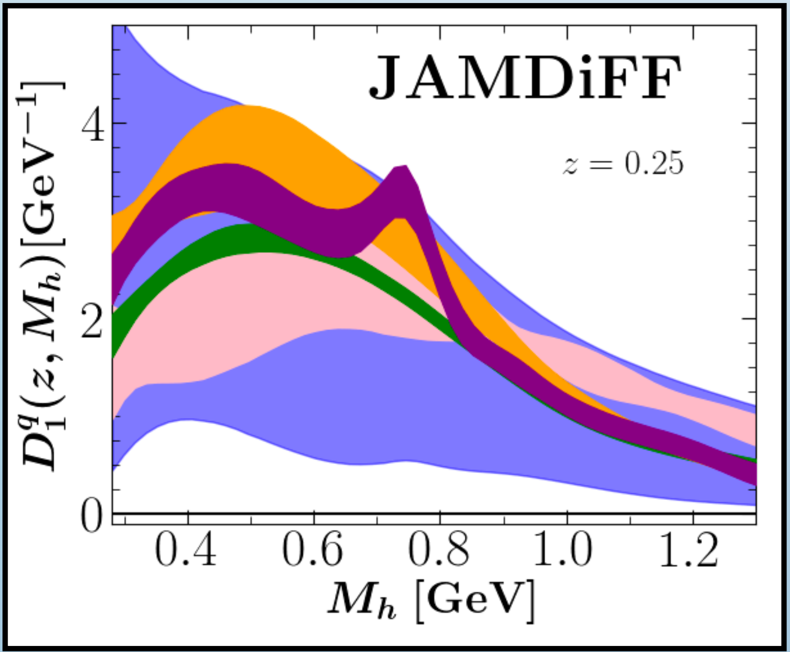
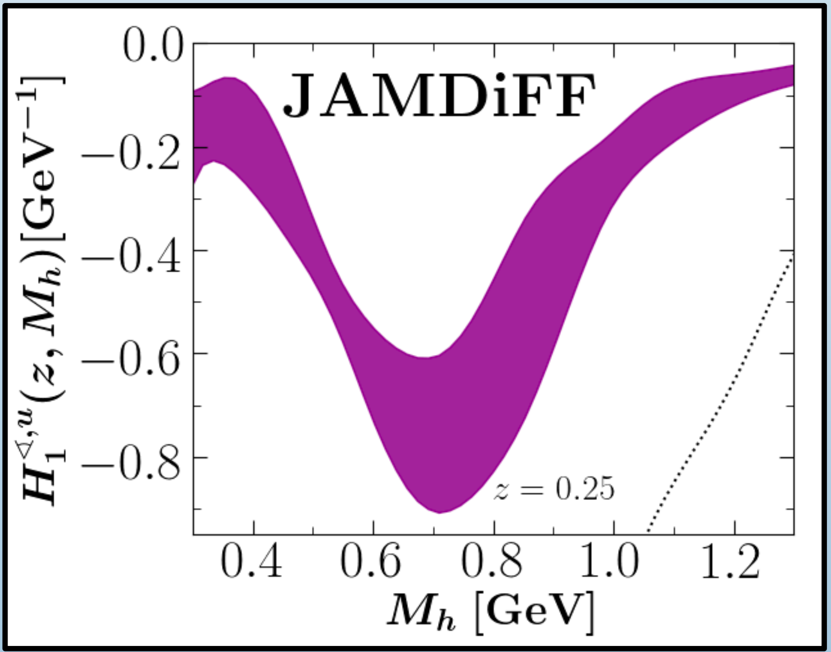


Large disagreements between three approaches…
Can this be solved?



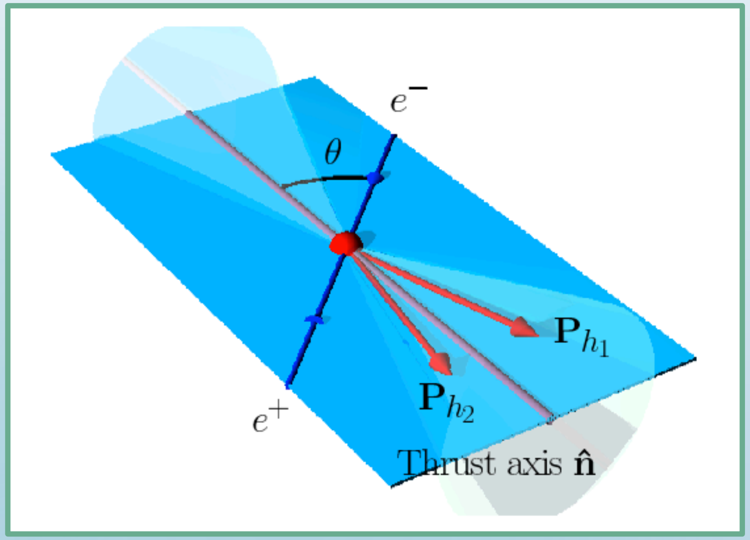
JAMDiFF
$z = 0.25$
$D_1^q(z, M_h)\,[\mathrm{GeV}^{-1}]$
M_h [GeV]



JAMDiFF
$z = 0.25$
$H_1^{\sphericalangle,u}(z, M_h)\,[\mathrm{GeV}^{-1}]$
M_h [GeV]

Observables for DiFFs

SIA Cross Section

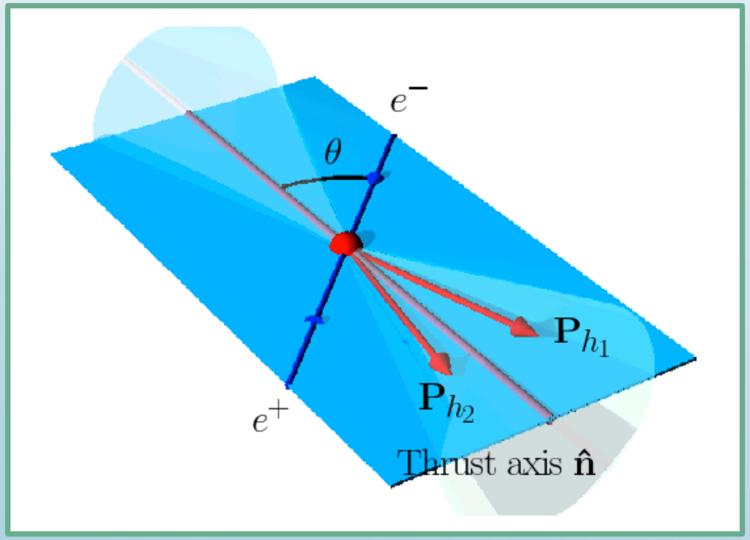


R. Seidl *et al.*, Phys. Rev. D **96**, no. 3, 032005 (2017)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}z\,\mathrm{d}M_h} = \frac{4\pi\alpha_{\mathrm{em}}^2}{s}\sum_q e_q^2\, D_1^q(z, M_h)$$

Observables for DiFFs

SIA Cross Section



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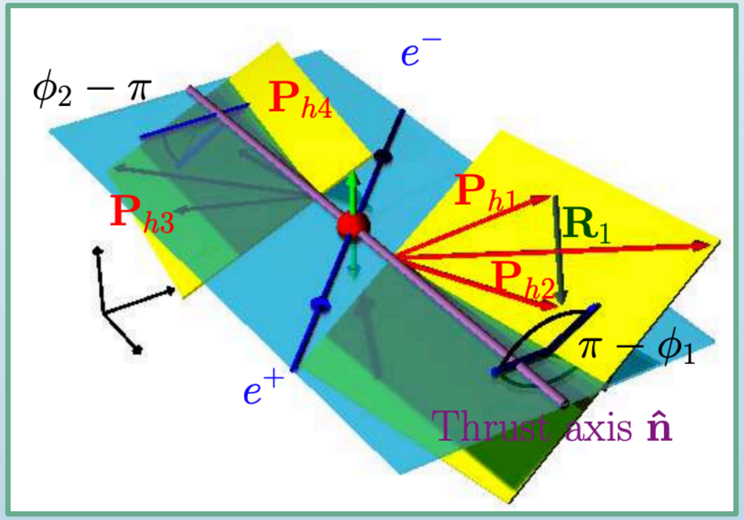
SIA Artru-Collins Asymmetry

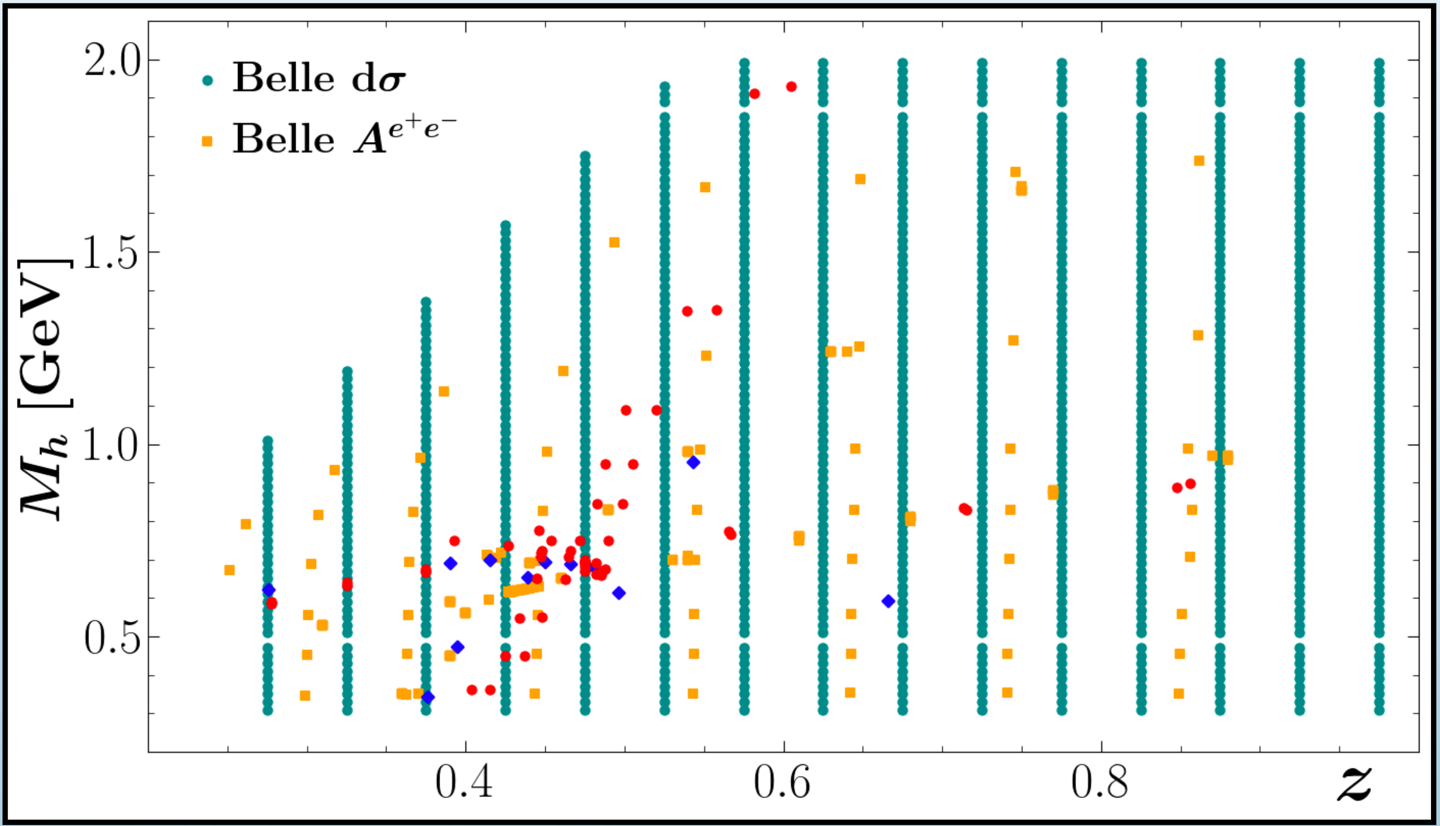


A. Vossen *et al.*, Phys. Rev. Lett. **107**, 072004 (2011)

$$A^{e^+e^-}(z, M_h, \bar{z}, \overline{M}_h) = \frac{\sin^2\theta \sum_q e_q^2\, H_1^{\sphericalangle,q}(z, M_h) H_1^{\sphericalangle,\bar{q}}(\bar{z}, \overline{M}_h)}{(1+\cos^2\theta)\sum_q e_q^2\, D_1^q(z, M_h) D_1^{\bar{q}}(\bar{z}, \overline{M}_h)}$$

Data for DiFFs

SIA cross section	Belle	1121 points
SIA Artru-Collins	Belle	183 points



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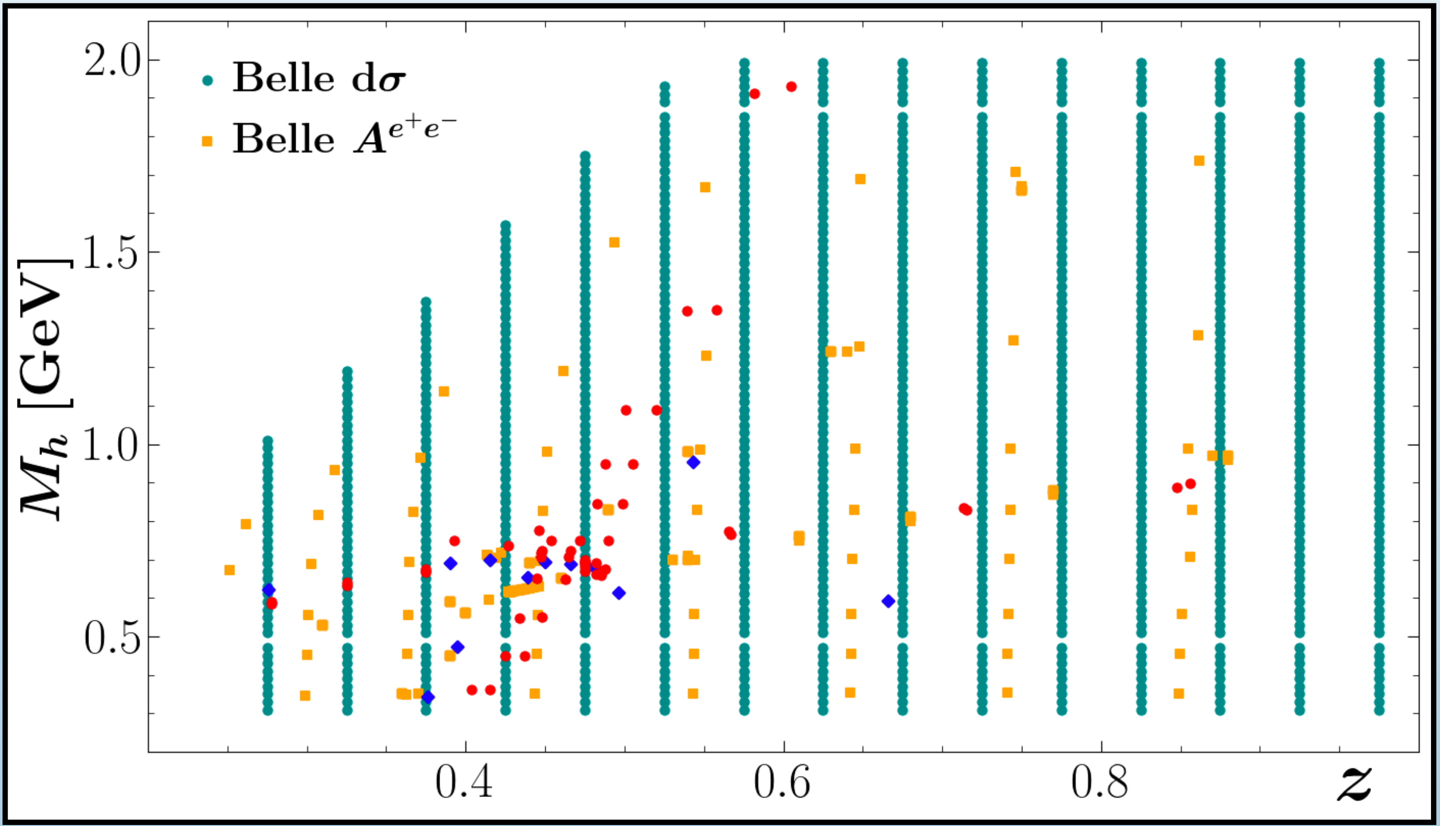

$\pi^+\pi^-$ DiFFs

$$D_1^u = D_1^d = D_1^{\bar{u}} = D_1^{\bar{d}},$$

$$D_1^s = D_1^{\bar{s}}, \quad D_1^c = D_1^{\bar{c}}, \quad D_1^b = D_1^{\bar{b}},$$

5 independent functions (w/ D_1^g)

[supplement with PYTHIA data]

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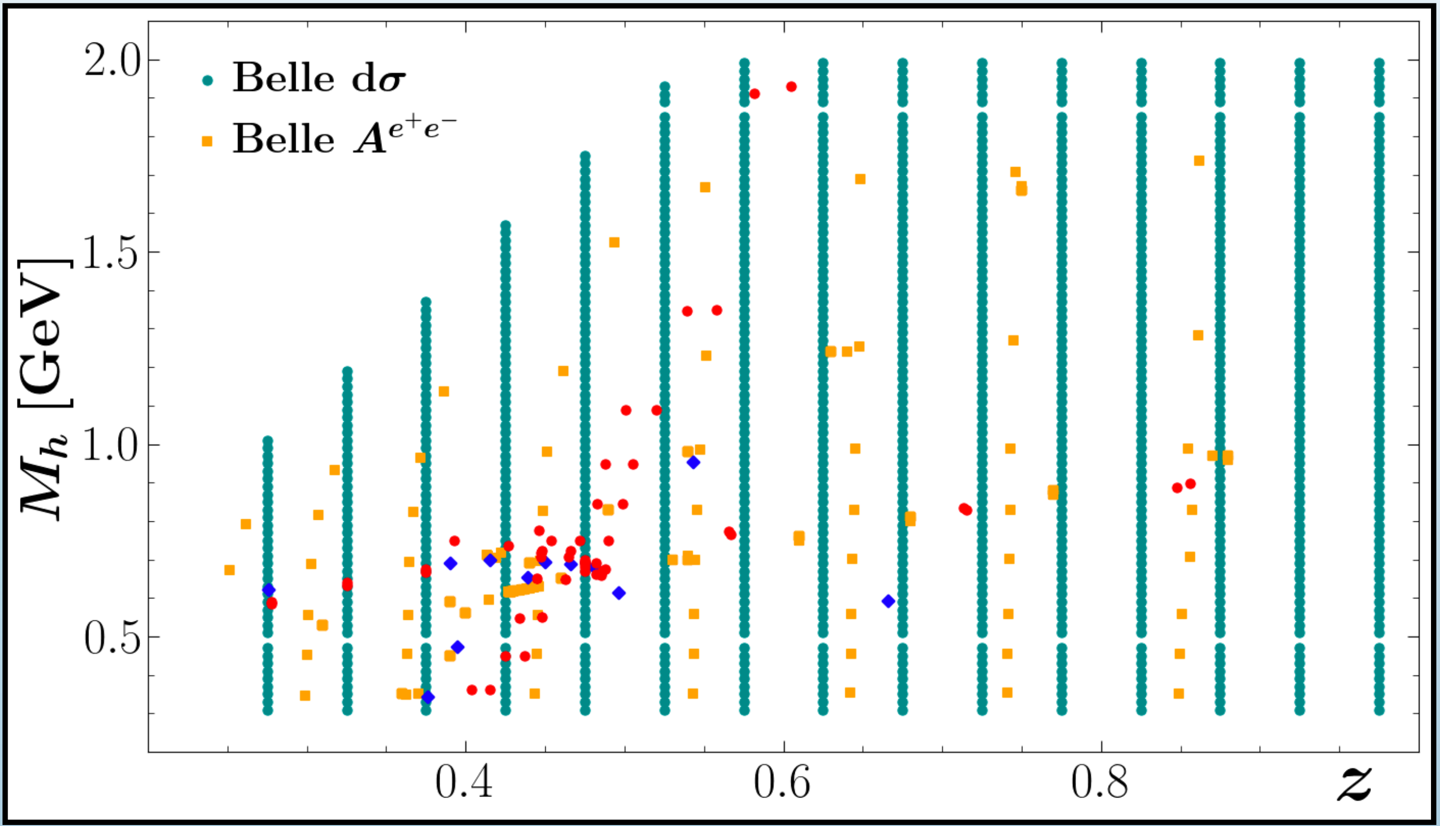

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[supplement with PYTHIA data]

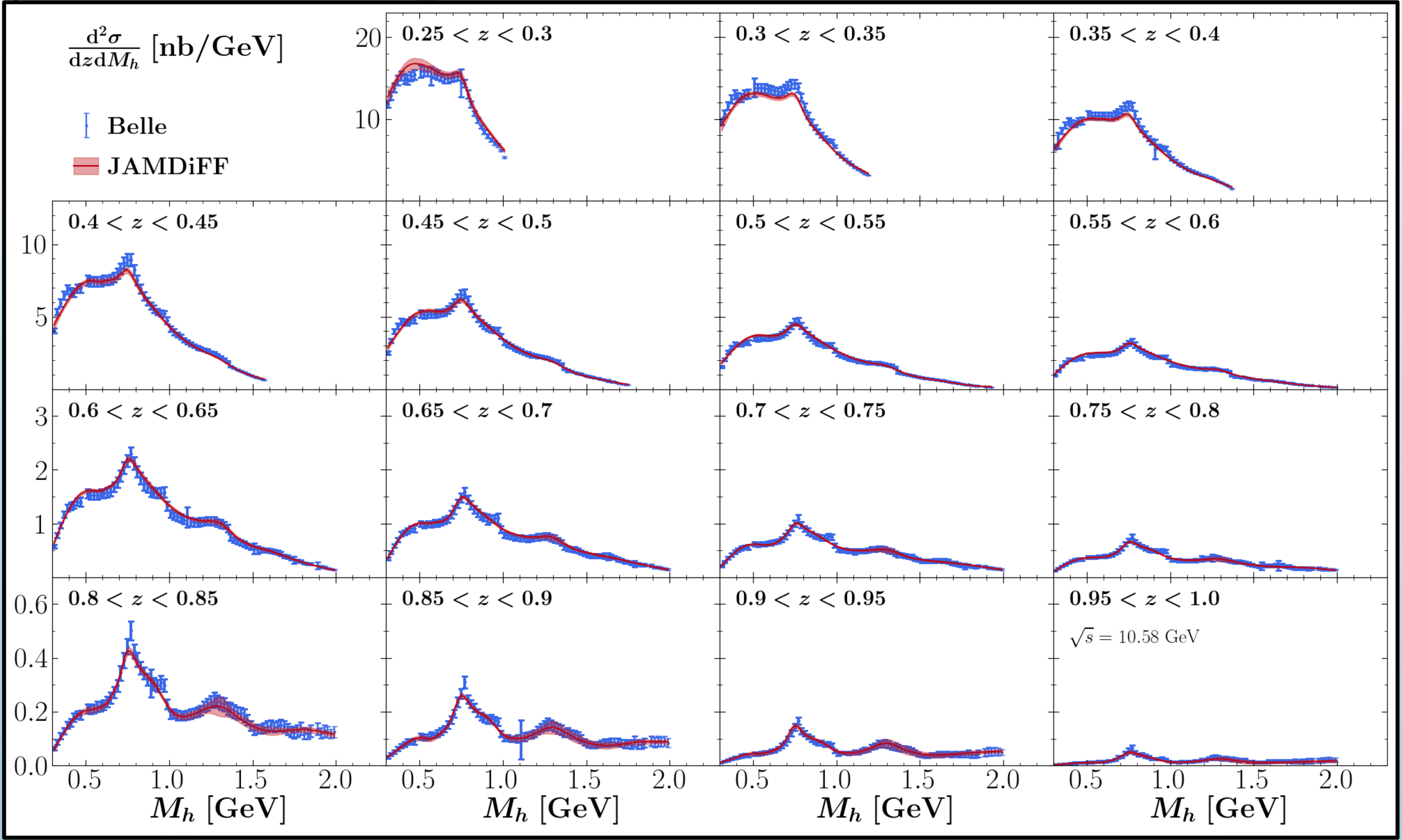
$$H_1^{\sphericalangle,u} = -H_1^{\sphericalangle,d} = -H_1^{\sphericalangle,\bar{u}} = H_1^{\sphericalangle,\bar{d}},$$

$$H_1^{\sphericalangle,s} = -H_1^{\sphericalangle,\bar{s}} = H_1^{\sphericalangle,c} = -H_1^{\sphericalangle,\bar{c}} = 0,$$

1 independent function

A. Courtoy et al., Phys. Rev. D **85**, 114023 (2012)

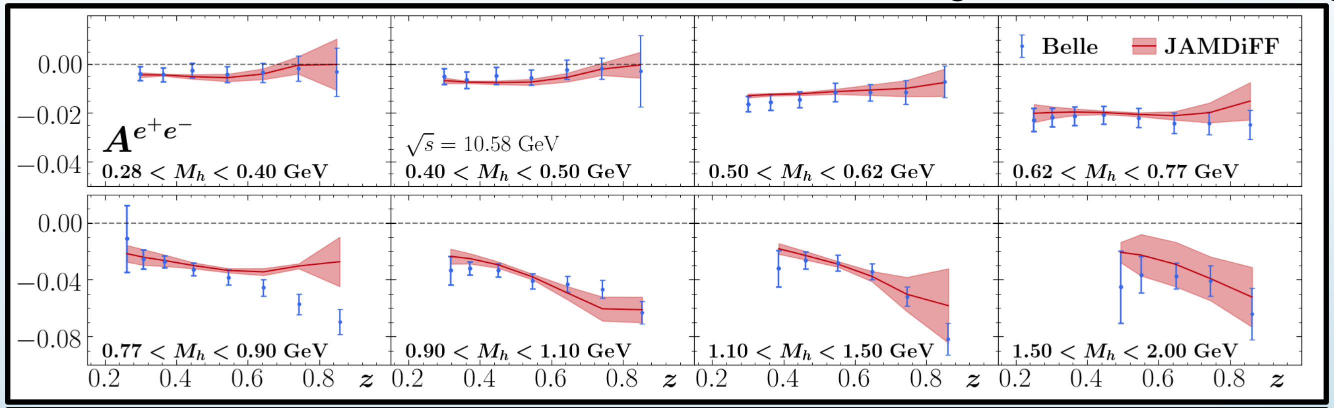
Quality of Fit (Unpolarized Cross Section)



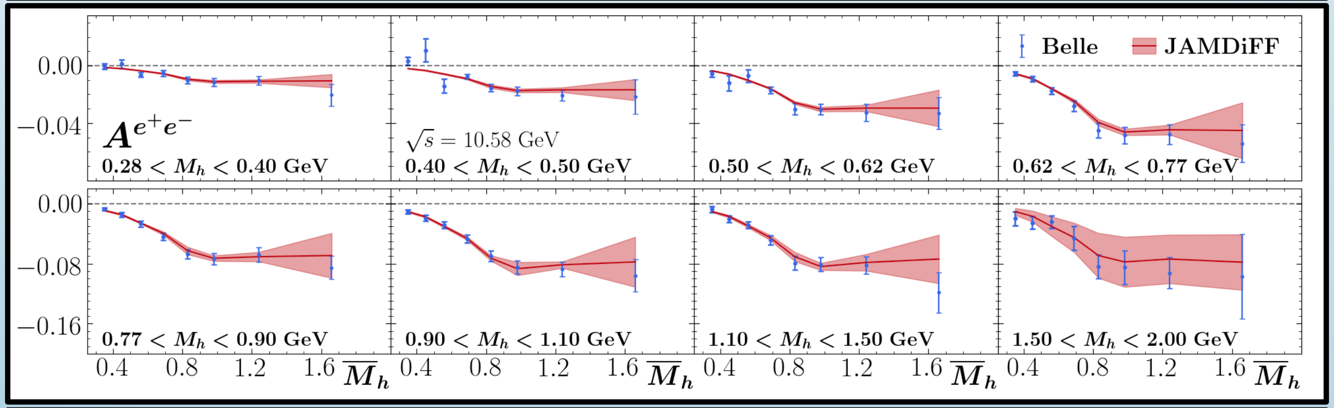
R. Seidl *et al.*, Phys. Rev. D **96**, 032005 (2017)

Quality of Fit (Artru-Collins Asymmetry)

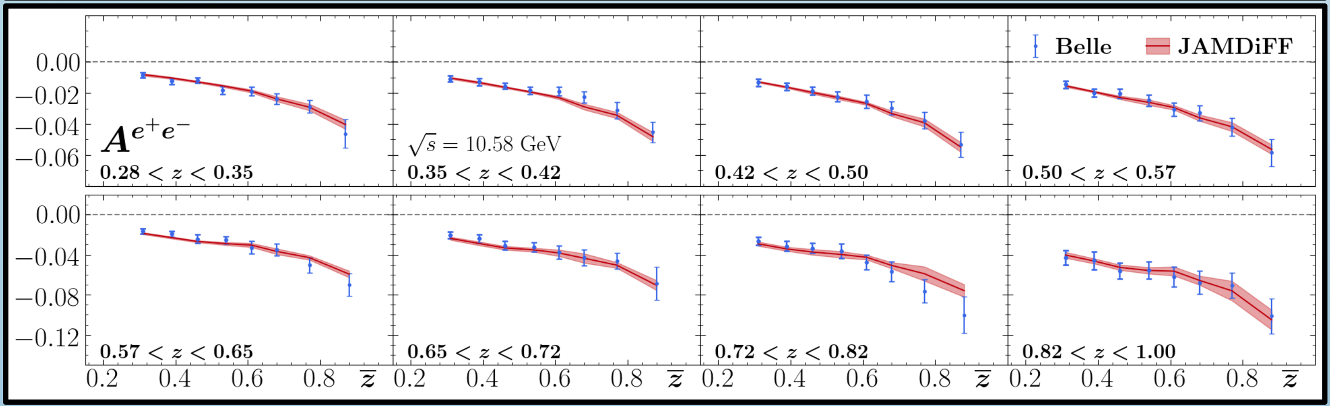
(z, M_h) binning



$(M_h, \overline{M}_h)$ binning

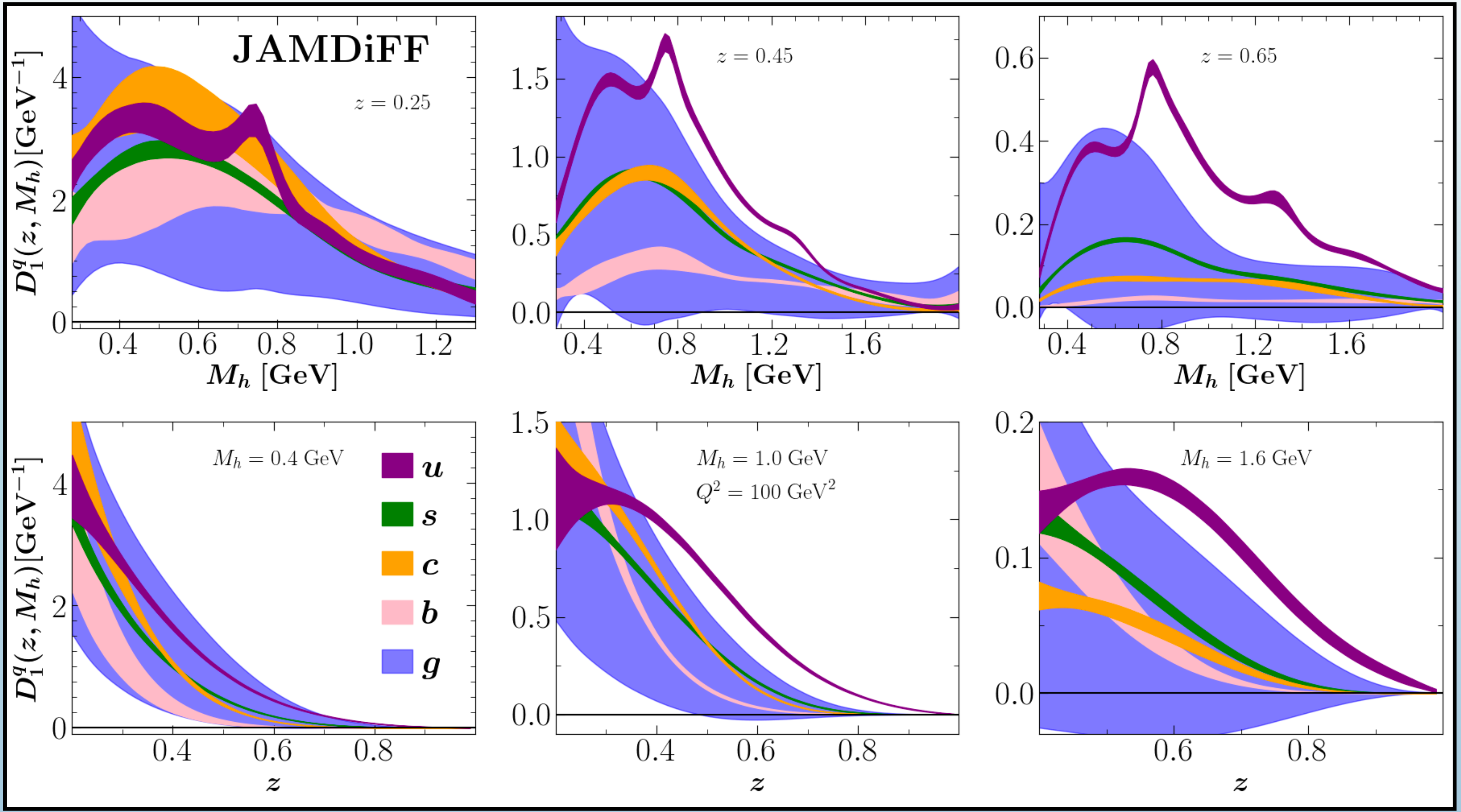


$(z, \bar{z})$ binning



A. Vossen *et al.*, Phys. Rev. Lett. **107**, 072004 (2011)

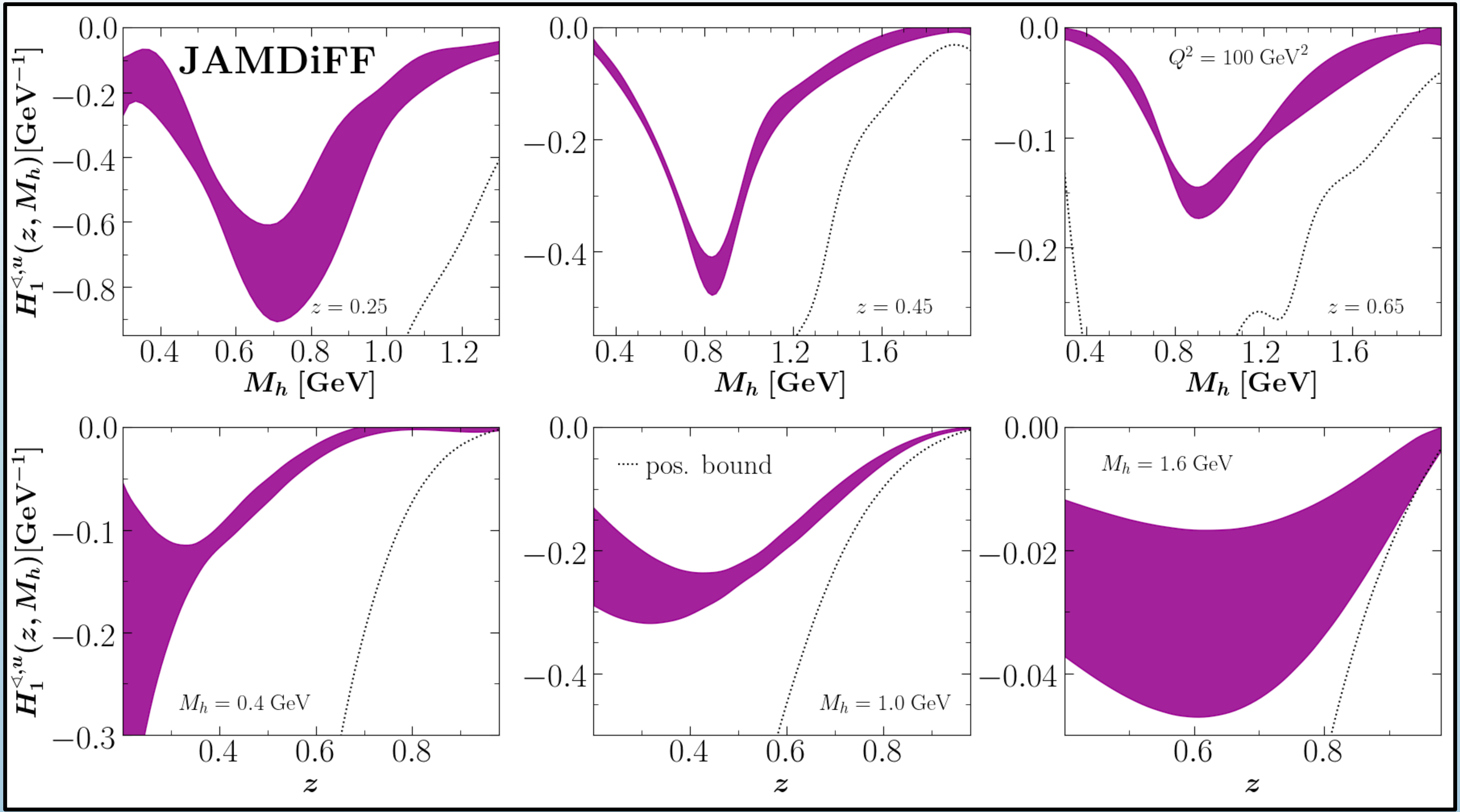
Extracted DiFFs



Bound: $D_1^q > 0$

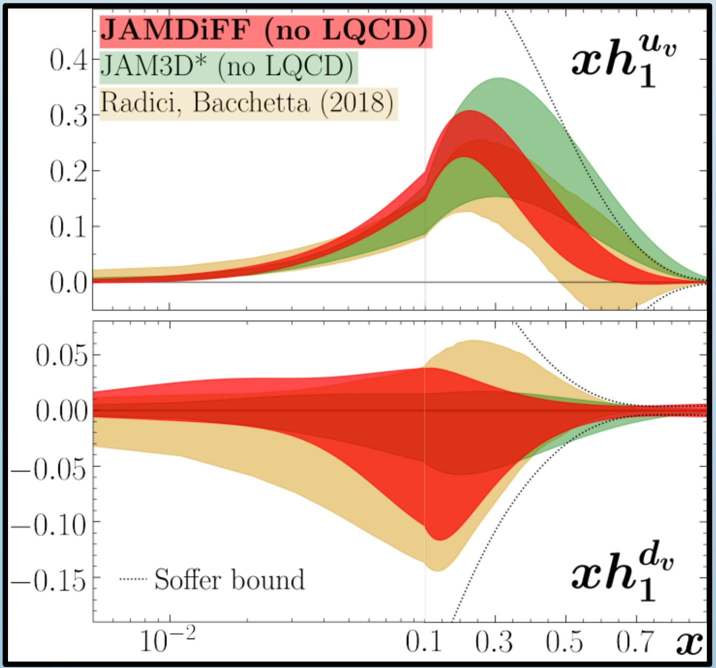
A. Bacchetta and M. Radici, Phys. Rev. D **67**, 094002 (2003)

Extracted IFFs

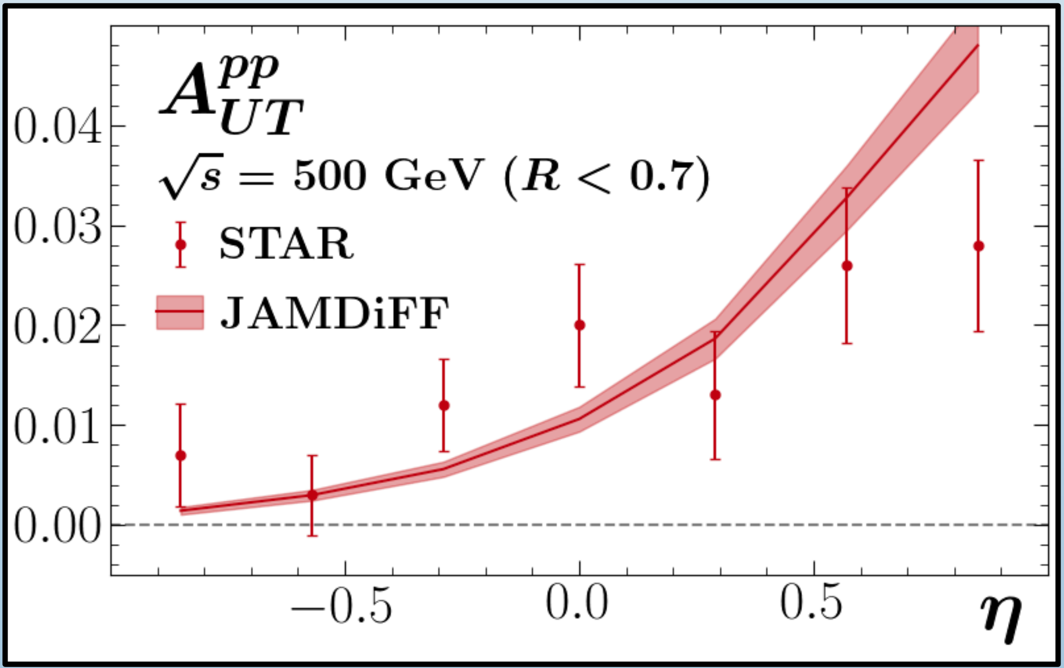


Bound:
$|H_1^{\sphericalangle,q}| < D_1^q$

A. Bacchetta and M. Radici, Phys. Rev. D **67**, 094002 (2003)



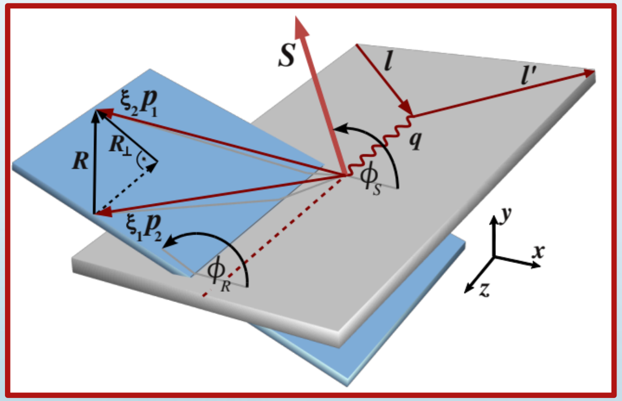
JAMDiFF (no LQCD)
JAM3D* (no LQCD)
Radici, Bacchetta (2018)
$xh_1^{u_v}$
$xh_1^{d_v}$
Soffer bound
x



A_{UT}^{pp}
$\sqrt{s}$ = 500 GeV ($R < 0.7$)
STAR
JAMDiFF
η

Observables for Transversity PDFs

SIDIS asymmetry (p and D)

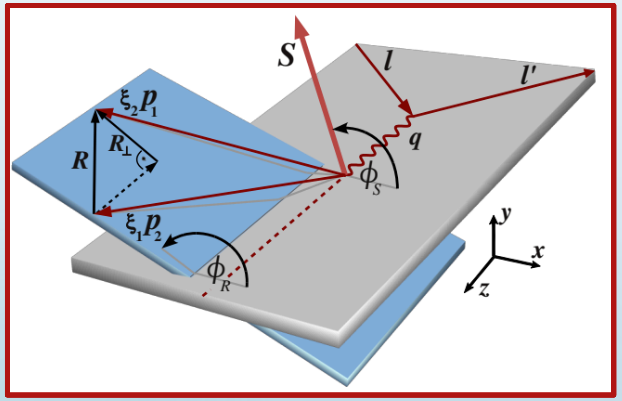


$$A_{UT}^{\text{SIDIS}} = c(y)\frac{\sum_q e_q^2\, h_1^q(x)\, H_1^{\sphericalangle,q}(z,M_h)}{\sum_q e_q^2\, f_1^q(x)\, D_1^q(z,M_h)}$$

C. Adolph *et al.*, Phys. Lett. B **713**, 10-16 (2012)

Observables for Transversity PDFs

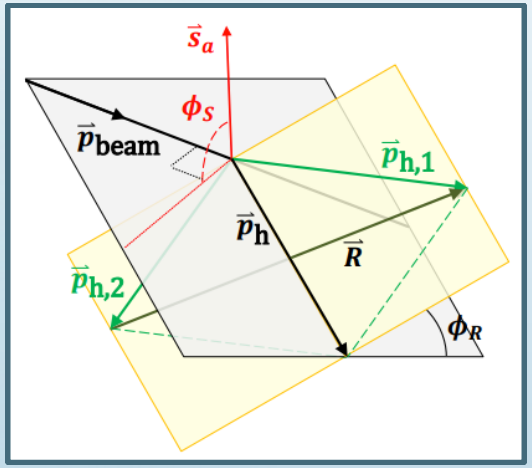
SIDIS asymmetry (*p* and *D*)



C. Adolph *et al.*, Phys. Lett. B **713**, 10-16 (2012)

$$A_{UT}^{\mathrm{SIDIS}} = c(y)\frac{\sum_q e_q^2\, h_1^q(x)\, H_1^{\sphericalangle,q}(z,M_h)}{\sum_q e_q^2\, f_1^q(x)\, D_1^q(z,M_h)}$$

pp Asymmetry



L. Adamczyk *et al.*, Phys. Rev. Lett. **115**, 242501 (2015)

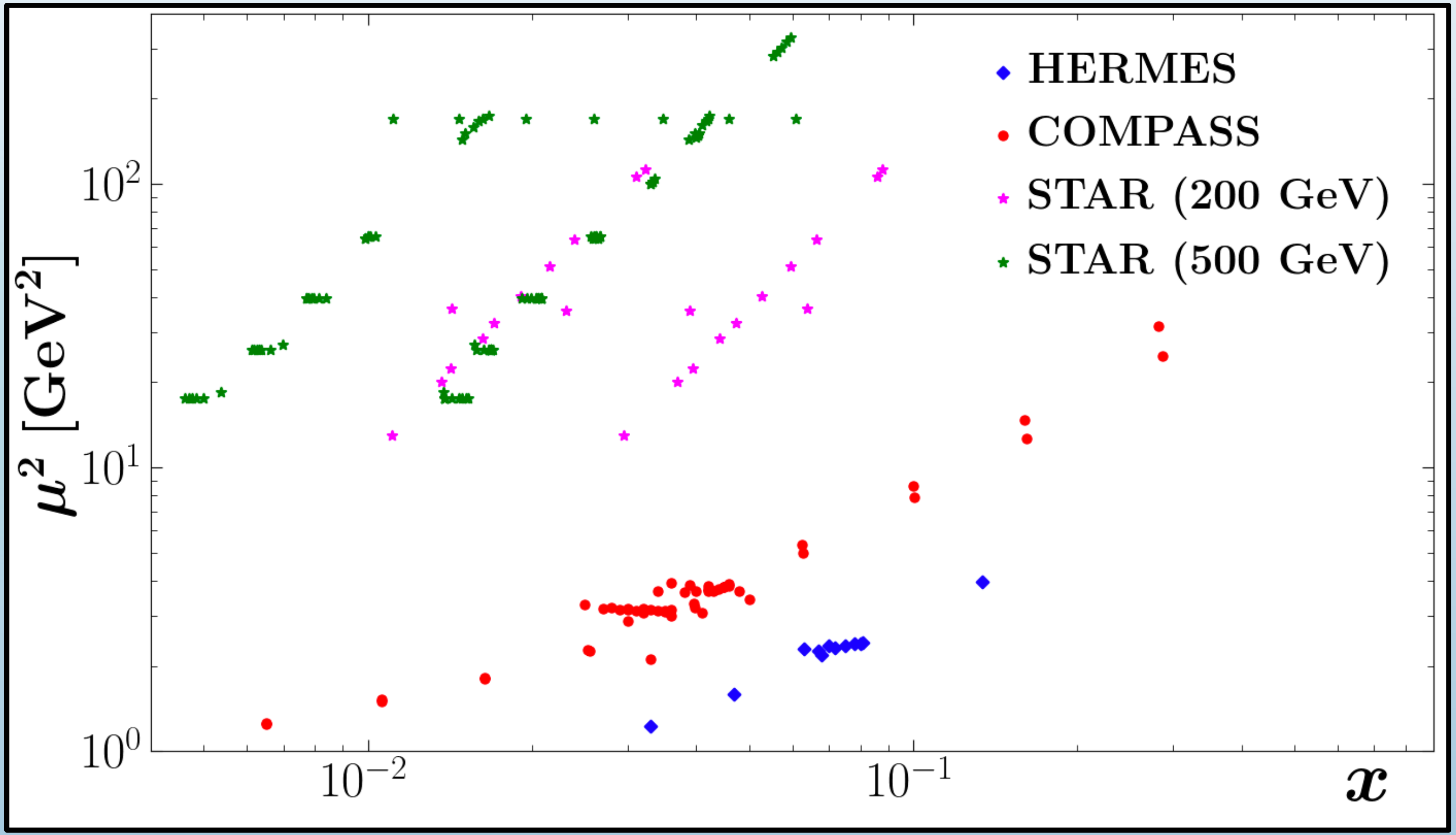
$$A_{UT}^{pp} = \frac{\mathscr{H}(M_h, P_{hT}, \eta)}{\mathscr{D}(M_h, P_{hT}, \eta)}$$

$$\mathscr{H}(M_h, P_{hT}, \eta) = 2P_{hT}\sum_i \sum_{a,b,c} \int_{x_a^{\min}}^1 \mathrm{d}x_a \int_{x_b^{\min}}^1 \frac{\mathrm{d}x_b}{z} f_1^a(x_a)\, h_1^b(x_b) \frac{\mathrm{d}\Delta\hat{\sigma}_{ab^\uparrow\to c^\uparrow d}}{\mathrm{d}\hat{t}} H_1^{\sphericalangle,c}(z, M_h)$$

$$\mathscr{D}(M_h, P_{hT}, \eta) = 2P_{hT}\sum_i \sum_{a,b,c} \int_{x_a^{\min}}^1 \mathrm{d}x_a \int_{x_b^{\min}}^1 \frac{\mathrm{d}x_b}{z} f_1^a(x_a)\, f_1^b(x_b) \frac{\mathrm{d}\hat{\sigma}_{ab\to cd}}{\mathrm{d}\hat{t}} D_1^c(z, M_h)$$

Data for PDFs

SIDIS (p, D)	COMPASS, HERMES	64 points
Proton-Proton	STAR	269 points



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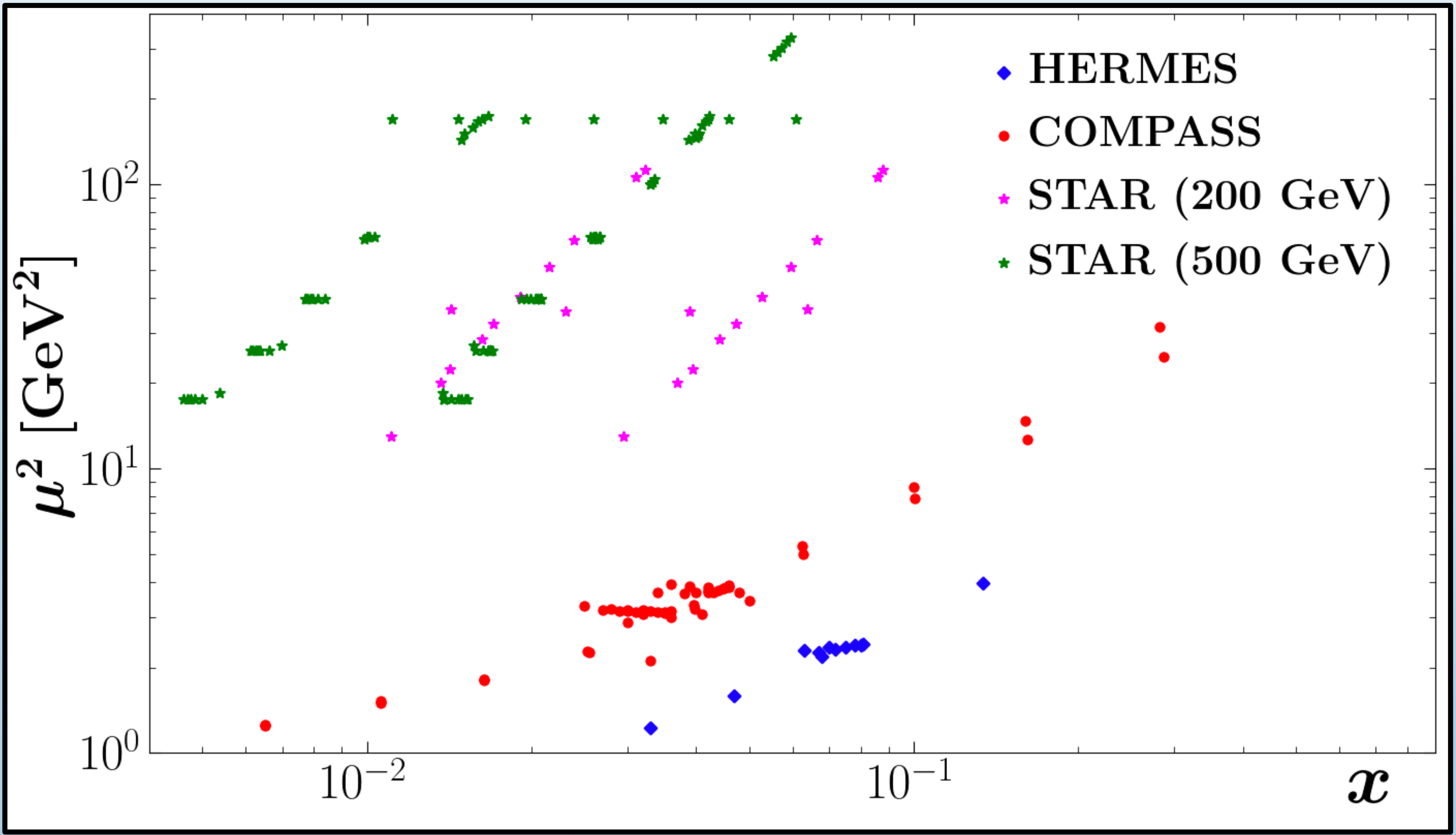

Parameterization Choices

3 independent observables
3 independent functions

$$h_1^{u_v}$$
$$h_1^{d_v}$$
$$h_1^{\bar{u}} = -h_1^{\bar{d}}$$

Data for PDFs

SIDIS (p, D)	COMPASS, HERMES	64 points
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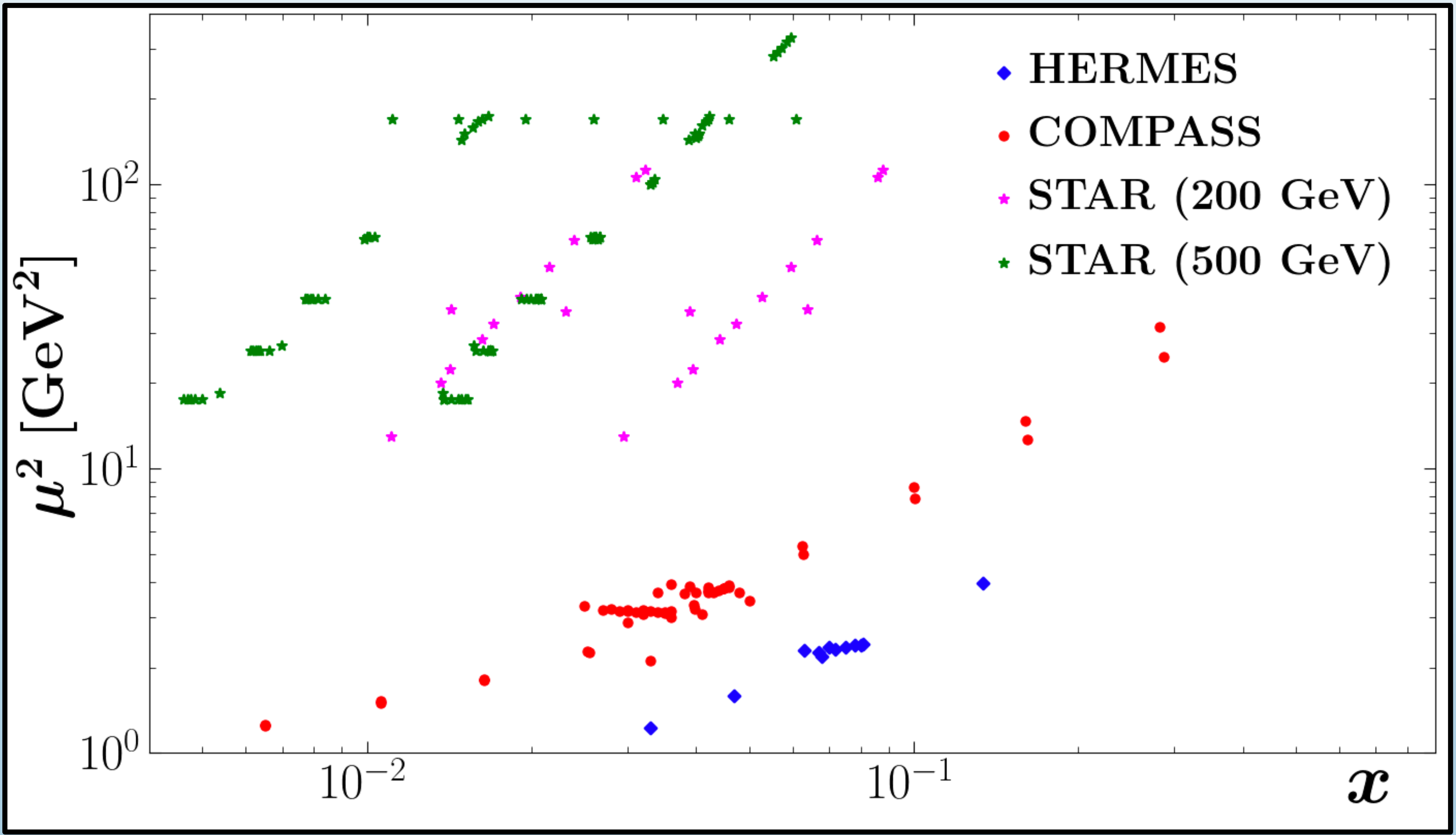

Parameterization Choices

3 independent observables
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$$h_1^{u_v}$$
$$h_1^{d_v}$$
$$h_1^{\bar{u}} = -\, h_1^{\bar{d}}$$

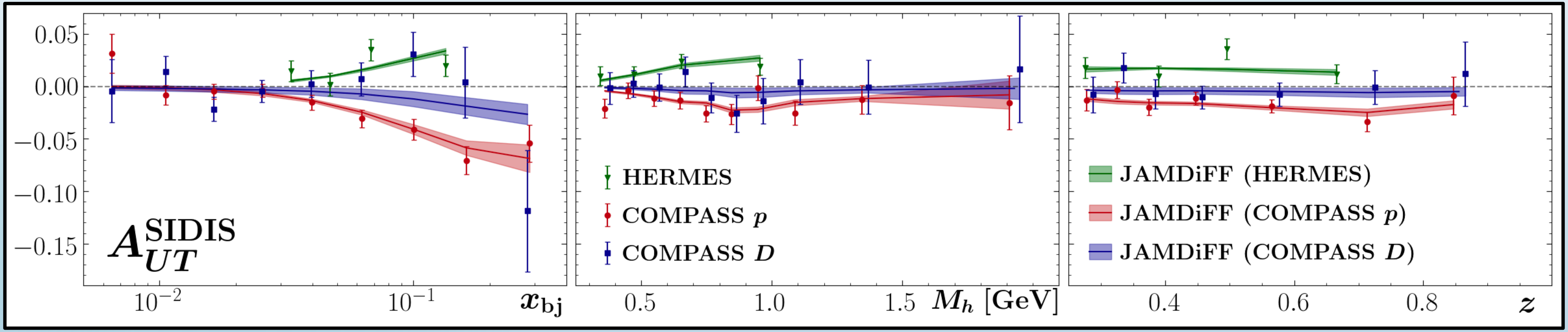
Prediction from large-N_c limit

P. V. Pobylitsa, arXiv:hep-ph/0301236 (2003)

Quality of Fit

Experiment	N_{dat}	χ^2_{red}
Belle (cross section)	1094	1.05
Belle (Artru-Collins)	183	0.78
HERMES	12	1.09
COMPASS (p)	26	0.75
COMPASS (D)	26	0.74
STAR (2015)	24	1.83
STAR (2018)	106	1.06
Total	1471	**1.02**

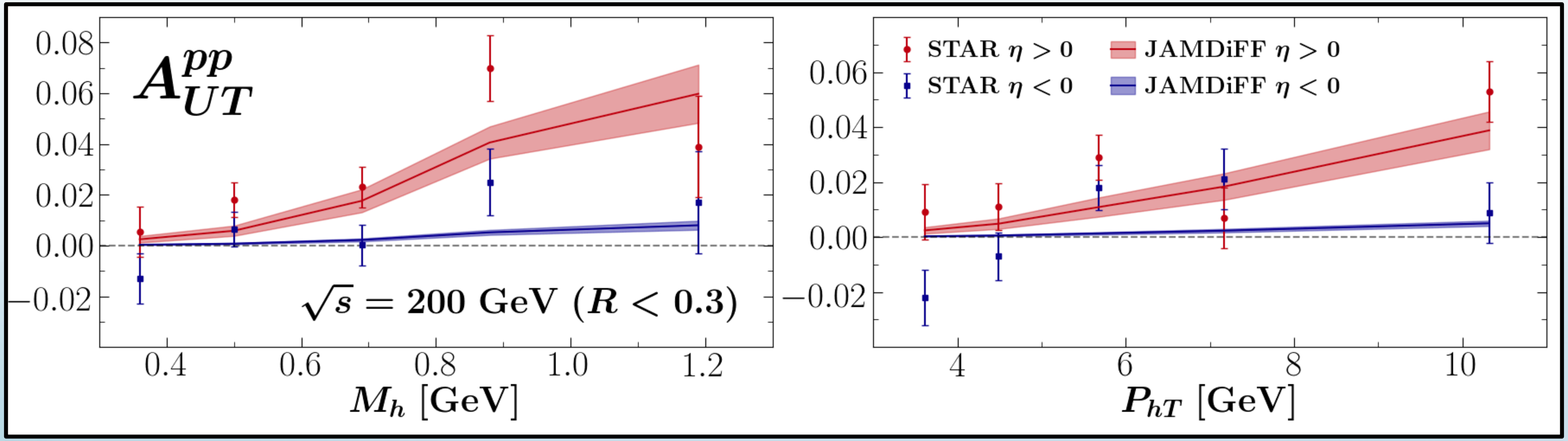
Quality of Fit (SIDIS)

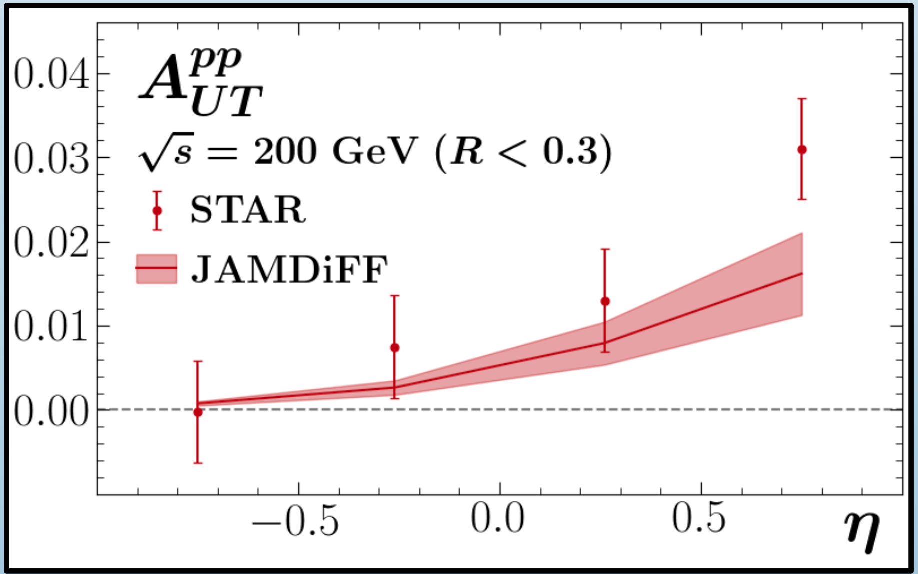


A. Airapetian *et al.*, JHEP **06**, 017 (2008)

COMPASS, arXiv:hep-ph/2301.02013 (2023)

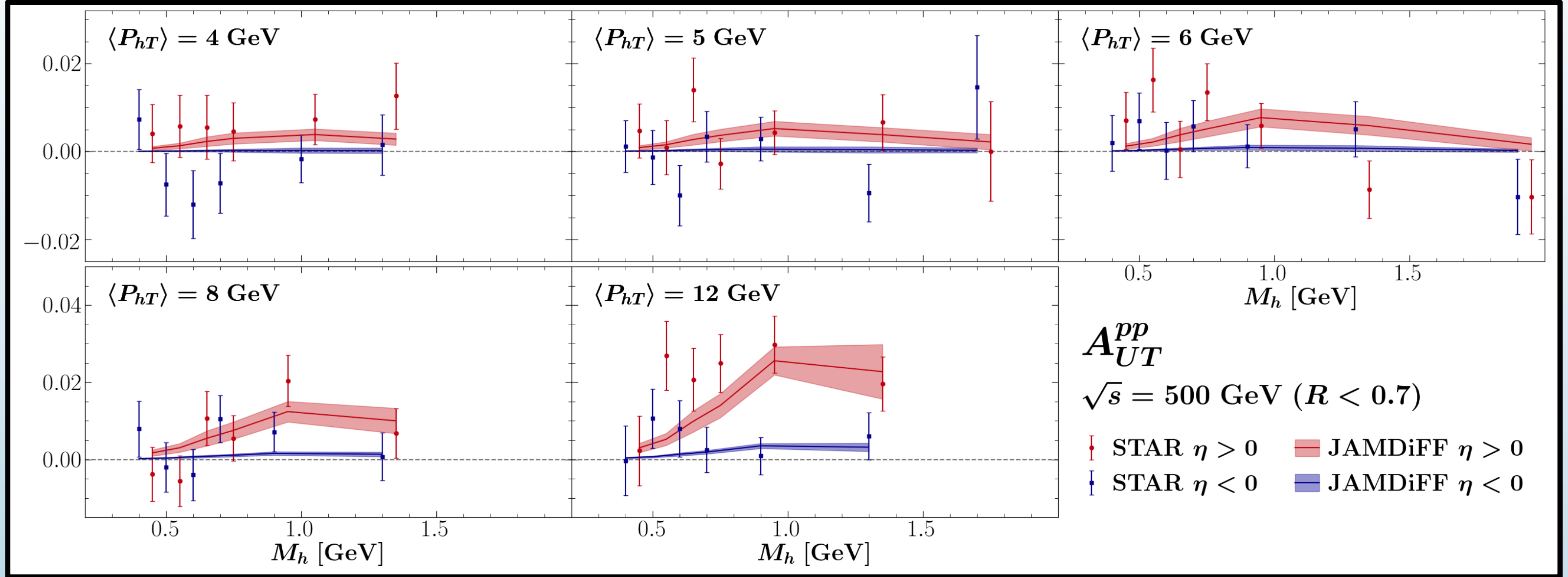
Quality of Fit (STAR $\sqrt{s} = 200$ GeV)

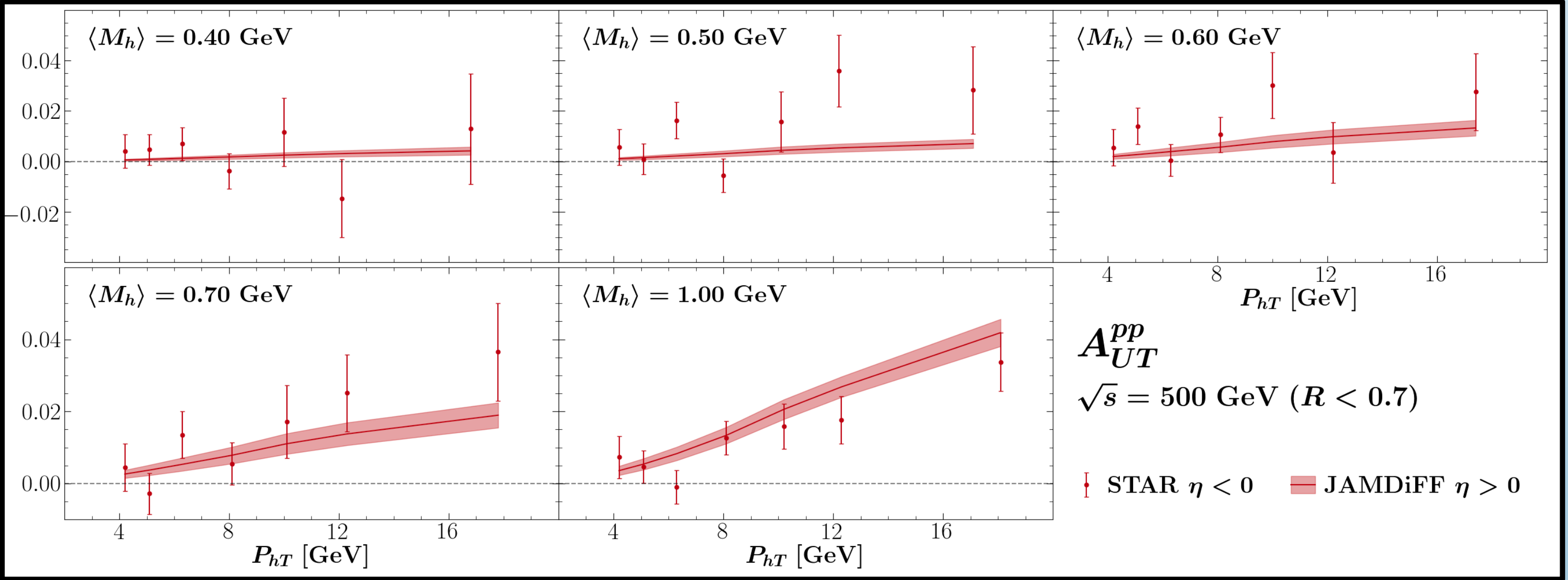


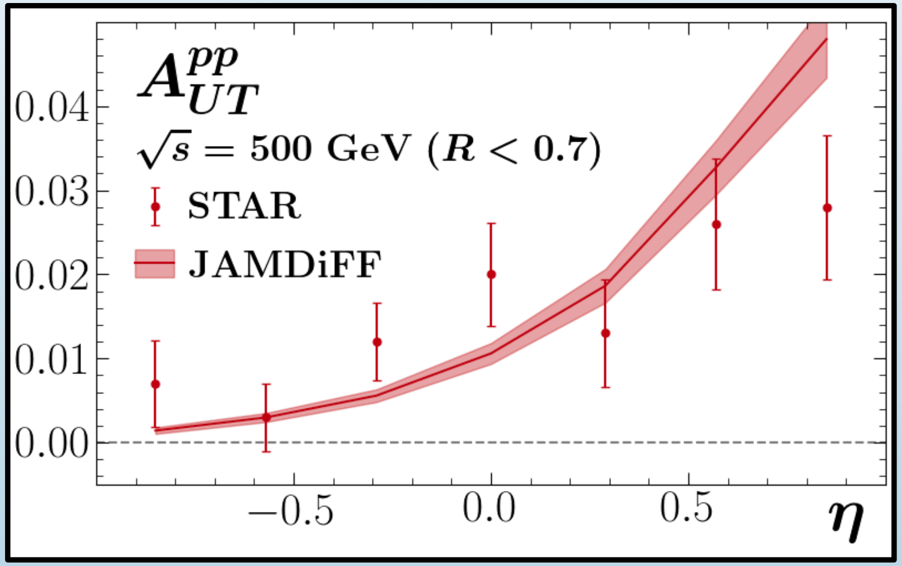


L. Adamczyk *et al.*, Phys. Rev. Lett. **115**, 24501 (2015)

Quality of Fit (STAR $\sqrt{s} = 500$ GeV)

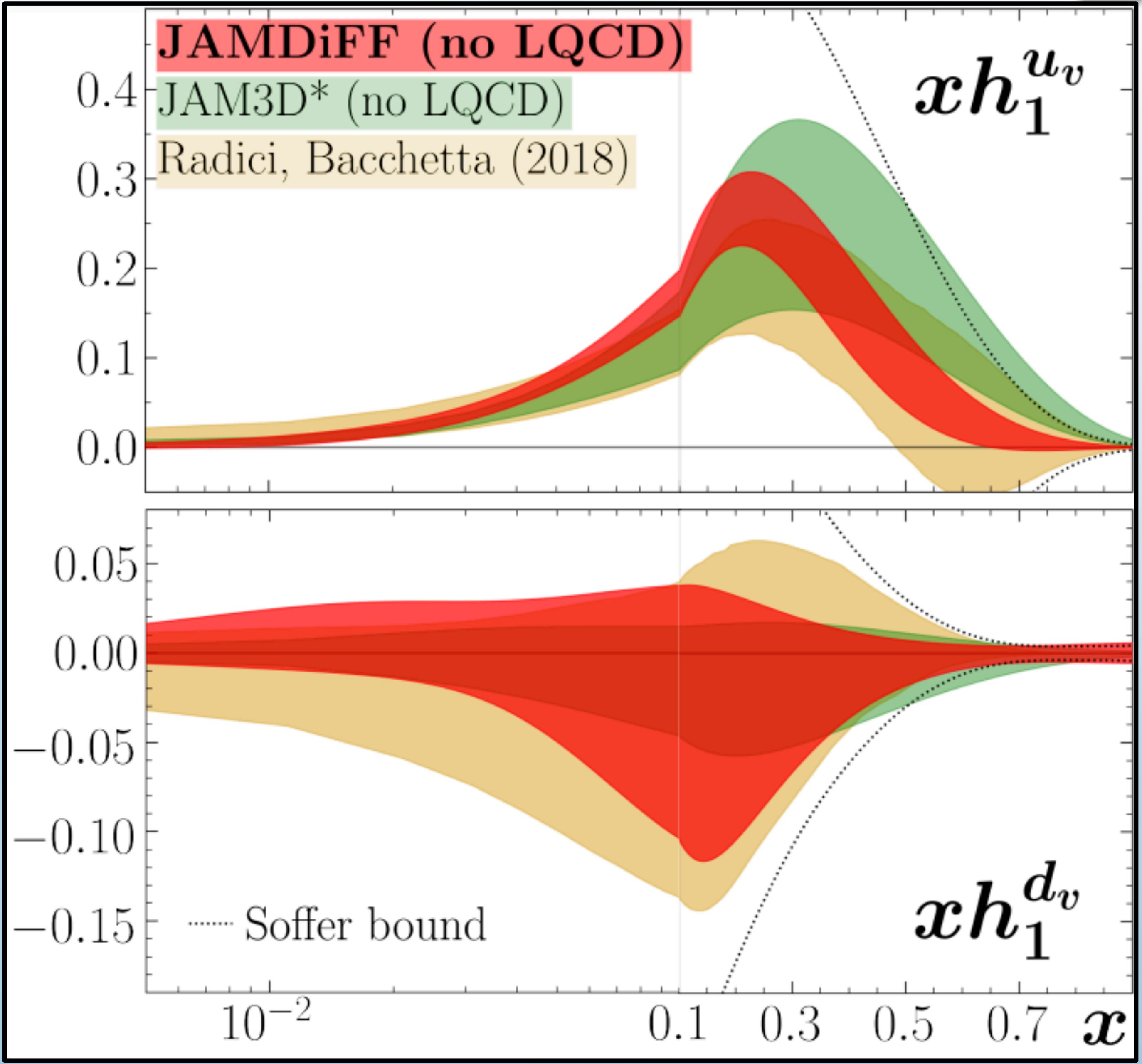






L. Adamczyk *et al.*, Phys. Rev. B **780**, 332-339 (2018)

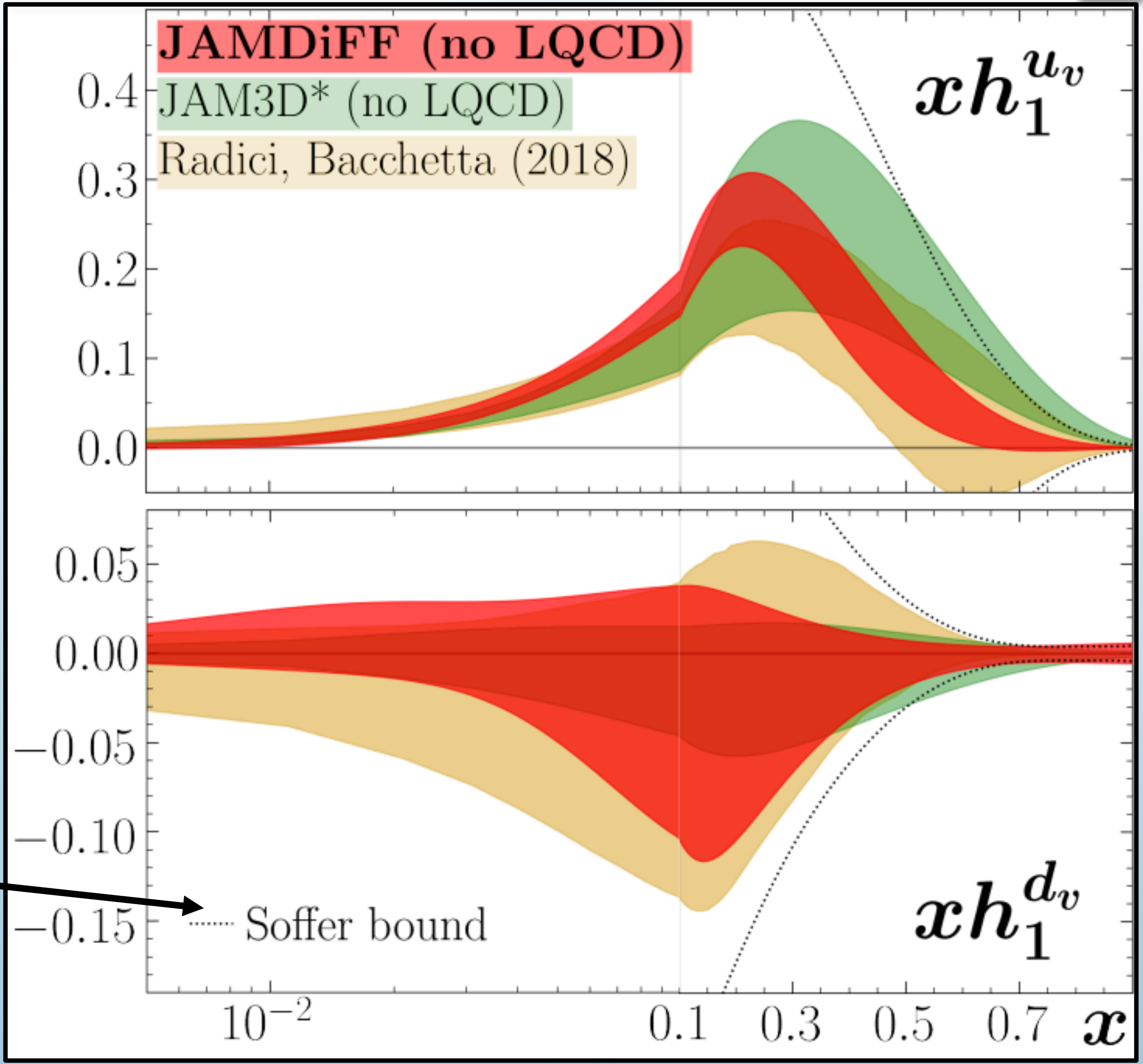
Transversity PDFs



Transversity PDFs

Soffer Bound: $|h_1^q| < \frac{1}{2}\left[f_1^q + g_1^q\right]$

J. Soffer, Phys. Rev. Lett. **74**, 1292-1294 (1995)

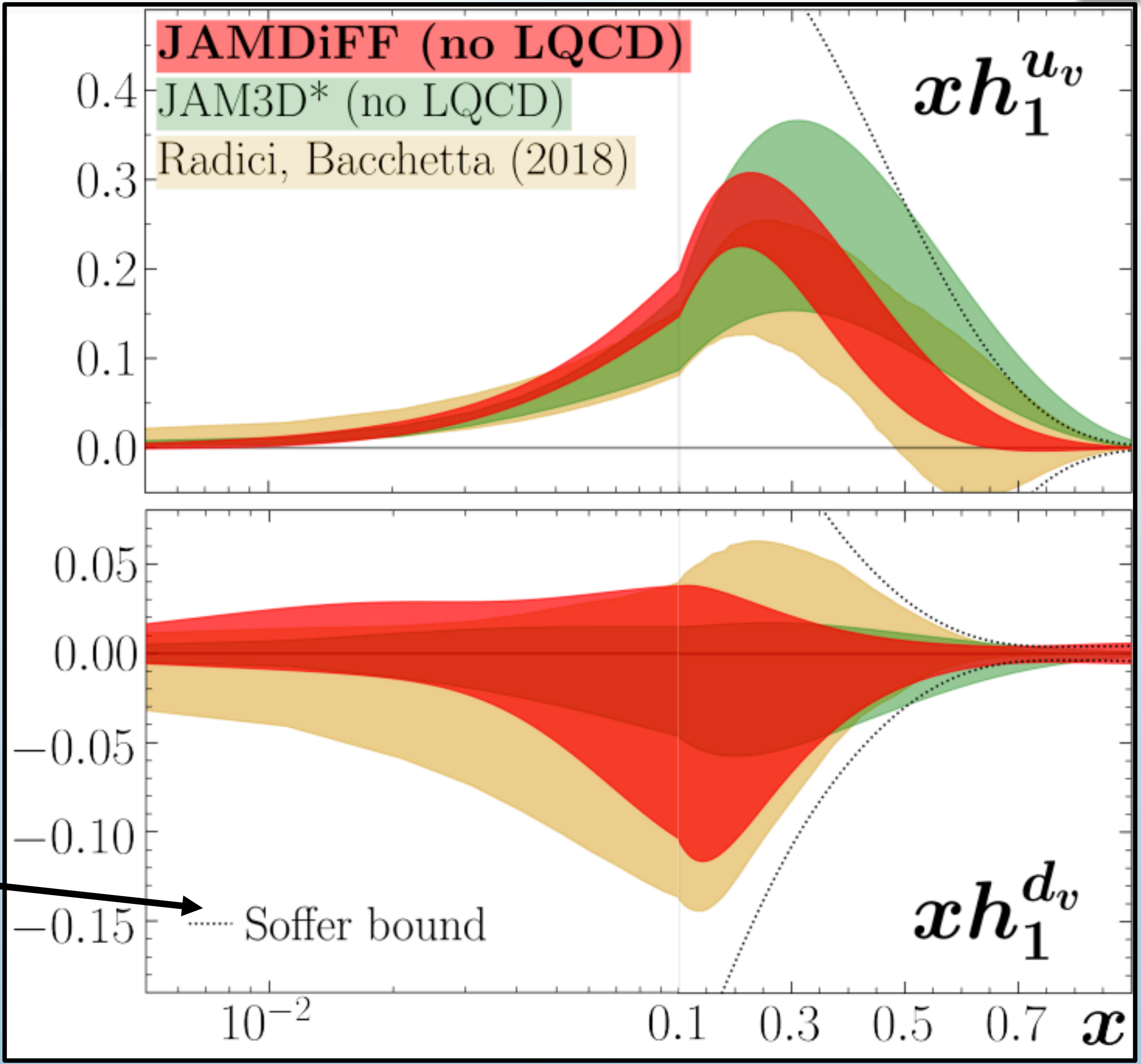


Transversity PDFs

JAM3D* = JAM3D-22 (no LQCD)
+ Antiquarks w/ $\bar{u} = -\bar{d}$
+ small-x constraint (see slide 23)

Soffer Bound: $|h_1^q| < \frac{1}{2}[f_1^q + g_1^q]$

J. Soffer, Phys. Rev. Lett. **74**, 1292-1294 (1995)



Transversity PDFs

JAM3D* = JAM3D-22 (no LQCD)
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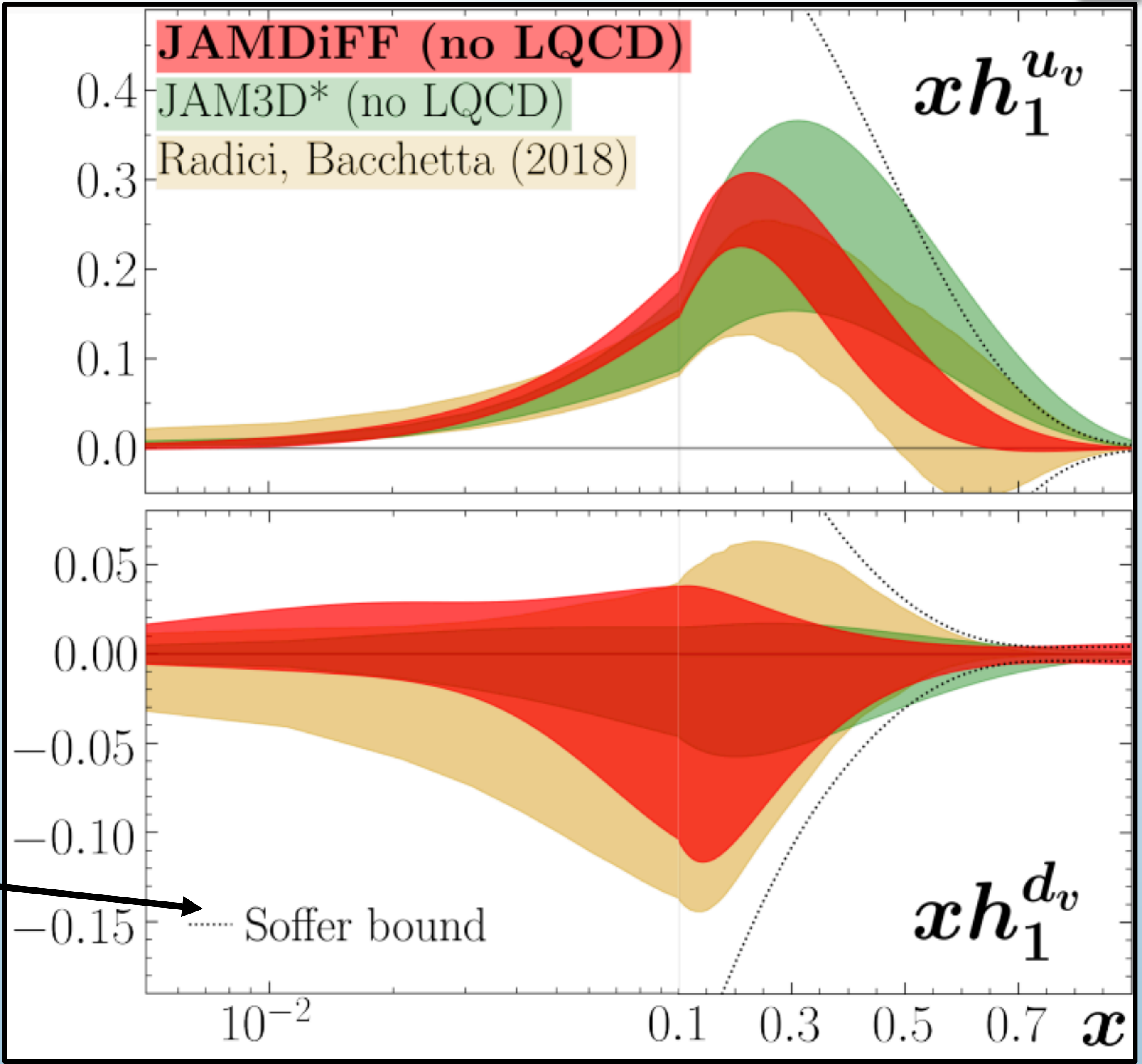
+ small-x constraint (see slide 23)

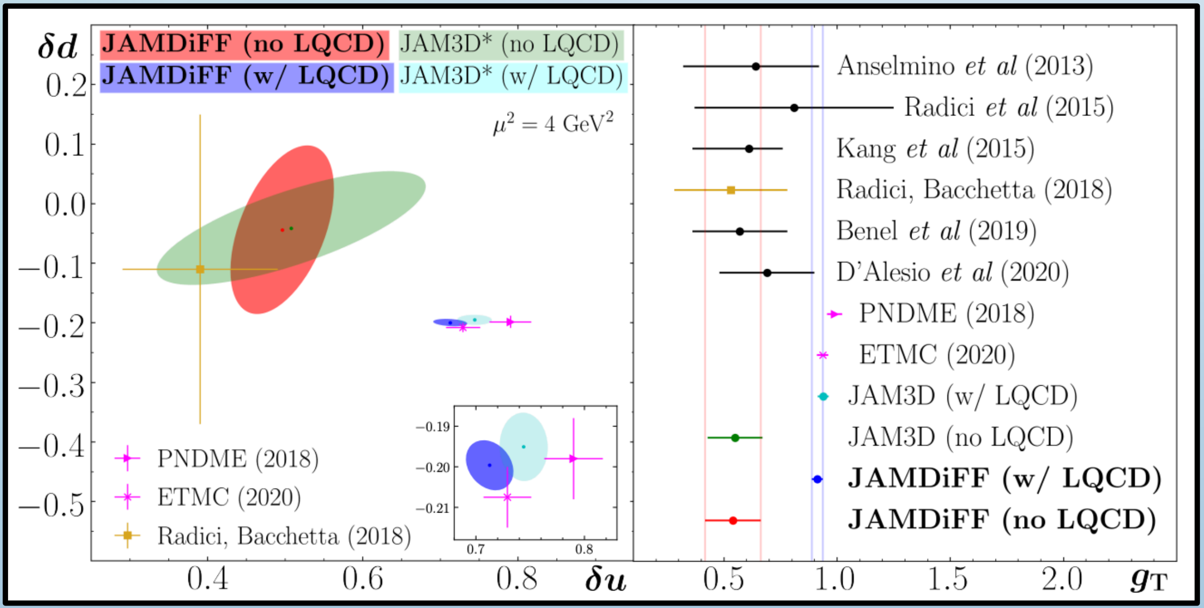
Agreement between all three analyses within errors

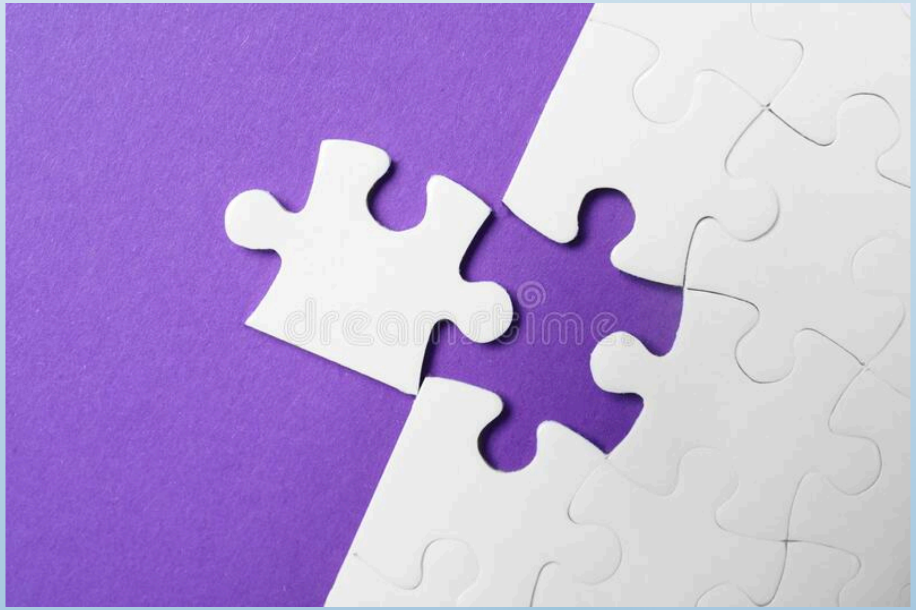
Soffer Bound: $|h_1^q| < \frac{1}{2}\left[f_1^q + g_1^q\right]$

J. Soffer, Phys. Rev. Lett. **74**, 1292-1294 (1995)



1. JAM Methodology
2. Extraction of DiFFs
3. Extraction of Transversity PDFs
4. **Extraction of Tensor Charges**
5. Conclusions and Outlook





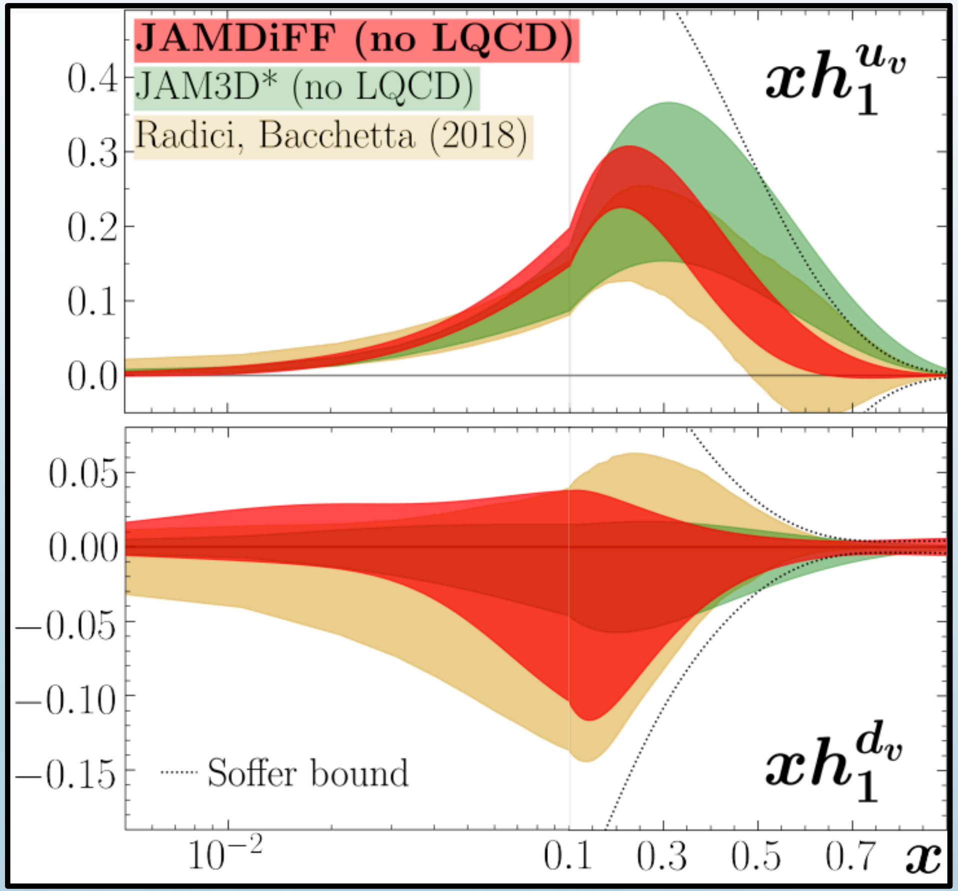
Controlling Extrapolation

$$\delta u \equiv \int_0^1 \mathrm{d}x (h_1^u - h_1^{\bar{u}}),$$

$$\delta d \equiv \int_0^1 \mathrm{d}x (h_1^d - h_1^{\bar{d}}),$$

$$g_T \equiv \delta u - \delta d,$$

Controlling Extrapolation



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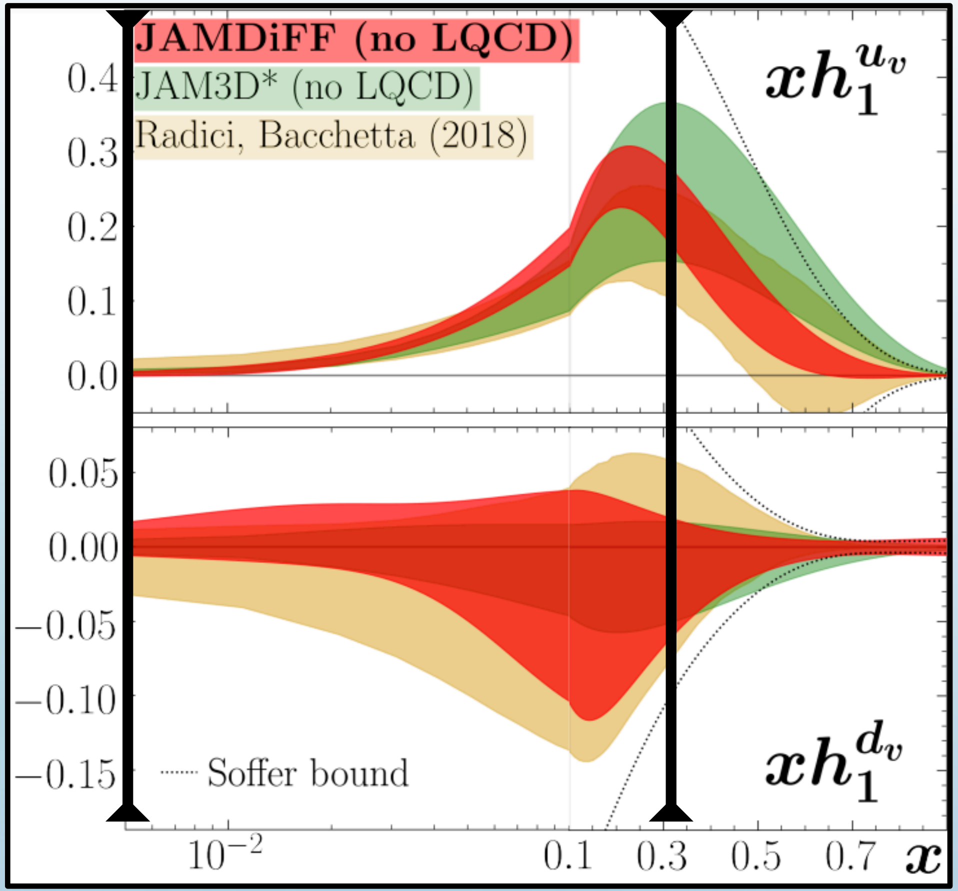


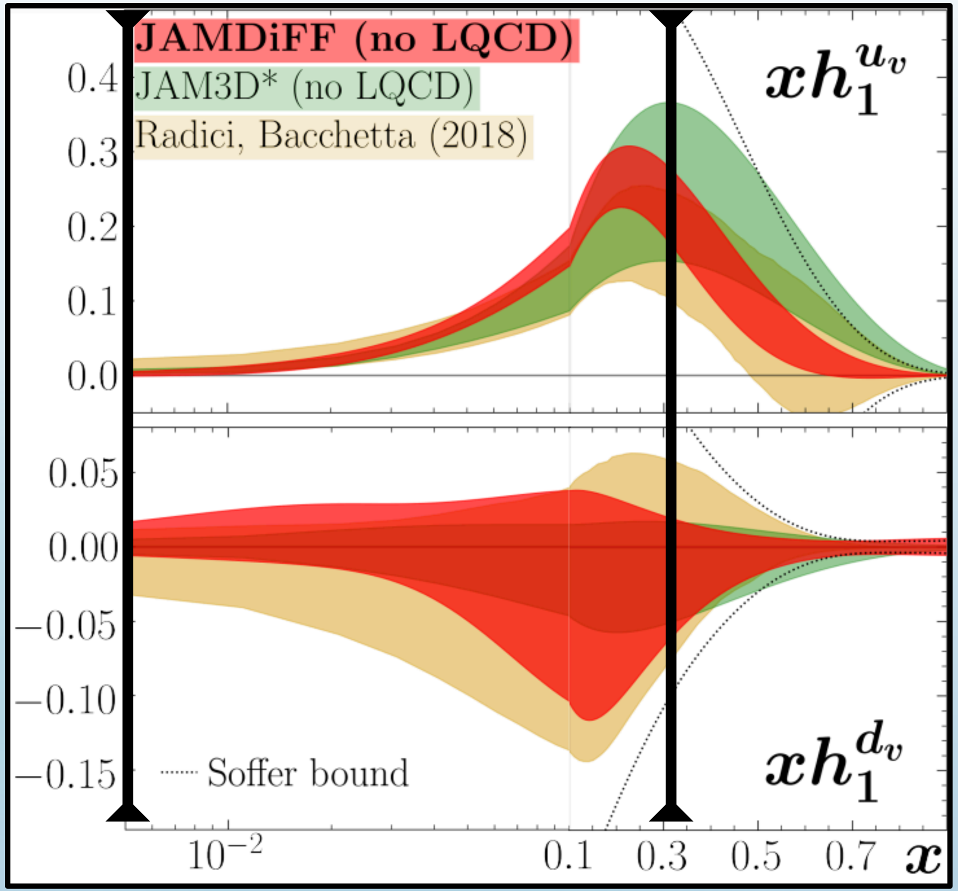
$$\delta u \equiv \int_0^1 \mathrm{d}x (h_1^u - h_1^{\bar{u}}),$$

$$\delta d \equiv \int_0^1 \mathrm{d}x (h_1^d - h_1^{\bar{d}}),$$

$$g_T \equiv \delta u - \delta d,$$

Measured Region

Controlling Extrapolation



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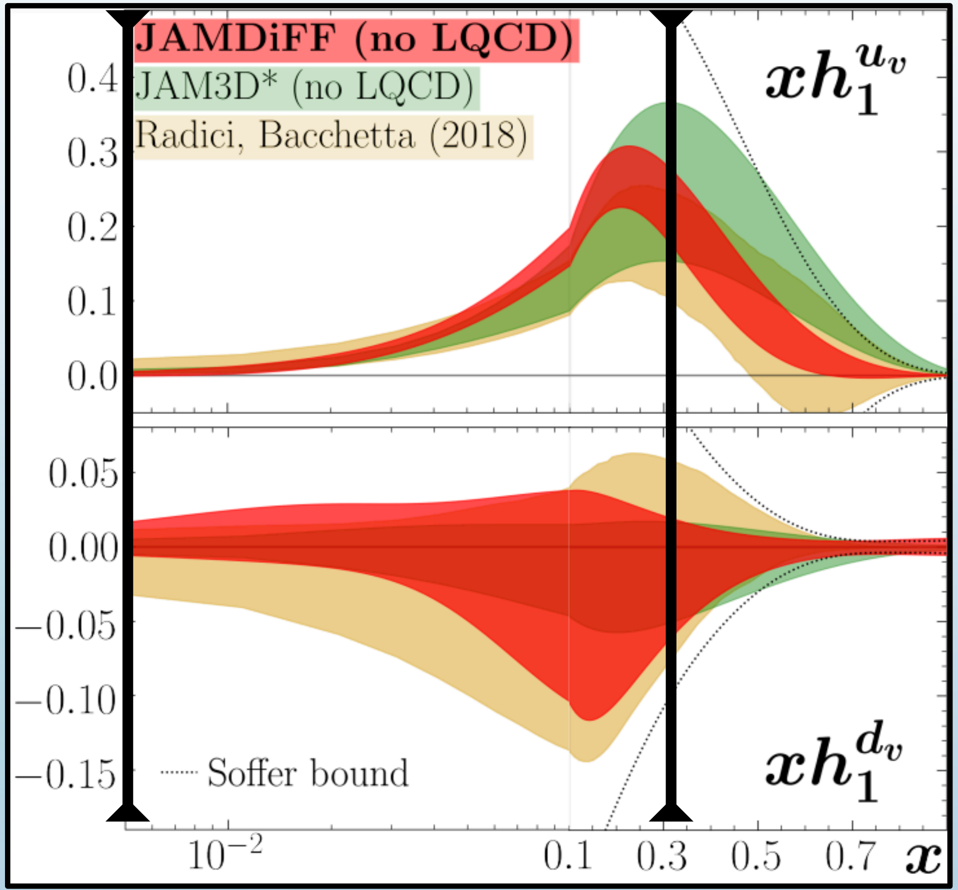
$$g_T \equiv \delta u - \delta d,$$

Large $x \gtrsim 0.3$

Soffer Bound: $|h_1^q| < \frac{1}{2}\left[f_1^q + g_1^q\right]$

J. Soffer, Phys. Rev. Lett. **74**, 1292-1294 (1995)

Controlling Extrapolation



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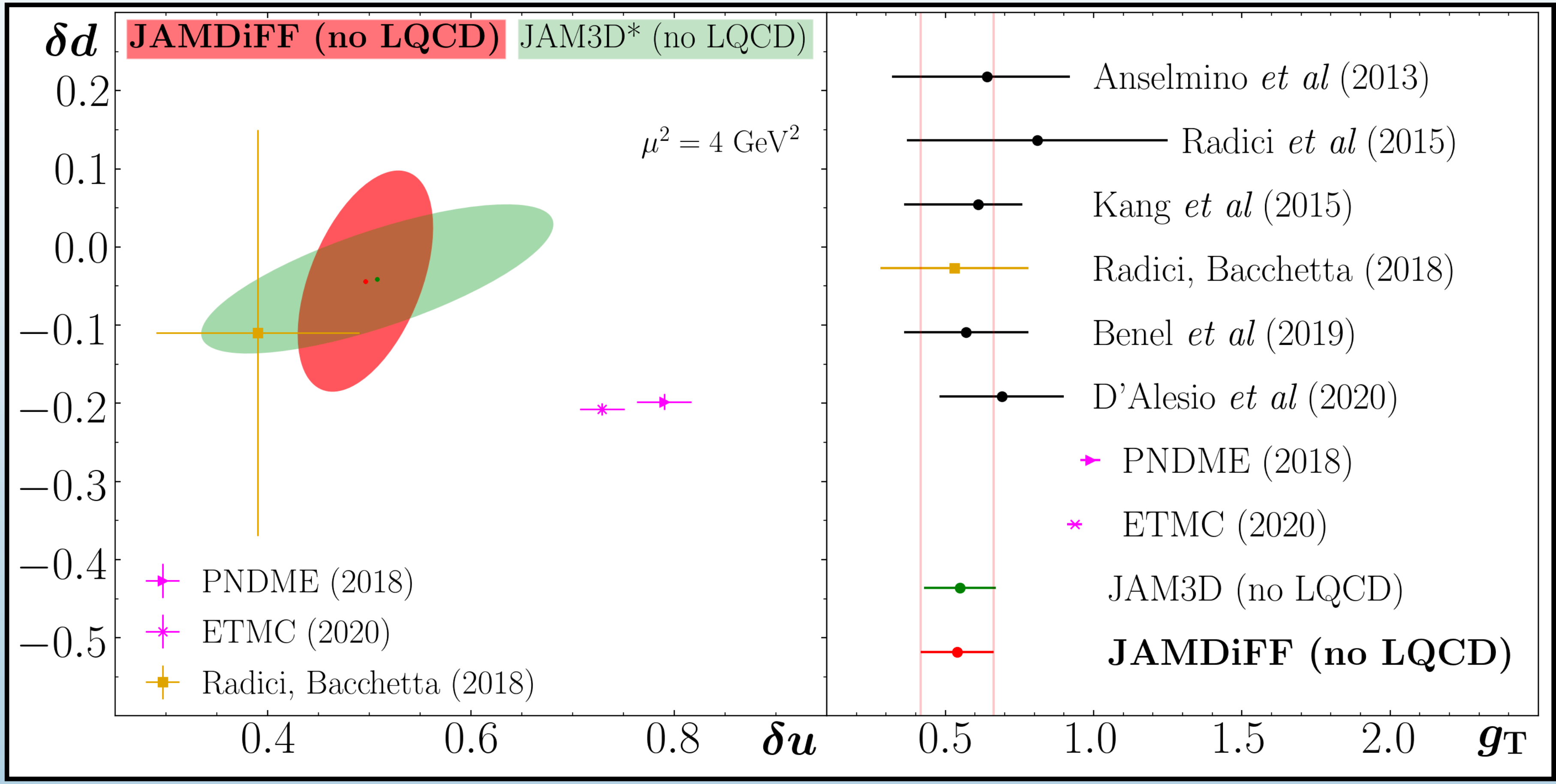
J. Soffer, Phys. Rev. Lett. **74**, 1292-1294 (1995)

Small $x \lesssim 0.005$

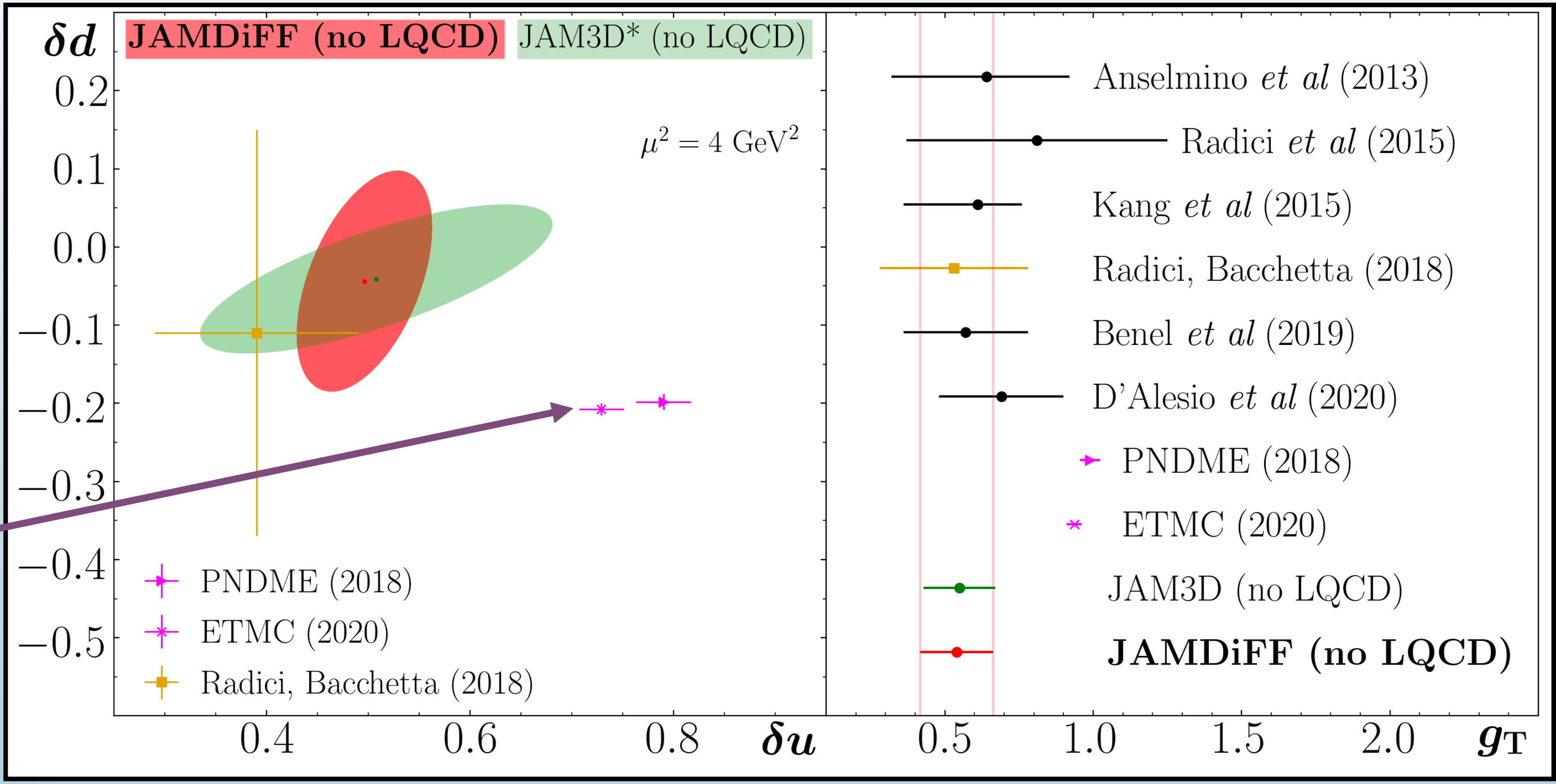
$$h_1^q \xrightarrow[x\to 0]{} x^{\alpha_q} \qquad \alpha_q = 1 - 2\sqrt{\frac{\alpha_s N_c}{2\pi}} \approx 0.17 \pm 0.085$$

Y. V. Kovchegov and M. D. Sievert, Phys. Rev. D **99**, 054033 (2019)

Tensor Charges



Tensor Charges

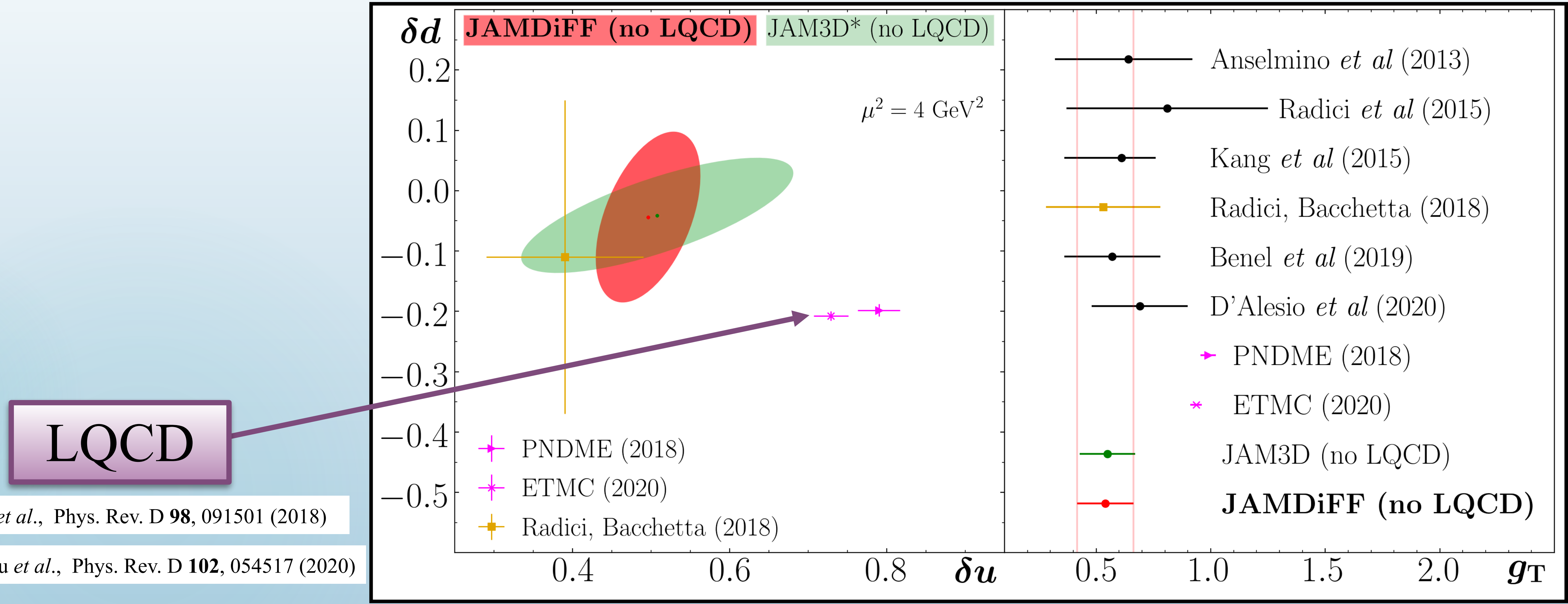


LQCD

R. Gupta *et al*., Phys. Rev. D **98**, 091501 (2018)

C. Alexandrou *et al*., Phys. Rev. D **102**, 054517 (2020)

Tensor Charges



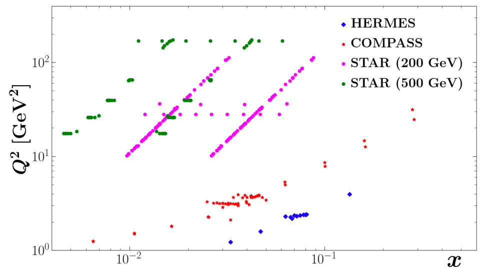
R. Gupta *et al*., Phys. Rev. D **98**, 091501 (2018)

C. Alexandrou *et al*., Phys. Rev. D **102**, 054517 (2020)

Consistent with RB18 and JAM3D* (no LQCD).
What happens if we include LQCD in the fit?

Experiment + Lattice + Theory

EXPERIMENT
(measured region)



THEORY
(unmeasured regions)

$$|h_1^q| < \frac{1}{2}[f_1^q + g_1^q]$$

$$\alpha_q = 1 - 2\sqrt{\frac{\alpha_s N_c}{2\pi}}$$

LATTICE
(full moments)

$$\delta u \equiv \int_0^1 \mathrm{d}x (h_1^u - h_1^{\bar{u}}),$$

$$\delta d \equiv \int_0^1 \mathrm{d}x (h_1^d - h_1^{\bar{d}}),$$

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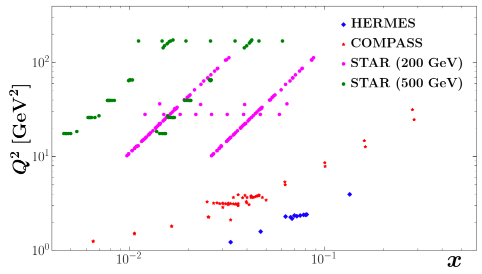
Experiment + Lattice + Theory

EXPERIMENT
(measured region)



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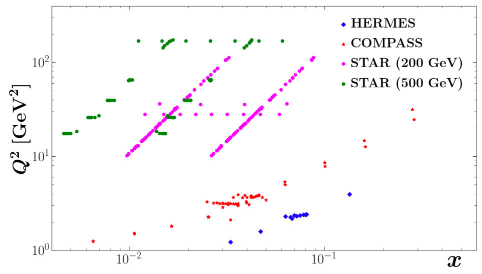
Presently, trivial to find compatibility between any two

Experiment + Lattice + Theory

EXPERIMENT
(measured region)



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Experiment + Lattice + Theory

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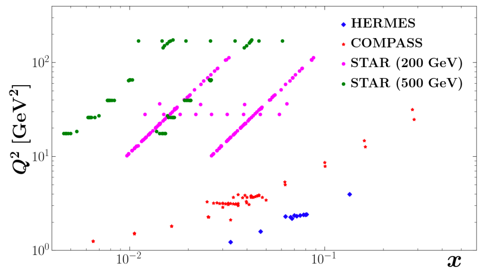

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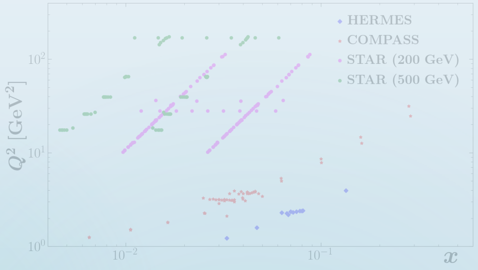
Presently, trivial to find compatibility between any two

Experiment + Lattice + Theory

EXPERIMENT
(measured region)



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Experiment + Lattice + Theory

EXPERIMENT
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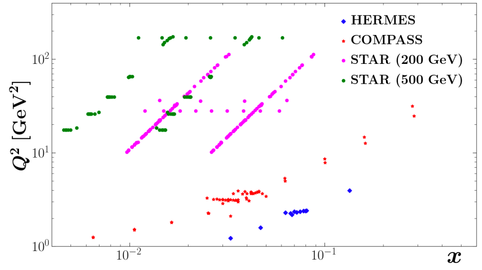

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$$g_T \equiv \delta u - \delta d,$$

Presently, trivial to find compatibility between any two

Only meaningful when all three are included

Quality of Fit

Experiment	N_{dat}	χ^2_{red} no LQCD	χ^2_{red} w/ LQCD
Belle (cross section)	1094	1.05	1.06
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COMPASS (D)	26	0.74	0.78
STAR (2015)	24	1.83	1.59
STAR (2018)	106	1.06	1.18
ETMC δu	1	—	0.55
ETMC δd	1	—	1.10
PNDME δu	1	—	8.20
PNDME δd	1	—	0.03
Total	1475	**1.02**	**1.05**

Quality of Fit

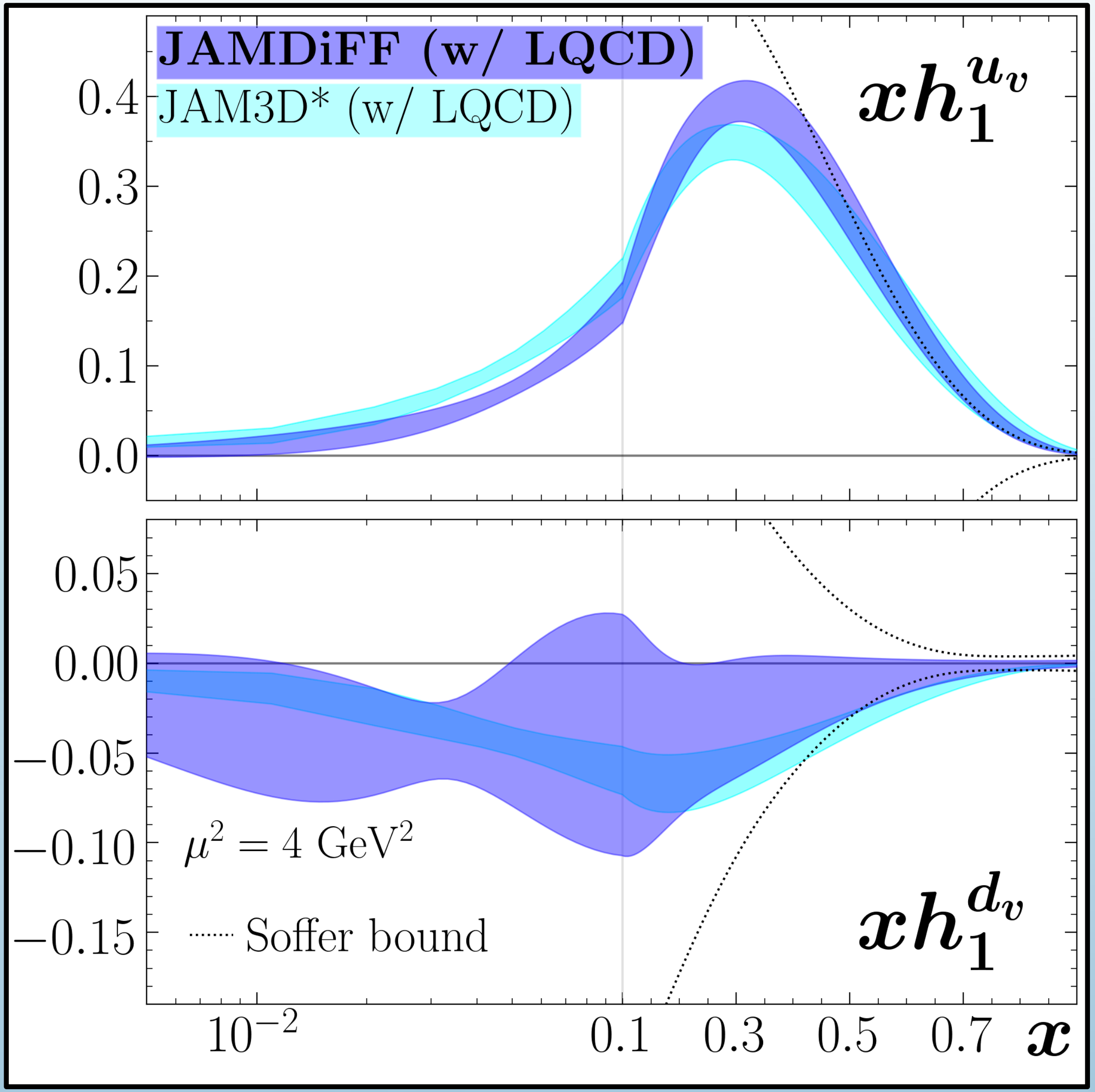
Physical Pion Mass

$N_f = 2 + 1 + 1$

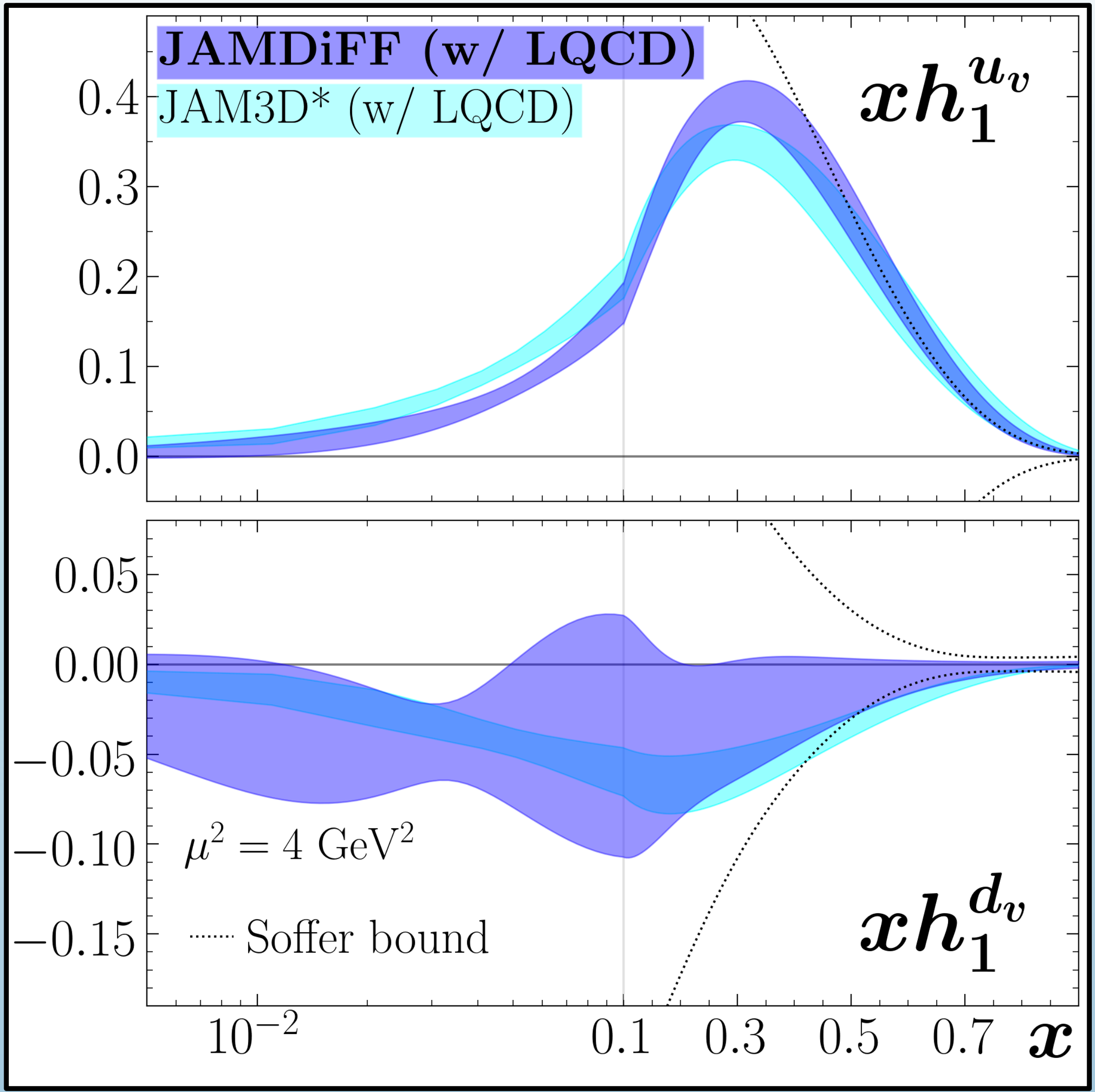
Use δu and δd instead of g_T

Experiment	N_{dat}	χ^2_{red} no LQCD	χ^2_{red} w/ LQCD
Belle (cross section)	1094	1.05	1.06
Belle (Artru-Collins)	183	0.78	0.78
HERMES	12	1.09	1.12
COMPASS (p)	26	0.75	1.25
COMPASS (D)	26	0.74	0.78
STAR (2015)	24	1.83	1.59
STAR (2018)	106	1.06	1.18
ETMC δu	1	—	0.55
ETMC δd	1	—	1.10
PNDME δu	1	—	8.20
PNDME δd	1	—	0.03
Total	1475	**1.02**	**1.05**

Transversity PDFs (w/ LQCD)

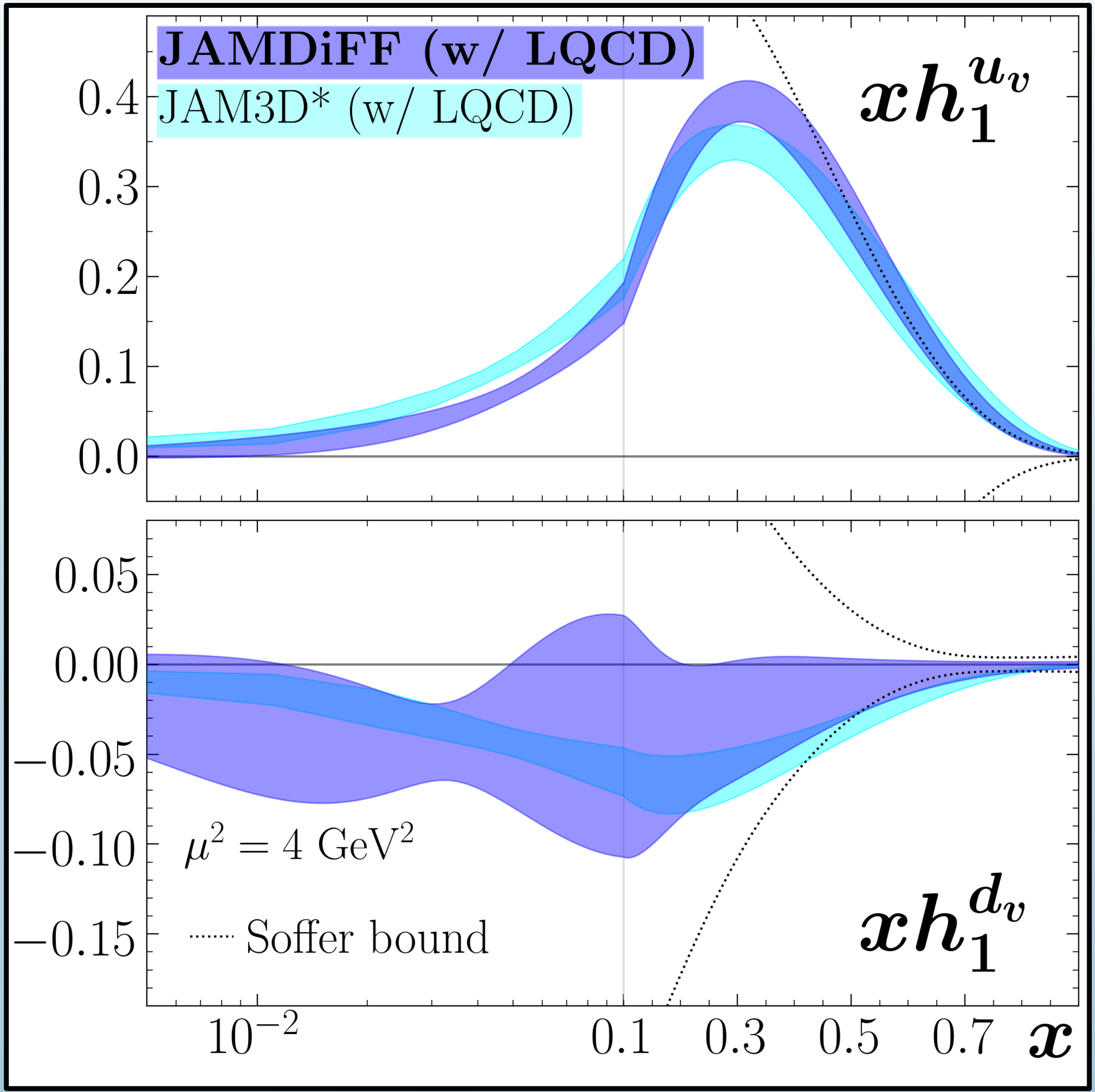


Transversity PDFs (w/ LQCD)



JAM3D* = JAM3D-22 (w/ LQCD)
+ Antiquarks w/ $\bar{u} = -\bar{d}$
+ small-x constraint (see slide 23)
+ $\delta u, \delta d$ from ETMC & PNDME
(instead of g_T from ETMC)

Transversity PDFs (w/ LQCD)



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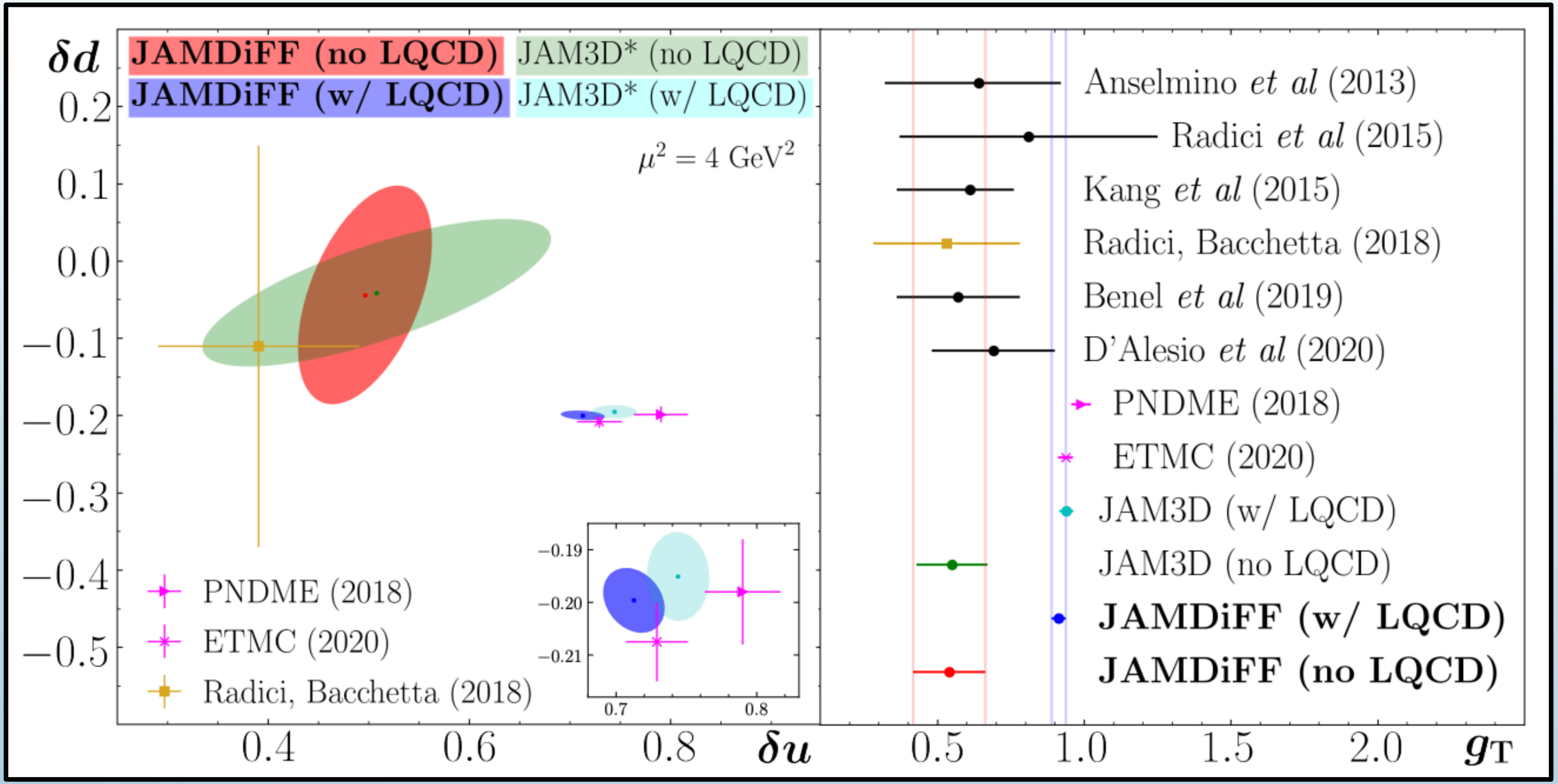
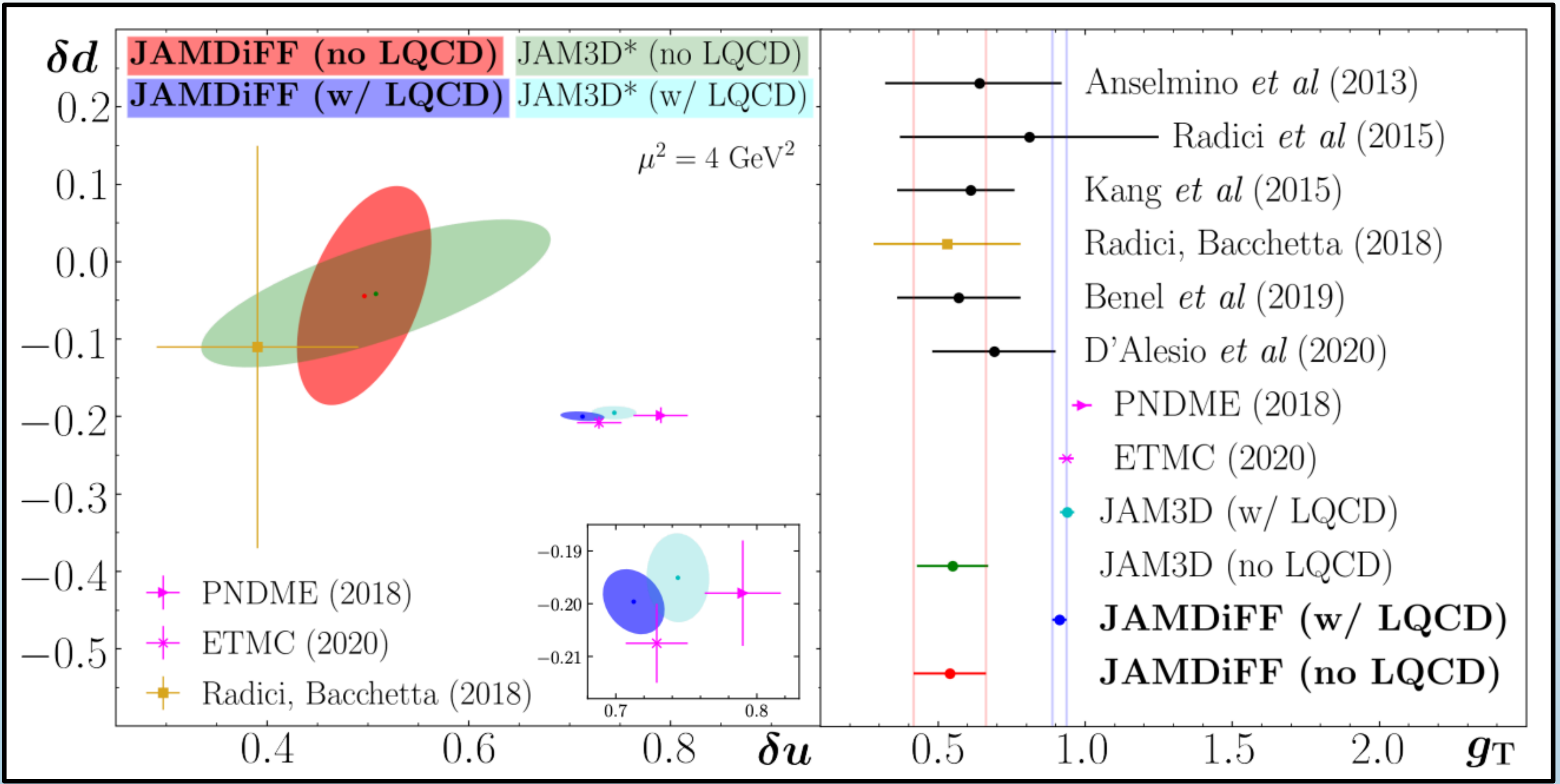
+ Antiquarks w/ $\bar{u} = -\bar{d}$
+ small-x constraint (see slide 23)
+ δu, δd from ETMC & PNDME
(instead of g_T from ETMC)

JAMDiFF (w/ LQCD) and JAM3D* (w/ LQCD) largely agree

Tensor Charges (w/ LQCD)

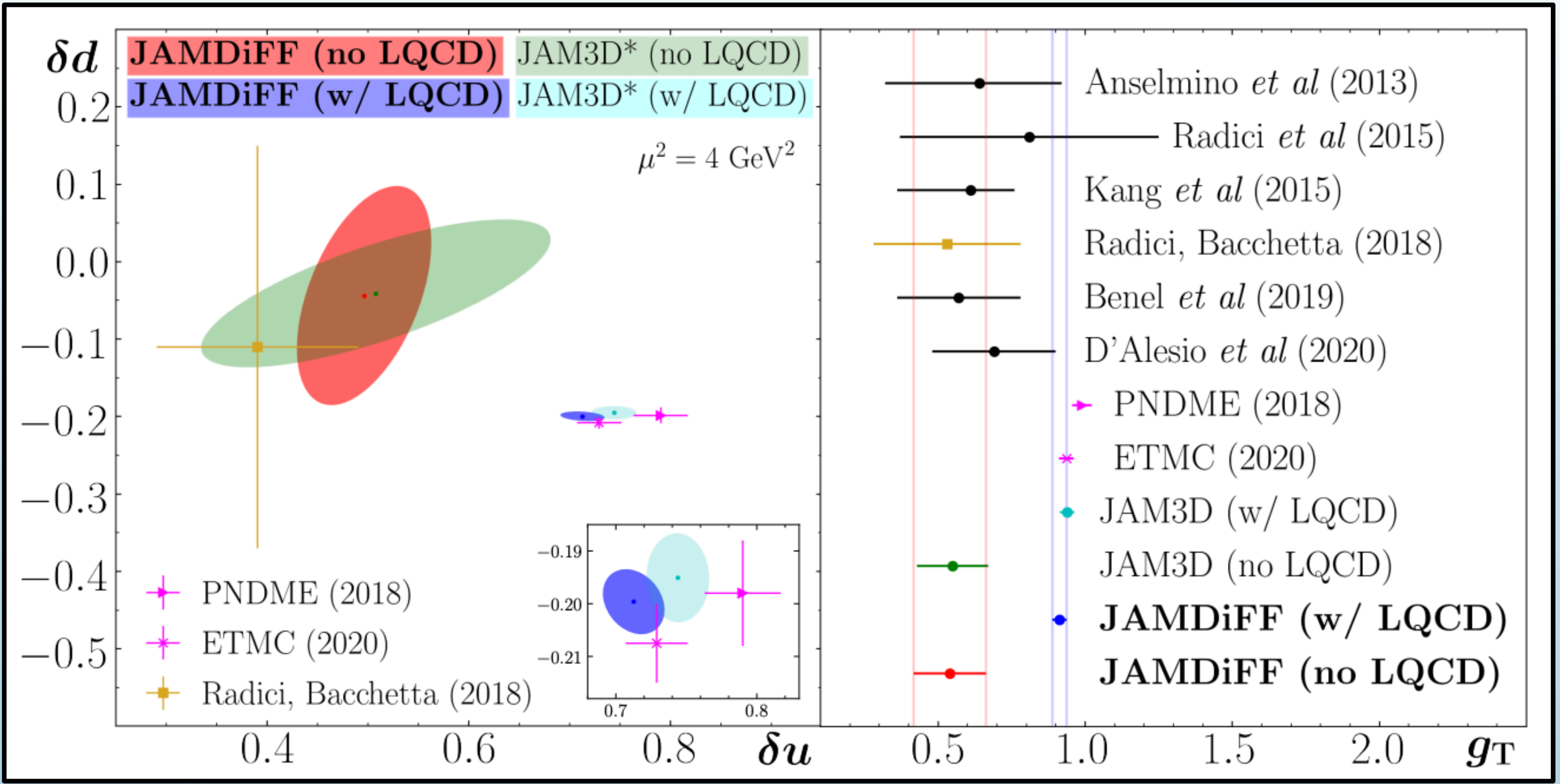


Tensor Charges (w/ LQCD)



Noticeable shift from including lattice data

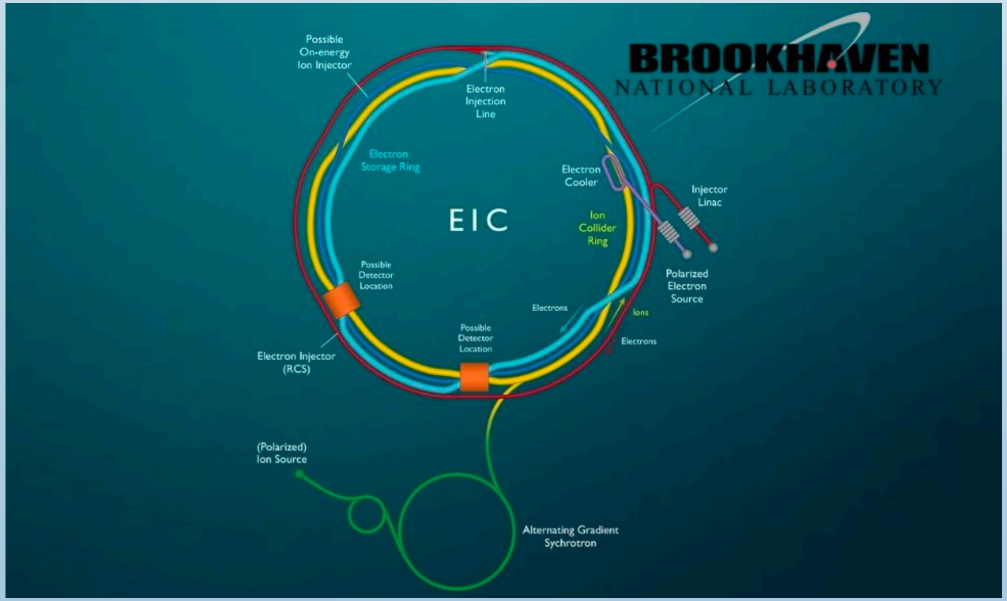
Tensor Charges (w/ LQCD)

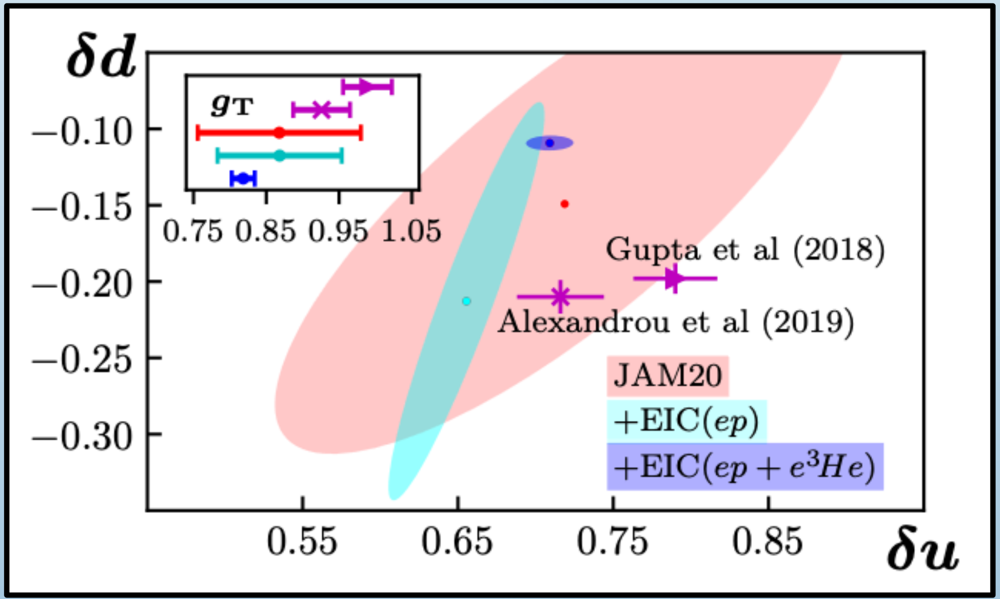


Noticeable shift from including lattice data

Likelihood function $\mathscr{L} = \exp(-\chi^2/2)$ does not guarantee that errors overlap

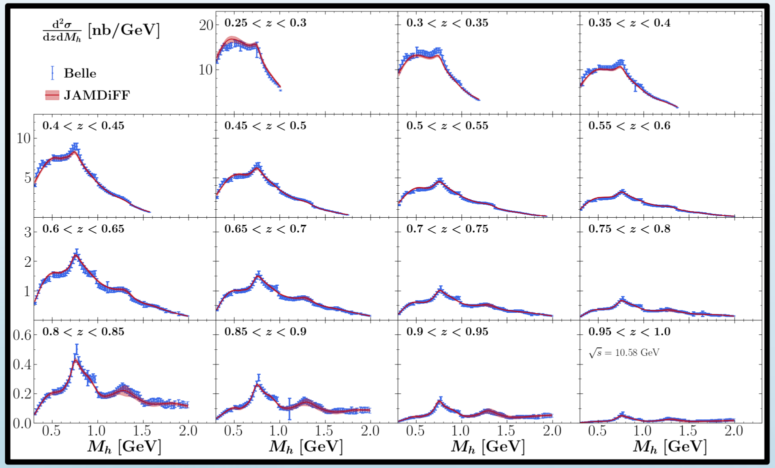
1. JAM Methodology
2. Extraction of DiFFs
3. Extraction of Transversity PDFs
4. Extraction of Tensor Charges
5. **Conclusions and Outlook**





Comprehensive Analysis of DiFFs and Transversity

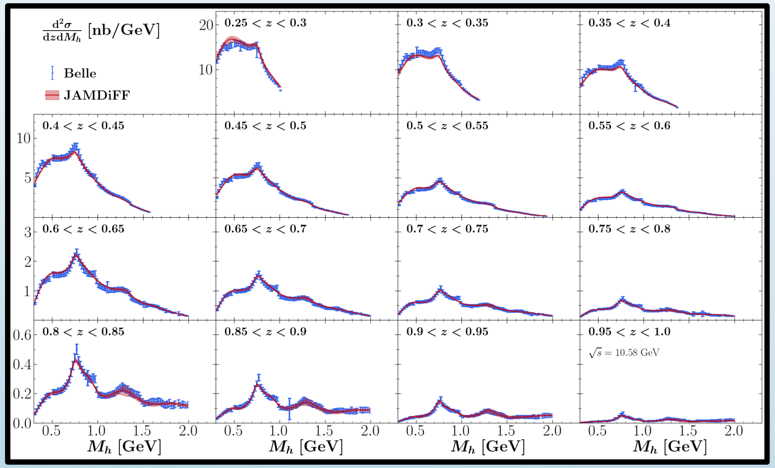
First inclusion of Belle cross section data

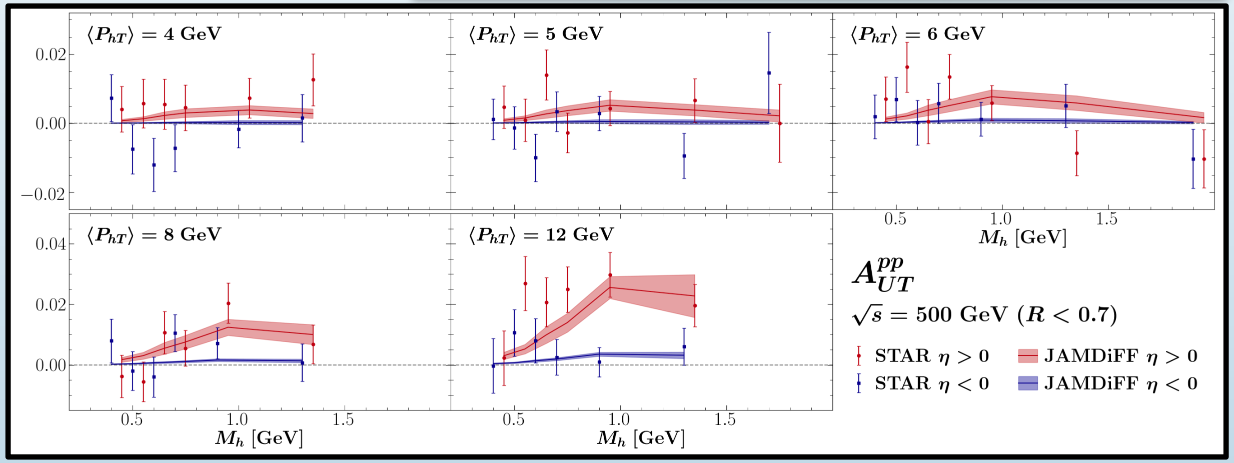


Comprehensive Analysis of DiFFs and Transversity

First inclusion of Belle cross section data

First inclusion of 500 GeV STAR data





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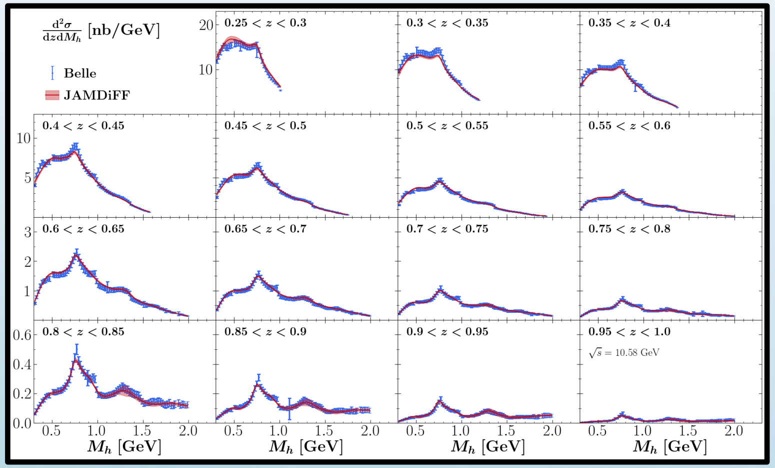
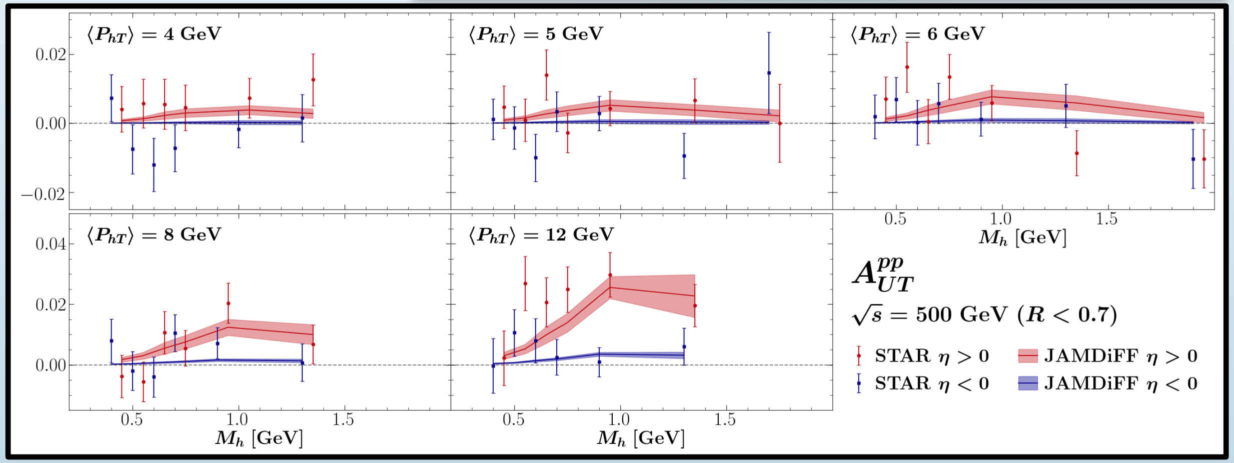
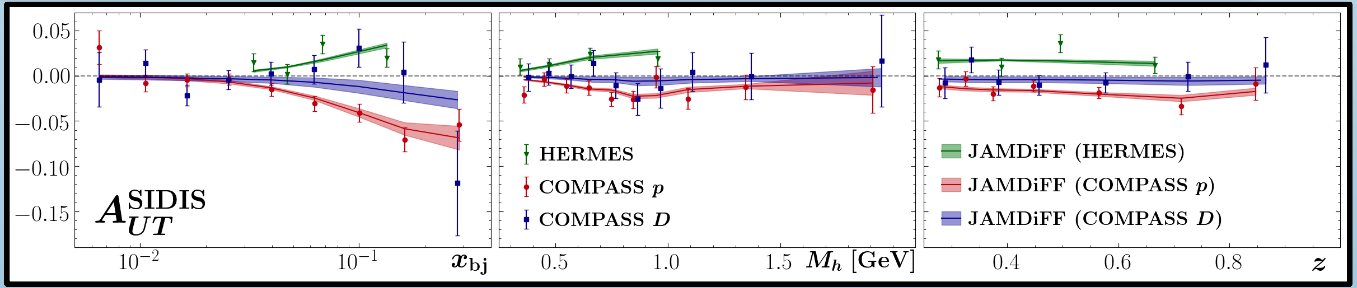


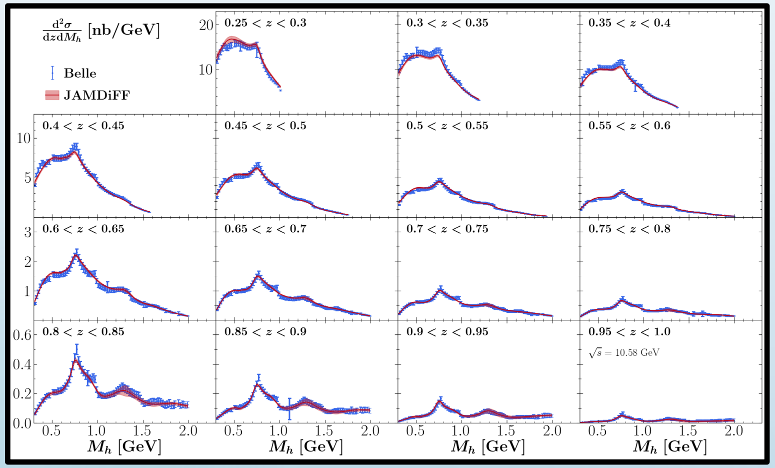


Utilized all binnings for Artru-Collins and SIDIS asymmetries

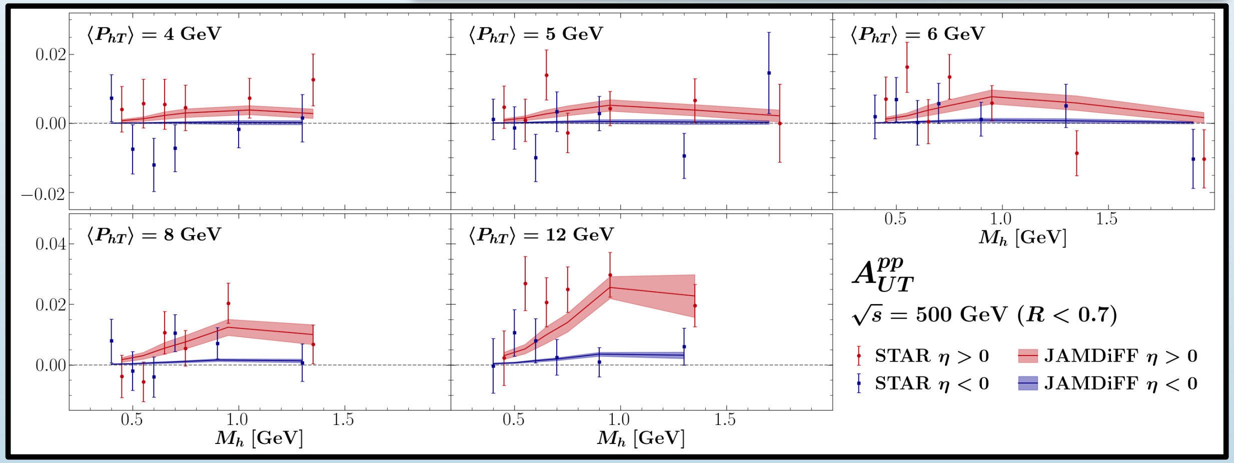


Comprehensive Analysis of DiFFs and Transversity

First inclusion of Belle cross section data



First inclusion of 500 GeV STAR data



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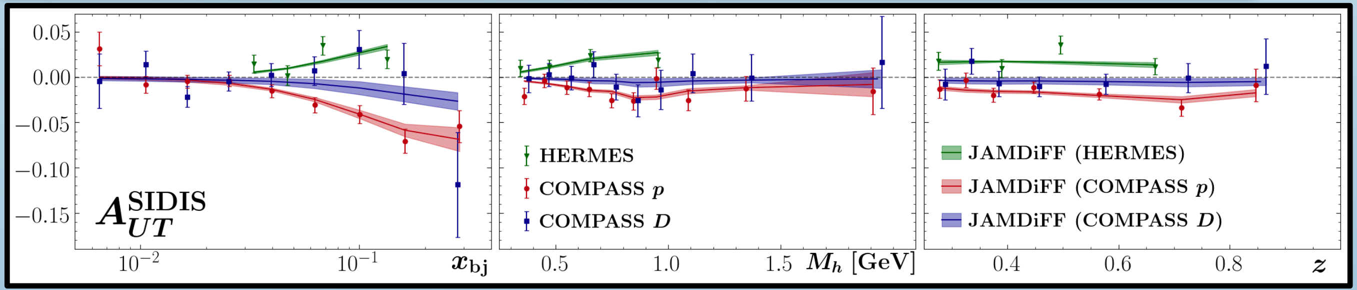
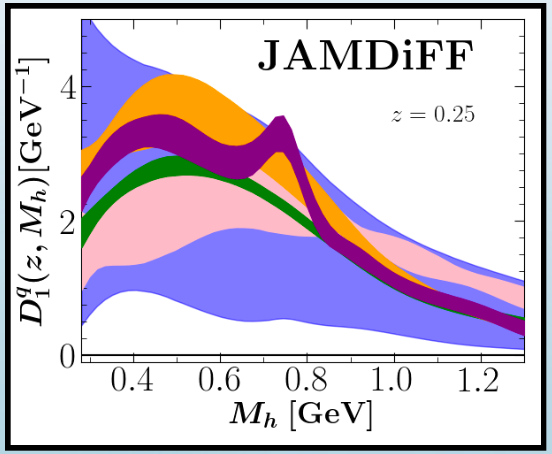
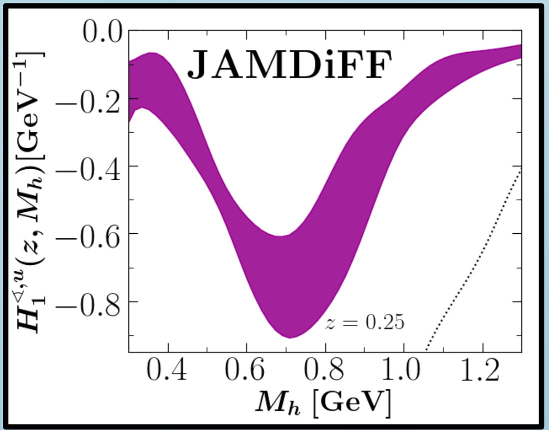


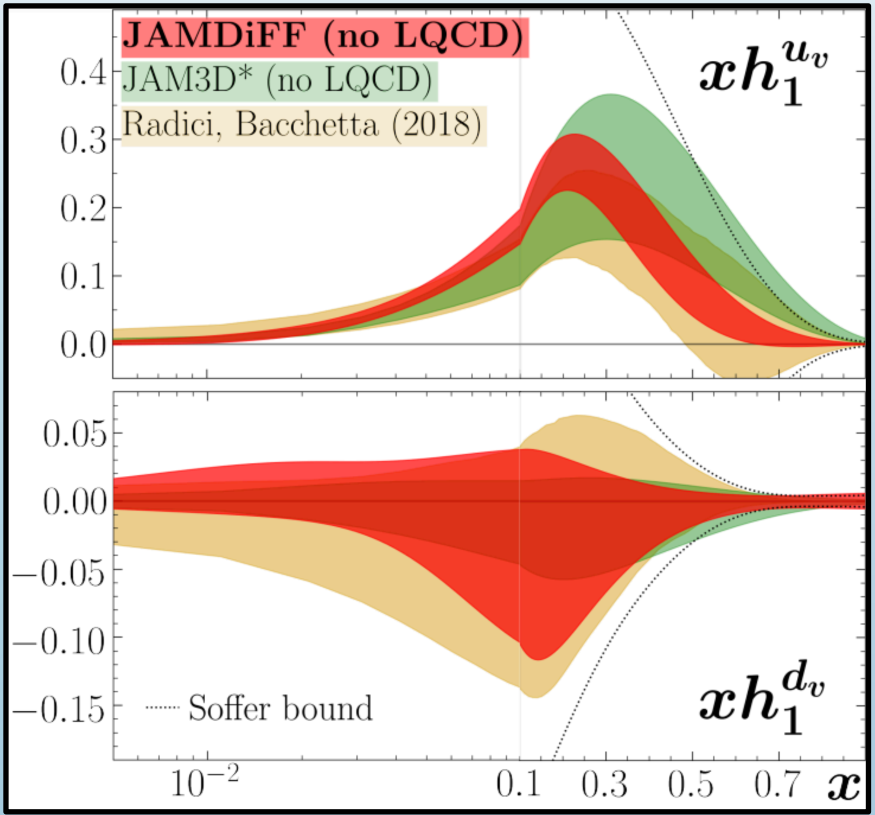
First simultaneous analysis of DiFFs and transversity PDFs

Conclusions

Simultaneous extraction of DiFFs and transversity PDFs



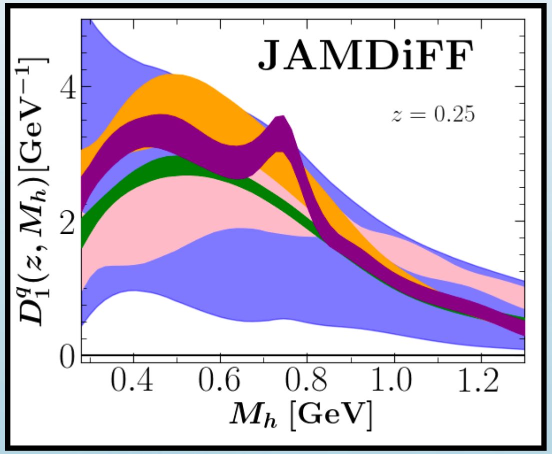


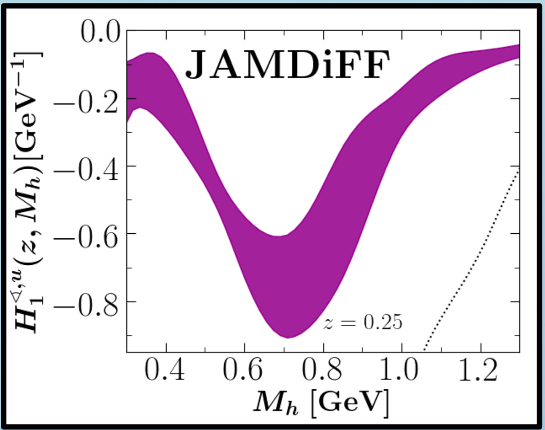


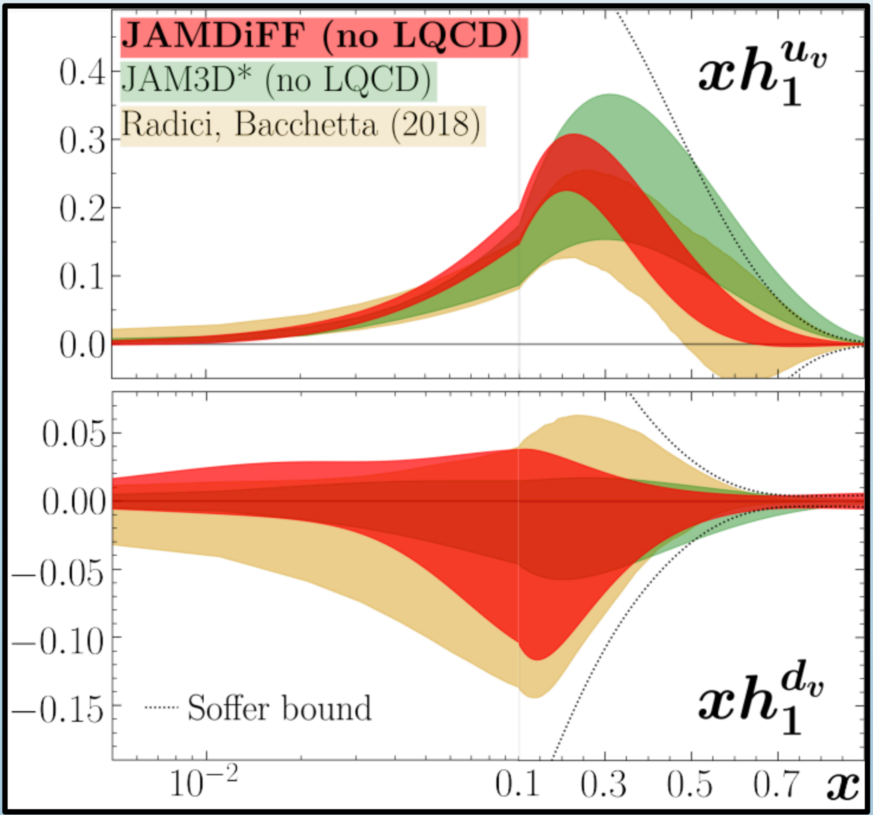
Conclusions

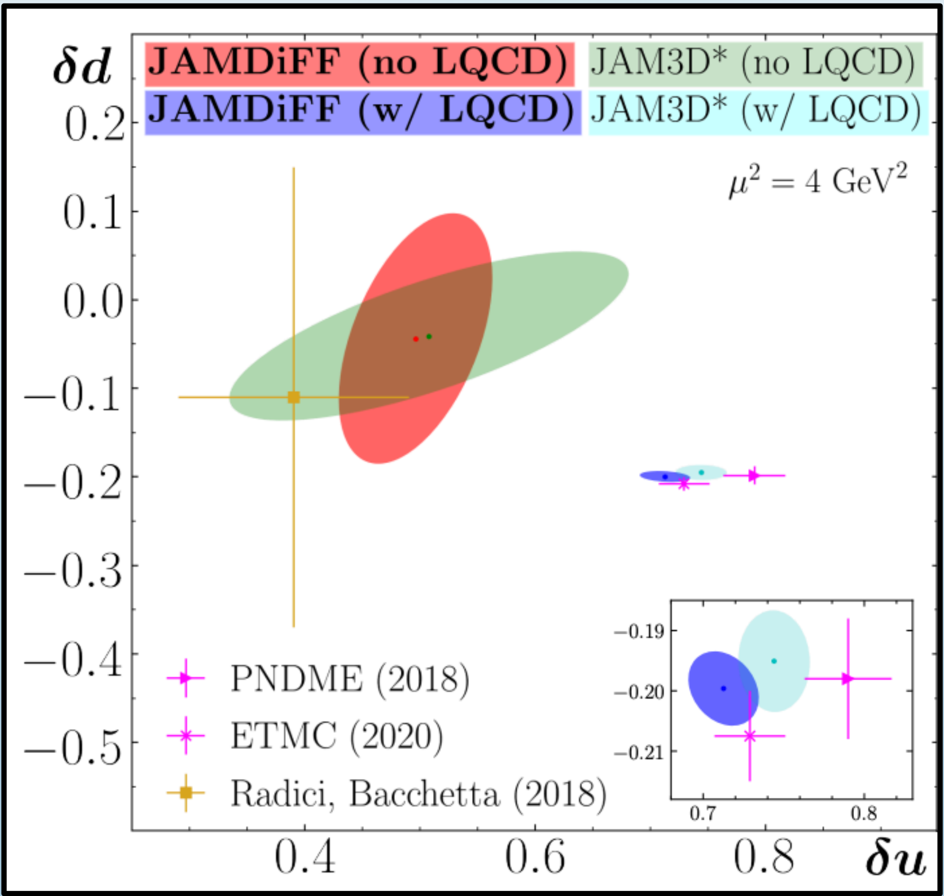
Simultaneous extraction of DiFFs and transversity PDFs

Universality of all available information on transversity









Outlook

More data from RHIC
Proton-proton cross section

Outlook

More data from RHIC
Proton-proton cross section

SIDIS multiplicities
from COMPASS

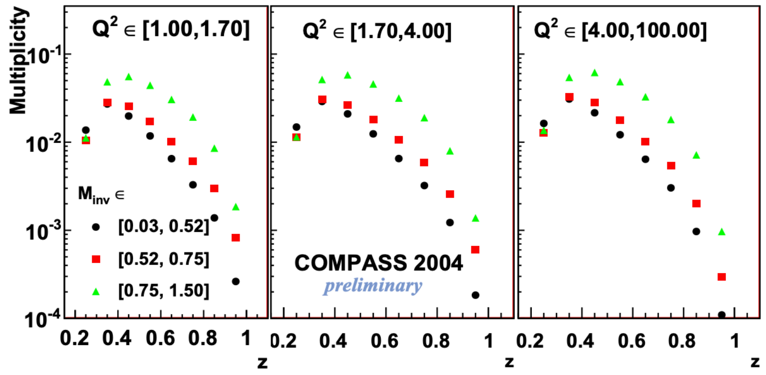


Figure 4: h^+h^- multiplicities measured versus z, M_{inv} and Q^2.

N. Makke, Phys. Part. Nucl. **45**, 138-140 (2014)

Outlook

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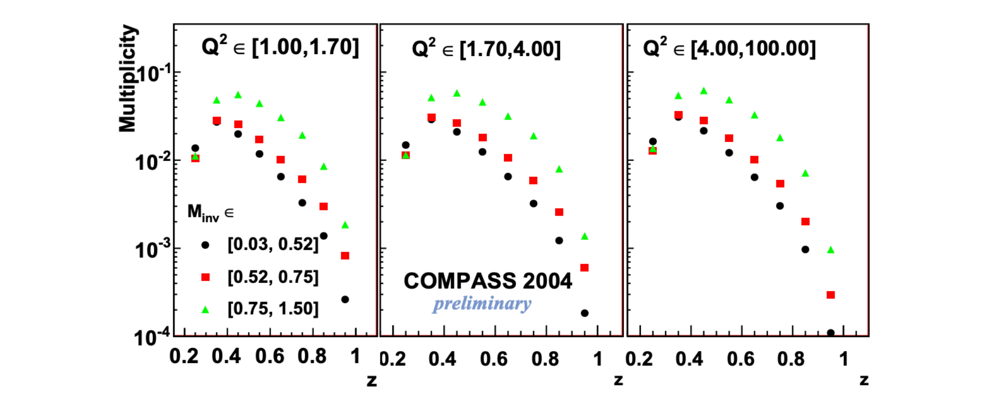

Figure 4: h^+h^- multiplicities measured versus z, M_{inv} and Q^2.

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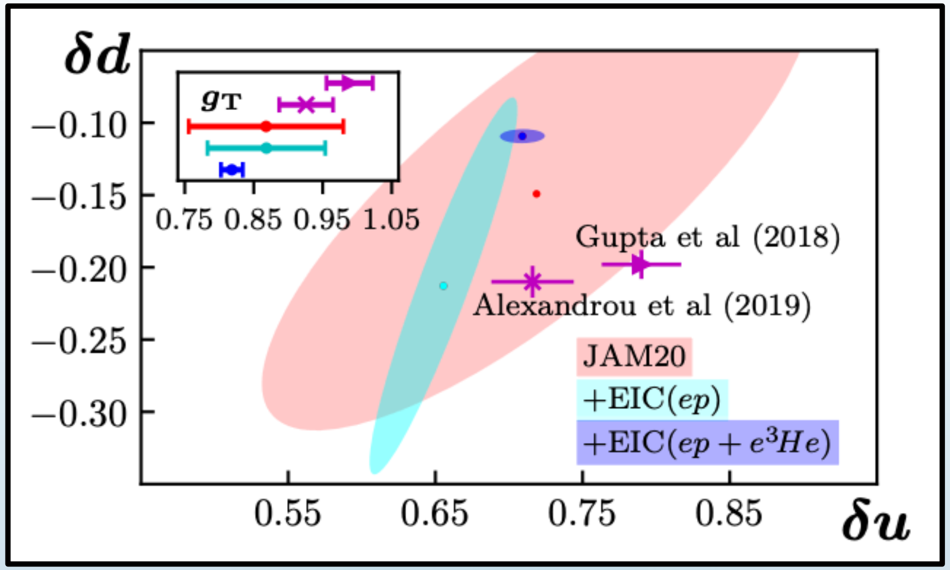
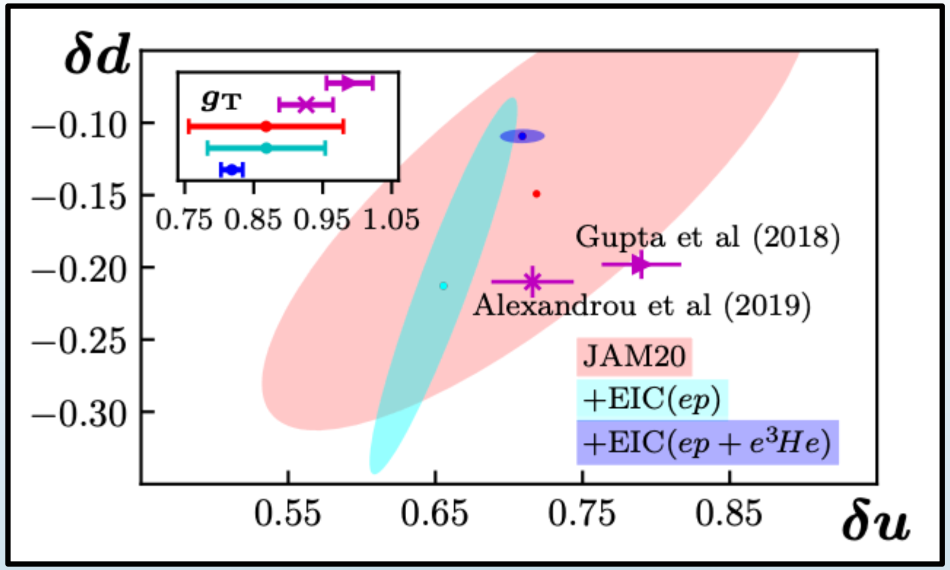
L. Gamberg *et al.*, Phys. Lett. B **816**, 136255 (2021)



EIC can provide new information

Outlook

L. Gamberg *et al.*, Phys. Lett. B **816**, 136255 (2021)



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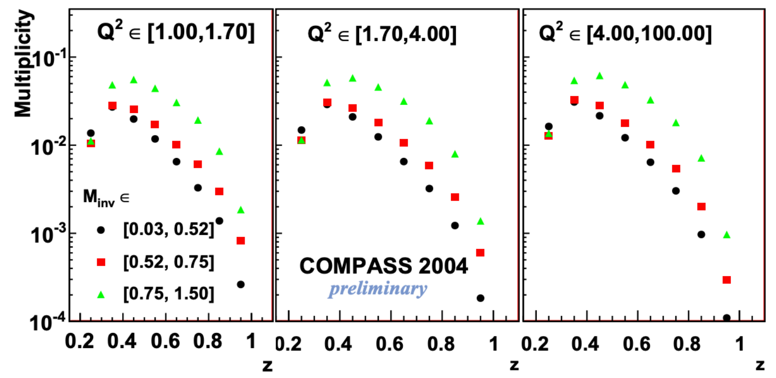


Figure 4: h^+h^- multiplicities measured versus z, M_{inv} and Q^2.

N. Makke, Phys. Part. Nucl. **45**, 138-140 (2014)

Simultaneous fit of DiFF channel + TMD channel + Lattice QCD

Collaboration

Andreas Metz

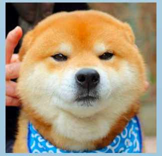
Nobuo Sato

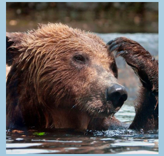
Daniel Pitonyak

Alexey Prokudin

Ralf Seidl

Thank you to Yiyu Zhou and Patrick Barry for helpful discussions





Extra Slides

Parameterize PDFs at input scale $Q_0^2 = m_c^2$

$$f_i(x) = Nx^\alpha(1-x)^\beta(1+\gamma\sqrt{x}+\eta x)$$

Parameterize PDFs at input scale $Q_0^2 = m_c^2$

$$f_i(x) = Nx^{\alpha}(1-x)^{\beta}(1+\gamma\sqrt{x}+\eta x)$$

Evolve PDFs using DGLAP

$$\frac{\mathrm{d}}{\mathrm{d}\ \ln(\mu^2)} f_i(x,\mu) = \sum_j \int_x^1 \frac{dz}{z} P_{ij}(z,\mu) f_j(\frac{x}{z},\mu)$$

Parameterize PDFs at input scale $Q_0^2 = m_c^2$

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Calculate Observables

$$d\sigma^{pp} = \sum_{ij} H_{ij}^{pp} \otimes f_i \otimes f_j$$

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$$d\sigma^{pp} = \sum_{ij} H_{ij}^{pp} \otimes f_i \otimes f_j$$

Mellin Space Techniques

$$\mathrm{d}\sigma^{pp} = \sum_{ijkl} \frac{1}{(2\pi i)^2} \int \mathrm{d}N \int \mathrm{d}M \tilde{f}_j(N,\mu_0)\tilde{f}_l(M,\mu_0) \otimes \left[x_1^{-N} x_2^{-M} \tilde{\mathcal{H}}_{ik}^{pp}(N,M,\mu) U_{ij}^S(N,\mu,\mu_0) U_{kl}^S(M,\mu,\mu_0)\right]$$

Parameterize PDFs at input scale $Q_0^2 = m_c^2$

$$f_i(x) = Nx^{\alpha}(1-x)^{\beta}(1+\gamma\sqrt{x}+\eta x)$$

Evolve PDFs using DGLAP

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$$\sigma = \sum_{ij} H_{ij} \otimes f_i \otimes f_j + \mathcal{O}(1/Q)$$

Experimentally measured cross-section

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"Hard part" (process dependent)
Cross-section at parton level
Calculated in perturbative QCD

Experimentally measured cross-section

"Soft part" (process independent)
Describes internal structure

$$\sigma = \sum_{ij} H_{ij} \otimes f_i \otimes f_j + \mathcal{O}(1/Q)$$

"Hard part" (process dependent)
Cross-section at parton level
Calculated in perturbative QCD

Now that the observables have been calculated…

$$\chi^2(\boldsymbol{a}) = \sum_{i,e} \left(\frac{d_{i,e} - \sum_k r_e^k \beta_{i,e}^k - T_{i,e}(\boldsymbol{a})/N_e}{\alpha_{i,e}} \right)^2 + \sum_k \left(r_e^k \right)^2 + \left(\frac{1 - N_e}{\delta N_e} \right)^2$$

Now that the observables have been calculated…

Data

$$\chi^2(\boldsymbol{a}) = \sum_{i,e} \left(\frac{d_{i,e} - \sum_k r_e^k \beta_{i,e}^k - T_{i,e}(\boldsymbol{a})/N_e}{\alpha_{i,e}} \right)^2 + \sum_k \left(r_e^k\right)^2 + \left(\frac{1 - N_e}{\delta N_e} \right)^2$$

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Uncorrelated Uncertainties

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Uncorrelated Uncertainties

Correlated Uncertainties

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Data

Theory

Normalization

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Uncorrelated Uncertainties

Correlated Uncertainties

Normalization Uncertainty

Now that we have calculated $\chi^2(\boldsymbol{a}, \text{data})$…

Likelihood Function

$$\mathcal{L}(\boldsymbol{a}, \text{data}) = \exp\left(-\frac{1}{2}\chi^2(\boldsymbol{a}, \text{data})\right)$$

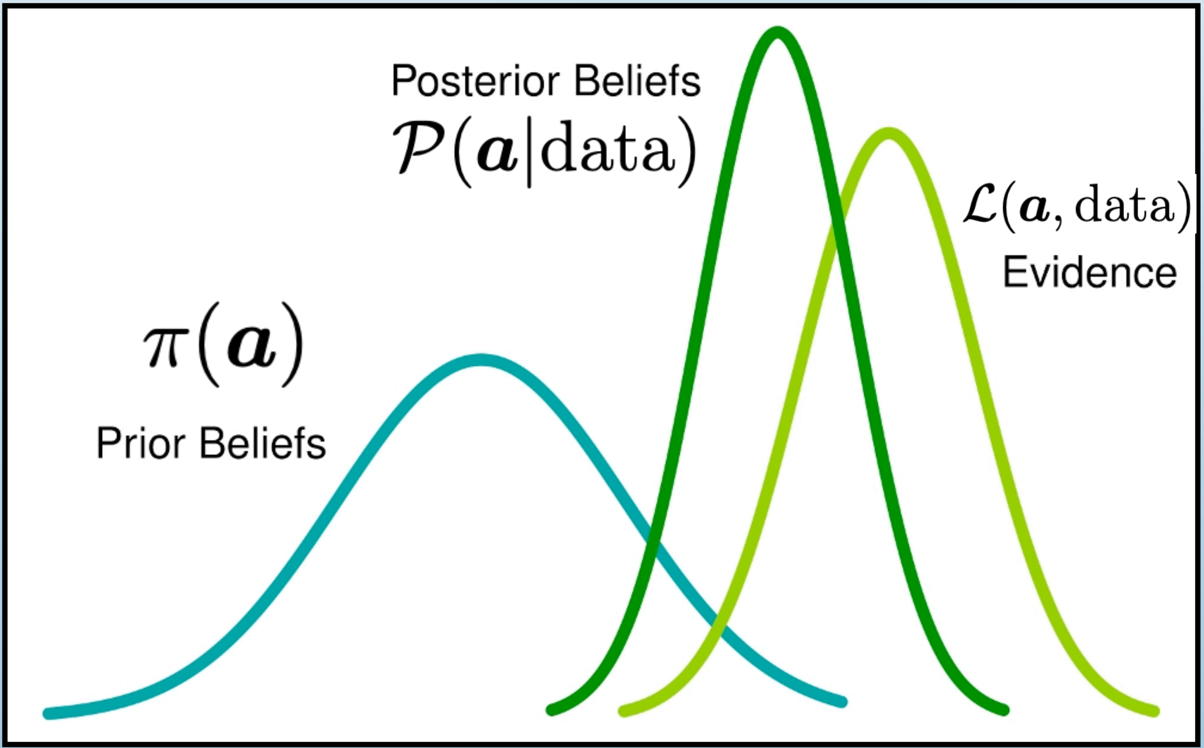
Now that we have calculated $\chi^2(\boldsymbol{a}, \text{data})$…

Likelihood Function

$$\mathcal{L}\left(\boldsymbol{a}, \text{data}\right) = \exp\left(-\frac{1}{2}\chi^2\left(\boldsymbol{a}, \text{data}\right)\right)$$

Bayes' Theorem

$$\mathcal{P}(\boldsymbol{a}|\text{data}) \sim \mathcal{L}(\boldsymbol{a}, \text{data})\, \pi(\boldsymbol{a})$$



$$\tilde{\sigma} = \sigma + N(0,1)\ \alpha$$

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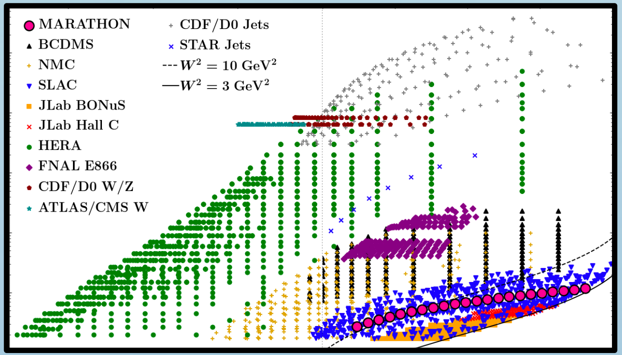
Data

Original Data



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Uncorrelated Uncertainties

Data

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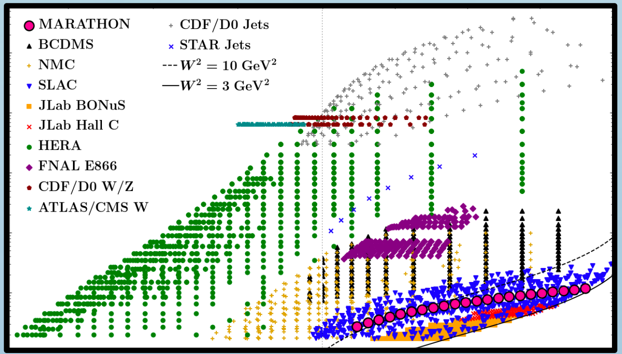


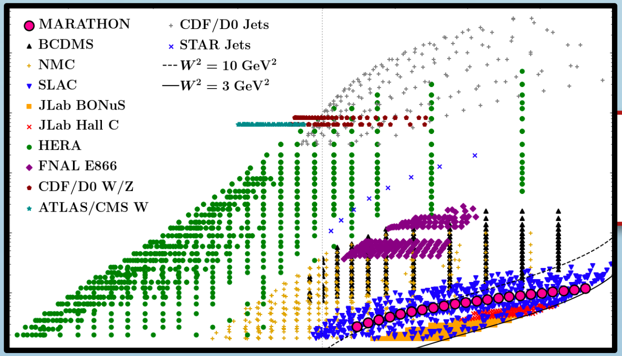
Pseudo-Data →

$$\tilde{\sigma} = \sigma + N(0,1)\ \alpha$$

← Uncorrelated Uncertainties

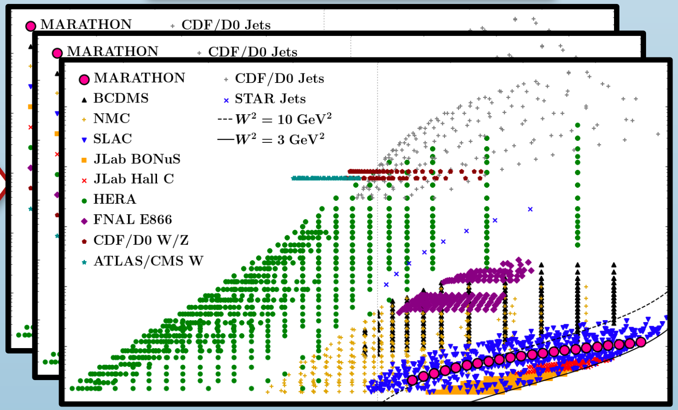
↑ Data

Original Data



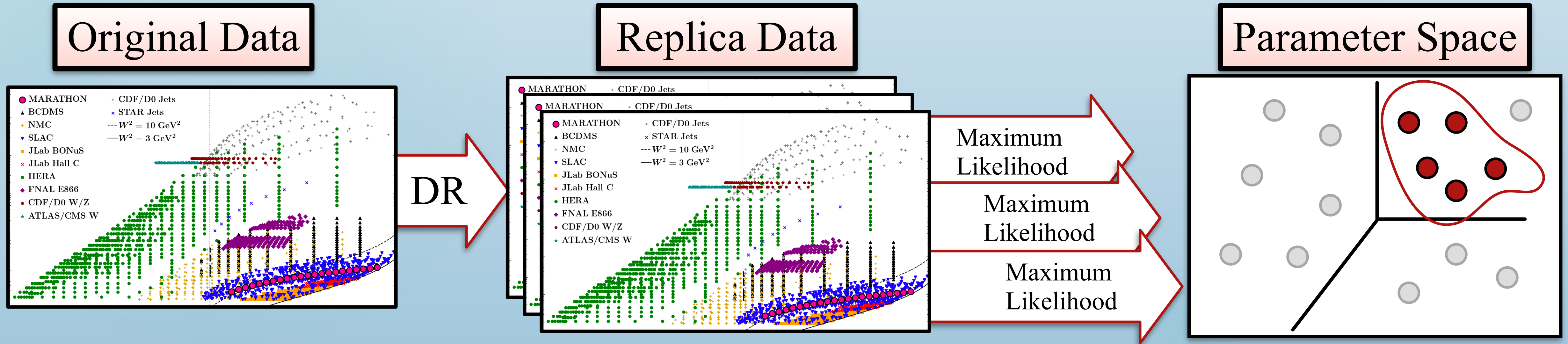
DR

Replica Data



Pseudo-Data → $$\tilde{\sigma} = \sigma + N(0,1)\ \alpha$$ ← Uncorrelated Uncertainties

↑ Data



For a quantity $O(\boldsymbol{a})$: (for example, a PDF at a given value of (x, Q^2))

$$E[O] = \int d^n a \; \rho(\boldsymbol{a} \,|\, data) \; O(\boldsymbol{a})$$

$$V[O] = \int d^n a \; \rho(\boldsymbol{a} \,|\, data) \; \left[O(\boldsymbol{a}) - E[O]\right]^2$$

Exact, but

$n = \mathcal{O}(100)$!

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Exact, but
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Build an MC ensemble

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$$E[O] \approx \frac{1}{N} \sum_k O(\boldsymbol{a}_k)$$

$$V[O] \approx \frac{1}{N} \sum_k \left[O(\boldsymbol{a}_k) - E[O]\right]^2$$

Average over k sets of the parameters (replicas)

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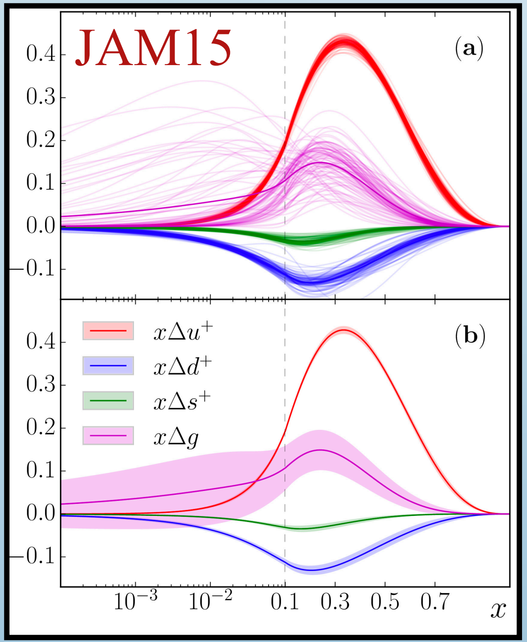
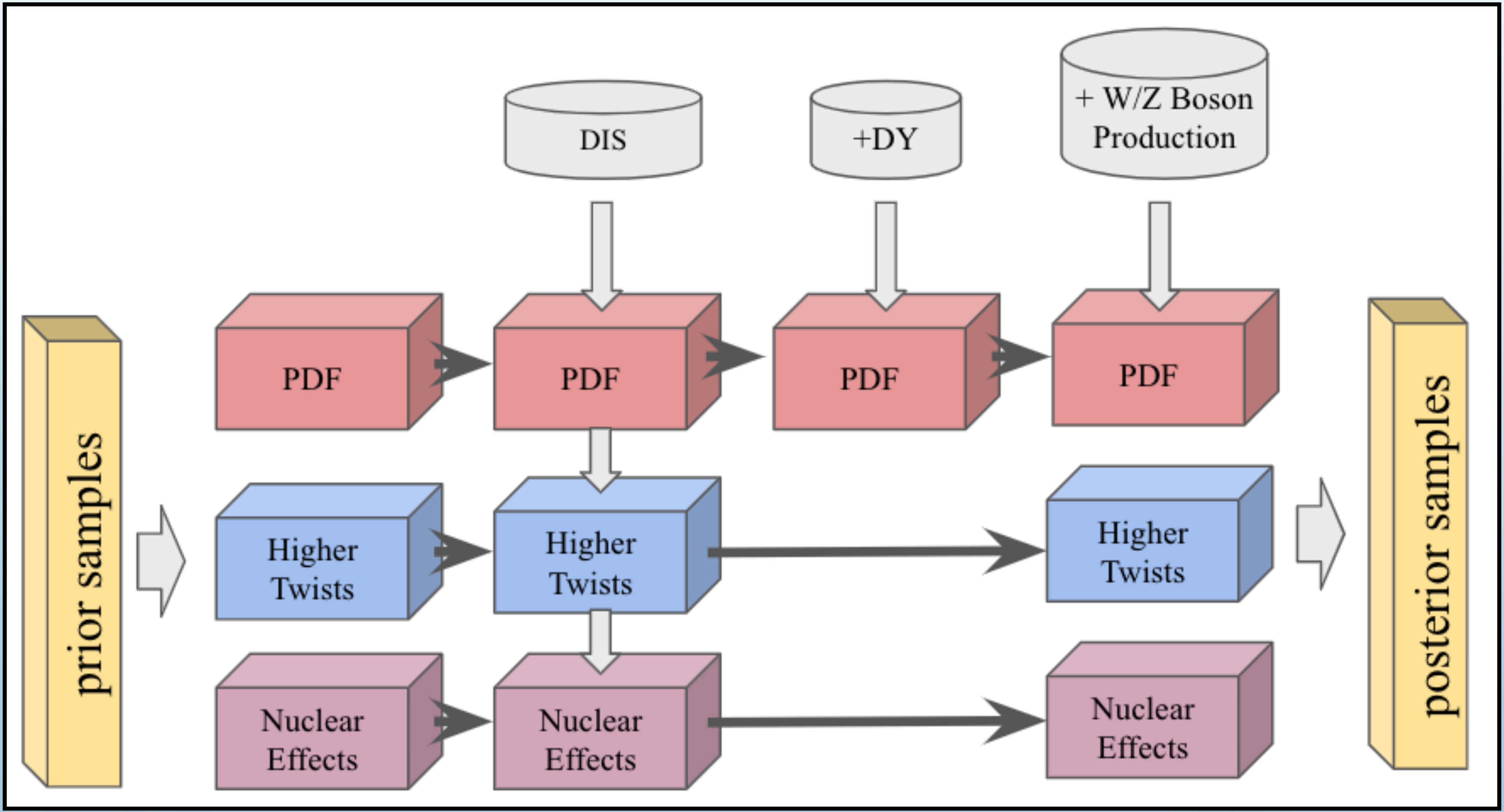
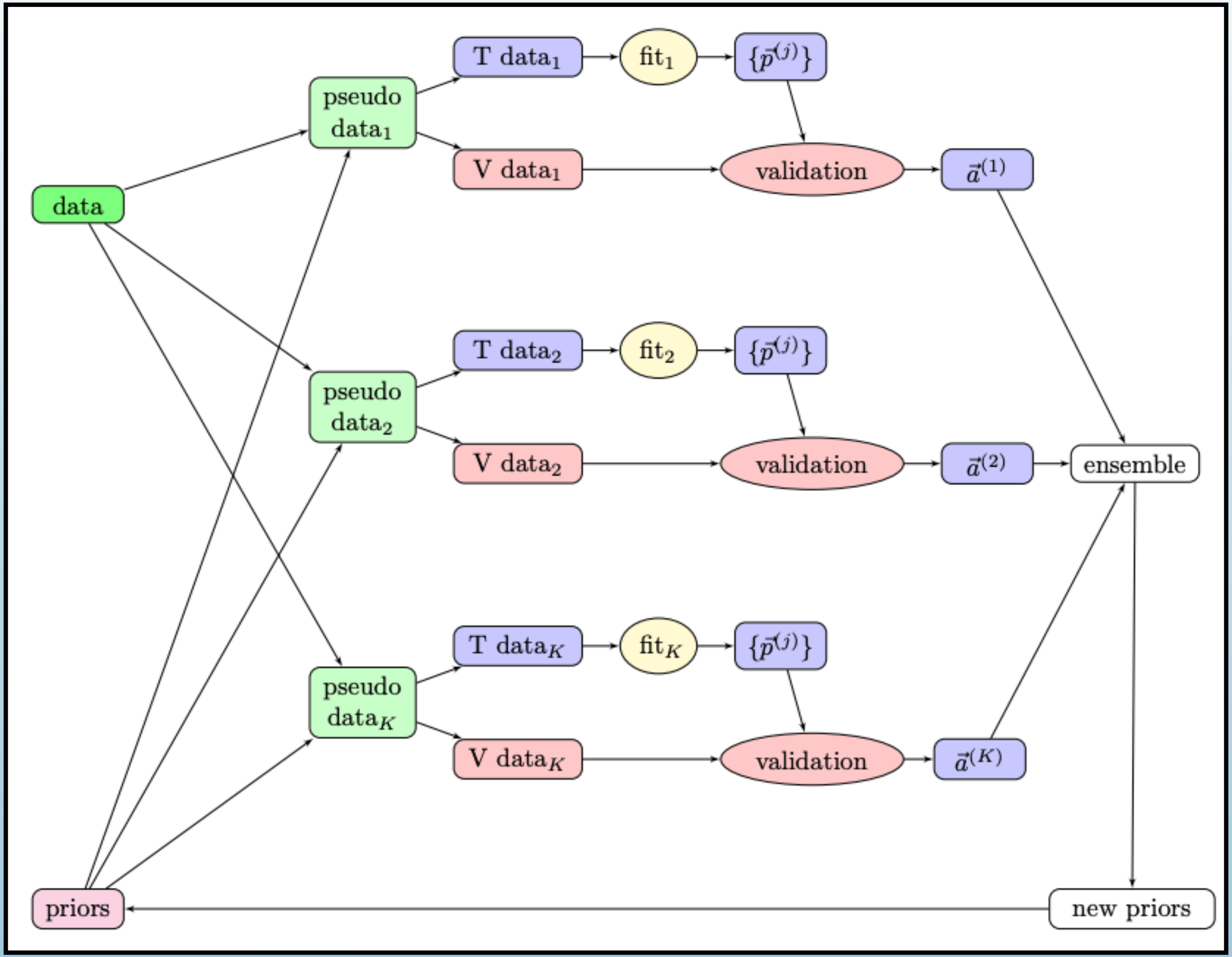
$$V[O] \approx \frac{1}{N}\sum_k \left[O(\boldsymbol{a}_k) - E[O]\right]^2$$

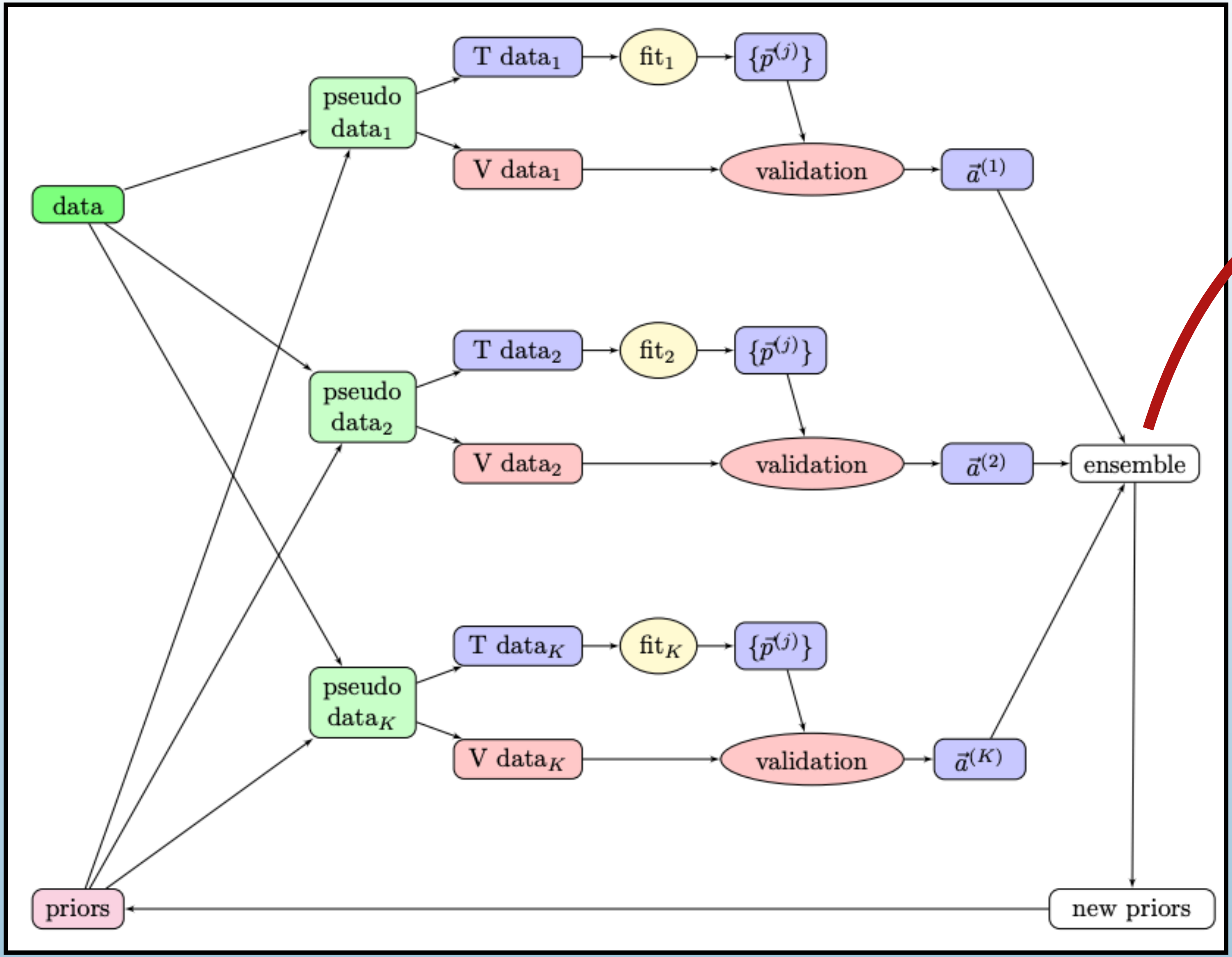
Average over k sets of the parameters (replicas)

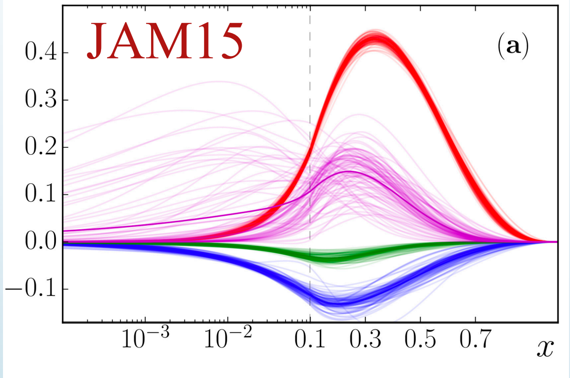


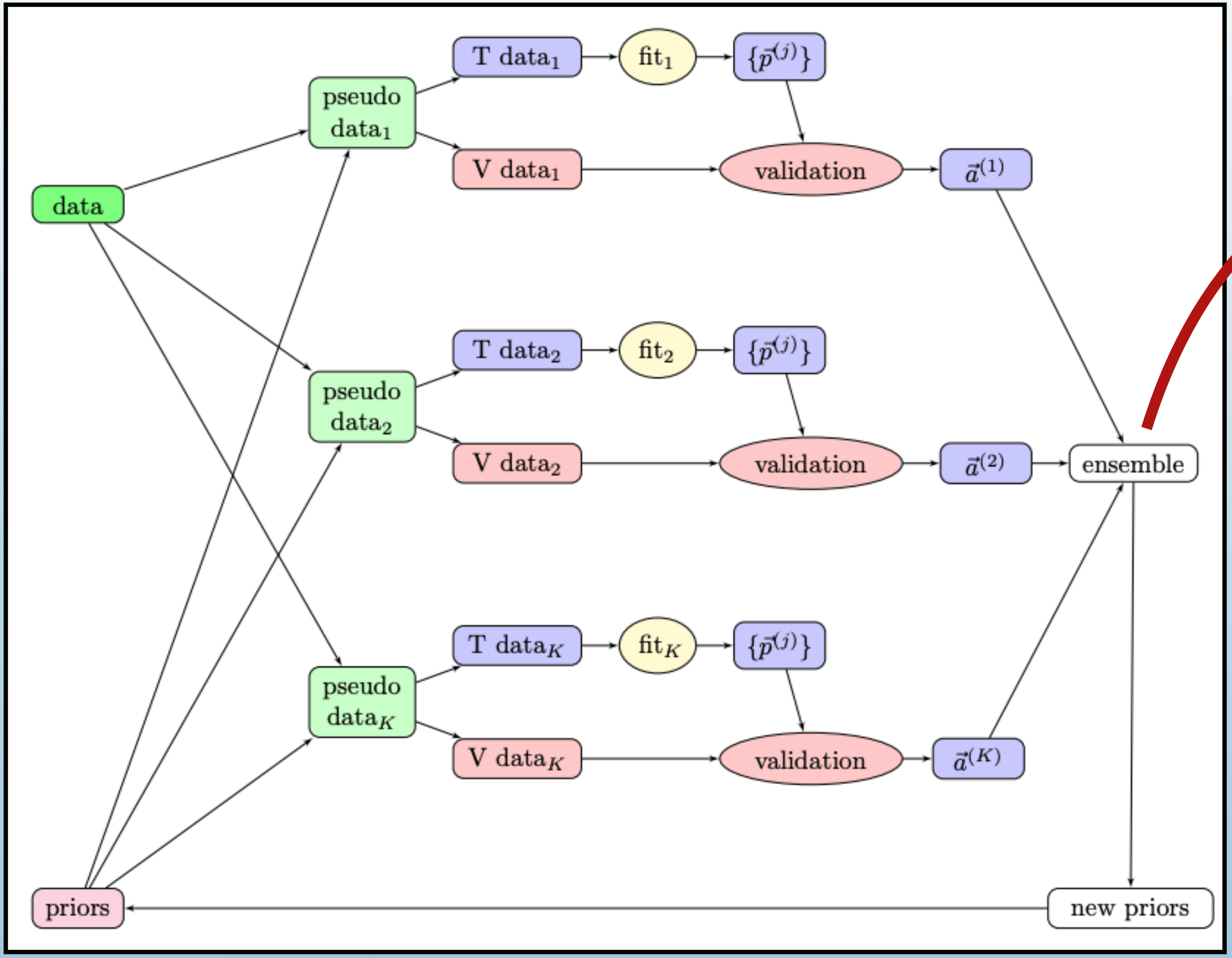

DIS
+DY
+ W/Z Boson Production
prior samples
PDF
PDF
PDF
PDF
Higher Twists
Higher Twists
Higher Twists
Nuclear Effects
Nuclear Effects
Nuclear Effects
posterior samples

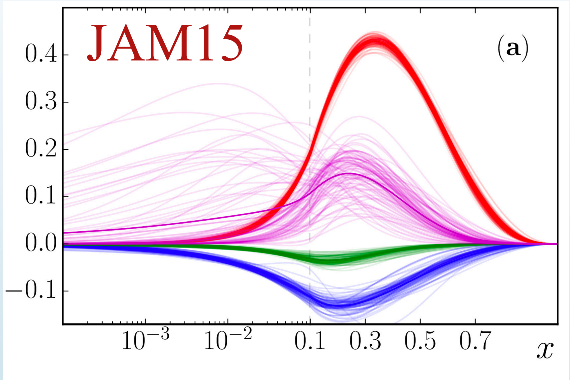


data
pseudo data$_1$
pseudo data$_2$
pseudo data$_K$
T data$_1$
V data$_1$
T data$_2$
V data$_2$
T data$_K$
V data$_K$
fit$_1$
fit$_2$
fit$_K$
$\{\vec{p}^{(j)}\}$
$\{\vec{p}^{(j)}\}$
$\{\vec{p}^{(j)}\}$
validation
validation
validation
$\vec{a}^{(1)}$
$\vec{a}^{(2)}$
$\vec{a}^{(K)}$
ensemble
new priors
priors


data
pseudo data$_1$
pseudo data$_2$
pseudo data$_K$
T data$_1$
V data$_1$
T data$_2$
V data$_2$
T data$_K$
V data$_K$
fit$_1$
fit$_2$
fit$_K$
$\{\vec{p}^{(j)}\}$
validation
$\vec{a}^{(1)}$
$\vec{a}^{(2)}$
$\vec{a}^{(K)}$
ensemble
priors
new priors


JAM15
(a)
0.4
0.3
0.2
0.1
0.0
−0.1
10^{-3}
10^{-2}
0.1
0.3
0.5
0.7
x

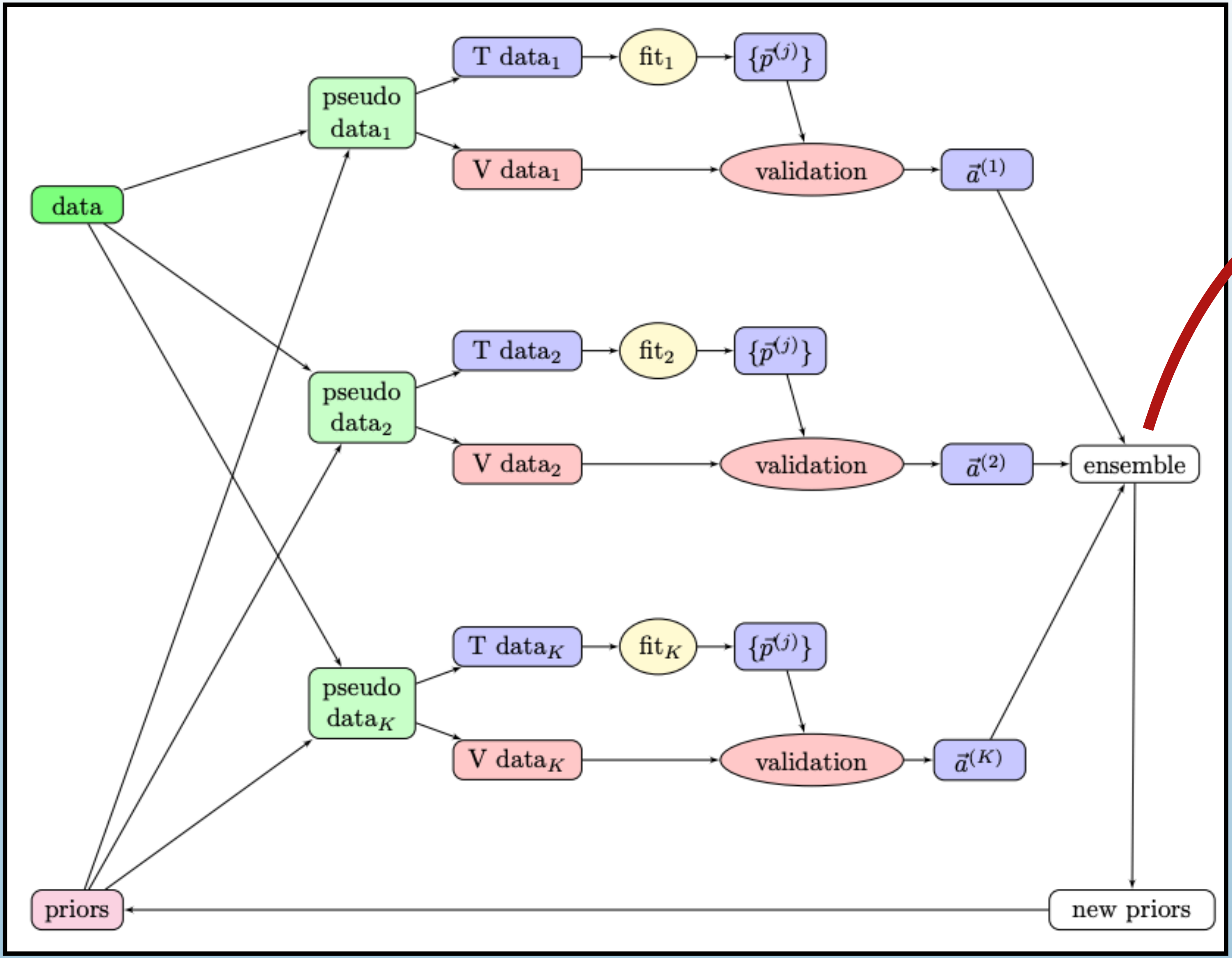


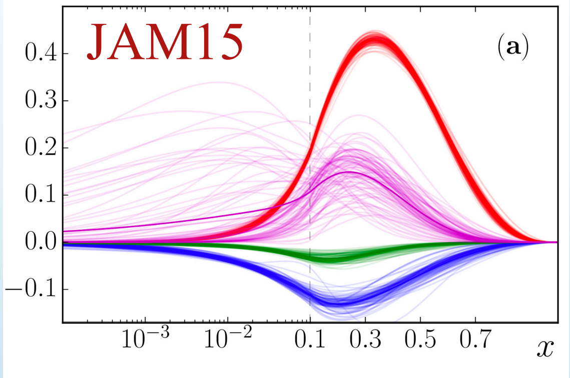


+

$$E[O] \approx \frac{1}{N} \sum_k O(\boldsymbol{a}_k)$$

$$V[O] \approx \frac{1}{N} \sum_k \left[O(\boldsymbol{a}_k) - E[O]\right]^2$$

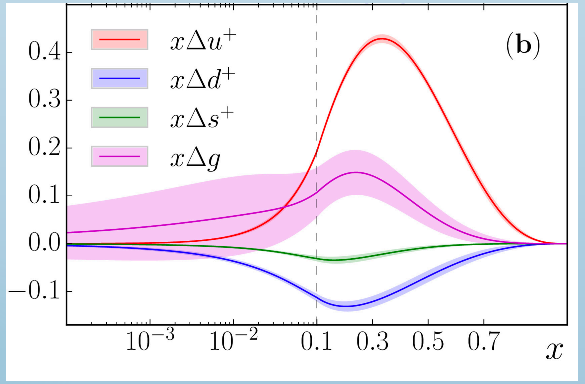




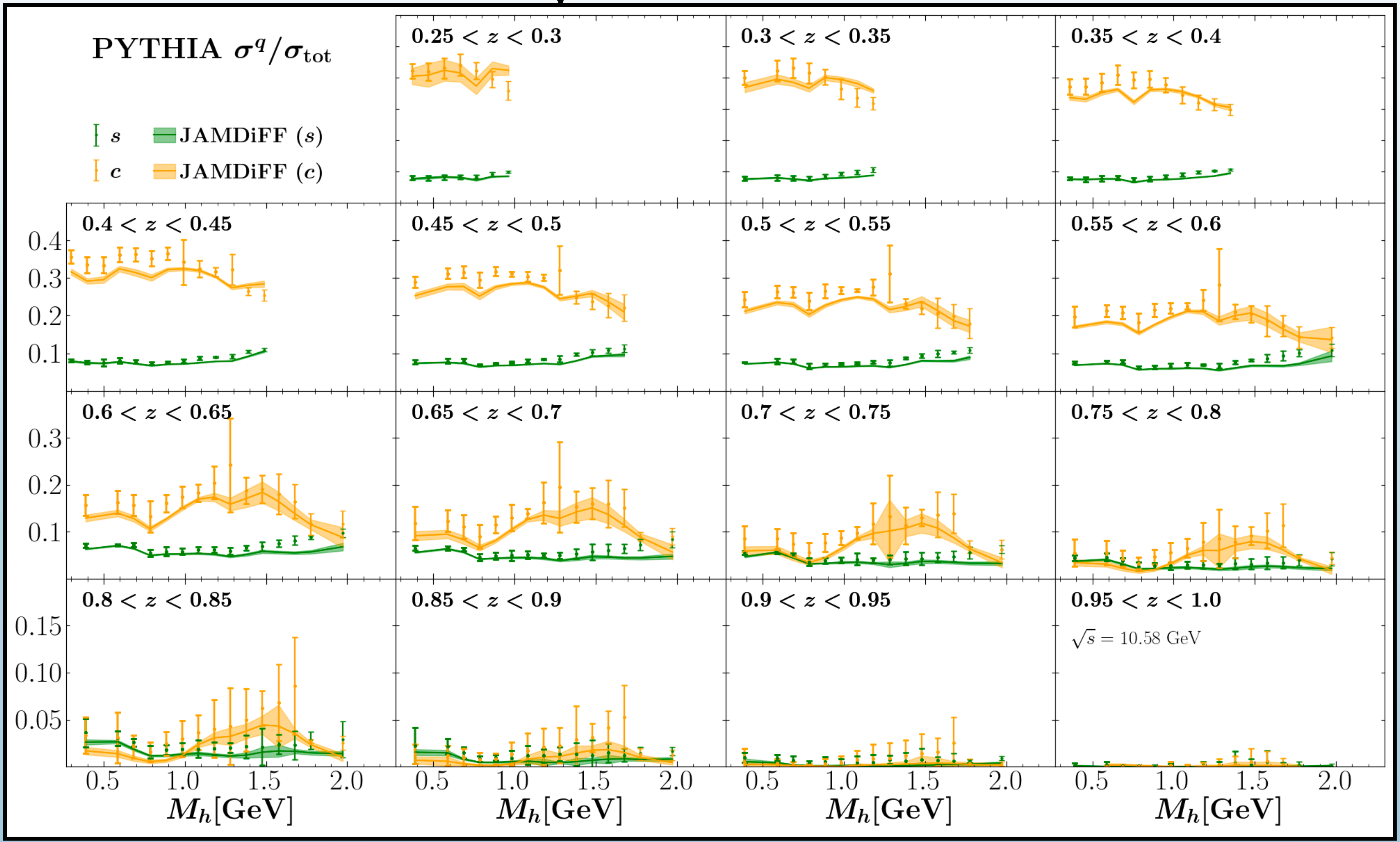
+

$$E[O] \approx \frac{1}{N}\sum_k O(\boldsymbol{a}_k)$$

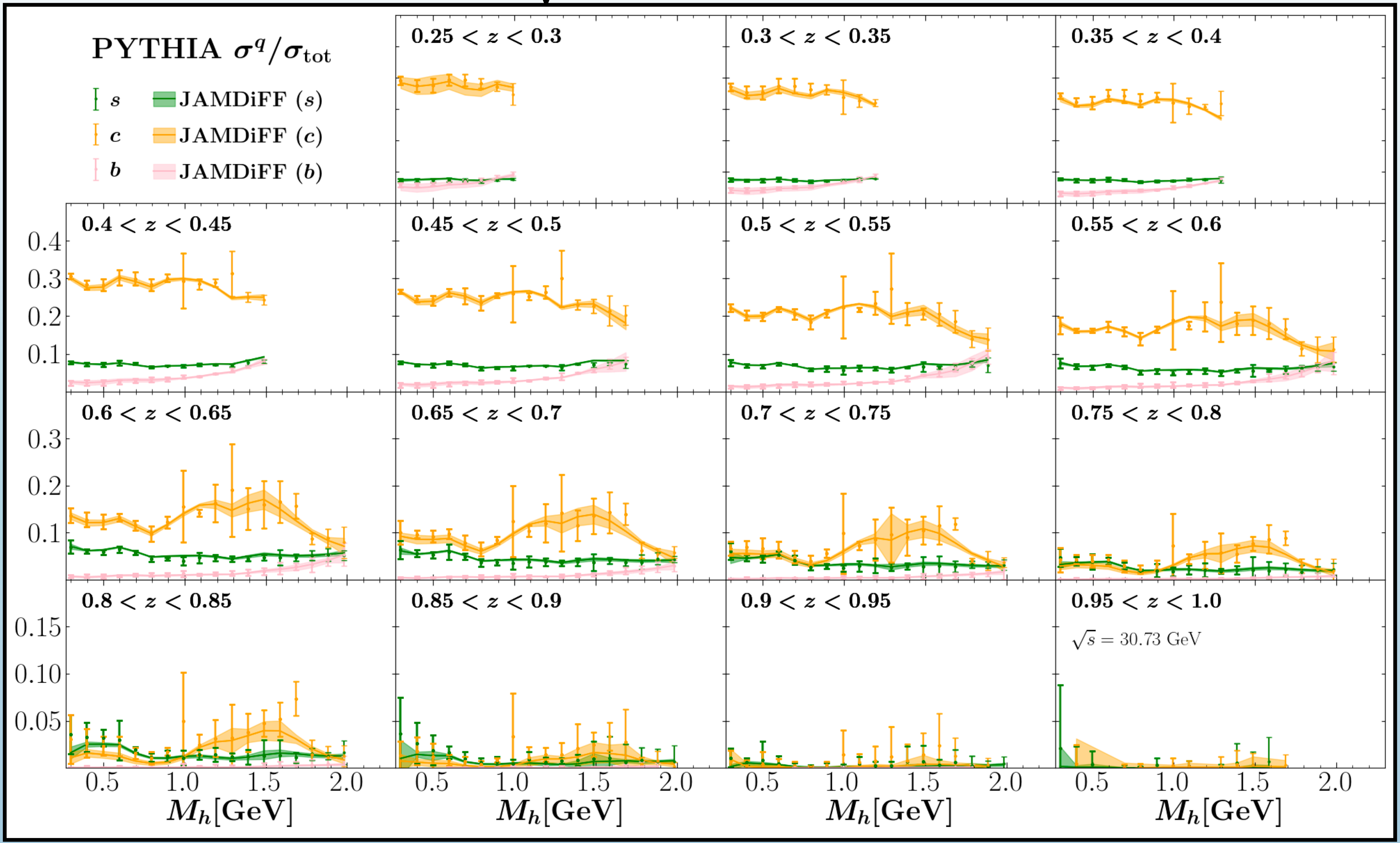
$$V[O] \approx \frac{1}{N}\sum_k \left[O(\boldsymbol{a}_k) - E[O]\right]^2$$



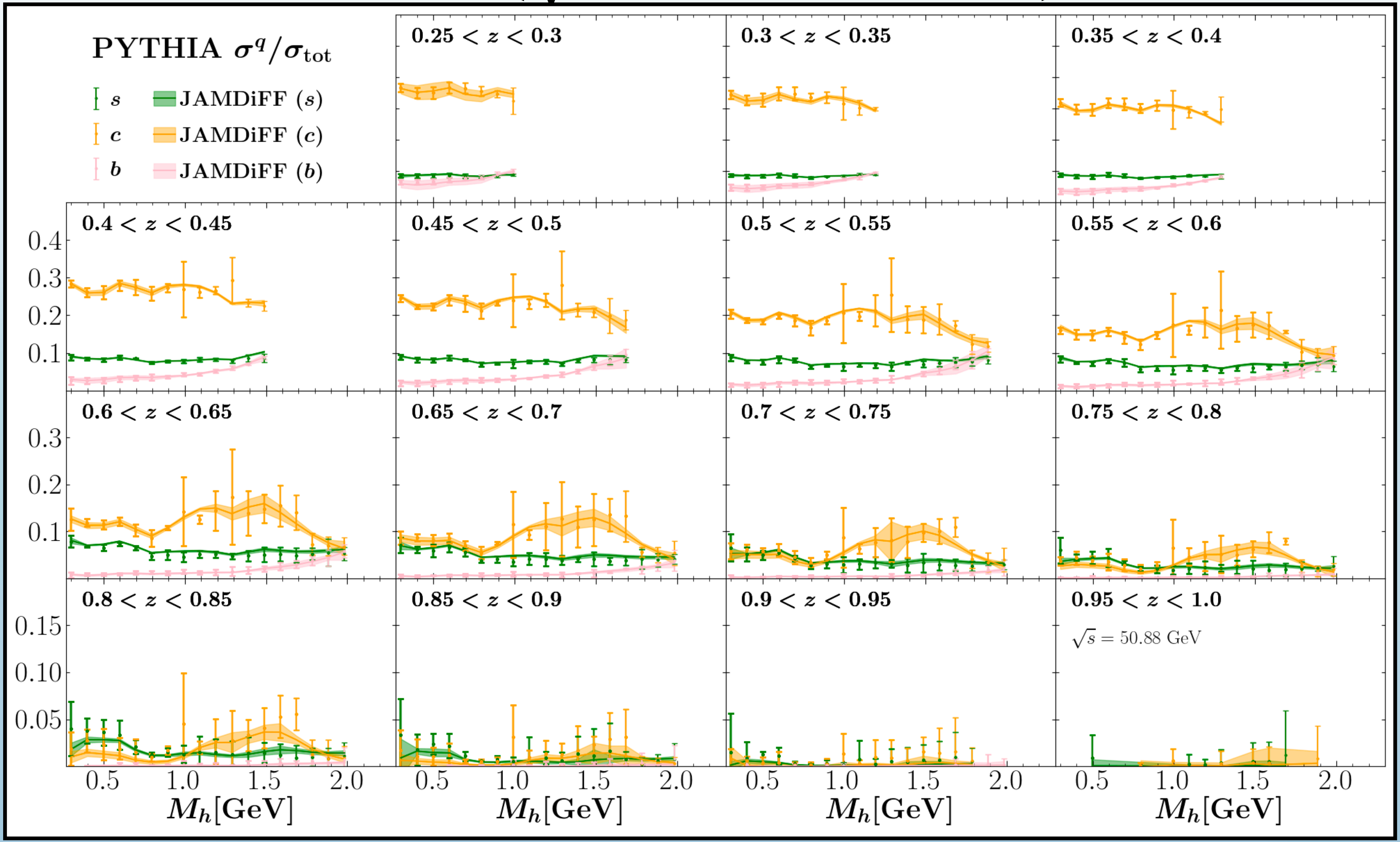
PYTHIA data ($\sqrt{s} = 10.58$ GeV)



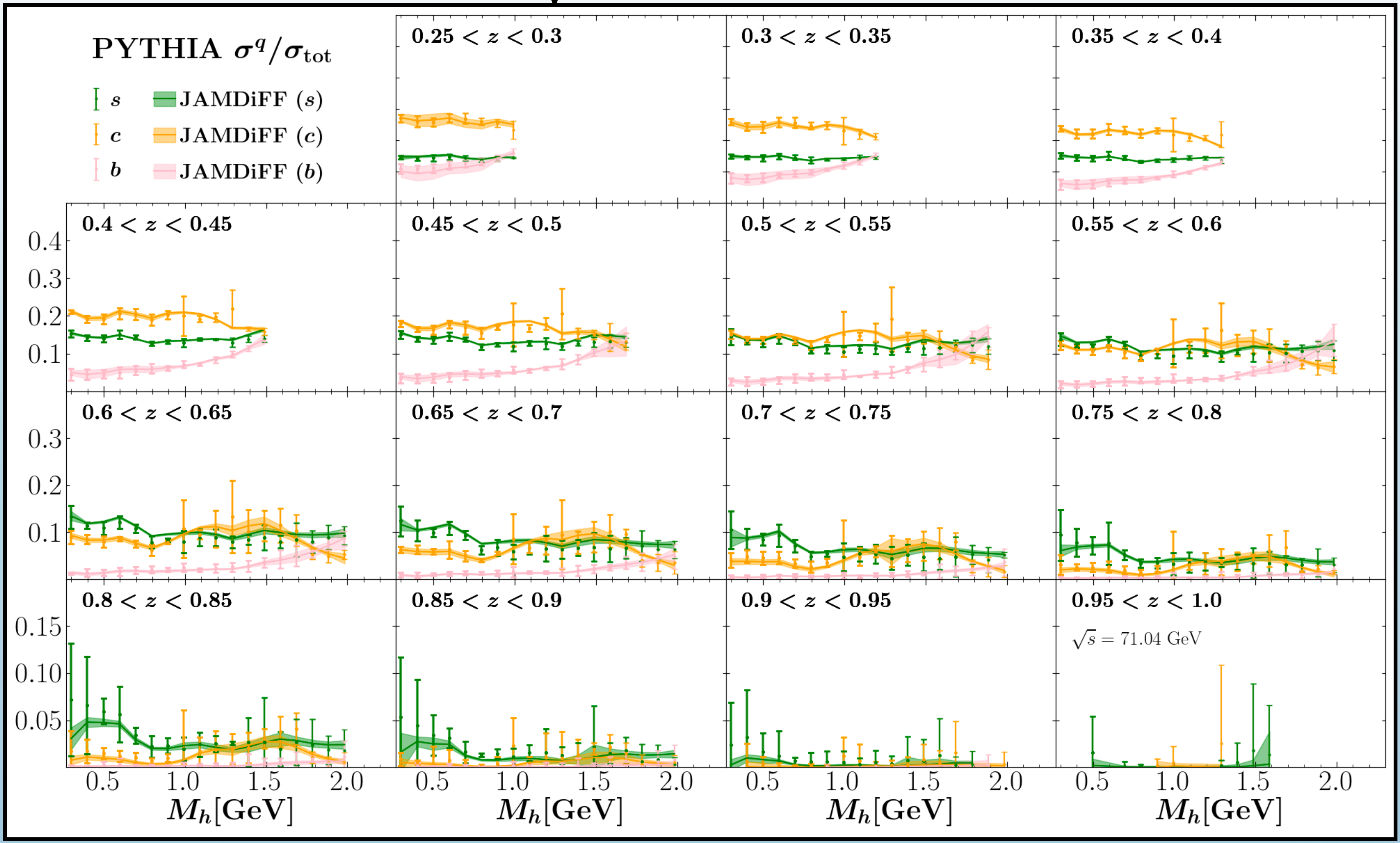
PYTHIA data ($\sqrt{s}$ = 30.73 GeV)



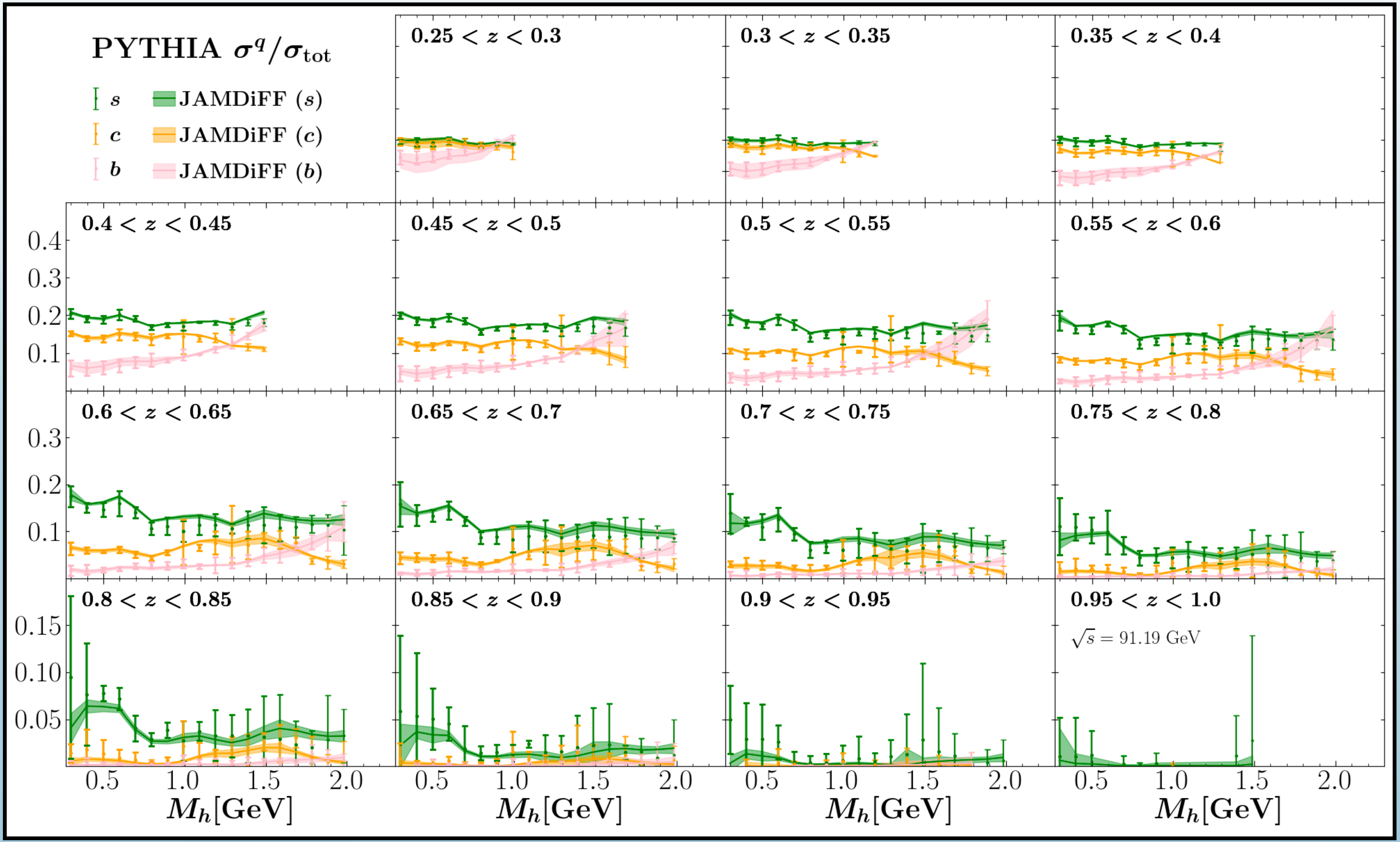
PYTHIA data ($\sqrt{s} = 50.88$ GeV)



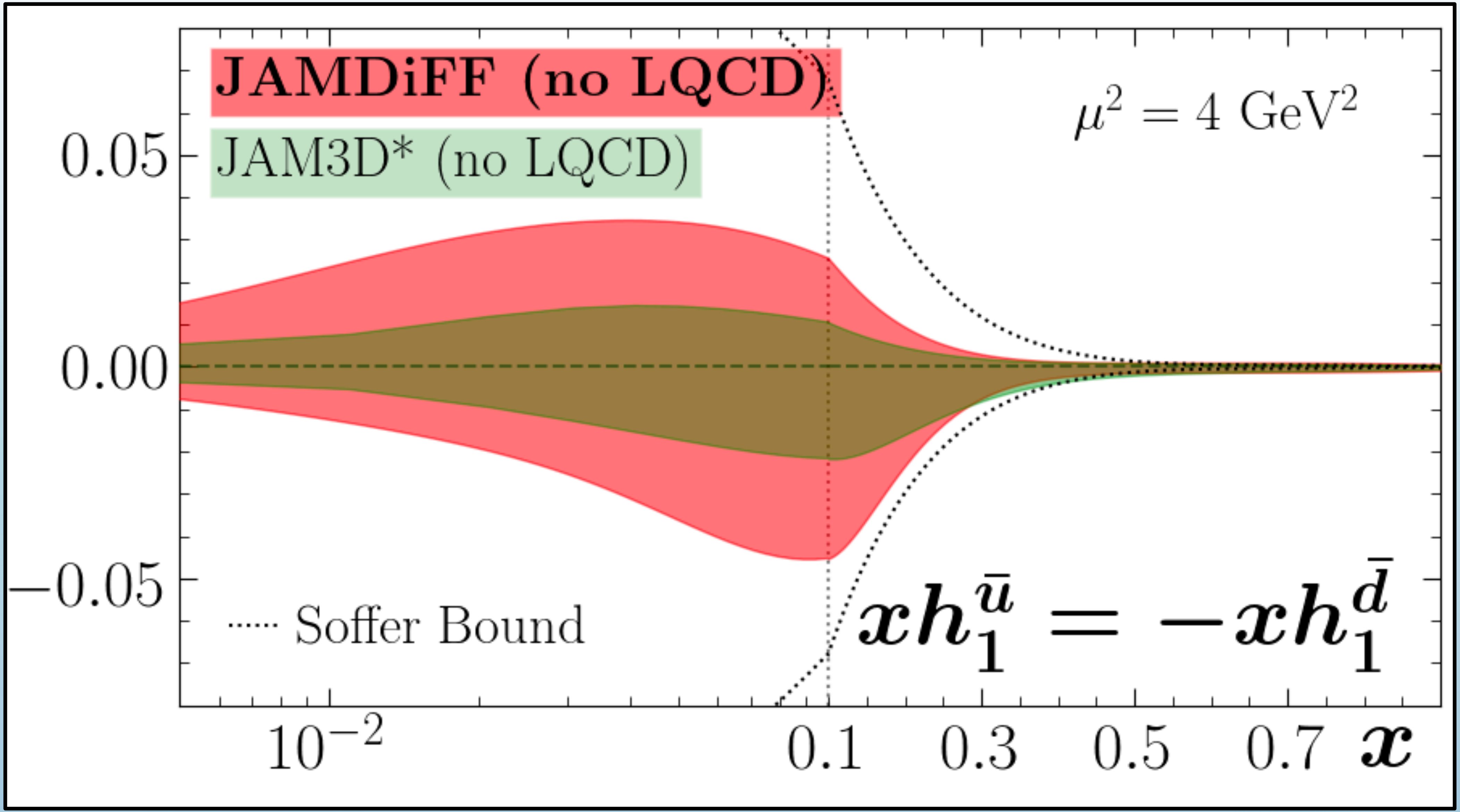
PYTHIA data ($\sqrt{s}$ = 71.04 GeV)



PYTHIA data ($\sqrt{s}$ = 91.19 GeV)



Transversity PDFs (antiquarks)



DiFF Parameterization

$$\mathbf{M}_h^u = [2m_\pi, 0.40, 0.50, 0.70, 0.75, 0.80, 0.90, 1.00, 1.20, 1.30, 1.40, 1.60, 1.80, 2.00]\ \text{GeV}.$$

$$D_1^q(z, \mathbf{M}_h^{q,i}) = \sum_{j=1,2,3} \frac{N_{ij}^q}{\mathcal{M}_{ij}^q} z^{\alpha_{ij}^q} (1-z)^{\beta_{ij}^q},$$

204 parameters for D_1

48 parameters for $H_1^{\sphericalangle}$

PDF Parameterization

$$h_1^{u_v}$$
$$h_1^{d_v}$$
$$h_1^{\bar{u}} = -h_1^{\bar{d}}$$

$$f(x,\mu_0^2) = \frac{N}{\mathcal{M}} x^\alpha (1-x)^\beta (1+\gamma\sqrt{x}+\eta x),$$

15 parameters for h_1

Tensor Charge Numbers

Fit	δu	δd	g_T
no LQCD	0.50(7)	-0.04(14)	0.54(12)
w/ LQCD	0.71(2)	-0.200(6)	0.91(2)