Simultaneous Global Analysis of Di-Hadron Fragmentation Functions and Transversity PDFs

Christopher Cocuzza

June 22, 2023
1. Introduction
2. Extraction of DiFFs
3. Extraction of Transversity PDFs
4. Extraction of Tensor Charges
5. Conclusions and Outlook
3-dimensional structure of nucleons:
• Parton distribution functions (PDFs)
• Fragmentation functions (FFs)
• Transverse momentum dependent distributions (TMDs)
• Generalized parton distributions (GPDs)
3-dimensional structure of nucleons:
• Parton distribution functions (PDFs)
• Fragmentation functions (FFs)
• Transverse momentum dependent distributions (TMDs)
• Generalized parton distributions (GPDs)

Collinear factorization in perturbative QCD
• Simultaneous determinations of PDFs, FFs, etc.
• Monte Carlo methods for Bayesian inference
Introduction

Hadron Structure

Global QCD Analysis
Introduction

LHC

Hadron Structure

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Introduction

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RHIC
Introduction

Hadron Structure

Global QCD Analysis

LHC

Jefferson Lab

RHIC
Introduction

LHC

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Hadron Structure

Global QCD Analysis

RHIC

USQCD
Introduction

Global QCD Analysis

Hadron Structure

RHIC

Jefferson Lab

LHC
Introduction

Global QCD Analysis

Hadron Structure

\[ \frac{d}{d \ln(\mu^2)} f_i(x, \mu) = \sum_i \int_x^1 \frac{dz}{z} P_{ij}(z, \mu) f_j \left( \frac{x}{z}, \mu \right) \]

Param. + Evolve + Factorization

\[ \sigma = \sum_{i,j} H_{ij} \otimes f_i \otimes f_j \]

RHIC

Jefferson Lab

USQCD

Extended Twisted Mass Collaboration

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Introduction

Global QCD Analysis

\[ \chi^2(a) = \sum_{i,e} \left( \frac{d_{i,e} - \sum_k r^k_{i,e} T_{i,e}(a)/N_c}{\alpha_{i,e}} \right)^2 + \sum_k (r^k)^2 + \left( \frac{1 - N_c}{\delta_{N_c}} \right)^2 \]

\[ \mathcal{L}(a, \text{data}) = \exp \left( -\frac{1}{2} \chi^2(a, \text{data}) \right) \]

\[ \mathcal{P}(a|\text{data}) \sim \mathcal{L}(a, \text{data}) \pi(a) \]

Hadron Structure

Param. + Evolve + Factorization

\[ \frac{d}{d \ln(\mu^2)} f_i(x, \mu) = \sum_j \int_x^1 \frac{dz}{z} P_{ij}(z, \mu) f_j \left( \frac{x}{z}, \mu \right) \]

\[ \sigma = \sum_{i,j} H_{ij} \otimes f_i \otimes f_j \]

\[ F_2^P \times 2^i \]

\[ Q^2 \text{ (GeV}^2\text{)} \]
Hadron Structure

Global QCD Analysis

\[ \sigma = \sum_{i,j} H_{ij} \otimes f_i \otimes f_j \]

\[ \frac{d}{d \ln(\mu^2)} f_i(x, \mu) = \sum_j \int_x^1 \frac{dz}{z} P_{ij}(z, \mu) f_j\left(\frac{x}{z}, \mu\right) \]

\[ \chi^2(a) = \sum_{i,e} \left( \frac{d_i - \sum_{k} \frac{x_i \mu_k}{\alpha_{i,k}} T_{i,e}(a)/N_k}{\alpha_{i,k}} \right)^2 + \sum_k (r_k)^2 + \left( \frac{1 - N_c}{\delta N_c} \right)^2 \]

\[ \chi^2 \text{ Minimization} \]

\[ \mathcal{L}(a, \text{data}) = \exp \left( -\frac{1}{2} \chi^2(a, \text{data}) \right) \]

\[ P(a|\text{data}) \sim \mathcal{L}(a, \text{data}) \pi(a) \]

Data Resampling

\[ \tilde{\sigma} = \sigma + N(0,1) \alpha \]
Approaches to Extract Transversity
Introduction

Approaches to Extract Transversity

Dihadron Frag.

- Radici + Bacchetta (RB18)
- Benel + Courtoy + Ferro-Hernandez (2020)

M. Radici and A. Bacchetta, Phys. Rev. Lett. 120, no. 19, 192001 (2018)
Introduction

Approaches to Extract Transversity

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- Benel + Courtoy + Ferro-Hernandez (2020)

TMD + Collinear Twist-3
- JAM3D

M. Radici and A. Bacchetta, Phys. Rev. Lett. 120, no. 19, 192001 (2018)

Approaches to Extract Transversity

**Dihadron Frag.**
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**TMD + Collinear Twist-3**
- JAM3D

**Lattice QCD**
- ETMC Collaboration
- PNDME Collaboration
- LHPC Collaboration

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**Introduction**

Dihadron Frag.

TMD + Collinear Twist-3

Lattice QCD
- C. Alexandrou et al., Phys. Rev. D **104**, no. 5, 054503 (2021)
JAM Global Analysis in the collinear DiFF Approach

First simultaneous extraction of $\pi^+\pi^- \text{ DiFFs } (D^q_1)$, IFFs $H^{\xi,q}_1$, and transversity PDFs $h^q_1$ at LO

Semi-Inclusive Annihilation

Semi-Inclusive Deep Inelastic Scattering

Proton-Proton Collisions


Tensor Charges

\[ \delta u \equiv \int_0^1 dx (h_1^u - h_1^{\bar{u}}), \]
\[ \delta d \equiv \int_0^1 dx (h_1^d - h_1^{\bar{d}}), \]
\[ g_T \equiv \delta u - \delta d, \]

Figure taken from D. Pitonyak
Tensor Charges

\[ \delta u \equiv \int_0^1 dx (h_1^u - h_1^{\bar{u}}), \]
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QCD Pheno for Transversity

Tensor Charges

Figure taken from D. Pitonyak

Anselmino, et al. (2007, 2009, 2013, 2015);
Goldstein, et al. (2014);
Kang, et al. (2016);
D’Alesio, et al. (2020);
Cammarota, et al. (2020);
Gamberg, et al. (2022)

Radici, et al. (2013, 2015, 2018);
Benel, et al. (2020);
Cocuzza, et al. (2023)
Introduction

Tensor Charges

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QCD Pheno for Transversity

Lattice QCD, Models

Anselmino, et al. (2007, 2009, 2013, 2015);
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Radici, et al. (2013, 2015, 2018);
Benel, et al. (2020);
Cocuzza, et al. (2023)

He, Ji (1995);
Barone, et al. (1997);
Schweitzer, et al. (2001);
Gamberg, Goldstein (2001);
Pasquini, et al. (2005);
Wakamatsu (2007);
Lorce (2009);
Gupta, et al. (2018);
Yamanaka, et al. (2018);
Hasan, et al. (2019);
Alexandrou, et al. (2019, 2023)
Yamanaka, et al. (2013);
Pitschmann, et al. (2015);
Xu, et al. (2015);
Wang, et al. (2018);
Liu, et al. (2019)
Tensor Charges

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\[ g_T \equiv \delta u - \delta d, \]

Herczeg (2001);
Erler, Ramsey-Musolf (2005);
Pospelov, Ritz (2005);
Severijns, et al. (2006);
Cirigliano, et al. (2013);
Courtoy, et al. (2015);
Yamanaka, et al. (2017);
Liu, et al. (2018);
Gonzalez-Alonso, et al. (2019)

Anselmino, et al. (2007, 2009, 2013, 2015);
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Xu, et al. (2015);
Wang, et al. (2018);
Liu, et al. (2019)
The Transverse Spin Puzzle?


Introduction
The Transverse Spin Puzzle?


**Introduction**

![Graph](image)
The Transverse Spin Puzzle?


Introduction
The Transverse Spin Puzzle?


Introduction
The Transverse Spin Puzzle?


RB18

JAM3D (no LQCD)

JAM3D (w/ LQCD)

Lattice (ETMC)
The Transverse Spin Puzzle?


Large disagreements between three approaches…
Can this be solved?
1. JAM Methodology
2. Extraction of DiFFs
3. Extraction of Transversity PDFs
4. Extraction of Tensor Charges
5. Conclusions and Outlook
Observables for DiFFs

SIA Cross Section

\[ \frac{d\sigma}{dz \, dM_h} = \frac{4\pi\alpha_{em}^2}{s} \sum_q e_q^2 D_1^q(z, M_h) \]
Observables for DiFFs

SIA Cross Section

\[ \frac{d\sigma}{dz \, dM_h} = \frac{4\pi\alpha^2_{em}}{s} \sum_q e_q^2 D_q^q(z, M_h) \]


SIA Artru-Collins Asymmetry

\[ A^{e^+e^-}(z, M_h, \bar{z}, \bar{M}_h) = \frac{\sin^2 \theta \sum_q e_q^2 H_1^{q,q}(z, M_h)H_1^{q,\bar{q}}(\bar{z}, \bar{M}_h)}{(1 + \cos^2 \theta) \sum_q e_q^2 D_1^q(z, M_h)D_1^q(\bar{z}, \bar{M}_h)} \]

Data for DiFFs

<table>
<thead>
<tr>
<th>SIA cross section</th>
<th>Belle</th>
<th>1121 points</th>
</tr>
</thead>
<tbody>
<tr>
<td>SIA Artru-Collins</td>
<td>Belle</td>
<td>183 points</td>
</tr>
</tbody>
</table>

Diagram showing data points for Belle.
Extraction of DiFFs

## Data for DiFFs

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</tr>
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</table>

### $\pi^+\pi^-$ DiFFs

\[
D_1^u = D_1^d = D_1^\bar{u} = D_1^\bar{d}, \\
D_1^s = D_1^\bar{s}, \\
D_1^c = D_1^\bar{c}, \\
D_1^b = D_1^\bar{b},
\]

5 independent functions (w/ $D_1^g$) [supplement with PYTHIA data]
## Data for DiFFs

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### π⁺π⁻ DiFFs

\[
D_1^u = D_1^d = D_1^{\bar{u}} = D_1^{\bar{d}},
\]

\[
D_1^s = D_1^{\bar{s}}, \quad D_1^c = D_1^{\bar{c}}, \quad D_1^b = D_1^{\bar{b}},
\]

5 independent functions (w/ \(D_1^g\))

[supplement with PYTHIA data]

\[
H_1^{\perp, u} = -H_1^{\perp, d} = -H_1^{\perp, \bar{u}} = H_1^{\perp, \bar{d}},
\]

\[
H_1^{\perp, s} = -H_1^{\perp, \bar{s}} = H_1^{\perp, c} = -H_1^{\perp, \bar{c}} = 0,
\]

1 independent function

Quality of Fit (Unpolarized Cross Section)

\[ \frac{d^2\sigma}{dzdM_h} \text{ [nb/GeV]} \]

- Belle
- JAMDiFF

\[ 0.25 < z < 0.3 \]
\[ 0.3 < z < 0.35 \]
\[ 0.35 < z < 0.4 \]
\[ 0.4 < z < 0.45 \]
\[ 0.45 < z < 0.5 \]
\[ 0.5 < z < 0.55 \]
\[ 0.55 < z < 0.6 \]
\[ 0.6 < z < 0.65 \]
\[ 0.65 < z < 0.7 \]
\[ 0.7 < z < 0.75 \]
\[ 0.75 < z < 0.8 \]
\[ 0.8 < z < 0.85 \]
\[ 0.85 < z < 0.9 \]
\[ 0.9 < z < 0.95 \]
\[ 0.95 < z < 1.0 \]
\[ \sqrt{s} = 10.58 \text{ GeV} \]

Quality of Fit (Artru-Collins Asymmetry)

\((z, M_h)\) binning

\((M_h, \overline{M}_h)\) binning

\((z, \overline{z})\) binning

A. Vossen et al.,
Extraction of DiFFs

Bound: $D^q > 0$

Bound: $|H_{1,q}^{\text{eff}}| < D_q^1$

1. JAM Methodology
2. Extraction of DiFFs
3. Extraction of Transversity PDFs
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Observables for Transversity PDFs

SIDIS asymmetry ($p$ and $D$)

\[ A_{UT}^{\text{SIDIS}} = c(y) \frac{\sum_q e_q^2 h_1^q(x) H_1^{4,q}(z, M_h)}{\sum_q e_q^2 f_1^q(x) D_1^q(z, M_h)} \]

**Extraction of DiFFs**

### Observables for Transversity PDFs

#### SIDIS asymmetry ($p$ and $D$)

\[ A_{UT}^{\text{SIDIS}} = c(y) \frac{\sum_q e_q^2 h_1^q(x) H_1^{A_1, q}(z, M_h)}{\sum_q e_q^2 f_1^q(x) D_1^q(z, M_h)} \]


#### pp Asymmetry

\[ A_{UT}^{pp} = \frac{\mathcal{H}(M_h, P_{hT}, \eta)}{\mathcal{D}(M_h, P_{hT}, \eta)} \]

Extraction of Transversity PDFs

Data for PDFs

<table>
<thead>
<tr>
<th>SIDIS (p, D)</th>
<th>COMPASS, HERMES</th>
<th>64 points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proton-Proton</td>
<td>STAR</td>
<td>269 points</td>
</tr>
</tbody>
</table>

![Graph depicting data points for different experiments]

- HERMES
- COMPASS
- STAR (200 GeV)
- STAR (500 GeV)
Data for PDFs

<table>
<thead>
<tr>
<th>SIDIS ($p, D$)</th>
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</table>

Parameterization Choices

- 3 independent observables
- 3 independent functions

\[
h_1^{u,v} = h_1^{d,v} = - h_1^{\bar{u}}
\]
Extraction of Transversity PDFs

Data for PDFs

<table>
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<th>SIDIS (p, D)</th>
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Parameterization Choices

3 independent observables
3 independent functions

\[
\begin{aligned}
    h_v^u &= h_v^d \\
    h_1^\bar{u} &= - h_1^\bar{d}
\end{aligned}
\]

Prediction from large-\(N_c\) limit

## Quality of Fit

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$N_{\text{dat}}$</th>
<th>$\chi^2_{\text{red}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belle (cross section)</td>
<td>1094</td>
<td>1.05</td>
</tr>
<tr>
<td>Belle (Artru-Collins)</td>
<td>183</td>
<td>0.78</td>
</tr>
<tr>
<td>HERMES</td>
<td>12</td>
<td>1.09</td>
</tr>
<tr>
<td>COMPASS ($p$)</td>
<td>26</td>
<td>0.75</td>
</tr>
<tr>
<td>COMPASS ($D$)</td>
<td>26</td>
<td>0.74</td>
</tr>
<tr>
<td>STAR (2015)</td>
<td>24</td>
<td>1.83</td>
</tr>
<tr>
<td>STAR (2018)</td>
<td>106</td>
<td>1.06</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>1471</strong></td>
<td><strong>1.02</strong></td>
</tr>
</tbody>
</table>
Quality of Fit (SIDIS)

\[ A_{UT}^{\text{SIDIS}} \]

- \( x_{\text{bj}} \) range: \( 10^{-2} \) to \( 1 \)
- \( M_h \) range: \( 0.5 \) to \( 1.5 \) GeV
- \( z \) range: \( 0.4 \) to \( 0.8 \)

- HERMES
- COMPASS \( p \)
- COMPASS \( D \)
- JAMDiFF (HERMES)
- JAMDiFF (COMPASS \( p \))
- JAMDiFF (COMPASS \( D \))

A. Airapetian et al., JHEP 06, 017 (2008)

Quality of Fit (STAR $\sqrt{s} = 200$ GeV)

Extraction of Transversity PDFs

Extraction of Transversity PDFs

Quality of Fit (STAR $\sqrt{s} = 500$ GeV)

$A^p_{UT} \quad \sqrt{s} = 500$ GeV ($R < 0.7$)

- STAR $\eta > 0$
- JAMDiFF $\eta > 0$
- STAR $\eta < 0$
- JAMDiFF $\eta < 0$

Transversity PDFs

Extraction of Transversity PDFs
Transversity PDFs

Soffer Bound: $|h_1^q| < \frac{1}{2} [f_1^q + g_1^q]$

Extraction of Transversity PDFs

Transversity PDFs

JAM3D* = JAM3D-22 (no LQCD)
+ Antiquarks w/ \( \bar{u} = -\bar{d} \)
+ small-\( x \) constraint (see slide 23)

Soffer Bound: \( |h_1^q| < \frac{1}{2} [f_1^q + g_1^q] \)

Transversity PDFs

\[ \text{JAM3D}^* = \text{JAM3D-22 (no LQCD)} \]
\[ + \text{Antiquarks w/ } \bar{u} = - \bar{d} \]
\[ + \text{small-}x \text{ constraint (see slide 23)} \]

Agreement between all three analyses within errors

Soffer Bound: \[ |h_1^q| < \frac{1}{2} [f_1^q + g_1^q] \]

1. JAM Methodology
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Controlling Extrapolation

\[ \delta u \equiv \int_0^1 dx (h_1^u - h_1^\bar{u}), \]

\[ \delta d \equiv \int_0^1 dx (h_1^d - h_1^\bar{d}), \]

\[ g_T \equiv \delta u - \delta d, \]
Controlling Extrapolation

\[ \delta u \equiv \int_0^1 dx (h_1^u - h_1^\bar{u}), \]
\[ \delta d \equiv \int_0^1 dx (h_1^d - h_1^\bar{d}), \]
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Controlling Extrapolation

\[ \delta u \equiv \int_0^1 dx (h_1^u - h_1^{\bar{u}}), \]

\[ \delta d \equiv \int_0^1 dx (h_1^d - h_1^{\bar{d}}), \]

\[ g_T \equiv \delta u - \delta d, \]

Measured Region
Controlling Extrapolation

\[ \delta u \equiv \int_0^1 dx (h_1^u - h_1^u), \]
\[ \delta d \equiv \int_0^1 dx (h_1^d - h_1^d), \]
\[ g_T \equiv \delta u - \delta d, \]

Soffer Bound: \( |h_1^q| < \frac{1}{2}[f_1^q + g_1^q] \)


Large \( x \geq 0.3 \)
Controlling Extrapolation

\[ x h_{1}^{u} \]

Small \( x \lesssim 0.005 \)

\[ \delta u \equiv \int_{0}^{1} dx (h_{1}^{u} - h_{1}^{\tilde{u}}), \]

\[ \delta d \equiv \int_{0}^{1} dx (h_{1}^{d} - h_{1}^{\tilde{d}}), \]

\[ g_{T} \equiv \delta u - \delta d, \]

\[ h_{1}^{q} \rightarrow x^{\alpha_{q}} \quad \alpha_{q} = 1 - 2 \sqrt{\frac{\alpha_{s} N_{c}}{2\pi}} \approx 0.17 \pm 0.085 \]

Large \( x \gtrsim 0.3 \)

Soffer Bound: \( |h_{1}^{q}| < \frac{1}{2} [f_{1}^{q} + g_{1}^{q}] \)


Extraction of Tensor Charges

Tensor Charges

\[
\delta d
\]

JAMDiFF (no LQCD)  JAM3D* (no LQCD)

\[ \mu^2 = 4 \text{ GeV}^2 \]

\( \delta u \)

0.4 0.6 0.8 0.5 1.0 1.5 2.0

\( g_T \)

Anselmino et al (2013)
Radici, Bacchetta (2018)
Benel et al (2019)
D’Alesio et al (2020)
PNDME (2018)
ETMC (2020)
JAM3D (no LQCD)
JAMDiFF (no LQCD)

Radici, Bacchetta (2018)
Tensor Charges

LQCD

C. Alexandrou et al., Phys. Rev. D 102, 054517 (2020)

Extraction of Tensor Charges

C. Alexandrou et al., Phys. Rev. D 102, 054517 (2020)
Extraction of Tensor Charges

Tensor Charges

Consistent with RB18 and JAM3D* (no LQCD).

What happens if we include LQCD in the fit?
Experiment + Lattice + Theory

**EXPERIMENT** (measured region)

**LATTICE** (full moments)

\[
\begin{align*}
\delta u &= \int_0^1 dx (h_1^u - h_1^d), \\
\delta d &= \int_0^1 dx (h_1^d - h_1^u), \\
g_T &= \delta u - \delta d,
\end{align*}
\]

**THEORY** (unmeasured regions)

\[
|h_1^q| < \frac{1}{2} [f_1^q + g_1^q]
\]

\[
\alpha_q = 1 - 2\sqrt{\frac{\alpha_s N_c}{2\pi}}
\]
Experiment + Lattice + Theory

EXPERIMENT
(measured region)

\[ q^2 \text{ (GeV)}^2 \]

LATTICE
(full moments)

\[
\delta u \equiv \int_0^1 dx (h_1^u - h_1^d), \\
\delta d \equiv \int_0^1 dx (h_1^d - h_1^d), \\
g_T \equiv \delta u - \delta d,
\]

THEORY
(unmeasured regions)

\[
|h_1^q| < \frac{1}{2} [f_1^q + g_1^q] \\
\alpha_q = 1 - 2\sqrt{\frac{\alpha_s N_c}{2\pi}}
\]

Presently, trivial to find compatibility between any two
**Experiment + Lattice + Theory**

**EXPERIMENT** (measured region)

Presently, trivial to find compatibility between any two

**LATTICE** (full moments)

\[
\delta u = \int_0^1 dx (h_1^u - h_1^q), \\
\delta d = \int_0^1 dx (h_1^d - h_1^q), \\
g_T \equiv \delta u - \delta d
\]

**THEORY** (unmeasured regions)

\[
|h_1^q| < \frac{1}{2} [f_1^q + g_1^q] \\
\alpha_q = 1 - 2 \sqrt{\frac{\alpha_s N_c}{2\pi}}
\]

Extraction of Tensor Charges
**Extraction of Tensor Charges**

**Experiment + Lattice + Theory**

**EXPERIMENT** (measured region)

**LATTICE** (full moments)

**THEORY** (unmeasured regions)

\[
| h^q_1 | < \frac{1}{2} [f^q_1 + g^q_1] \\
\alpha_q = 1 - 2 \sqrt{\frac{\alpha_s N_c}{2\pi}}
\]

Presently, trivial to find compatibility between any two
Experiment + Lattice + Theory

Presently, trivial to find compatibility between any two

**Extraction of Tensor Charges**

**EXPERIMENT**
(measured region)

| $h_1^q$ | $< \frac{1}{2} [f_1^q + g_1^q]$ |

**LATTICE**
(full moments)

$\delta u \equiv \int_0^1 dx (h_1^u - h_1^d)$,

$\delta d \equiv \int_0^1 dx (h_1^d - h_1^d)$,

$g_T \equiv \delta u - \delta d$,

**THEORY**
(unmeasured regions)

$\alpha_q = 1 - 2\sqrt{\frac{\alpha_s N_c}{2\pi}}$

$|h_1^q| < \frac{1}{2} [f_1^q + g_1^q]$
Experiment + Lattice + Theory

**Extraction of Tensor Charges**

- **EXPERIMENT** (measured region)
- **LATTICE** (full moments)
- **THEORY** (unmeasured regions)

\[
| h_1^q | < \frac{1}{2} \left[ f_1^q + g_1^q \right]
\]

\[
\alpha_q = 1 - 2 \sqrt{\frac{\alpha_s N_c}{2\pi}}
\]

Presently, trivial to find compatibility between any two

Only meaningful when all three are included
## Quality of Fit

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$N_{\text{dat}}$</th>
<th>$\chi^2_{\text{red}}$</th>
<th>LQCD</th>
<th>w/ LQCD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belle (cross section)</td>
<td>1094</td>
<td>1.05</td>
<td>1.06</td>
<td></td>
</tr>
<tr>
<td>Belle (Artru-Collins)</td>
<td>183</td>
<td>0.78</td>
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<td></td>
</tr>
<tr>
<td>HERMES</td>
<td>12</td>
<td>1.09</td>
<td>1.12</td>
<td></td>
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<tr>
<td>COMPASS ($p$)</td>
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<tr>
<td>ETMC $\delta u$</td>
<td>1</td>
<td>—</td>
<td>—</td>
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</tr>
<tr>
<td>ETMC $\delta d$</td>
<td>1</td>
<td>—</td>
<td>—</td>
<td>1.10</td>
</tr>
<tr>
<td>PNDME $\delta u$</td>
<td>1</td>
<td>—</td>
<td>—</td>
<td>8.20</td>
</tr>
<tr>
<td>PNDME $\delta d$</td>
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<td>—</td>
<td>—</td>
<td>0.03</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>1475</td>
<td><strong>1.02</strong></td>
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Physical Pion Mass

$N_f = 2 + 1 + 1$

Use $\delta u$ and $\delta d$ instead of $g_T$
Transversity PDFs (w/ LQCD)

$JAMDiFF$ (w/ LQCD)

$JAM3D^*$ (w/ LQCD)

$x h_{1}^{u,v}$

$x h_{1}^{d,v}$

$\mu^2 = 4 \text{ GeV}^2$

---

Soffer bound
Extraction of Tensor Charges

Transversity PDFs (w/ LQCD)

JAM3D* = JAM3D-22 (w/ LQCD)
+ Antiquarks w/ $\bar{u} = -\bar{d}$
+ small-$x$ constraint (see slide 23)
+ $\delta u$, $\delta d$ from ETMC & PNDME
  (instead of $g_T$ from ETMC)
Transversity PDFs (w/ LQCD)

JAM3D* = JAM3D-22 (w/ LQCD) + Antiquarks w/ $\bar{u} = -\bar{d}$ + small-$x$ constraint (see slide 23) + $\delta u, \delta d$ from ETMC & PNDME (instead of $g_T$ from ETMC)

JAMDiFF (w/ LQCD) and JAM3D* (w/ LQCD) largely agree
Tensor Charges (w/ LQCD)

Extraction of Tensor Charges

$$\delta d$$

$0.2$

$0.1$

$0.0$

$-0.1$

$-0.2$

$-0.3$

$-0.4$

$-0.5$

$\delta u$

$0.4$

$0.6$

$0.8$

$0.5$

$1.0$

$1.5$

$2.0$

$g T$

JAMDiFF (no LQCD)

JAMDiFF (w/ LQCD)

JAM3D* (no LQCD)

JAM3D* (w/ LQCD)

Anselmino et al (2013)


Radici, Bacchetta (2018)

Benel et al (2019)

D’Alesio et al (2020)

PNDME (2018)

ETMC (2020)

JAM3D (w/ LQCD)

JAM3D (no LQCD)

JAMDiFF (w/ LQCD)

JAMDiFF (no LQCD)

$\mu^2 = 4 \text{ GeV}^2$
Extraction of Tensor Charges

Tensor Charges (w/ LQCD)

Noticeable shift from including lattice data
Extraction of Tensor Charges (w/ LQCD)

Noticeable shift from including lattice data

Likelihood function $\mathcal{L} = \exp(-\chi^2/2)$ does not guarantee that errors overlap
1. JAM Methodology
2. Extraction of DiFFs
3. Extraction of Transversity PDFs
4. Extraction of Tensor Charges
5. Conclusions and Outlook
Comprehensive Analysis of DiFFs and Transversity

First inclusion of Belle cross section data
Conclusions and Outlook

Comprehensive Analysis of DiFFs and Transversity

First inclusion of Belle cross section data

First inclusion of 500 GeV STAR data
Comprehensive Analysis of DiFFs and Transversity

First inclusion of Belle cross section data

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Utilized all binnings for Artru-Collins and SIDIS asymmetries
Conclusions and Outlook

Comprehensive Analysis of DiFFs and Transversity

First simultaneous analysis of DiFFs and transversity PDFs

First inclusion of Belle cross section data

First inclusion of 500 GeV STAR data

Utilized all binnings for Artru-Collins and SIDIS asymmetries

First simultaneous analysis of DiFFs and transversity PDFs
Conclusions

Simultaneous extraction of DiFFs and transversity PDFs
Conclusions

Simultaneous extraction of DiFFs and transversity PDFs

Universality of all available information on transversity
Outlook

More data from RHIC
Proton-proton cross section
Outlook

More data from RHIC
Proton-proton cross section

SIDIS multiplicities from COMPASS

Figure 4: $h^+h^-$ multiplicities measured versus $z$, $M_{\text{inv}}$ and $Q^2$.

Outlook

More data from RHIC
Proton-proton cross section

SIDIS multiplicities
from COMPASS

EIC can provide new
information

Figure 4: \( h^+h^- \) multiplicities measured versus \( z \), \( M_{\text{inv}} \) and \( Q^2 \).


More data from RHIC
Proton-proton cross section

SIDIS multiplicities
from COMPASS

EIC can provide new information

Simultaneous fit of DiFF channel + TMD channel + Lattice QCD

Figure 4: $h^+h^-$ multiplicities measured versus $z$, $M_{inv}$ and $Q^2$.


Thank you to Yiyu Zhou and Patrick Barry for helpful discussions.
Extra Slides
Parameterize PDFs at input scale $Q_0^2 = m_c^2$

$$f_i(x) = N x^\alpha (1 - x)^\beta (1 + \gamma \sqrt{x} + \eta x)$$
Parameterize PDFs at input scale $Q_0^2 = m_c^2$

$$f_i(x) = N x^\alpha (1 - x)^\beta (1 + \gamma \sqrt{x} + \eta x)$$

Evolve PDFs using DGLAP

$$\frac{d}{d \ln(\mu^2)} f_i(x, \mu) = \sum_j \int_x^1 \frac{dz}{z} P_{ij}(z, \mu) f_j\left(\frac{x}{z}, \mu\right)$$
Parameterize PDFs at input scale $Q_0^2 = m_c^2$

$$f_i(x) = N x^\alpha (1 - x)^\beta (1 + \gamma \sqrt{x + \eta x})$$

Evolve PDFs using DGLAP

$$\frac{d}{d \ln(\mu^2)} f_i(x, \mu) = \sum_j \int_x^1 \frac{dz}{z} P_{ij}(z, \mu) f_j \left( \frac{x}{z} \right)$$

Calculate Observables

$$d\sigma^{pp} = \sum_{ij} H_{ij}^{pp} \otimes f_i \otimes f_j$$
Parameterize PDFs at input scale $Q_0^2 = m_c^2$

$$f_i(x) = N x^\alpha (1 - x)^\beta (1 + \gamma \sqrt{x + \eta x})$$

Evolve PDFs using DGLAP

$$\frac{d}{d \ln(\mu^2)} f_i(x, \mu) = \sum_j \int_x^1 \frac{dz}{z} P_{ij}(z, \mu) f_j\left(\frac{x}{z}, \mu\right)$$

Calculate Observables

$$d\sigma^{pp} = \sum_{ijkl} \frac{1}{(2\pi i)^2} \int dN \int dM \hat{\mathcal{H}}_{ik}^{pp}(N, M, \mu) U^{S}_{ij}(N, \mu, \mu_0) U^{S}_{kl}(M, \mu, \mu_0)$$

Mellin Space Techniques
Parameterize PDFs at input scale $Q_0^2 = m_c^2$

$$f_i(x) = Nx^\alpha (1 - x)^\beta (1 + \gamma \sqrt{x} + \eta x)$$

Evolve PDFs using DGLAP

$$\frac{d}{d \ln(\mu^2)} f_i(x, \mu) = \sum \int_1^1 \frac{dz}{z} P_{ij}(z, \mu) f_j \left( \frac{x}{z}, \mu \right)$$

Calculate Observables

$$d\sigma_{pp} = \sum \frac{1}{(2\pi i)^2} \int dN \int dM \tilde{f}_j(N, \mu_0) \tilde{f}_l(M, \mu_0)$$

$$\otimes \left[ x_1^{-N} x_2^{-M} \mathcal{H}_{ik}^{pp}(N, M, \mu) U_{ij}^S(N, \mu, \mu_0) U_{kl}^S(M, \mu, \mu_0) \right]$$

Mellin Space Techniques
\[ \sigma = \sum_{ij} H_{ij} \otimes f_i \otimes f_j + \mathcal{O}(1/Q) \]
Experimentally measured cross-section

\[ \sigma = \sum_{ij} H_{ij} \otimes f_i \otimes f_j + \mathcal{O}(1/Q) \]
Experimentally measured cross-section

\[ \sigma = \sum_{ij} H_{ij} \otimes f_i \otimes f_j + \mathcal{O}(1/\mathcal{Q}) \]

“Hard part” (process dependent)
Cross-section at parton level
Calculated in perturbative QCD
Experimentally measured cross-section

“Soft part” (process independent)
Describes internal structure

\[ \sigma = \sum_{ij} H_{ij} \otimes f_i \otimes f_j + \mathcal{O}(1/Q) \]

“Hard part” (process dependent)
Cross-section at parton level
Calculated in perturbative QCD
Now that the observables have been calculated...

\[ \chi^2(\mathbf{a}) = \sum_{i,e} \left( \frac{d_{i,e} - \sum_k r^k_e \beta_{i,e}^{k_e} - T_{i,e}(\mathbf{a})/N_e}{\alpha_{i,e}} \right)^2 + \sum_k (r^k_e)^2 + \left( \frac{1 - N_e}{\delta N_e} \right)^2 \]
Now that the observables have been calculated...

\[
\chi^2(a) = \sum_{i,e} \left( \frac{d_{i,e} - \sum_k r^k r_{i,e} - T_{i,e}(a)/N_e}{\alpha_{i,e}} \right)^2 + \sum_k (r^k)^2 + \left( \frac{1 - N_e}{\delta N_e} \right)^2
\]
Now that the observables have been calculated…

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\chi^2(a) = \sum_{i,e} \left( \frac{d_{i,e} - \sum_k r^k_{i,e} \beta^k_{i,e} - T_{i,e}(a)/N_e}{\alpha_{i,e}} \right)^2 + \sum_k (r^k_e)^2 + \left( \frac{1 - N_e}{\delta N_e} \right)^2
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Now that the observables have been calculated...

\[ \chi^2(a) = \sum_{i,e} \left( \frac{d_{i,e} - \sum_k r_e^k \beta_{i,e} - T_{i,e}(a)/N_e}{\alpha_{i,e}} \right)^2 + \sum_k (r_e^k)^2 + \left( \frac{1 - N_e}{\delta N_e} \right)^2 \]

- Data
- Theory
- Uncorrelated Uncertainties
- Correlated Uncertainties
Now that the observables have been calculated...

\[
\chi^2(a) = \sum_{i,e} \left( \frac{d_{i,e} - \sum_k r^k_{i,e} \beta^k_{i,e} - T_{i,e}(a)/N_e}{\alpha_{i,e}} \right)^2 + \sum_k (r^k_e)^2 + \left( \frac{1 - N_e}{\delta N_e} \right)^2
\]
Now that we have calculated $\chi^2(a, \text{data})$...

$$\mathcal{L}(a, \text{data}) = \exp \left( -\frac{1}{2} \chi^2(a, \text{data}) \right)$$
Now that we have calculated $\chi^2(a, \text{data})$...

**Likelihood Function**

$$\mathcal{L}(a, \text{data}) = \exp \left( -\frac{1}{2} \chi^2(a, \text{data}) \right)$$

**Bayes’ Theorem**

$$\mathcal{P}(a|\text{data}) \sim \mathcal{L}(a, \text{data}) \pi(a)$$
\[ \tilde{\sigma} = \sigma + N(0,1) \alpha \]
\[ \tilde{\sigma} = \sigma + N(0,1) \alpha \]

Original Data

Uncorrelated Uncertainties

Data
\[ \tilde{\sigma} = \sigma + \mathcal{N}(0,1) \alpha \]
\[ \tilde{\sigma} = \sigma + N(0,1) \alpha \]
For a quantity $O(a)$: (for example, a PDF at a given value of $(x, Q^2)$)

\[
E[O] = \int d^n a \, \rho(a \mid data) \, O(a)
\]

\[
V[O] = \int d^n a \, \rho(a \mid data) \, [O(a) - E[O]]^2
\]

Exact, but

\[n = \mathcal{O}(100)!\]
For a quantity $O(a)$: (for example, a PDF at a given value of $(x, Q^2)$)

$$E[O] = \int d^n a \, \rho(a \mid data) \, O(a)$$

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Exact, but

$n = \mathcal{O}(100)$!

Build an MC ensemble
For a quantity $O(\mathbf{a})$: (for example, a PDF at a given value of $(x, Q^2)$)

$$E[O] = \int d^n \mathbf{a} \, \rho(\mathbf{a} \mid \text{data}) \, O(\mathbf{a})$$

$$V[O] = \int d^n \mathbf{a} \, \rho(\mathbf{a} \mid \text{data}) \, [O(\mathbf{a}) - E[O]]^2$$

Exact, but $n = \mathcal{O}(100)!$

Build an MC ensemble

$$E[O] \approx \frac{1}{N} \sum_k O(\mathbf{a}_k)$$

$$V[O] \approx \frac{1}{N} \sum_k [O(\mathbf{a}_k) - E[O]]^2$$

Average over $k$ sets of the parameters (replicas)
For a quantity $O(a)$: (for example, a PDF at a given value of $(x, Q^2)$)

\[
E[O] = \int d^n a \, \rho(a \mid data) \, O(a)
\]

\[
V[O] = \int d^n a \, \rho(a \mid data) \, [O(a) - E[O]]^2
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\]

Average over $k$ sets of the parameters (replicas)
\[ E[O] \approx \frac{1}{N} \sum_{k} O(a_k) \]
\[ V[O] \approx \frac{1}{N} \sum_{k} \left[ O(a_k) - E[O] \right]^2 \]
\[ E[O] \approx \frac{1}{N} \sum_k O(a_k) \]
\[ V[O] \approx \frac{1}{N} \sum_k \left[ O(a_k) - E[O] \right]^2 \]
PYTHIA data ($\sqrt{s} = 10.58$ GeV)
PYTHIA data ($\sqrt{s} = 30.73$ GeV)
PYTHIA data ($\sqrt{s} = 50.88$ GeV)
PYTHIA data ($\sqrt{s} = 71.04$ GeV)
PYTHIA data ($\sqrt{s} = 91.19$ GeV)
Transversity PDFs (antiquarks)

\[ x h_1^{-u} = -x h_1^{-d} \]

\[ \mu^2 = 4 \text{ GeV}^2 \]
DiFF Parameterization

\[
M_{h_h}^u = [2m_\pi, 0.40, 0.50, 0.70, 0.75, 0.80, 0.90, 1.00, 1.20, 1.30, 1.40, 1.60, 1.80, 2.00] \text{ GeV}.
\]

\[
D_1^q(z, M_{h_h}^{q,i}) = \sum_{j=1,2,3} \frac{N_{ij}^q}{M_{ij}^q} z^{\alpha_{ij}^q} (1 - z)^{\beta_{ij}^q},
\]

204 parameters for \( D_1 \)
48 parameters for \( H_1^{A} \)
PDF Parameterization

\[
\begin{align*}
  h_{1u}^v &= h_{1d}^v \\
  h_1^a &= -h_1^a
\end{align*}
\]

\[
f(x, \mu_0^2) = \frac{N}{\mathcal{M}} \ x^\alpha (1 - x)^\beta (1 + \gamma \sqrt{x} + \eta x),
\]

15 parameters for \(h_1\)
### Tensor Charge Numbers

<table>
<thead>
<tr>
<th>Fit</th>
<th>$\delta u$</th>
<th>$\delta d$</th>
<th>$g_T$</th>
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<tbody>
<tr>
<td>no LQCD</td>
<td>0.50(7)</td>
<td>-0.04(14)</td>
<td>0.54(12)</td>
</tr>
<tr>
<td>w/ LQCD</td>
<td>0.71(2)</td>
<td>-0.200(6)</td>
<td>0.91(2)</td>
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