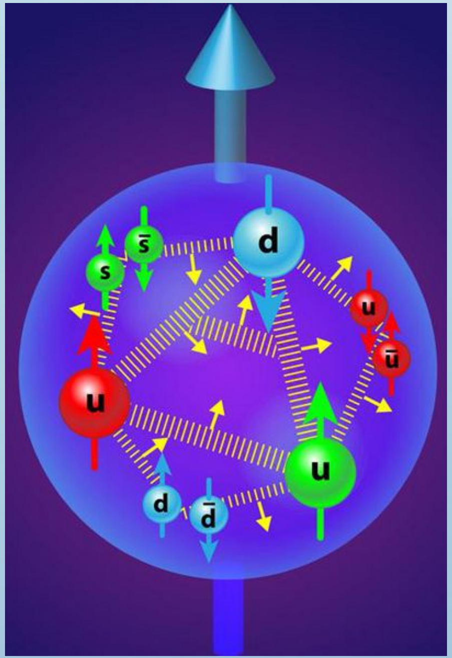
Simultaneous Global Analysis of Di-Hadron Fragmentation Functions and Transversity PDFs

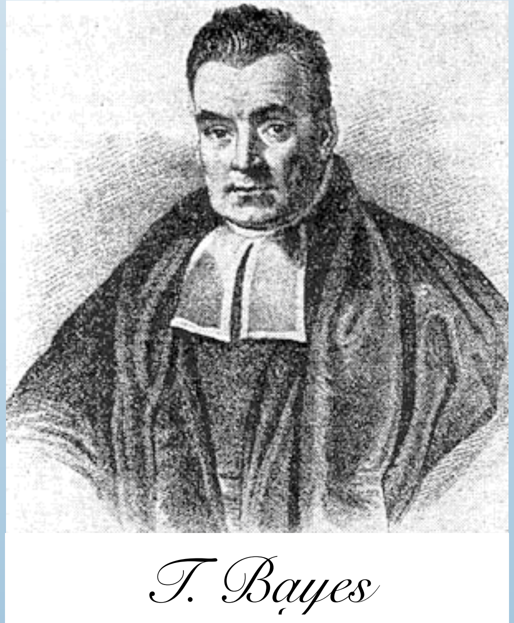
Christopher Cocuzza

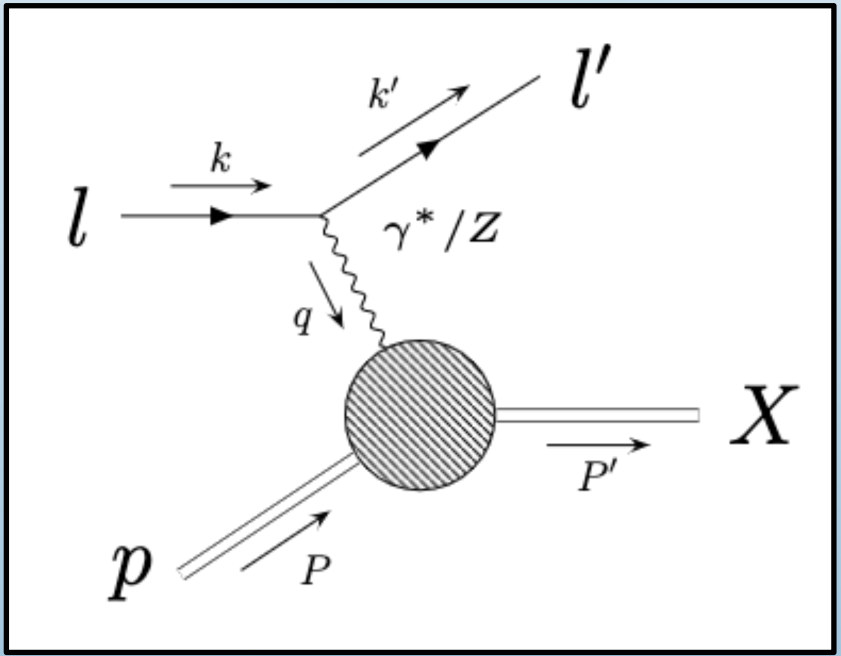
March 30, 2023

DIS23
MICHIGAN STATE UNIVERSITY
https://pa.msu.edu/conf/DIS2023

1. Introduction
2. Extraction of DiFFs
3. Extraction of Transversity PDFs
4. Extraction of Tensor Charges
5. Conclusions and Outlook



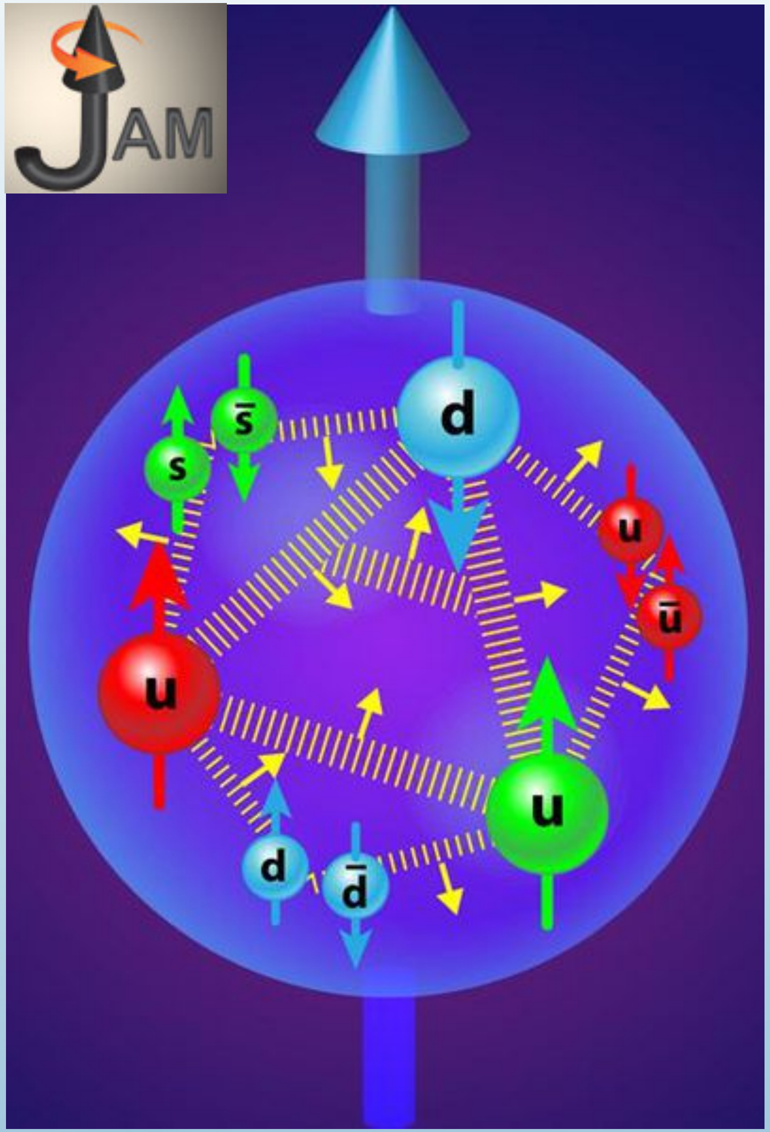




JAM Collaboration

3-dimensional structure of nucleons:

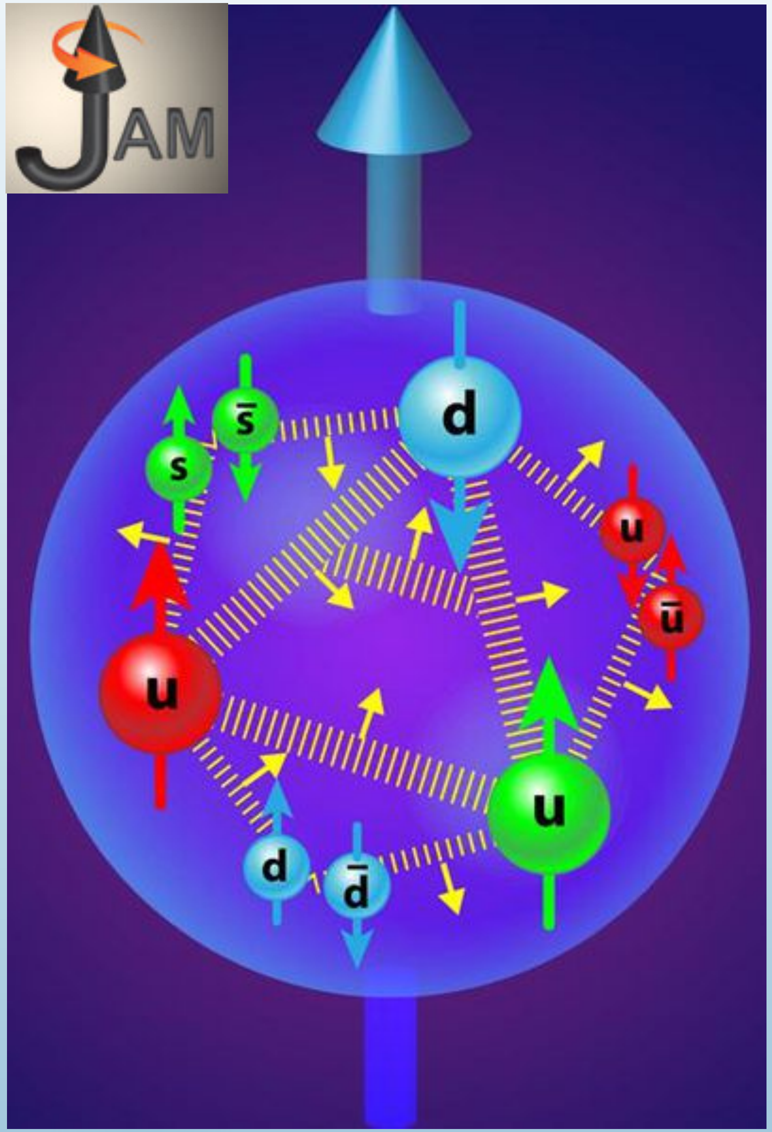
- Parton distribution functions (PDFs)
- Fragmentation functions (FFs)
- Transverse momentum dependent distributions (TMDs)
- Generalized parton distributions (GPDs)



JAM Collaboration

3-dimensional structure of nucleons:
- Parton distribution functions (PDFs)
- Fragmentation functions (FFs)
- Transverse momentum dependent distributions (TMDs)
- Generalized parton distributions (GPDs)

- Collinear factorization in perturbative QCD
- Simultaneous determinations of PDFs, FFs, etc.
- Monte Carlo methods for Bayesian inference



Hadron Structure

Global QCD Analysis

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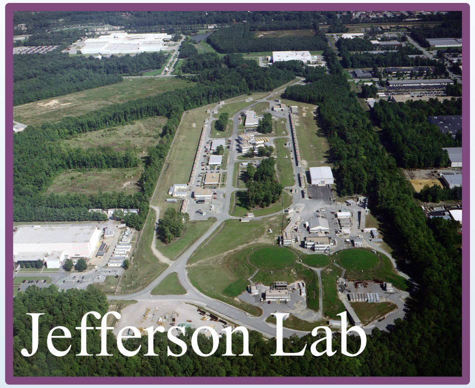
Global QCD Analysis

Introduction



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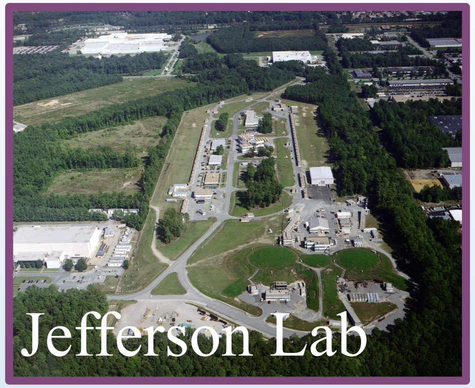
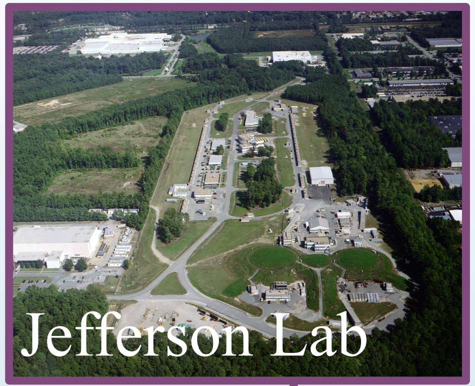


Hadron Structure

Global QCD Analysis

USQCD

Extended Twisted Mass Collaboration



Hadron Structure

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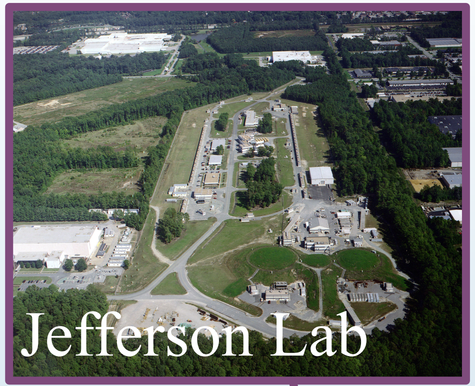


Hadron Structure

$$\frac{\mathrm{d}}{\mathrm{d}\ \ln(\mu^2)} f_i(x,\mu) = \sum_j \int_x^1 \frac{dz}{z} P_{ij}(z,\mu) f_j(\frac{x}{z},\mu)$$

Param. + Evolve + Factorization

$$\sigma = \sum_{i,j} H_{ij} \otimes f_i \otimes f_j$$

Global QCD Analysis

USQCD

Extended Twisted Mass Collaboration

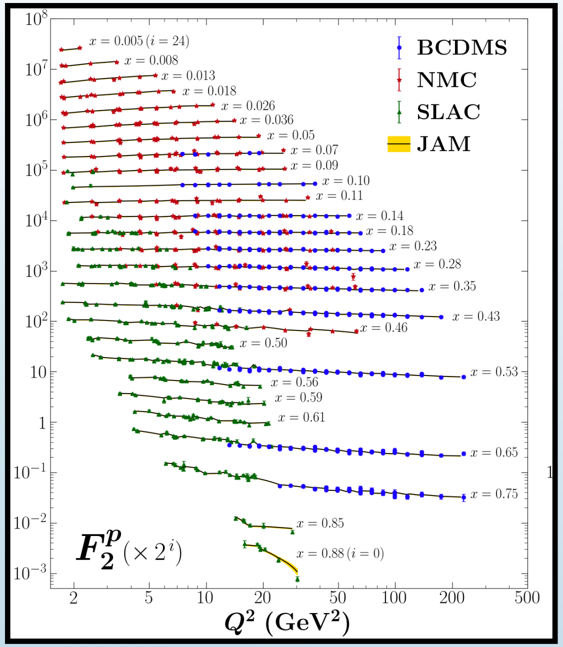
Introduction

$$\chi^2(\boldsymbol{a}) = \sum_{i,e}\left(\frac{d_{i,e} - \sum_k r_e^k \beta_{i,e}^k - T_{i,e}(\boldsymbol{a})/N_e}{\alpha_{i,e}}\right)^2 + \sum_k \left(r_e^k\right)^2 + \left(\frac{1-N_e}{\delta N_e}\right)^2$$

χ^2 Minimization

$$\mathcal{L}\left(\boldsymbol{a}, \text{data}\right) = \exp\left(-\frac{1}{2}\chi^2\left(\boldsymbol{a}, \text{data}\right)\right)$$

$$\mathcal{P}(\boldsymbol{a}|\text{data}) \sim \mathcal{L}(\boldsymbol{a}, \text{data})\,\pi(\boldsymbol{a})$$



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Global QCD Analysis

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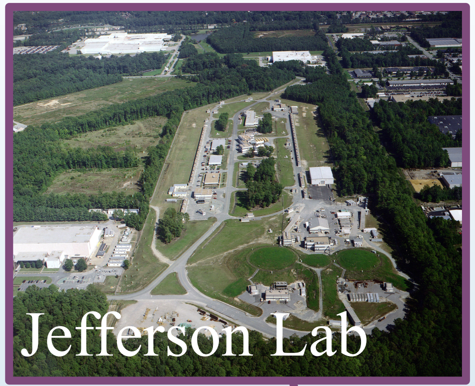
Introduction



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Global QCD Analysis

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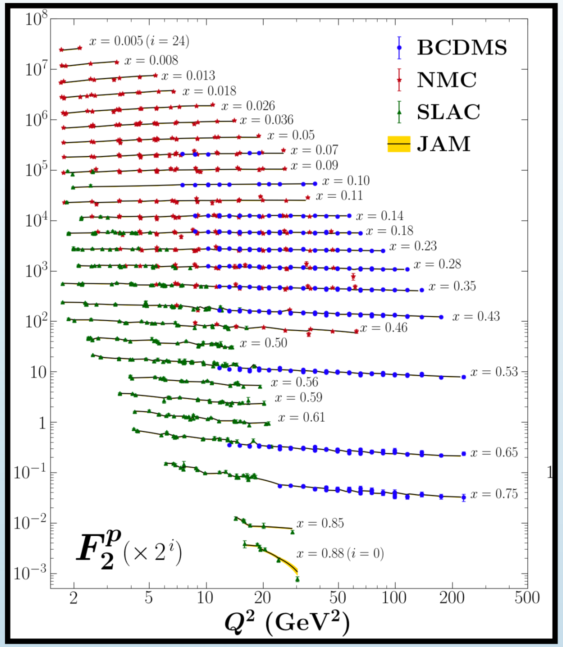
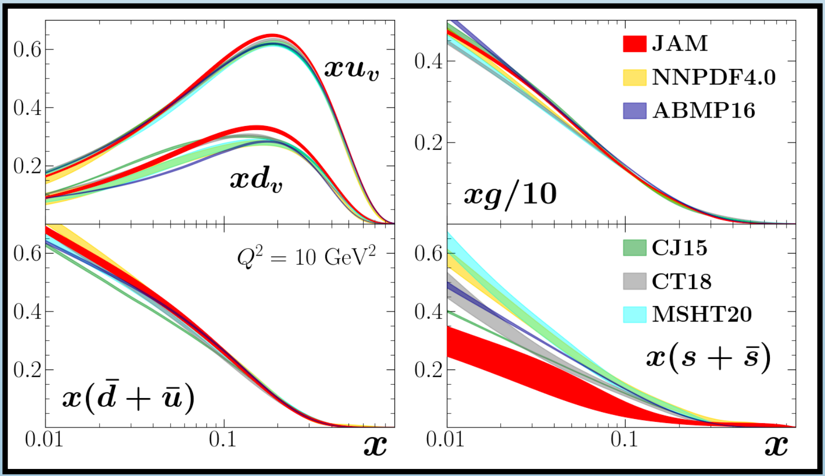
$$\mathcal{P}(\boldsymbol{a}|\text{data}) \sim \mathcal{L}(\boldsymbol{a}, \text{data})\, \pi(\boldsymbol{a})$$



Data Resampling

$$\tilde{\sigma} = \sigma + N(0,1)\ \alpha$$



Approaches to Extract Transversity

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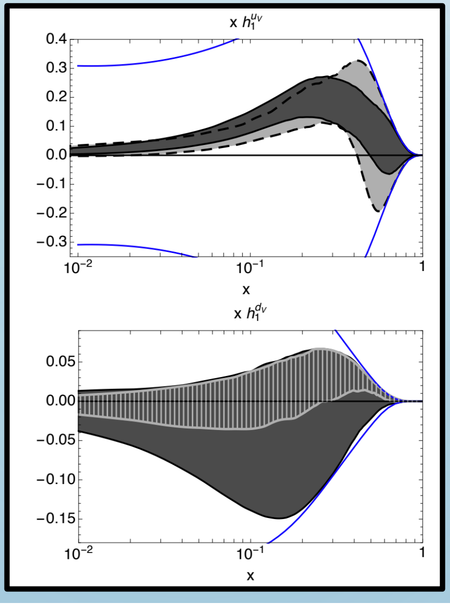
Di-Hadron Frag.

- Radici + Bacchetta (RB18)
- Benel + Courtoy + Ferro-Hernandez (2020)



M. Radici and A. Bacchetta,
Phys. Rev. Lett. **120**, no. 19, 192001 (2018)

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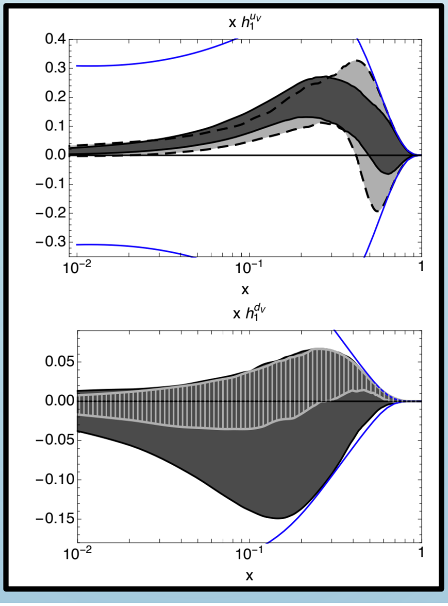

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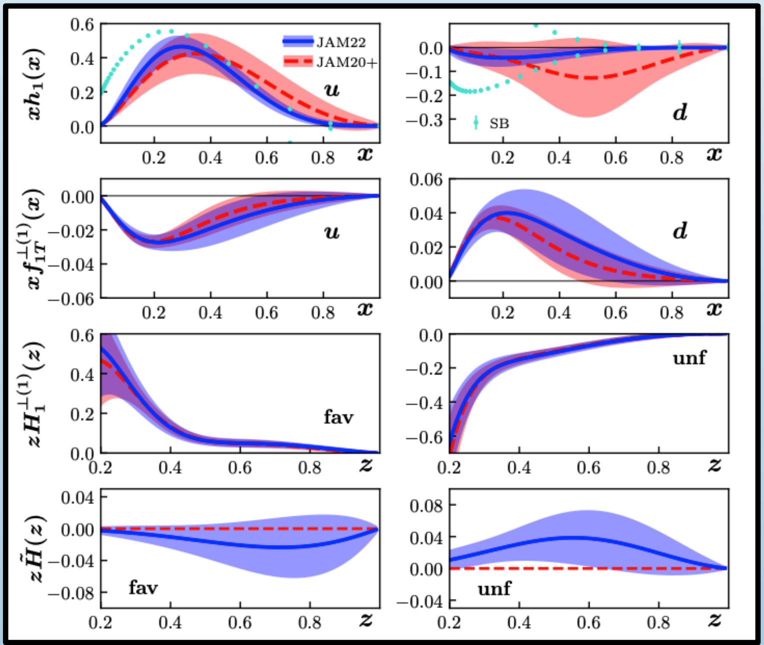
TMD + Collinear Twist-3

- JAM3D



L. Gamberg *et al.*, Phys. Rev. D **106**, no. 3, 034014 (2022)

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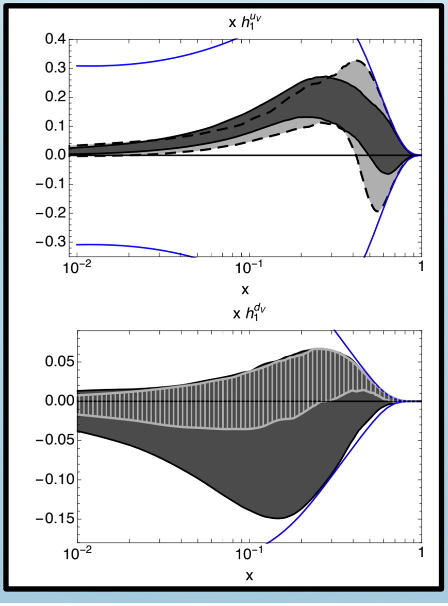

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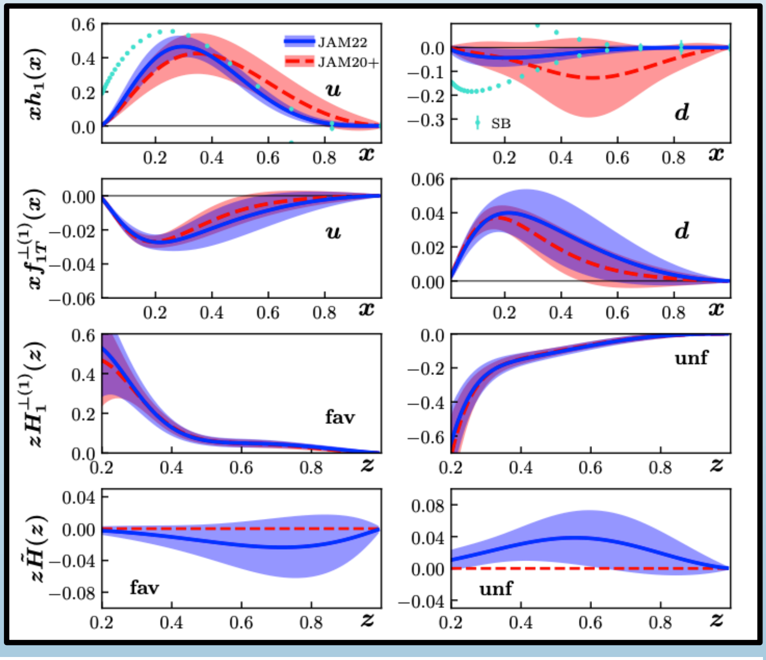
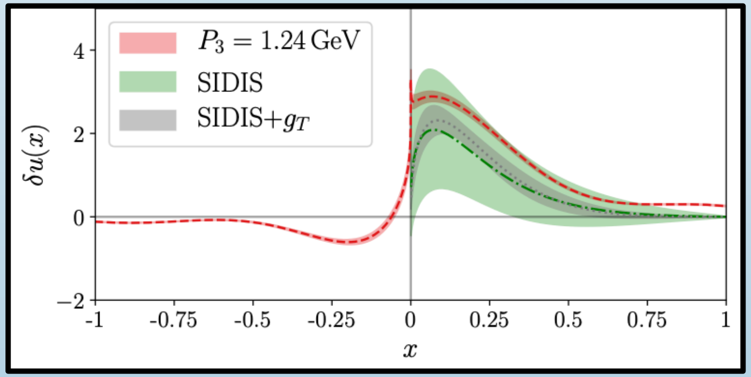


L. Gamberg *et al.*, Phys. Rev. D **106**, no. 3, 034014 (2022)

Lattice QCD

- ETMC Collaboration
- PNDME Collaboration
- Hasan *et al.*

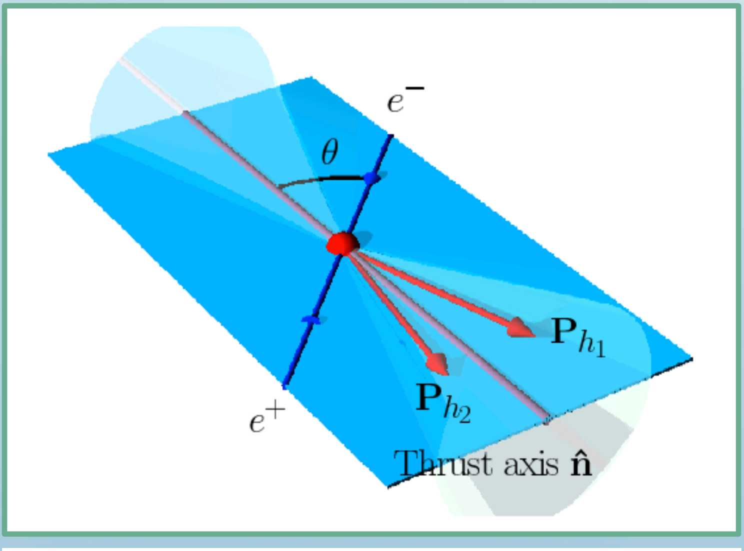


C. Alexandrou *et al.*, Phys. Rev. D **104**, no. 5, 054503 (2021)

JAM Global Analysis in the collinear DiFF Approach

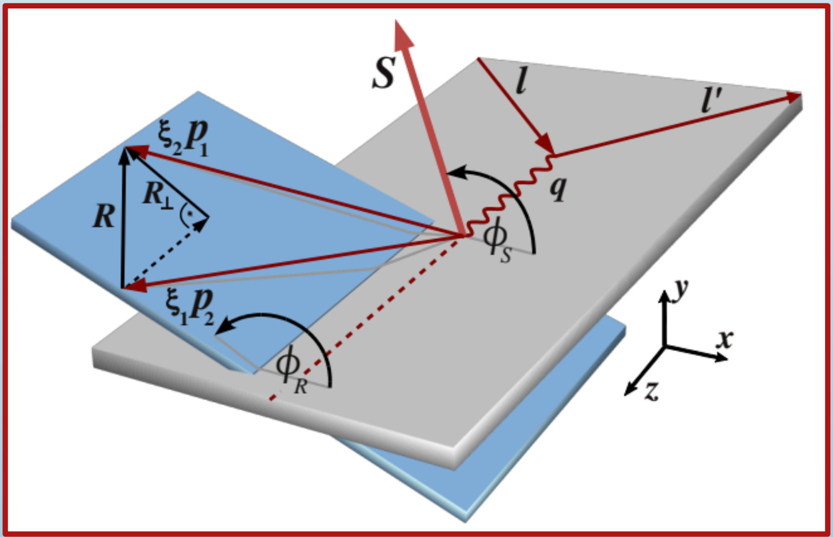
First *simultaneous* extraction of $\pi^+\pi^-$ DiFFs (D_1^q), IFFs ($H_1^{\sphericalangle,q}$), and transversity PDFs (h_1^q) at LO

Semi-Inclusive Annihilation



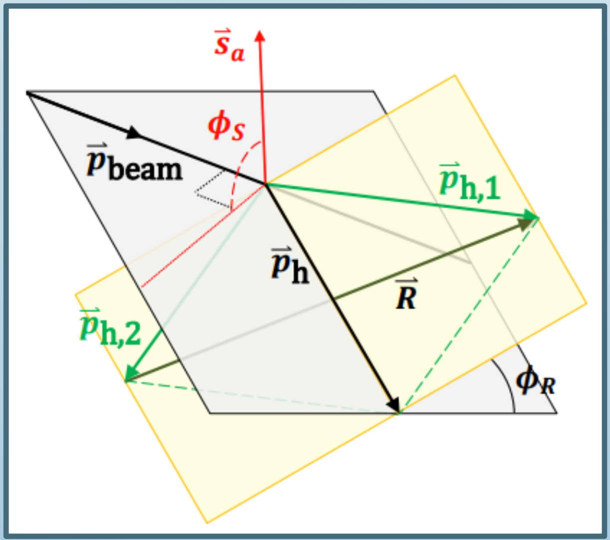
R. Seidl *et al.*, Phys. Rev. D **96**, no. 3, 032005 (2017)

Semi-Inclusive Deep Inelastic Scattering



C. Adolph *et al.*, Phys. Lett. B **713**, 10-16 (2012)

Proton-Proton Collisions



L. Adamczyk *et al.*, Phys. Rev. Lett. **115**, 242501 (2015)

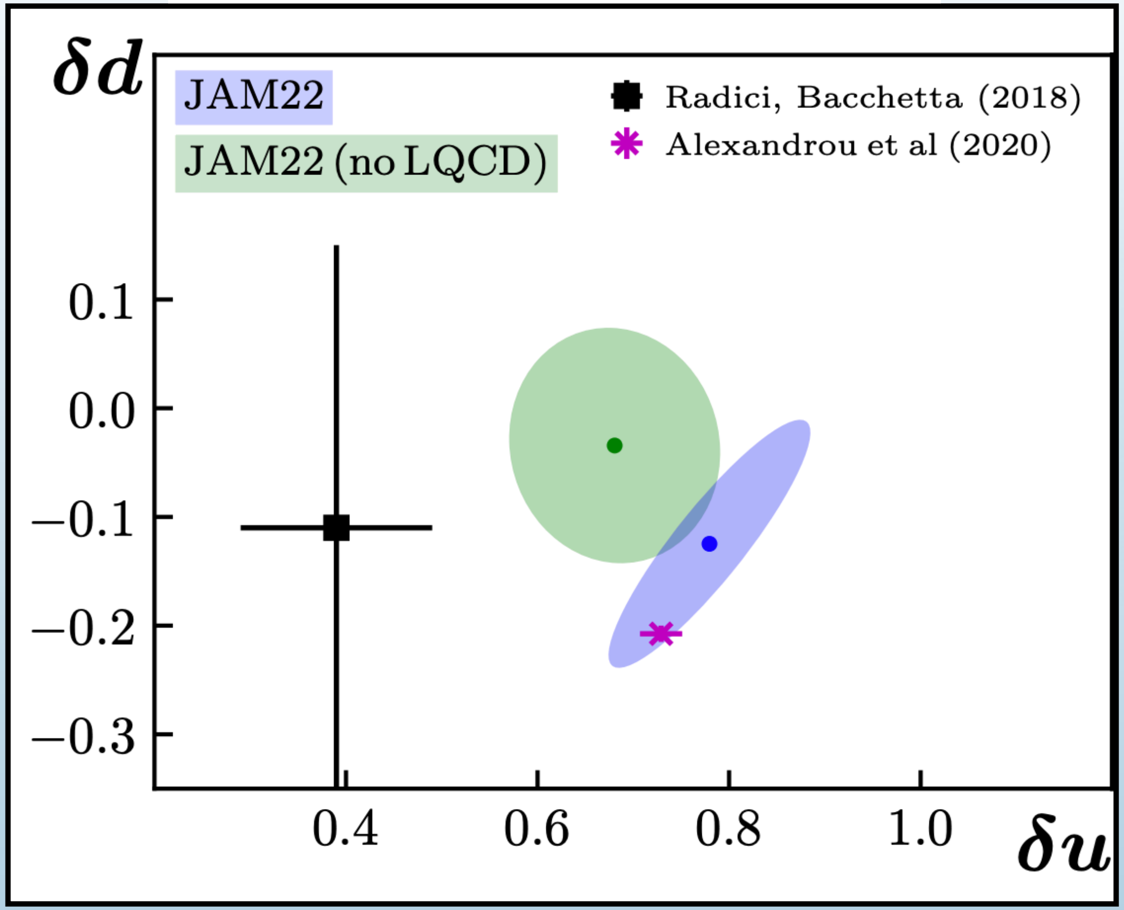
The Transverse Spin Puzzle?

$$\delta u \equiv \int_0^1 \mathrm{d}x(h_1^u - h_1^{\bar{u}}),$$

$$\delta d \equiv \int_0^1 \mathrm{d}x(h_1^d - h_1^{\bar{d}}),$$

$$g_T \equiv \delta u - \delta d,$$

L. Gamberg *et al.*, Phys. Rev. D **106**, no. 3, 034014 (2022)



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RB18

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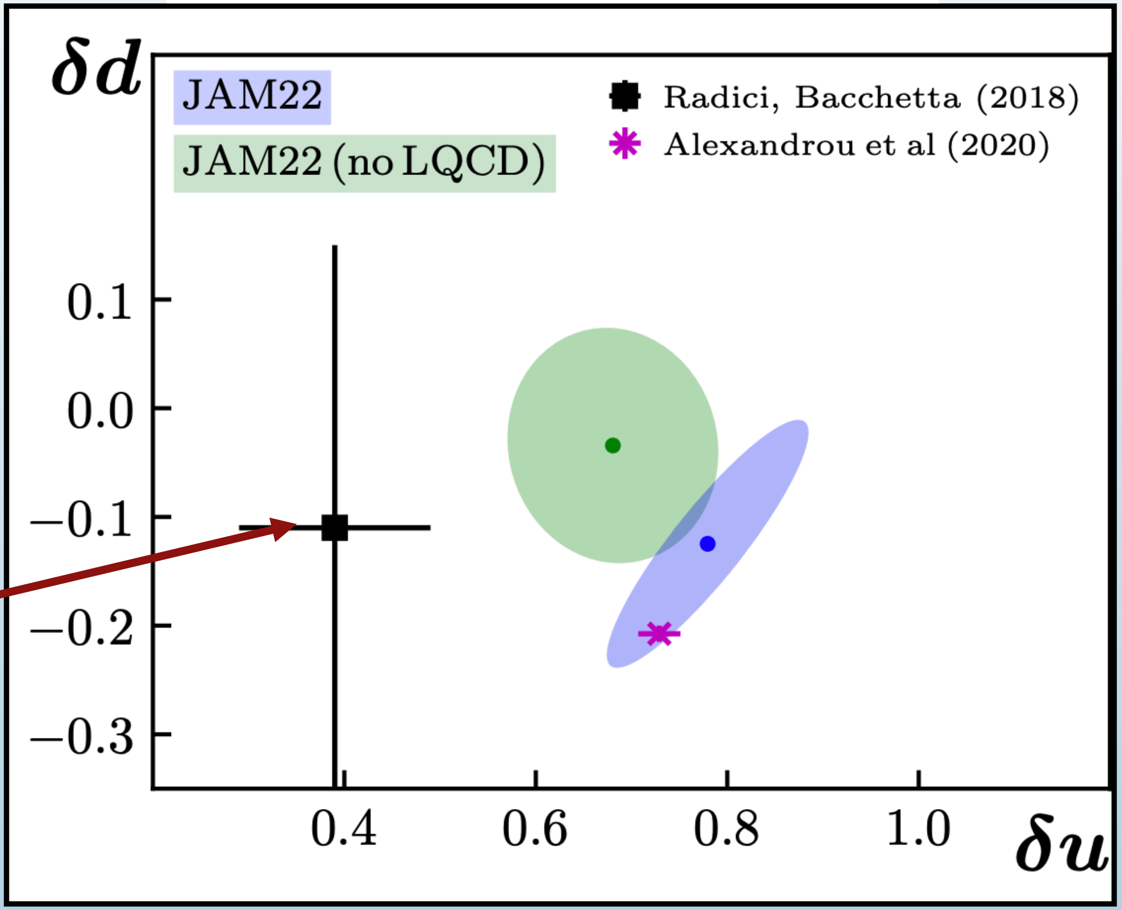

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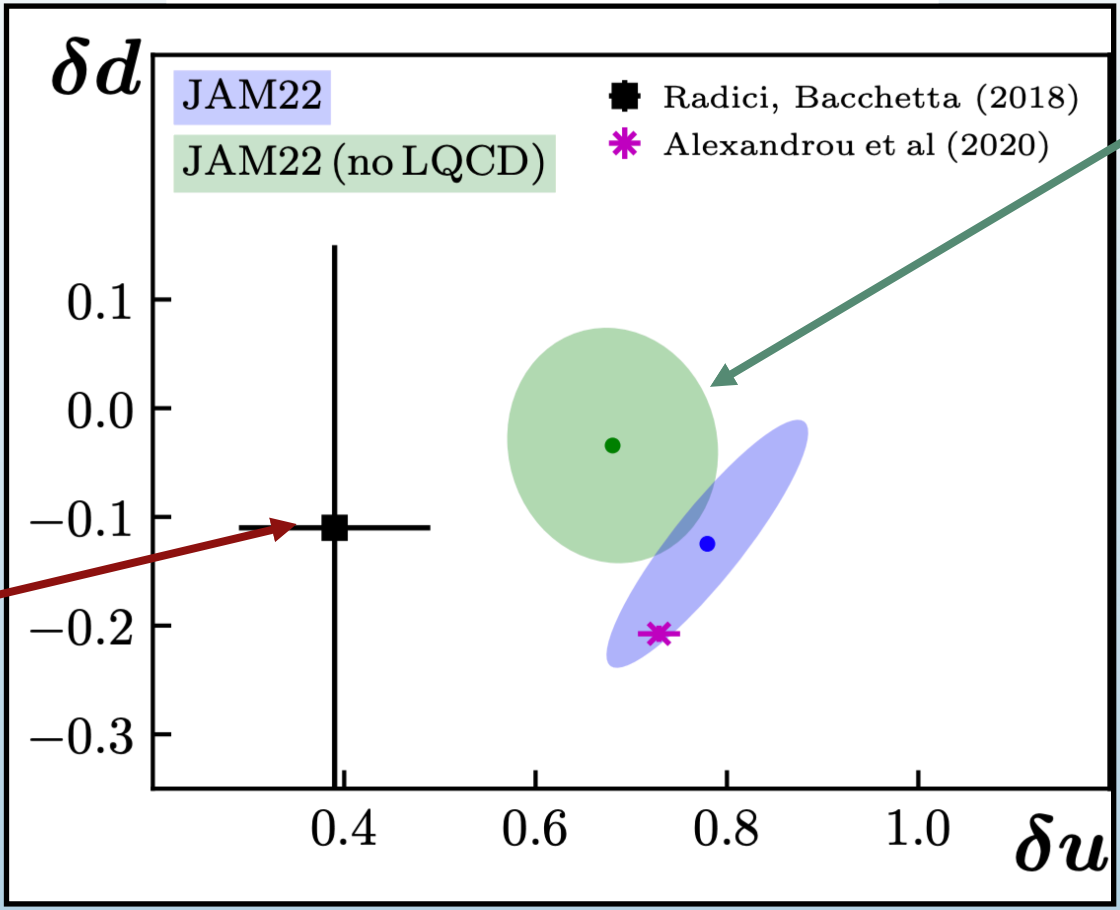

JAM3D (no LQCD)

RB18

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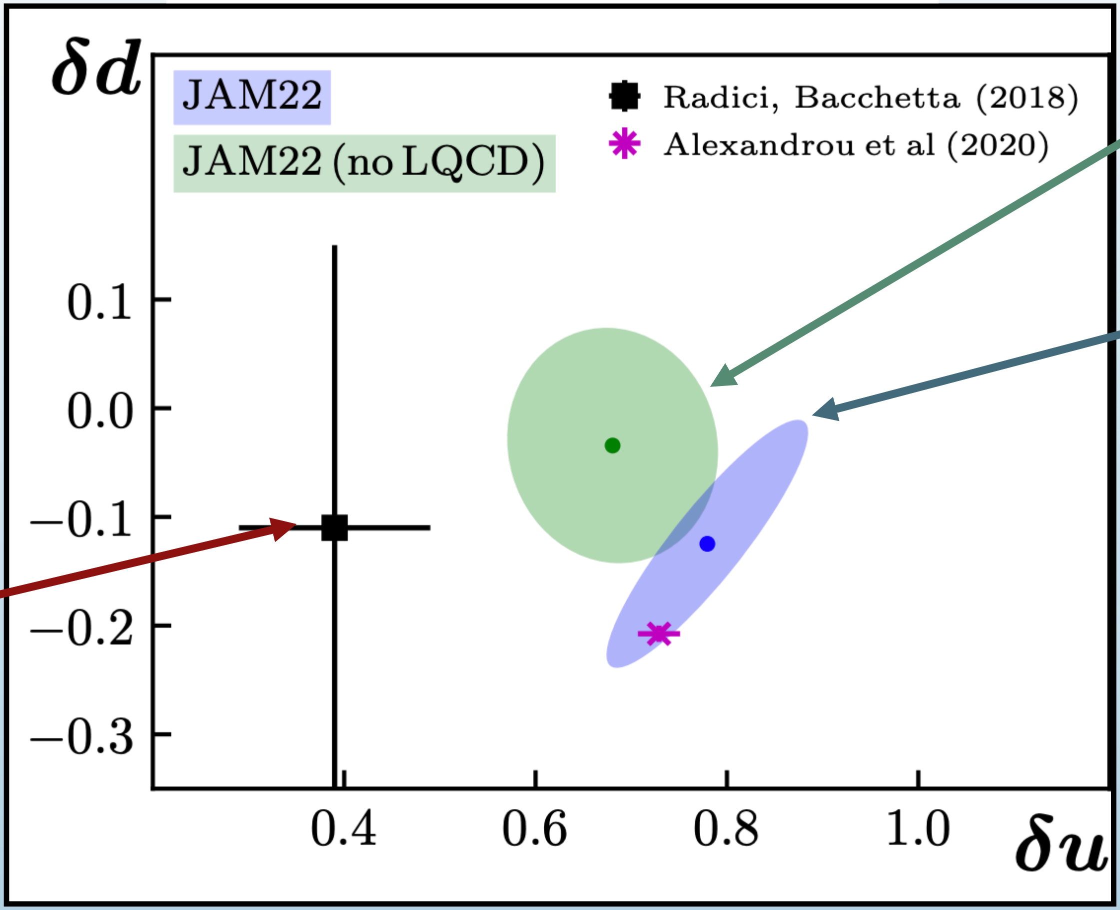

JAM3D (no LQCD)

JAM3D (w/ LQCD)

RB18

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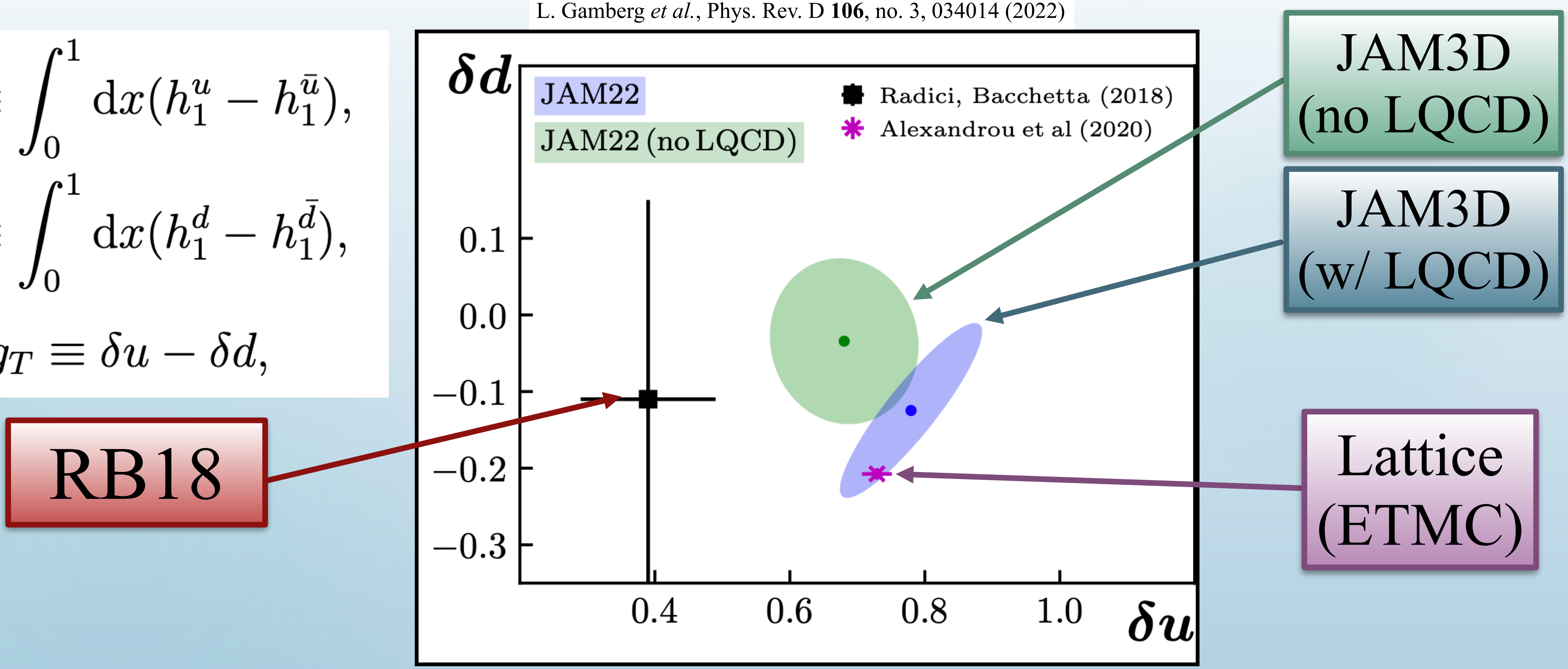

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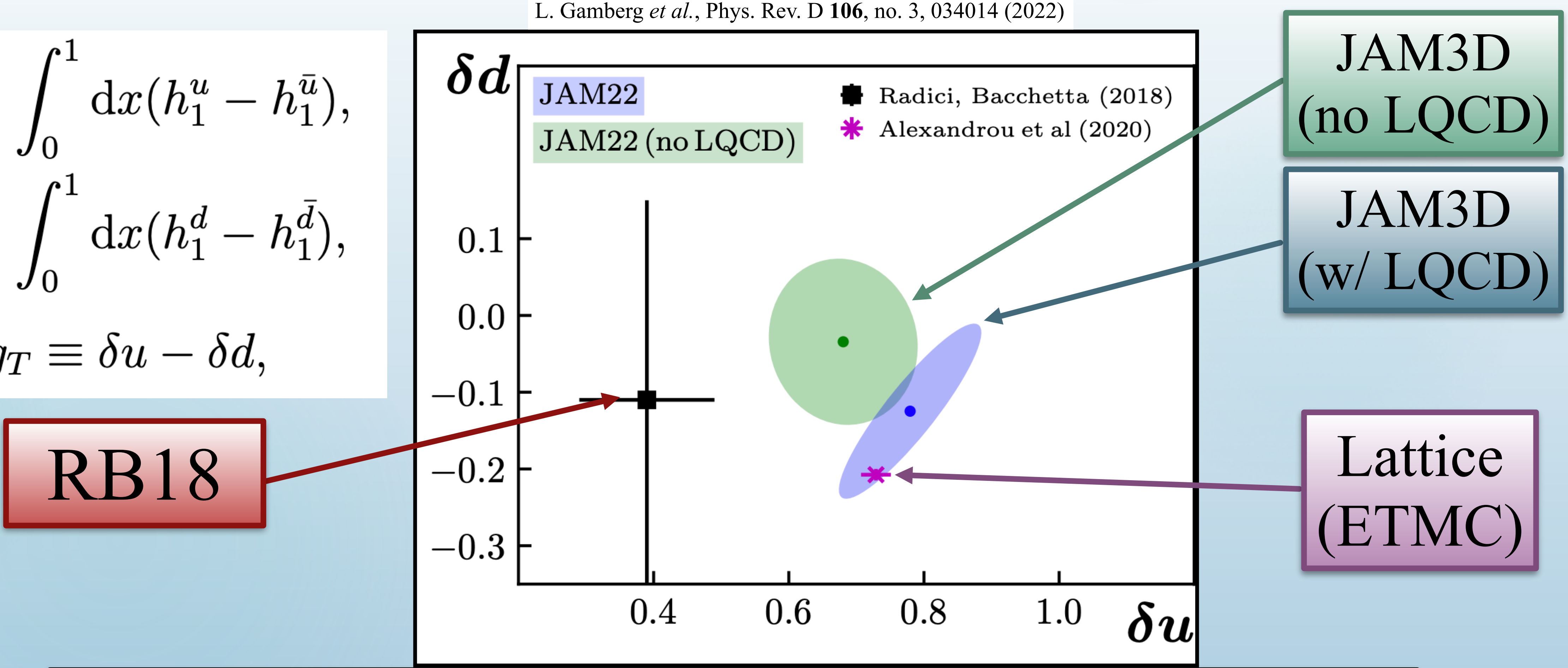
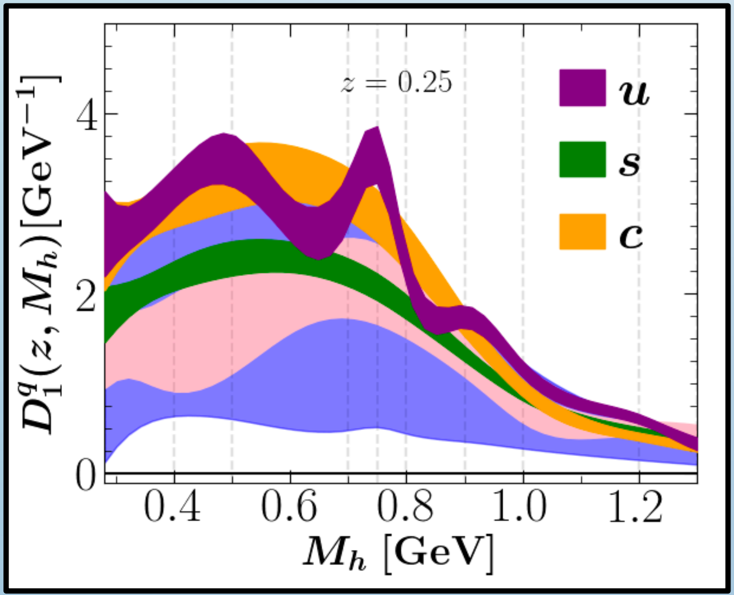
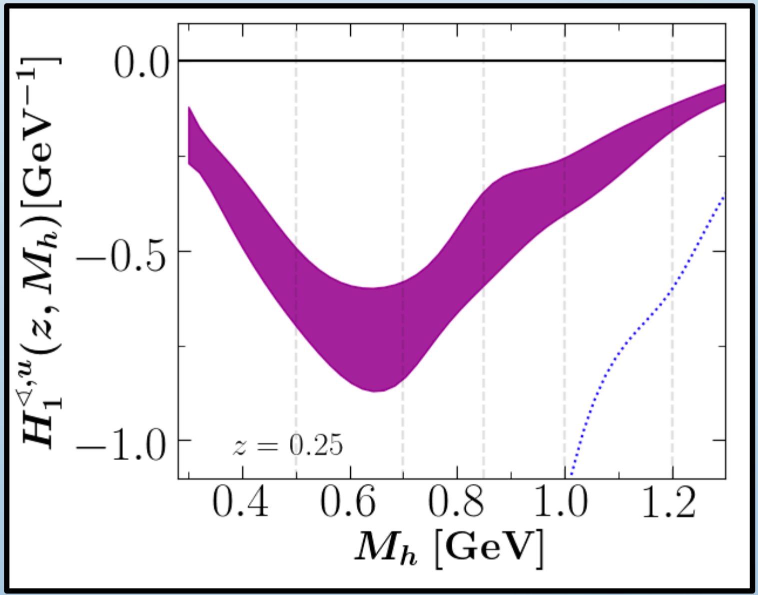


Large disagreements between three approaches…
Can this be solved?

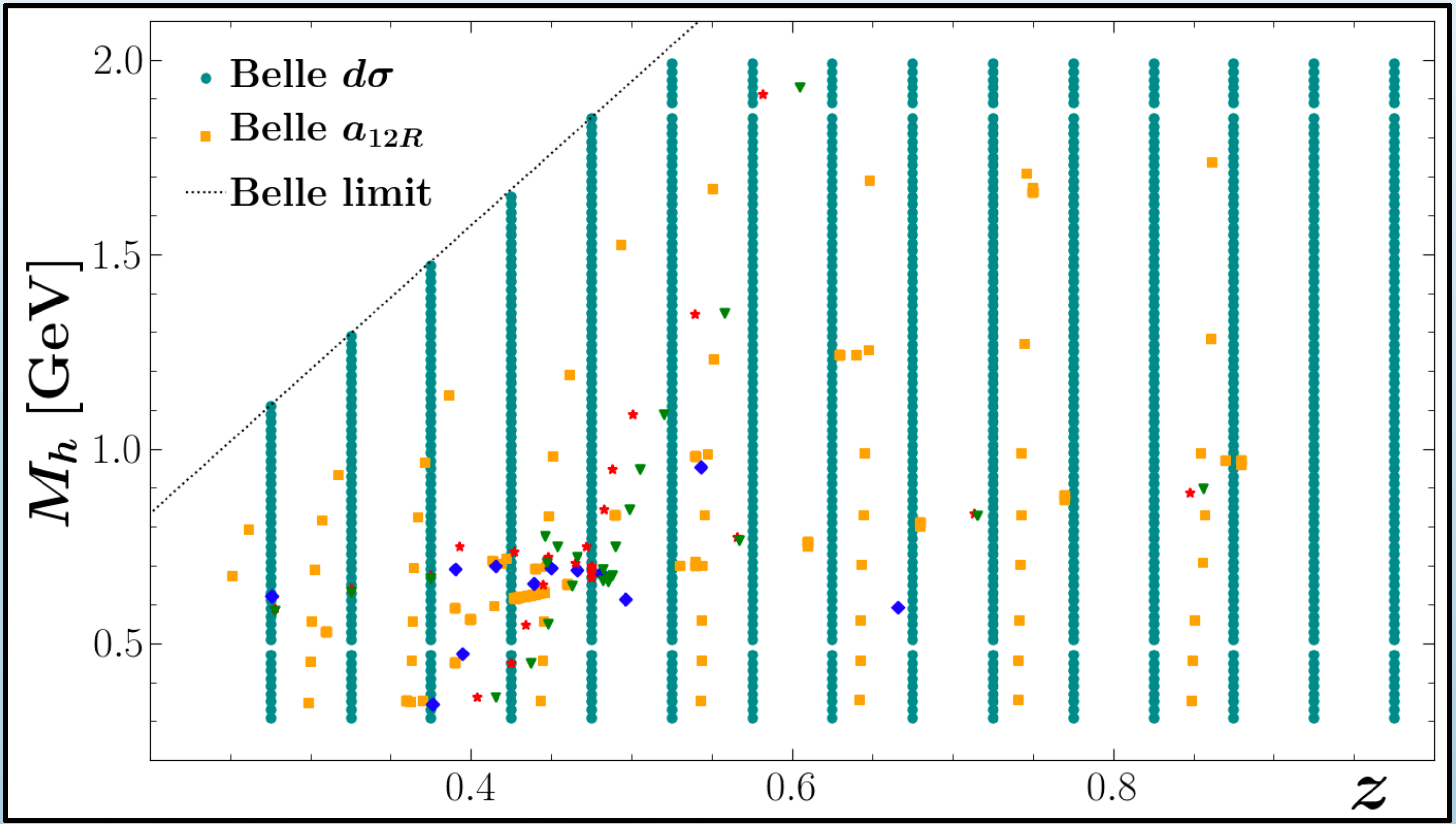
1. JAM Methodology
2. Extraction of DiFFs
3. Extraction of Transversity PDFs
4. Extraction of Tensor Charges
5. Conclusions and Outlook





Data for DiFFs

SIA cross section	Belle	1121 points
SIA Artru-Collins	Belle	183 points



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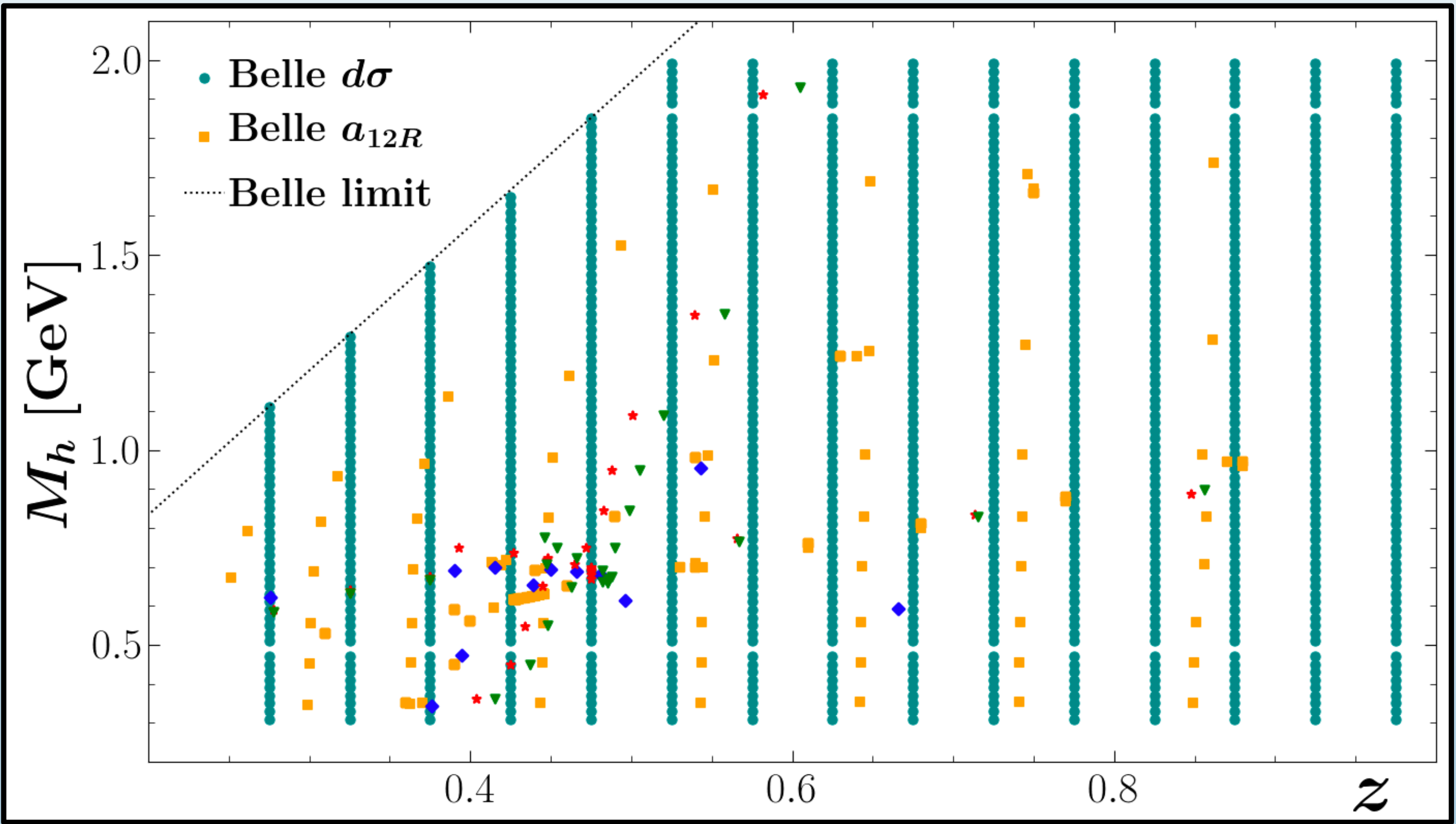

$\pi^+\pi^-$ DiFFs

$$D_1^u = D_1^d = D_1^{\bar{u}} = D_1^{\bar{d}},$$

$$D_1^s = D_1^{\bar{s}}, \quad D_1^c = D_1^{\bar{c}}, \quad D_1^b = D_1^{\bar{b}},$$

5 independent functions (w/ D_1^g)

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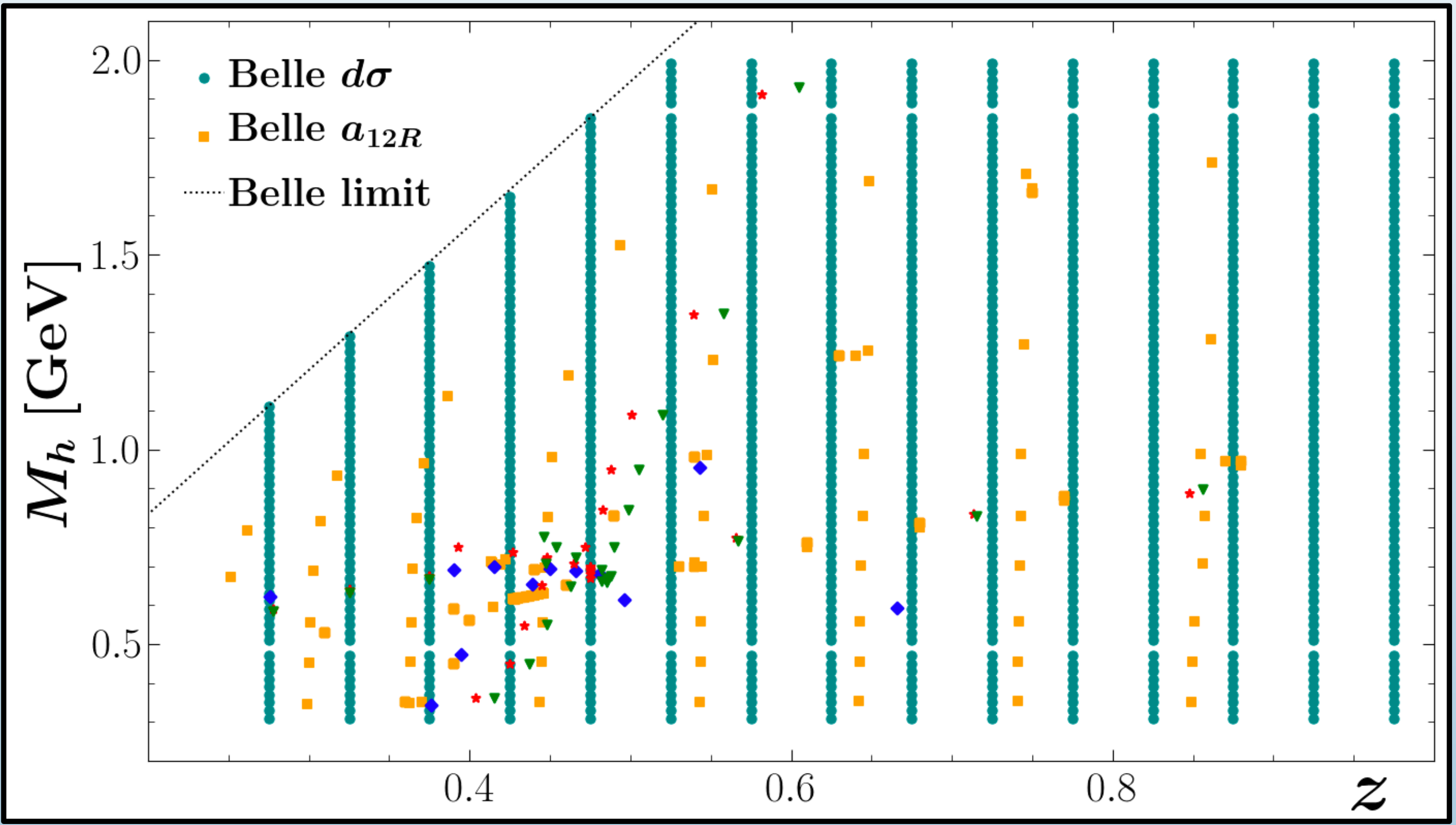

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5 independent functions (w/ D_1^g)

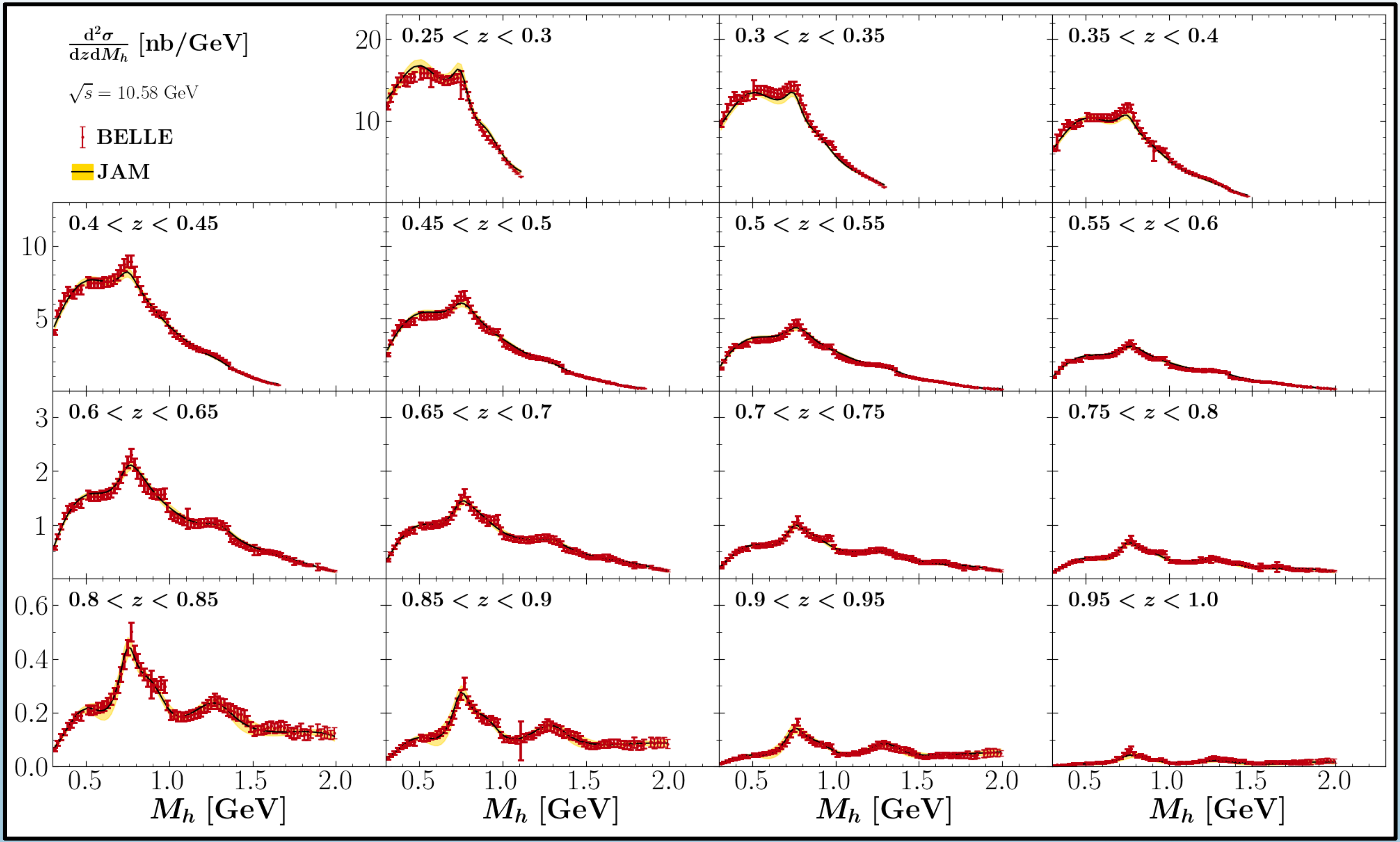
$$H_1^{\sphericalangle,u} = -H_1^{\sphericalangle,d} = -H_1^{\sphericalangle,\bar{u}} = H_1^{\sphericalangle,\bar{d}},$$

$$H_1^{\sphericalangle,s} = -H_1^{\sphericalangle,\bar{s}} = H_1^{\sphericalangle,c} = -H_1^{\sphericalangle,\bar{c}} = 0,$$

1 independent function

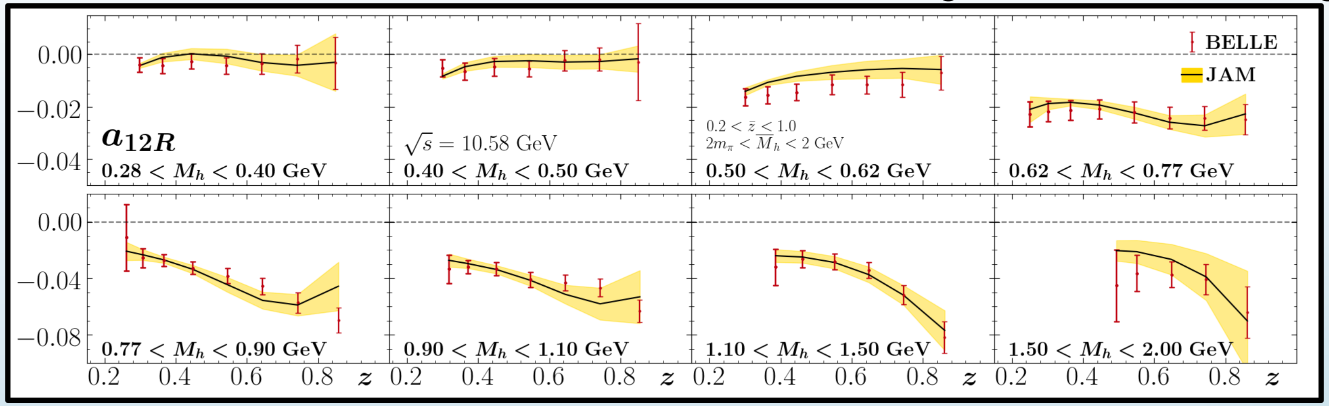
A. Courtoy et al., Phys. Rev. D **85**, 114023 (2012)

Quality of Fit (Unpolarized Cross Section)

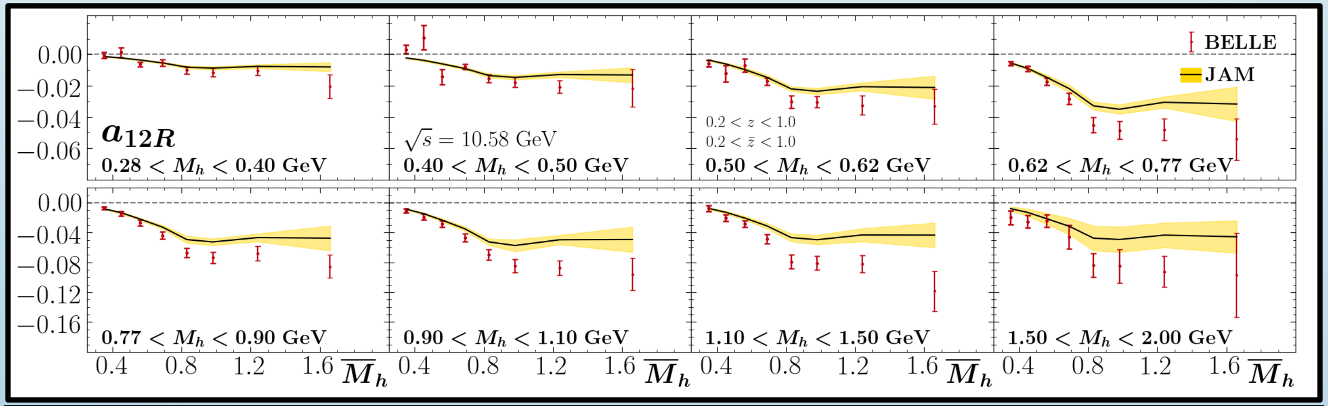


Quality of Fit (Artru-Collins Asymmetry)

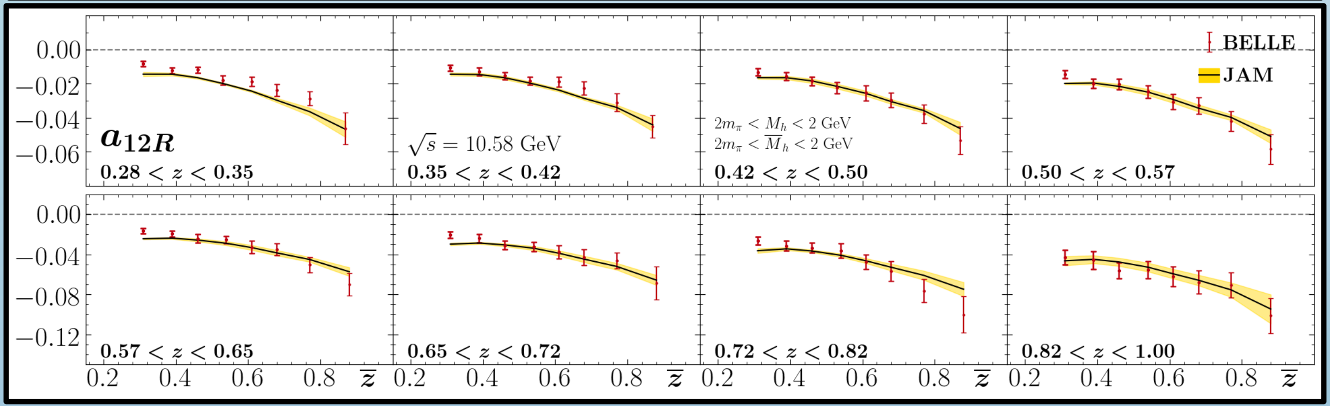
(z, M_h) binning



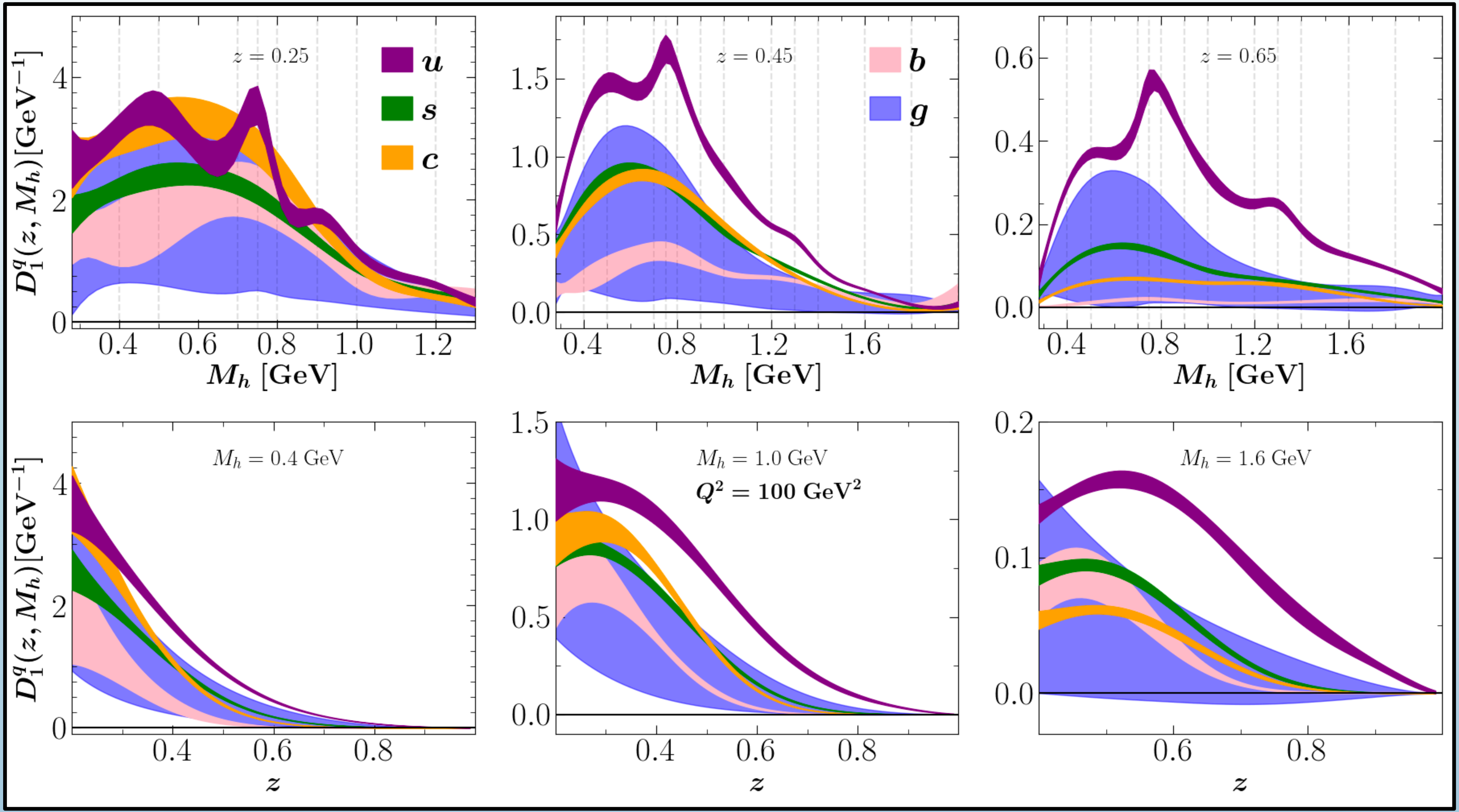
$(M_h, \overline{M}_h)$ binning



$(z, \bar{z})$ binning



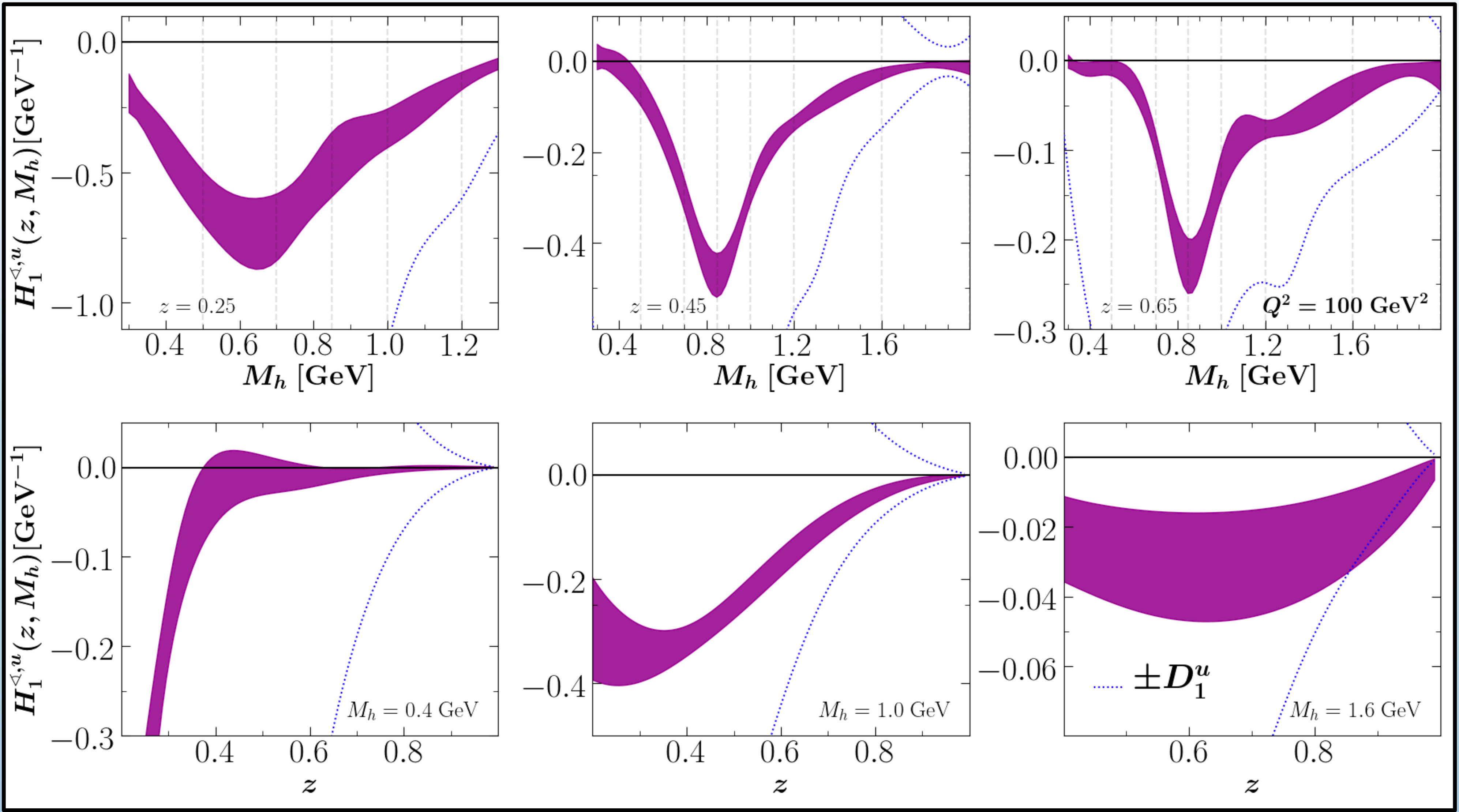
Extracted DiFFs



Bound: $D_1^q > 0$

A. Bacchetta and M. Radici, Phys. Rev. D **67**, 094002 (2003)

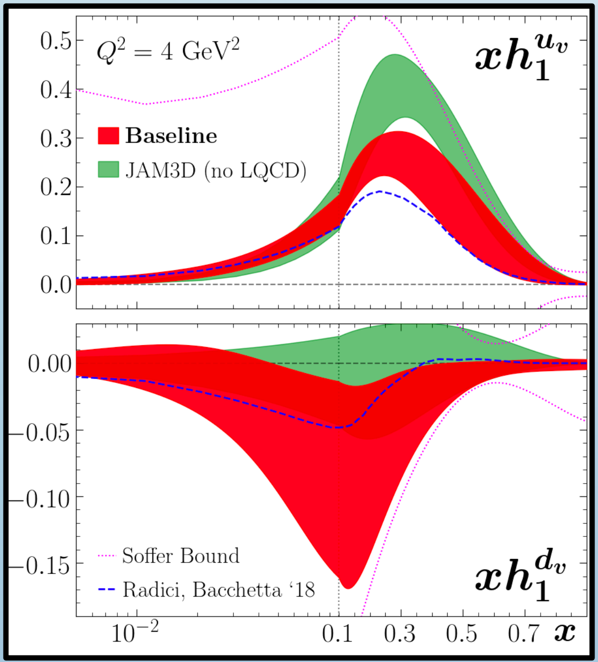
Extracted IFFs



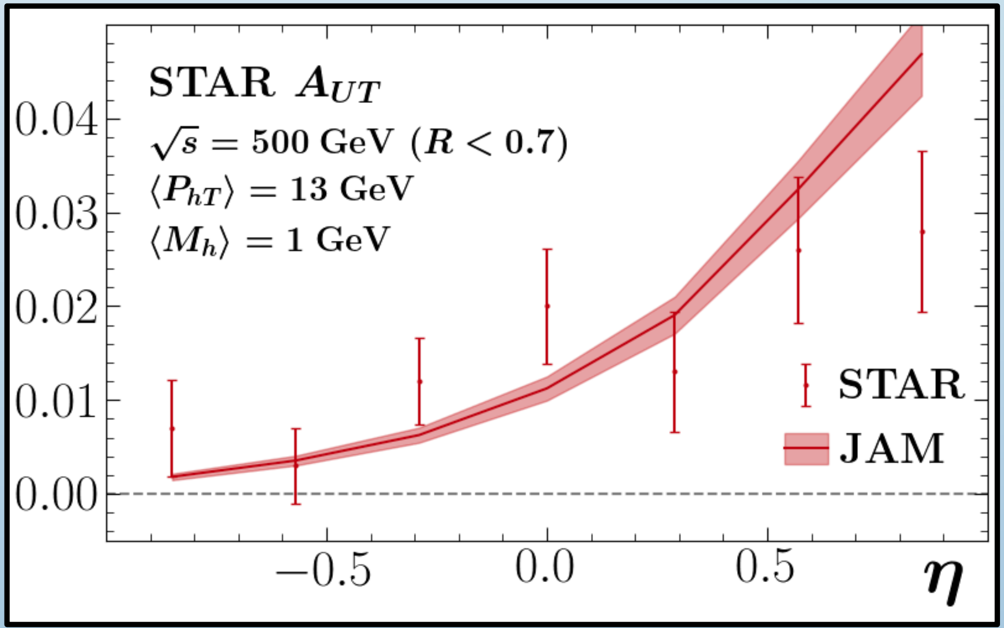
Bound:

$$|H_1^{\sphericalangle,q}| < D_1^q$$

A. Bacchetta and M. Radici, Phys. Rev. D **67**, 094002 (2003)



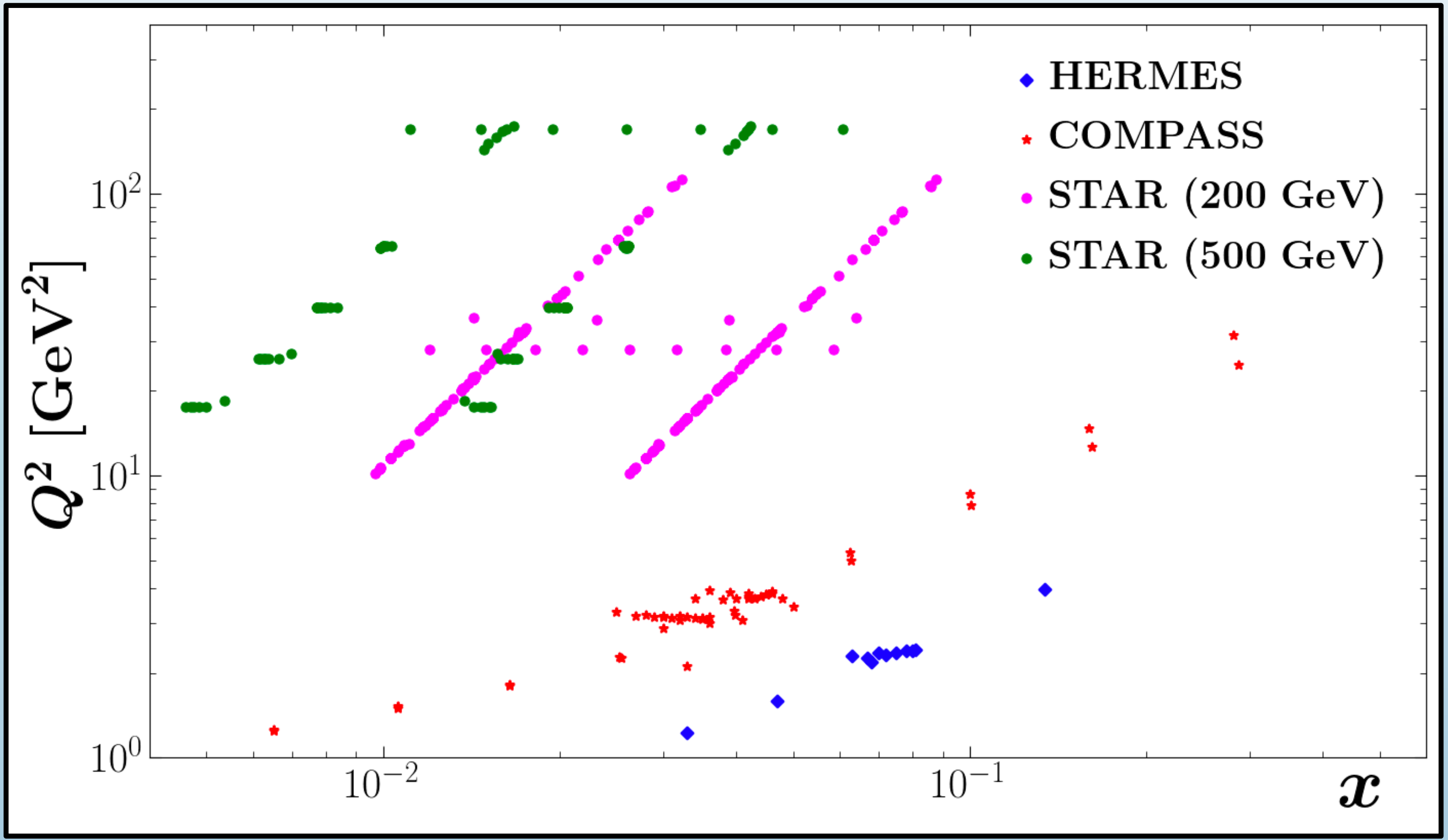
$Q^2 = 4$ GeV^2
$xh_1^{u_v}$
Baseline
JAM3D (no LQCD)
Soffer Bound
Radici, Bacchetta '18
$xh_1^{d_v}$
0.5
0.4
0.3
0.2
0.1
0.0
0.00
−0.05
−0.10
−0.15
10^{-2}
0.1
0.3
0.5
0.7
x



STAR A_{UT}
$\sqrt{s} = 500$ GeV ($R < 0.7$)
$\langle P_{hT} \rangle = 13$ GeV
$\langle M_h \rangle = 1$ GeV
STAR
JAM
0.04
0.03
0.02
0.01
0.00
−0.5
0.0
0.5
η

Data for PDFs

SIDIS (p, D)	COMPASS, HERMES	64 points
Proton-Proton	STAR	269 points



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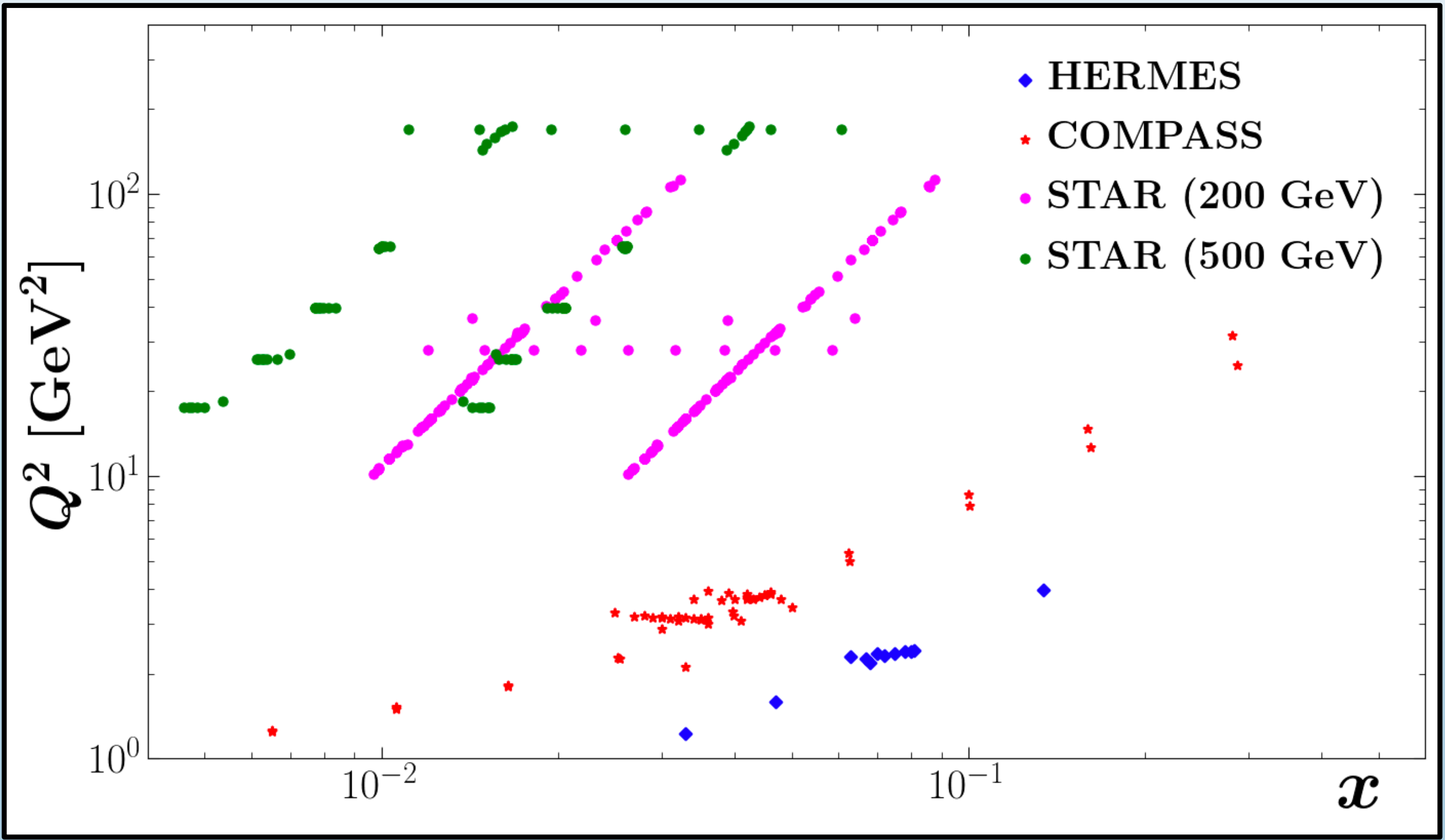

Parameterization Choices

3 independent observables
3 independent functions

$$h_1^{u_v}$$
$$h_1^{d_v}$$
$$h_1^{\bar{u}} = -h_1^{\bar{d}}$$

Data for PDFs

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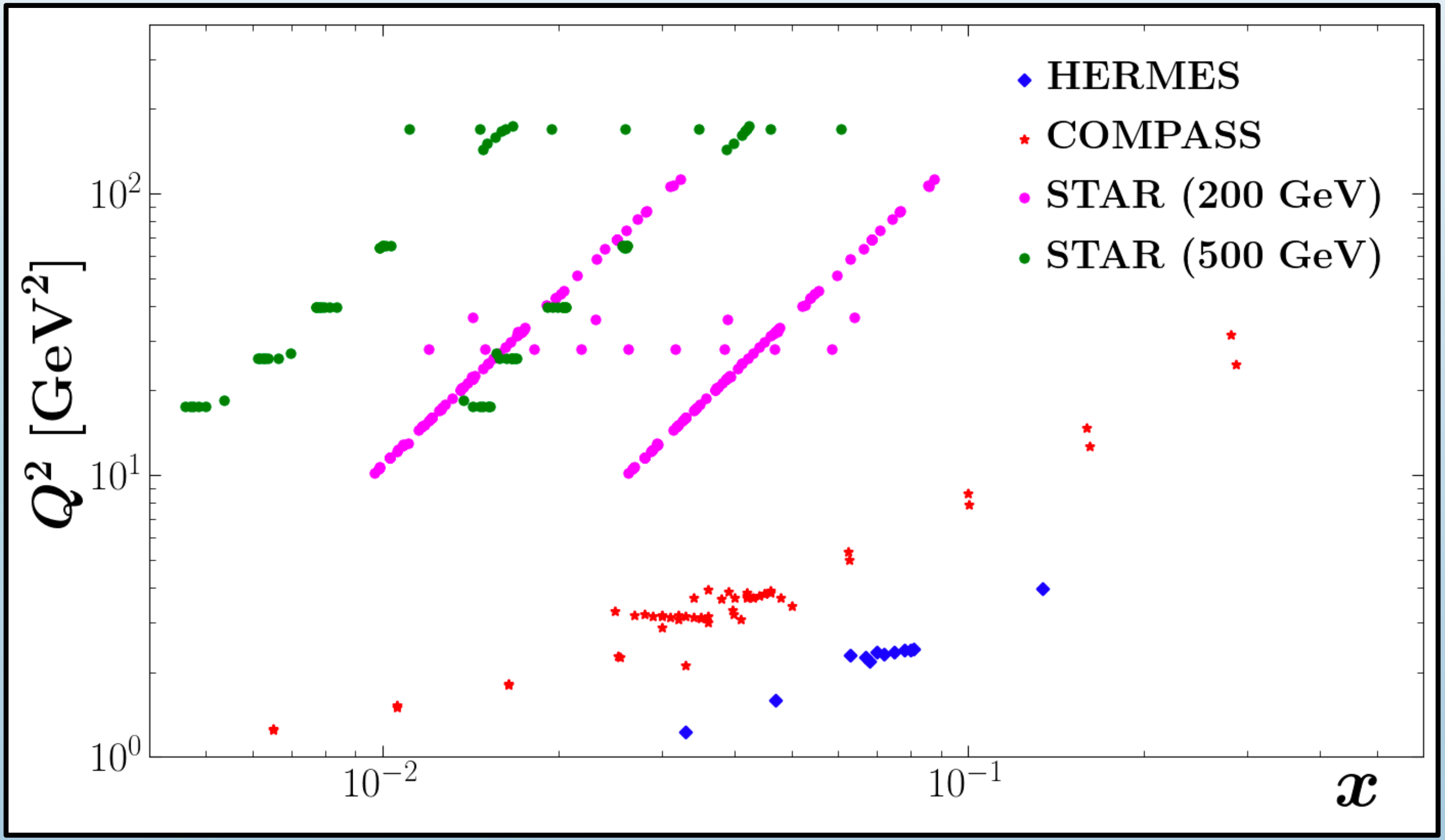


Parameterization Choices

3 independent observables
3 independent functions

$$h_1^{u_v}$$
$$h_1^{d_v}$$
$$h_1^{\bar{u}} = -h_1^{\bar{d}}$$

Prediction from large-N_c limit

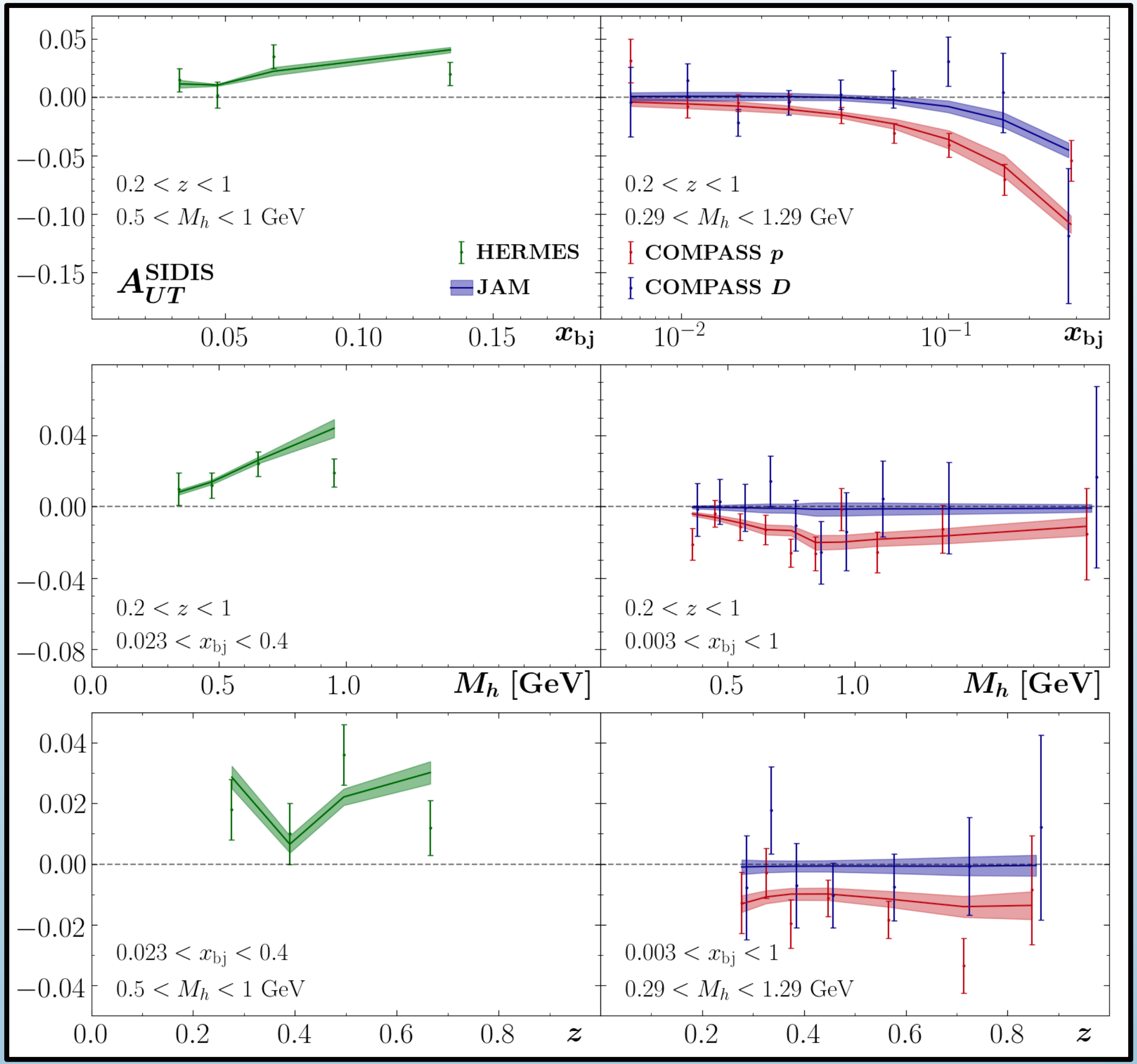
P. V. Pobylitsa, arXiv:hep-ph/0301236 (2003)

Quality of Fit (SIDIS)

x binning

M_h binning

z binning



A. Airapetian *et al.*, JHEP **06**, 017 (2008)

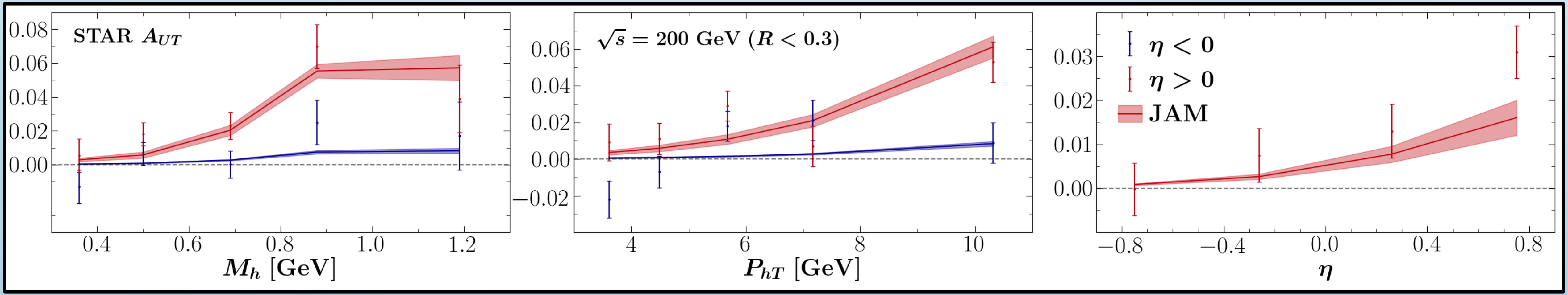
COMPASS, arXiv:hep-ph/2301.02013 (2023)

Quality of Fit (STAR $\sqrt{s} = 200$ GeV)

M_h binning

P_{hT} binning

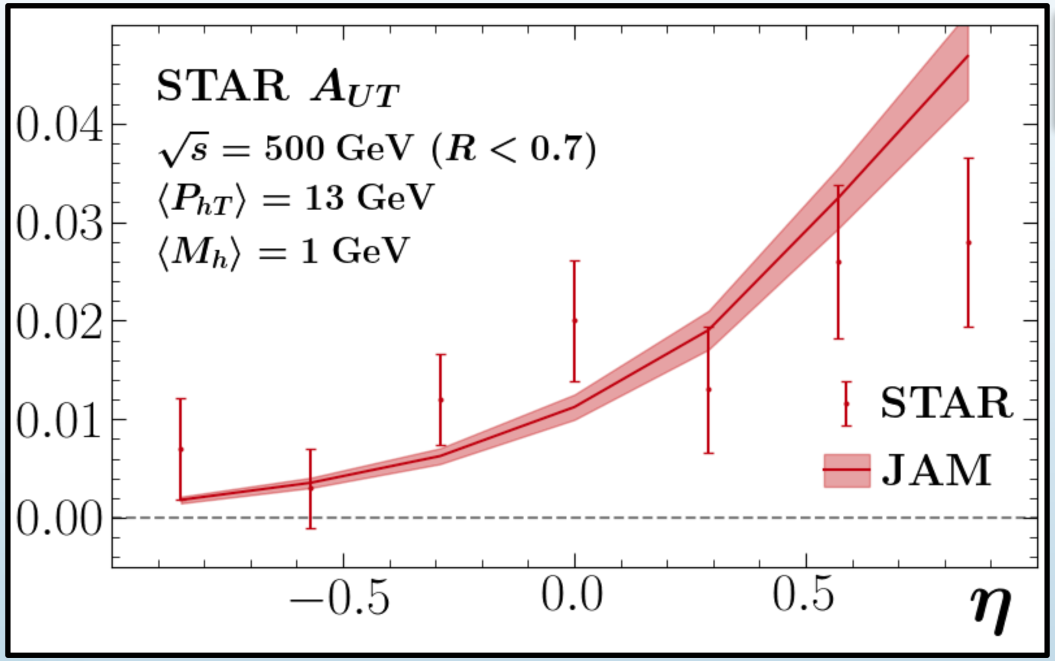
η binning



L. Adamczyk *et al.*, Phys. Rev. Lett. **115**, 24501 (2015)

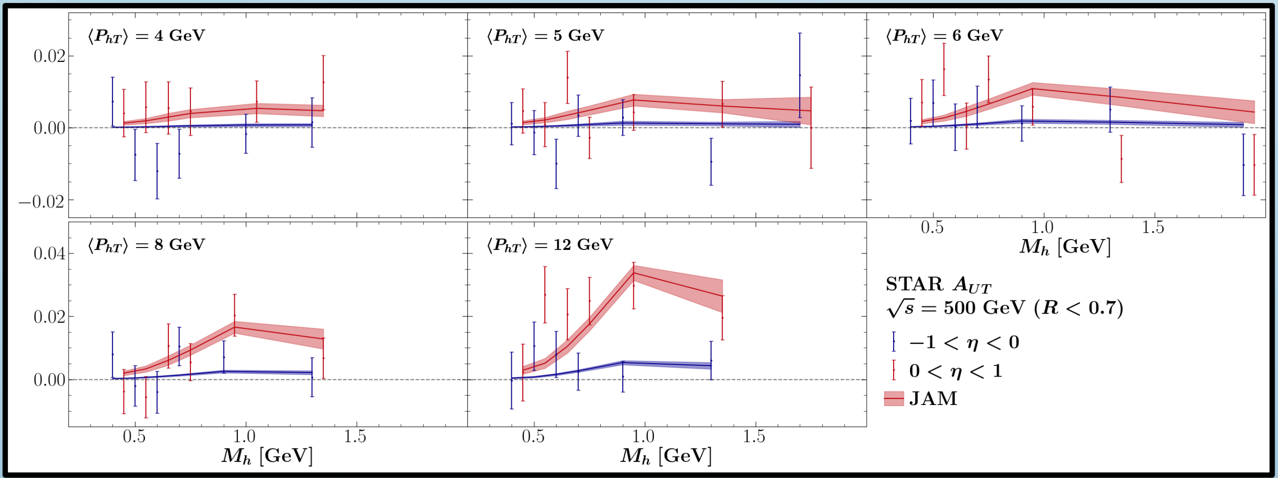
Quality of Fit (STAR $\sqrt{s} = 500$ GeV)

η binning

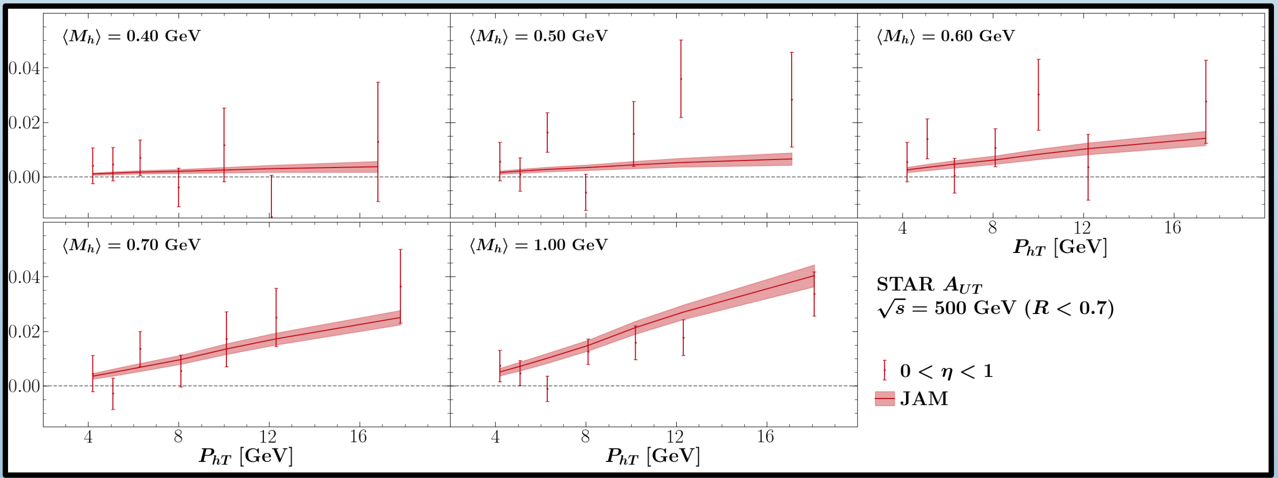


L. Adamczyk *et al.*, Phys. Rev. B **780**, 332-339 (2018)

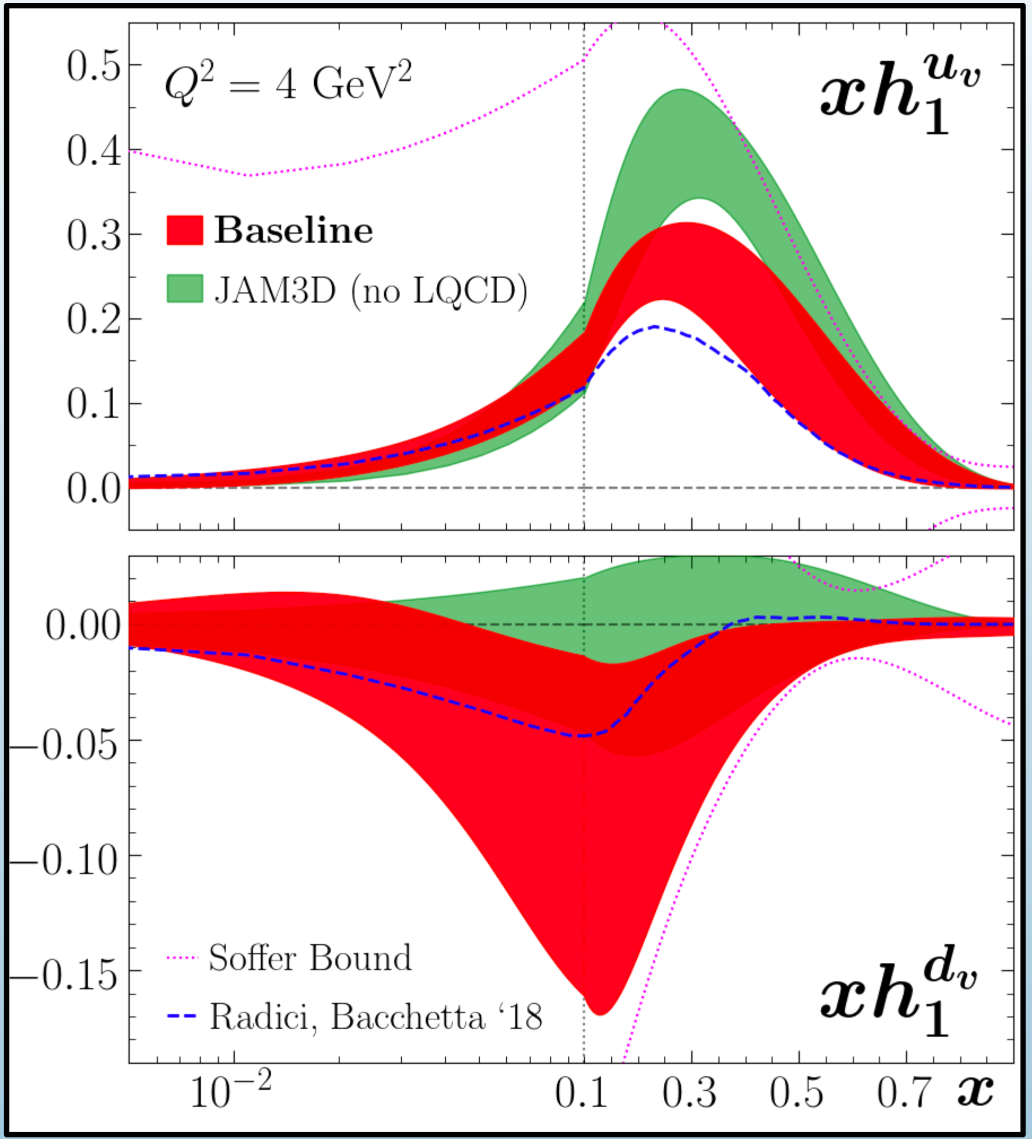
M_h binning



P_{hT} binning



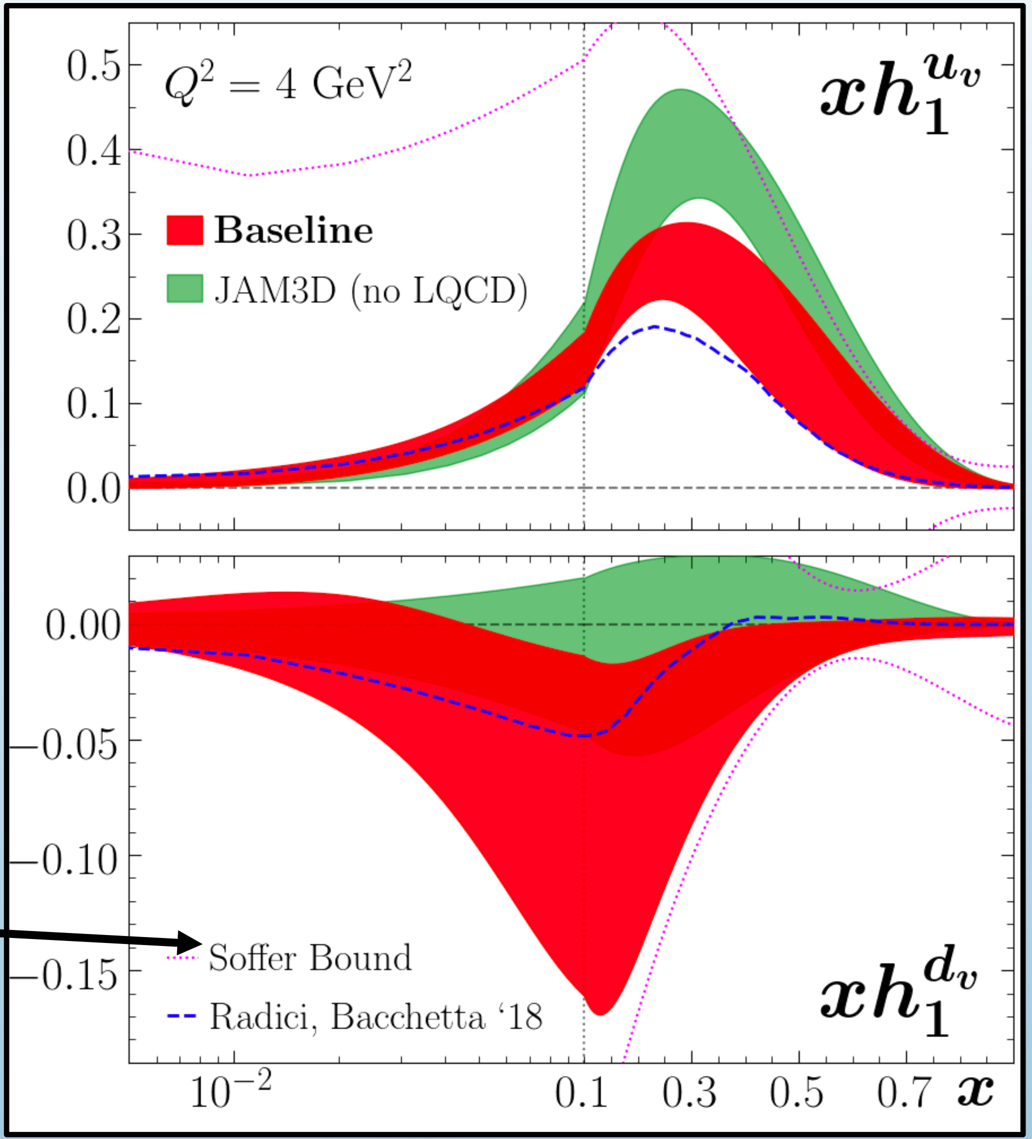
Transversity PDFs



Transversity PDFs

Soffer Bound: $|h_1^q| < \frac{1}{2}\left[f_1^q + g_1^q\right]$

J. Soffer, Phys. Rev. Lett. **74**, 1292-1294 (1995)



Transversity PDFs

Agreement with RB18

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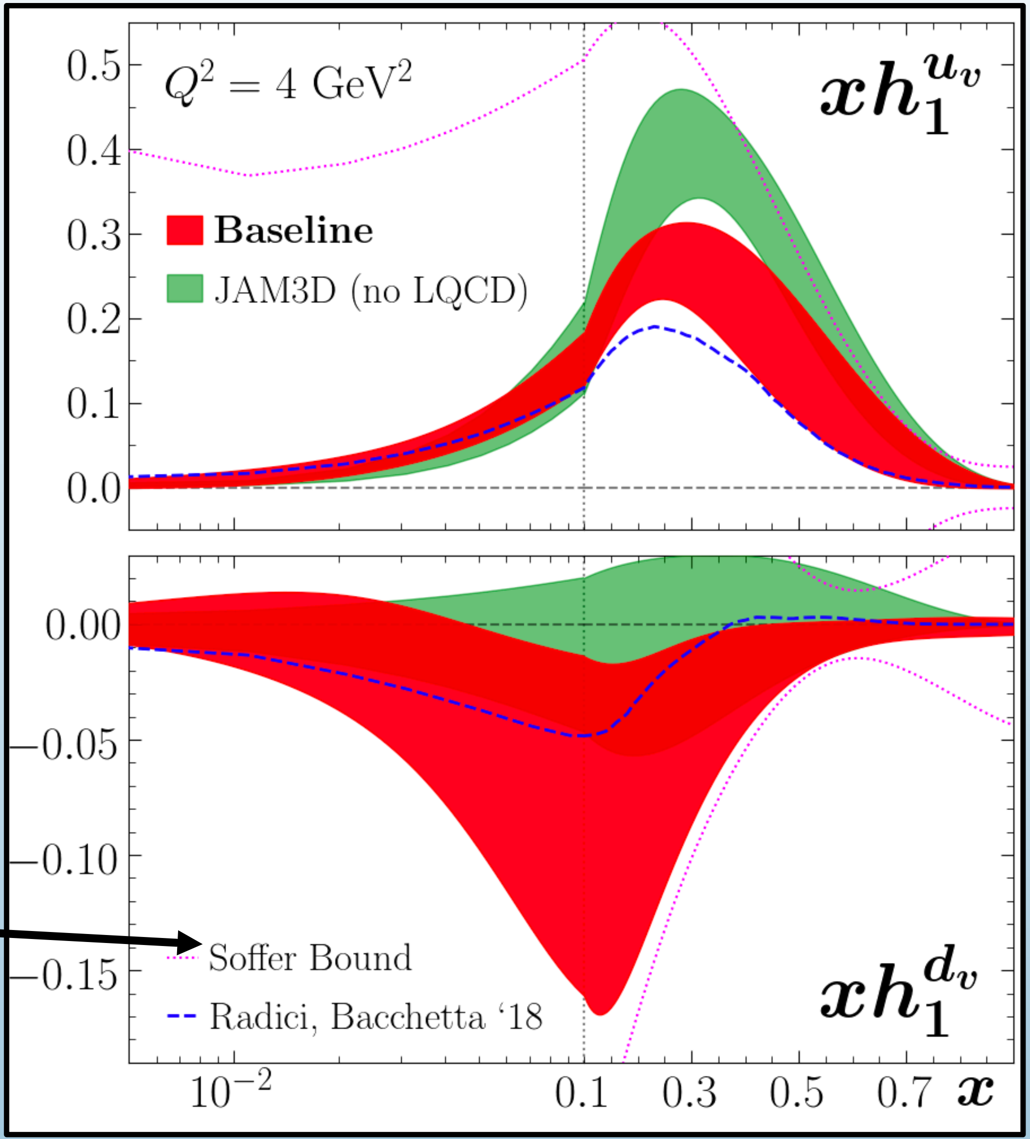


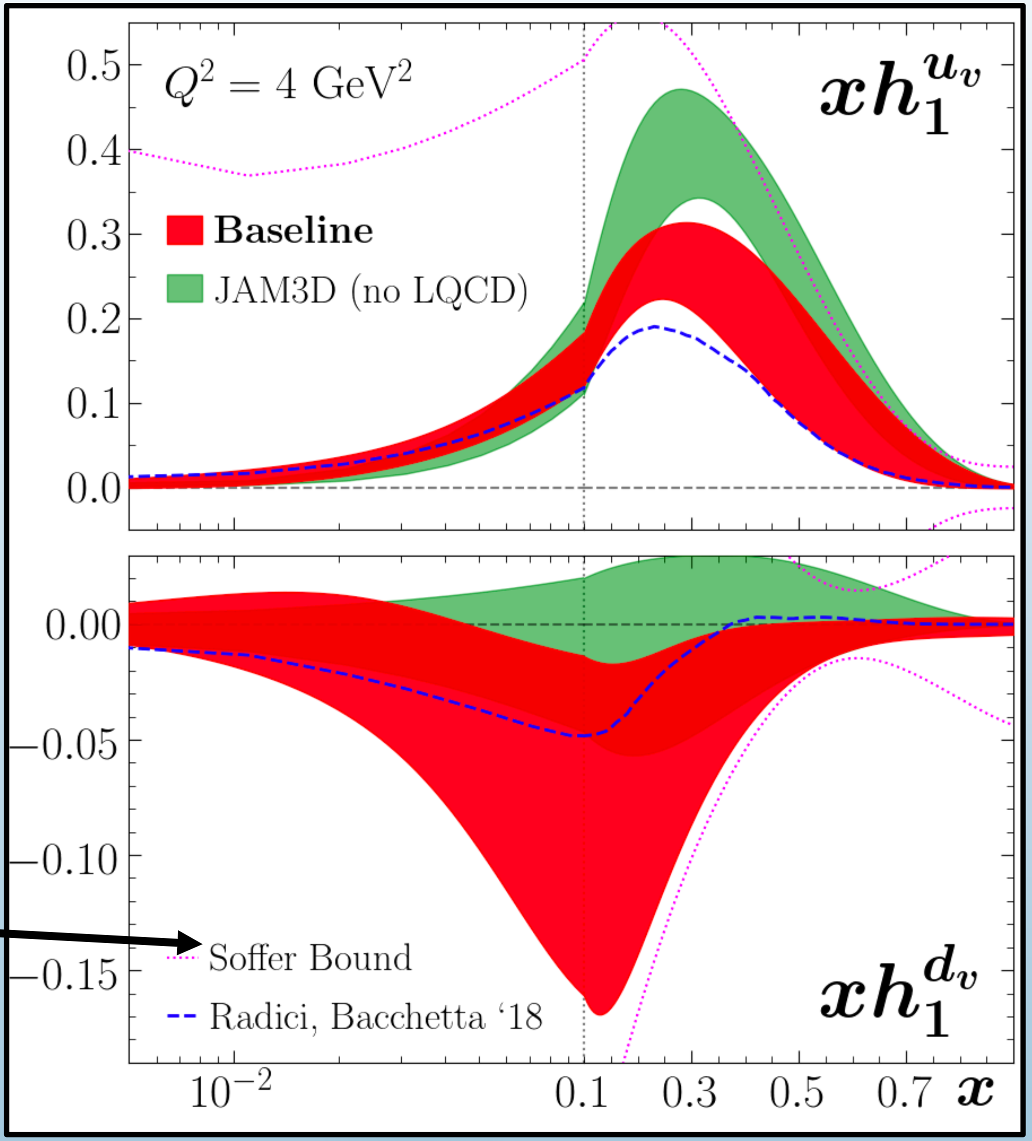
Transversity PDFs

Agreement with RB18

Result for u_v smaller than result from JAM3D (no LQCD)

Soffer Bound: $|h_1^q| < \frac{1}{2}[f_1^q + g_1^q]$

J. Soffer, Phys. Rev. Lett. **74**, 1292-1294 (1995)



1. JAM Methodology
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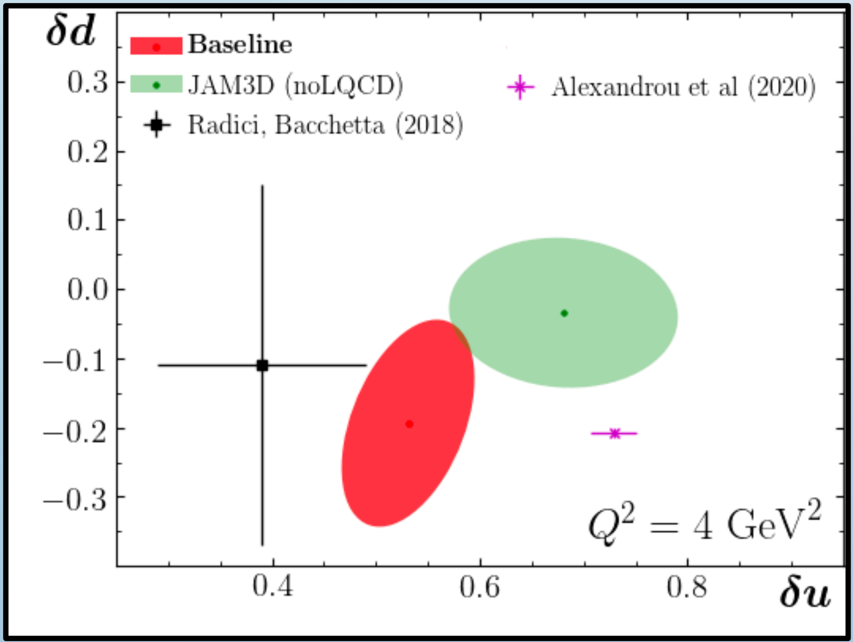
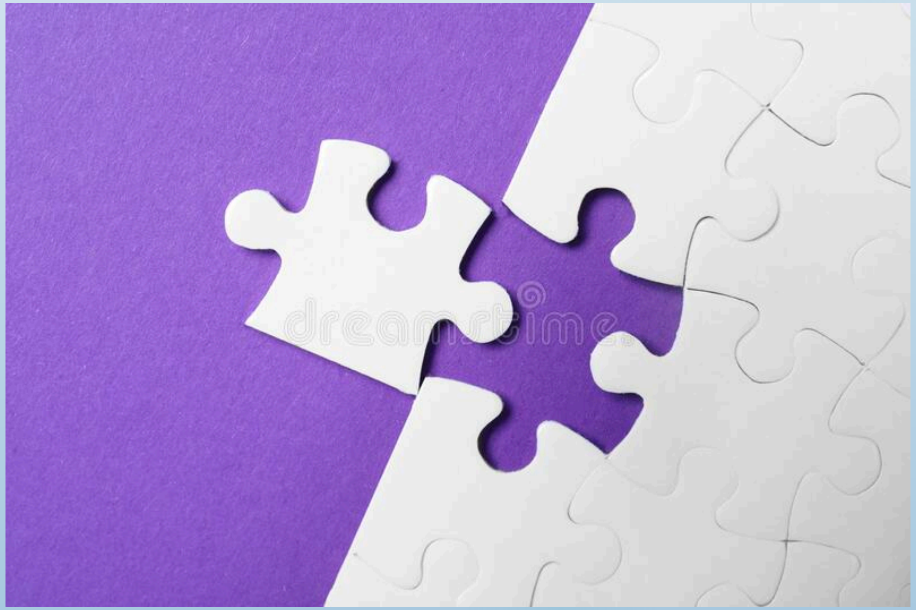




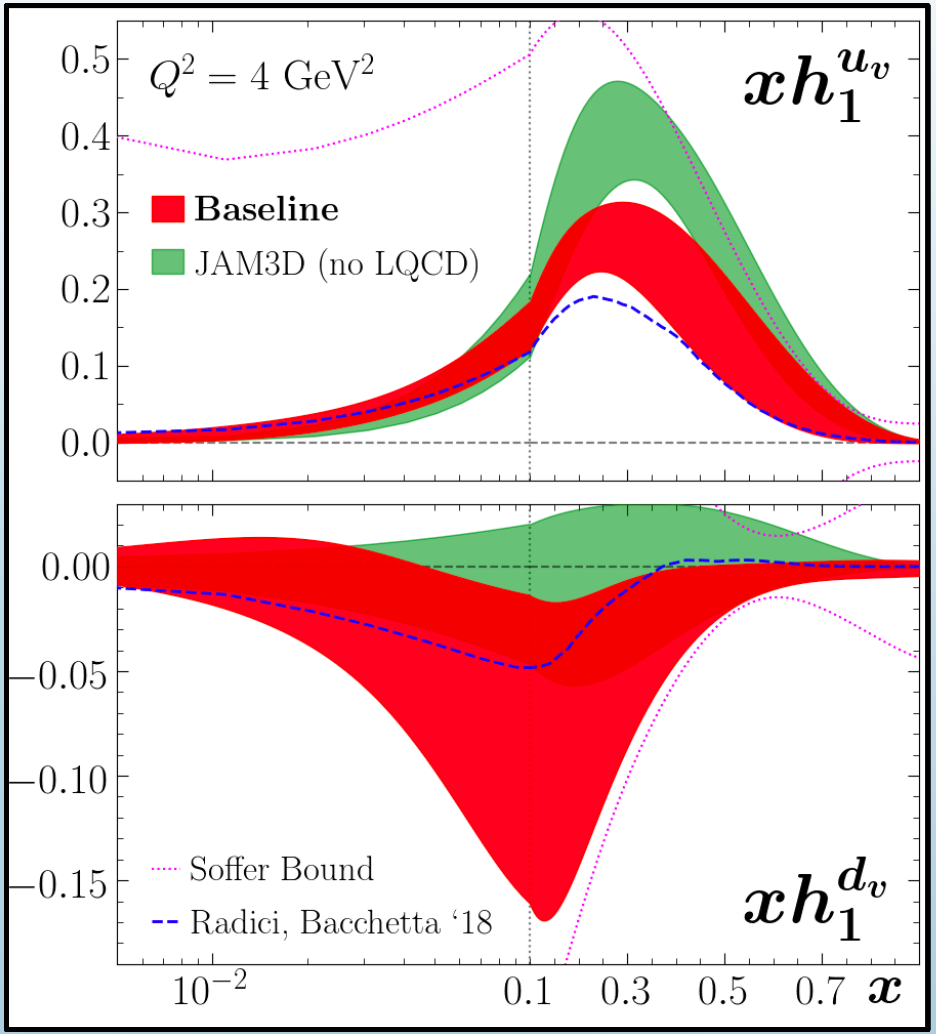
Controlling Extrapolation

$$\delta u \equiv \int_0^1 \mathrm{d}x (h_1^u - h_1^{\bar{u}}),$$

$$\delta d \equiv \int_0^1 \mathrm{d}x (h_1^d - h_1^{\bar{d}}),$$

$$g_T \equiv \delta u - \delta d,$$

Controlling Extrapolation



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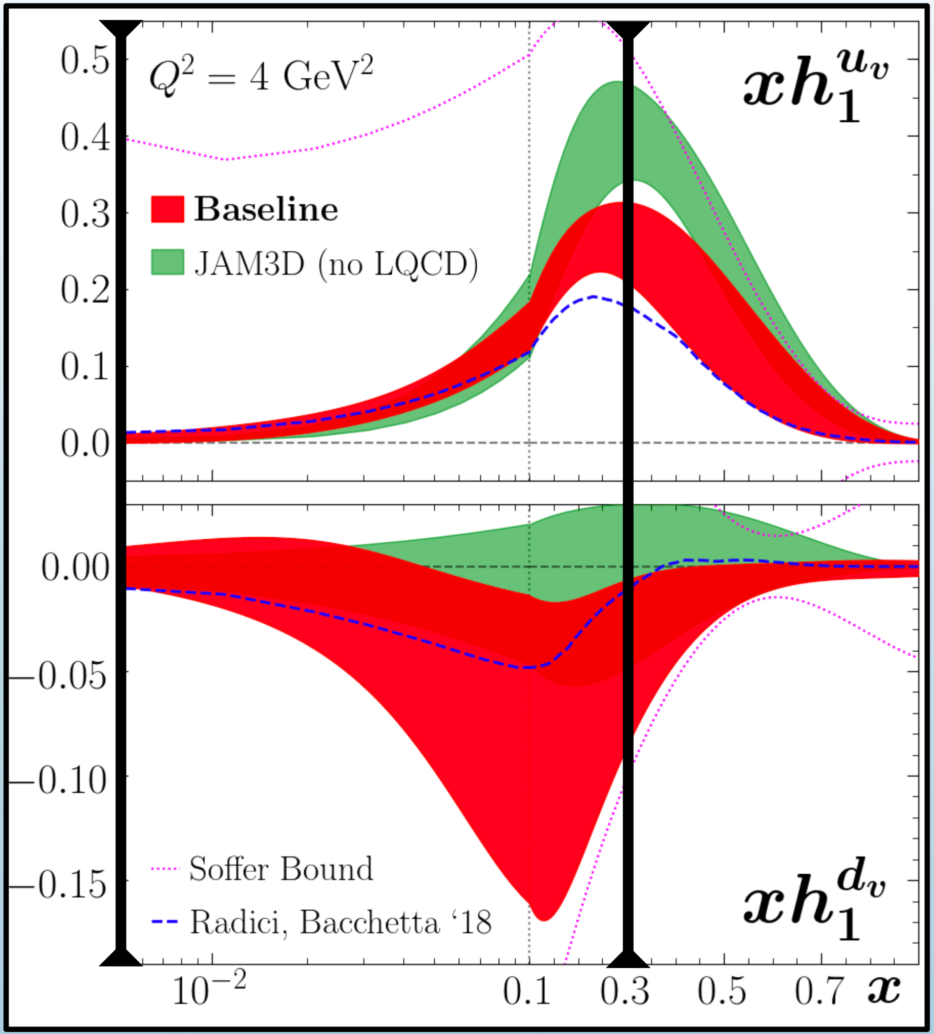

Measured Region

$$\delta u \equiv \int_0^1 \mathrm{d}x(h_1^u - h_1^{\bar{u}}),$$

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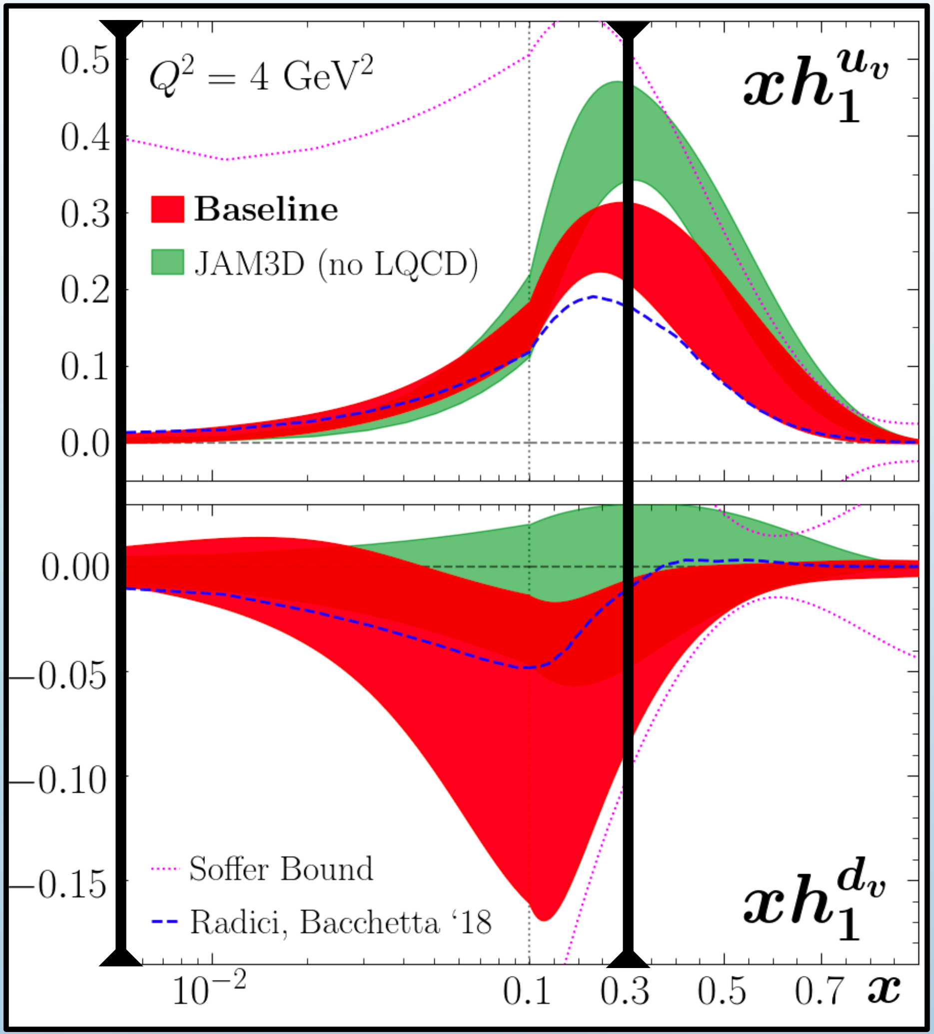
Controlling Extrapolation



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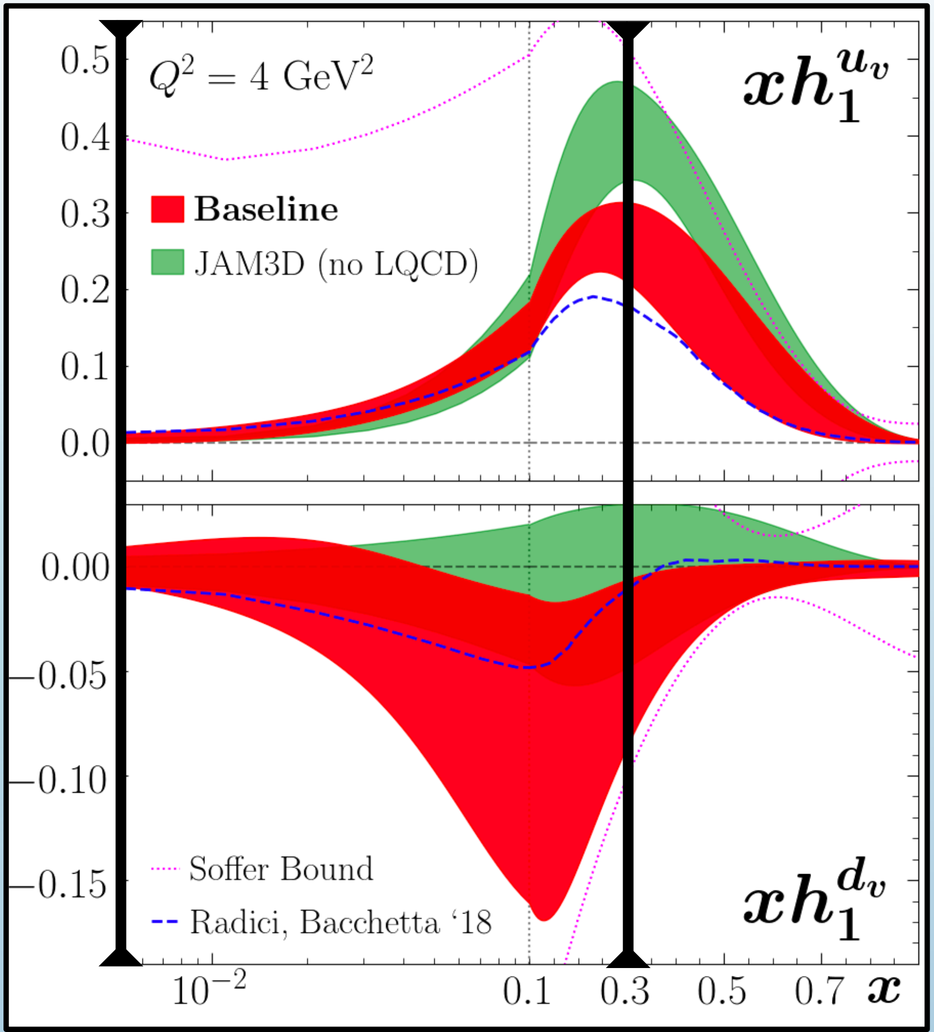
Large $x \gtrsim 0.3$

Soffer Bound: $|h_1^q| < \frac{1}{2}\left[f_1^q + g_1^q\right]$

J. Soffer, Phys. Rev. Lett. **74**, 1292-1294 (1995)

Controlling Extrapolation



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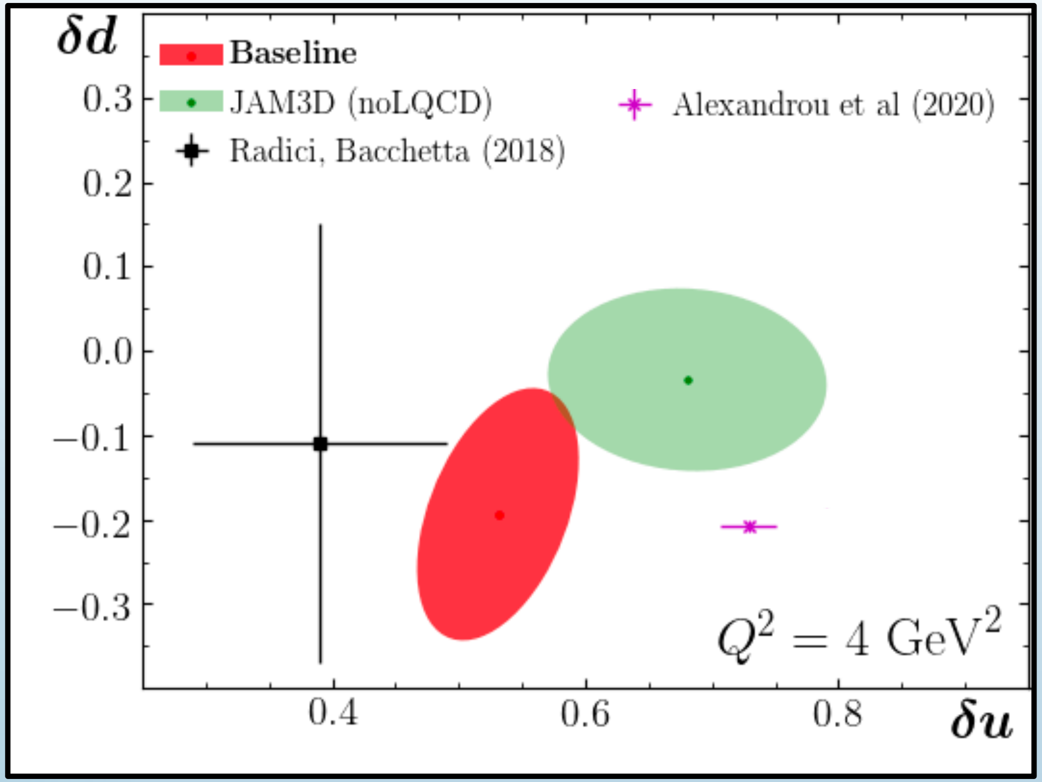
J. Soffer, Phys. Rev. Lett. **74**, 1292-1294 (1995)

Small $x \lesssim 0.005$

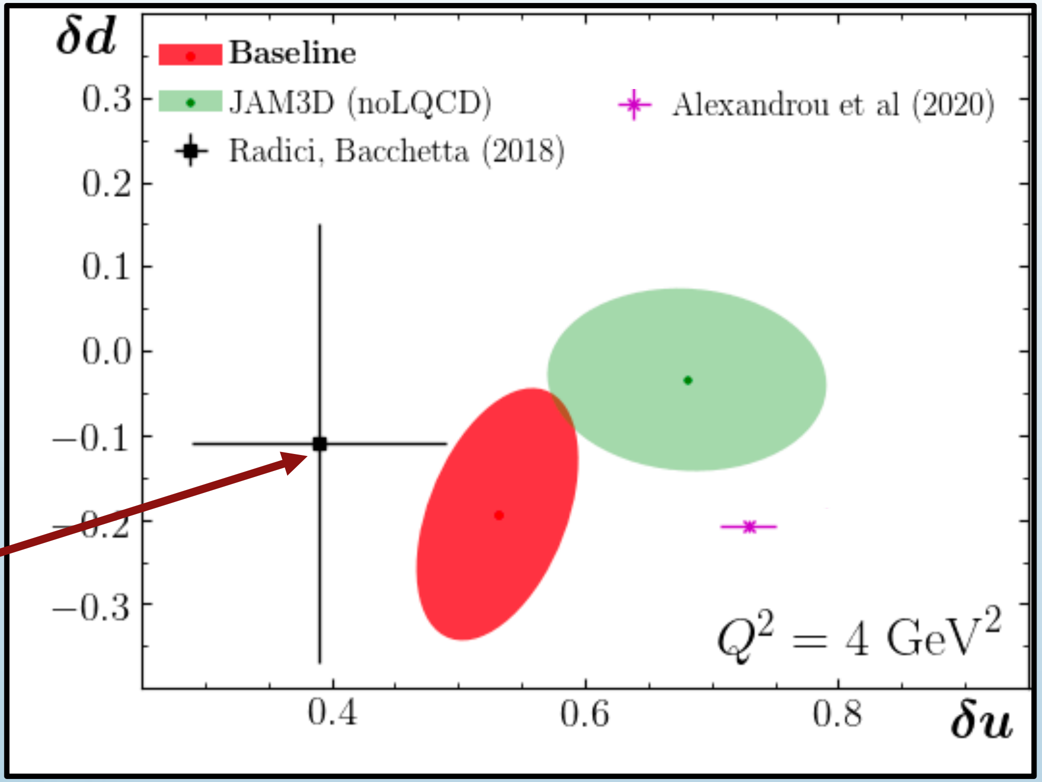
$$h_1^q \xrightarrow[x\to 0]{} x^{\alpha_q} \qquad \alpha_q = 1 - 2\sqrt{\frac{\alpha_s N_c}{2\pi}} \approx 0.17 \pm 50\,\%$$

Y. V. Kovchegov and M. D. Sievert, Phys. Rev. D **99**, 054033 (2019)

Tensor Charges (no lattice in fit)

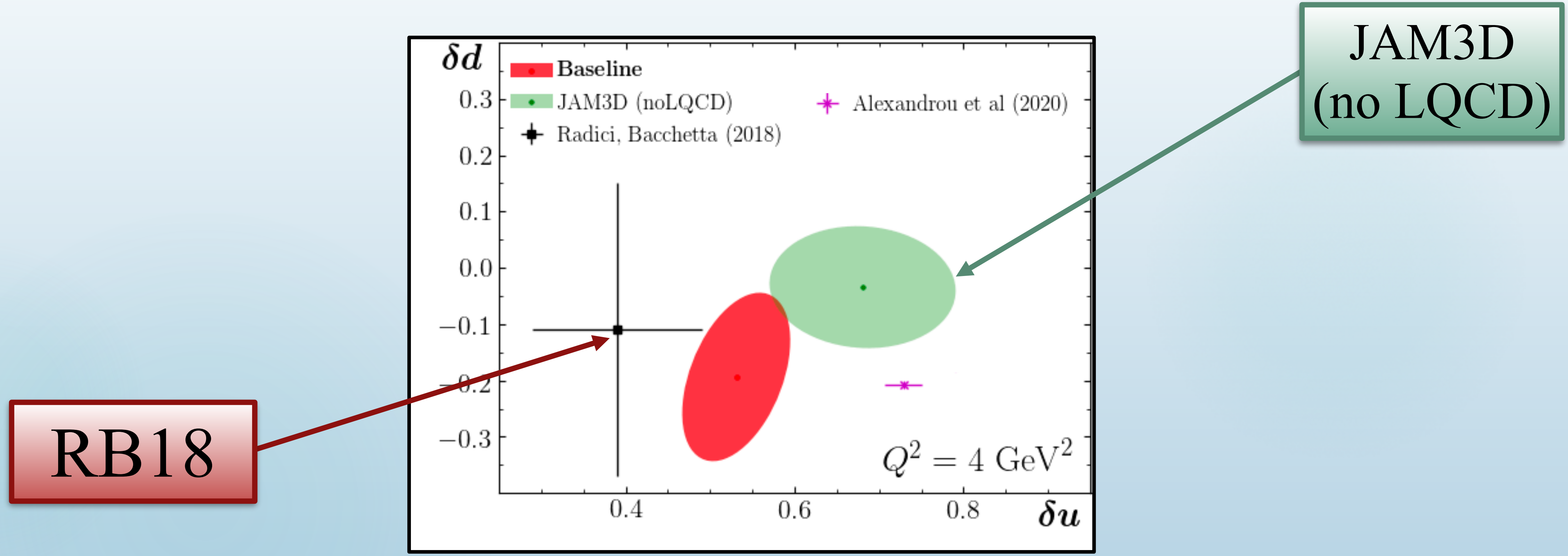


Tensor Charges (no lattice in fit)

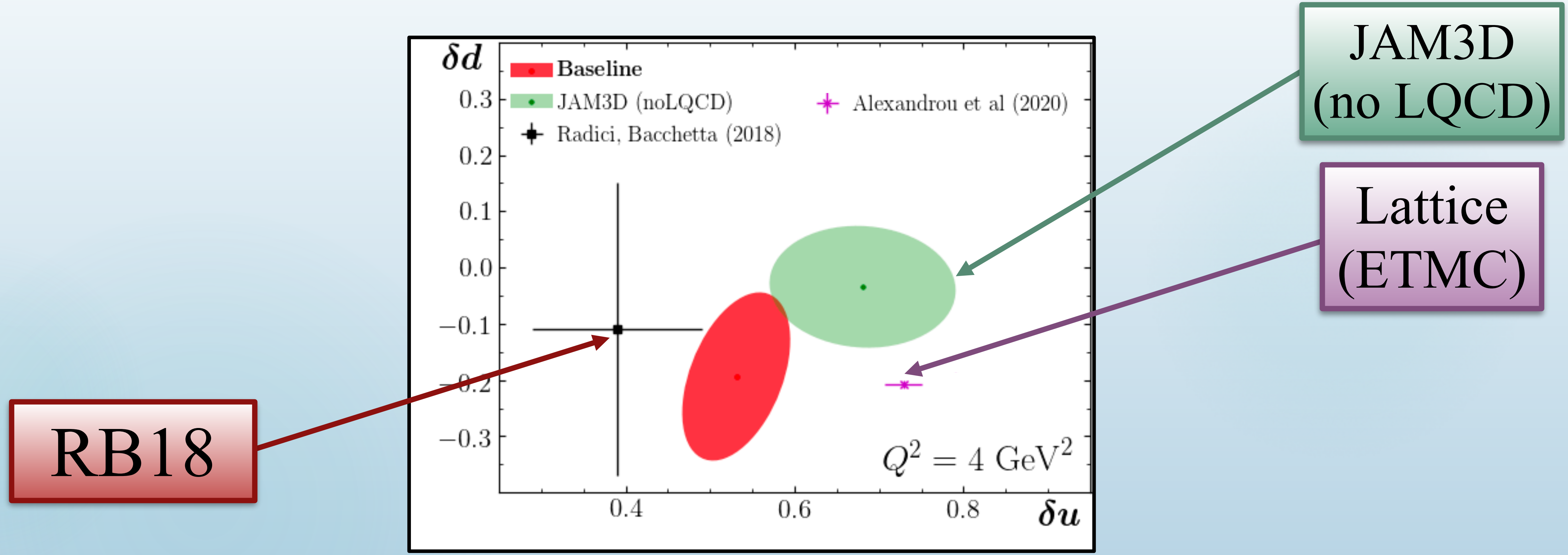


RB18

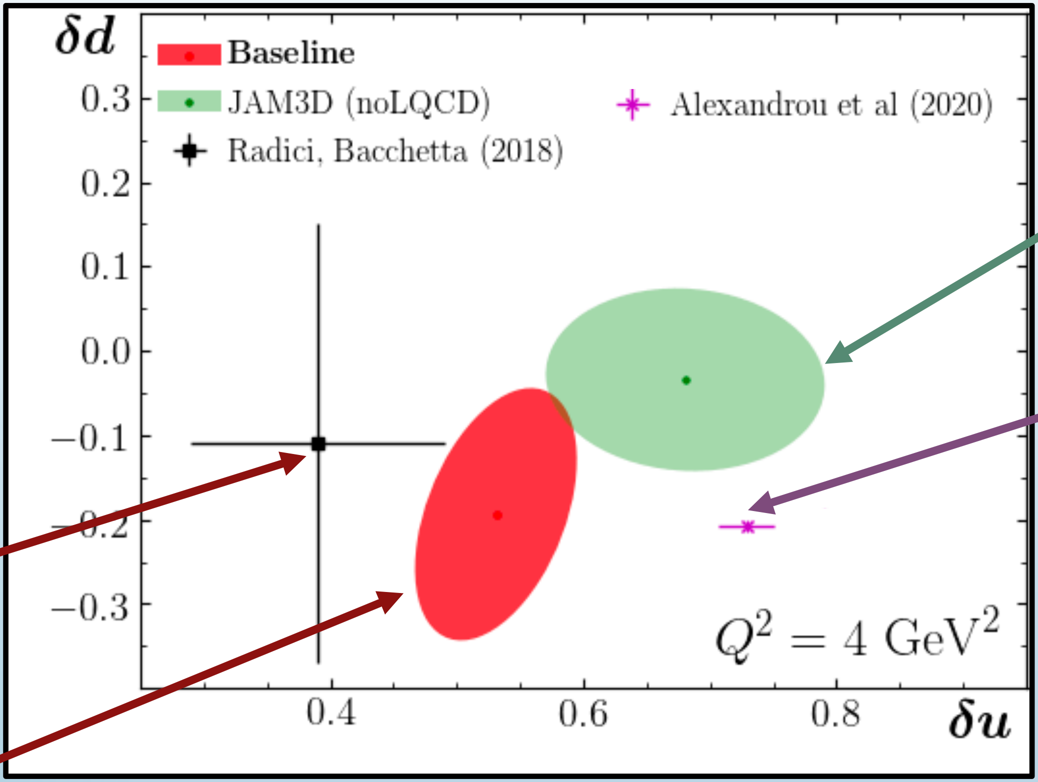
Tensor Charges (no lattice in fit)



Tensor Charges (no lattice in fit)



Tensor Charges (no lattice in fit)



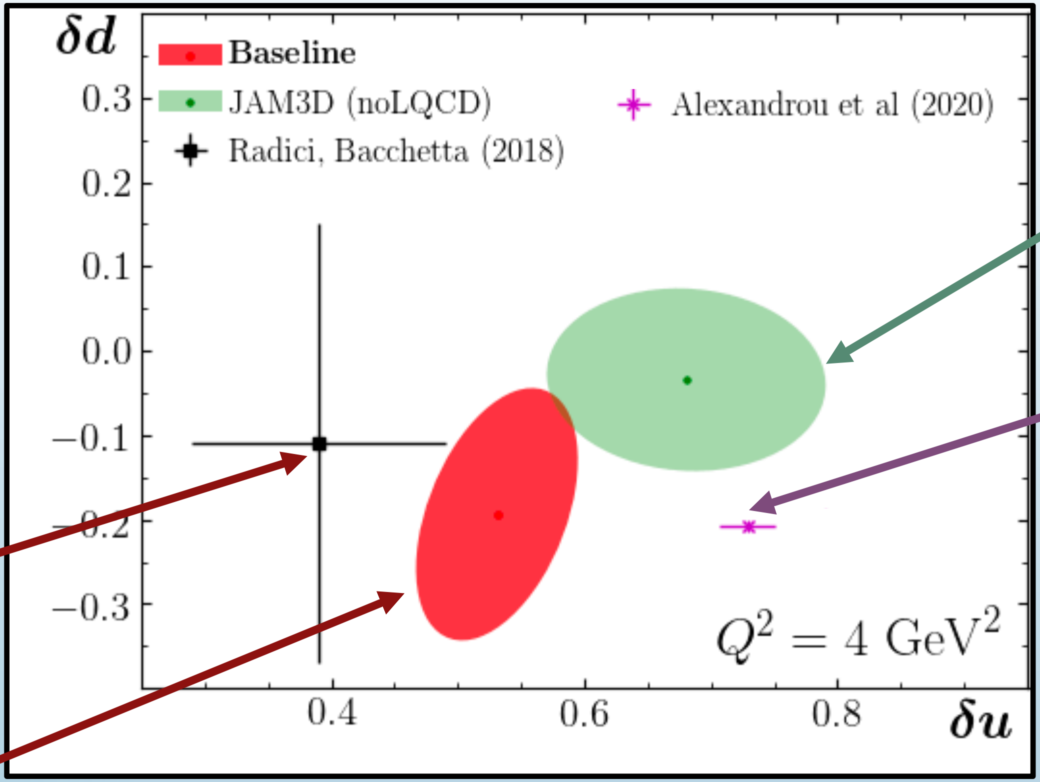
JAM3D (no LQCD)

Lattice (ETMC)

RB18

This analysis

Tensor Charges (no lattice in fit)



JAM3D (no LQCD)

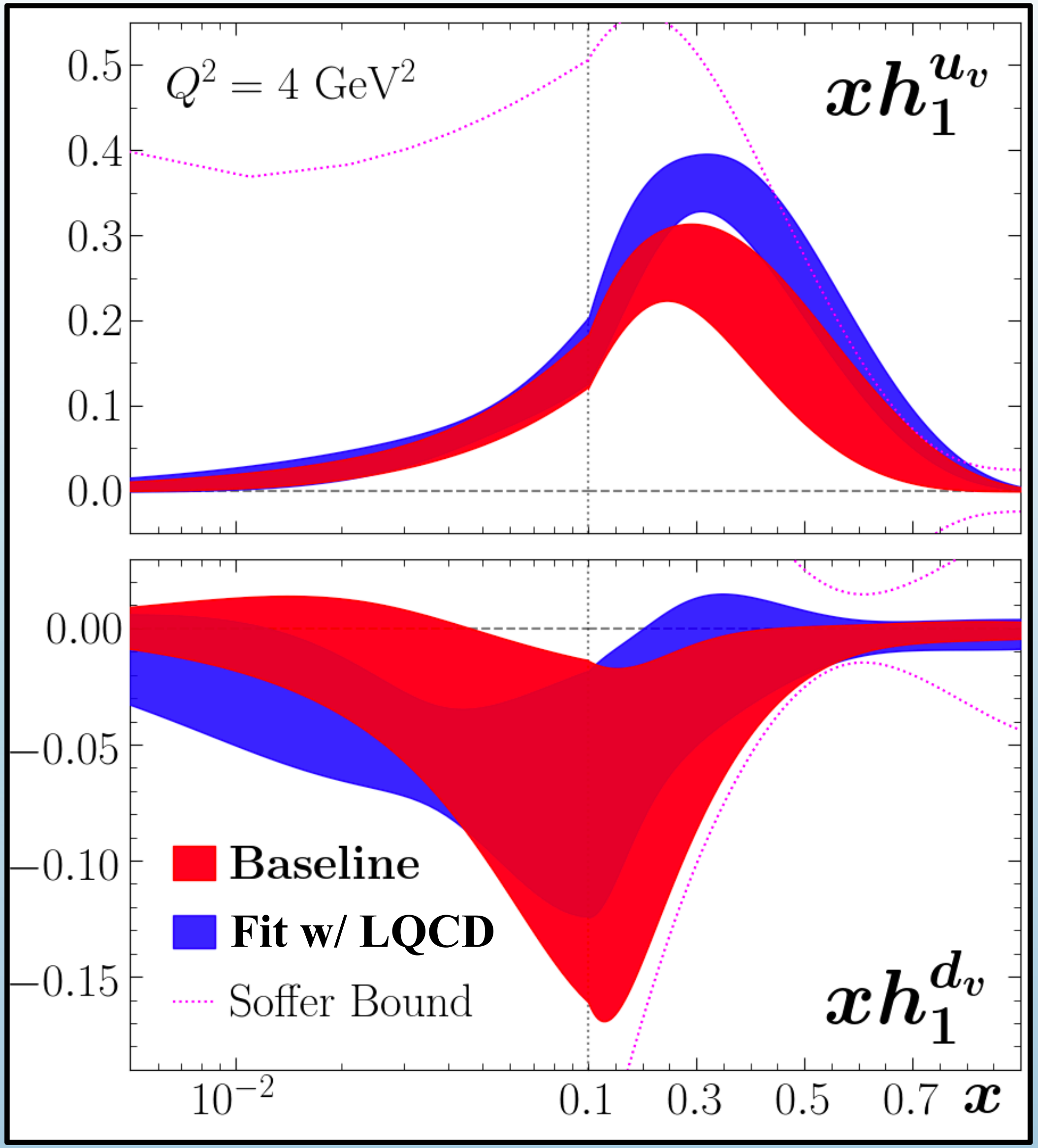
Lattice (ETMC)

RB18

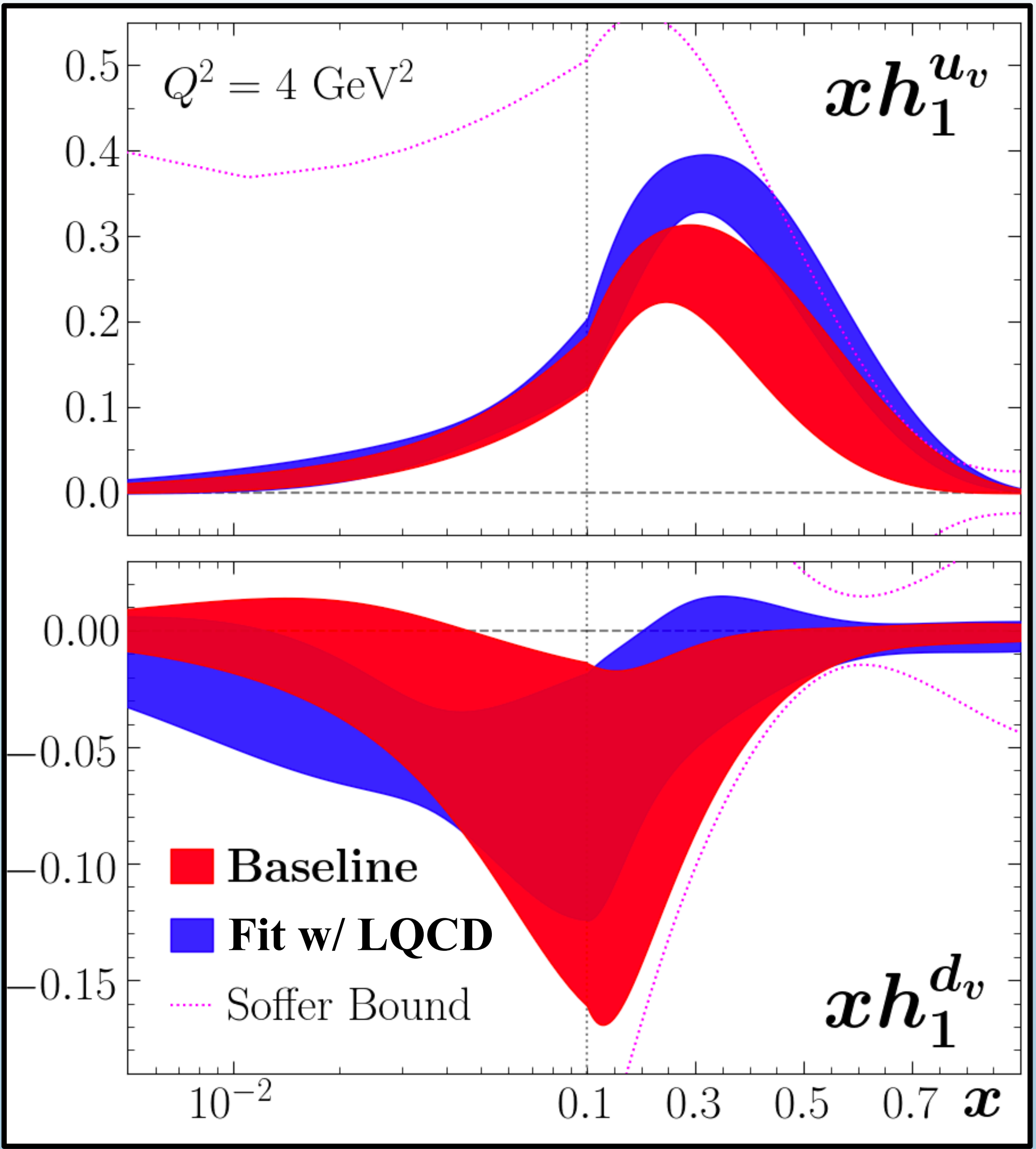
This analysis

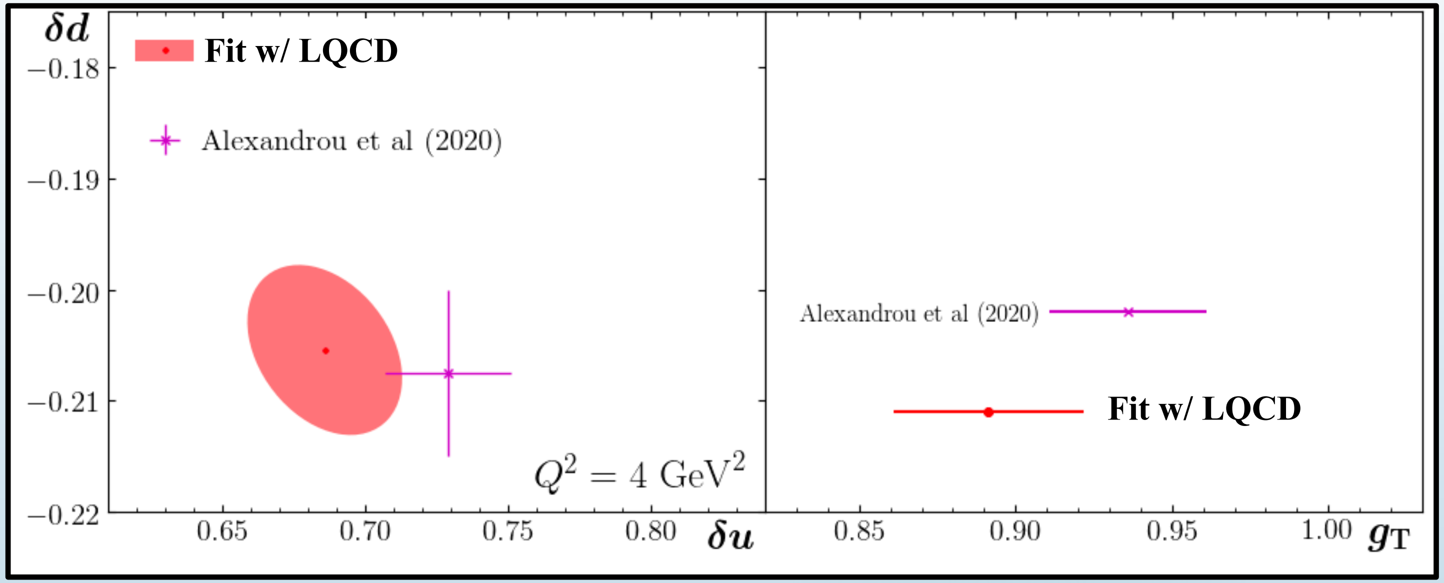
Consistent with RB18 and JAM3D (no LQCD).
What happens if lattice is included in our fit?

Tensor Charges (with lattice in fit)

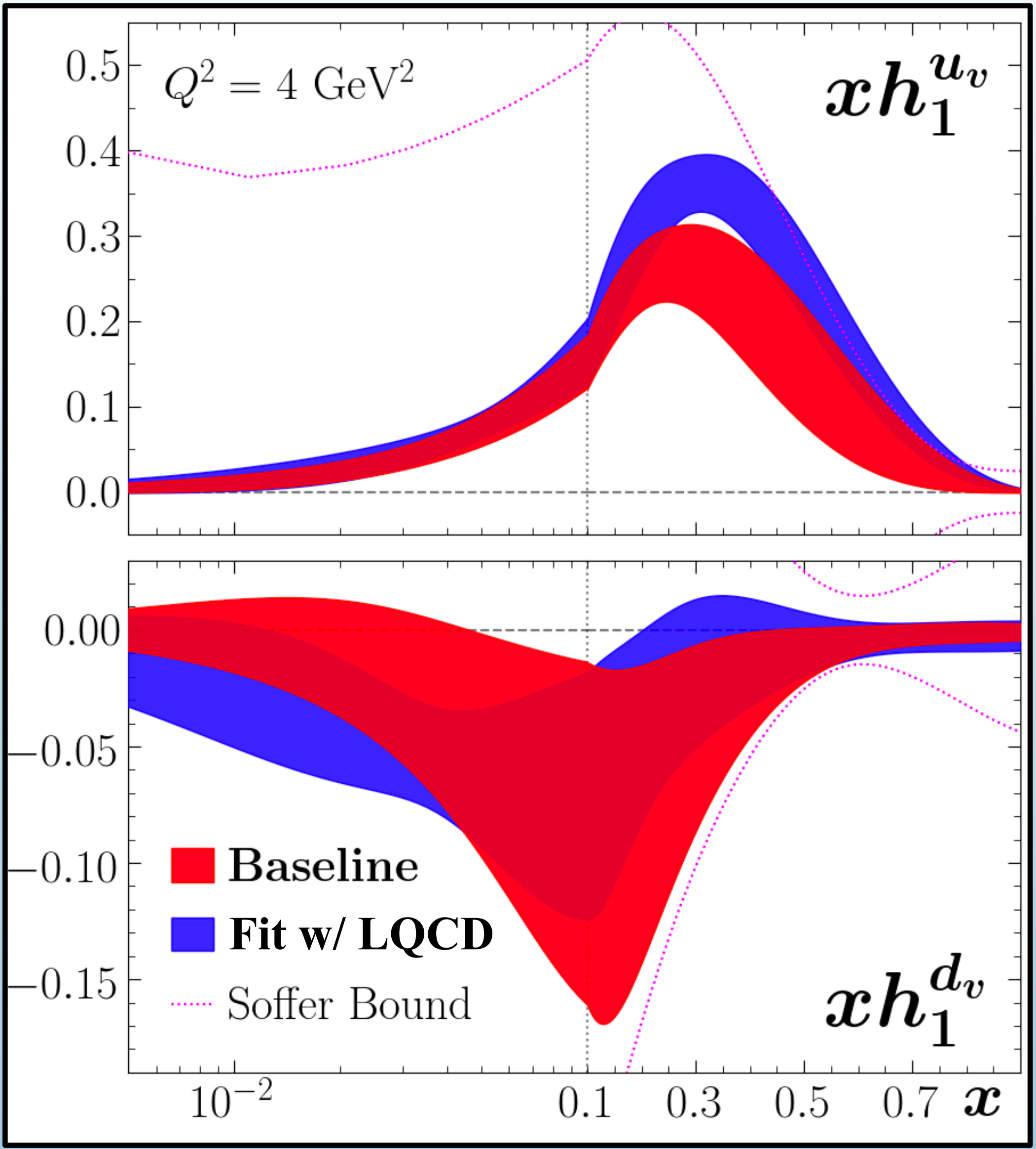


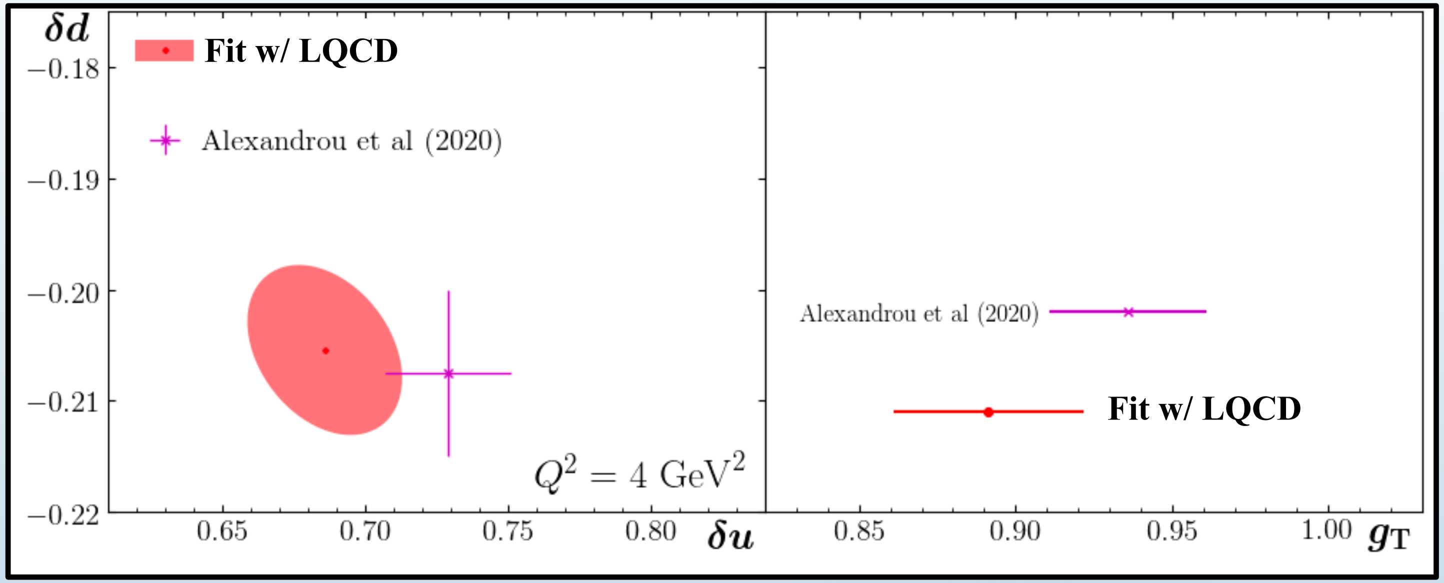
Tensor Charges (with lattice in fit)





Tensor Charges (with lattice in fit)

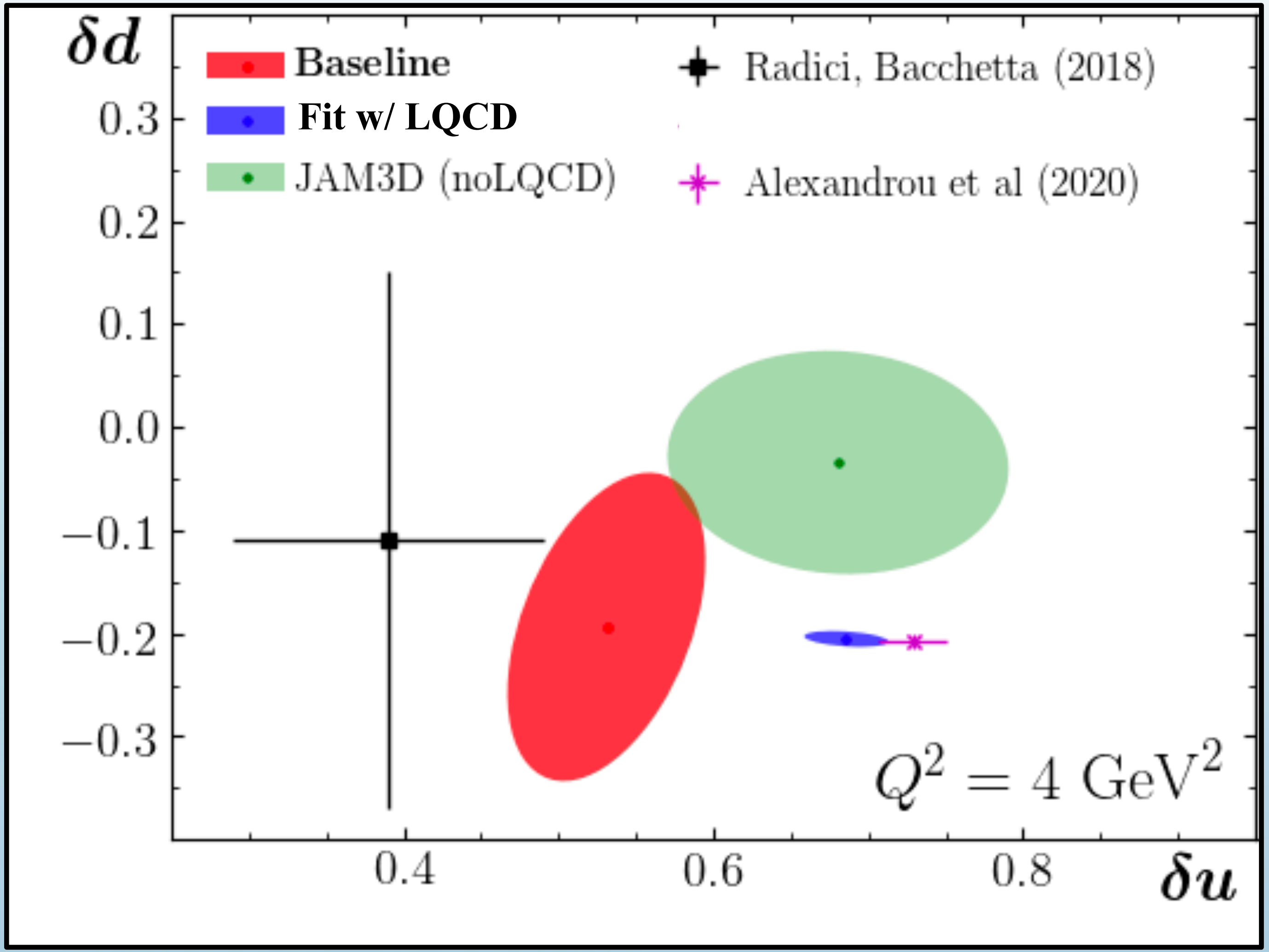




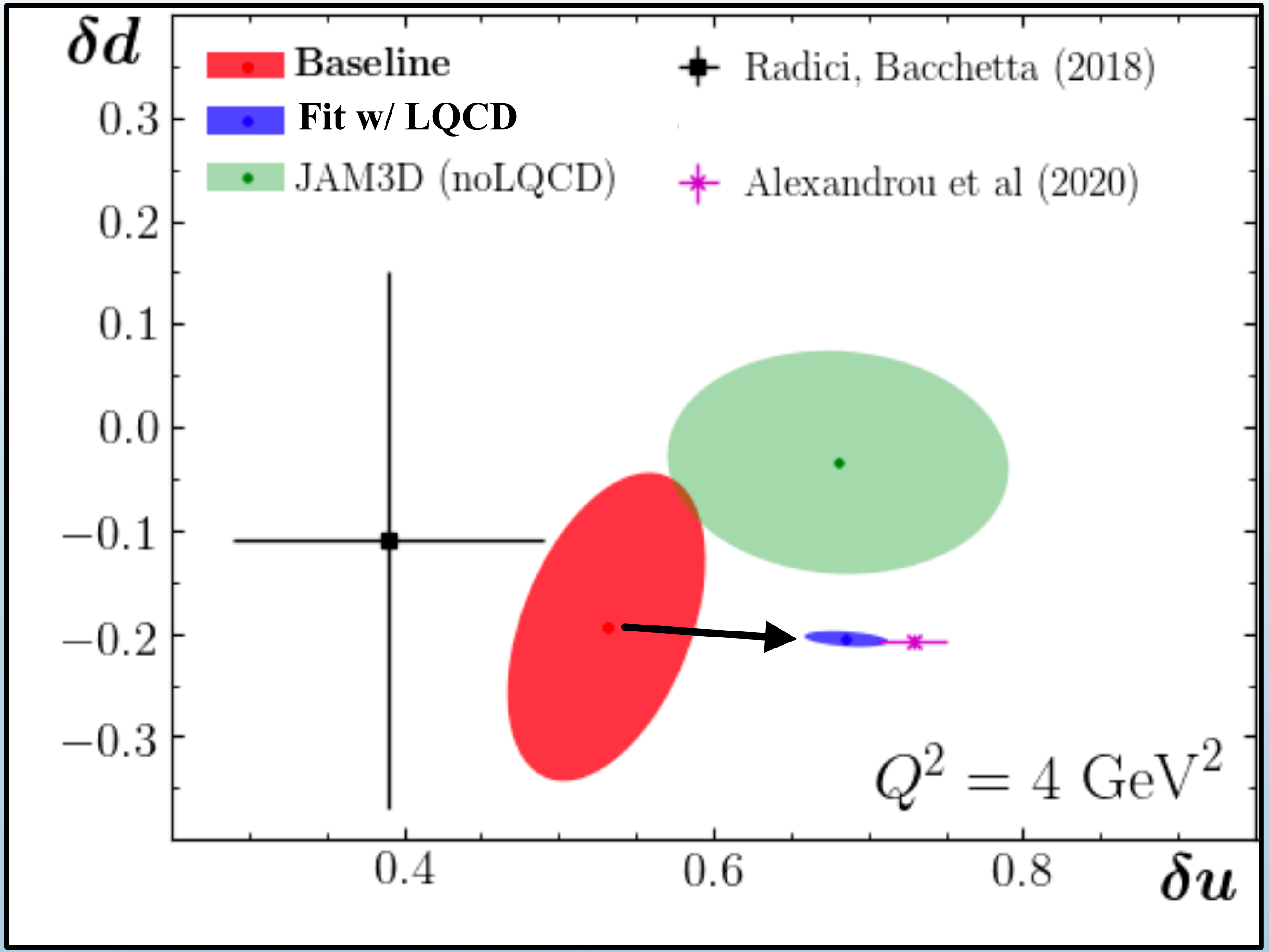
Fit is able to accommodate lattice data quite well!

Global χ^2 without lattice: 1.11
Global χ^2 with lattice: 1.15

Tensor Charges (before and after lattice)

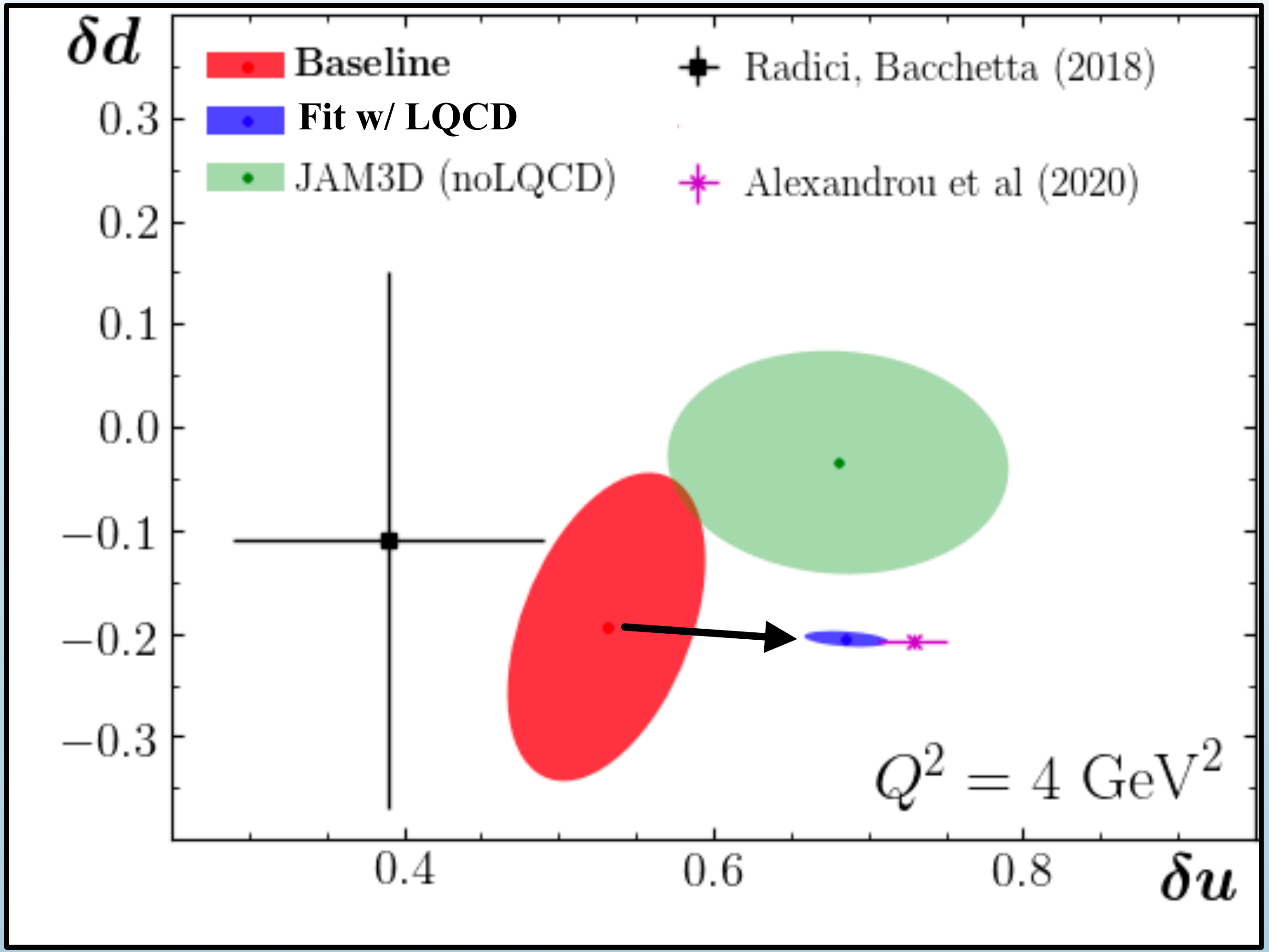


Tensor Charges (before and after lattice)



Noticeable shift from including lattice data

Tensor Charges (before and after lattice)



Noticeable shift from including lattice data

Experimental data has only weak preference, as it is not directly sensitive to the full moment

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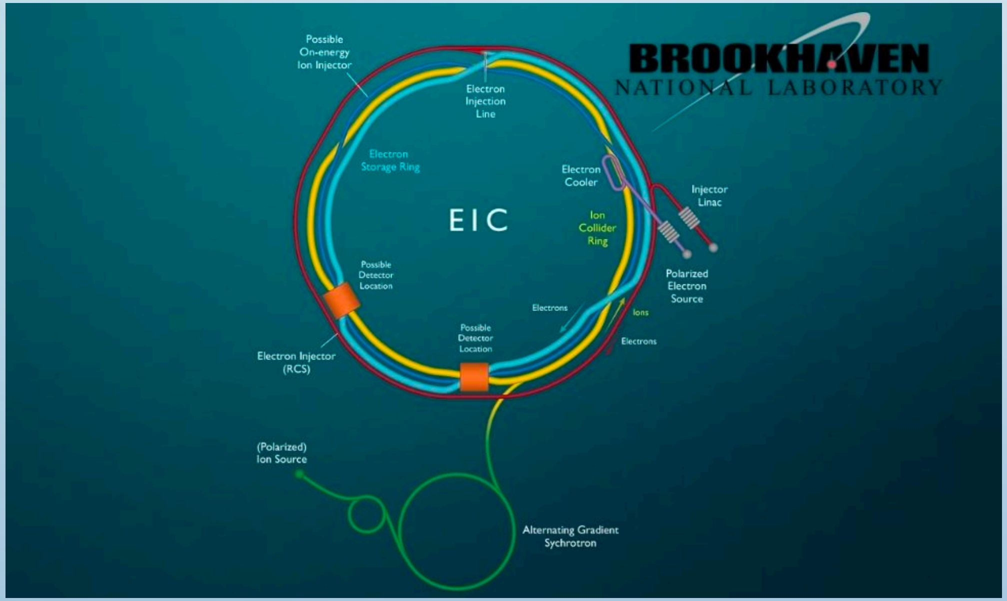
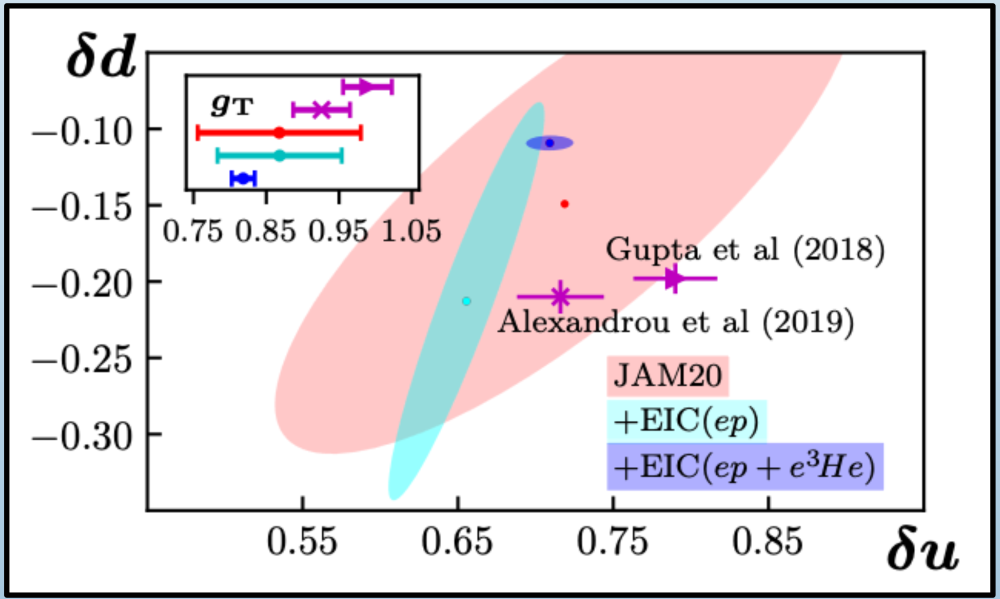
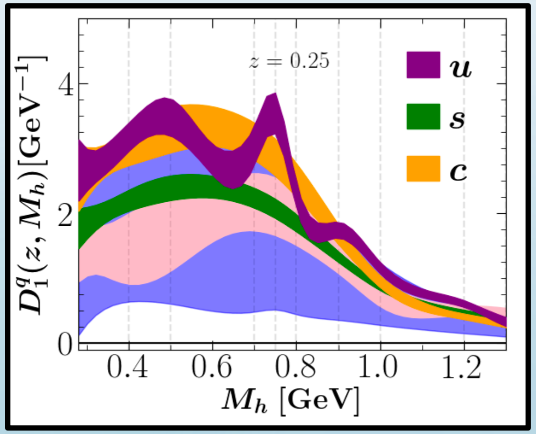
5. **Conclusions and Outlook**

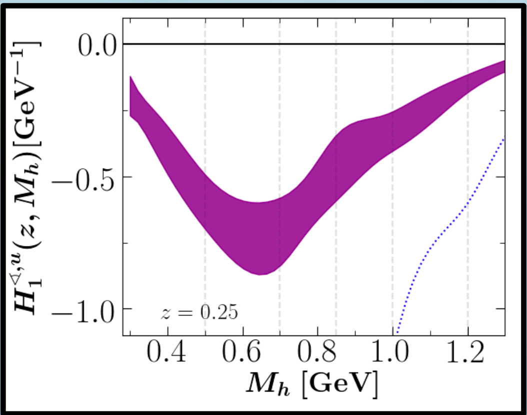


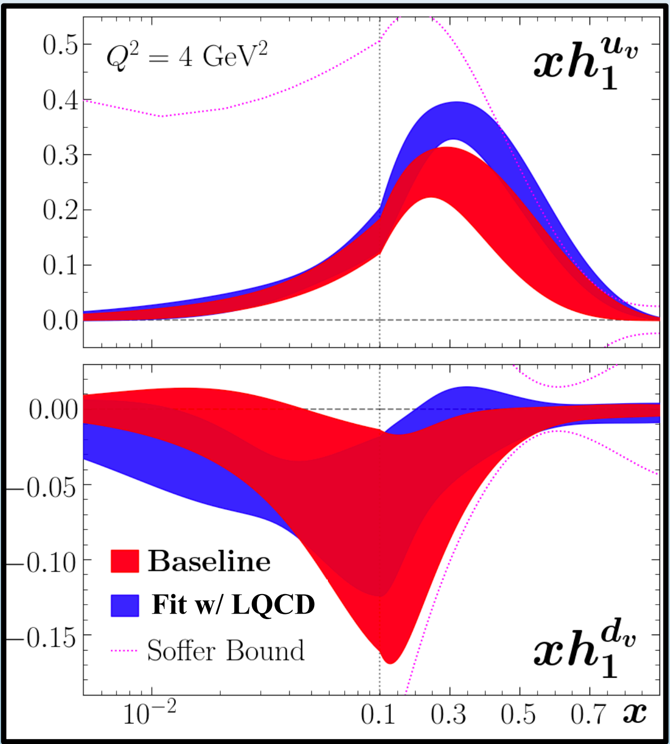


Conclusions

Simultaneous extraction of DiFFs and transversity PDFs







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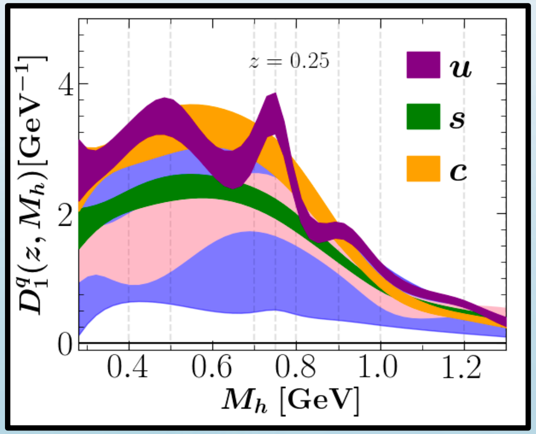
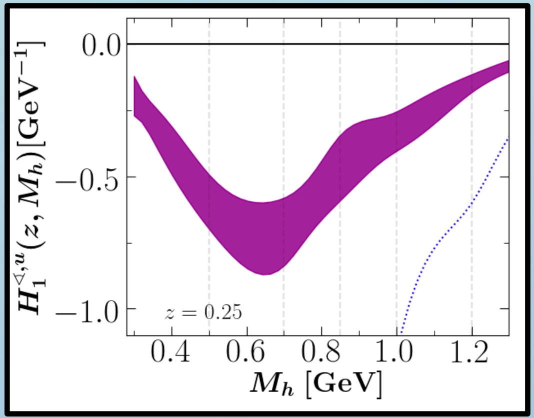
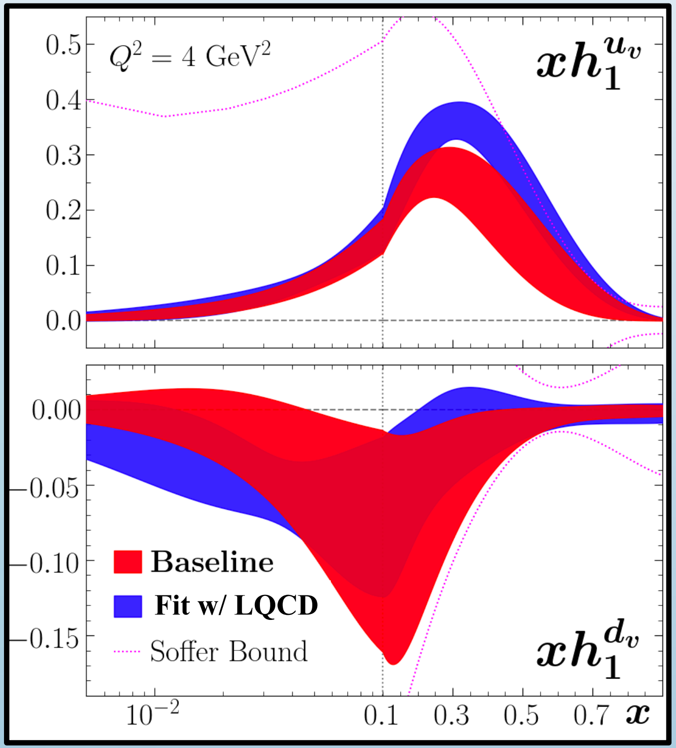
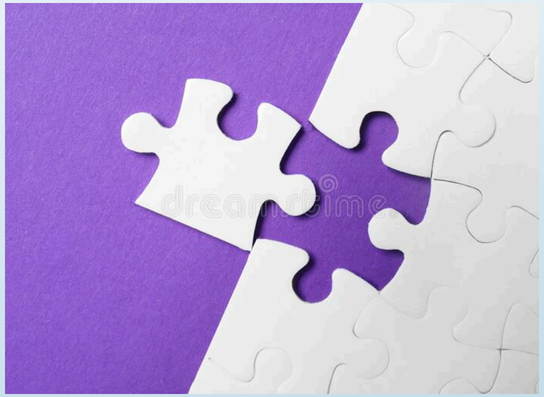






Transverse spin puzzle?



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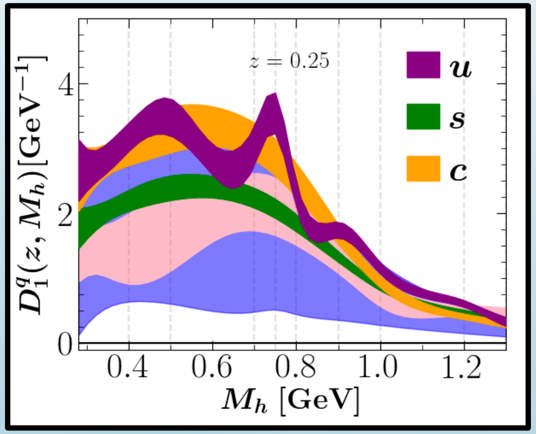

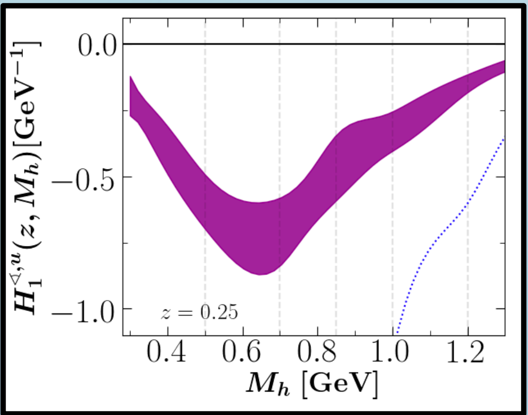

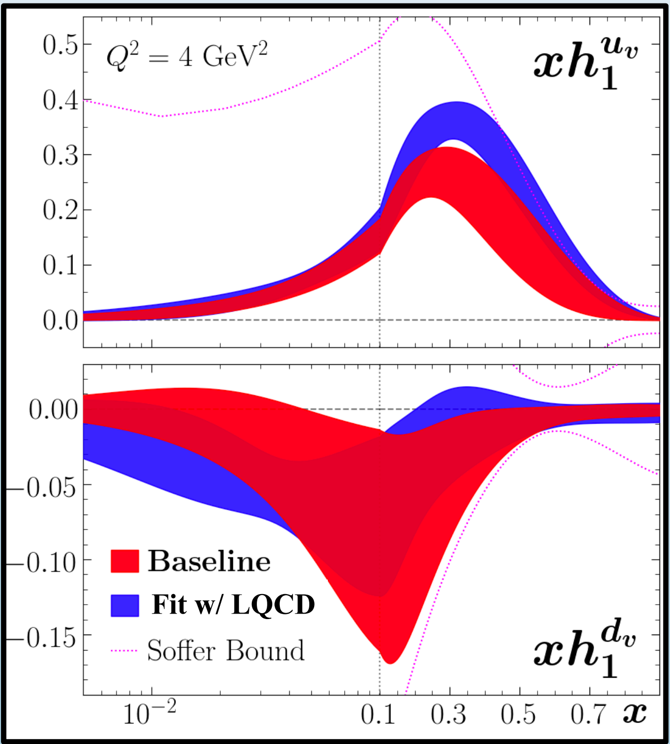

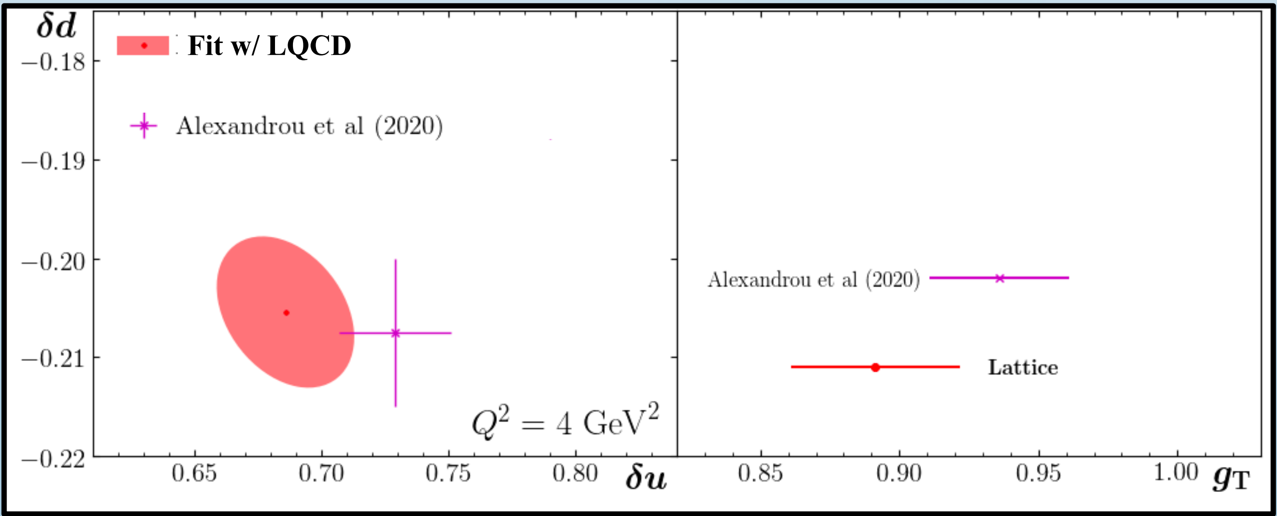

Outlook

More data from RHIC

See talks by **Babu Pokhrel**
and **Navagyan Ghimire** (right after this!)

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SIDIS multiplicities from COMPASS

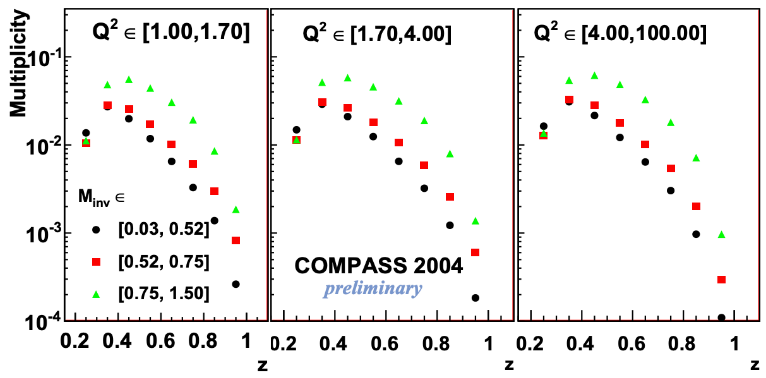
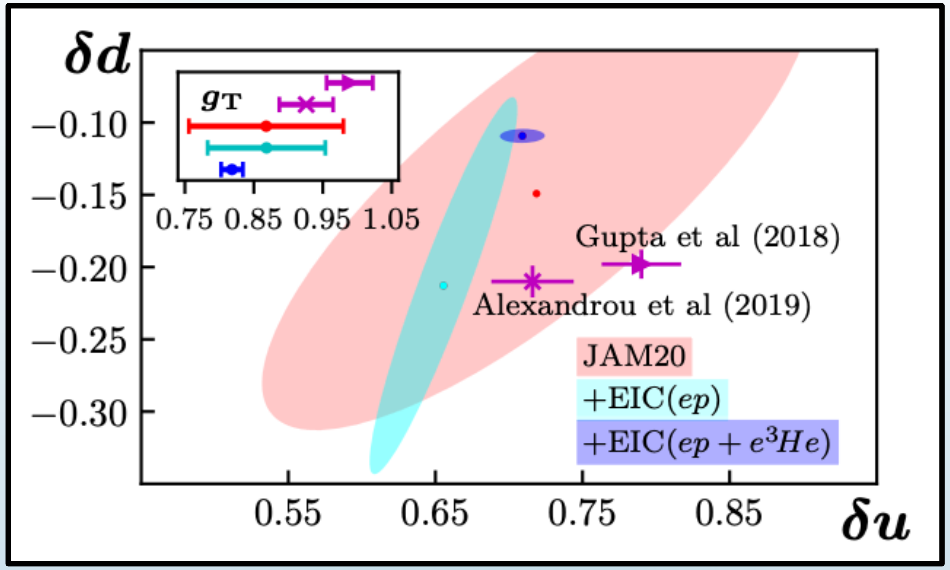


Figure 4: h^+h^- multiplicities measured versus z, M_{inv} and Q^2.

N. Makke, Phys. Part. Nucl. **45**, 138-140 (2014)

Outlook

L. Gamberg *et al.*, Phys. Lett. B **816**, 136255 (2021)



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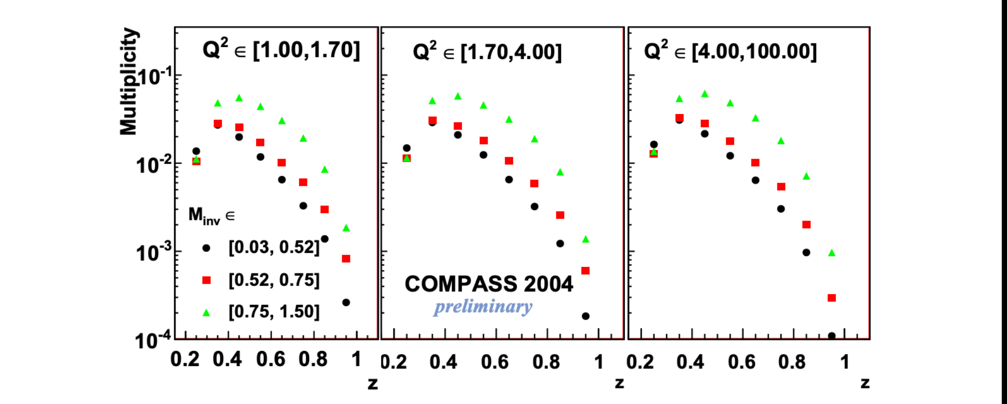
SIDIS multiplicities from COMPASS

EIC can provide new information



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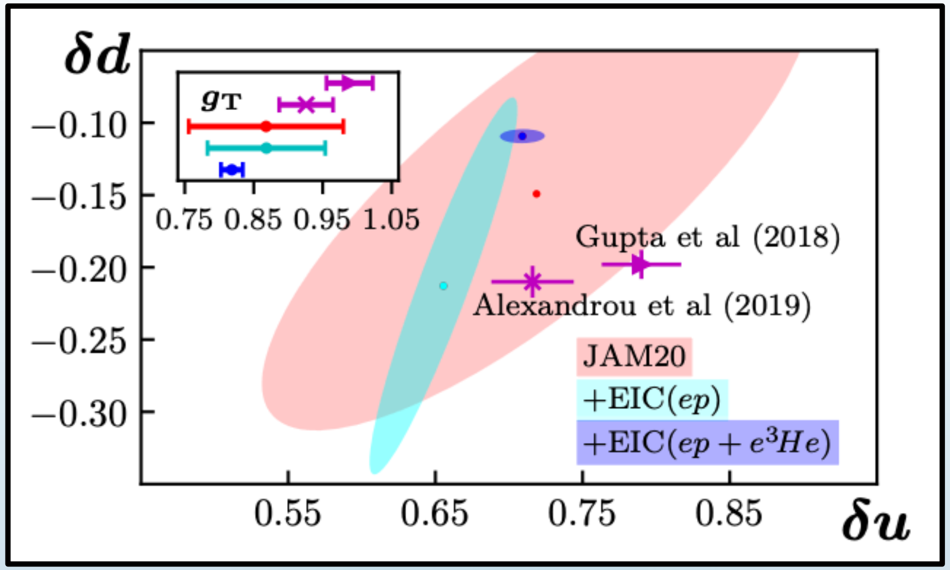

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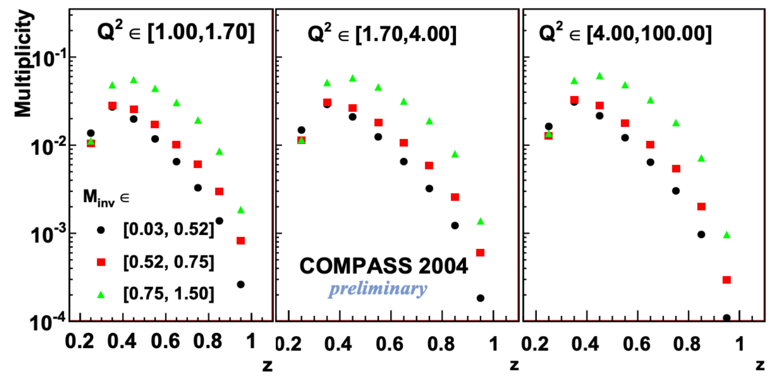

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N. Makke, Phys. Part. Nucl. **45**, 138-140 (2014)

Simultaneous fit of DiFF channel + TMD channel + Lattice QCD

Collaboration

Andreas Metz

Wally Melnitchouk

Alexey Prokudin

Ralf Seidl

Nobuo Sato

Daniel Pitonyak

Extra Slides

Parameterize PDFs at input scale $Q_0^2 = m_c^2$

$$f_i(x) = Nx^{\alpha}(1-x)^{\beta}(1+\gamma\sqrt{x}+\eta x)$$

Parameterize PDFs at input scale $Q_0^2 = m_c^2$

$$f_i(x) = Nx^{\alpha}(1-x)^{\beta}(1+\gamma\sqrt{x}+\eta x)$$

Evolve PDFs using DGLAP

$$\frac{\mathrm{d}}{\mathrm{d}\ \ln(\mu^2)} f_i(x,\mu) = \sum_j \int_x^1 \frac{dz}{z} P_{ij}(z,\mu) f_j(\frac{x}{z},\mu)$$

Parameterize PDFs at input scale $Q_0^2 = m_c^2$

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Calculate Observables

$$d\sigma^{pp} = \sum_{ij} H_{ij}^{pp} \otimes f_i \otimes f_j$$

Parameterize PDFs at input scale $Q_0^2 = m_c^2$

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Mellin Space Techniques

$$\mathrm{d}\sigma^{pp} = \sum_{ijkl} \frac{1}{(2\pi i)^2} \int \mathrm{d}N \int \mathrm{d}M\, \tilde{f}_j(N,\mu_0)\tilde{f}_l(M,\mu_0)$$
$$\otimes \left[x_1^{-N} x_2^{-M} \tilde{\mathcal{H}}_{ik}^{pp}(N,M,\mu) U_{ij}^S(N,\mu,\mu_0) U_{kl}^S(M,\mu,\mu_0)\right]$$

Parameterize PDFs at input scale $Q_0^2 = m_c^2$

$$f_i(x) = N x^{\alpha}(1-x)^{\beta}(1+\gamma\sqrt{x}+\eta x)$$

Evolve PDFs using DGLAP

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$$\sigma = \sum_{ij} H_{ij} \otimes f_i \otimes f_j + \mathcal{O}(1/Q)$$

Experimentally measured cross-section

$$\sigma = \sum_{ij} H_{ij} \otimes f_i \otimes f_j + \mathcal{O}(1/Q)$$

Experimentally measured cross-section

$$\sigma = \sum_{ij} H_{ij} \otimes f_i \otimes f_j + \mathcal{O}(1/Q)$$

“Hard part” (process dependent)
Cross-section at parton level
Calculated in perturbative QCD

Experimentally measured cross-section

"Soft part" (process independent)
Describes internal structure

$$\sigma = \sum_{ij} H_{ij} \otimes f_i \otimes f_j + \mathcal{O}(1/Q)$$

"Hard part" (process dependent)
Cross-section at parton level
Calculated in perturbative QCD

Now that the observables have been calculated…

$$\chi^2(\boldsymbol{a}) = \sum_{i,e} \left(\frac{d_{i,e} - \sum_k r_e^k \beta_{i,e}^k - T_{i,e}(\boldsymbol{a})/N_e}{\alpha_{i,e}} \right)^2 + \sum_k \left(r_e^k\right)^2 + \left(\frac{1 - N_e}{\delta N_e} \right)^2$$

Now that the observables have been calculated…

Data

$$\chi^2(\boldsymbol{a}) = \sum_{i,e} \left(\frac{d_{i,e} - \sum_k r_e^k \beta_{i,e}^k - T_{i,e}(\boldsymbol{a})/N_e}{\alpha_{i,e}} \right)^2 + \sum_k \left(r_e^k\right)^2 + \left(\frac{1 - N_e}{\delta N_e} \right)^2$$

Now that the observables have been calculated…

Data → $d_{i,e}$

Theory → $T_{i,e}(\boldsymbol{a})$

$$\chi^2(\boldsymbol{a}) = \sum_{i,e} \left(\frac{d_{i,e} - \sum_k r_e^k \beta_{i,e}^k - T_{i,e}(\boldsymbol{a})/N_e}{\alpha_{i,e}} \right)^2 + \sum_k \left(r_e^k\right)^2 + \left(\frac{1 - N_e}{\delta N_e} \right)^2$$

Now that the observables have been calculated…

Data

Theory

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Uncorrelated Uncertainties

Now that the observables have been calculated…

Data

Theory

$$\chi^2(\boldsymbol{a}) = \sum_{i,e} \left(\frac{d_{i,e} - \sum_k r_e^k \beta_{i,e}^k - T_{i,e}(\boldsymbol{a})/N_e}{\alpha_{i,e}} \right)^2 + \sum_k \left(r_e^k\right)^2 + \left(\frac{1 - N_e}{\delta N_e} \right)^2$$

Uncorrelated Uncertainties

Correlated Uncertainties

Now that the observables have been calculated…

Data

Theory

Normalization

$$\chi^2(\boldsymbol{a}) = \sum_{i,e} \left(\frac{d_{i,e} - \sum_k r_e^k \beta_{i,e}^k - T_{i,e}(\boldsymbol{a})/N_e}{\alpha_{i,e}} \right)^2 + \sum_k \left(r_e^k\right)^2 + \left(\frac{1 - N_e}{\delta N_e} \right)^2$$

Uncorrelated Uncertainties

Correlated Uncertainties

Normalization Uncertainty

Now that we have calculated $\chi^2(\boldsymbol{a}, \text{data})$…

Likelihood Function

$$\mathcal{L}\left(\boldsymbol{a}, \text{data}\right) = \exp\left(-\frac{1}{2}\chi^2\left(\boldsymbol{a}, \text{data}\right)\right)$$

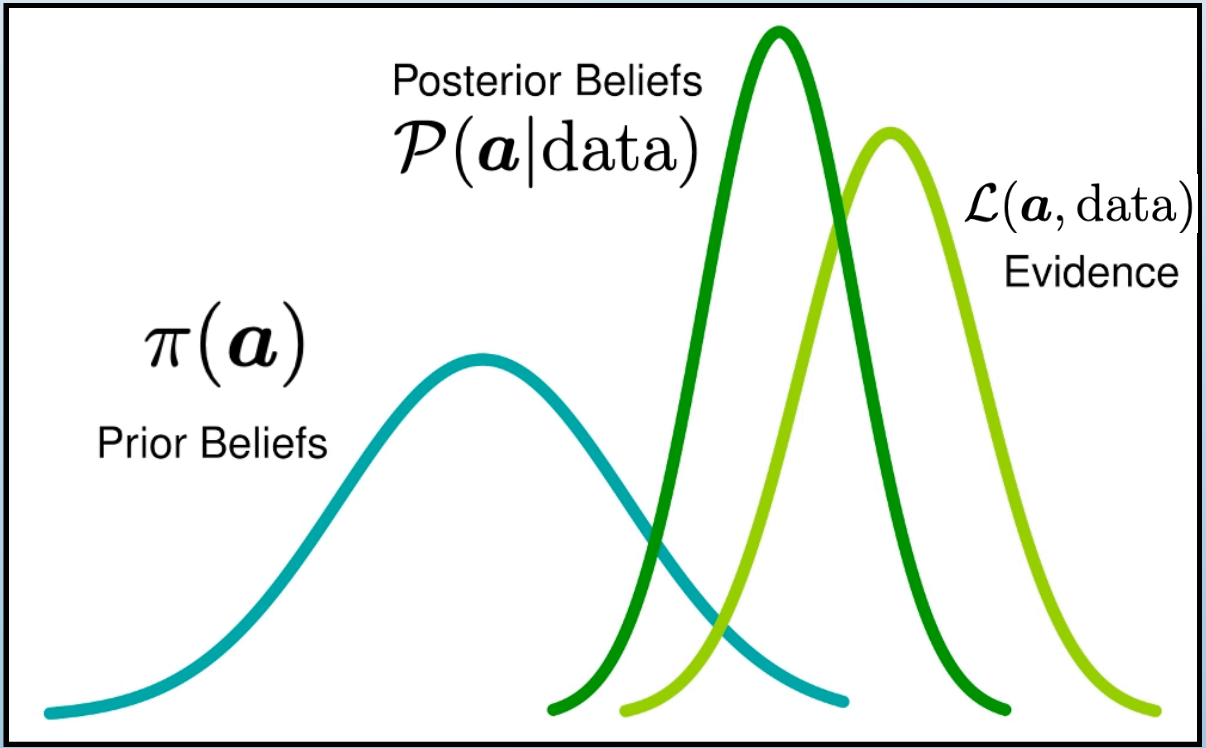
Now that we have calculated $\chi^2(\boldsymbol{a}, \text{data})$…

Likelihood Function

$$\mathcal{L}(\boldsymbol{a}, \text{data}) = \exp\left(-\frac{1}{2}\chi^2(\boldsymbol{a}, \text{data})\right)$$

Bayes' Theorem

$$\mathcal{P}(\boldsymbol{a}|\text{data}) \sim \mathcal{L}(\boldsymbol{a}, \text{data})\,\pi(\boldsymbol{a})$$



$$\tilde{\sigma} = \sigma + N(0,1)\ \alpha$$

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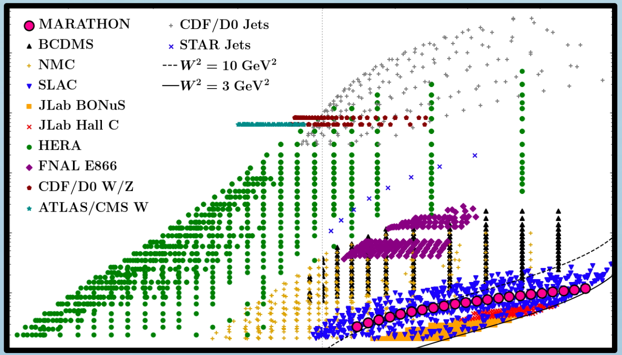
Data

Original Data

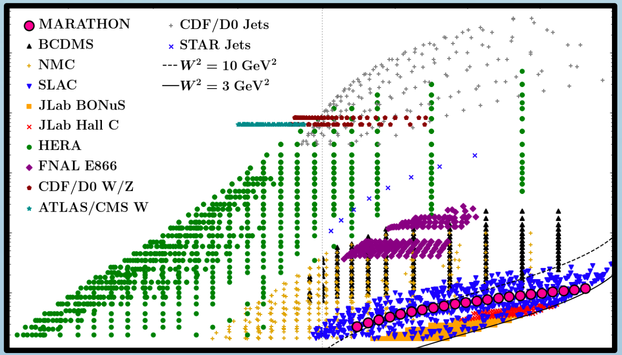


$$\tilde{\sigma} = \sigma + N(0,1)\ \alpha$$

Uncorrelated Uncertainties

Data

Original Data



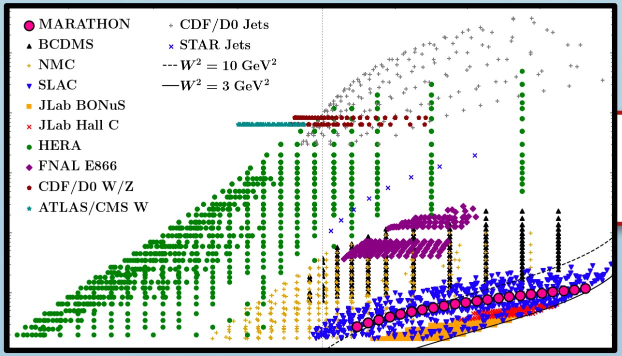
Pseudo-Data →

$$\tilde{\sigma} = \sigma + N(0,1)\ \alpha$$

← Uncorrelated Uncertainties

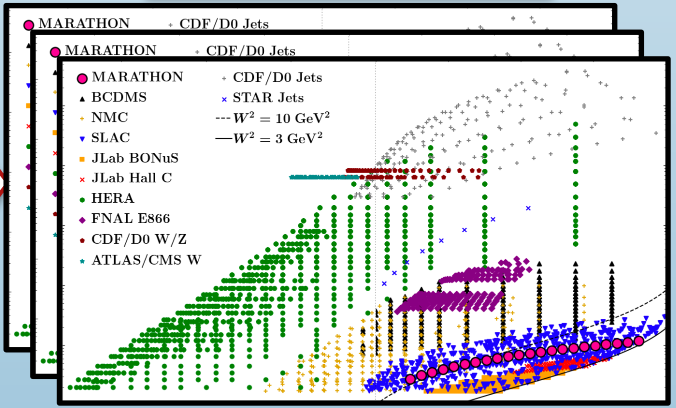
↑ Data

Original Data



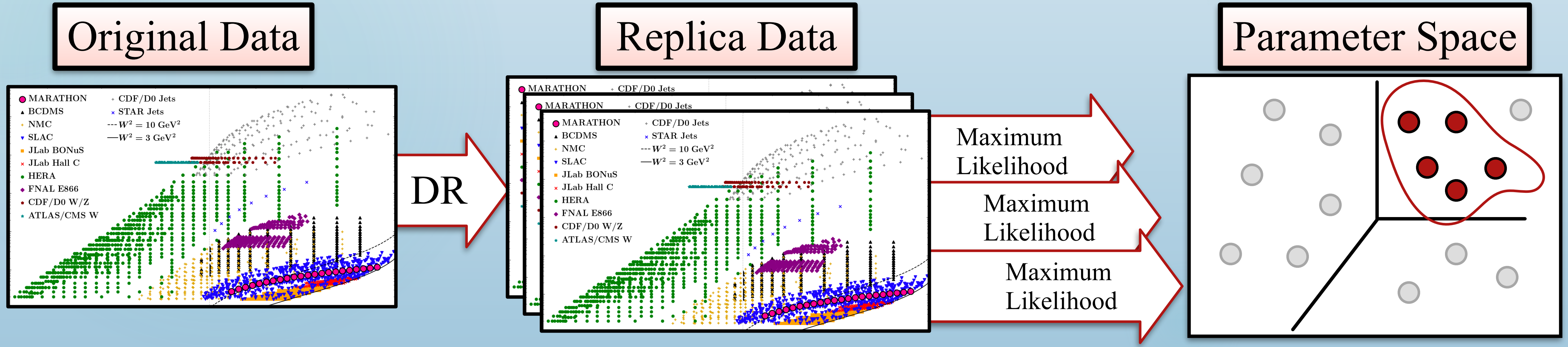
DR

Replica Data



Pseudo-Data → $$\tilde{\sigma} = \sigma + N(0,1)\ \alpha$$ ← Uncorrelated Uncertainties

↑ Data



For a quantity $O(\boldsymbol{a})$: (for example, a PDF at a given value of (x, Q^2))

$$E[O] = \int d^n a \; \rho(\boldsymbol{a} \,|\, data) \; O(\boldsymbol{a})$$

$$V[O] = \int d^n a \; \rho(\boldsymbol{a} \,|\, data) \; \left[O(\boldsymbol{a}) - E[O]\right]^2$$

Exact, but
$n = \mathcal{O}(100)$!

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Build an MC ensemble

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Build an MC ensemble

$$E[O] \approx \frac{1}{N} \sum_k O(\boldsymbol{a}_k)$$

$$V[O] \approx \frac{1}{N} \sum_k \left[O(\boldsymbol{a}_k) - E[O]\right]^2$$

Average over k sets of the parameters (replicas)

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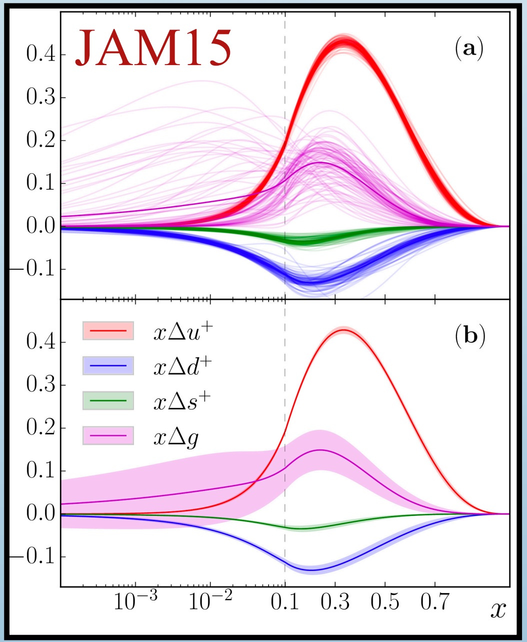
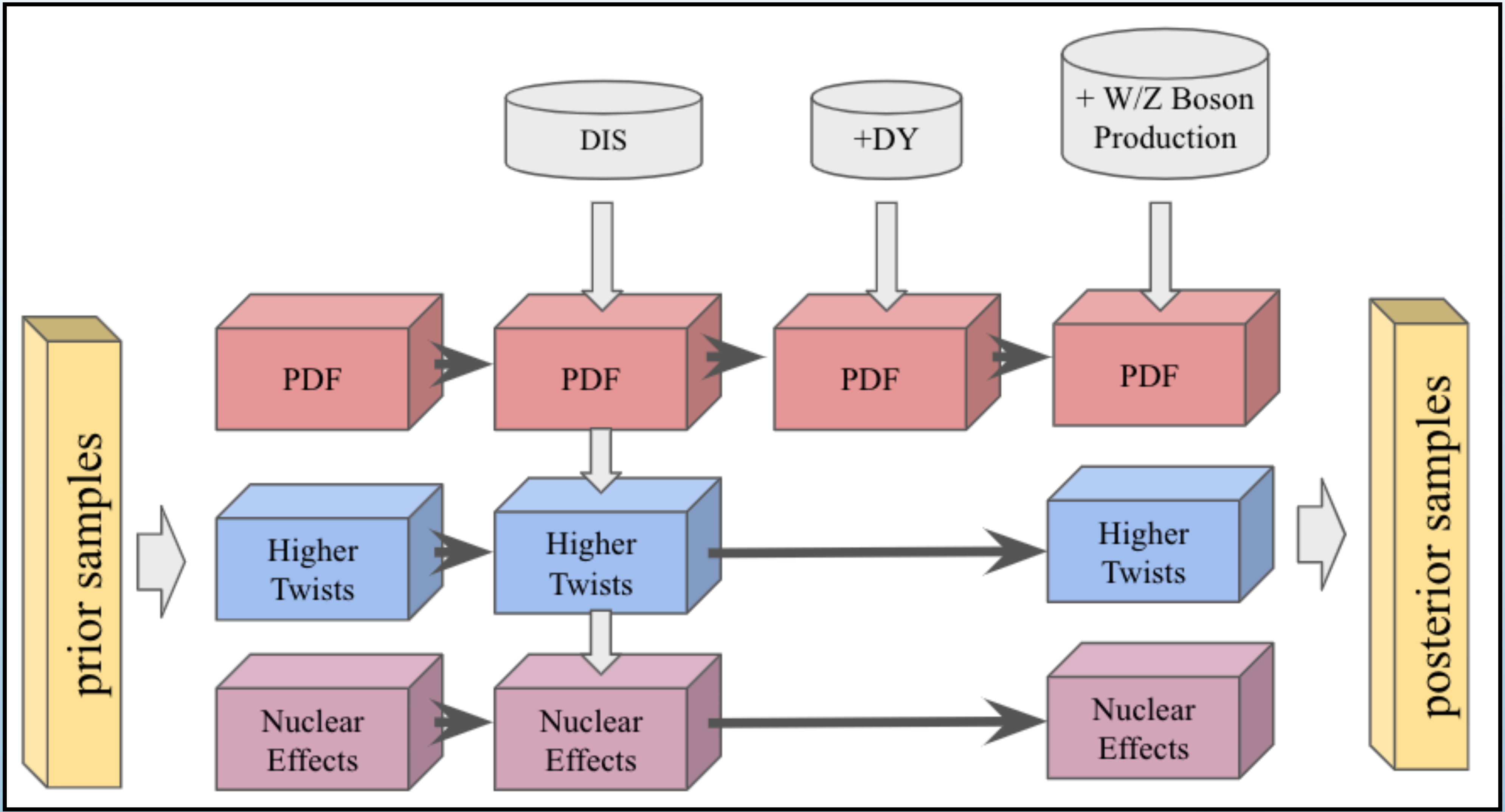
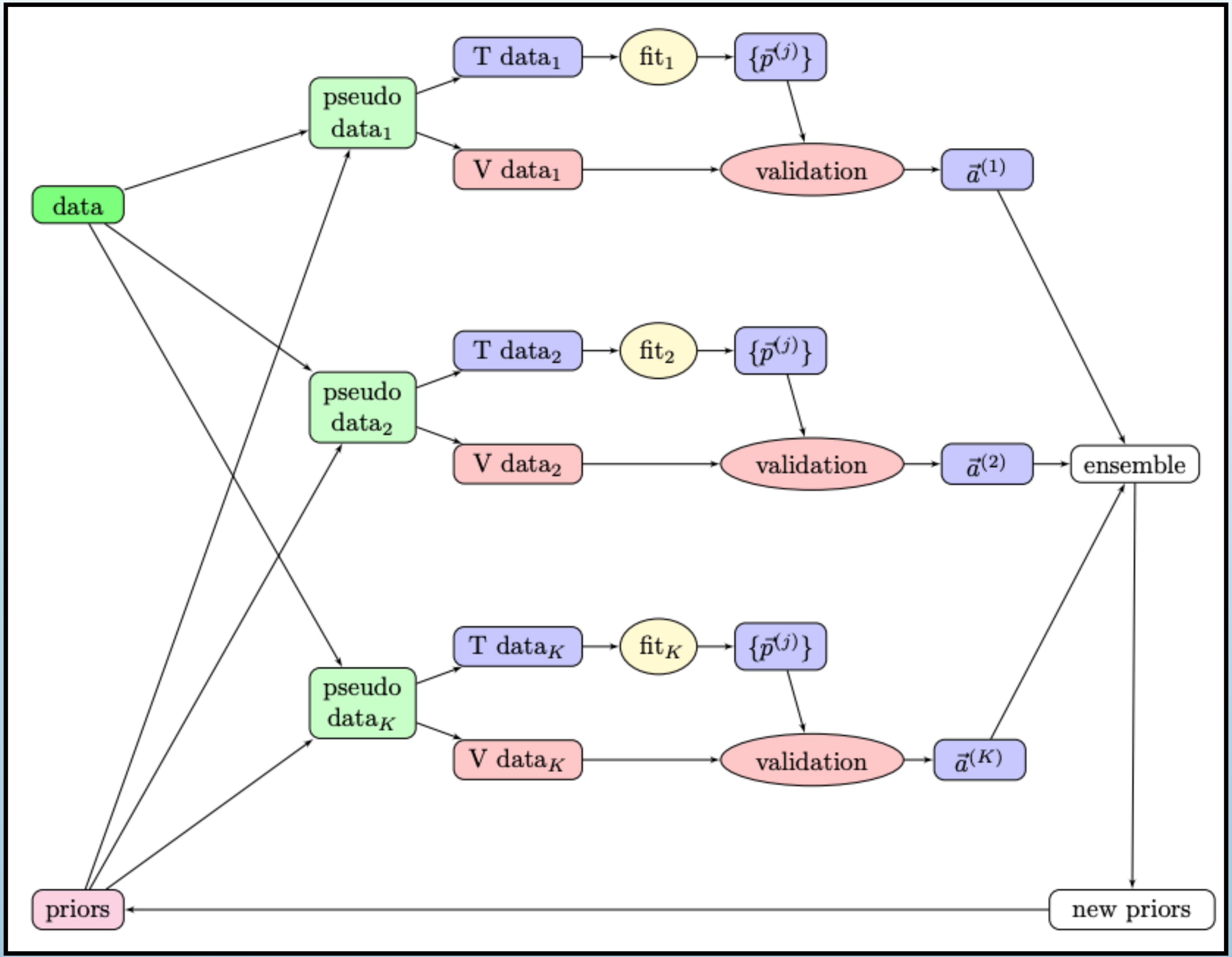
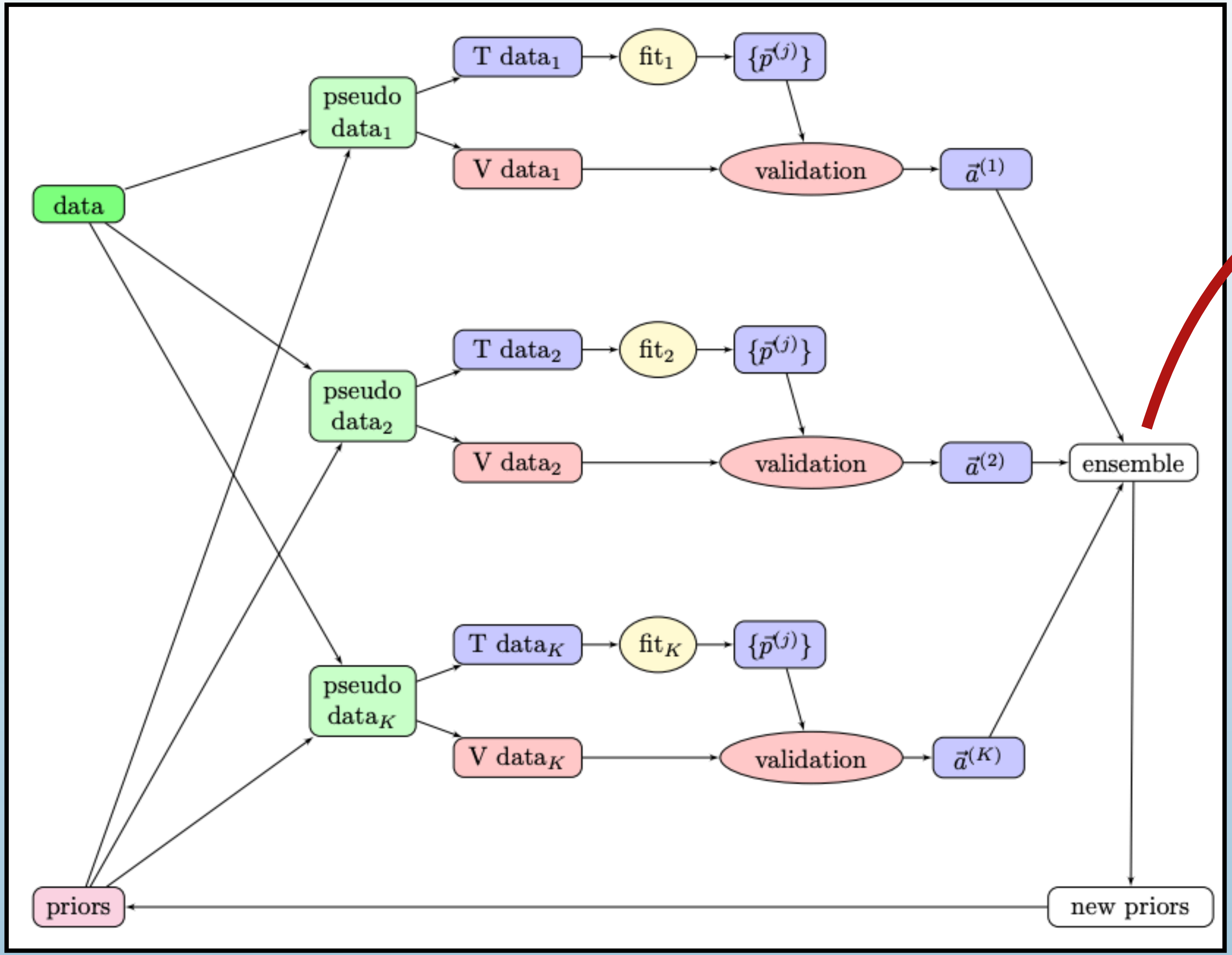
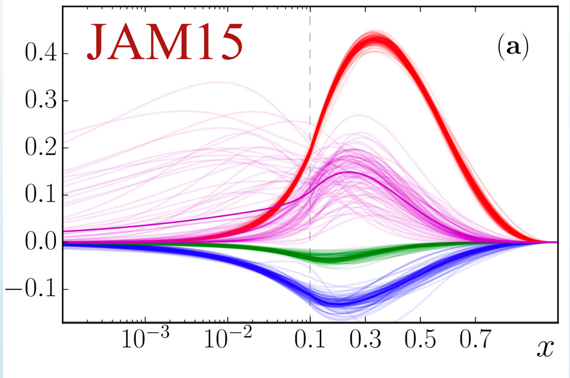


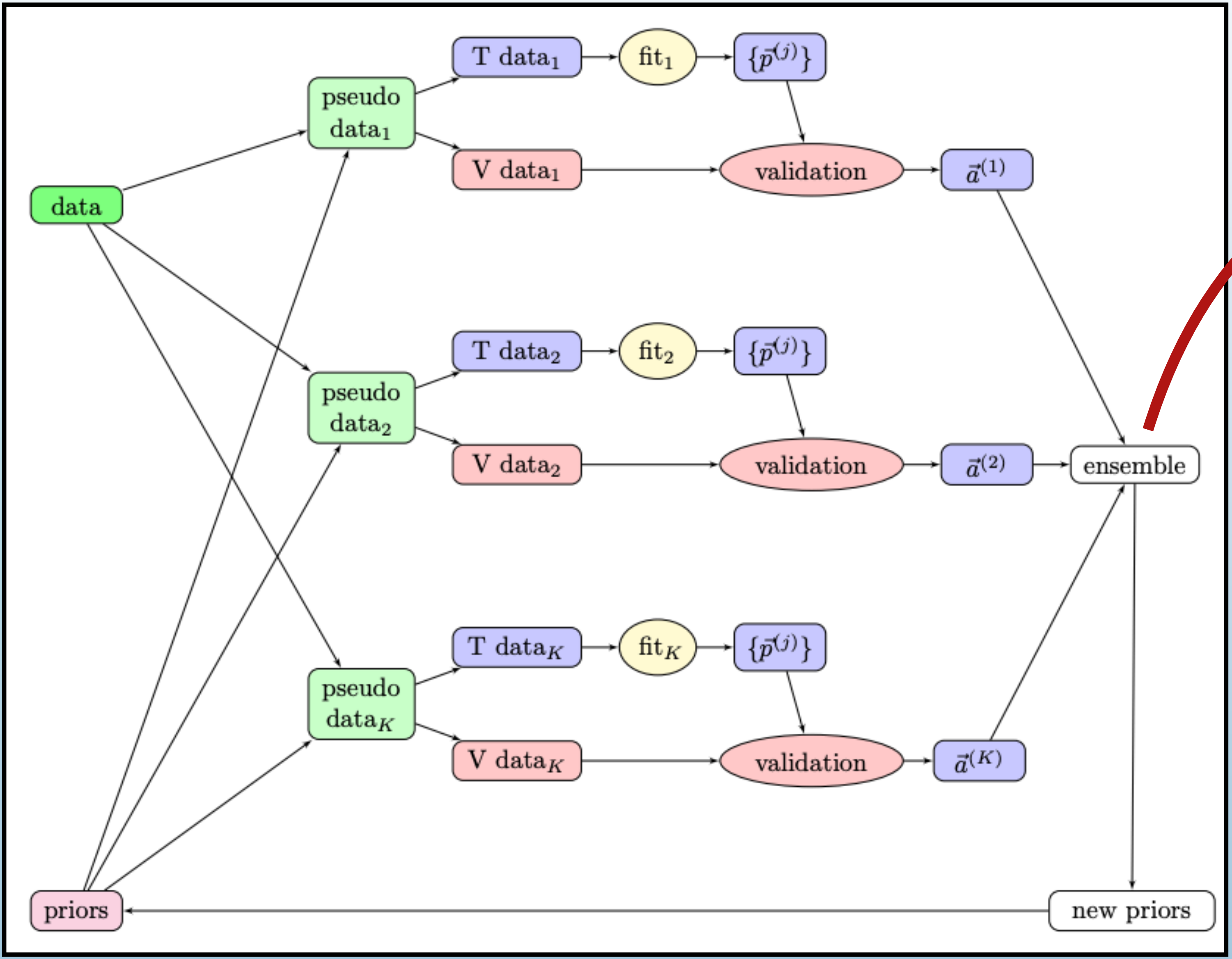

DIS
+DY
+ W/Z Boson Production
prior samples
PDF
PDF
PDF
PDF
Higher Twists
Higher Twists
Higher Twists
Nuclear Effects
Nuclear Effects
Nuclear Effects
posterior samples

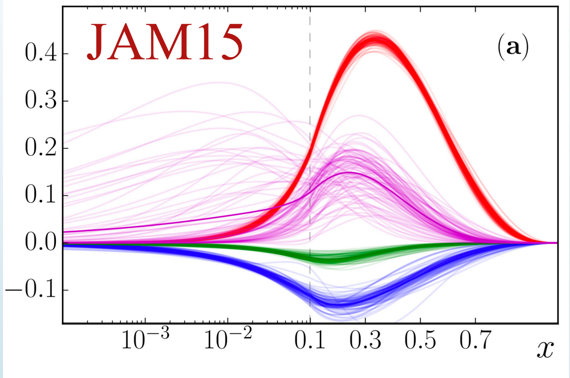


data
pseudo data$_1$
pseudo data$_2$
pseudo data$_K$
T data$_1$
V data$_1$
T data$_2$
V data$_2$
T data$_K$
V data$_K$
fit$_1$
fit$_2$
fit$_K$
$\{\vec{p}^{(j)}\}$
$\{\vec{p}^{(j)}\}$
$\{\vec{p}^{(j)}\}$
validation
validation
validation
$\vec{a}^{(1)}$
$\vec{a}^{(2)}$
$\vec{a}^{(K)}$
ensemble
new priors
priors


data
pseudo data$_1$
pseudo data$_2$
pseudo data$_K$
T data$_1$
V data$_1$
T data$_2$
V data$_2$
T data$_K$
V data$_K$
fit$_1$
fit$_2$
fit$_K$
$\{\vec{p}^{(j)}\}$
validation
$\vec{a}^{(1)}$
$\vec{a}^{(2)}$
$\vec{a}^{(K)}$
ensemble
new priors
priors


JAM15
(a)
0.4
0.3
0.2
0.1
0.0
−0.1
10^{-3}
10^{-2}
0.1
0.3
0.5
0.7
x





$$E[O] \approx \frac{1}{N}\sum_k O(\boldsymbol{a}_k)$$

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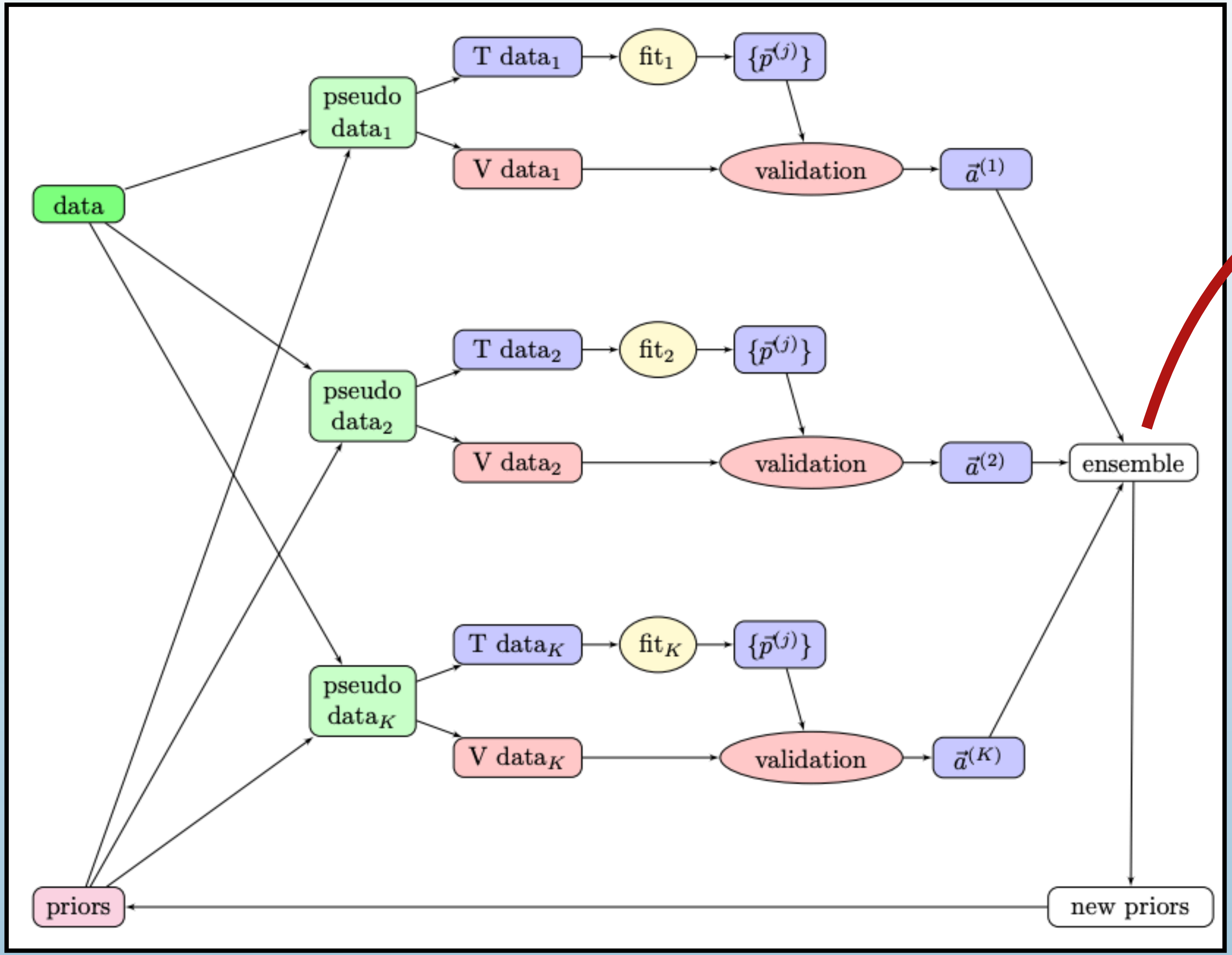

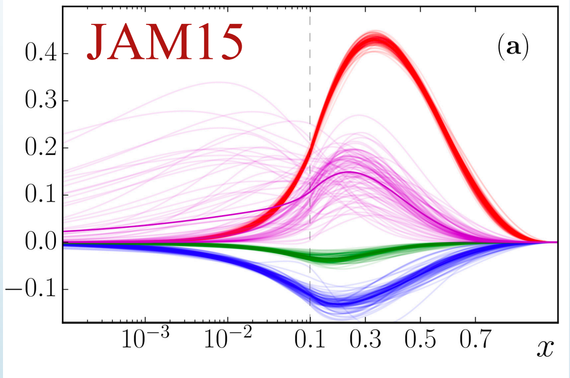

+

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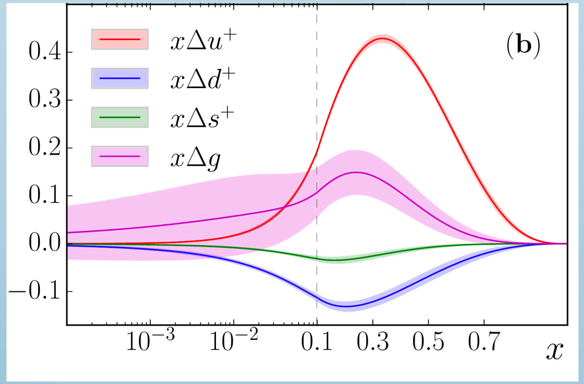
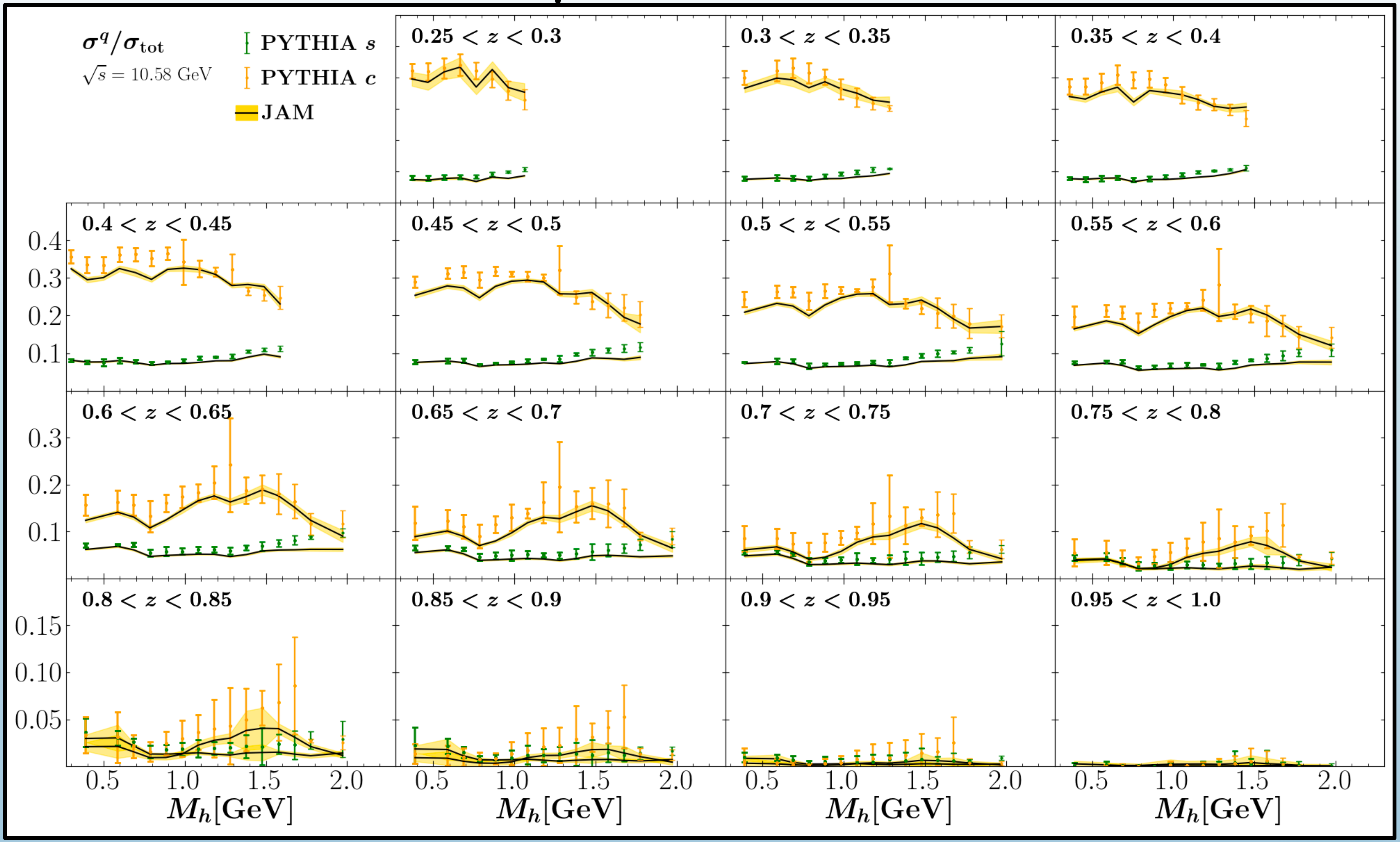
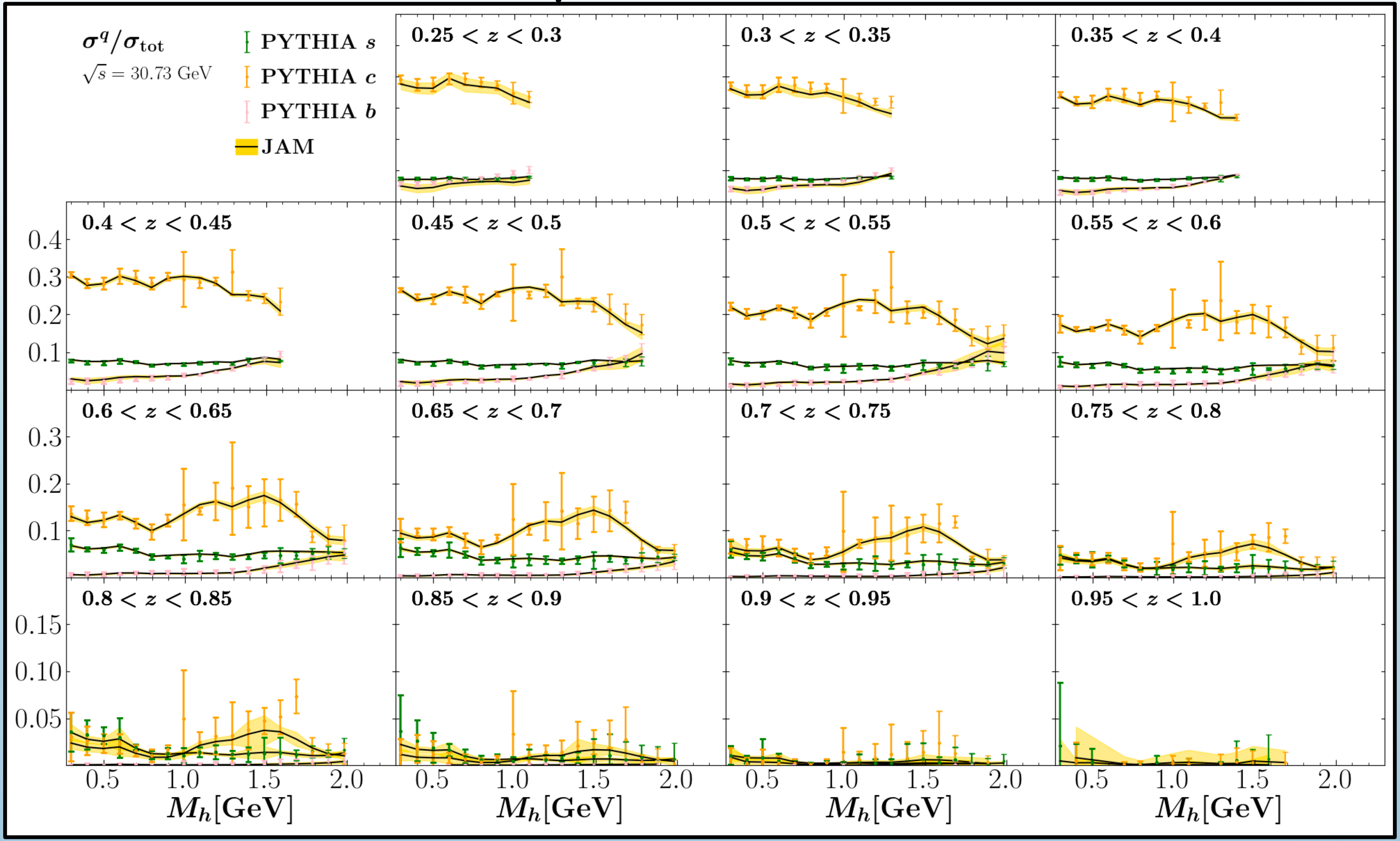


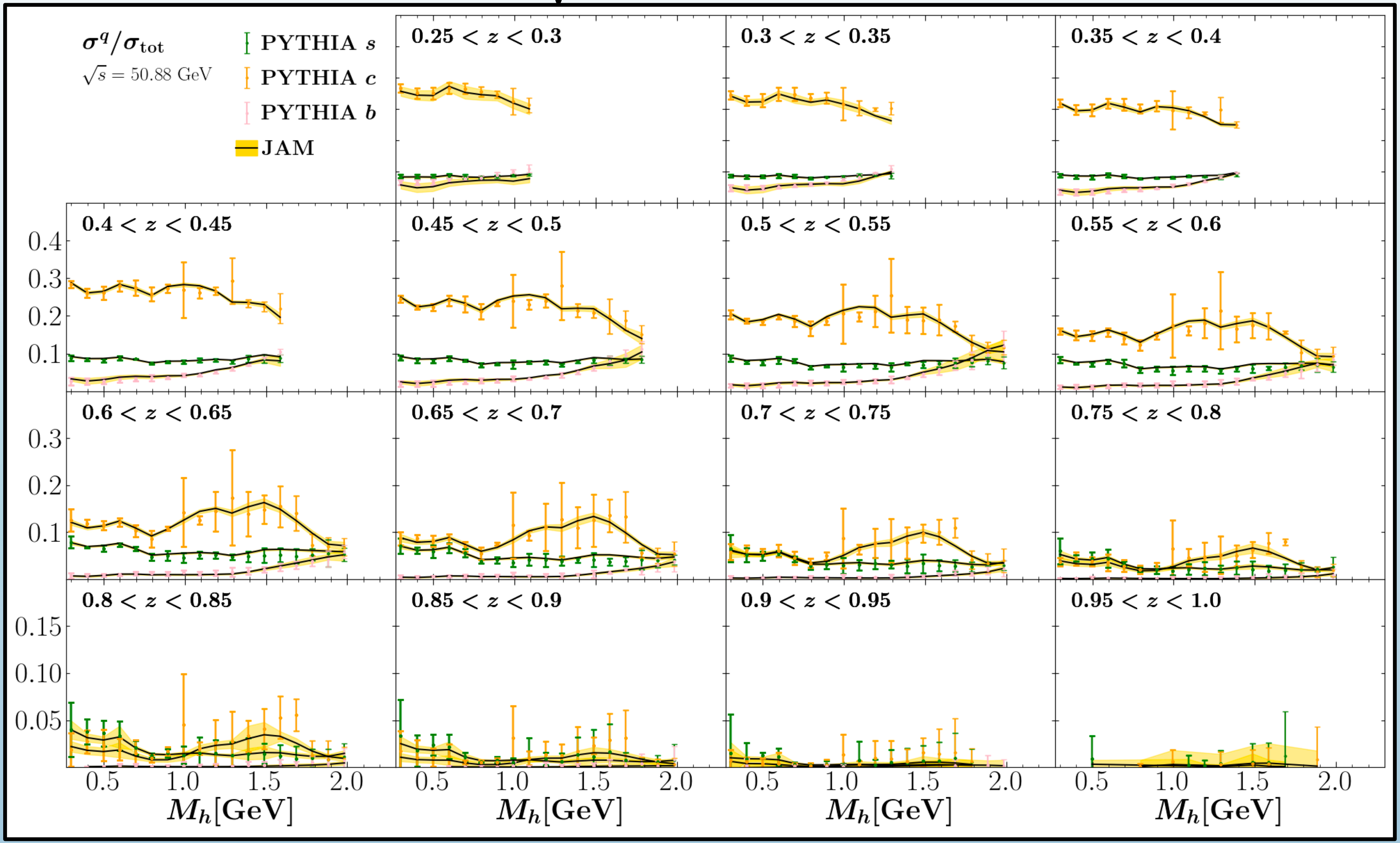
PYTHIA data ($\sqrt{s} = 10.58$ GeV)



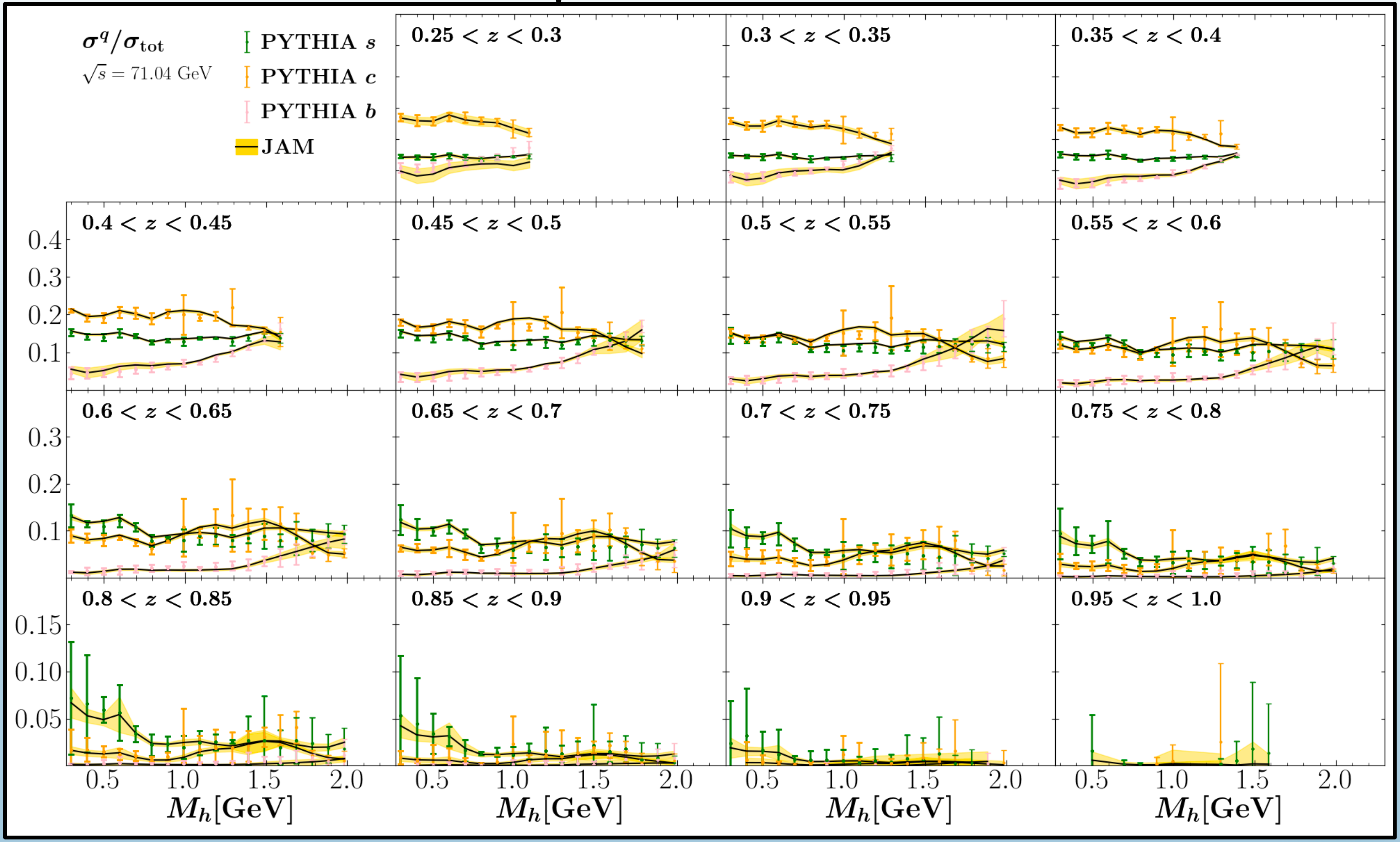
PYTHIA data ($\sqrt{s} = 30.73$ GeV)



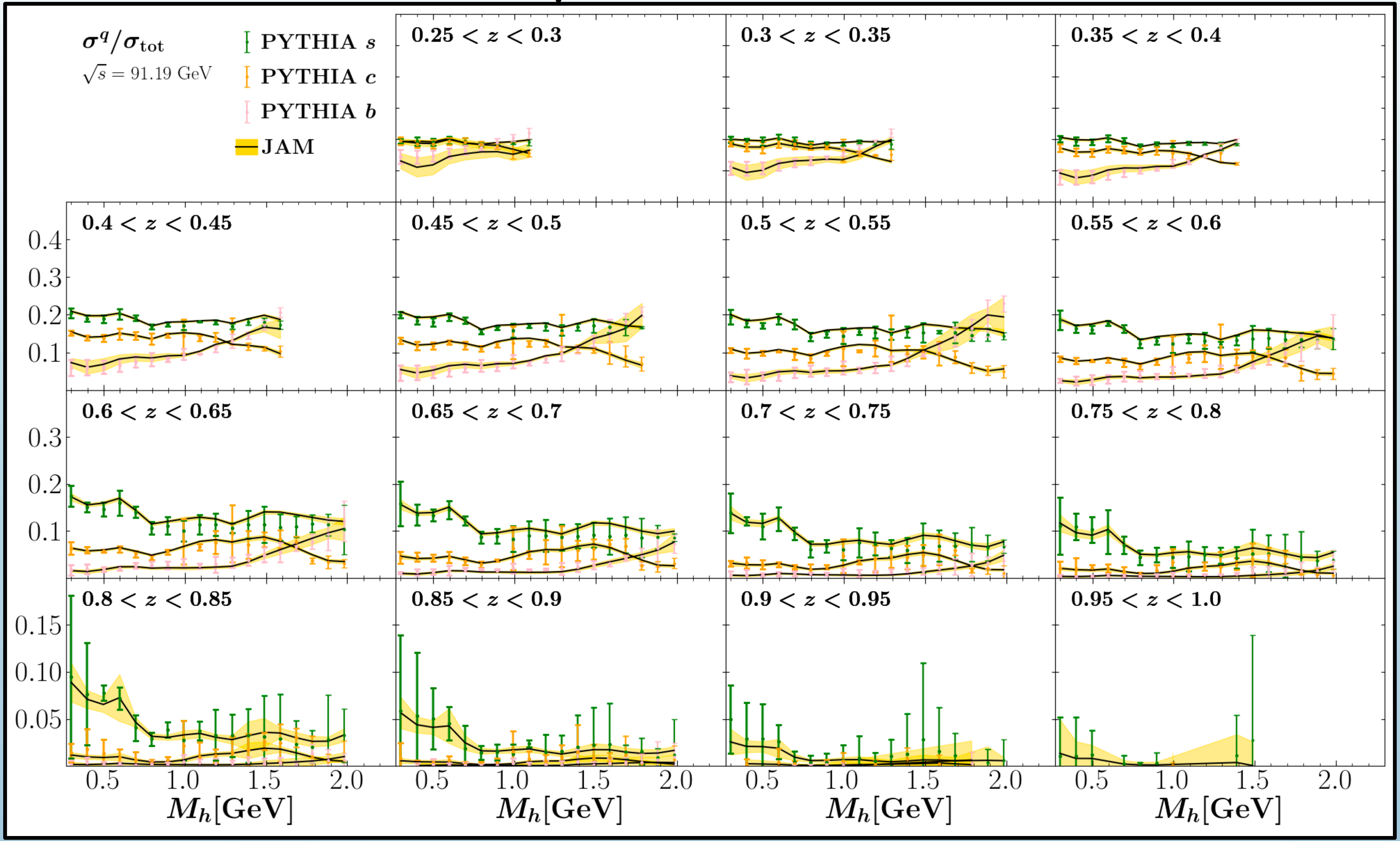
PYTHIA data ($\sqrt{s} = 50.88$ GeV)



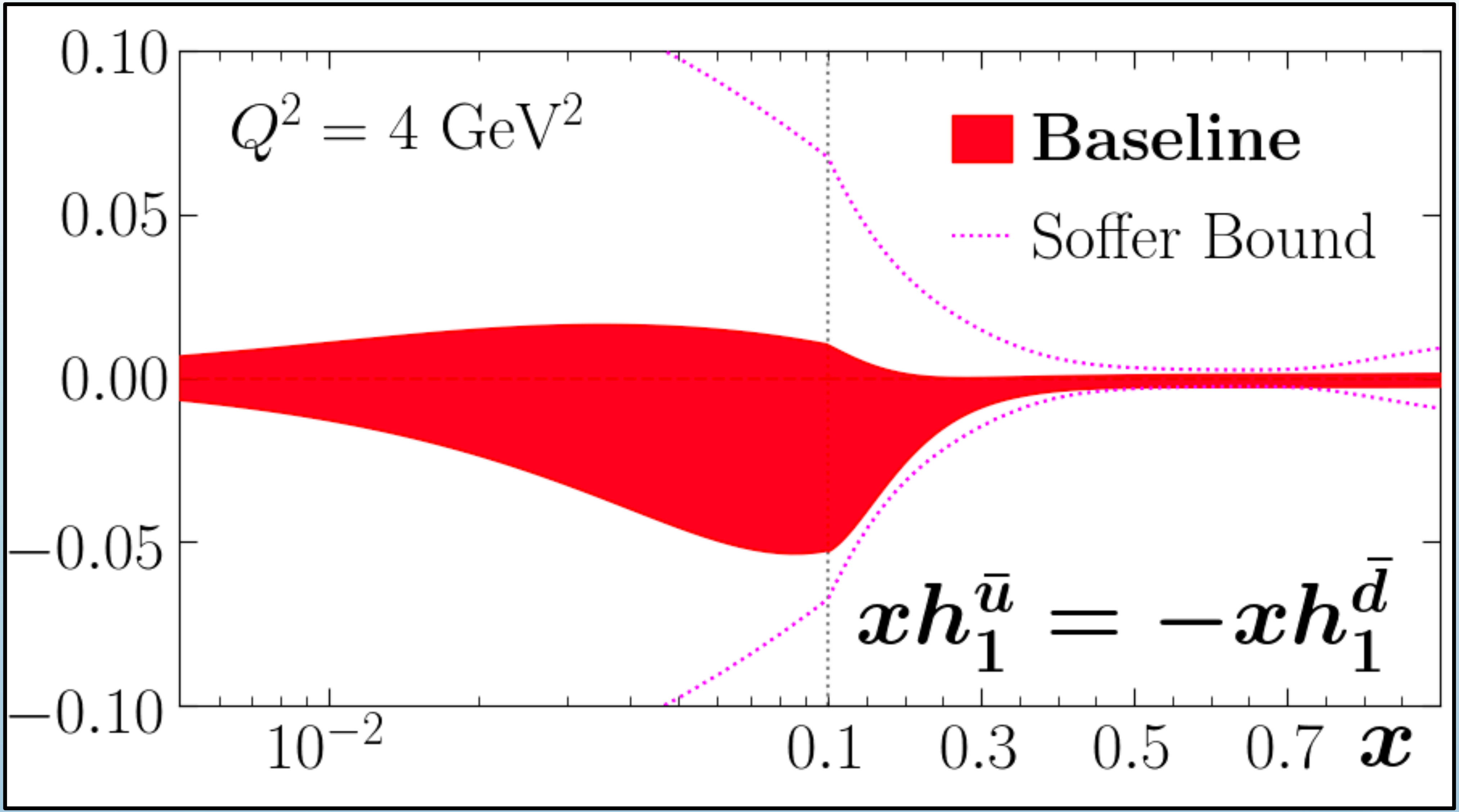
PYTHIA data ($\sqrt{s}$ = 71.04 GeV)



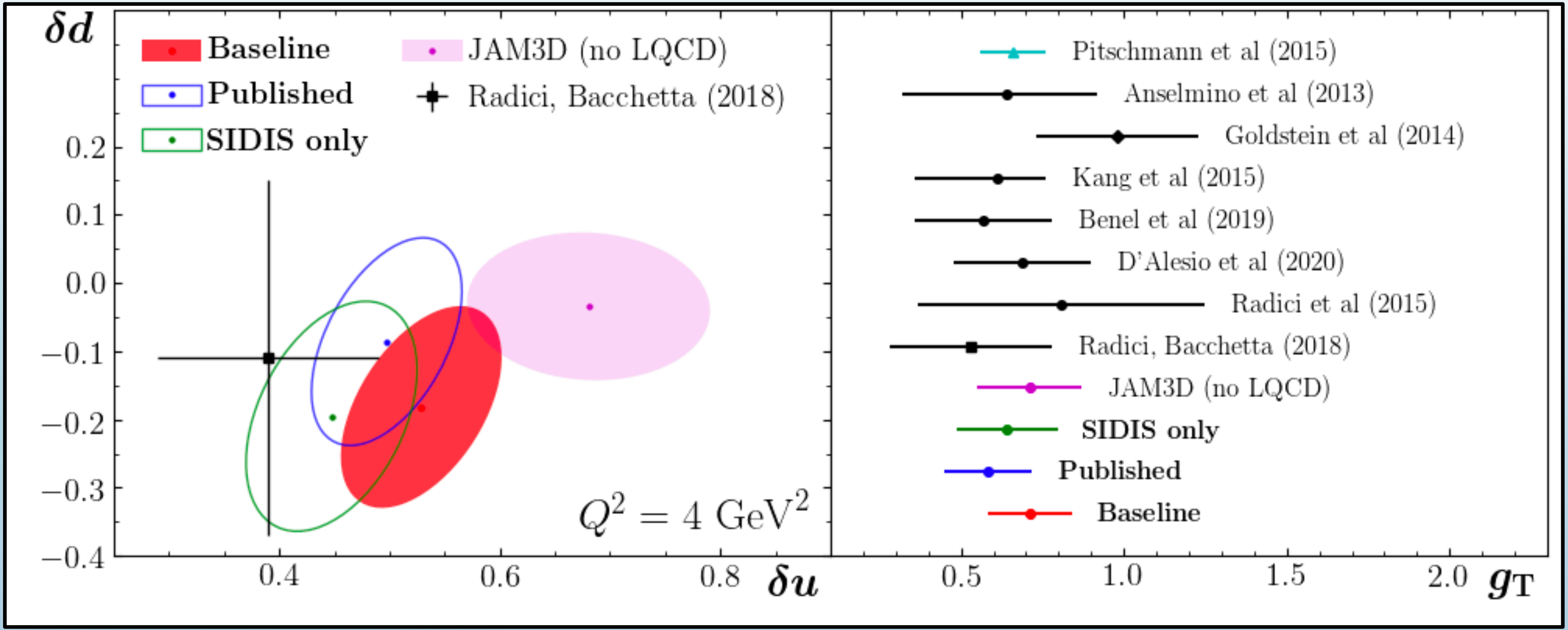
PYTHIA data ($\sqrt{s} = 91.19$ GeV)



Transversity PDFs (antiquarks)



Tensor Charges (Different Datasets)



DiFF Parameterization

$$\mathbf{M}_h^u = [2m_\pi, 0.40, 0.50, 0.70, 0.75, 0.80, 0.90, 1.00, 1.20, 1.30, 1.40, 1.60, 1.80, 2.00]\ \mathrm{GeV}.$$

$$D_1^q(z, \mathbf{M}_h^{q,i}) = \sum_{j=1,2,3} \frac{N_{ij}^q}{\mathcal{M}_{ij}^q} z^{\alpha_{ij}^q} (1-z)^{\beta_{ij}^q},$$

204 parameters for D_1
48 parameters for $H_1^{\sphericalangle}$

PDF Parameterization

$$h_1^{u_v}$$
$$h_1^{d_v}$$
$$h_1^{\bar{u}} = -h_1^{\bar{d}}$$

$$f(x, \mu_0^2) = \frac{N}{\mathcal{M}} x^{\alpha}(1-x)^{\beta}(1+\gamma\sqrt{x}+\eta x),$$

15 parameters for h_1

χ^2 Tables

experiment	observable	binning	$N_{\rm dat}$	$\chi^2_{\rm red}$	fitted norm.
Belle [2]	$\frac{d^2\sigma}{dz\,dM_h}$	z, M_h	1121	1.24	0.992(20)
Belle [3]	a_{12R}	z, M_h	55	0.53	—
		$M_h, \overline{M}_h$	64	3.43	
		$z, \bar{z}$	64	1.54	
HERMES [5]	$A_{UT}^{\rm HERMES}$	$x_{\rm bj}$	4	1.84	1.101(43)
		M_h	4	1.27	
		z	4	1.74	
COMPASS (p) [53]	$A_{UT}^{\rm COMPASS}$	$x_{\rm bj}$	9	0.88	0.994(4)
		M_h	10	1.12	
		z	7	1.58	
COMPASS (D) [53]	$A_{UT}^{\rm COMPASS}$	$x_{\rm bj}$	9	1.20	1.002(5)
		M_h	10	0.39	
		z	7	0.47	
STAR [6] $\sqrt{s} = 200$ GeV $R < 0.3$	A_{UT}^{pp}	$M_h, \eta < 0$	5	2.54	0.982(17)
		$M_h, \eta > 0$	5	1.52	
		$P_{hT}, \eta < 0$	5	0.92	
		$P_{hT}, \eta > 0$	5	1.05	
		η	4	1.72	
STAR [25] $\sqrt{s} = 500$ GeV $R < 0.7$	A_{UT}^{pp}	$M_h, \eta < 0$	32	0.78	1.078(27)
		$M_h, \eta > 0$	32	1.16	
		$P_{hT}, \eta > 0$	35	1.09	
		η	7	1.57	
STAR [76] $\sqrt{s} = 200$ GeV $R < 0.3$ PRELIMINARY	A_{UT}^{pp}	$M_h, \eta < 0$	31	0.94	0.955(16)
		$M_h, \eta > 0$	31	1.25	
		$P_{hT}, \eta < 0$	29	0.85	
		$P_{hT}, \eta > 0$	29	1.05	
		η	9	2.06	
Total			1627	**1.29**	

experiment	$N_{\rm dat}$	Lattice	Baseline
HERMES [5]	12	1.92	1.62
COMPASS (p) [53]	26	1.28	1.16
COMPASS (D) [53]	26	0.71	0.69
STAR (2015) [6]	24	1.62	1.54
STAR (2018) [25]	106	1.09	1.05
STAR (PRELIM) [76]	129	1.09	1.10
ETMC δu [46]	1	4.04	—
ETMC δd [46]	1	0.15	—
Total	325	**1.15**	**1.11**