# Tomography of pions and protons via transverse momentum dependent distribution


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w/ Patrick Barry, Eric Moffat, Alexei Prokudin
Wally Melnitchouk, Nobuo Sato, Daniel Pitonyak
"JAM 3-D"

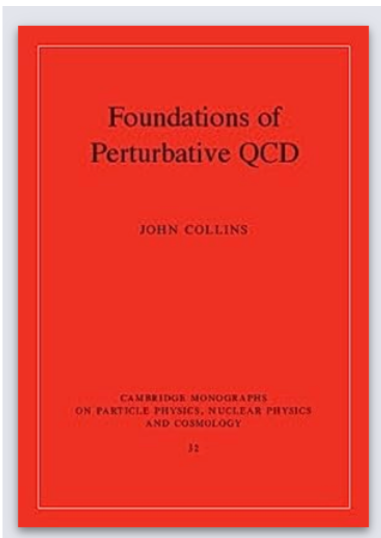

TMDs: Towards a Synergy between Lattice QCD and Global Analyses
*BNL-EIC Theory Institute & CFNS*
Stony Brook 06/21-23/2023

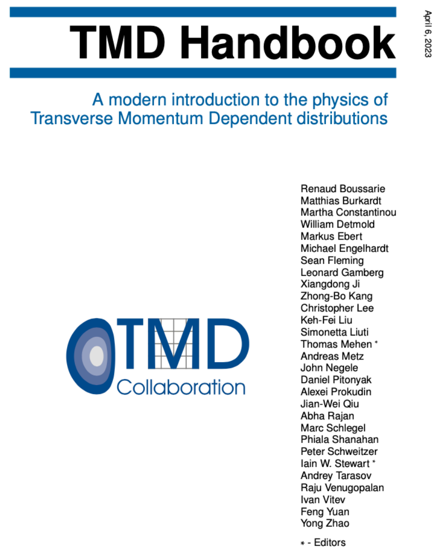

U.S. DEPARTMENT OF ENERGY | Office of Science

TMD Collaboration

Here we focus on TMDs and novel properties of the transverse separation of quark fields as a function of their longitudinal momenta for the proton and pion, giving deeper insights into color confined systems that emerge from QCD.

**Aybat Rogers 2011 PRD**
**Collins 2011 red book**
**Collins Rogers 2015 PRD**
**TMD Handbook arXiv https://arxiv.org/abs/2304.03302**





TMD Handbook

April 6, 2023

A modern introduction to the physics of
Transverse Momentum Dependent distributions

TMD Collaboration

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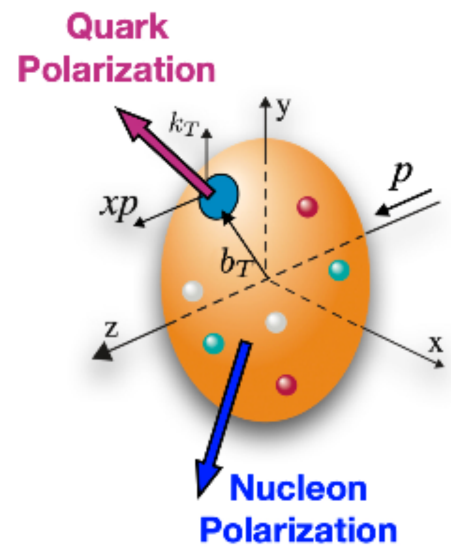



Figure 1.1: Illustration of the momentum and spin variables probed by TMD parton distributions.

# Outline of talk

- Intro Pion $\pi$ structure
- Analysis framework: provides motivation to perform simultaneous fit of
  - pion & proton TMD pdfs
  - w/ pion collinear pdfs

  from DY & LN data within JAM QCD analysis

- Pheno
- Results

***...with an eye toward lattice & TMDs***

# Motivation of my talk

- We explore the impact on JAM 21 pion pdfs extracted from a simultaneous fit of low energy fixed target $p_T$ dependent DY & collinear $\pi$ -nuclear cross section data

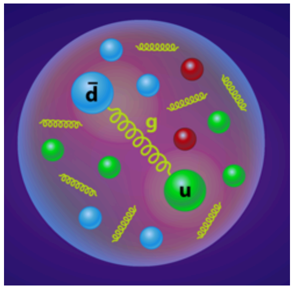
In particular:

- First we carry out a fit of the non-perturbative parameters of the pion TMD from the available data.
- As a second step we open up the fit of both collinear pion pdf parameters along with non-perturbative parameters.
- As a final step, we perform a fit of the $p_T$ integrated and $p_T$ dependent data to carry out a simultaneous fit of the pion collinear pdfs and pion TMDs.
This constitutes a first such study.
- We also compare the impact of various scenarios for describing non-perturbative content of the TMD contribution.

**https://arxiv.org/abs/2302.01192**

# Intro

# The Pion as bound state QCD




- Pion plays a central/"outsized" role in hadron physics
- @ low energy, as nearly massless $\bar{q}q$ bound state Goldstone boson is a critical ingredient for understanding dynamical $\chi$ symmetry breaking from small current quark masses

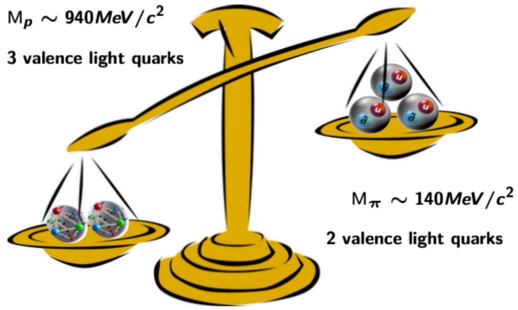
Pion pole condition from Bethe-Salpeter

$$m_\pi^2 = \frac{m}{2\,G_\pi\,M\,I(m_\pi^2)}$$

Gell Mann Reiner Oaks Relation

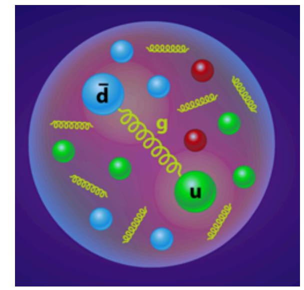
$$f_\pi^2\,m_\pi^2 = \frac{1}{2}\,(m_u + m_d)\,\langle \bar{u}u + \bar{d}d \rangle$$

★ Mass without mass" bulk of pion mass due to QCD quantum fluctuations of $\bar{q}q$ pairs, gluons, & energy associated with quarks moving at close to speed of light

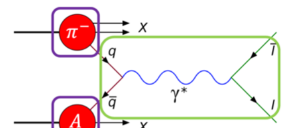


# The Pion recent progress

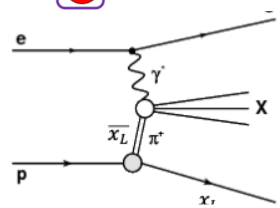
@ high energies pion's partonic structure unfolded/revealed from DY process as predicted from Collinear Factorization → momentum distributions, $f_{i/\pi}(x,\mu)$



$$\frac{\mathrm{d}\sigma^{\mathrm{DY}}}{\mathrm{d}Q^2\mathrm{d}y} = \sum_{a,b}\int dx_a\, dx_b\, H^{\mathrm{DY}}_{a,b}(x_a, x_b, y, Q^2, \mu^2)\, f_{a/A}(x_a,\mu^2)\, f_{b/B}(x_b,\mu^2)$$




$$\frac{\mathrm{d}^3\sigma^{\mathrm{LN}}}{\mathrm{d}x_B\,\mathrm{d}Q^2\,\mathrm{d}x_L} = \frac{4\pi\alpha^2}{x_B\, Q^4}\left(1 - y_e + \frac{y_e^2}{2}\right) F_2^{\mathrm{LN}}(x_B, Q^2, x_L)$$




**Jefferson Lab Angular Momentum (JAM)**

Barry, Sato, Melnitchouk, C.-R. Ji PRL 2018

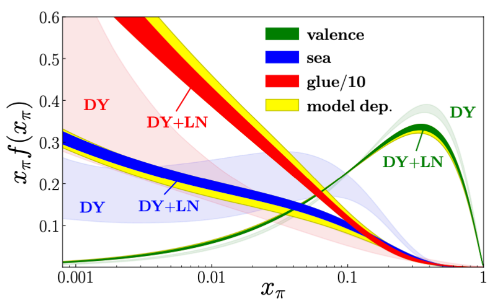
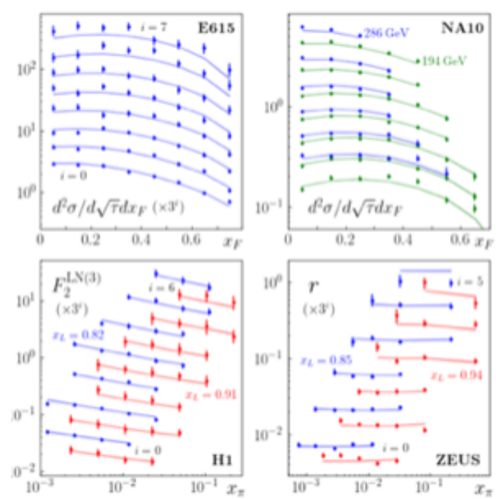


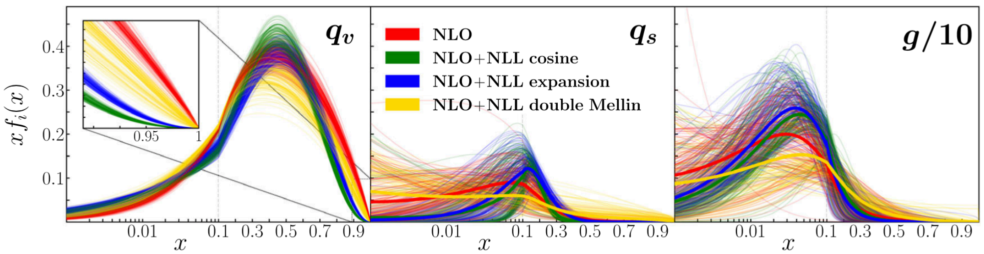

FIG. 2. Pion valence (green), sea quark (blue) and gluon (red, scaled by 1/10) PDFs versus $x_\pi$ at $Q^2 = 10$ GeV$^2$, for the full DY + LN (dark bands) and DY only (light bands) fits. The bands represent $1\sigma$ uncertainties, as defined in the standard Monte Carlo determination of the uncertainties [42] from the experimental errors. The model dependence of the fit is represented by the outer yellow bands.




- Top row – Drell-Yan
- Bottom row Leading neutron
- Good agreement with data
- $\chi^2_{\mathrm{npts}} = 0.979$

# The Pion PDFs NLL resummation & Lattice progress

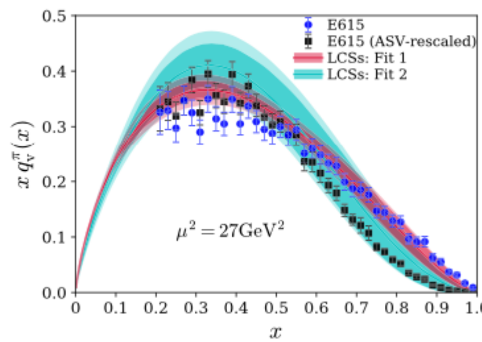
Barry, C.R. Ji, Sato, Melnitchouk, PRL 2021
global analysis w/ **threshold resummation**



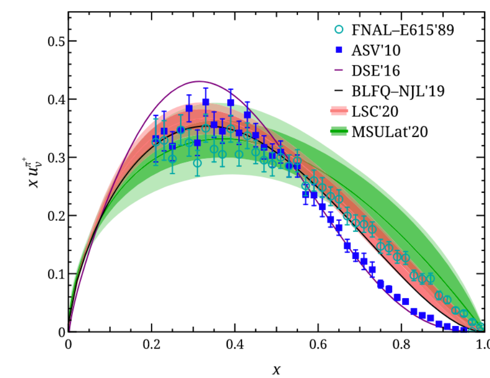

NLO + NLL cosine (green), expansion (blue), and double Mellin (gold) analyses. Inset in the left panel magnifies the very large-x region. The central values of the sea quark and gluon posterior samples are indicated by solid lines.

## Lattice

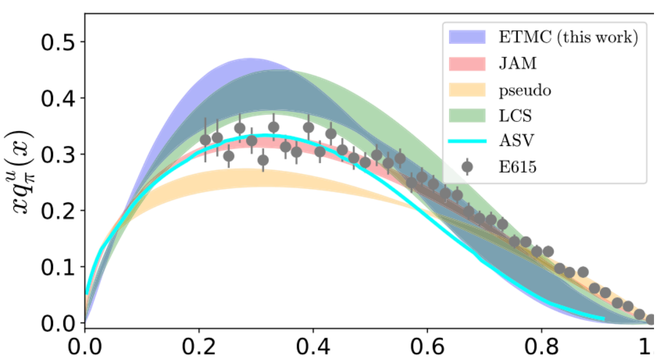
Sufian et al. PRD 102 (2020)
Had-strcture/JLab



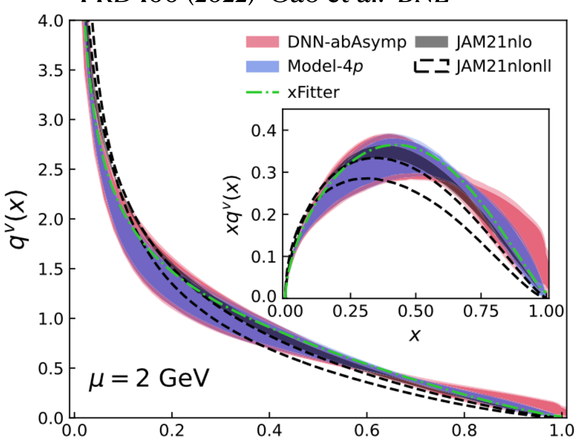

Huey-Wen Lin, et al. PRD 103 (2021)



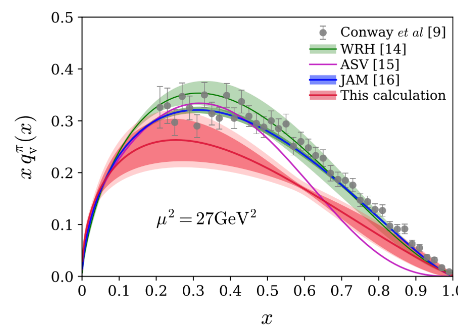

PRD 104 (2021) Alexandrou et al. EMT




PRD 106 (2022) Gao et al. BNL



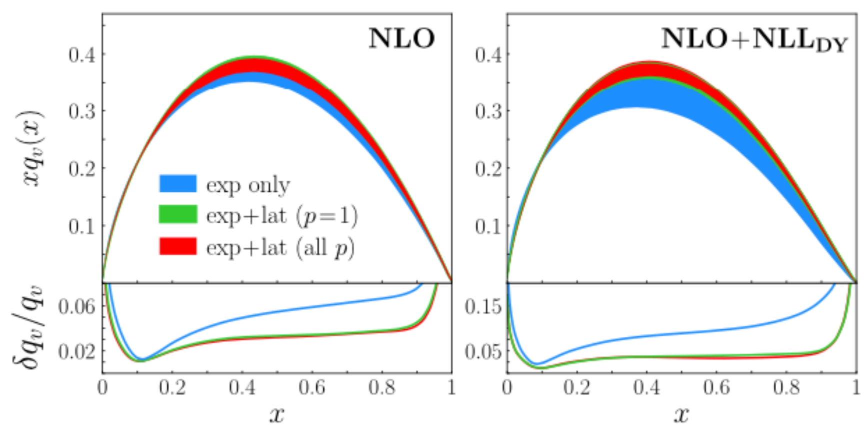

Bálint Joó, et al. PRD 100 (2019)
Had-struc /JLab



# The Pion PDFs  progress

First analysis of kind, where experimental data on pion-nucleus Drell-Yan and LN electro-production reactions supplemented by **lattice QCD data**; reduced Ioffe time pseudo-distributions to constrain the PDFs of the pion.

**PRD 105 (2022) Barry et al. JAM/Had-struct.**

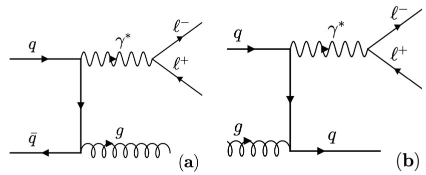



Valence quark distributions (top) extracted from experimental data alone (blue), combined with the p=1 lattice data (green), and combined with all the lattice data (red) for the NLO (left) and NLO & NLLDY (right) cases,  along with the relative uncertainties (bottom). The bands represent a 1σ uncertainty level.

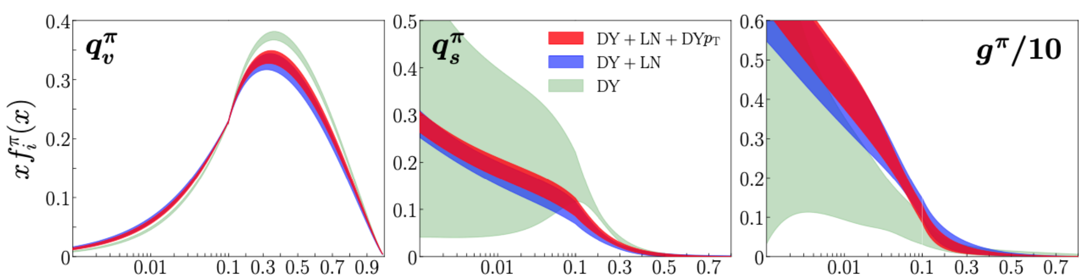
# Extended fit to include "large" $p_T$ Drell Yan data

Cao et al *Phys.Rev.D* 103 (2021), added transverse momentum dependent DY data in a globalQCD analysis of large transverse momentum $p_T \sim Q$ dominated hard QCD radiation

$$\frac{d\sigma^{\mathrm{DY}}}{dQ^2 dy dp_T} = \sum_{a,b} \int \mathrm{d}x_a\, \mathrm{d}x_b\, H^{\mathrm{FO}}_{a,b}(x_a, x_b, y, p_{\mathrm{T}}, Q^2, \mu^2)\, f_{a/A}(x_a, \mu^2)\, f_{b/B}(x_b, \mu^2)$$




The inclusion of $p_{\mathrm{T}}$-dependent data only slightly reduce uncertainties of the gluon distribution at large $x$ & impacts on other distributions negligible




**Understanding how these contrasting manifestations of the same $\bar{q}q$ bound state arise dynamically at different energy scales from first principles remains a major challenge in QCD**

# Extend fit to include "low" $p_T$ Drell Yan data

- We consider impact on collinear pion pdfs from $p_T(\equiv q_T) \sim k_T \ll Q$ "TMD" region (*n.b.* smaller statistical uncertainties on the data)
  - Pion induced DY scattering processes provide possibility to extract TMDs of the pion and nucleon when the cross section is kept differential in the transverse momentum of the produced lepton pair
  - Factorized according to the framework of Collins-Soper-Sterman (CSS)

## Factorization for low Drell-Yan $q_T \sim k_T \ll Q$

$W$-term, valid for low $q_T$

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}\tau \mathrm{d}Y \mathrm{d}q_T^2} = \frac{4\pi^2\alpha^2}{9\tau S^2} \sum_q H_{q\bar{q}}(Q^2,\mu) \int \mathrm{d}^2 b_T\, e^{ib_T\cdot q_T} \times \tilde{f}_{q/\pi}(x_\pi, b_T, \mu, Q^2)\, \tilde{f}_{\bar{q}/A}(x_A, b_T, \mu, Q^2)$$

- Factorization commonly carried out in $b = (b^-, b^+, \mathbf{b}_T)$ space
- Bare Fourier transform (FT) TMD commonly carried out in $b = (b^-, b^+, \mathbf{b}_T)$ space

$$\tilde{f}_{q/\mathcal{N}}(x, b_T) = \int \frac{\mathrm{d}b^-}{4\pi} e^{-ixP^+b^-} \mathrm{Tr}\left[\langle \mathcal{N} \,|\, \bar{\psi}_q(b)\gamma^+ \mathcal{W}(b,0)\psi_q(0) \,|\, \mathcal{N} \rangle\right]$$

- $\mathbf{b}_T$ is the Fourier conjugate to the intrinsic transverse momentum of quarks $k_T$

# The Pion TMDs: factorization, renormalization, and evolution

*"Bare" TMD Factorization Parton Model*

✦Mulders Tangerman NPB1995, Boer Mulders PRD 1997

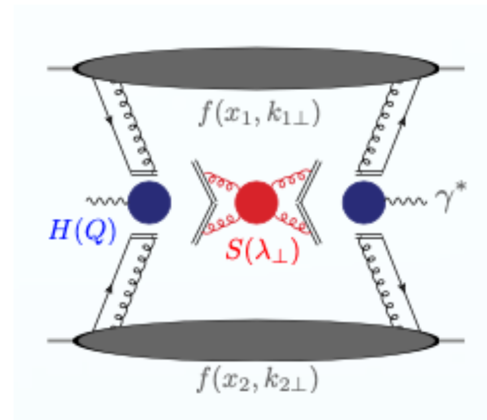
$$q_T \sim k_T \ll Q$$

Leading Quark TMDPDFs — Quark Spin

| | Quark Polarization | | |
|---|---|---|---|
| | Un-Polarized (U) | Longitudinally Polarized (L) | Transversely Polarized (T) |
| U | $f_1$ = Unpolarized | | $h_1^\perp$ = − Boer-Mulders |

$$f(x, k_T, s_T) = \frac{1}{2}\left[ f_1^\pi(x, k_T^2) + \frac{s_T^i \epsilon^{ij} k_T^j}{m_\pi} h_1^{\pi\,\perp}(x, k_T^2) \right]$$

- Factorization carried out Fourier $b_T$ space FT TMDs $\tilde{f}(x, b_T)$
- Real QCD need QFT definitions of TMDs LC & UV divergences reflected in the CS & RG Eqs. $\tilde{f}(x, b_T; \mu, \zeta)$
- TMD Evolution depends on rapidity $\zeta$ and RGE scales $\mu$




*TMD Factorization*

✦*Collins Soper Sterman NPB 1985*

✦*Ji Ma Yuan PRD PLB …2004, 2005*

✦*Aybat Rogers PRD 2011*

✦*Collins 2011 Cambridge Press*

✦*Echevarria, Idilbi, Scimemi JHEP 2012, …*

✦*SCET Becher & Neubert, 2011 EJPC*

$$\tilde{f}_{q/\mathcal{N}}(x, b_T) = \int \frac{db^-}{4\pi} e^{-ixP^+b^-} \mathrm{Tr}\left[\langle \mathcal{N} \,|\, \bar{\psi}_q(b)\gamma^+ \mathcal{W}(b,0)\psi_q(0) \,|\, \mathcal{N} \rangle\right]$$

**Aybat Rogers 2011 PRD**
**Collins 2011 red book**
**Collins Rogers 2015 PRD**
**TMD Handbook https://arxiv.org/abs/2304.03302**

$$\tilde{f}_{i/p}(x, \mathbf{b}_T, \mu, \zeta) = \lim_{\epsilon \to 0} Z_{\mathrm{uv}}(\mu, \zeta, \epsilon) \lim_{y_B \to -\infty} \frac{\tilde{f}^{0\,(\mathrm{u})}_{i/p}(x, \mathbf{b}_T, \epsilon, y_B, xP^+)}{\sqrt{\tilde{S}^0_{n_A(2y_n) n_B(2y_B)}(b_T, \epsilon, 2y_n - 2y_B)}}$$

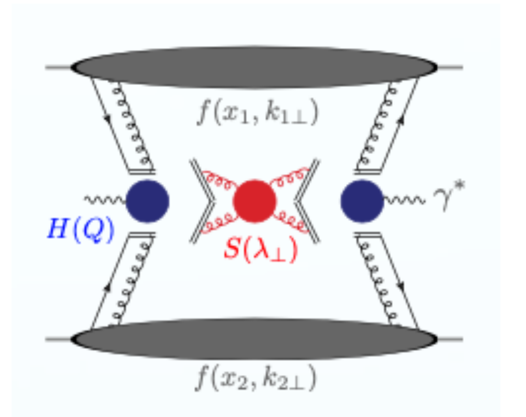
# Renormalization and TMD Evolution- $\{\zeta, \mu\}$

- Collins Soper Eq. $$\frac{\partial \ln \tilde{f}_{j/H}(x, b_T; \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T, \mu)$$

$$\tilde{K}(b_T, \mu) \equiv \frac{1}{2} \frac{\partial}{\partial y_n} \ln \frac{S(b_T, y_n, -\infty)}{S(b_T, y_n, -\infty)}$$

- RGE for C.S. kernel $$\frac{d\tilde{K}(b_T; \mu)}{d \ln \mu} = -\gamma_k(\alpha_s(\mu))$$

- RGE for TMD $$\frac{d \ln \tilde{f}_{j/H}(x, b_T; \mu, \zeta)}{d \ln \mu} = -\gamma_F(\alpha_s(\mu), \zeta/\mu)$$



# Evolution Renormalization and TMD $\{\zeta, \mu\}$

$$\tilde{f}_{i/P}(x, \mathbf{b}_T, \mu, \zeta) = \tilde{f}_{i/P}(x, \mathbf{b}_T, \mu_0, \zeta_0)\ \exp\left\{\int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'}\gamma_q\left[\alpha_s(\mu'); \zeta_0/\mu'^2\right]\right\}\exp\left\{\tilde{K}(b_T;\mu)\ln\sqrt{\frac{\zeta}{\zeta_0}}\right\}$$

Integral extends from $b = 0$ to $\infty$ one cannot avoid using parton densities and $\tilde{K}$ in the non-perturbative large-$b_T$ region

# Evolution Renormalization and TMD $\{\zeta, \mu\}$

$$\tilde{f}_{i/P}(x, \mathbf{b}_T, \mu, \zeta) = \tilde{f}_{i/P}(x, \mathbf{b}_T, \mu_0, \zeta_0)\ \exp\left\{\int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \gamma_q\left[\alpha_s(\mu'); \zeta_0/\mu'^2\right]\right\} \exp\left\{\tilde{K}(b_T; \mu) \ln\sqrt{\frac{\zeta}{\zeta_0}}\right\}$$

Integral extends from $b = 0$ to $\infty$ one cannot avoid using parton densities and $\tilde{K}$ in the non-perturbative large-$b_T$ region

- At small $b_T$ , the TMD PDF can be described in terms of its OPE:

$$\tilde{f}_{q/\mathcal{N}}(x, b_T; \mu, \zeta_F) = \sum_j \int_x^1 \frac{d\xi}{\xi} \tilde{C}_{q/j}(x/\xi, b_T; \mu, \zeta_F) f_{q/\mathcal{N}}(\xi; \mu) + \mathcal{O}((\Lambda_{\mathrm{QCD}} b_T)^a)$$

- where $\tilde{C}$ are the Wilson coefficients, and $f_{q/\mathcal{N}}$ is the collinear PDF
- Breaks down when $b_T$ gets large

# Evolution Renormalization and TMD $\{\zeta,\mu\}$

$$\tilde{f}_{i/P}(x,\mathbf{b}_T,\mu,\zeta)=\tilde{f}_{i/P}(x,\mathbf{b}_T,\mu_0,\zeta_0)\ \exp\left\{\int_{\mu_0}^{\mu}\frac{d\mu'}{\mu'}\gamma_q\left[\alpha_s(\mu');\zeta_0/\mu'^2\right]\right\}\exp\left\{\tilde{K}(b_T;\mu)\ln\sqrt{\frac{\zeta}{\zeta_0}}\right\}$$

Integral extends from $b=0$ to $\infty$ one cannot avoid using parton densities and $\tilde{K}$ in the non-perturbative large-$b_T$ region

- Introduction of non-perturbative functions @large $b_T$ freeze to $b_{max}$ chosen to transition from perturbative to nonperturbative physics

$$b_*(b_T)\equiv\sqrt{\frac{b_T^2}{1+b_T^2/b_{\mathrm{max}}^2}}$$

$$\frac{\tilde{f}_{i/p}(x,\mathbf{b}_T,\mu,\zeta)}{\tilde{f}_{i/p}(x,\mathbf{b}_*,\mu,\zeta)}=\frac{\tilde{f}_{i/p}(x,\mathbf{b}_T,\mu_0,\zeta_0')}{\tilde{f}_{i/p}(x,\mathbf{b}_*,\mu_0,\zeta_0')}\exp\left[\ln\sqrt{\frac{\zeta}{\zeta_0'}}\left(\tilde{K}(b_T,\mu)-\tilde{K}(b_*,\mu)\right)\right]$$
$$=\exp\left[-g_{i/p}(x,b_T)\right]\exp\left[-\ln\sqrt{\frac{\zeta}{\zeta_0'}}g_k(b_T;b_{\mathrm{max}})\right],$$

$$g_k(b_T;b_{\mathrm{max}})=\tilde{K}(b_*,\mu_0)-\tilde{K}(b_T,\mu_0)$$

# CSS evolution *F.T.*-TMD B.C. OPE & $b_*$ prescript.

$$\{\zeta, \mu\} \qquad \rightarrow \zeta = Q^2, \quad \mu = \mu_Q \sim Q$$

## TMD PDF within the $b_*$ prescription

$$\mathbf{b}_*(\mathbf{b}_T) \equiv \frac{\mathbf{b}_T}{\sqrt{1 + b_T^2/b_{\max}^2}}.$$

Low-$b_T$: perturbative
high-$b_T$: non-perturbative

$$\tilde{f}_{q/\mathcal{N}(A)}(x, b_T, \mu_Q, Q^2) = \boxed{(C \otimes f)_{q/\mathcal{N}(A)}(x; b_*)}$$

$$\times \exp\left\{ \boxed{-g_{q/\mathcal{N}(A)}(x, b_T) - g_K(b_T) \ln \frac{Q}{Q_0}} - \boxed{S(b_*, Q_0, Q, \mu_Q)} \right\}$$

Relates the TMD at small-$b_T$ to the **collinear** PDF
$\Rightarrow$ TMD is sensitive to collinear PDFs

$g_{q/\mathcal{N}(A)}$: intrinsic non-perturbative structure of the TMD
$g_K$: universal non-perturbative Collins-Soper kernel

Controls the perturbative evolution of the TMD

Collins, Soper, Sterman, NPB **250**, 199 (1985).

Aybat Rogers 2011 PRD
Collins 2011 red book
Collins Rogers 2015 PRD
TMD Handbook arXiv https://arxiv.org/abs/2304.03302

$W - term$

# TMD factorization in Drell-Yan

- In small-$q_T$ region, use the Collins-Soper-Sterman (CSS) formalism and $b_*$ prescription

Can these data constrain the pion collinear PDF?

$$\frac{d\sigma}{dQ^2\, dy\, dq_T^2} = \frac{4\pi^2\alpha^2}{9Q^2 s} \sum_{j,j_A,j_B} H^{\rm DY}_{j\bar{j}}(Q, \mu_Q, a_s(\mu_Q)) \int \frac{d^2\boldsymbol{b}_T}{(2\pi)^2} e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}$$
$$\times e^{-g_{j/A}(x_A, b_T; b_{\max})} \int_{x_A}^1 \frac{d\xi_A}{\xi_A} \boxed{f_{j_A/A}(\xi_A;\mu_{b_*})}\, \tilde{C}^{\rm PDF}_{j/j_A}\left(\frac{x_A}{\xi_A}, b_*; \mu^2_{b_*}, \mu_{b_*}, a_s(\mu_{b_*})\right)$$
$$\times e^{-g_{\bar{j}/B}(x_B, b_T; b_{\max})} \int_{x_B}^1 \frac{d\xi_B}{\xi_B} f_{j_B/B}(\xi_B;\mu_{b_*})\, \tilde{C}^{\rm PDF}_{\bar{j}/j_B}\left(\frac{x_B}{\xi_B}, b_*; \mu^2_{b_*}, \mu_{b_*}, a_s(\mu_{b_*})\right)$$
$$\times \exp\left\{-g_K(b_T; b_{\max}) \ln\frac{Q^2}{Q_0^2} + \tilde{K}(b_*;\mu_{b_*}) \ln\frac{Q^2}{\mu^2_{b_*}} + \int_{\mu_{b_*}}^{\mu_Q} \frac{d\mu'}{\mu'}\left[2\gamma_j(a_s(\mu')) - \ln\frac{Q^2}{(\mu')^2}\gamma_K(a_s(\mu'))\right]\right\}$$

Perturbative pieces

Non-perturbative pieces

Non-perturbative piece of the CS kernel

# Summary of details of analysis framework

- Nuclear TMD model linear combination of bound protons and neutrons
  - Include an additional $A$-dependent nuclear parameter
- We use the MAP collaboration's parametrization for non-perturbative TMDs
  - Only tested parametrization flexible enough to capture features of $Q$ bins
- Perform a **simultaneous global analysis** of pion TMD and collinear PDFs, with proton (nuclear) TMDs
  - Include both $q_T$-dependent and collinear pion data and fixed-target $pA$ data

# Summary of details of analysis framework

## Perturbative accuracy used in our fits

| $\gamma_K\,(\alpha_s(\mu))$ | $\beta[\alpha_s(\mu)]$ | $\gamma_q\,(\alpha_s(\mu);1)$ | $\tilde{K}(\bar{b}_T;1/\bar{b}_T)$ | $\tilde{C}_{j/j'}$ | accuracy | accuracy (SCET) |
|---|---|---|---|---|---|---|
| — | — | — | — | 0 | QPM | |
| 1 | 1 | — | — | 0 | LO-LL | LL |
| 2 | 2 | 1 | 1 | 0 | LO-NLL | NLL |
| 3 | 3 | 2 | 2 | 0 | LO-NNLL | |
| 2 | 2 | 1 | 1 | 1 | NLO-NLL | NLL′ |
| 3 | 3 | 2 | 2 | 1 | NLO-NNLL | NNLL |
| 3 | 3 | 2 | 2 | 2 | NNLO-NNLL | NNLL′ |
| 4 | 4 | 3 | 3 | 2 | NNLO-$N^3$LL | $N^3$LL |
| 4 | 4 | 3 | 3 | 3 | $N^3$LO-$N^3$LL | $N^3$LL′ |

The collinear observables we use at NLO

# Nuclear TMD PDFs

- The TMD factorization allows for the description of a quark inside a nucleus to be $\tilde{f}_{q/A}$
- However, the intrinsic non-perturbative structure will in-principle change from nucleus-to-nucleus
- Want to model these in terms of protons and neutrons as we don't have enough observables to separately parametrize different nuclei

# Nuclear TMD PDFs – working hypothesis

- We must model the nuclear TMD PDF from proton

$$\tilde{f}_{q/A}(x, b_T, \mu, \zeta) = \frac{Z}{A}\tilde{f}_{q/p/A}(x, b_T, \mu, \zeta) + \frac{A-Z}{A}\tilde{f}_{q/n/A}(x, b_T, \mu, \zeta)$$

- Each object on the right side independently obeys the CSS equation
  - **Assumption** that the bound proton and bound neutron follow TMD factorization
- Make use of isospin symmetry in that $u/p/A \leftrightarrow d/n/A$, etc.

# Building of the nuclear TMD PDF

- Then taking into account the intrinsic non-perturbative, we model the flavor-dependent pieces of the TMD PDF as

$$
\begin{aligned}
(C \otimes f)_{u/A}(x) e^{-g_{u/A}(x,b_T)} \rightarrow\ & \frac{Z}{A}(C \otimes f)_{u/p/A}(x) e^{-g_{u/p/A}(x,b_T)} \\
& + \frac{A-Z}{A}(C \otimes f)_{d/p/A}(x) e^{-g_{d/p/A}(x,b_T)}
\end{aligned}
$$

and

$$
\begin{aligned}
(C \otimes f)_{d/A}(x) e^{-g_{d/A}(x,b_T)} \rightarrow\ & \frac{Z}{A}(C \otimes f)_{d/p/A}(x) e^{-g_{d/p/A}(x,b_T)} \\
& + \frac{A-Z}{A}(C \otimes f)_{u/p/A}(x) e^{-g_{u/p/A}(x,b_T)}.
\end{aligned}
$$

# Nuclear TMD parametrization

- Specifically, we include a parametrization similar to Alrashed, et al., Phys. Rev. Lett **129**, 242001 (2022).

$$g_{q/\mathcal{N}/A} = g_{q/\mathcal{N}} \left(1 - a_{\mathcal{N}} \left(A^{1/3} - 1\right)\right)$$

- Where $a_{\mathcal{N}}$ is an additional parameter to be fit

# Summary of details of analysis framework

- Nuclear TMD model linear combination of bound protons and neutrons
  - Include an additional $A$-dependent nuclear parameter
- We use the MAP collaboration's parametrization for non-perturbative TMDs
  - Only tested parametrization flexible enough to capture features of $Q$ bins
- Perform a **simultaneous global analysis** of pion TMD and collinear PDFs, with proton (nuclear) TMDs
  - Include both $q_T$-dependent and collinear pion data and fixed-target $pA$ data

# MAP parametrization See Chiara's talk

- A recent work from the MAP collaboration (arXiv:2206.07598) used a complicated form for the non-perturbative function

$$f_{1\,NP}(x,\boldsymbol{b}_T^2;\zeta,Q_0) = \frac{g_1(x)\,e^{-g_1(x)\frac{\boldsymbol{b}_T^2}{4}} + \lambda^2\,g_{1B}^2(x)\left[1-g_{1B}(x)\frac{\boldsymbol{b}_T^2}{4}\right]e^{-g_{1B}(x)\frac{\boldsymbol{b}_T^2}{4}} + \lambda_2^2\,g_{1C}(x)\,e^{-g_{1C}(x)\frac{\boldsymbol{b}_T^2}{4}}}{g_1(x)+\lambda^2\,g_{1B}^2(x)+\lambda_2^2\,g_{1C}(x)}\left[\frac{\zeta}{Q_0^2}\right]^{\boxed{g_K(\boldsymbol{b}_T^2)/2}}, \quad (38)$$

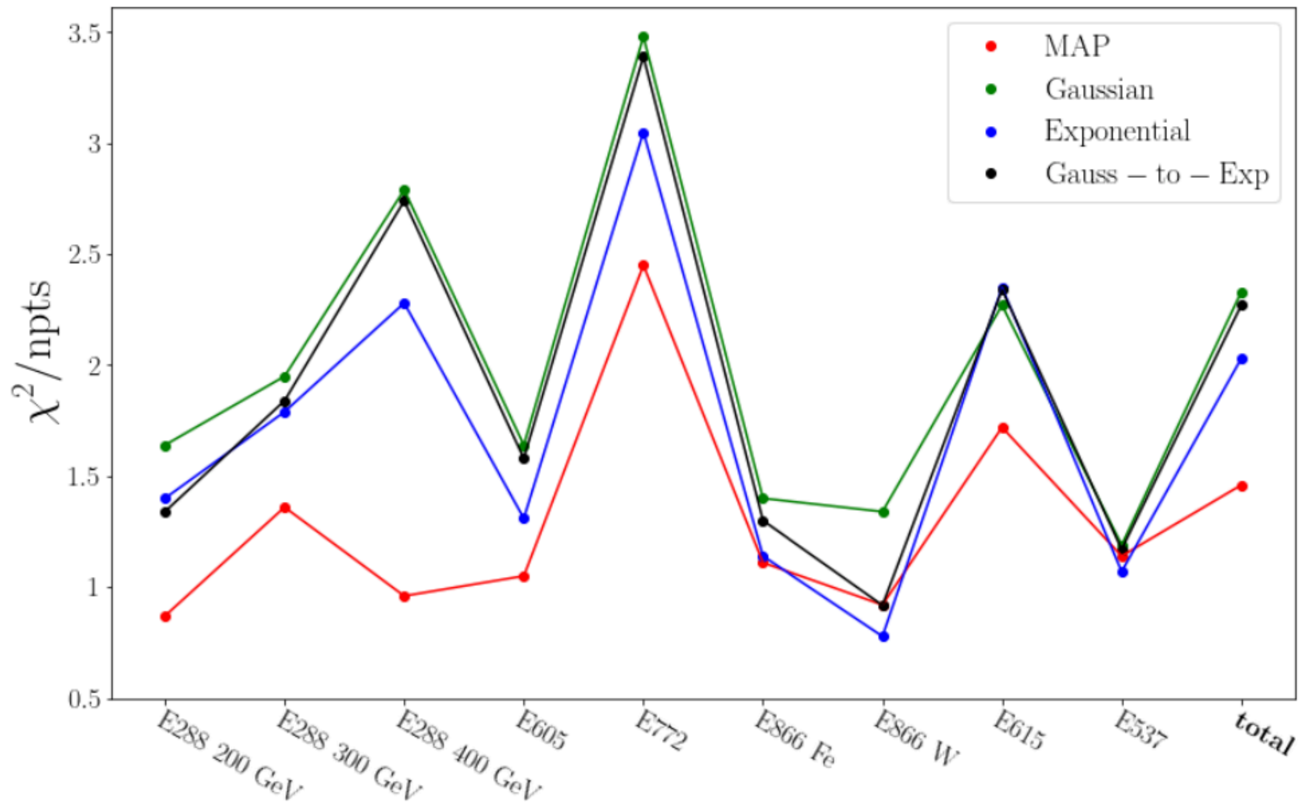
$$g_{\{1,1B,1C\}}(x) = N_{\{1,1B,1C\}}\,\frac{x^{\sigma_{\{1,2,3\}}}\,(1-x)^{\alpha^2_{\{1,2,3\}}}}{\hat{x}^{\sigma_{\{1,2,3\}}}\,(1-\hat{x})^{\alpha^2_{\{1,2,3\}}}},$$

$$\boxed{g_K(\boldsymbol{b}_T^2) = -g_2^2\,\frac{\boldsymbol{b}_T^2}{2}}$$

Universal CS kernel

# Resulting $\chi^2$ for each parametrization

- MAP gives best overall
- How significant?



# Perform the Monte Carlo

- We use the MAP parametrization
- Now, we can include the pion collinear PDF and its collinear datasets
- Include an additional 225 collinear data points
- Simultaneously extract
    1. Pion TMD PDFs
    2. Pion collinear PDFs
    3. Proton TMD PDFs
    4. Nuclear dependence
    5. Non-perturbative CS kernel

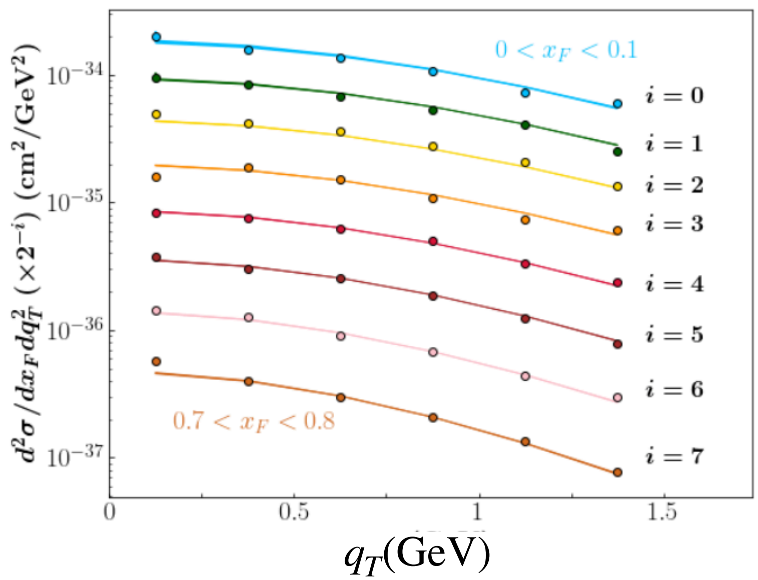
# Pheno-Aspects of the fit

This analysis both $q_T$ dependent and collinear data, and are consequently able to, first time simultaneously extract the pion's TMD and collinear PDFs

## Data and theory agreement

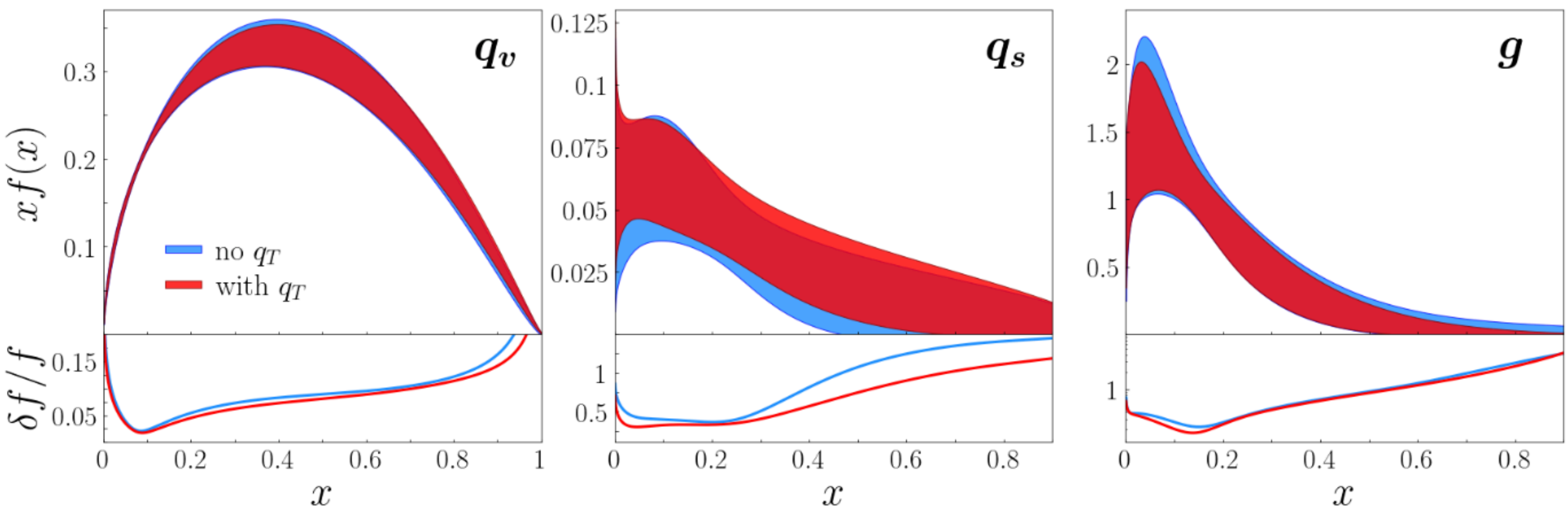
- Fit both $pA$ and $\pi A$ DY data and achieve good agreement to both

| Process | Experiment | $\sqrt{s}$ GeV | $\chi^2$/np | $Z$-score |
|---|---|---|---|---|
| $q_T$-integr. DY | E615 [37] | 21.8 | 0.86 | 0.76 |
| $\pi W \to \mu^+\mu^- X$ | NA10 [38] | 19.1 | 0.54 | 2.27 |
| | NA10 [38] | 23.2 | 0.91 | 0.18 |
| Leading neutron | H1 [73] | 318.7 | 0.36 | 4.61 |
| $ep \to e'nX$ | ZEUS [74] | 300.3 | 1.48 | 2.16 |
| $q_T$-dep. $pA$ DY | E288 [67] | 19.4 | 0.93 | 0.25 |
| $pA \to \mu^+\mu^- X$ | E288 [67] | 23.8 | 1.33 | 1.54 |
| | E288 [67] | 24.7 | 0.95 | 0.23 |
| | E605 [68] | 38.8 | 1.07 | 0.39 |
| | E772 [69] | 38.8 | 2.41 | 5.74 |
| | E866 $(Fe/Be)$ [70] | 38.8 | 1.07 | 0.29 |
| | E866 $(W/Be)$ [70] | 38.8 | 0.89 | 0.11 |
| $q_T$-dep. $\pi A$ DY | E615 [37] | 21.8 | 1.61 | 2.58 |
| $\pi W \to \mu^+\mu^- X$ | E537 [71] | 15.3 | 1.11 | 0.57 |
| Total | | | 1.15 | 2.55 |



# Pheno-Aspects of the fit

## Extracted pion PDFs




- The small-$q_T$ data do not constrain much the PDFs

By def., the TMDPDF is a $2-D$ number density dependent on $x$ & $b_T$ “joint” “probability distribution”

$$\tilde{f}_{q/\mathcal{N}}(x, b_T; \mu, \zeta)$$

Here we study the probability distribution in $b_T$ for a given $x$: this is a quantity in which describes the ratio of the $2-D$ density to the integrated or $b_T$—independent density; that is dependent on “$b_T$ given $x$ ”
$\longrightarrow$ “conditional density”

$$\equiv \tilde{f}_{q/\mathcal{N}}(b_T | x; \mu, \zeta)$$

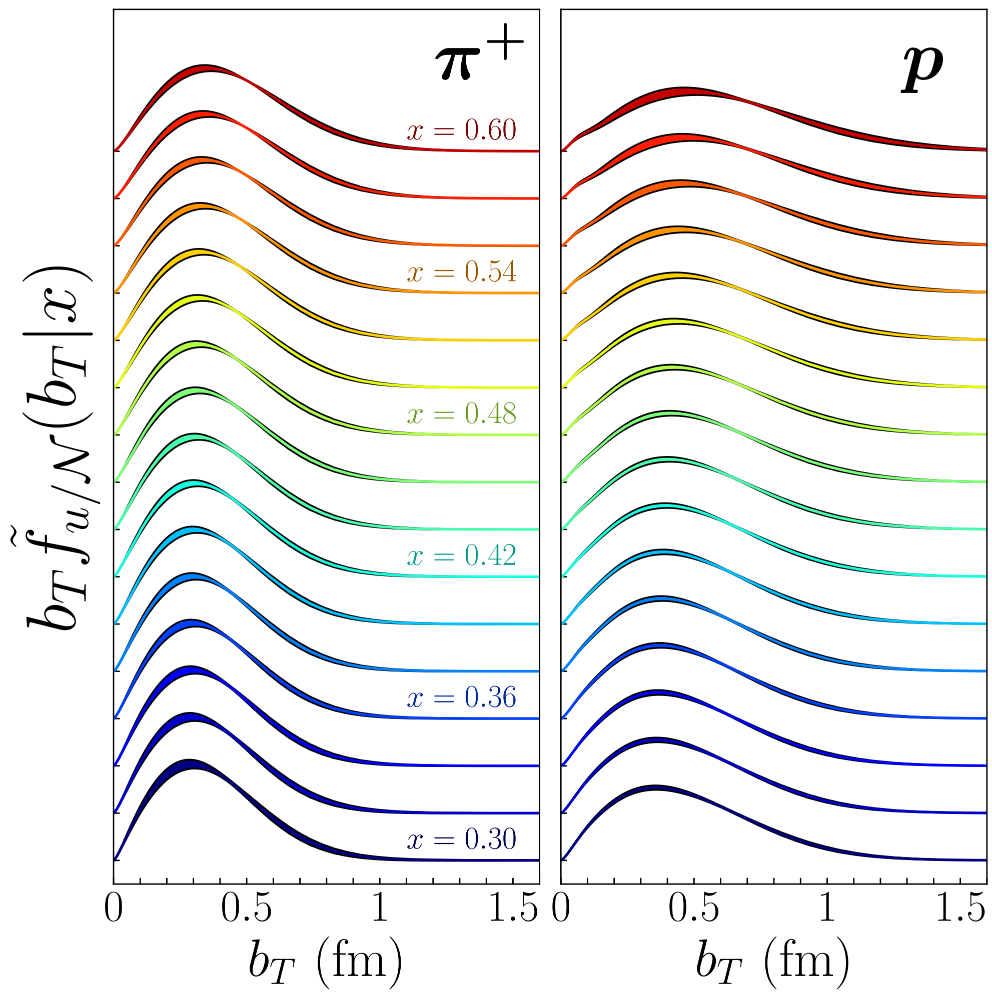
Naturally follow from Bayes’ theorem define a conditional density on $b_T$ — given $x$

$$\tilde{f}(x, b_T; \mu, \zeta) = \tilde{f}(b_T | x; \mu, \zeta) f(x, \mu)$$

Operationally :

$$\boxed{\tilde{f}_{q/\mathcal{N}}(b_T | x; \mu, \zeta) = \frac{\tilde{f}_{q/\mathcal{N}}(x, b_T; \mu, \zeta)}{\int d^2\boldsymbol{b}_T \tilde{f}_{q/\mathcal{N}}(x, b_T; \mu, \zeta)}}$$

# Resulting TMD PDFs of proton & pion




$$\tilde{f}_{q/\mathcal{N}}(b_T|x;\mu,\zeta) = \frac{\tilde{f}_{q/\mathcal{N}}(x, b_T;\mu,\zeta)}{\int d^2\boldsymbol{b}_T \tilde{f}_{q/\mathcal{N}}(x, b_T;\mu,\zeta)}$$

- Shown in $x$ range $x \in [0.3, 0.6]$ where $\pi$ and $p$ are both constrained: each TMDpdf show w/ $1\sigma$ uncertainty band from analysis
- $u$-quark in $\pi$ is narrower than $u$-quark in $p$ & "both" become wider w/increasing $x$

To make quantitative comparison btwn hadron distributions we consider the average $b_T$ as a function of $x$

defined as

$$\langle b_T|x\rangle_{q/\mathcal{N}} = \int d^2\boldsymbol{b}_T\, b_T\, \tilde{f}_{q/\mathcal{N}}(b_T|x;Q,Q^2)$$

# “Average $b_T$ “

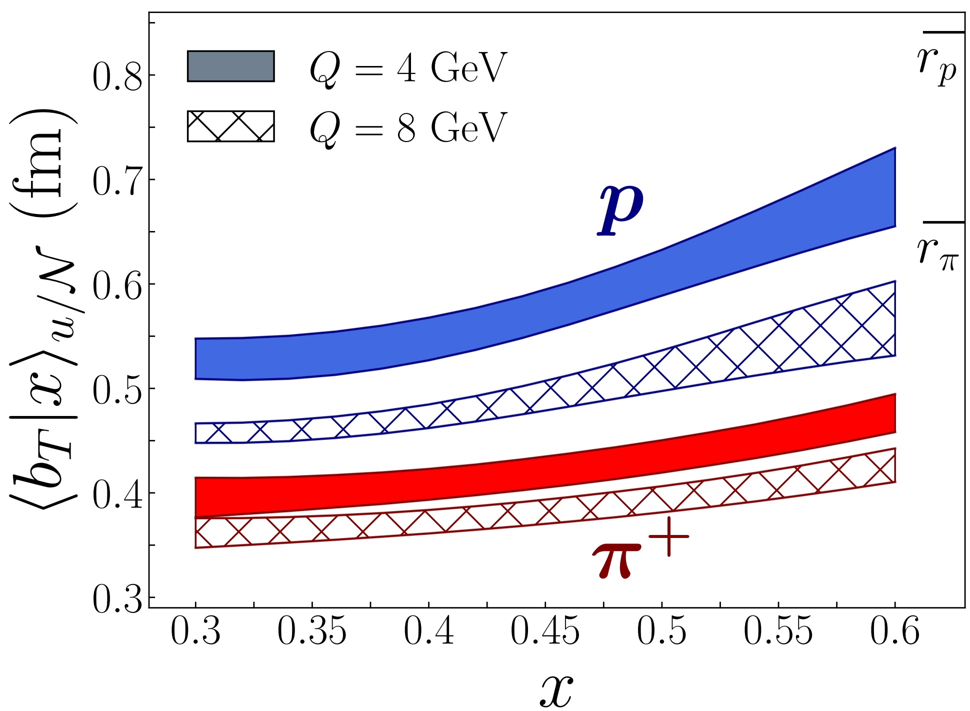
The conditional expectation value of $b_T$ for a given $x$

$$\langle b_T | x \rangle_{q/\mathcal{N}} = \int d^2\boldsymbol{b}_T \, b_T \, \tilde{f}_{q/\mathcal{N}}(b_T | x; Q, Q^2)$$

Gives a measure of the transverse correlation in coordinate space of the quark in a hadron for a given $x$

# The conditional expectation value of $b_T$ for a given $x$

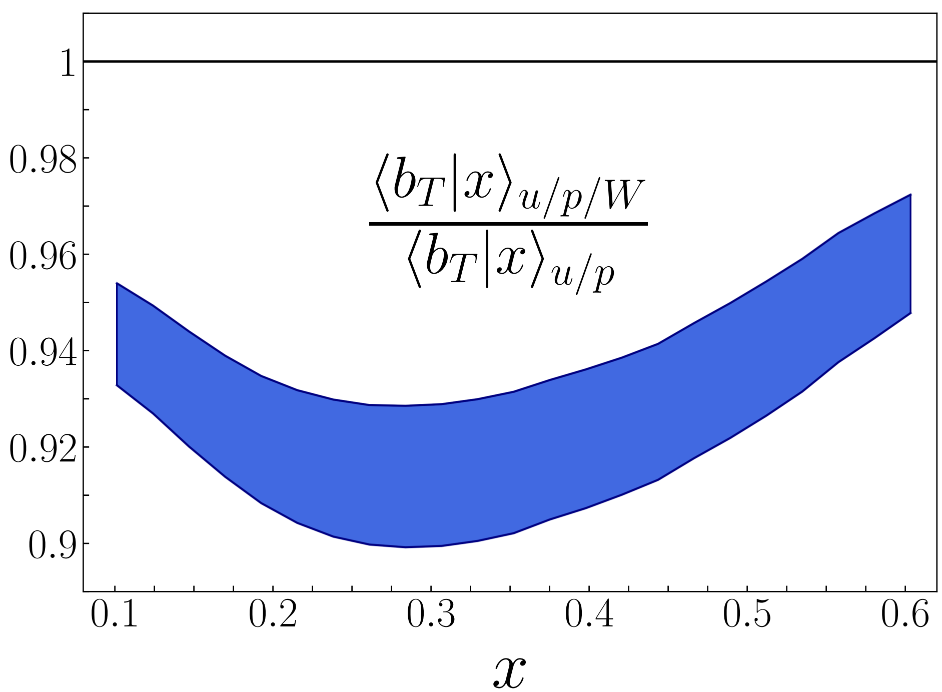
$$\langle b_T|x\rangle_{q/\mathcal{N}} = \int d^2\boldsymbol{b}_T\, b_T\, \tilde{f}_{q/\mathcal{N}}(b_T|x;Q,Q^2)$$




- On av. ~20% reduction of $u$-quark transverse correlations in pion relative to proton within a $(5.3 - 7.5)\,\sigma$ confidence level
- Decreases as $x$ decreases
- Interestingly: charge radius of the pion is also about 20% smaller than that of the proton, using the nominal PDG values, $r_p$= 0.8409(4) fm, $r_\pi$ = 0.659(4) fm) [PDG]
- Also, within each hadron, the average spatial separation of quark fields in the transverse direction does not exceed its charge radius, as shown on the right edge of fig.
- As $x \to 1$ phase space for the transverse motion $k_T$ of partons becomes smaller, since most of the momentum is along the light-cone direction, thus one expects an increase in the transverse correlations in $b_T$ space
- & as $Q$ increases more glue is radiated, which makes TMDpdf wider in $k_T$ & therefore narrow in $b_T$ space

***both quantitatively confirmed in figure***

# The effect of nuclear environment on quark correlations inside nucleon
# EMC effect for conditional $\langle b_T | x \rangle$ av




- taking the ratio of $\langle b_T | x \rangle$ for a bound proton in a nucleus to that of a free proton
- Find analogous suppression at $x \sim 0.3$ similar to that found for collinear distributions "transverse EMC" effect
- Results are consistent with Alrasheed et al. PRL 129 (2022) extend beyond by looking at $x$ dependence of non-perturbative transverse structure within a simultaneous collinear & TMD QCD global analysis

# Summary

- We have presented a comprehensive analysis of $\pi$ and $P$ TMD PDFs at N2LL perturbative precision using fixed-target DY data.
- For the first time used both $q_T$ -integrated and $q_T$ - differential DY data, as well as *LN* measurements, to simultaneously extract $\pi$ collinear and TMD PDFs and $P$ TMD PDFs.
- The combined analysis, including an exploration of the nuclear dependence of TMDs, allows us to perform a detailed comparison of $\pi$ and $P$ TMDs and to study the similarities and differences of their transverse spatial & momentum dependence
- We have determined conclusively that the transverse correlations of quarks in a pion are ≈ 20% smaller than those in a proton
- The observed characteristic decrease of the average separation of quark fields for decreasing x may indicate the influence of dynamical chiral symmetry breaking [102].
- We also found evidence for a transverse EMC effect, as discussed earlier by Alrashed et al. [73].

# Outlook

- Exploration of the quark transverse correlations in pions and protons can be extended to other hadrons, such as kaons and neutrons, in the near future, when the tagged SIDIS programs at Jefferson Lab and the EIC become available.
- Include lattice data from collinear $\pi$ pdfs

- Also, studying in various NP schemes NP schemes, flavor dependence, …

  ★ Future entails the study of matching low and hi $p_{\mathrm{T}}$ data

# Extras

# Explored nonperturbative parametrization of TMDs

- First perform single fits of these data to explore various aspects
- Many types of parametrizations have been used in the past
- For the "intrinsic" non-perturbative TMD, we perform fits with each of the following

**Gaussian**

$$\exp(-g_{q/N}(x, b_T)) = \exp\left(-g_q(x, A)\, b_T^2\right) ,$$

**Exponential**

$$\exp(-g_{q/N}(x, b_T)) = \exp\left(-g_q(x, A)\, b_T\right) ,$$

**Gaussian-to-Exponential**

$$\exp(-g_{q/N}(x, b_T)) = \exp\left(-g_q(x, A)\, \frac{b_T^2}{\sqrt{1 + B_{NP}(x) b_T^2}}\right) ,$$

# Parametrizations

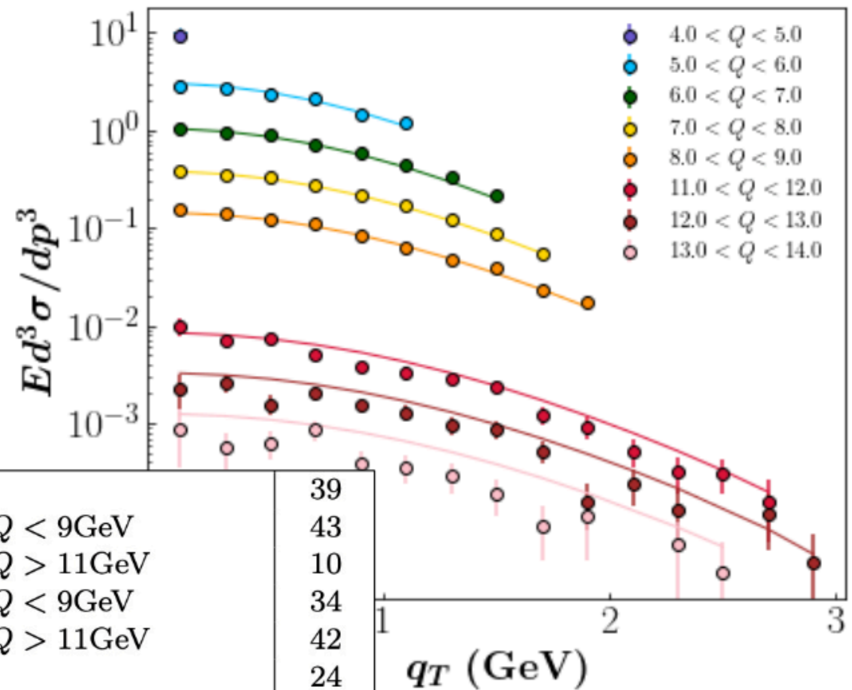
- We can test whether or not the $x$-dependence is important for these functions (it is!)
- For these $g_q$ functions, we have the following

$$g_q(x, A) = |g^q + g_2^q x + g_3^q (1-x)^2|(1 + g_1(A^{1/3} - 1)) \,,$$
$$B_{NP}(x) = b_{NP} x^2 \,,$$

- 4 free parameters for each scheme (5 for Gaussian-to-Exponential)
- We may also open up these for each flavor in the proton ($u$, $d$, and $sea$) and for the pion ($val$, $sea$)

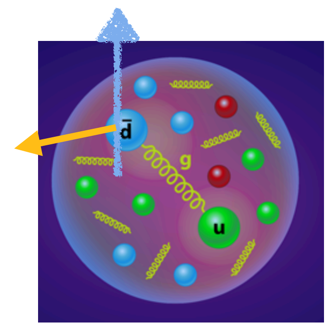
# Problem describing data

- The E288 400 GeV data are difficult to describe the same above and below the $\Upsilon$ resonance
- Theory overpredicts data when $Q > 11\text{GeV}$
- Could treat as separate datasets – separate normalizations:



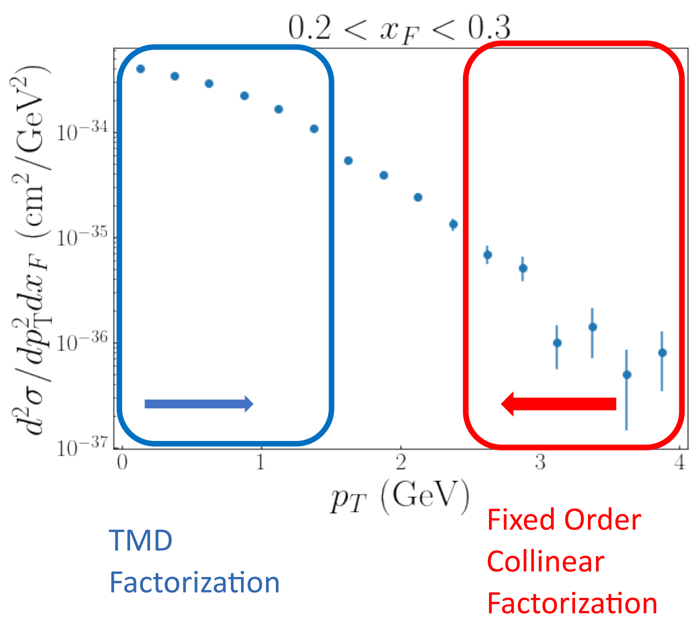

| | |
|---|---|
| E228-200 | 39 |
| E228-300 $Q < 9\text{GeV}$ | 43 |
| E228-300 $Q > 11\text{GeV}$ | 10 |
| E228-400 $Q < 9\text{GeV}$ | 34 |
| E228-400 $Q > 11\text{GeV}$ | 42 |
| E772 | 24 |
| E605 $Q < 9\text{GeV}$ | 21 |
| E605 $Q > 11\text{GeV}$ | 32 |

# “More granular” $p_T \sim k_T \ll Q$ access to the Pion TMDs




To describe the transverse momentum “region” $p_T \sim k_T \ll Q$ is the regime of TMDs of the pion

Requires fitting “region” $p_T \sim k_T \ll Q$ differential pion-induced Drell-Yan cross section




***TMD Factorization***

$$\frac{\mathrm{d}\sigma^W}{\mathrm{d}Q^2\,\mathrm{d}x_F\,\mathrm{d}p_\mathrm{T}^2} = \int \frac{\mathrm{d}^2\boldsymbol{b}_\mathrm{T}}{(2\pi)^2}\, e^{i\boldsymbol{p}_\mathrm{T}\cdot\boldsymbol{b}_\mathrm{T}}\tilde{W}(x_F, b_T, Q)$$

$$\tilde{W}(x_F, b_T, Q) = \sum_j H^{\mathrm{DY}}_{j\bar{j}}(Q, \mu, a_s(\mu))\tilde{f}_{j/A}(x_A, b_\mathrm{T}; \zeta_A, \mu)\tilde{f}_{\bar{j}/B}(x_B, b_\mathrm{T}; \zeta_B, \mu)$$