QCD Global Analyses of Transverse Single-Spin Asymmetries: Single-Hadron and Dihadron Observables

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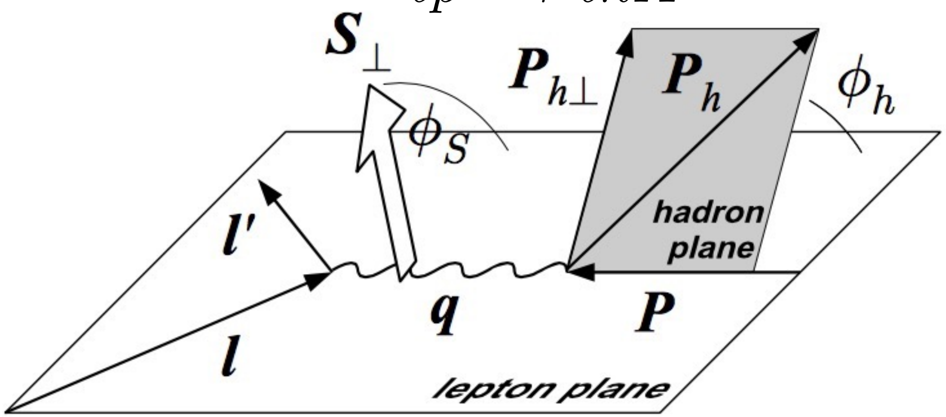
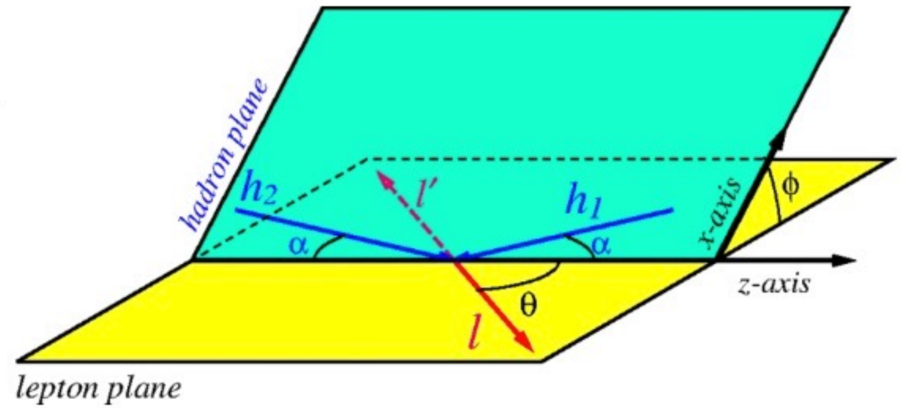
Background:
TSSAs for Single-Hadron Fragmentation

$$\ell p^{\uparrow} \to \ell h X$$



$$\{\pi, p\} p^{\uparrow} \to \{\ell^+\ell^-, W^{\pm}, Z\} X$$



$$F_{UT}^{\sin(\phi_h - \phi_S)} = \mathcal{C}\left[-\frac{\hat{h}\cdot\vec{k}_T}{M} \boldsymbol{f_{1T}^{\perp}} D_1\right]$$

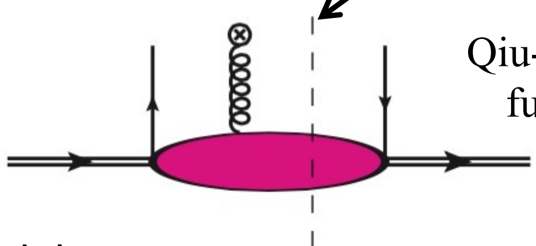
$$F_{TU}^{\sin\phi} = \mathcal{C}\left[-\frac{\hat{h}\cdot\vec{k}_{aT}}{M_a} \boldsymbol{f_{1T}^{\perp}}\, \bar{f}_1\right]$$

TMD/Collins-Soper-Sterman (CSS) Evolution

OPE

Sudakov exponentials (gluon radiation)

$$\tilde{f}_{1T}^{\perp(1)}(x, b_T; Q^2, \mu_Q) \sim F_{FT}(x, x; \mu_{b_*}) \exp\left[-S_{pert}(b_*(b_T); \mu_{b_*}, Q, \mu_Q) - S_{NP}^{f_{1T}^{\perp}}(b_T, Q)\right]$$



Qiu-Sterman function

$$g_{f_{1T}^{\perp}}(x, b_T) + g_K(b_T)\ln(Q/Q_0)$$

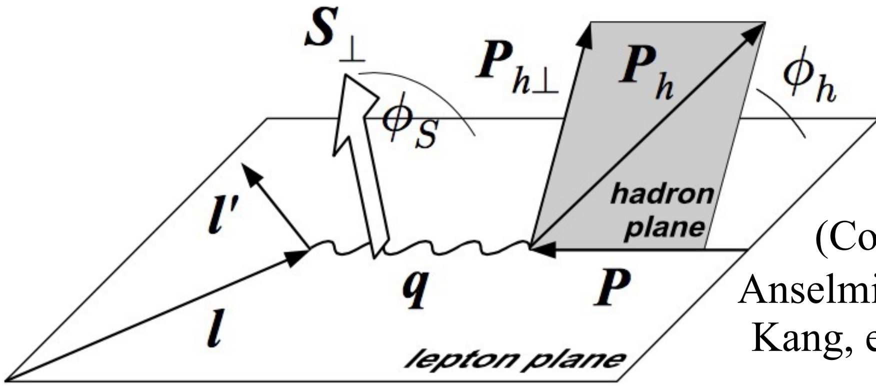
(Aybat, et al. (2012); Bury, et al. (2021); Echevarria, et al. (2014, 2021))

Parton model

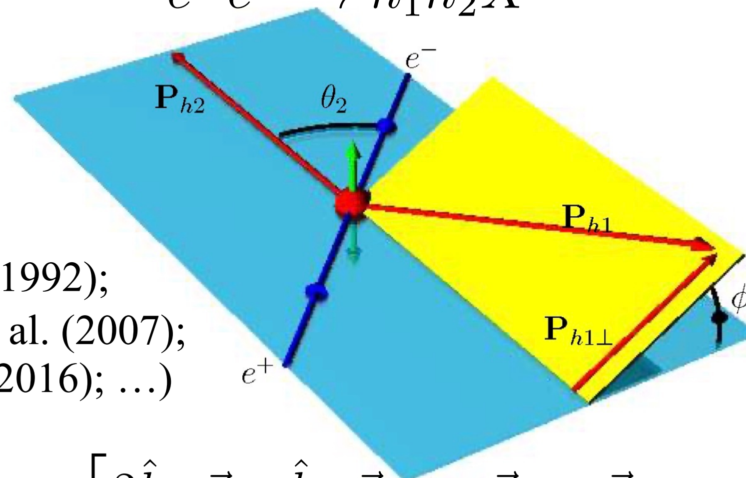
$$\pi F_{FT}(x, x) = \int d^2\vec{k}_T\, \frac{k_T^2}{2M^2} f_{1T}^{\perp}(x, k_T^2) \equiv f_{1T}^{\perp(1)}(x)$$

(Boer, Mulders, Pijlman (2003))

$\ell N^{\uparrow} \to \ell\, h\, X$



$e^+e^- \to h_1 h_2 X$



(Collins (1992); Anselmino, et al. (2007); Kang, et al. (2016); …)

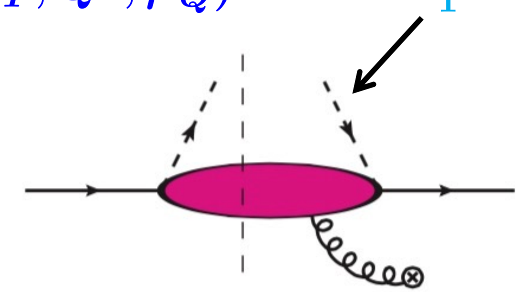
$$F_{UT}^{\sin(\phi_h+\phi_S)} = \mathcal{C}\left[-\frac{\hat{h}\cdot\vec{p}_\perp}{M_h}\boldsymbol{h_1 H_1^\perp}\right] \qquad F_{UU}^{\cos(2\phi_0)} = \mathcal{C}\left[\frac{2\hat{h}\cdot\vec{p}_{a\perp}\,\hat{h}\cdot\vec{p}_{b\perp} - \vec{p}_{a\perp}\cdot\vec{p}_{b\perp}}{M_a M_b}\,\boldsymbol{H_1^\perp \bar{H}_1^\perp}\right]$$

TMD/Collins-Soper-Sterman (CSS) Evolution

OPE — Sudakov exponentials (gluon radiation)

$$\boldsymbol{\tilde{h}_1(x, b_T; Q^2, \mu_Q)} \quad\sim\quad \boldsymbol{h_1(x; \mu_{b_*})}\exp\left[-S_{pert}(b_*(b_T);\mu_{b_*},Q,\mu_Q) - S_{NP}^{h_1}(b_T,Q)\right]$$

$$\boldsymbol{\tilde{H}_1^{\perp(1)}(z, b_T; Q^2, \mu_Q)} \quad\sim\quad \boldsymbol{H_1^{\perp(1)}(z; \mu_{b_*})}\exp\left[-S_{pert}(b_*(b_T);\mu_{b_*},Q,\mu_Q) - S_{NP}^{H_1^\perp}(b_T,Q)\right]$$



qgq fragmentation function

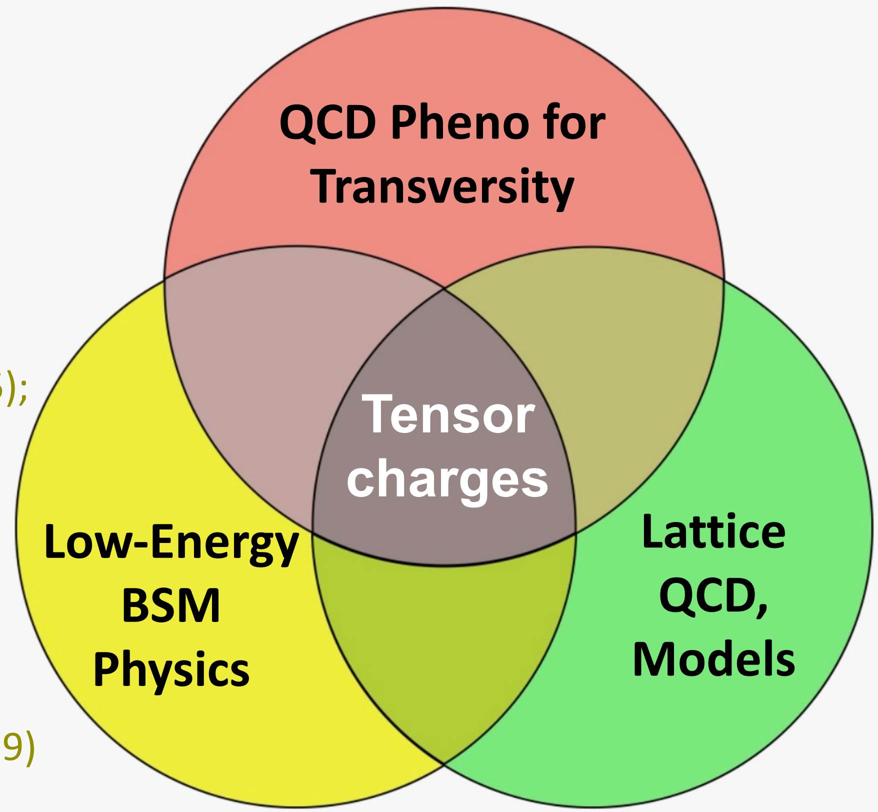
Parton model

$$h_1(x) = \int d^2\vec{k}_T\, h_1(x, \vec{k}_T^2)$$

$$H_1^{\perp(1)}(z) = z^2\int d^2\vec{p}_\perp \frac{p_\perp^2}{2M_h^2} H_1^\perp(z, z^2p_\perp^2)$$

$$\delta q \equiv \int_0^1 dx\,[h_1^q(x) - h_1^{\bar{q}}(x)] \qquad g_T \equiv \delta u - \delta d$$

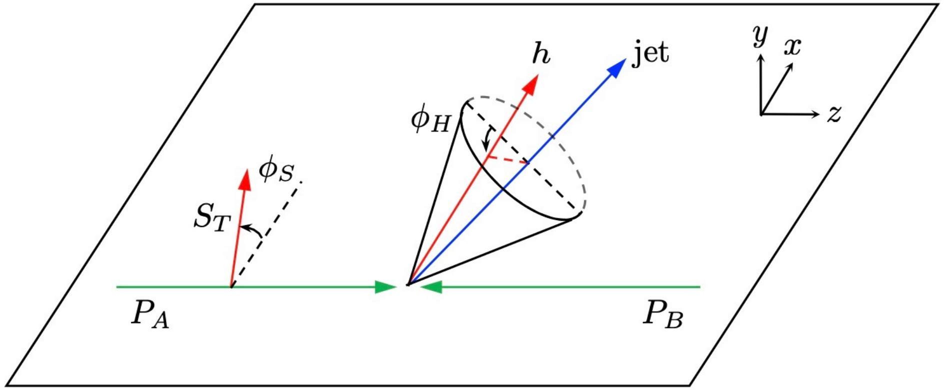
Anselmino, et al. (2007, 2009, 2013, 2015);
Goldstein, et al. (2014); Kang, et al. (2016); Radici, et al. (2013, 2015, 2018);
Benel, et al. (2020); D'Alesio, et al. (2020); Cammarota, et al. (2020);
Gamberg, et al. (2022); Cocuzza, et al. (2023)



Herczeg (2001);
Erler, Ramsey-Musolf (2005);
Pospelov, Ritz (2005);
Severijns, et al. (2006);
Cirigliano, et al. (2013);
Courtoy, et al. (2015);
Yamanaka, et al. (2017);
Liu, et al. (2018);
Gonzalez-Alonso, et al. (2019)

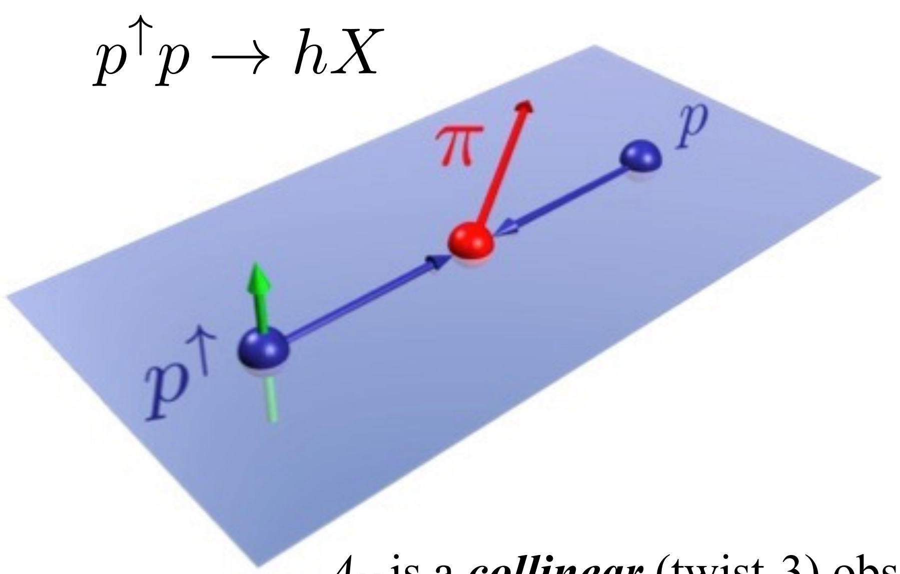
He, Ji (1995);
Barone, et al. (1997);
Schweitzer, et al. (2001);
Gamberg, Goldstein (2001);
Pasquini, et al. (2005);
Wakamatsu (2007);
Lorce (2009);
Gupta, et al. (2018);
Yamanaka, et al. (2018);
Hasan, et al. (2019);
Alexandrou, et al. (2019, 2023);
Yamanaka, et al. (2013);
Pitschmann, et al. (2015);
Xu, et al. (2015);
Wang, et al. (2018);
Liu, et al. (2019)

$$p^{\uparrow} p \to (h\, jet)\, X$$



(Yuan (2008); D'Alesio, Murgia, Pisano (2017); Kang, Prokudin, Ringer, Yuan (2017), …)

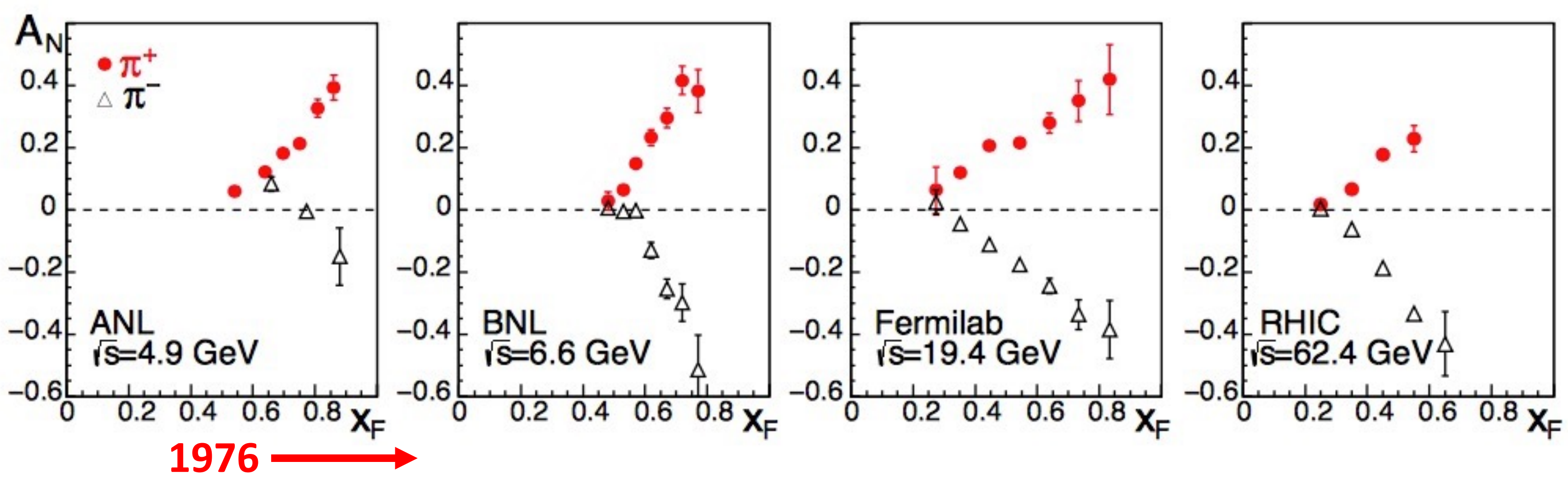
$$F_{UT}^{\sin(\phi_S-\phi_H)} \sim H^{\rm Collins}_{ab\to c}(\hat{s},\hat{t},\hat{u}) \otimes \boldsymbol{h_1^a(x_1)} \otimes f_1^b(x_2) \otimes (j_\perp/(z_h M_h))\boldsymbol{H_1^{\perp h/c}(z_h, j_\perp^2)}$$



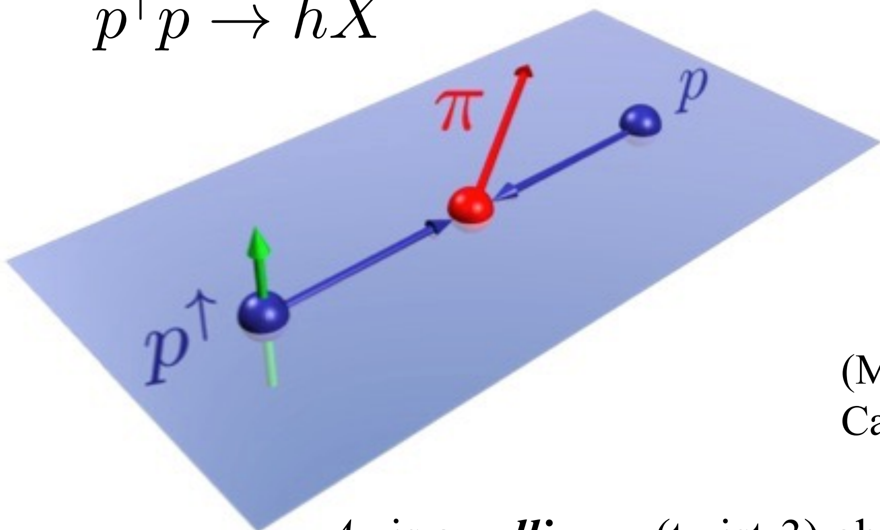
$$d\Delta\sigma(S_T) \sim \underbrace{H_{QS} \otimes f_1 \otimes \boldsymbol{F_{FT}} \otimes D_1}_{\text{Qiu-Sterman term}}$$

$$+ \underbrace{H_F \otimes f_1 \otimes \boldsymbol{h_1} \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{\tilde{H}}\right)}_{\text{Fragmentation term}}$$

A_N is a ***collinear*** (twist-3) observable



$$p^{\uparrow}p \to hX$$

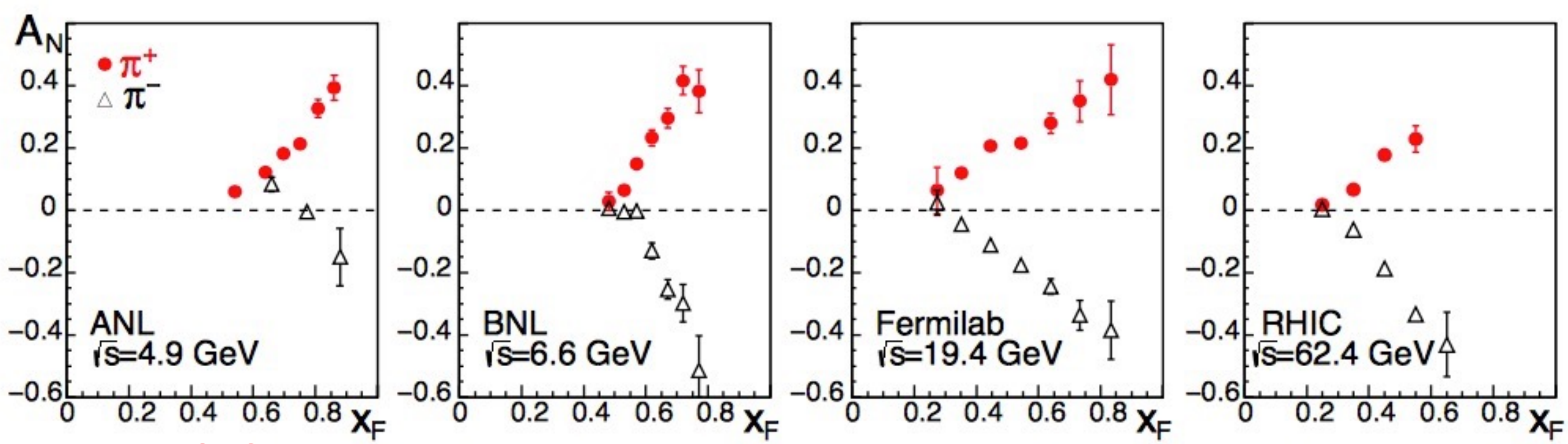


$$d\Delta\sigma(S_T) \sim \underbrace{H_{QS} \otimes f_1 \otimes \boldsymbol{F_{FT}} \otimes D_1}_{\text{Qiu-Sterman term}}$$

$$+ \underbrace{H_F \otimes f_1 \otimes \boldsymbol{h_1} \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{\tilde{H}}\right)}_{\text{Fragmentation term}}$$

(Metz, DP (2012); Kanazawa, et al. (2014); Cammarota, et al. (2020); Gamberg, et al. (2017, 2022))

A_N is a ***collinear*** (twist-3) observable



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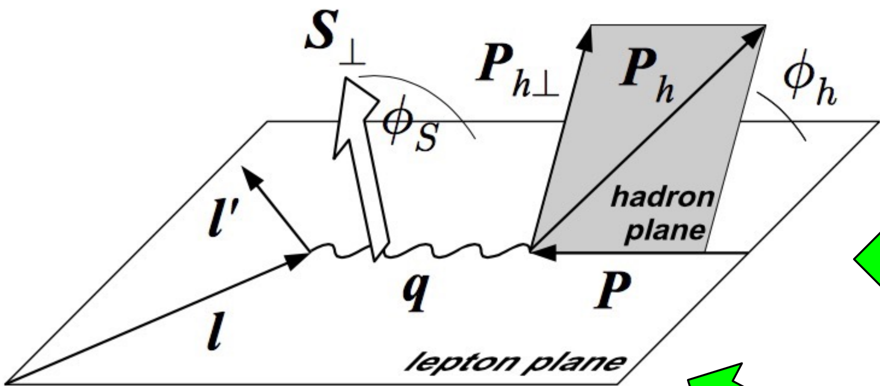
Updated QCD Global Analysis of TSSAs for Single-Hadron Fragmentation

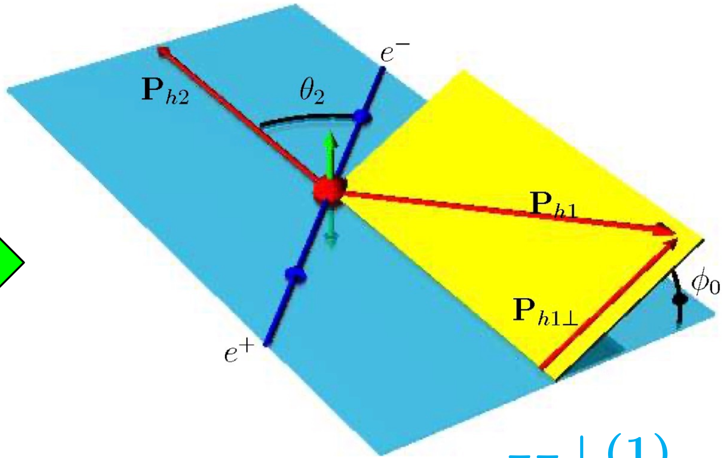
Gamberg, Malda, Miller, DP, Prokudin, Sato, PRD **106**, 034014 (2022)

User-friendly jupyter notebook to calculate functions and asymmetries:
https://colab.research.google.com/github/pitonyak25/jam3d_dev_lib/blob/main/JAM3D_Library.ipynb

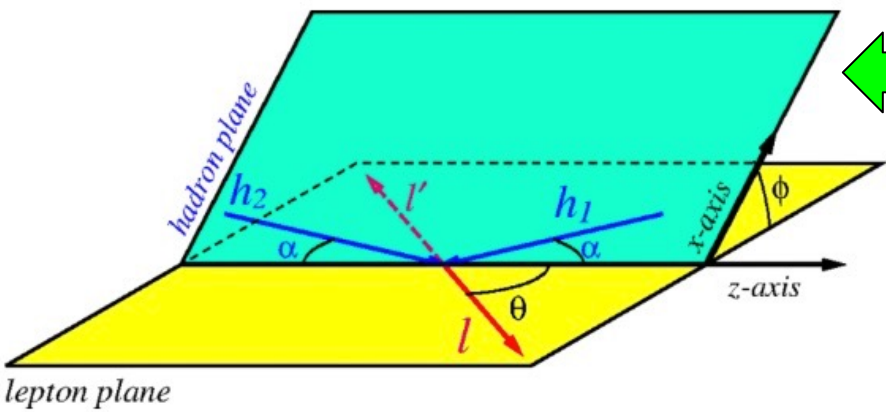
LHAPDF tables available (thanks to C. Cocuzza):
https://github.com/pitonyak25/jam3d_dev_lib/tree/main/LHAPDF_tables



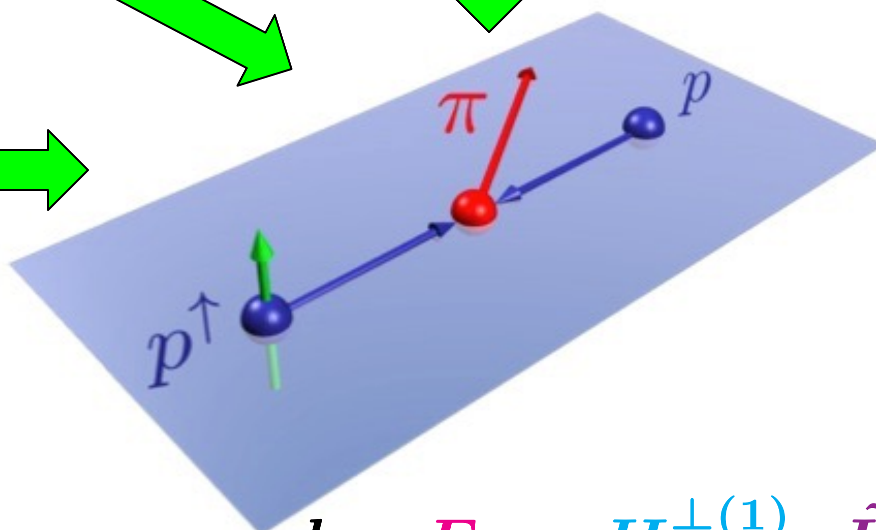
h_1, F_{FT}, $H_1^{\perp(1)}$, $\tilde{H}$



$H_1^{\perp(1)}$



F_{FT}



h_1, F_{FT}, $H_1^{\perp(1)}$, $\tilde{H}$

- Analyze TSSAs in SIDIS, Drell-Yan, e^+e^- annihilation, and proton-proton collisions and extract

$$h_1(x),\ F_{FT}(x,x),\ H_1^{\perp(1)}(z),\ \tilde{H}(z)$$

along with the relevant transverse momentum widths for the Sivers, transversity, and Collins functions: $\langle k_T^2\rangle_{f_{1T}^\perp}, \langle k_T^2\rangle_{h_1}, \langle p_\perp^2\rangle_{H_1^\perp}^{fav}, \langle p_\perp^2\rangle_{H_1^\perp}^{unf}$

- We use a Gaussian ansatz: $F^q(x,k_T^2) \sim F^q(x)e^{-k_T^2/\langle k_T^2\rangle}$ where

$$F^q(x) = \frac{N_q\, x^{a_q}(1-x)^{b_q}(1+\gamma_q\, x^{\alpha_q}(1-x)^{\beta_q})}{\mathrm{B}[a_q+2, b_q+1] + \gamma_q \mathrm{B}[a_q+\alpha_q+2, b_q+\beta_q+1]}$$

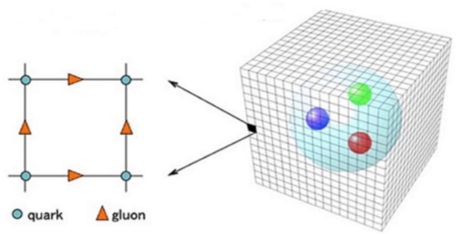
NB: $\{\gamma, \alpha, \beta\}$ only used for Collins function

- DGLAP-type evolution for the collinear functions analogous to Duke & Owens (1984): double-log Q^2-dependent term explicitly added to the parameters

- Additional data/constraints included in the fit compared to 2020:
  - Collins and Sivers effects (3D-binned) SIDIS data from HERMES (2020)
  - $A_{UT}^{\sin\phi_S}$ data (x and z projections only) from HERMES (2020)

$$\int d^2\vec{P}_{hT}\, F_{UT}^{\sin\phi_S} = -\frac{x}{z}\sum_q e_q^2\, \frac{2M_h}{Q} h_1^{q/N}(x)\, \tilde{H}^{h/q}(z)$$

  - Lattice data on g_T at the physical pion mass from ETMC (Alexandrou, et al. (2019))

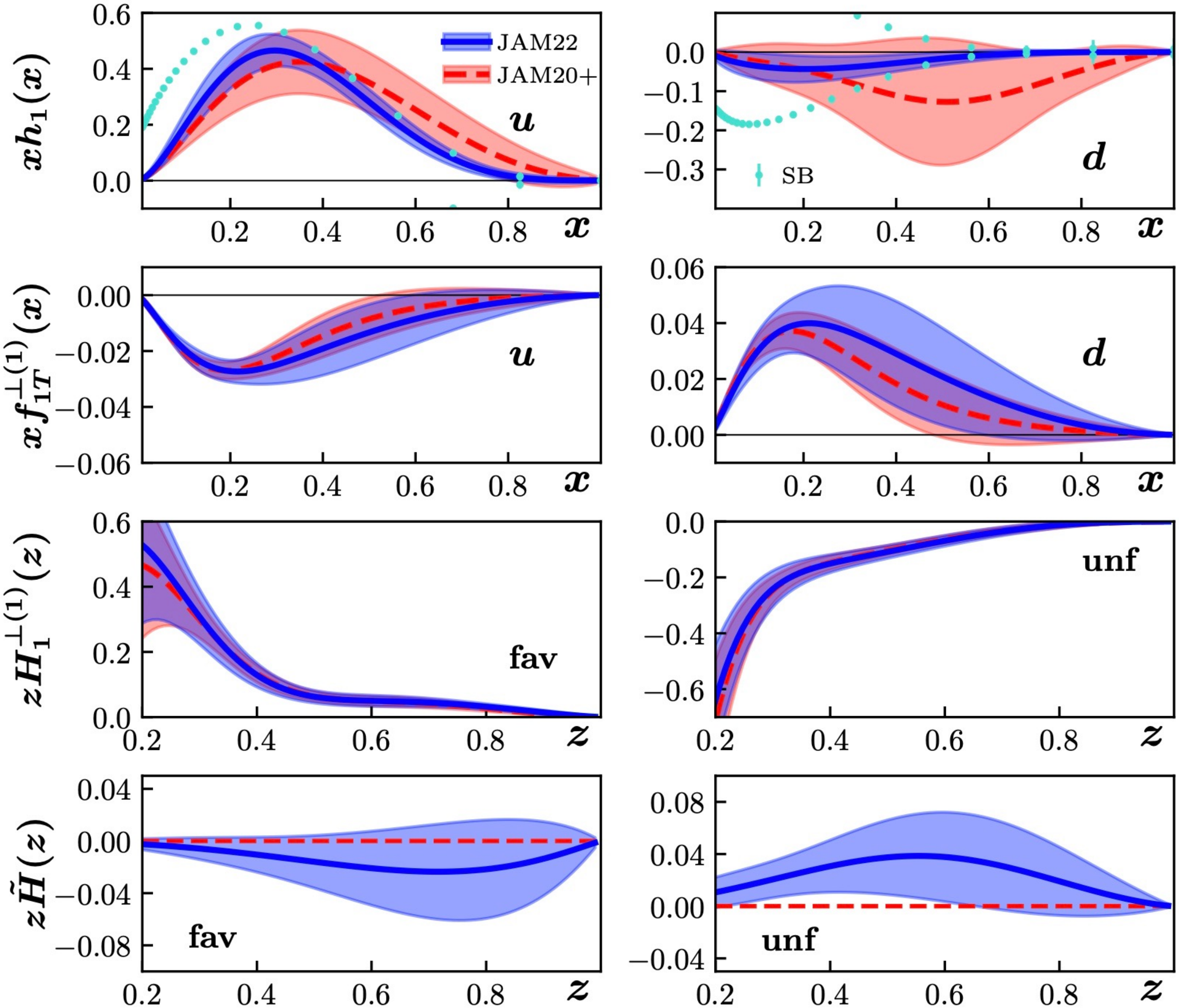


  - Imposing the Soffer bound on transversity: $|h_1^q(x)| \leq \frac{1}{2}(f_1^q(x) + g_1^q(x))$

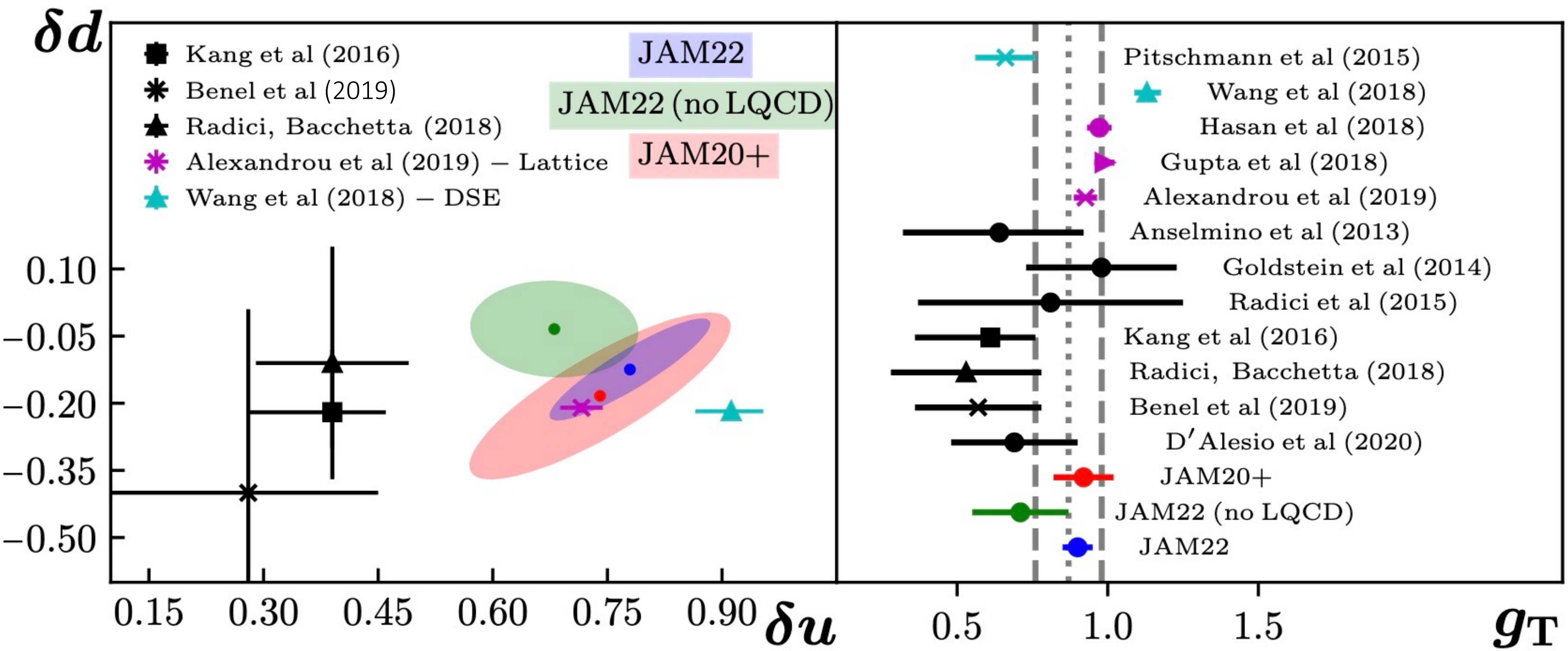
Generate "data" (central value and 1-σ uncertainty) using recent simultaneous fit of f_1 and g_1 from Cocuzza, et al. (2022) and add to the χ^2 if SB is violated by more than the uncertainty in the data

$$\chi^2/N_{\text{pts.}} = 647/634 = 1.02$$

Observable	Reactions	Non-Perturbative Function(s)	χ^2/npts
$A_{UT}^{\sin(\phi_h-\phi_S)}$	$e+(p,d)^\uparrow \to e+(\pi^+,\pi^-,\pi^0)+X$	$f_{1T}^\perp(x,\vec{k}_T^2)$	$182.9/166 = 1.10$
$A_{UT}^{\sin(\phi_h+\phi_S)}$	$e+(p,d)^\uparrow \to e+(\pi^+,\pi^-,\pi^0)+X$	$h_1(x,\vec{k}_T^2), H_1^\perp(z,z^2\vec{p}_T^2)$	$181.0/166 = 1.09$
$^*A_{UT}^{\sin\phi_S}$	$e+p^\uparrow \to e+(\pi^+,\pi^-,\pi^0)+X$	$h_1(x), \tilde{H}(z)$	$18.6/36 = 0.52$
$A_{UC/UL}$	$e^+ + e^- \to \pi^+\pi^-(UC,UL)+X$	$H_1^\perp(z,z^2\vec{p}_T^2)$	$154.9/176 = 0.88$
$A_{T,\mu^+\mu^-}^{\sin\phi_S}$	$\pi^- + p^\uparrow \to \mu^+\mu^- + X$	$f_{1T}^\perp(x,\vec{k}_T^2)$	$6.92/12 = 0.58$
$A_N^{W/Z}$	$p^\uparrow + p \to (W^+,W^-,Z)+X$	$f_{1T}^\perp(x,\vec{k}_T^2)$	$30.8/17 = 1.81$
A_N^π	$p^\uparrow + p \to (\pi^+,\pi^-,\pi^0)+X$	$h_1(x), F_{FT}(x,x) = \frac{1}{\pi}f_{1T}^{\perp(1)}(x), H_1^{\perp(1)}(z), \tilde{H}(z)$	$70.4/60 = 1.17$
Lattice g_T	——	$h_1(x)$	$1.82/1 = 1.82$



First direct information from experiment on $\tilde{H}(z)$



- Dihadron (e.g., Radici, Bacchetta (2018); Benel, Courtoy, Ferro-Hernandez (2019)) and TMD analyses that only include e^+e^- and SIDIS Collins effect data (e.g., Kang, et al. (2016)), are generally below the lattice values for g_T and δu
- Note that because of the SB, one initially finds JAM3D-22 has more tension with lattice, but this does *not* imply phenomenology and lattice are incompatible – one can only fully answer this by including lattice data in the analysis
- **Once the the lattice g_T data point is included, we find the non-perturbative functions can accommodate it *and still describe the experimental data well***

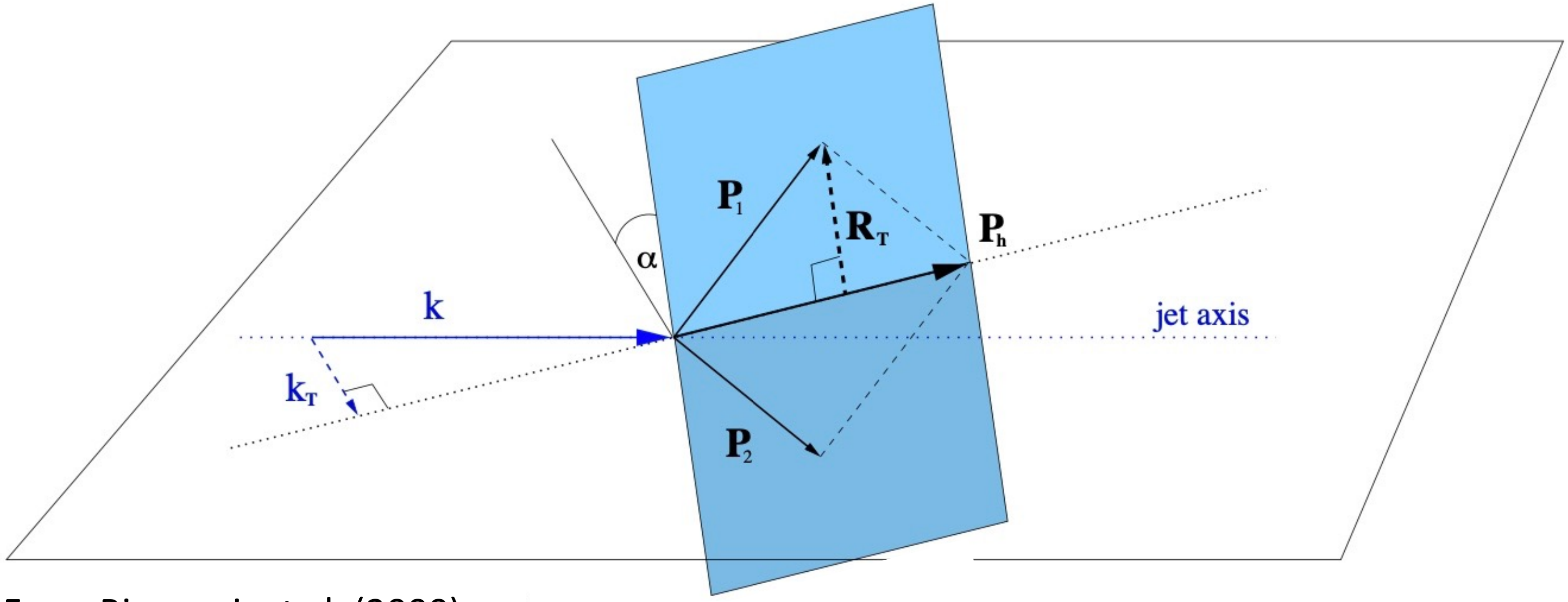
Background:
TSSAs for Dihadron Fragmentation

Bianconi, et al. (2000); Bacchetta, Radici (2003, 2004), …

$$P_h = P_1 + P_2 \qquad R = (P_1 - P_2)/2 \qquad z = z_1 + z_2 \qquad \zeta = (z_1 - z_2)/z$$

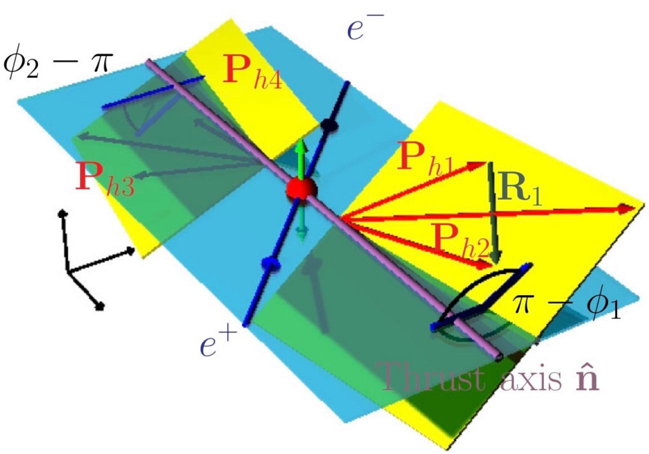
$$P_1 = \left(\frac{M_1^2 + \vec{R}_T^{\,2}}{(1+\zeta)P_h^-}, \frac{1+\zeta}{2} P_h^-, \vec{R}_T \right) \qquad P_2 = \left(\frac{M_2^2 + \vec{R}_T^{\,2}}{(1-\zeta)P_h^-}, \frac{1-\zeta}{2} P_h^-, -\vec{R}_T \right)$$

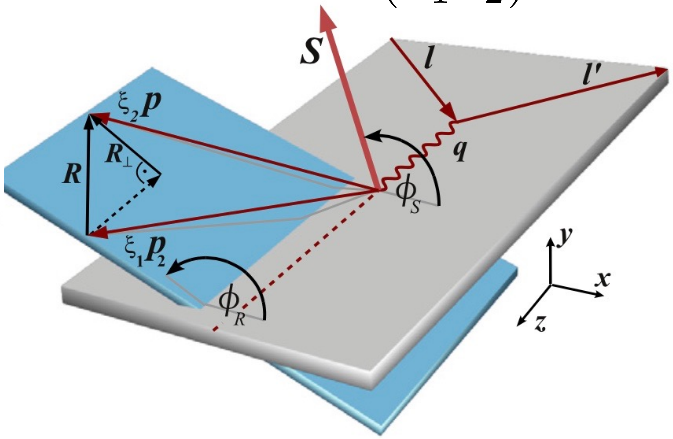
$$\vec{R}_T^{\,2} = \frac{1-\zeta^2}{4} M_h^2 - \frac{1-\zeta}{2} M_1^2 - \frac{1+\zeta}{2} M_2^2$$

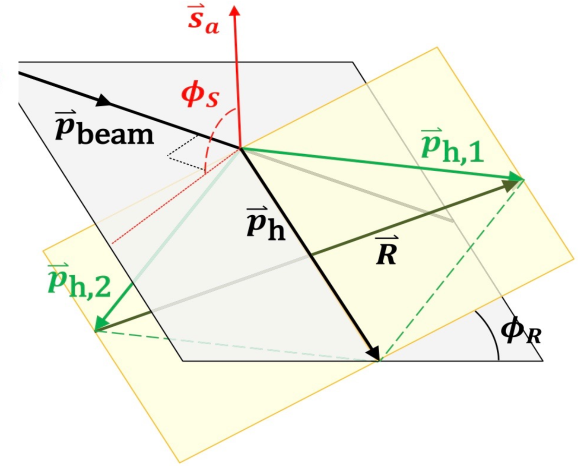


From Bianconi, et al. (2000)

$$e^+e^- \to (h_1 h_2)(\bar{h}_1 \bar{h}_2)\, X \qquad \ell\, N^\uparrow \to \ell\, (h_1 h_2)\, X \qquad p^\uparrow p \to (h_1 h_2)\, X$$







(Collins, et al. (1994); Bianconi, et al. (2000); Bacchetta, Radici (2003, 2004); Courtoy, et al. (2012); Matevosyan, et al. (2018); Radici, et al. (2013, 2015, 2018); Benel, et al. (2020), …)

$$a_{12R} = \frac{\sin^2\theta_2 \sum_q e_q^2\, H_1^{\sphericalangle,q}(z, M_h)\, H_1^{\sphericalangle,\bar{q}}(\bar{z}, \overline{M}_h)}{(1+\cos^2\theta_2)\sum_q e_q^2\, D_1^q(z, M_h)\, D_1^{\bar{q}}(\bar{z}, \overline{M}_h)}$$

Artru-Collins asymmetry

Note: D_1 can be constrained using data on $d\sigma/dzdM_h$ from BELLE (2017)

$$\frac{d\sigma}{dz\, dM_h} = \frac{4\pi N_c \alpha_{\rm em}^2}{3Q^2} \sum_q e_q^2\, D_1^q(z, M_h)$$

$$A_{UT}^{\sin(\phi_R+\phi_S)} = \frac{\sum_q e_q^2\, h_1^q(x)\, H_1^{\sphericalangle,q}(z, M_h)}{\sum_q e_q^2\, f_1^q(x)\, D_1^q(z, M_h)}$$

$$A_{UT}^{\sin(\phi_R-\phi_S)} \sim \frac{\frac{d\Delta\hat{\sigma}_{a^\uparrow b\to c^\uparrow}}{d\hat{t}} \otimes h_1^a(x_a) \otimes f_1^b(x_b) \otimes H_1^{\sphericalangle,c}(z, M_h)}{\frac{d\hat{\sigma}_{ab\to c}}{d\hat{t}} \otimes f_1^a(x_a) \otimes f_1^b(x_b) \otimes D_1^c(z, M_h)}$$

New Definition of DiFFs with a Number Density Interpretation

DP, Cocuzza, Metz, Prokudin, Sato, arXiv:2305.XXXXX

$$\Delta^{h_1 h_2/q}(z, \vec{k}_T; P_1, P_2) = \sum_X \int \frac{d\xi^+ \, d^2\vec{\xi}_T}{(2\pi)^3} \, e^{i(k^-\xi^+ - \vec{k}_T\cdot\vec{\xi}_T)} \, \langle 0| \, \mathcal{W}_1(\infty, \xi) \, \psi_q(\xi^+, 0^-, \vec{\xi}_T) \, |P_1, P_2; X\rangle \times \langle P_1, P_2; X| \, \bar{\psi}_q(0^+, 0^-, \vec{0}_T)\mathcal{W}_2(0, \infty) \, |0\rangle$$

Bianconi, et al. (2000)

$$D_1^{h_1 h_2/q,\mathrm{BBJR}}(z, \zeta, \vec{k}_T^{\,2}, \vec{R}_T^{\,2}, \vec{k}_T \cdot \vec{R}_T) = \frac{\mathbf{1}}{\mathbf{4}\boldsymbol{z}} \mathrm{Tr}\Big[\Delta^{h_1 h_2/q}(z, \vec{k}_T; \vec{P}_1, \vec{P}_2)\gamma^-\Big]$$

Does ***not*** allow sum rules to be derived that justify a number density interpretation

$$\Delta^{h_1 h_2/q}(z, \vec{k}_T; P_1, P_2) = \sum_X \int \frac{d\xi^+ \, d^2\vec{\xi}_T}{(2\pi)^3} \, e^{i(k^- \xi^+ - \vec{k}_T \cdot \vec{\xi}_T)} \langle 0 | \, \mathcal{W}_1(\infty, \xi) \, \psi_q(\xi^+, 0^-, \vec{\xi}_T) \, | P_1, P_2; X \rangle \times \langle P_1, P_2; X | \, \bar{\psi}_q(0^+, 0^-, \vec{0}_T) \mathcal{W}_2(0, \infty) \, | 0 \rangle$$

Bianconi, et al. (2000)

$$D_1^{h_1 h_2/q, \mathrm{BBJR}}(z, \zeta, \vec{k}_T^{\,2}, \vec{R}_T^{\,2}, \vec{k}_T \cdot \vec{R}_T) = \frac{\mathbf{1}}{\mathbf{4z}} \mathrm{Tr}\left[\Delta^{h_1 h_2/q}(z, \vec{k}_T; \vec{P}_1, \vec{P}_2)\gamma^-\right]$$

Does ***not*** allow sum rules to be derived that justify a number density interpretation

DP, Cocuzza, Metz, Prokudin, Sato (2023)

$$D_1^{h_1 h_2/q}(z, \zeta, \vec{k}_T^{\,2}, \vec{R}_T^{\,2}, \vec{k}_T \cdot \vec{R}_T) = \frac{\boldsymbol{z}}{\mathbf{32\pi^3(1-\zeta^2)}} \mathrm{Tr}\left[\Delta^{h_1 h_2/q}(z, \vec{k}_T; P_1, P_2)\gamma^-\right]$$

Does allow for a number density interpretation

$$\Delta^{h_1 h_2/q}(z,\vec{k}_T;P_1,P_2) = \sum\!\!\!\!\!\!\!\int_X \frac{d\xi^+\, d^2\vec{\xi}_T}{(2\pi)^3}\, e^{i(k^-\xi^+ - \vec{k}_T\cdot\vec{\xi}_T)}\,\langle 0|\,\mathcal{W}_1(\infty,\xi)\,\psi_q(\xi^+,0^-,\vec{\xi}_T)\,|P_1,P_2;X\rangle \times \langle P_1,P_2;X|\,\bar{\psi}_q(0^+,0^-,\vec{0}_T)\mathcal{W}_2(0,\infty)\,|0\rangle$$

DP, Cocuzza, Metz, Prokudin, Sato (2023)

$$D_1^{h_1 h_2/q}(z,\zeta,\vec{k}_T^{\,2},\vec{R}_T^{\,2},\vec{k}_T\cdot\vec{R}_T) = \frac{z}{32\pi^3(1-\zeta^2)}\mathrm{Tr}\Big[\Delta^{h_1 h_2/q}(z,\vec{k}_T;P_1,P_2)\gamma^-\Big]$$

$$\sum_{h_1}\sum_{h_2}\int dz\, d\zeta\, d^2\vec{k}_T\, d^2\vec{R}_T\, D_1^{h_1 h_2/q}(z,\zeta,\vec{k}_T^{\,2},\vec{R}_T^{\,2},\vec{k}_T\cdot\vec{R}_T) = \underbrace{\mathcal{N}(\mathcal{N}-1)}$$

Total number of *hadron pairs* produced when the parton fragments

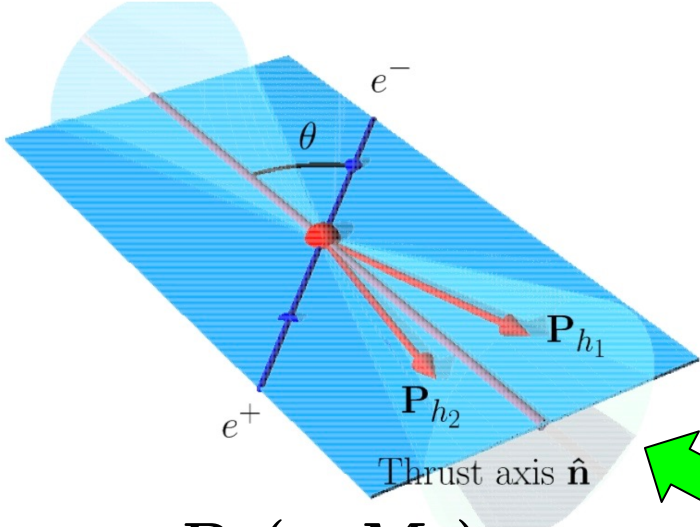
($\mathcal{N}$ is the total number of hadrons produced when the parton fragments)

DiFFs extracted from experiment will now have a clear physical meaning: they are densities in the momentum variables for the number of hadron pairs (h_1, h_2) fragmenting from the parton

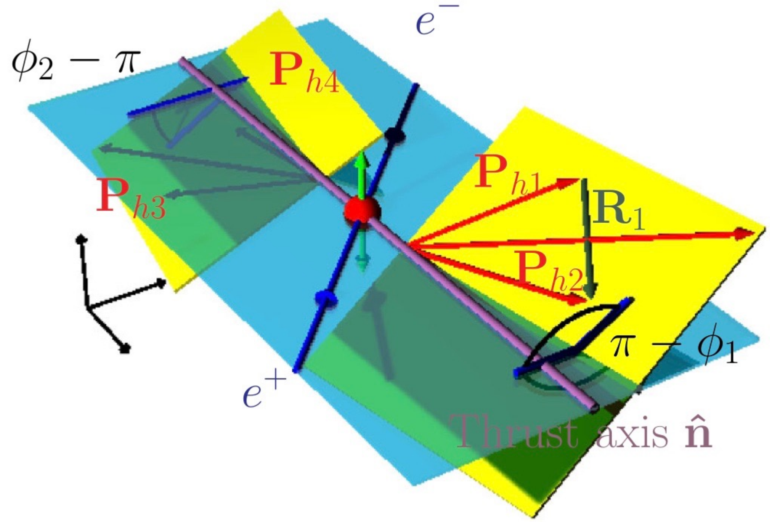
QCD Global Analysis of TSSAs for Dihadron Fragmentation

Cocuzza, Metz, DP, Prokudin, Sato, Seidl, arXiv:2306.XXXXX

JAM Jefferson Lab Angular Momentum Collaboration

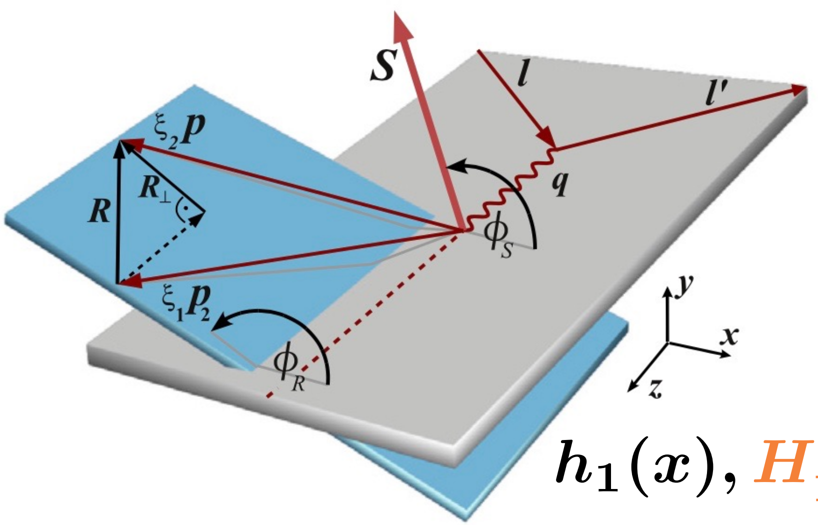


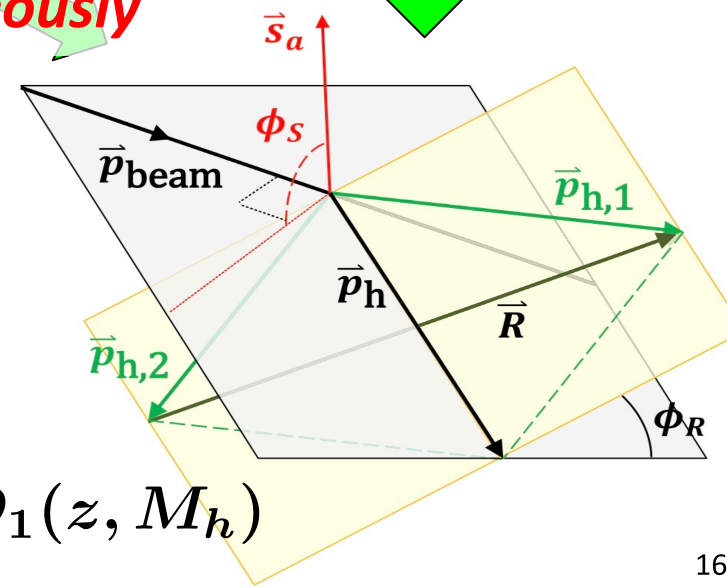
$D_1(z, M_h)$



$H_1^{\sphericalangle}(z, M_h), D_1(z, M_h)$

DiFFs and transversity PDFs extracted simultaneously





$h_1(x), H_1^{\sphericalangle}(z, M_h), D_1(z, M_h)$

- Analyze TSSAs for $\pi^+\pi^-$ production in e^+e^- annihilation, SIDIS, and proton-proton collisions and extract

$$\boldsymbol{h_1(x)}, \boldsymbol{H_1^{\sphericalangle}(z, M_h)}, \boldsymbol{D_1(z, M_h)}^*$$

*Also need data from PYTHIA for flavor separation and to constrain the gluon $D_1(z,M_h)$

- We use the following functional form for the transversity PDFs u_v, d_v, and $\bar{u} = -\bar{d}$ (from large-N_c limit (Pobylitsa (2003))) and impose the Soffer bound

$$F(x) \sim N x^{\alpha}(1-x)^{\beta}(1+\gamma\sqrt{x}+\delta x)$$

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$$F(x) \sim N x^{\alpha} (1-x)^{\beta} (1 + \gamma\sqrt{x} + \delta x)$$

Use constraint from small-x asymptotics (Kovchegov, Sievert (2019))

$$\alpha \xrightarrow{x \to 0} 1 - 2\sqrt{\frac{\alpha_s N_c}{2\pi}} \quad \Longrightarrow \quad \alpha = 0.170 \pm 0.085$$

50% uncertainty due to unaccounted for $1/N_c$ and NLO corrections

- Analyze TSSAs for $\pi^+\pi^-$ production in e^+e^- annihilation, SIDIS, and proton-proton collisions and extract

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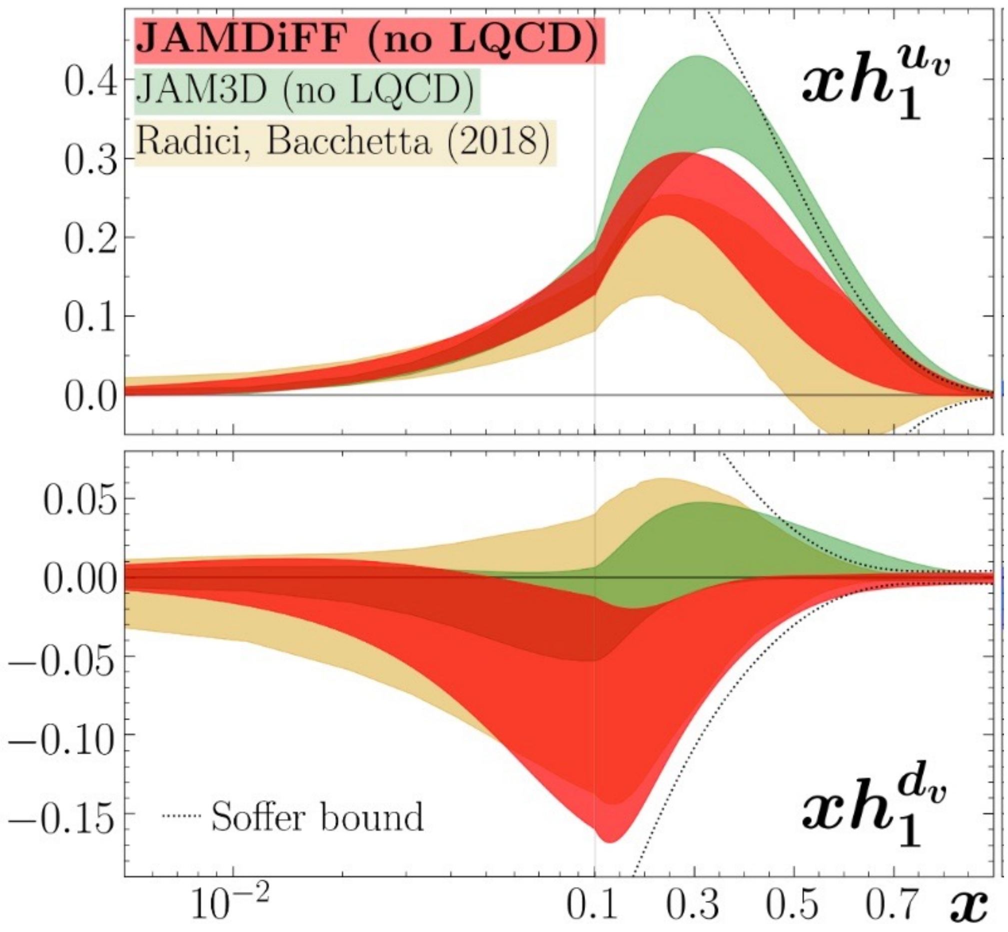
- The DiFFs $D_1(z, M_h)$, $H_1^{\sphericalangle}(z, M_h)$ use the same functional form as above ($x \to z$) for the z dependence, which is repeated on a grid in M_h (more finely spaced around the resonances) and interpolated to obtain the function value at any M_h

- Perform the analysis with and without LQCD data for the tensor charges $\boldsymbol{\delta} u, \boldsymbol{\delta} d$ from ETMC (Alexandrou, et al. (2019)) and PNDME (Gupta, et al. (2018)) (physical pion mass and 2+1+1 flavors)

Experiment	N_{dat}	χ^2/N_{dat} no LQCD	χ^2/N_{dat} w/ LQCD
Belle (cross section) [90]	1121	1.24	1.30
Belle (Artru-Collins) [91]	183	1.89	1.92
HERMES [92]	12	1.62	2.08
COMPASS (p) [93]	26	1.16	1.29
COMPASS (D) [93]	26	0.69	0.71
STAR (2015) [94]	24	1.54	1.68
STAR (2018) [62]	106	1.05	1.12
STAR (PRELIM) [63]	129	1.10	1.15
ETMC δu [30]	1	—	0.17
ETMC δd [30]	1	—	0.57
PNDME δu [25]	1	—	6.73
PNDME δd [25]	1	—	0.15
Total	1631	**1.29**	**1.35**



- JAMDiFF (no LQCD) finds agreement with Radici, Bacchetta (2018) with a slightly larger u_v function at larger x
- The u_v function for JAMDiFF (no LQCD) is smaller than JAM3D (no LQCD) but the d_v function is larger in magnitude

Experiment	N_{dat}	χ^2/N_{dat} no LQCD	χ^2/N_{dat} w/ LQCD
Belle (cross section) [90]	1121	1.24	1.30
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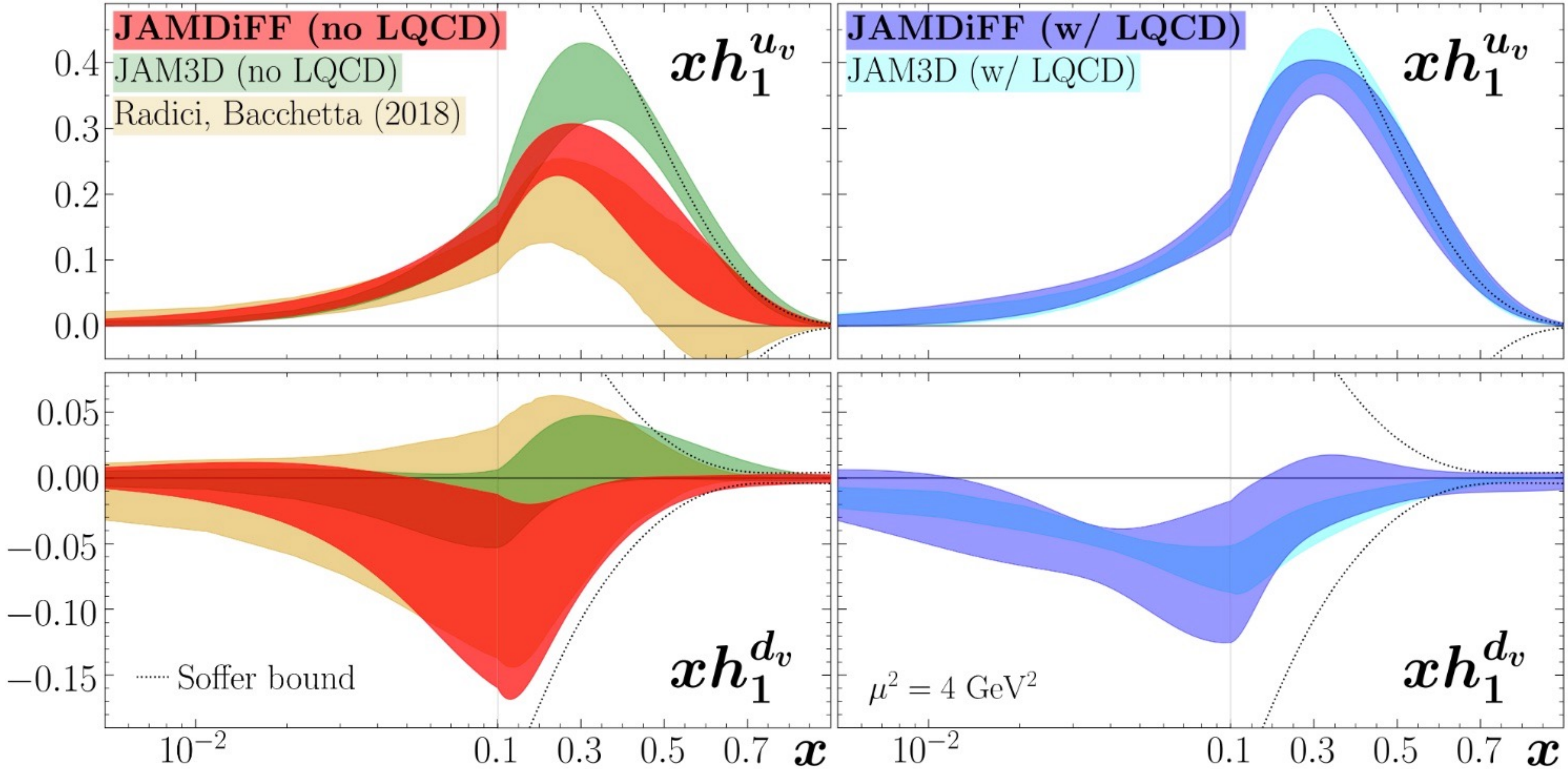
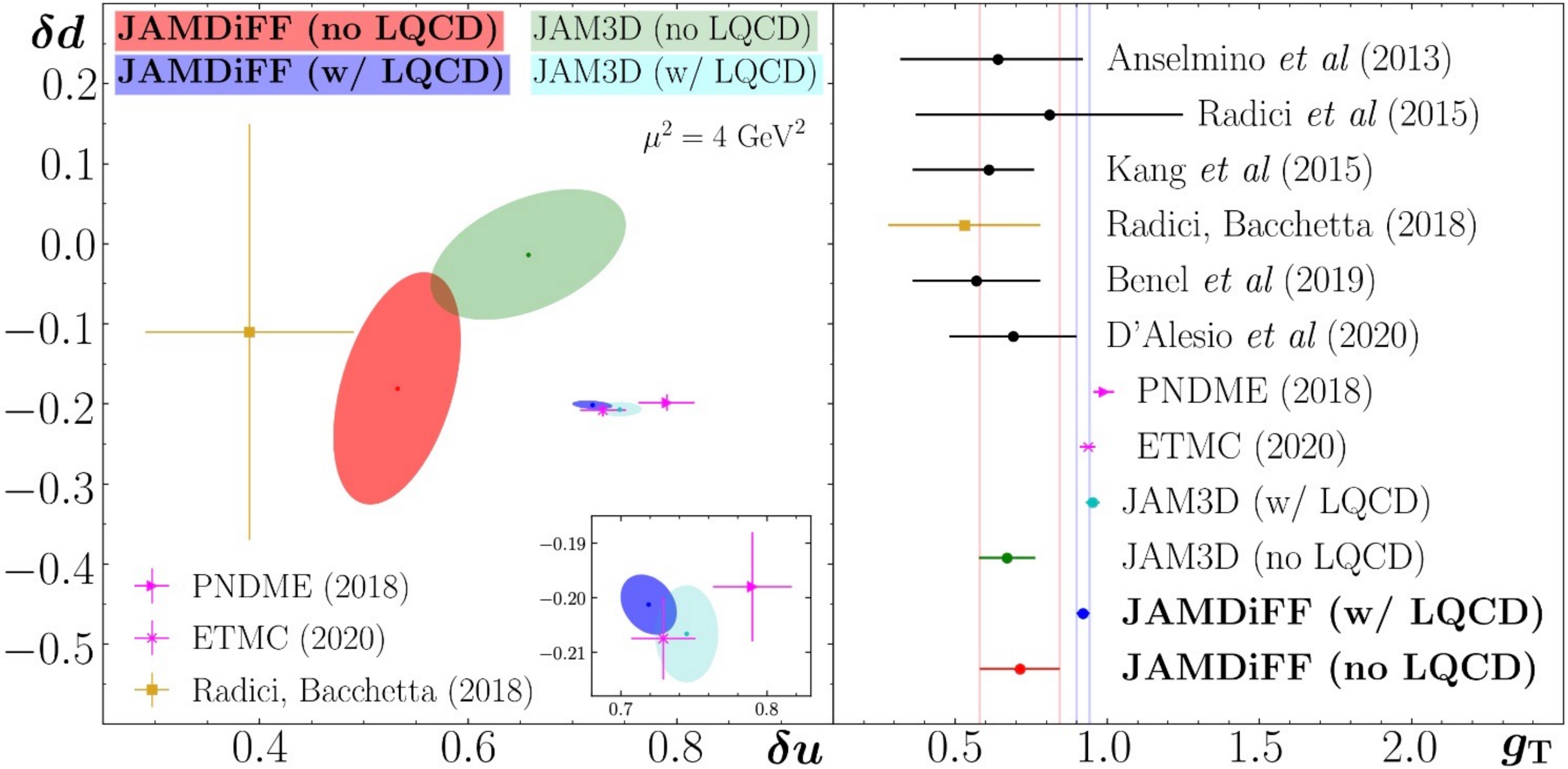


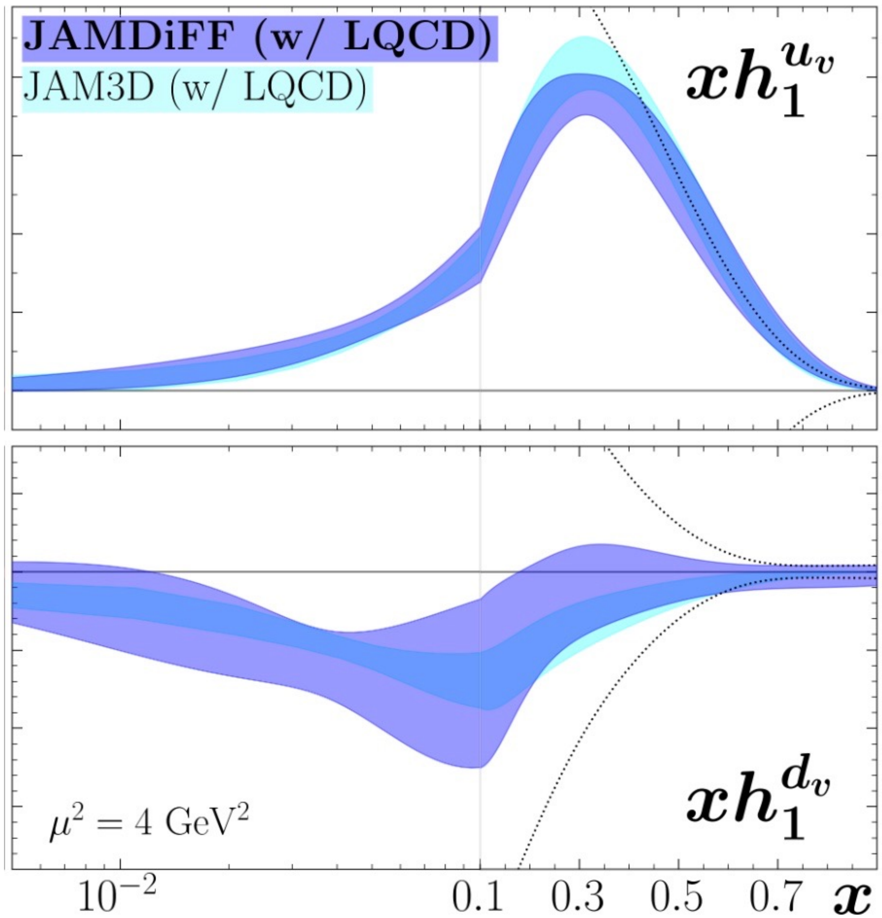
- JAMDiFF (w/LQCD) agrees with JAM3D (w/LQCD*) - nontrivial since the lattice data only constrains the full moment of the transversity PDFs

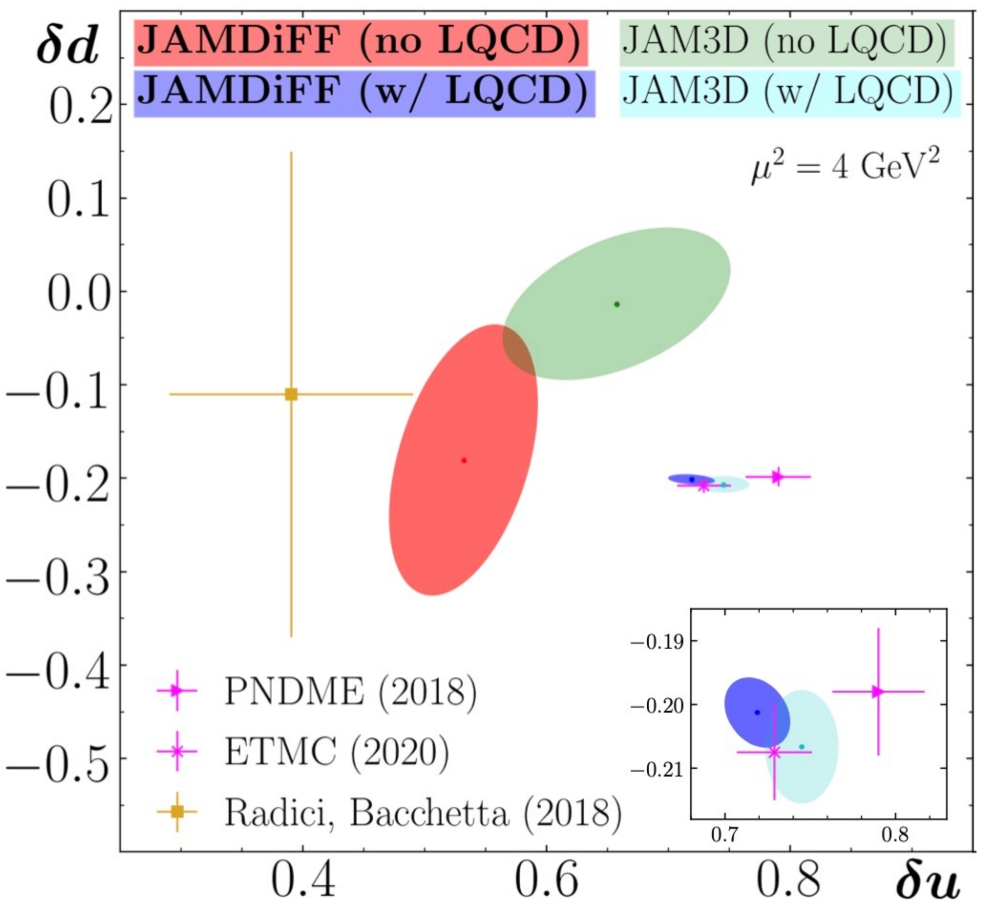
*The JAM3D (w/LQCD) analysis is slightly modified from the published version: antiquarks are now included (with $\bar{u} = -\bar{d}$) and $\delta u, \delta d$ from ETMC and PNDME are both included in the fit (rather than just g_T from ETMC)



- JAMDiFF (no LQCD) agrees within errors with JAM3D (no LQCD) and Radici, Bacchetta (2018) for the tensor charges
- Similar to the JAM3D analysis, **JAMDiFF also finds compatibility with lattice once that data is included in the fit,** ***and can still describe the experimental data well***

- Possible explanation for the surprising shift (~3-4σ difference with lattice to a ~0.3-2σ difference) in the phenomenological values of the tensor charges once LQCD data is included in the analysis:
  - The experimental measurements are sensitive to the *x*-dependence of the transversity PDFs, not the full moment like the lattice data.
  - When integrating over *x*, small but consistent changes in the PDF as a function of *x* can accumulate into a large change in the tensor charge while not significantly affecting the description of the experimental data.
  - Before drawing a conclusion about the compatibility between LQCD tensor charges and experimental data, one needs first to include both in the analysis.
  - One should only be concerned if the description of the lattice data remains poor even after its inclusion and/or if the description of the experimental data suffers significantly.





JAM3D and JAMDiFF agree on the x-dependence of transversity and also can successfully include lattice QCD data on the tensor charges in the analyses, thus showing the universal nature of all available information on transversity and the tensor charges of the nucleon

Summary and Outlook

Summary

- We have performed separate QCD global analyses of TSSAs in TMD/collinear twist-3 single-hadron observables and in dihadron fragmentation measurements, also studying the role of lattice QCD in our fits

- We have introduced a ***new definition of dihadron fragmentation functions that is consistent with a number density interpretation***

- Quantities of particular interest are the tensor charges of the nucleon - they are fundamental properties of the nucleon that have connections to QCD phenomenology, *ab initio* lattice QCD computations, model calculations, and low-energy beyond the Standard Model studies (e.g., beta decay, EDM)

Recent analyses by the JAM Collaboration show agreement between single-hadron and dihadron approaches for extracting transversity as well as compatibility with lattice QCD tensor charges, thus showing the universal nature of all this information

Outlook

- Further refinements/improvements:
  - TMD/collinear twist-3: include lattice tensor charge data and hadron-in-jet Collins effect measurements with CSS evolution in the analysis, …
  - Dihadron: other groups including lattice tensor charge data; unpolarized *pp* cross section measurements to better constrain $D_1^g(z, M_h)$, …
  - "Universal" analysis where TMD/collinear twist-3 ***and*** dihadron measurements are fit simultaneously
  - Incorporate proper small-x evolution for transversity (Kovchegov, Sievert (2019))
  - Using pseudo-PDF or quasi-PDF approaches, lattice can now compute $h_1(x)$ (Egerer, et al. (2021); Alexandrou, et al. (2022)) - eventually can include data into phenomenology (more constraining than the tensor charge data)